Direct Measurement of Thermo-Optic Coefficients in Coatings by Photothermal Spectroscopy

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Matt Abernathy Presenting

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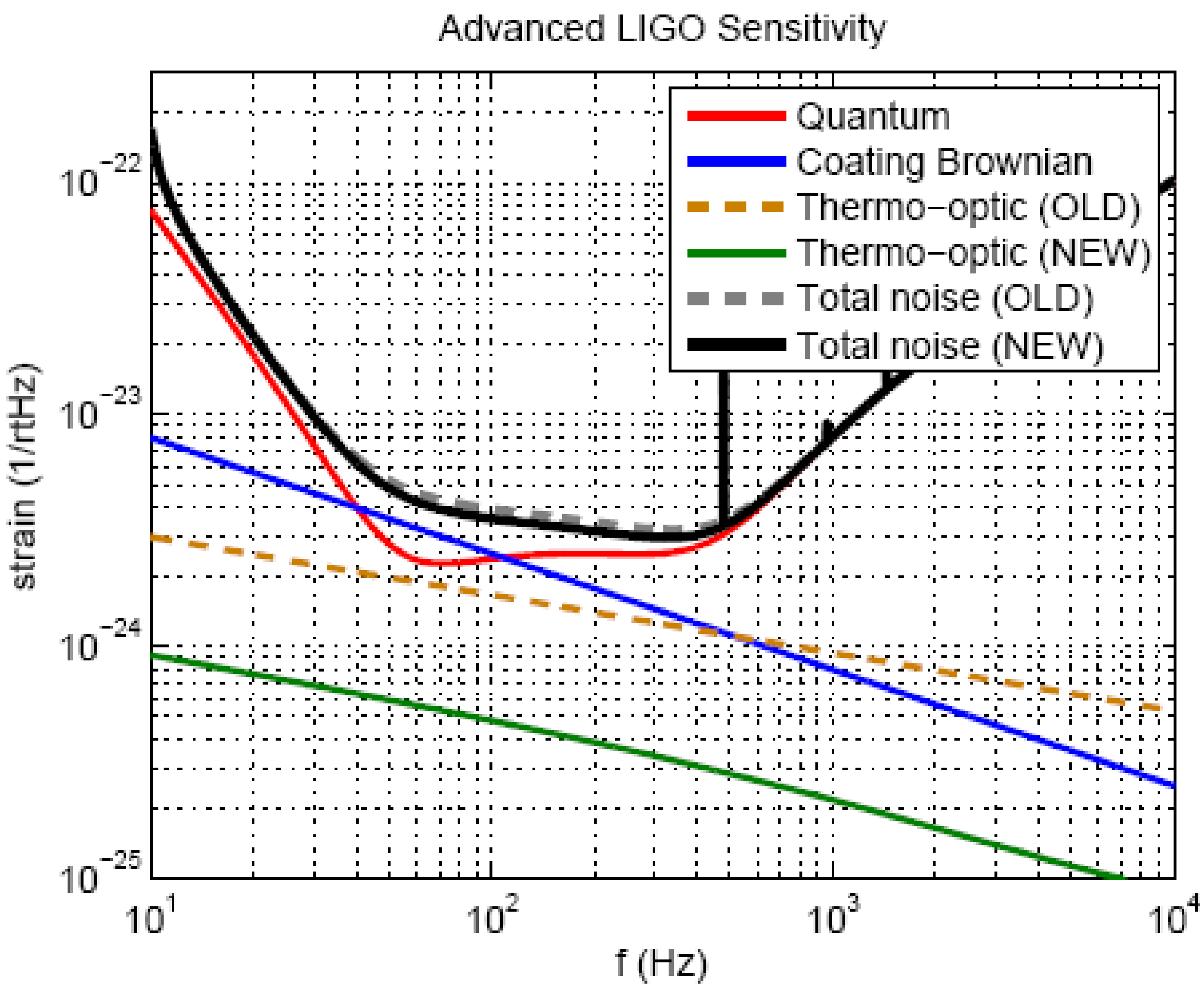
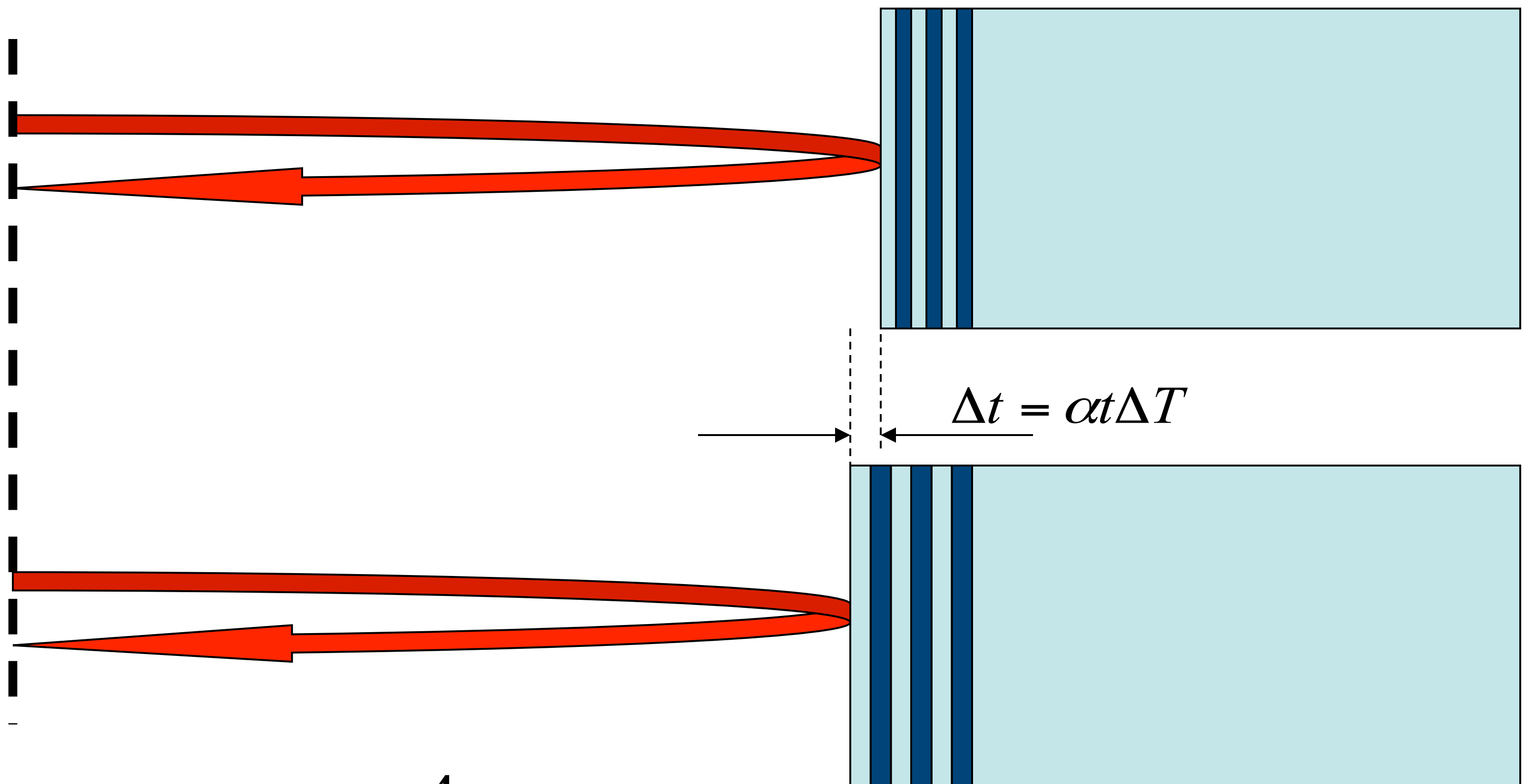
The AdLIGO Noise Curve



Source: Evans et al, LIGO-P080071-00

Thermo-optic Noise: TO = TE + TR

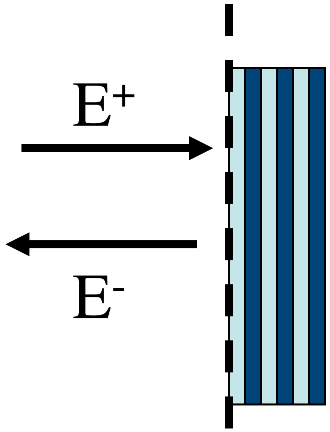
- **<u>Thermo-Elastic (TE):</u>** Mirror's surface expands into probe beam. By convention, negative dφ/dT



$$\Delta\varphi_{TE} = -\frac{4\pi}{\lambda}\Delta t$$

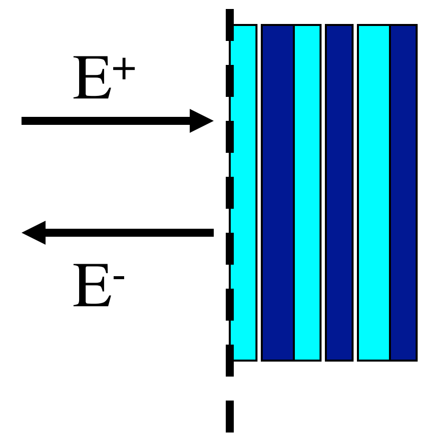
Thermo-optic Noise: TO = TE + TR

- **Thermo-Refractive (TR):** Coating layers deviate from λ/4 condition – due to both physical expansion and change in index of refraction. To first order, this manifests as a change in the phase of the reflected beam.



Quarter-wave stack:

