LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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LLO MC2 Violin Mode Q

Mark Barton

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of the LIGO Laboratory.

California Institute of Technology
LIGO Project – MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – NW22-295
185 Albany St
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland WA 99352
Phone 509-372-8106
Fax 509-372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone 225-686-3100
Fax 225-686-7189

http://www.ligo.caltech.edu/

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1 Introduction

1.1 Purpose and Scope

This is a writeup of a quick calculation to understand the measured violin mode Q of the MC2 suspension at LLO but is applicable to all HSTS. The interesting conclusion is that the violin mode Q is dominated by thermoelastic damping which has its peak nearby.

1.2 References

LLO alog entries 4470, 4472

T070101: Dissipation Dilution

T080096: Wire Attachment Points and Flexure Corrections

LIGO-T0900435: HAM Small Triple Suspension (HSTS) Final Design Document

LIGO-D020700: HSTS Overall Assembly

Cumming et al., Design and development of the advanced LIGO monolithic fused silica suspension, Class. Quantum Grav. 29 (2012) 035003.

1.3 Version history

8/28/12: -v1.

2 Measurement

The violin mode of one of the wires supporting the optic in the MC2 was measured by Keiko Kokeyama, with advice from Gaby Gonzalez and Peter Fritschel. See LLO alog entries 4470 and 4472. The frequency and Q were 631.55 Hz and $2.3\text{x}10^5$.

3 Theory

To see whether this was reasonable, the frequency and Q were calculated using the Mathematica model of the suspension, specifically case `{"mark.barton", "20120120hsts"}` of the `TripleLite2` model. This is equivalent to the Matlab parameter set `^/trunk/Common/MatlabTools/TripleModel_Production/hstsopt_metal.m` revision 2007 which has given a good fit with measured TFs. However since neither the Mathematica nor Matlab models includes violin modes explicitly calculating these was a matter of using numerical values from the parameter sets in general formulae as described below.

The frequency of a violin mode is approximately

$$f_n = \frac{n}{2(L-2a)}\sqrt{\frac{T}{\rho_L}} = \frac{n}{2(L-2a)}\sqrt{\frac{T}{\rho_V A}}\ \text{c}$$

where $n = 1,2,3...$ is the mode number, L is the wire length, $\rho_L = \rho_V A$ is the linear mass density (ρ_V is the volume mass density and A is the wire area), $T = mg/4$ is tension (m is optic mass and g is gravity; 4 is the number of wires) and a is a flexure correction defined (T080096) as

$$a = \sqrt{\frac{YI}{T}}$$

where Y is the Young's Modulus and I (known as `M` in the model code) is the second moment of area of the wire, equal to $\pi r^4/4$ for a wire of circular cross-section. The flexure correction reflects the fact that a wire of non-zero intrinsic stiffness does not bend sharply at the clamp point but, effectively at least, at an effective point a away.

The Q of the violin mode depends on the material damping factor ϕ and the dissipation dilution factor D. The damping factor is modeled as a frequency-independent structural term plus a thermoelastic term:

$$\phi(f) = \phi_{struct} + \phi_{thermo} = \phi_{struct} + \frac{YT_W}{\rho_V C}\left(\alpha - \frac{\sigma\beta}{Y}\right)^2 \frac{2\pi f \tau \Delta}{1+\left(2\pi f \tau\right)^2)}$$

where (e.g., Cumming et al.)

$$\tau = 0.0732 C r^2 \rho_V / \kappa$$

is a time constant for heat diffusion across the wire (C is heat capacity and κ is heat conductivity) T_W is temperature, α is linear expansion, $\beta = \frac{1}{Y}\frac{dY}{dT_W}$, and $\sigma = T/A$ is stress.

Because the energy in a violin mode is stored in second-order stress changes of the elastic material, dissipation dilution is applicable (T070101) and the quality factor Q is not just $1/\phi$ for the material, but D/ϕ where

