Modeling Magnetic Coupling to Suspensions Part 1: The Physics and Designed Mitigation

G1200524-v2 J. Kissel

Physics 202 Freshman Year Magnetism

In the presence of an external static (independent of time/frequency) Magnet Field, the force felt on a Magnet is:





Physics 402 **Junior Year Magnetism**

The Magnetic Field is a function of the Magnetic Vector Potential

 $\vec{B} = \vec{\nabla} \times \vec{A}$

Vector Potential is defined by integrating over a local **Current Density** as observed from a **Point** some **Distance**

 $\vec{A}(\vec{R}) = \frac{\mu_0}{4\pi} I \oint \frac{1}{|\vec{R} - \vec{r}|} d\vec{\ell}$

If sufficiently far away, you can make the **Multipole** Approximation

 $r \ll R$

with which you can see what G12terms2dominate the Potential

$$\frac{1}{|\vec{R} - \vec{r}|} \approx \frac{1}{R} + \frac{\vec{R} \cdot \vec{r}}{R^3} + \frac{3(\vec{R} \cdot \vec{r})^2 - R^2 r^2}{R^5} + \dots$$

$$\frac{1}{R} = \frac{1}{R} + \frac{r}{R^2} + \frac{r^2}{R^3} + \frac{r^3}{R^4} + \frac{r^3}{R^4} + \dots$$



Physics 402 Junior Year Magnetism

But, Maxwell says there're no magnetic $\vec{\nabla} \bullet \vec{B} = 0$ monopoles

So the dominant term left is the Dipole

$$\vec{A}_{DIPOLE}(\vec{R}) = \frac{\mu_0}{4\pi} \left[\frac{(\vec{\mu} \times \vec{R})}{R^3} \right] \propto \frac{r_{\mu}^2}{R^2}$$

assume $r_{ext} \approx r_{\nabla} \approx r_{\mu} = r << R$

Where we've defined the magnetic dipole moment

$$\vec{\mu} = I \oint (\hat{R} \bullet \vec{r}) d\ell = I \int d\vec{a} = I \vec{A} \propto r_{\mu}^{2}$$

Which means (after "some math") the Magnetic Dipole Field is

$$\vec{B}_{DIP}(\vec{R}) = \vec{\nabla} \times \vec{A}_{DIP} = \frac{\mu_0}{4\pi} \frac{1}{R^3} \left[3(\vec{\mu} \cdot \hat{R})\hat{R} - \vec{\mu} \right] \propto \frac{1}{R} \cdot \frac{r_{\mu}^2}{R^2} \propto \frac{r_{\mu}^2}{R^3}$$
So, an **External Field**, assuming it's also a dipole, acting on a
dipole moment feels a **Force**
(where the **Gradient** is over the coordinates $\vec{F}_{\mu}^{DIP} = (\vec{\mu} \cdot \vec{\nabla})\vec{B}_{ext}^{DIP} \propto \left(r_{\mu}^2 \cdot \frac{1}{r_{\nabla}}\right) \frac{r_{ext}^2}{R^3} \propto \frac{r^3}{R^3}$

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Physics 502 Senior Magnetism

Thus far we've assumed a static field, created by a constant current. What if that current is a function of time?

$$\vec{I}(t) = \vec{I}_0 \cos\left[\omega(t - \frac{|\vec{R} - \vec{r}|}{c})\right]$$

Electromagnetic waves!

$$\vec{A}(t) = \frac{\mu_0}{4\pi} I_0 \oint \frac{\cos\left[\omega(t - \frac{|\vec{R} - \vec{r}|}{c})\right]}{|\vec{R} - \vec{r}|} d\vec{\ell}$$

After "some math," the dominate **dipole** field (in spherical coordinates) [Cowan, 1968]

$$B_{r} = \frac{2\mu_{0}A\cos\theta}{4\pi}I_{0}\left[\frac{1}{R^{3}}\cos[\omega(t-R/c)] - \frac{\omega}{R^{2}c}\sin[\omega(t-R/c)]\right]$$
$$B_{\theta} = \frac{\mu_{0}A\sin\theta}{4\pi}I_{0}\left[\left(\frac{1}{R^{3}} - \frac{\omega^{2}}{rc^{2}}\right)\cos[\omega(t-R/c)] - \frac{\omega}{R^{2}c}\sin[\omega(t-R/c)]\right]$$
$$E_{\phi} = \frac{\mu_{0}A\sin\theta}{4\pi}I_{0}\left[\frac{\omega^{2}}{Rc}\cos[\omega(t-R/c)] + \frac{\omega^{2}}{R}\sin[\omega(t-R/c)]\right]$$

$$\vec{B}(t) = \vec{\nabla} \times \vec{A}(t)$$
$$\vec{E}(t) = \frac{\partial \vec{A}(t)}{\partial t}$$

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Physics 502 LIGO Magnetism

We have time dependent External Fields all over!











And magnets all over for local electromagnetic control!

How are we not screwed?