$$E^- = E^+ r e^{i\varphi} \cong -E^+$$

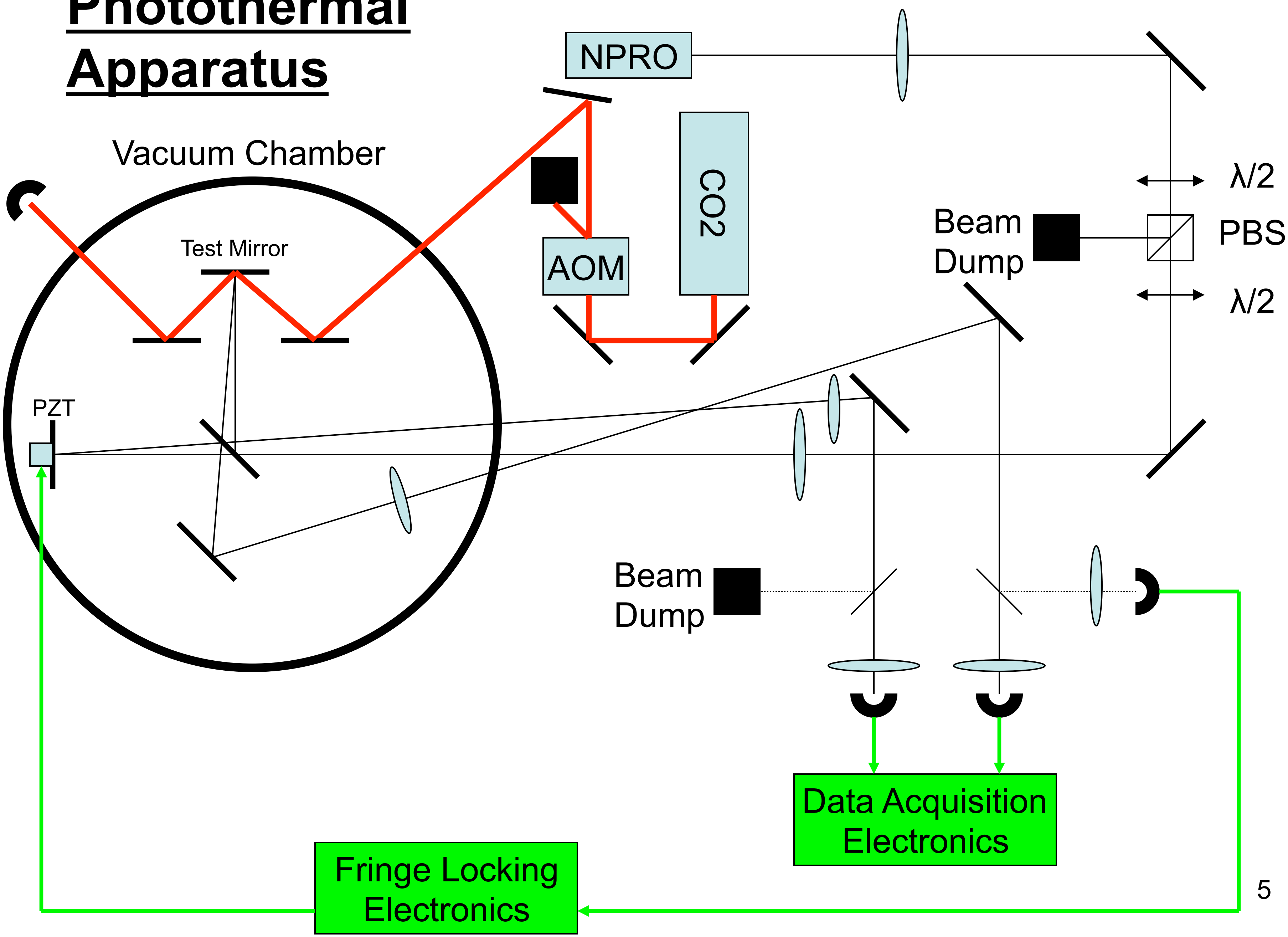


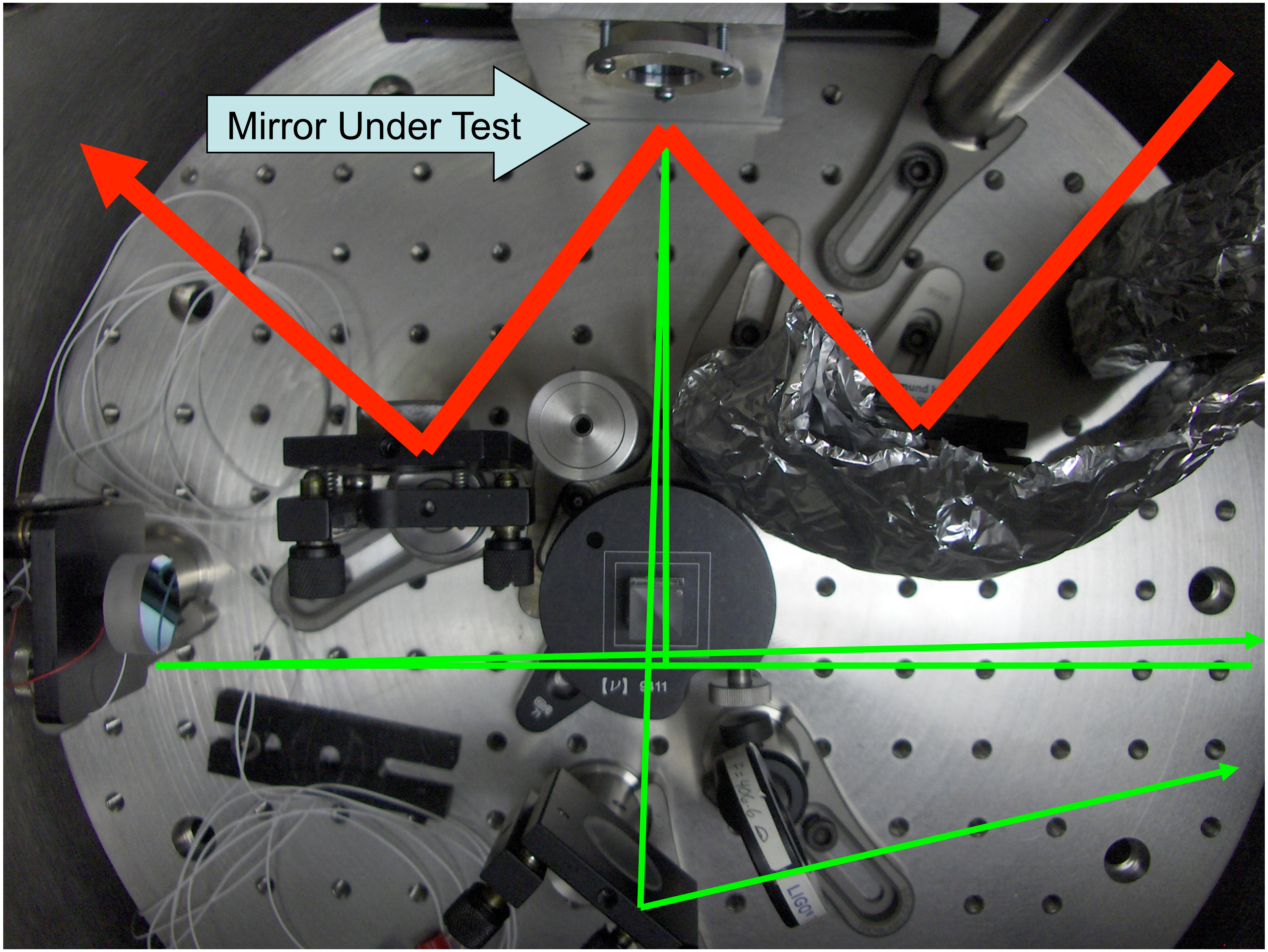
After expansion, index change:

$$E^- = E^+ r' e^{i\varphi'} \cong E^+ r e^{i(\varphi + \Delta\varphi_{TR})}$$

$$= -E^+ e^{i\Delta\varphi_{TR}}$$

Photothermal Apparatus




Mirror Under Test

Expected Signal: Canonical Form

$$\Delta\phi = \Delta\phi_{TE} + \Delta\phi_{TR}$$

$$= \frac{P_0}{A\kappa}\left[\alpha \frac{4\pi}{\lambda}\frac{a}{\omega}e^{i(\omega t + 90^\circ)} + \beta_{eff}\sqrt{\frac{a}{\omega}}e^{i(\omega t - 45^\circ)}\right]$$

Substrate CTE

Expected Signal: Canonical Form

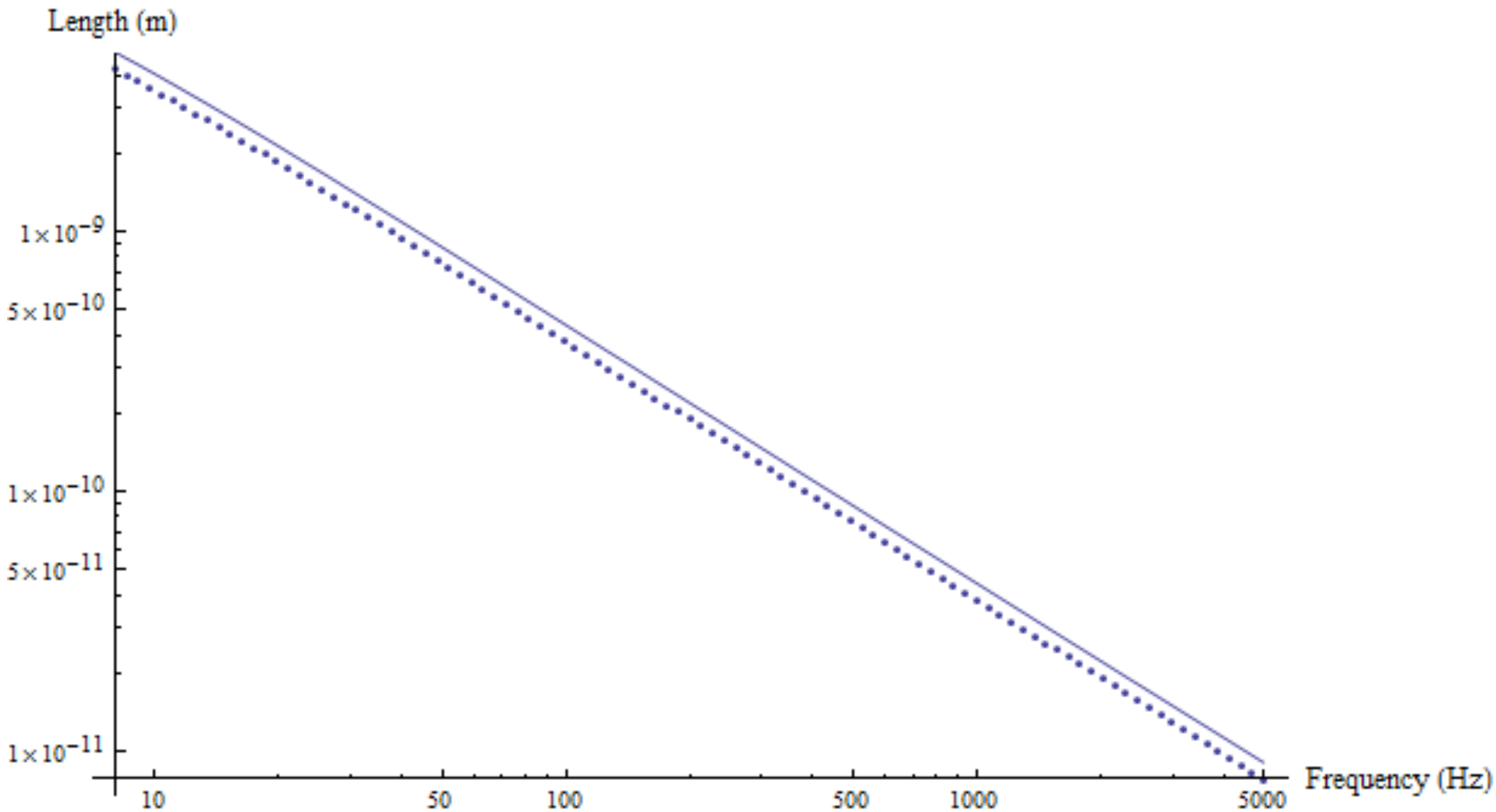
$$\Delta\phi = \Delta\phi_{TE} + \Delta\phi_{TR}$$

$$= \frac{P_0}{A\kappa}\left[\alpha \frac{4\pi}{\lambda}\frac{a}{\omega}e^{i(\omega t+90^\circ)} + \beta_{eff}\sqrt{\frac{a}{\omega}}e^{i(\omega t-45^\circ)}\right]$$

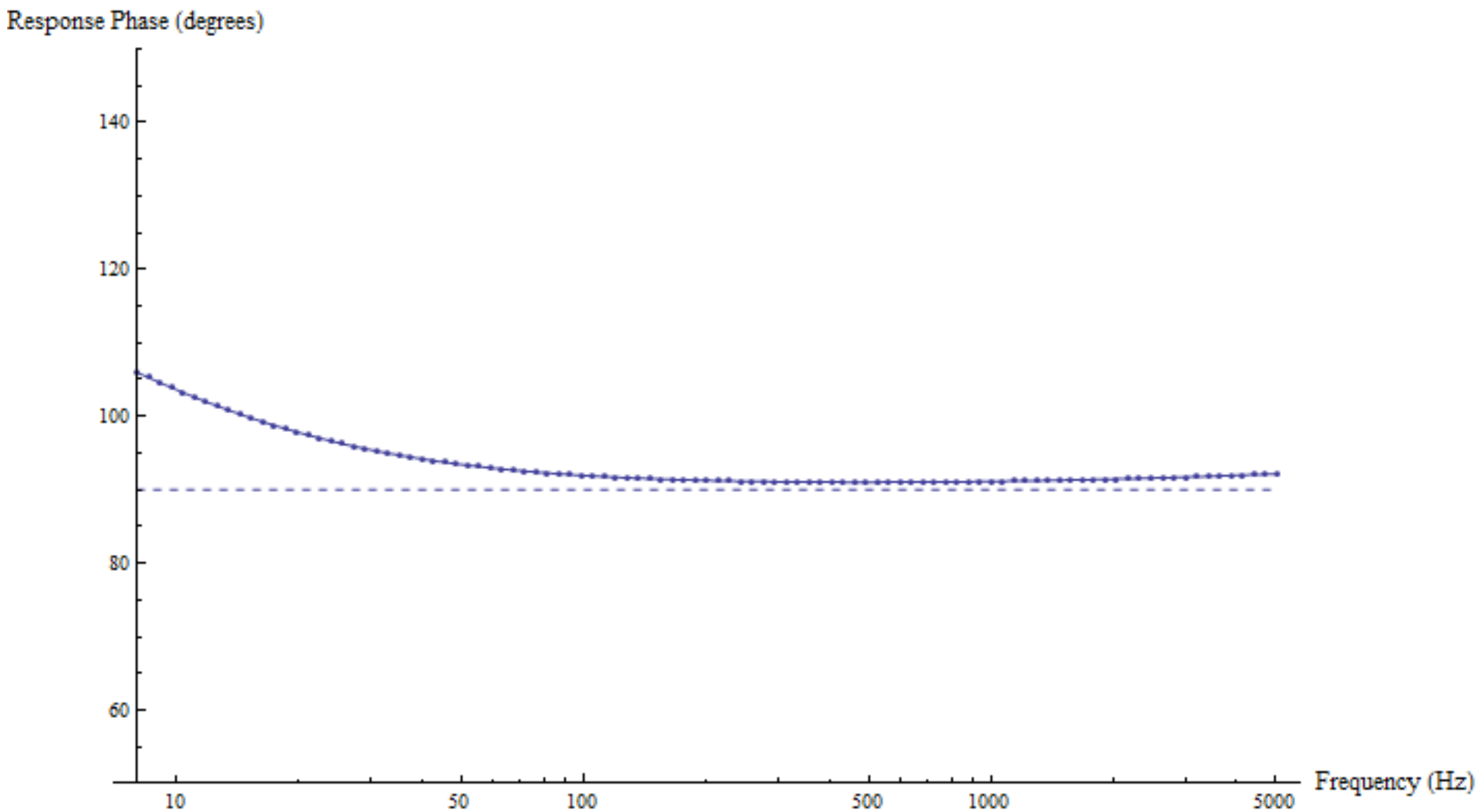
Substrate CTE

Coating properties (including coating CTE effects)

