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LIGO-T1200418-v3 *ADVANCED LIGO* 19 April 2013

LLO MC2 Violin Mode Q

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**Table of Contents**

1 Introduction 3

1.1 Purpose and Scope 3

1.2 References 3

1.3 Version history 3

2 Measurement 3

3 Theory 3

4 Model parameter values 6

5 Results 6

5.1 Frequencies and tensions 7

5.2 Damping 9

6 Conclusion 11

7 Appendix 14

# Introduction

## Purpose and Scope

This is a writeup of a calculation to understand the measured violin mode Q’s of the MC2 suspension at LLO but much of the background is applicable to all suspensions.

## References

LLO alog entries [4470](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=4470), [4472](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=4472), [5097](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5097), [5280](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5280)

G. Cagnoli et al., Phys. Lett. A 255 (1999), p230

[T0900415](https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=5084): Upper Limit to Suspension Thermal Noise from LIGO 1 and Implications for Wire Suspensions in Advanced LIGO

T070101: [Dissipation Dilution](https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=27812)

T080096: [Wire Attachment Points and Flexure Corrections](https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=10955)

LIGO-T0900435: [HAM Small Triple Suspension (HSTS) Final Design Document](https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=5424)

LIGO-D020700: [HSTS Overall Assembly](https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=6873)

Cumming et al., Design and development of the advanced LIGO monolithic fused silica suspension, Class. Quantum Grav. 29 (2012) 035003.

## Version history

8/28/12: -v1.

10/31/12: -v2, with analysis of data on additional wires and modes taken by Keiko and posted 10/29 in LLO alog [5097](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5097).

4/19/13: -v3. Various refinements suggested by Jeff K, including a calculation of net thermal noise. Incorporate refitted Q’s from LLO alog 5280.

# Measurement

The violin mode of one of the wires supporting the optic in the MC2 was measured by Keiko Kokeyama, with advice from Gaby Gonzalez and Peter Fritschel. See LLO alog entries [4470](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=4470) and [4472](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=4472). The frequency and Q were 631.55 Hz and 2.3x105. See -v1 of this document for the analysis of that data point.

Later, Keiko measured the frequencies and Q’s of the first three modes of all four wires. See LLO alog entry [5097](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5097). This data is reproduced in Table 2 (Section 5) below.

Later again, Keiko reprocessed the ringdown curves from alog [5097](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5097) using an improved Q-fitting procedure to produce a new data set in alog [5280](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5280), reproduced in Table 6 (Section 6) below. However it turns out that there is probably some problem with this reanalysis because it implies unphysical damping parameters.

# Theory

## Mode frequencies

To see whether these measurements were reasonable, the frequency and Q were calculated using the Mathematica model of the suspension, specifically case {"mark.barton", "20120120hstsMC2damp"} of the TripleLite2 model. This is equivalent to the Matlab parameter set ^/trunk/Common/MatlabTools/TripleModel\_Production/hstsopt\_metal.m revision 2007 which has given a good fit with measured TFs. It also includes modifications, used below, for optionally assigning a separate damping function on each of the four final wires, so as to allow net pendulum mode thermal noise to be calculated from fitted parameters on the respective wires. However since neither the Mathematica nor Matlab models includes violin modes explicitly, calculating these was a matter of using numerical values from the parameter sets in general formulae as described below.

Per Eq. 2.67 of Fletcher and Rossing, to second order in small quantities, the frequency of a violin mode is



(Their  has been renamed  to avoid confusion with the thermodynamic material property  used below.)

Here is the mode number, and



 ,

is the frequency of a wire without bending stiffness but the same length , tension and mass per length .

The dimensionless quantity  (formerly) is



where is the radius of gyration of the wire,  is the Young’s Modulus, and  is the cross-sectional area, but it is closely related to the usual flexure length, defined (T080096) as



**

Here,  is the second moment of area of the wire in the bending direction, equal to  in any direction for a wire of circular cross-section. (The moments of area of the bottom wires in the longitudinal and transverse directions are called M31 and M32 in the model code.)