$$D = \frac{2a}{L}\left(1 + \frac{a(n\pi)^2}{2L}\right)$$

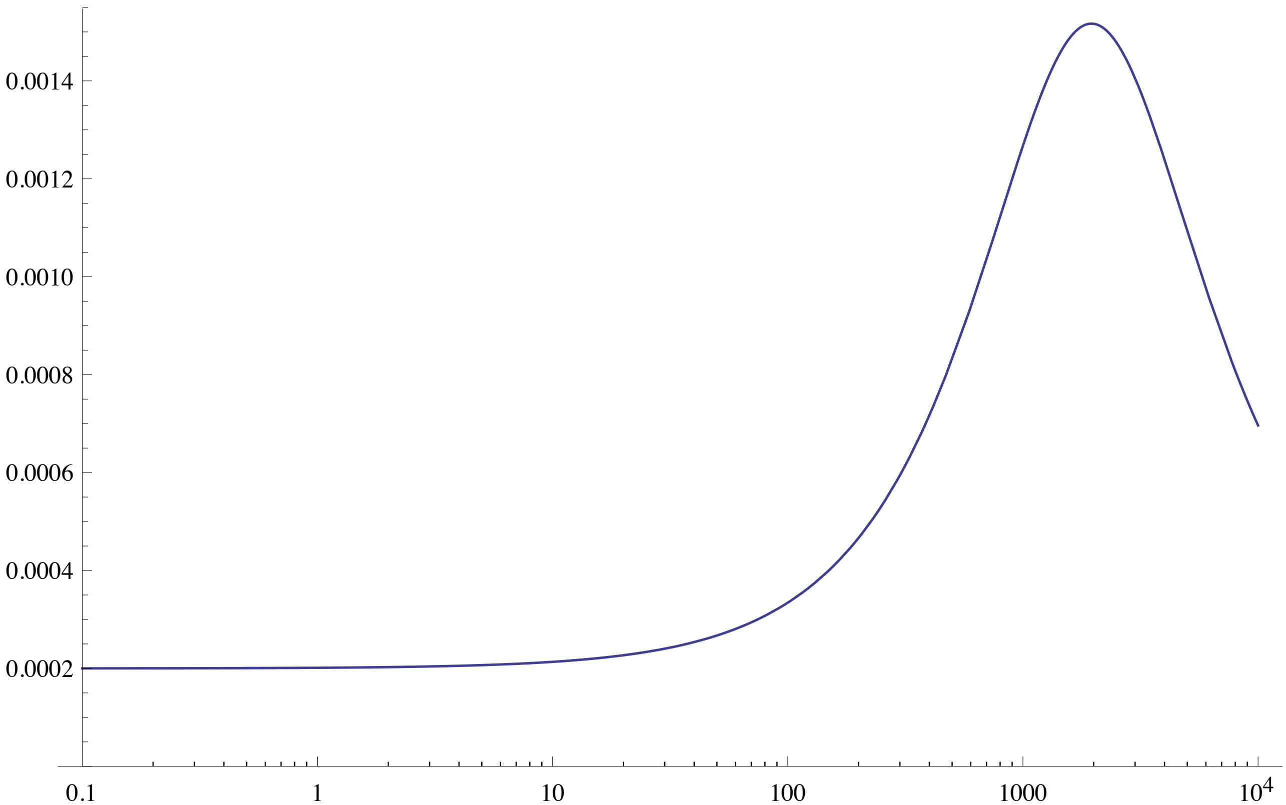
4 Results

Using values from the model in the above frequency formula (see Appendix) gives f = 650.55 Hz, some 29 Hz or 4.5% high, which is fair but not as good as one might have hoped. Parameters that could plausibly be off include wire length `l3`, radius `r3`, and density `rhosteel`.

`l3` was 0.22 m in the final design document (T0900435-v2) and the EASM file of the top level assembly (D020700-v1), but could conceivably have changed if the design of the wire jig didn't quite match the design of the bottom stage, or if there's some subtle difference between the metal and glass builds. The wire was nominally 0.047" in diameter, 597 µm in radius, but could conceivably have been slightly off depending on the tolerance and whether it was in fact in spec. A generic value of 7800 kg/m^3 was used for carbon steel, but some sources suggest 7850 for piano wire specifically, e.g., https://sites.google.com/site/lorenkoehlerwebsite/piano-craft/piano-wire .

The thermoelastic term turns out to contribute the bulk of the damping in the frequency range of the violin modes. See Figure 1.

Figure 1: Loss angle as a function of frequency for the wire. The thermoelastic peak is visible on the right.



At the violin mode frequency, the total ϕ is 9.9x10^{-4}, almost 5 times the structural term, 2x10^{-4}. Together with a D of 0.00503, this gives an effective ϕ of 4.97x10^{-6} or a Q of 2.01x10^{5}, which is quite close to the measured value of 2.3x10^{5}.

5 Conclusion

The measured values of frequency and Q are both quite close to predicted values, although because the parameters that go into the frequency are more accurately measurable, the discrepancy is more significant, and some more work could usefully be done to understand it.

6 Appendix

In the PDF version of this report, a printout of the Mathematica notebook contain the calculation will be appended.

Calculation of TripleLite2 model with violin mode stuff (for T1200418-v1)

■ Setup

Switches to enable loading of previously saved results instead of recalculating from scratch

In[2]:=
```
useprecomputed = True; (* set to True to use saved results from precomputed subdirectory *)
If[useprecomputed,
    exceptdamping = False, (* False by default, True to recalculate just damping-dependent stuff*)
    exceptdamping = True (* DON"T CHANGE *)
];
```

In[4]:=
```
loadcasefromuser["ASUS3L2ModelCaseDefn.m"];
```

In[5]:=
```
modelcase
```

Out[5]= {mark.barton, 20120120hsts}

In[6]:=
```
modelcasecomment
```

Out[6]= Equivalent to Jeff K's hstsopt_metal.m revision 2007 of 1/19/12.