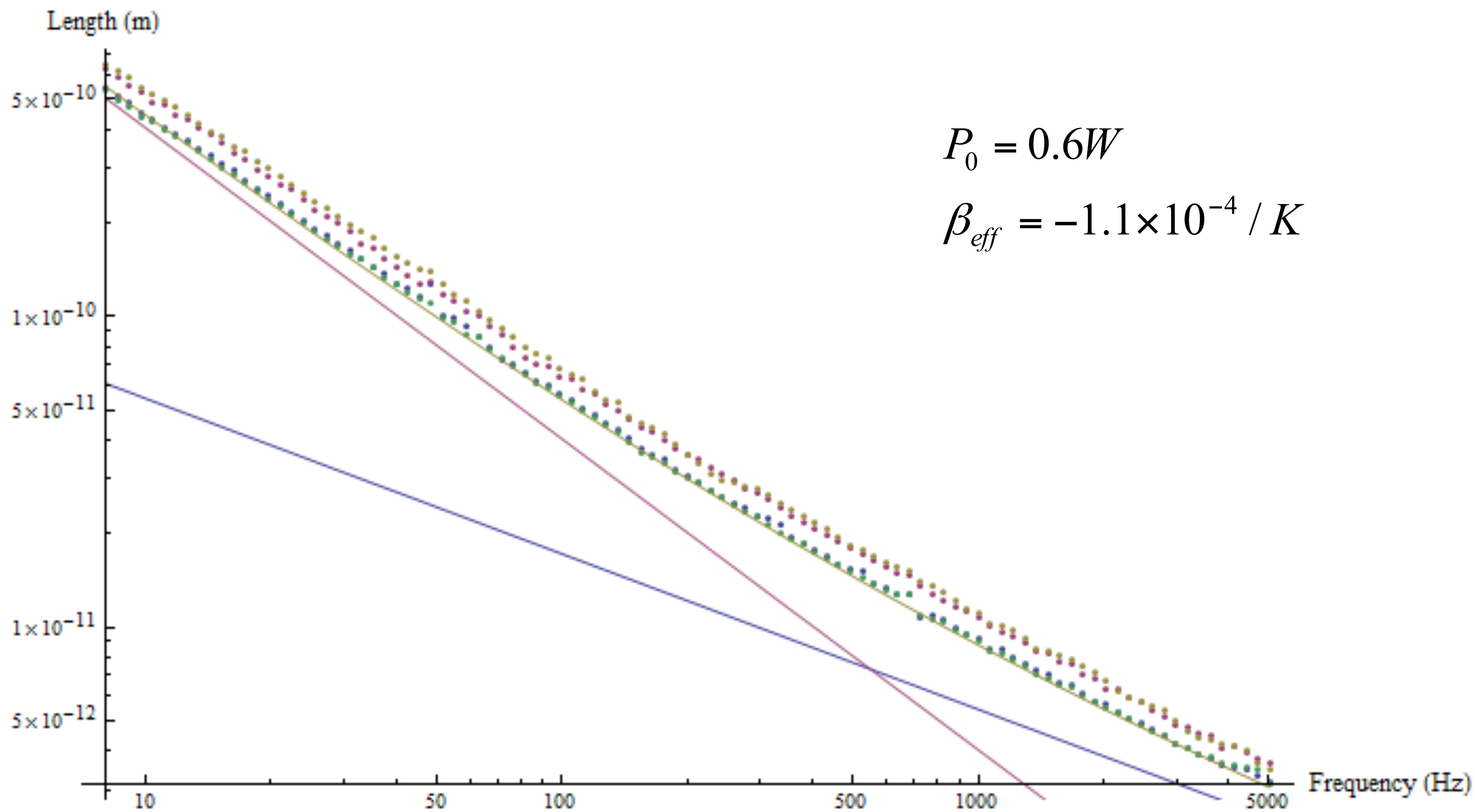
Sapphire Substrate
Response Magnitude



Sapphire Substrate
Response Phase



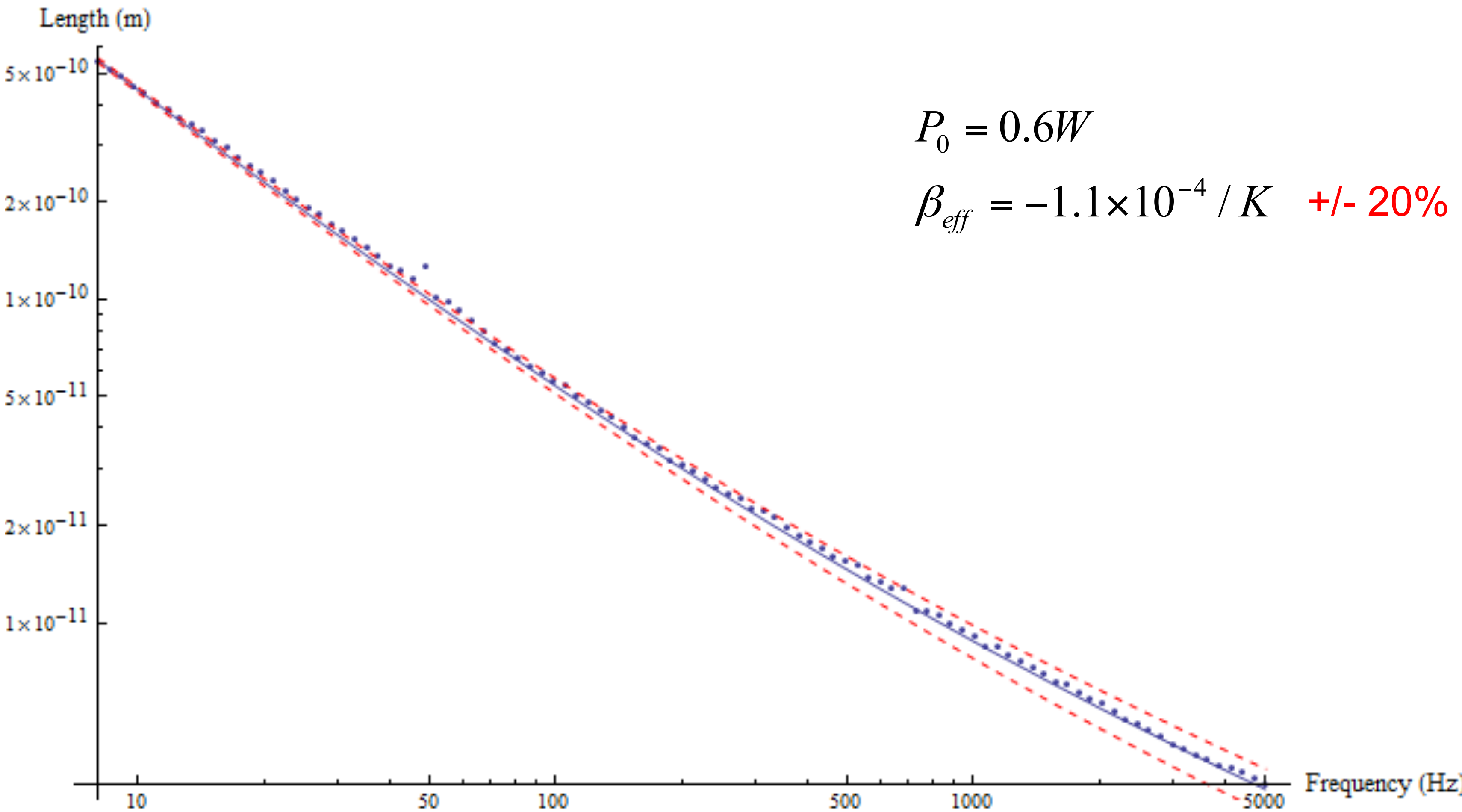
Silica Substrate Response Magnitude



$P_0 = 0.6W$

$\beta_{eff} = -1.1\times10^{-4} / K$

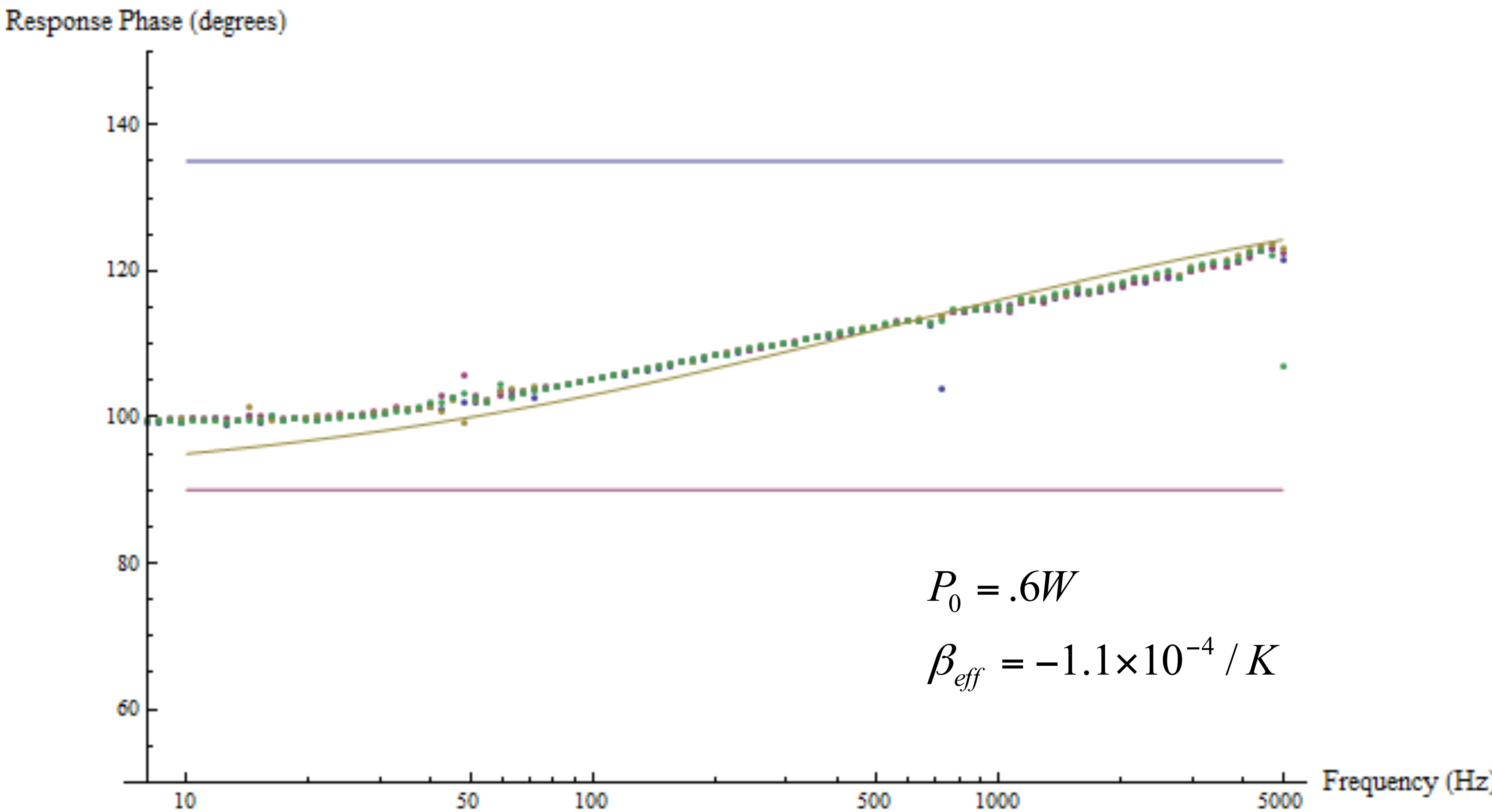
Silica Substrate
Response Magnitude



$P_0 = 0.6W$

$\beta_{eff} = -1.1\times10^{-4} / K$ +/- 20%

Recent Results: Silica Substrate



Combined TE/TR Results

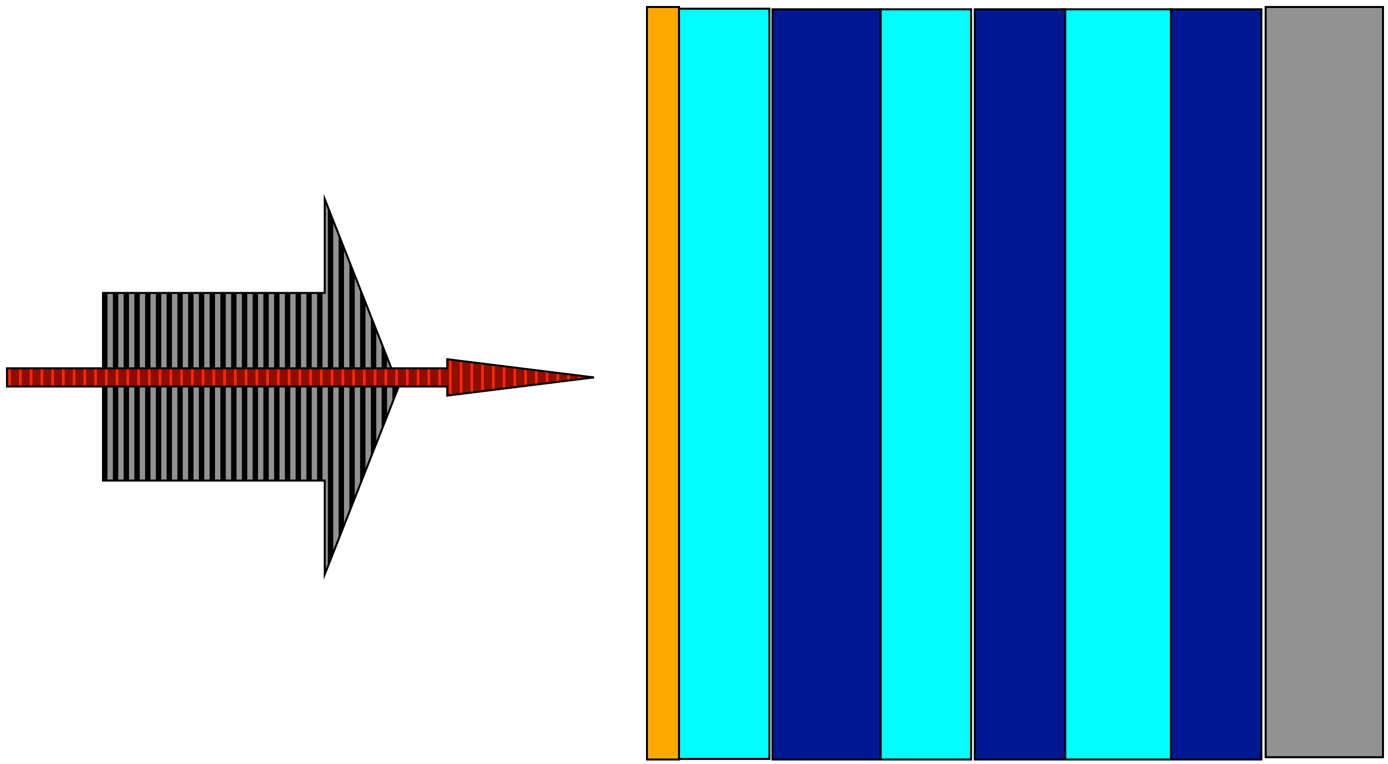
- QWL

$$\beta_{eff} = -(1.18 \pm .03) \times 10^{-4} / K$$

- Bragg

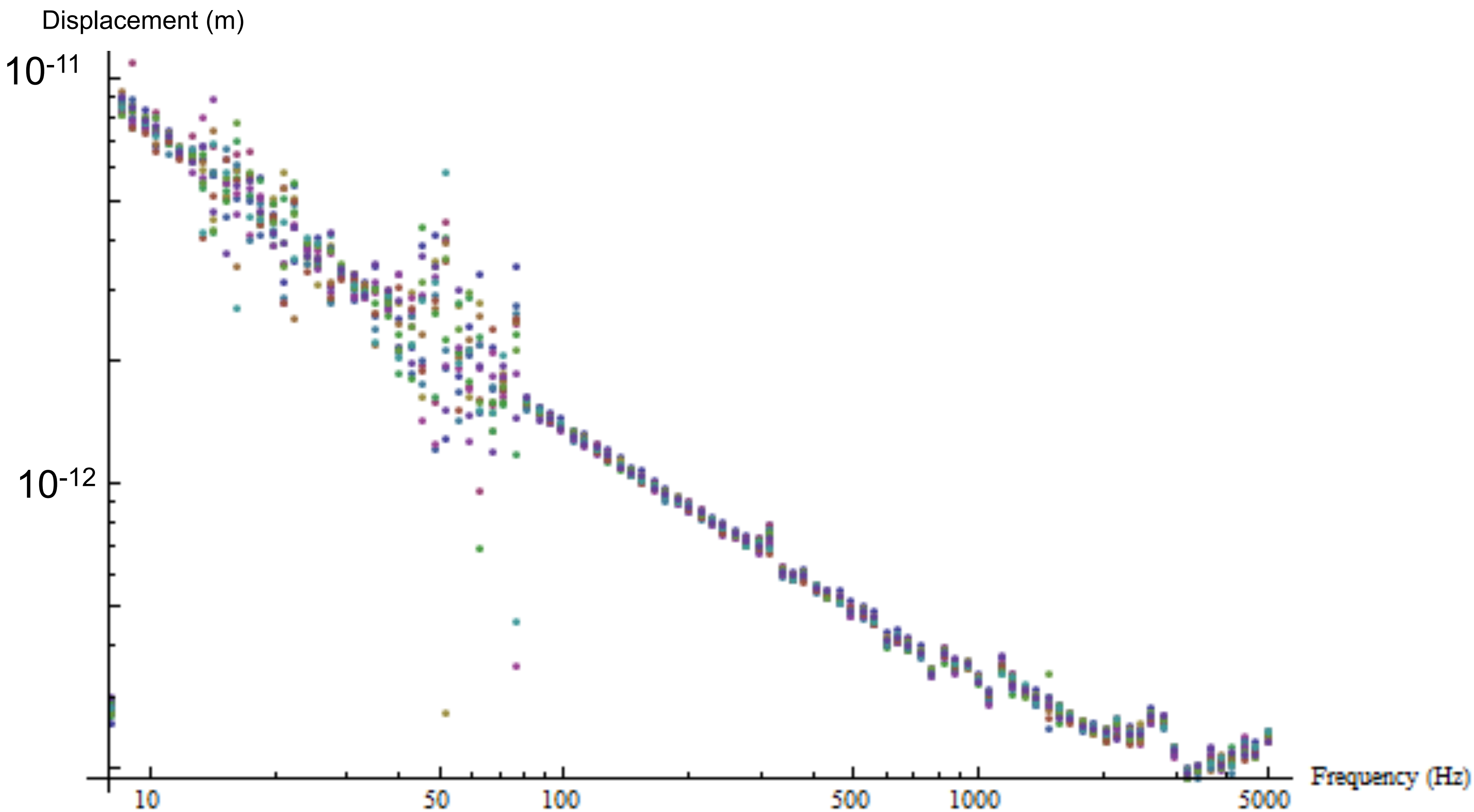
$$\beta_{eff} = -(1.08 \pm .04) \times 10^{-4} / K$$

Gold coatings for pure TE measurements



Challenge: 80% CO_2 absorption drops down to 0.5% CO_2 absorption.

Much lower SNR



Gold Coated "TE alone" Results

- QWL

$$\beta_{eff} = -(3.3 \pm 0.2) \times 10^{-4} / K$$

- Bragg

$$\beta_{eff} = -(3.1 \pm 0.2) \times 10^{-4} / K$$

Extracting Values

For quarter-wavelength coatings

$$\beta_{eff} = -21.7\alpha_L - 13.9\alpha_H + 4.48\beta_L + 1.41\beta_H - 10.6\alpha_{Cr}$$

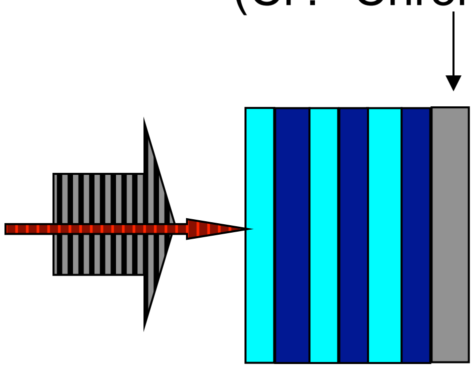
For 1/8-3/8 coatings

$$\beta_{eff} = -30.4\alpha_L - 6.21\alpha_H + 6.87\beta_L + 1.18\beta_H - 10.6\alpha_{Cr}$$

For quarter-wavelength TE only

$$\beta_{eff} = -28\alpha_L - 17\alpha_H - 10.6\alpha_{Cr}$$

(Cr? Chromium.)



For 1/8-3/8 coatings TE only

$$\beta_{eff} = -40\alpha_L - 8.4\alpha_H - 10.6\alpha_{Cr}$$

The Measurement Matrix