It is convenient and instructive to put the above formula in terms of :

**

This makes it obvious that to first order in  (≈ 0.00248 for the HSTS) the effect is simply to shorten the wire by one flexure length  at each end for all harmonics. This is consistent with the fact that a wire of non-zero bending stiffness does not bend sharply at the clamp point but along a curve that for most purposes gives the effect of a pivot  away from the attachment point. In addition, there is also a tiny shortening  second order in both  and mode number . The plain  term disappears because it turns out to be an artifact of doing the expansion in the numerator rather than the denominator, i.e.,



## Damping

The  of the violin mode depends on the material damping factor  and the dissipation dilution factor . The damping factor is modeled as a frequency-independent structural term  (Cagnoli et al. 1999; also T0900415) plus a thermoelastic term:







where (e.g., Cumming et al.)



is a time constant for heat diffusion across the wire ( is heat capacity and  is heat conductivity), and



is twice the thermoelastic damping at the peak frequency ( is temperature,  is linear expansion, , and  is stress). The magic number 0.0732 is a geometrical factor for wires of cylindrical shape, equal to  where  is the first zero of the derivative of the first Bessel function of the first kind:





Because the energy in a violin mode is stored in second-order stress changes of the elastic material, dissipation dilution is applicable (T070101) and the quality factor  is not just  for the material, but  where



Again there is a higher order term proportional to , which turns out to be significant.

# Model parameter values

The following table gives symbol names and values for key parameters from the “production” HSTS model as of 1/20/2012 through the date of this report, which aims to be a good approximation to a generic HSTS suspension and has given good fits to measured transfer functions. In some cases, e.g., the optic mass m3, a value specific to L1:MC2 is available and this has been noted in the table and used where appropriate in the subsequent analysis. The model can be found in the SUS SVN at

^/trunk/Common/MathematicaModels/TripleLite2/mark.barton/20120120hsts

Table 1: Key parameter values from Mathematica model “20120120hsts”

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter (Theory) | Parameter (Mathematica) | Value (SI Units) | Note |
|  | m3 | 2.889 | optic mass (generic HSTS value) |
|  | m3 | 2.90099 | optic mass, L1:MC2 with prisms |
|  | l3 | 0.22 | wire length |
|  | Y3==Ysteel | 2.119\*10^11 | Young’s modulus |
|  | r3 | 0.0000597 | wire radius |
|  | flex3 | 0.000546237 | flexure length (generic HSTS value) |
|  | M31 | 9.97671\*10^-18 | wire second moment of area |
|  | betasteel | -2.5\*10^-4 | logarithmic rate of change of Young’s modulus with temperature |
|  | alphasteel | 12\*10^-6 | thermal expansion coefficient |
|  | rhosteel | 7800 | density |
|  | Csteel | 486 | heat capacity |
|  | phisteel | 2\*10^-4 | structural component of phi |
|  | taufibre | 0.0000813372 | thermoelastic time constant |
|  | deltafibre | 0.00263381 | thermoelastic half maximum phi |
| (n=1) | D1 | 0.00502663 | dissipation dilution (n=1) |
| (n=2) | D2 | 0.00520916 | dissipation dilution (n=2) |
| (n=3) | D3 | 0.00551338 | dissipation dilution (n=3) |

# Results

The raw data from LLO alog [5097](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5097) (initial ringdown fitting algorithm) is given in Table 2.

Table 2: Raw data from initial ringdown fitting algorithm

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Wire | f1 (Hz) | Q1 | f2 (Hz) | Q2 | f3 (Hz) | Q3 |
| #1 | 631.56 | 247105 | 1263.3 | 163943 | 1895.19 | 141496 |
| #2 | 644.977 | 251198 | 1290.09 | 172007 | 1935.45 | 150543 |
| #3 | 660.305 | 249889 | 1320.62 | 187364 | 1981.46 | 177172 |
| #4 | 671.023 | 229827 | 1342.16 | 175060 | 2013.56 | 169019 |

## Frequencies and tensions

Using values from the model in the frequency formula of Section 1 gives f = 650.55 Hz. This represented a mystery in -v1 of this report when the only frequency value was for wire #1, i.e., 631.56 Hz or a 4% discrepancy. However with the luxury of frequency values for all four wires (Table 2) it becomes obvious that the problem was simply that there is a spread of values, presumably due to uneven tensions, and that the theoretical value is in the centre of the cluster.

To make this precise, the tension in each wire was inferred from the n=1 mode frequency using the above equation for with - see Table 3. The total tension over 4 wires is 28.4791 N (frequency is proportional to the square root of tension and could not haave been sensibly averaged or added). This is within 0.07% of the total load force from a mass of 2.90099 kg, i.e., 28.4587 N, which is excellent agreement.



Table 3: Tensions inferred from measured fundamental violin mode frequencies

|  |  |  |  |
| --- | --- | --- | --- |
| Wire number | f1 (Hz) | Tension (N) | % difference from mean |
| #1 | 631.56 | 6.67533 | -6.24 |
| #2 | 644.977 | 6.96346 | -2.20 |
| #3 | 660.305 | 7.30009 | +2.53 |
| #4 | 671.023 | 7.54019 | +5.90 |
| Sum |  | 28.4791 |  |
| MC2mass\*g |  | 28.4587 |  |

Besides the sum, there are three other linear combinations of interest, which are related to the pitch, roll, and “pringle” forces on the optic:

“pitch” = BL+BR-FL-FR- (BL = back left, etc)

“roll” = FL+BL-FR-BR

“pringle” = FL+BR-FR-BL

Because the modes are measured by exciting the whole mass, it is difficult to identify which wire position corresponds to which frequency and the wire numbers have been assigned purely by ascending order of frequency. Thus we need to consider multiple possible mappings from wire number to position.