■ Pre-flight check

In[7]:=
```
Reset[All]
```

In[8]:=
```
Calculate[constval];
```

Wire length

In[9]:=
```
l3 /. constval
```

Out[9]= 0.22

Young’s modulus

In[10]:=
```
Y3 /. constval
```

Out[10]= 2.119×10^{11}

Wire radius

In[11]:=
```
r3 /. constval
```

Out[11]= 0.0000597

Wire tension

In[12]:=
```
tension = m3 * g / nw3 /. constval
```

Out[12]= 7.08527

Wire second moment of area

In[13]:=
```
M31 /. constval (* a.k.a. I *)
```

Out[13]= 9.97671×10^{-18}

Fundamental violin mode for wires above mass 3 (the optic)

In[14]:=
```
f31 = Sqrt[(m3 * g / c3 / nw3) / (rhosteel * A3)] / (2 * (l3 - 2 * flex3)) /. constval
```

Out[14]= 650.565

Giles’ formula for D

In[15]:=
```
D1 = (2 / l3) * Sqrt[Y2 * M31 / tension] *
     (1 + (1 / 2 / l3) * (Pi * n) ^2 * Sqrt[Y2 * M31 / tension]) /. constval /. n → 1
```

Out[15]= 0.00502663

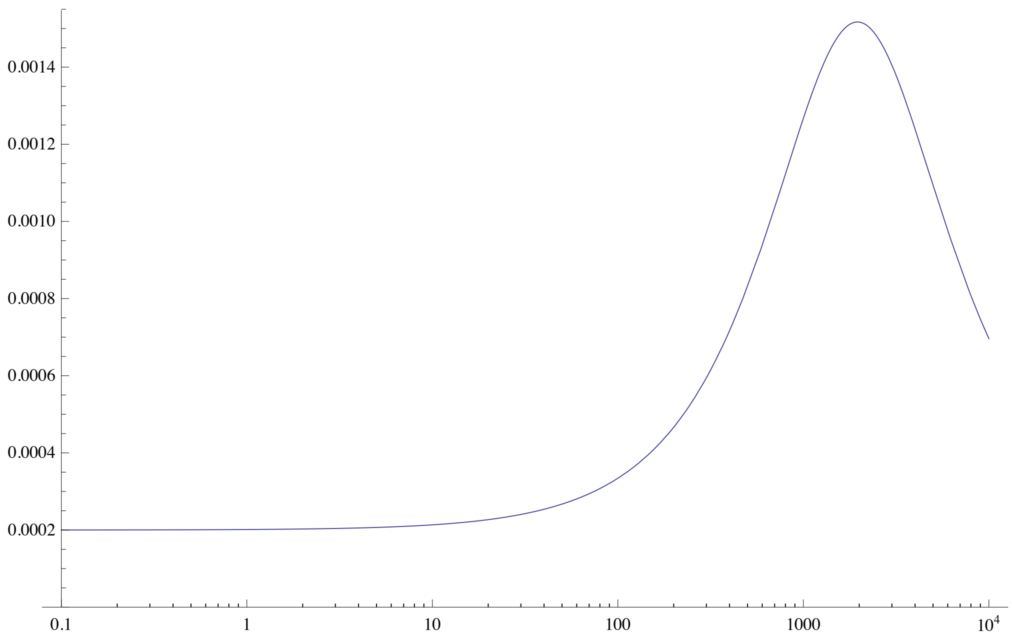
Structural component of wire damping

In[16]:=
```
phisteel /. constval
```

Out[16]= 0.0002

In[17]:=
```
Qstruct = 1 / (D1 * phisteel) /. constval
```
Out[17]= 994 702.

phi(f) - note thermoelastic peak in the vicinity of the violin modes (damping tag is “fibreatype” but model really does have wire parameters)

In[18]:=
```
LogLinearPlot[damping[imag, fibreatype][f] /. constval,
 {f, 0.1, 10 000}, PlotRange → {0, Automatic}]
```
Out[18]=


phi at violin mode

In[20]:=
```
damping[imag, fibreatype][f31] /. constval
```
Out[20]= 0.000988515

phi at violin mode

In[48]:=
```
D1 * damping[imag, fibreatype][f31] /. constval
```
Out[48]= 4.96945×10^{-6}

Q with thermoelastic

In[19]:=
```
1 / (D1 * damping[imag, fibreatype][f31]) /. constval
```
Out[19]= 201 252.