$$\begin{bmatrix} \beta_{eff}^{QWL} \\ \beta_{eff}^{Bragg} \\ \beta_{eff}^{TE-QWL} \\ \beta_{eff}^{TE-Bragg} \\ \beta_{eff}^{TE-QWLThinCr} \end{bmatrix} = \begin{bmatrix} -21.6 & -14.1 & 4.5 & 1.4 & -10.6 \\ -30.4 & -6.22 & 6.9 & 1.18 & -10.6 \\ -28 & -17 & 0 & 0 & -10.6 \\ -40 & -8.4 & 0 & 0 & -10.6 \\ -28 & -17 & 0 & 0 & -1.5 \end{bmatrix} \begin{bmatrix} \alpha_L \\ \alpha_H \\ \beta_L \\ \beta_H \\ \alpha_{Cr} \end{bmatrix}$$

Which we invert to get…

The Parameter Estimation Matrix

$$\begin{bmatrix} \alpha_L \\ \alpha_H \\ \beta_L \\ \beta_H \\ \alpha_{Cr} \end{bmatrix} = \begin{bmatrix} 0 & 0 & .041 & -.038 & -.023 \\ 0 & 0 & -.059 & .063 & -.031 \\ -.27 & .32 & .21 & -.26 & .028 \\ 1.6 & -1.0 & -1.5 & .90 & .08 \\ 0 & 0 & -.11 & 0 & .11 \end{bmatrix} \begin{bmatrix} \beta_{eff}^{QWL} \\ \beta_{eff}^{Bragg} \\ \beta_{eff}^{TE-QWL} \\ \beta_{eff}^{TE-Bragg} \\ \beta_{eff}^{TE-QWLThinCr} \end{bmatrix}$$

Our Results…

Our Measurements of α

SiO2 – Low Index

- $2.1 \times 10^{-6}\ K^{-1}$
  - Cetinorgu et al, Applied Optics 48, 4536 (2009)
- $5.1 \times 10^{-7}\ K^{-1}$
  - Crooks et al, CQG (2004)
- $5.5 \times 10^{-7}\ K^{-1}$
  - Braginsky et al, Phys Lett A 312, 244 (2003)

$(5.5 \pm 1.2) \times 10^{-6}\ K^{-1}$

Ta2O5 – High Index

- $+ 4.4 \times 10^{-6}\ K^{-1}$
  - Cetinorgu et al, Applied Optics 48, 4536 (2009)
- $+ 3.6 \times 10^{-6}\ K^{-1}$
  - Crooks et al, CQG (2004)
- $- 4.4 \times 10^{-5}\ K^{-1}$
  - MN Inci, J Phys D 37, 3151 (2004)
- $+ 5 \times 10^{-6}\ K^{-1}$
  - Braginsky et al, arXiv: gr-qc/0304100v1 (2003)

$(8.9 \pm 1.8) \times 10^{-6}\ K^{-1}$

Our Measurements of β

<u>SiO2 – Low Index</u>

- $8x10^{-6}$ K^{-1}
  - GWINC v2 ("Braginsky")