If the tensions are plotted as a function of wire number, as in Figure 1, they fall on a straight line to a good approximation, that is, they are equally spaced. This is a striking pattern but the significance of it is not clear and it may be a coincidence.

Figure 1: Tensions inferred from measured fundamental violin mode frequencies (plotted)



The “pringle” imbalance (between the sums over the two diagonals) can be arbitrarily large because the optic is very stiff and resists it. However it is not possible to assign four equally spaced tensions to the corners so that *only* the “pringle” imbalance is large. In some order or other, the three linear combinations will be 0.048 N, 0.528 N and 1.201 N, i.e., one small and two large.

The small imbalance is presumably “roll”. There is a 0.5° horizontal wedge which gives a left-right COM shift of 0.17 mm. This creates a torque of 0.00482 N.m, which is respectably close to 0.048 N acting over a lever arm of n5 = 0.08 m, i.e., 0.0038 N.m.

The mid-sized imbalance is *probably* “pitch”. A 0.093 mm displacement forward or backward of the prism (slightly less than the gluing accuracy goal of 0.1 mm) would give a slightly smaller torque than in roll, of 0.264 N.m, but the lever arm is smaller (sl = 0.005 m), so this would correspond to a 0.528 N force imbalance. Matching the largest imbalance of 1.2 N with a 0.21 mm prism offset is not totally unthinkable but neither especially plausible.

There aren’t enough clues to determine the sign of pitch, but we can constrain roll: the optic is installed with the thick side of the wedge on the right as viewed from the back (upside down with respect to the substrate drawing). This gives a positive roll torque in standard SUS coordinates, so we need “roll”<0 as defined above to balance it, i.e., FR+BR>FL+BL.

All this implies either the following arrangement or its front-back mirror:

Table 4: Mapping of wire numbers to corners

Note that the front-back assignment is not constrained and could be flipped.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | | | | Left-right sum | Front-back difference (“pitch”) | Pitch torque (N.m) |
| FL #1 631.56 Hz | 6.67533 N | FR #3 660.305 Hz | 7.30009 N | 13.97542 N | -0.52823 N | -0.00264 N.m |
| BL #4 671.023 Hz | 7.54019 N | BR #2 644.977 Hz | 6.96346 N | 14.50365 N |  | |
| Front-back sum: | 14.21552 N |  | 14.26355 N |  | | |
| Right-left difference (“roll”): | -0.04803 N |  | |
| Roll torque | -0.00384 N.m |

## Damping

The stock damping function included with the model has two terms, one for structural and one for thermoelastic damping. The thermoelastic term turns out to contribute the bulk of the damping in the frequency range of the violin modes but the structural term is not negligible. See Figure 2.

Putting in entirely stock values, at the nominal violin mode frequency of 650.56, the total  is 9.9x10-4, almost 5 times the structural term, 2x10-4. Together with a of 0.00503, this gives an effective  of 4.97x10-6 or a of 2.01x105. Interestingly the measured range of 2.3x105 to 2.5x105 is very much in the same ballpark but slightly *better* (lower , higher ).



This discrepancy persists when the stock damping function is used but allowance is made for the uneven tensions. Tension affects the mode Q in three ways: (i) via the thermoelastic , which has a (small, negative) term proportional to stress, (ii) via pushing the frequency to a different point on the damping function, which is moderately steeply increasing in the range of the first three violin modes, and (ii) via the dissipation dilution factor, which falls (more dilution) with tension. In fact the last effect dominates and the net falls with frequency. See the small coloured dots in Figure 3, which lie above the larger coloured dots representing the measured values. However within each cluster (n=1, n=2, n=3) both the measured points and the stock theory with tension correction show the same characteristic downward trend with frequency from the dependency of the dissipation dilution on tension.



Figure 2: Bending loss angle (before dissipation dilution) as a function of frequency for the wire. The thermoelastic peak is visible on the right.





Since three data points (for three different harmonics) are available per wire, we can potentially say something about three different parameters of the damping function, the obvious candidates of interest being the structural and the thermoelastic and . The Mathematica function FindRoot[] was used to find the parameter values that would reproduce the observed Q’s in conjunction with dissipation dilution incorporating the tensions derived earlier. (FindFit[] and NSolve[] were tried first but even when fairly narrow constraints on the sensible solutions were provided, they failed to give curves passing through or even particularly near the data.)