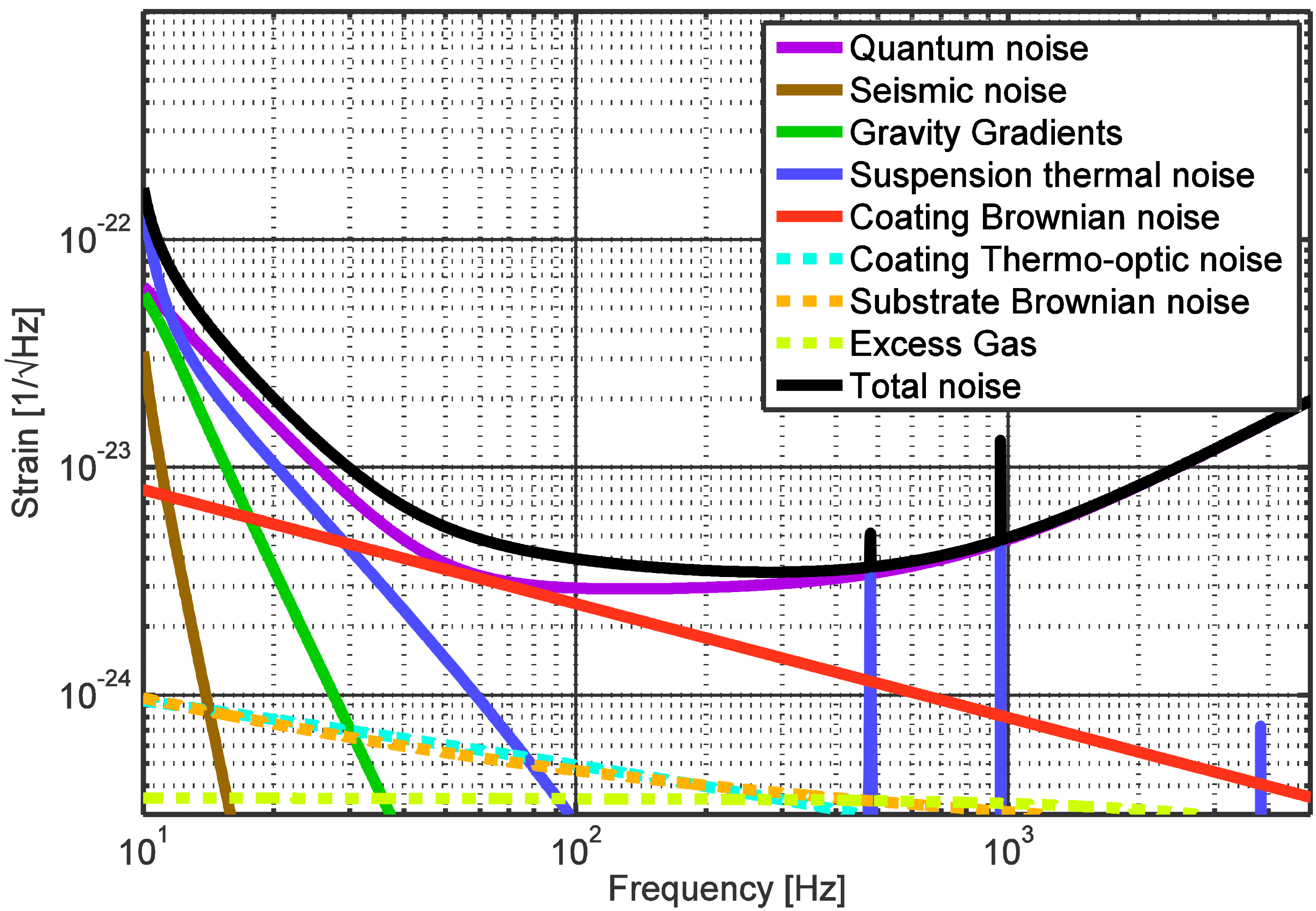
$(1.9 \pm 8.0)x10^{-6}$ K^{-1}

<u>Ta2O5 – High Index</u>

- $1.21x10^{-4}$ K^{-1}
  - MN Inci, J Phys D 37, 3151 (2004)
- $6x10^{-5}$ K^{-1} *
  - Gretarsson, LIGO-G080151-00-Z (2008) *Assumes α

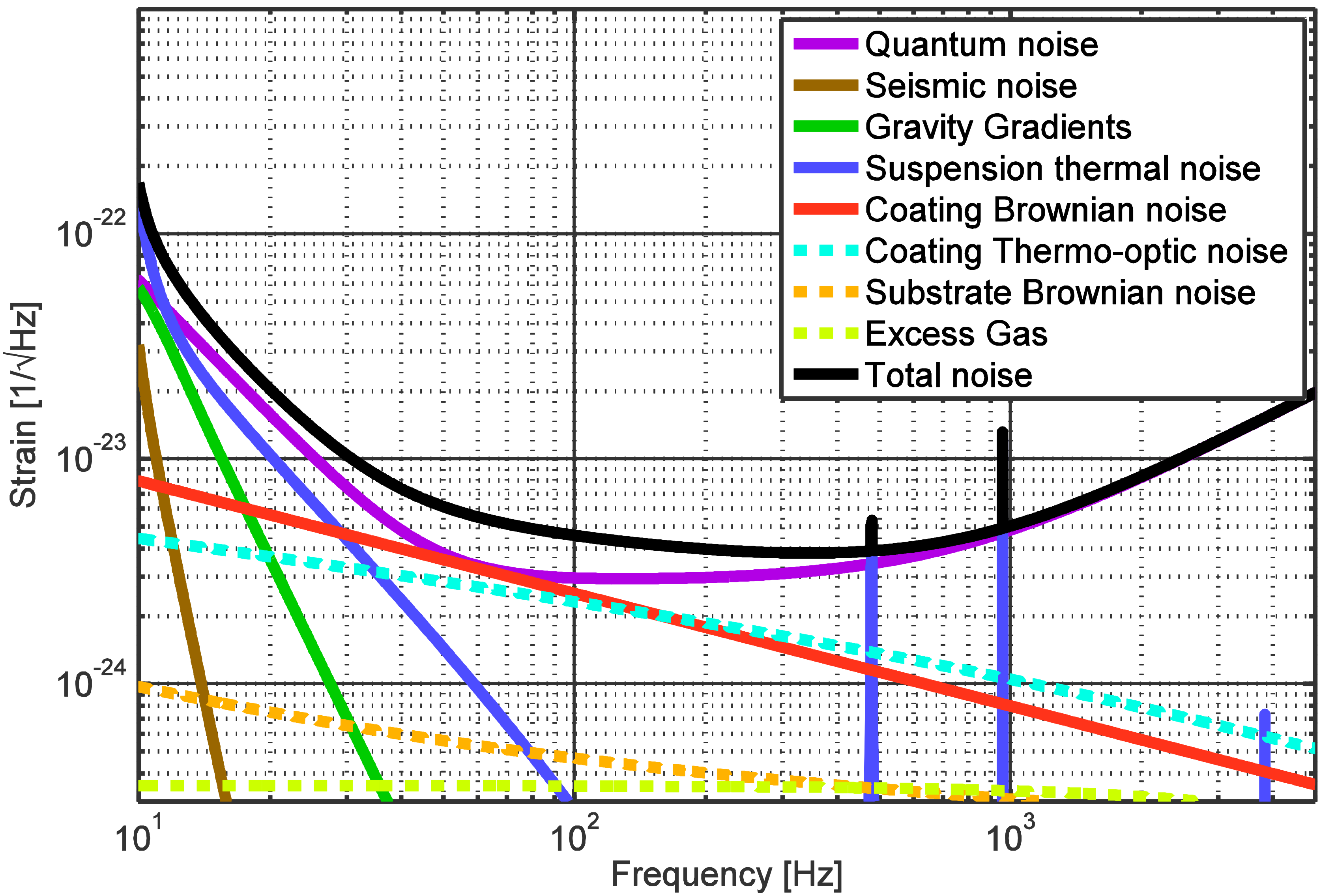
$(1.2 \pm 0.4)x10^{-4}$ K^{-1}

AdLIGO Baseline (GWINC v3)



AdLIGO with Our Parameters

Disclaimer: This Is Not an AdLIGO Prediction



Conclusions

- Measuring these parameters is non-trivial, but we have demonstrated a technique, and reported initial results
- We have the ability to measure <u>exactly</u> what AdLIGO needs
- Thermo-optic noise, and these parameters in particular, could be critical and need further study for future generations of gravitational wave detectors

Future Directions

- Characterize and reduce systematic errors
- Perform measurements on AdLIGO coatings with Cr layers (or at the very least Ion Beam Sputtered coatings and Ti:Ta2O5 coatings)
- Look at measurements of other materials and geometries

Acknowledgements

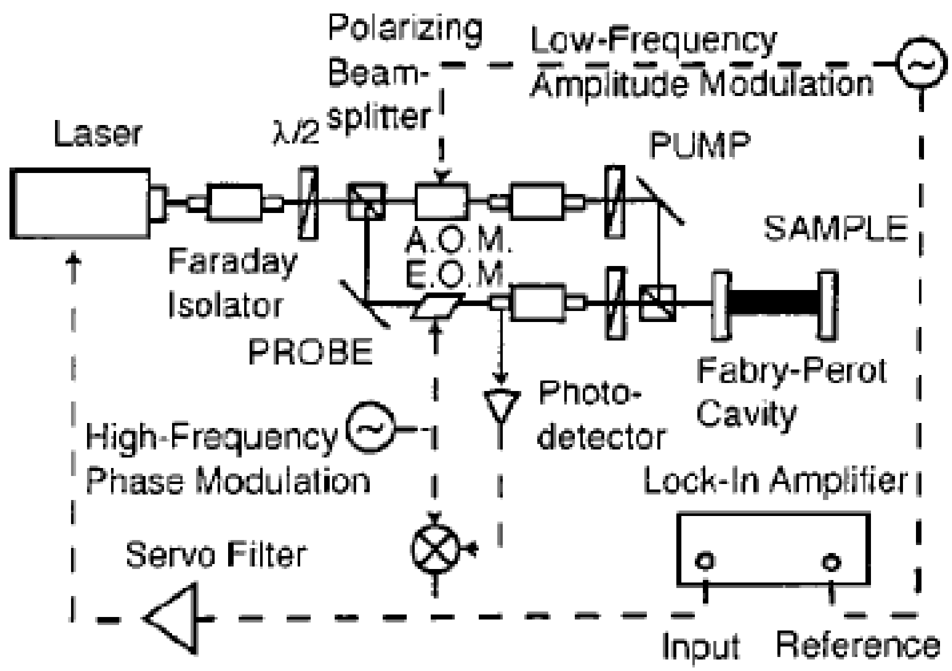
- Greg Ogin
- Ken Libbrecht, Eric Black
- Eric Gustafson
- Caltech LIGO-X, Akira Villar
- Family and friends
- LIGO and the NSF
  - Award PHY-0757058

Questions?

Supplimentary Slides follow

Measuring α: Cavity Assisted Photothermal Spectroscopy

- Probe locked to cavity
- Pump derived from probe laser chopped to cyclically heat cavity end mirror
- Sensitivity to mirror expansion proportional to Finesse
- Heating power in cavity proportional to Finesse
- Sample coated with gold to enhance absorption



$$\Rightarrow Signal \propto (Finesse)^2$$

Black et al, J Appl Phys 95, 7655 (2004)

Details of the two terms:

- Thermo-Elastic:

$$\left(\frac{\partial \varphi}{\partial T}\right)_{TE} = -\frac{4\pi}{\lambda}\alpha_{eff}t$$

Negative phase

- Thermo-Refractive:

$$\left(\frac{\partial \phi}{\partial T}\right)_{TR} \cong \pi \frac{\left(\frac{\partial n_H}{\partial T} + \alpha_H n_H\right) + \left(\frac{\partial n_L}{\partial T} + \alpha_L n_L\right) \bullet \left[2(n_H / n_L)^2 - 1\right]}{n_H^2 - n_L^2}$$

Positive phase

Evans et al, Physical Review D 78, 102003 (2008)

Theory: Assumptions

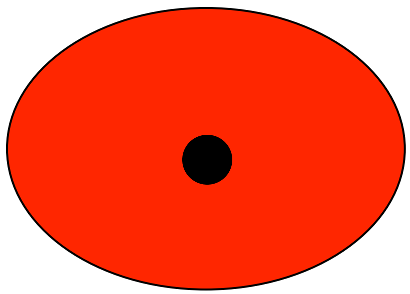
- The scale of periodic thermal disturbances (a "thermal wavelength") is much smaller than our heating spot

$$w_{pump} > w_{probe} >> \lambda_{thermal}$$

- The coating thickness is smaller than a thermal wavelength

$$\lambda_{thermal} > t_{coating}$$

Together, these give us a 1-D problem where the thermal dynamics are all determined by the properties of the substrate.



Theory: Heat Equation Solutions

- The heat equation becomes

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial z^2} \qquad a \equiv \frac{\kappa}{C_p \rho}$$

- With solutions

$$u(t,z) = Ce^{i(\omega t - kz)}$$

$$k = \frac{\sqrt{2}}{2}(1-i)\sqrt{\omega / a} = e^{-i45^\circ}\sqrt{\omega / a}$$

Theory: Boundary Condition

- Our boundary condition gives C(ω)

$$-\kappa \left.\frac{\partial u}{\partial z}\right|_{z=0} = \frac{P_0}{A} e^{i\omega t} \quad \Rightarrow \quad C = \frac{P_0}{A\kappa} \frac{e^{-i45^\circ}}{\sqrt{\omega / a}}$$

$$u(t,x) = \frac{P_0}{A\kappa} \frac{e^{-i45^\circ}}{\sqrt{\omega / a}} e^{i(\omega t - kz)}$$

Expected Signal - A Coherent Sum of…

$$\phi_{TR}(t) = \left(\frac{\partial \phi_c}{\partial u}\right) u(t,0) = \beta_{eff} \frac{P_0 \sqrt{a}}{A\kappa} \frac{1}{\sqrt{\omega}} e^{i(\omega t - 45^\circ)}$$

$$\Delta\phi_{TE} = -\frac{4\pi}{\lambda}\int \alpha u(z)dz = -\frac{4\pi}{\lambda}\alpha \frac{P_0}{A\kappa(\omega / a)} e^{i(\omega t - 90^\circ)}$$

$$\phi_{TE}(t) = \frac{4\pi}{\lambda}\alpha \frac{P_0 a}{A\kappa}\frac{1}{\omega} e^{i(\omega t - 270^\circ)} = \frac{4\pi}{\lambda}\alpha \frac{P_0 a}{A\kappa}\frac{1}{\omega} e^{i(\omega t + 90^\circ)}$$

Expected Signal: Canonical Form

$$\Delta\phi = \Delta\phi_{TE} + \Delta\phi_{TR}$$

$$= \frac{P_0}{A\kappa}\left[\alpha \frac{4\pi}{\lambda}\frac{a}{\omega} e^{i(\omega t + 90^\circ)} + \beta_{eff}\sqrt{\frac{a}{\omega}} e^{i(\omega t - 45^\circ)}\right]$$

(Reminder)

- Thermo-Elastic:

$$\left(\frac{\partial \varphi}{\partial T}\right)_{TE} = -\frac{4\pi}{\lambda}\alpha_{eff}t$$

Negative phase

- Thermo-Refractive:

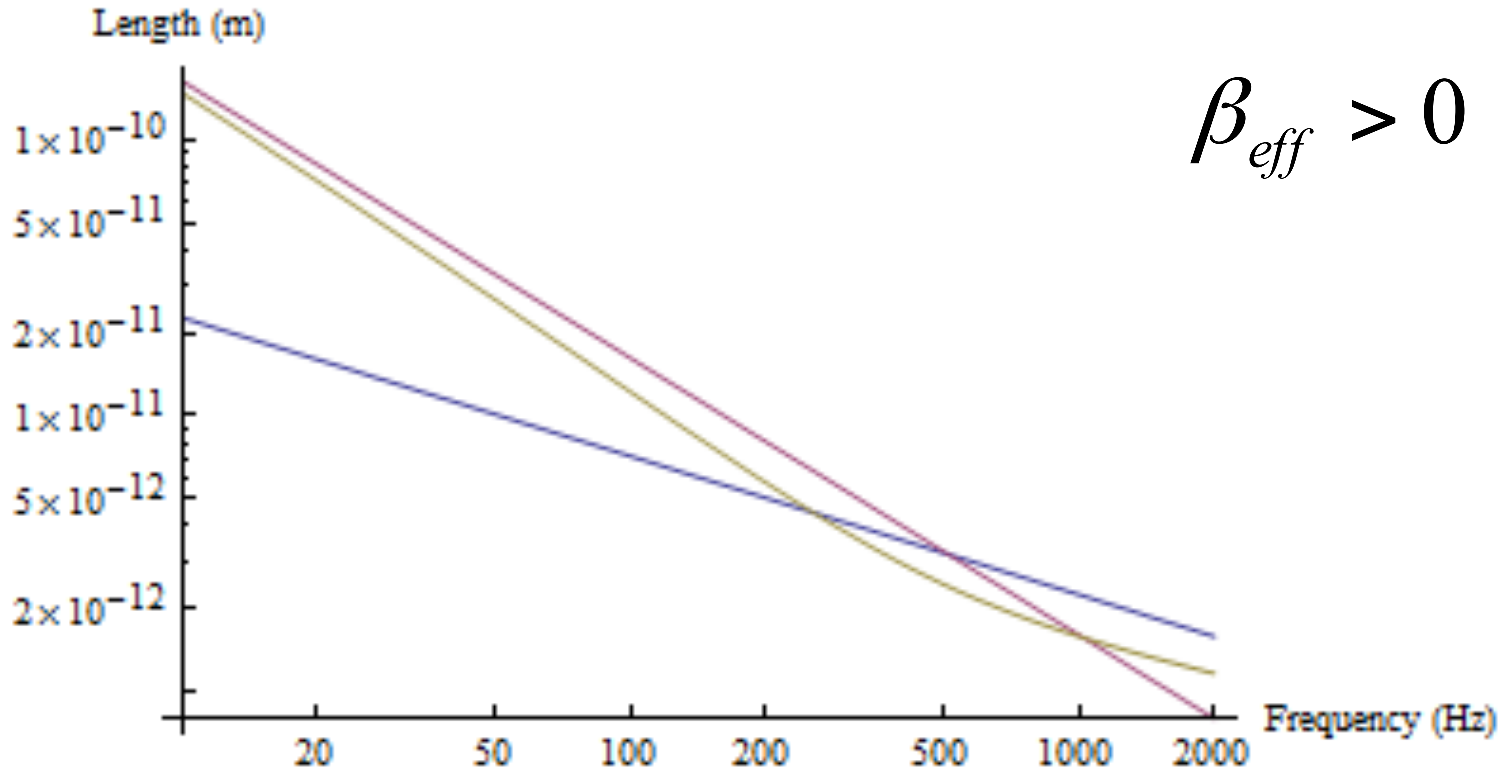
$$\left(\frac{\partial \phi}{\partial T}\right)_{TR} \cong \pi \frac{\left(\frac{\partial n_H}{\partial T} + \alpha_H n_H\right) + \left(\frac{\partial n_L}{\partial T} + \alpha_L n_L\right) \bullet \left[2(n_H / n_L)^2 - 1\right]}{n_H^2 - n_L^2}$$

Positive phase

Evans et al, Physical Review D 78, 102003 (2008)

Expected Signal: Canonical Form

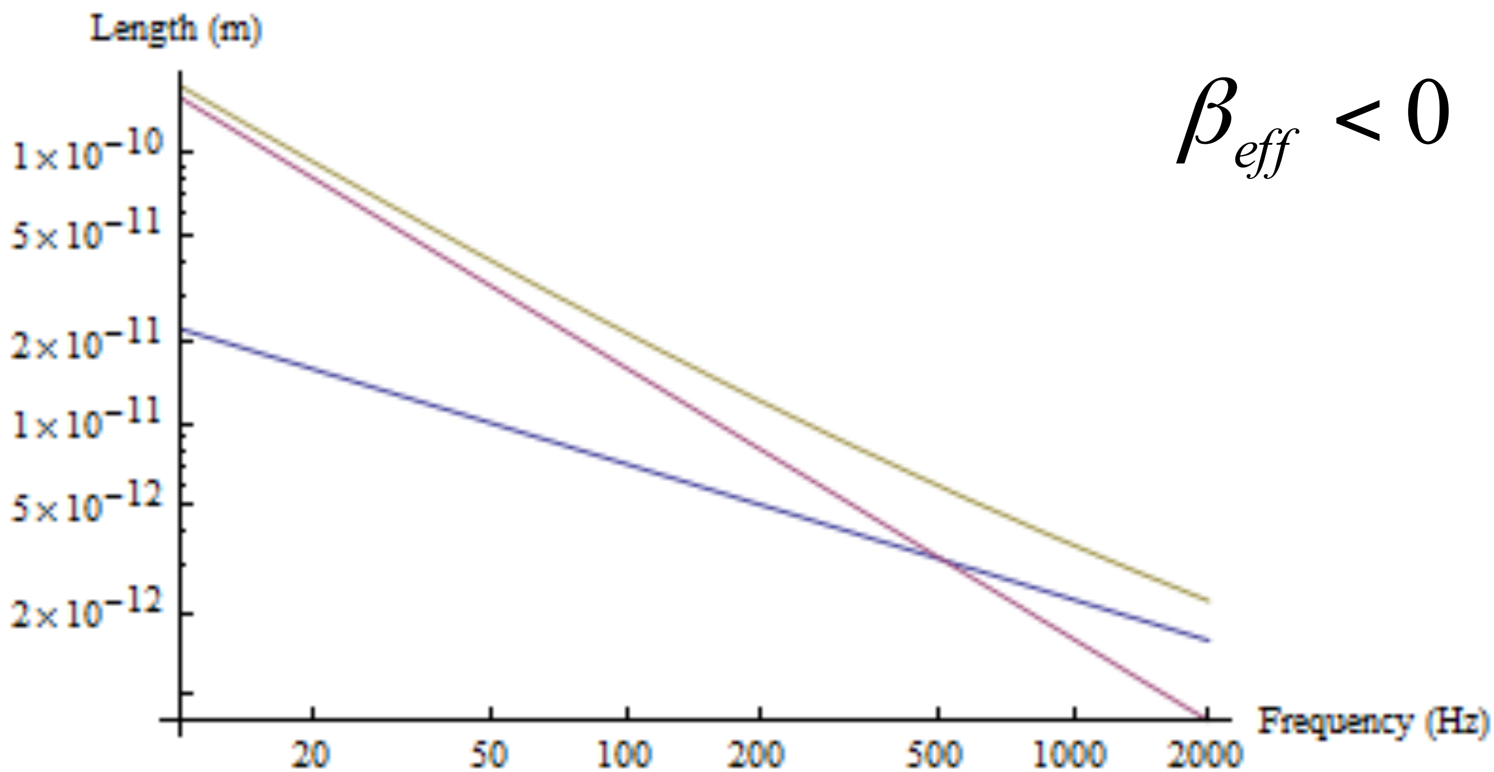
$$\Delta\ell = \frac{P_0}{A\kappa}\left[\alpha\frac{a}{\omega}e^{i(\omega t+90^\circ)} + \beta_{eff}\frac{\lambda}{4\pi}\sqrt{\frac{a}{\omega}}e^{i(\omega t-45^\circ)}\right]$$



$$\beta_{eff} > 0$$

Expected Signal: Canonical Form

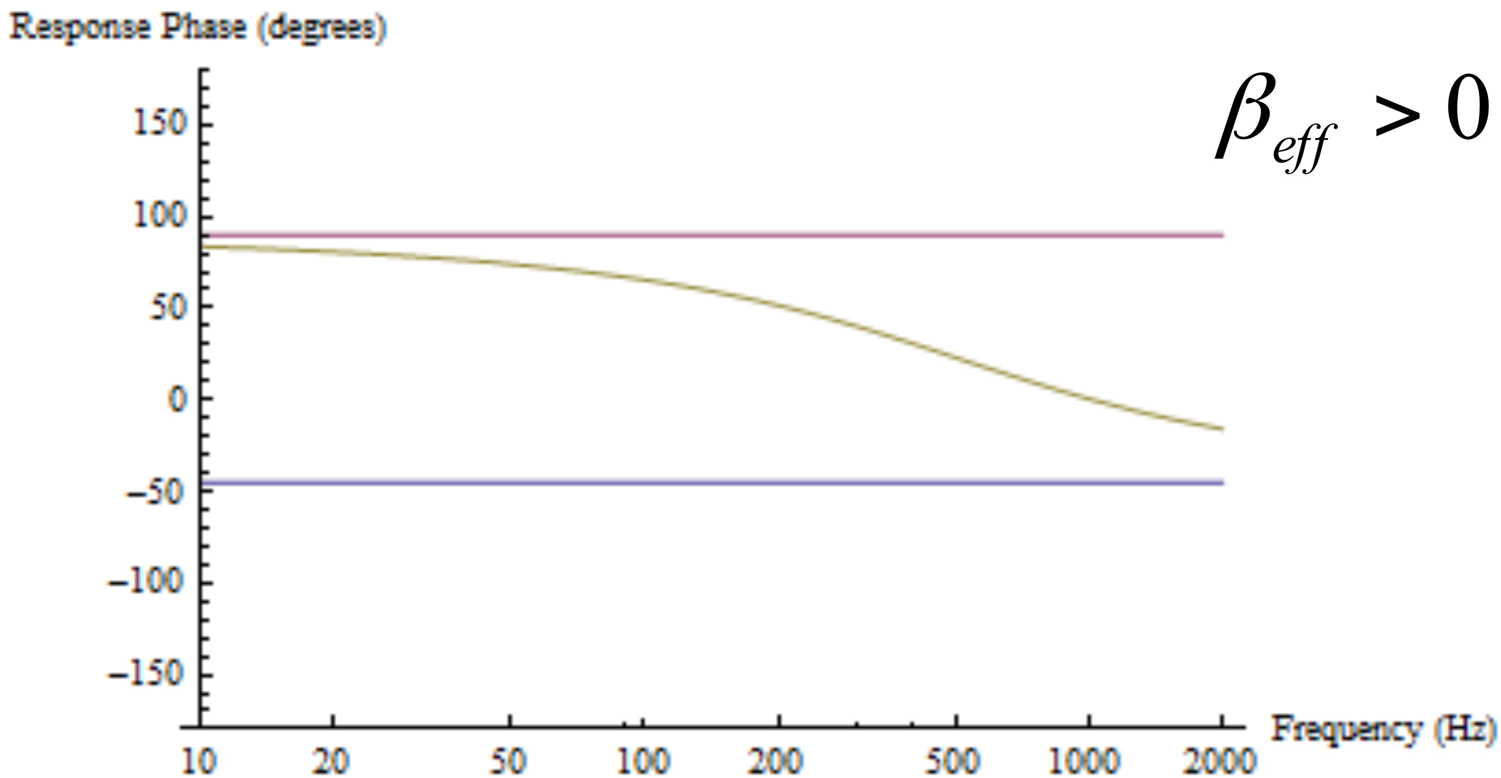
$$\Delta \ell = \frac{P_0}{A\kappa}\left[\alpha \frac{a}{\omega} e^{i(\omega t + 90^\circ)} + \beta_{eff} \frac{\lambda}{4\pi}\sqrt{\frac{a}{\omega}} e^{i(\omega t - 45^\circ)}\right]$$