The parameter values obtained are listed in Table 5, together with the stock values for comparison. Curves based on the parameters have been plotted in Figure 3, and do in fact pass precisely through the data points they’re based on. As an aid to the eye, the mode number has been treated as a continuous function of frequency and interpolated so as to give a smooth curve that can be compared to the underlying damping function. Despite the fit being based on only three points on a somewhat unrepresentative section of the curve (the upward slope just to the left of the thermoelastic peak) the consistency of the fitted parameters and the naturalness of the fitted curves is quite good. The fitted values of structural phi are clustered around the stock value of 2x10-4 (mean 2.06x10-4), whereas the fitted values of thermoelastic are clustered around about 71% of the stock value.



Figure 3: Fitted damping curves with Q’s from original ringdown fitting algorithm

Black dots are prediction from stock model. Wire #1=red, #2=blue, #3=green,#4=brown. Large coloured dots are measured 1/Q. Small dots are predictions using stock values for all parameters except wire tensions. Dashed curves use fitted parameters (the mode number n has been interpolated to give a continuous curve).



Table 5: Fitted damping parameters with Q’s from original ringdown fitting algorithm



# Refitted data

Later, Keiko reanalyzed the original ringdown curves with a different fitting algorithm that was hoped to be better. See LLO alog [5280](https://alog.ligo-la.caltech.edu/aLOG/index.php?callRep=5280). The new data is reproduced in Table 6. The frequencies are the same (and thus all the above conclusions about tensions) but the Qs are somewhat different, especially for the n=3 mode.

Table 6: Raw data with Q’s from “improved” fitting algorithm

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Wire number | f1 (Hz) | Q1 | f2 (Hz) | Q2 | f3 (Hz) | Q3 |
| #1 | 631.56 | 234598 | 1263.3 | 160933 | 1895.19 | 195142 |
| #2 | 644.977 | 238817 | 1290.09 | 165250 | 1935.45 | 178511 |
| #3 | 660.305 | 239867 | 1320.62 | 167053 | 1981.46 | 174990 |
| #4 | 671.023 | 229072 | 1342.16 | 164167 | 2013.56 | 177825 |

This rather disturbs the subsequent curve fitting of ,  and . The ’s and ’s are substantially larger than before (the thermoelastic peak frequency is lower), and the  values come out negative, as in Table 7 and Figure 4. Since a negative  is unphysical, there is presumably something wrong with the new algorithm, and we persist with the original Q’s.

Table 7: Fitted damping parameters with Q’s from “improved” ringdown fitting algorithm



Figure 4: Fitted damping curves with Q’s from “improved” ringdown fitting algorithm

Black dots are prediction from stock model. Wire #1=red, #2=blue, #3=green,#4=brown. Large coloured dots are measured 1/Q. Small dots are predictions using stock values for all parameters except wire tensions. Dashed curves use fitted parameters (the mode number n has been interpolated to give a continuous curve).



# Effect on net thermal noise

To examine the effect that different amounts of damping at different corners would have, the stock Mathematica model was altered to allow different damping functions on each of the four lower wires, and the fitted parameters from the initial Q’s were plugged in. The modified model is in the SUS SVN at

^/trunk/Common/MathematicaModels/TripleLite2/mark.barton/20120120hstsMC2damp

The results are compared with those from the stock model in Figure 5 and Table 8. In fact the thermal noise is almost identical up to 30 Hz and about 11% lower at 1000 Hz.

Figure 5: Net pendulum thermal noise



Table 8: Net thermal noise at representative frequencies



# Conclusion

The observed frequencies tell a consistent story. The tensions implied by the frequencies sum to almost exactly the known payload force, and the “roll” imbalance matches the expected value based on the horizontal wedge. It’s difficult to say definitively which of the two remaining imbalances is “pitch” and which is “pringle”, but the smaller of the two is what would be expected if the prisms were at the limit of their 0.1 mm front-back gluing tolerance and so is probably pitch. Conversely the larger is an additional factor two, consistent with the fact that there is little constraint on the “pringle” imbalance.

The damping from the first ringdown fitting algorithm ties up quite well with the model and what small discrepancy there is (of order 25%) is (i) in the advantageous direction and (ii) better explained by the thermoelastic term (rather than the structural term) being low.

The Q’s from the second ringdown fitting algorithm imply unphysical values of , and we propose to ignore them.

To the extent the initial Q values can be trusted, the effect on thermal noise at the optic is very small (<11%) and in the favorable direction compared to the stock model.

# Appendix

In the PDF version of this report, a printout of the Mathematica notebook containing the calculation will be appended.

calculationcalculations will be appended.