$$\beta_{eff} < 0$$

Expected Signal: Canonical Form

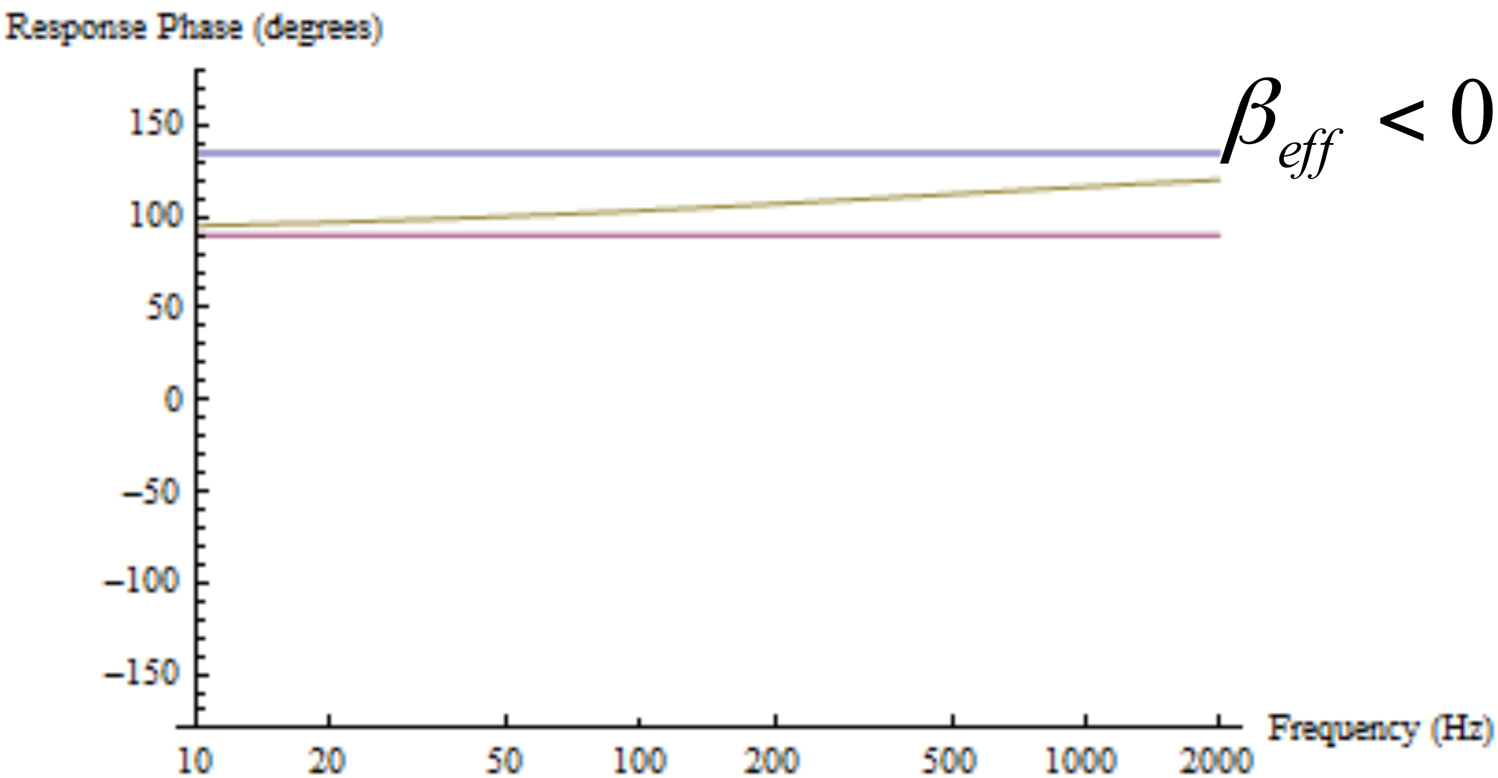
$$\Delta\ell = \frac{P_0}{A\kappa}\left[\alpha \frac{a}{\omega} e^{i(\omega t + 90^\circ)} + \beta_{eff} \frac{\lambda}{4\pi}\sqrt{\frac{a}{\omega}} e^{i(\omega t - 45^\circ)}\right]$$



$$\beta_{eff} > 0$$

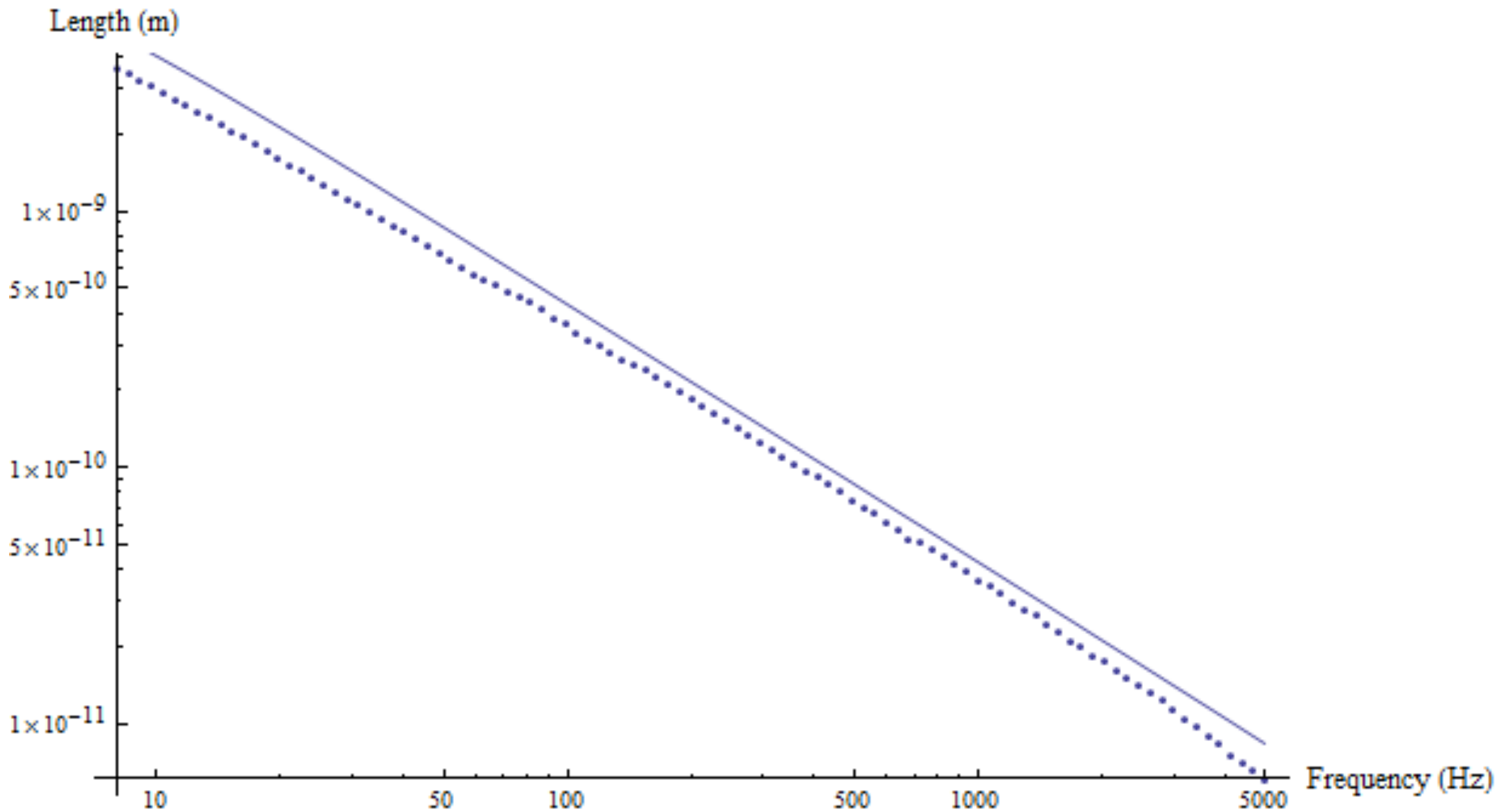
Expected Signal: Canonical Form

$$\Delta\ell = \frac{P_0}{A\kappa}\left[\alpha\frac{a}{\omega}e^{i(\omega t+90^\circ)} + \beta_{eff}\frac{\lambda}{4\pi}\sqrt{\frac{a}{\omega}}e^{i(\omega t-45^\circ)}\right]$$

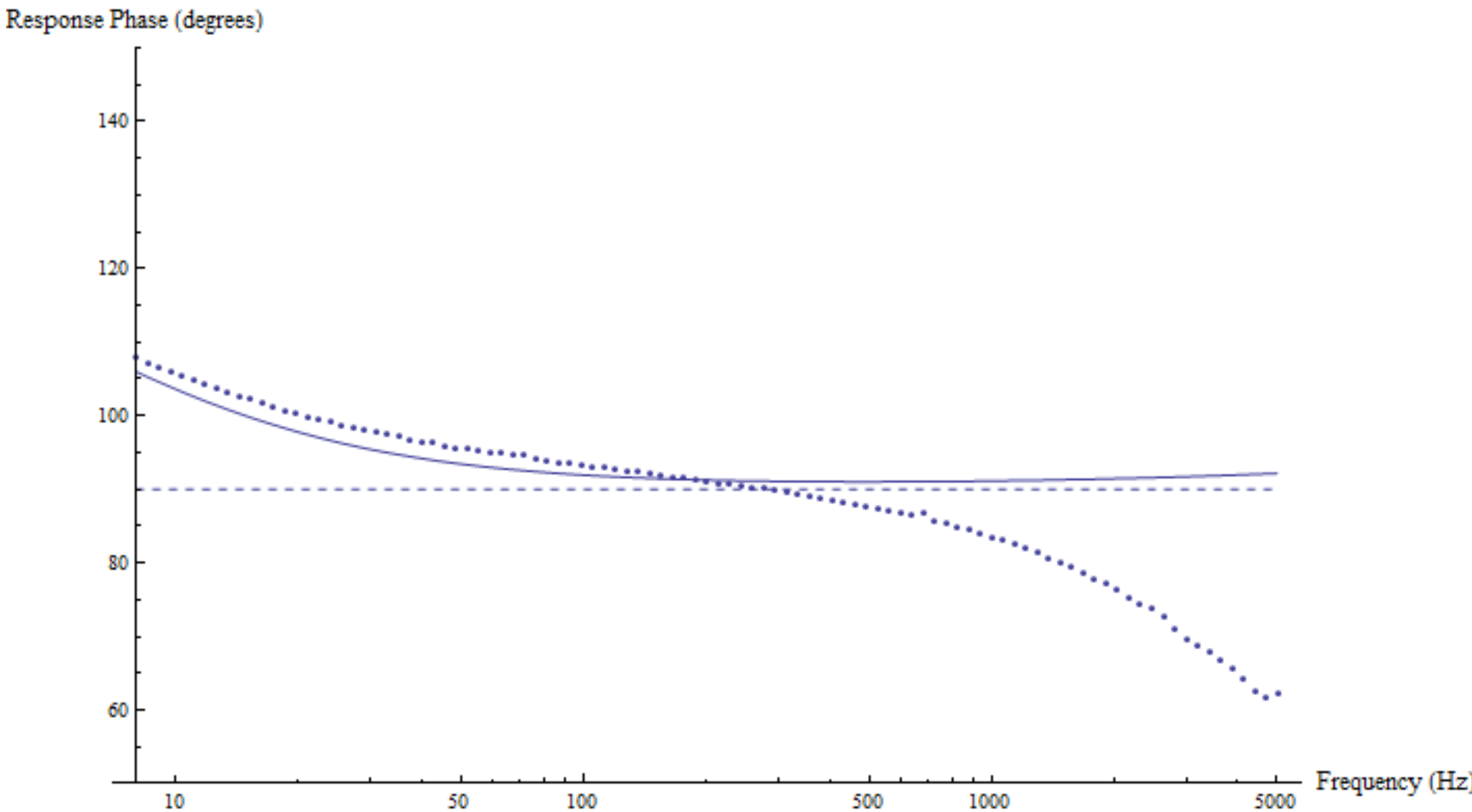


$\beta_{eff} < 0$

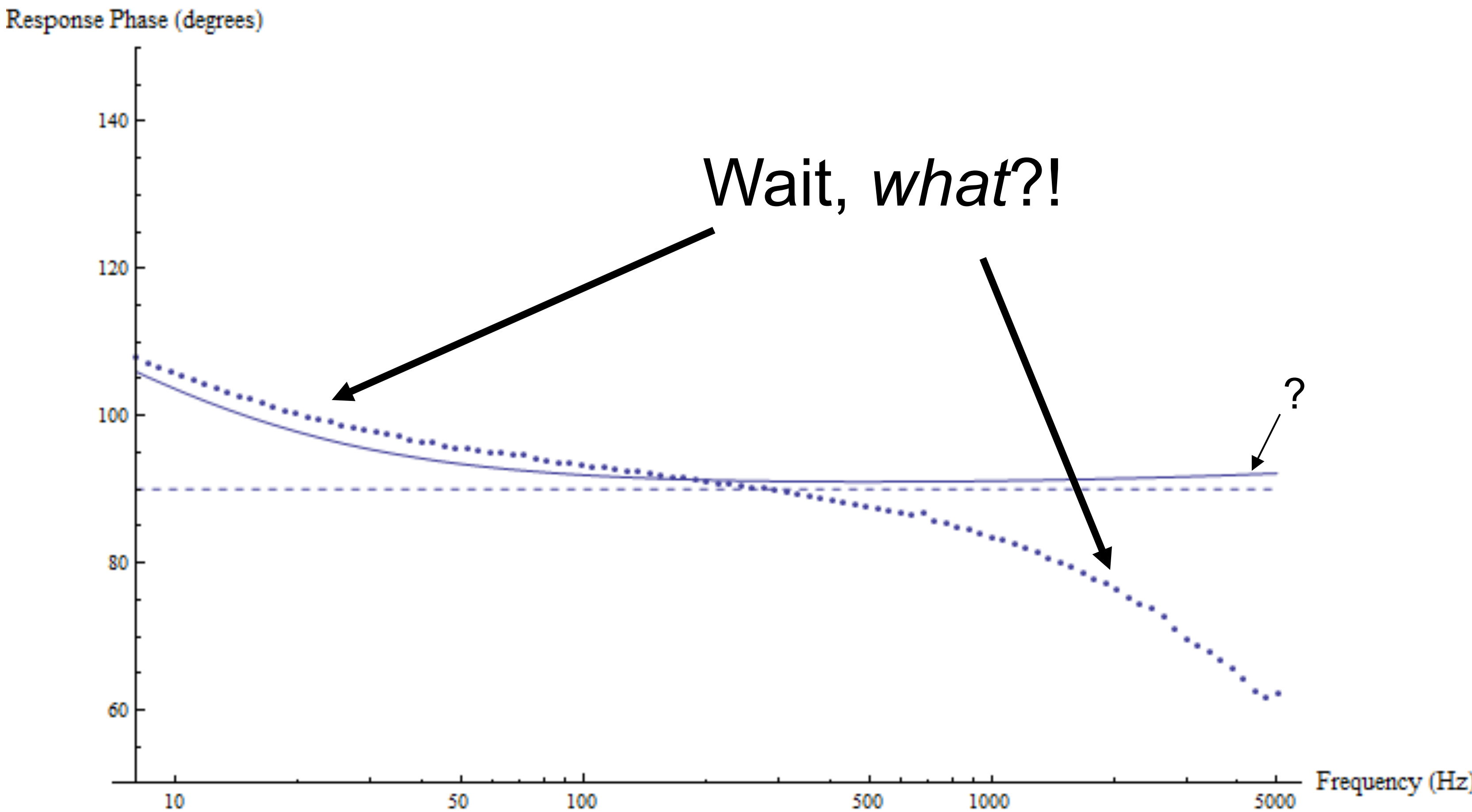
Recent Results: Sapphire Substrate Response Magnitude



Recent Results: Sapphire Substrate Response Phase

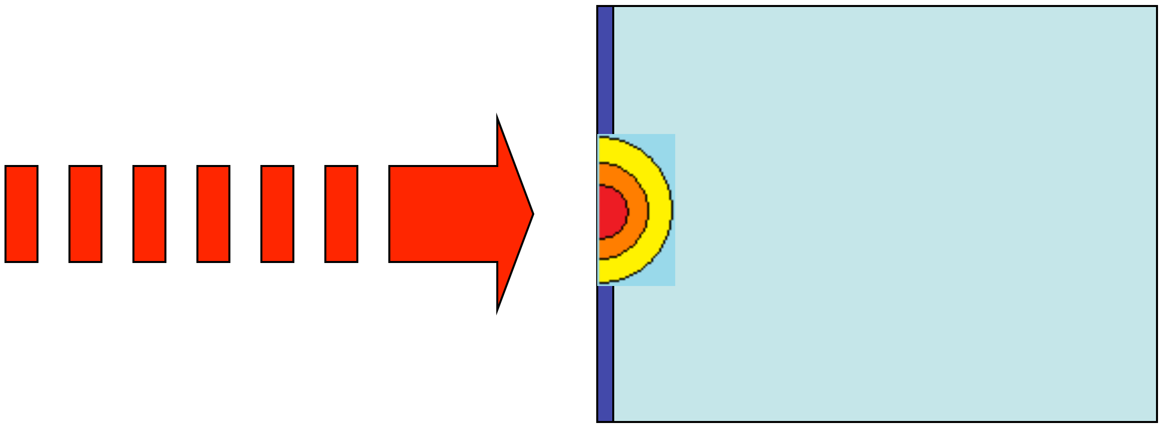


Recent Results: Sapphire Substrate Response Phase



Sapphire: Long Thermal Wavelength

$w_{pump} \approx \lambda_{thermal}$ really means we have a 3-D problem (axially symmetric), "plane thermal waves" don't work



"Cerdonio"-type solution

- Green's function on the surface of a half-space

$$G(t,\vec{x},\tau,\vec{x}') = \left[\frac{2}{4\pi a(t-\tau)}\right]^{3/2} e^{\frac{-\left|\vec{x}-\vec{x}'\right|^2}{4a(t-\tau)}}$$

- Forced sinusoidally with a Gaussian profiled beam

$$f(x,y,z,t) = \frac{P_0}{\rho C \pi r_0^2 z_0} e^{-\left(\frac{x^2+y^2}{r_0^2}\right)} e^{i\omega t} \qquad 0 < z < z_0$$

Then all you have to do is…

- Integrate

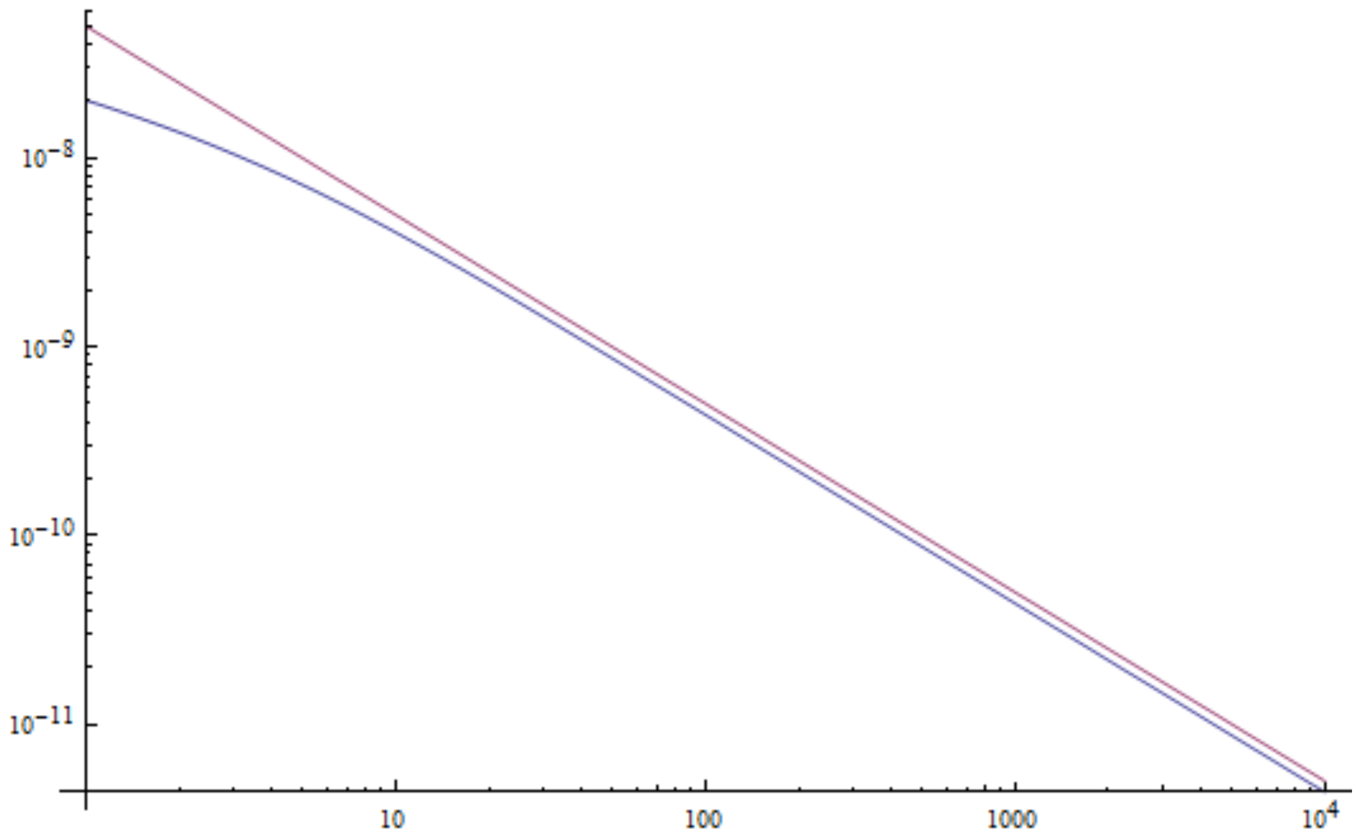
$$u(t,x,y,z)=\int_{-\infty}^{t}d\tau\int_{0}^{z_0}dz'\int_{-\infty}^{\infty}dy'\int_{-\infty}^{\infty}dx'2\left[\frac{1}{4\pi\alpha(t-\tau)}\right]^{3/2}e^{\frac{-|\vec{x}-\vec{x}'|^2}{4\alpha(t-\tau)}}\frac{P_0}{\rho C\pi r_0^2 z_0}e^{-\left(\frac{x'^2+y'^2}{r_0^2}\right)}e^{i\omega\tau}$$

- and again.

$$l(t)=a\int_{0}^{\infty}u(0,0,z,t)dz$$

Thanks Mathematica

$$l(t) = a\frac{P_0}{\rho C\pi}e^{i\omega t}\left[-\frac{ie^{ir_0^2\omega/4\alpha}}{4\alpha}\left(\pi - i\cdot ExpIntegralEi\left[-ir_0^2\omega/4\alpha\right]\right)\right]$$



Thanks Mathematica

$$l(t) = a\frac{P_0}{\rho C\pi}e^{i\omega t}\left[-\frac{ie^{ir_0^2\omega/4\alpha}}{4\alpha}\left(\pi - i\cdot ExpIntegralEi\left[-ir_0^2\omega/4\alpha\right]\right)\right]$$