We arrange the **magnets** to form higher order **Multipoles...**





Then we can define the next several multipole moments

$$\mu_{i} \equiv \frac{1}{2} \int_{V} r_{j} J_{i} dv \qquad \mu_{ij} \equiv \frac{2}{3} \int_{V} r_{j} r_{k} J_{i} dv \qquad \mu_{ijk} \equiv \frac{2}{5} \int_{V} r_{j} r_{k} r_{\ell} J_{i} dv$$

$$\propto r^{2} \qquad \propto r^{3} \qquad \propto r^{4}$$

$$\text{dipole} \qquad \text{quadrapole} \qquad \text{octapole}$$

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Physics 702 Р **Graduate Magnetism** In the presense of an Slowly Varying External Field $B_{k}(\vec{R}) = B_{k}^{(0)} + (\nabla_{\ell}B_{k})r_{\ell} + \frac{1}{2}(\nabla_{m}\nabla_{\ell}B_{k})r_{\ell}r_{m} + \frac{1}{6}(\nabla_{m}\nabla_{\ell}B_{k})r_{\ell}r_{m} + \dots$ $R - \kappa$) "it can be shown" that the Force on the static current distribution is $F_i = \mu_j \nabla_j B_i + \frac{1}{2} \mu_{jk} \nabla_k \nabla_j B_i + \frac{1}{6} \mu_{jk\ell} \nabla_\ell \nabla_\ell \nabla_k \nabla_j B_i + \dots$ $\propto r_{\mu}^{2} \frac{1}{r_{\mu}} \frac{r_{ext}^{2}}{R^{3}} + r_{\mu}^{3} \frac{1}{r_{\mu}^{2}} \frac{r_{ext}^{3}}{R^{4}} + r_{\mu}^{4} \frac{1}{r_{\mu}^{3}} \frac{r_{ext}^{4}}{R^{5}} + \dots$ $\propto \frac{r^3}{R^3} + \frac{r^4}{R^4} + \frac{r^5}{R^5} + \dots$ assume $r_{ext} \approx r_{\nabla} \approx r_{\mu} = r << R$ dipole quadrapole octapole

Up to this point, we've worked to get to the ultra general case where we assume the **current density** is arbitrarily complicated which, by itself, has all of these **moments** that interact with the **external field**. Let's get back to **LIGO**.

Physics 702 LIGO Magnetism

We are free to set up the **current density** (i.e. the **magnets**) exactly how we want it, and create **ideal multipole moments** with our magnets, that, in the far field (r<<R), behave like higher order **moments** only, and therefore couple less to external fields (the force falls off with distance faster)



Take various combinations of **ideal dipoles** with identical **dimensions** and **magnetic**



Physics 502 LIGO Magnetism

Look at a QUAD Top Mass, for example...



Physics 702 LIGO Magnetism

Or an ISI "Coarse" Actuator (ST1 on HAM and BSC), for example...

There are two counter-wound coils to create a quadrapole Also, they're shielded in the "X" and "Y" direction with iron housing







Sect. 5.13 Effect of a Circular Hole in a Perfectly Conducting Plane 203



Modeling Magnetic Coupling to Suspensions Part 2: The Real World and a "Simple" Model

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Physics 2012 The Stuff that isn't in Textbooks

But, as you might guess the real world is not so "pretty."





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Physics 2012 The Stuff that isn't in Textbooks

Take the "quadrupole" that we mentioned earlier:



If the **magnets** are mismatched by α , there is a **residual dipole moment**, μ' equivalent to one of the individual dipole, reduced by the mismatch,

$$\mu' = (1+\alpha)\mu - \mu = \alpha\mu$$

and the force experienced in a external dipole now goes as F_i

$$F_i \propto \alpha \frac{r^3}{R^3} + \frac{r^4}{R^4} + \dots$$

Magnetic Force on Multipoles in Dipole Field $r = 0.05 [m], \alpha = 5\%$ 10 $\alpha * (r^{3}/R^{3})$ (r^{4}/R^{4}) 10⁻² (r^{5}/R^{5}) 10⁻³⊧ 10⁻⁴⊧ Force [arb.] 10⁻⁶ dipole 10^{-7} 10^{-8} octapole quadrapole 10⁻⁹ , 10⁻¹ 10° 10^{1} Radius [m]

and the (reduced) **dipole term dominates** again in the **far field**

A "Simple" Model The Preposition

Six, BSC-ISI Stage 1 Actuators are roughly 1m away from the TOP mass(es) of a QUAD

Assuming the ISI actuators generate stray fields when working to isolate ST1,

How much can they **drive** before the **magnetic force** on the **residual dipole moment** of the **TOP mass(es)** shorts the mechanical & controlled isolation from ST2 and the TOP stage suspension?





A "Simple" Model The History

Requirements:

–From **T010007**, 4.2.7 (Barton, Robertson, Fritschel, Shoemaker, Willems)

"... We require that [technical noise] be ... 10% [the] amplitude of ... pendulum thermal noise. Sources include ... ambient magnetic field fluctuations at the magnetic actuators... ."

-From E050159, Sect. 1.1.4.5 (D. Coyne)

"... max magnetic field and gradient, at ... 10 cm below the optics table, due to SEI Actuators ... shall be < 10 [pT/rtHz] and 20 [pT.m⁻¹/rtHz] at 10 Hz, varying as 1/f."

Previous work:

From T050105 (K. Strain)

- -Used requirements from E050159 for input field
- -Assumed (former baseline design of) ECD magnets on tablecloth
- -Only estimated at 10 Hz
- -Was before final BSC-ISI design
- -Assumed Top mass was 0.2 m away
- -Used VtoV * 0.001 Top to Test TF
- -"Largest allowable dipole moment is 1 [A.m²]"







A "Simple" Model The Culprits (with some grains of salt) B_{ctrl}



F_{mag} A "Simple" Model The Culprits (with some grains of salt)



Conclusions

 Magnetic coupling between SUS magnets and external fields could be a problem, even with what mitigation is in place

- "Simple" model showed one possible source is close but
 - Still not a great model, and is still over simplified
 - There's a lot that could be improved
 - We need some better / more direct measurements*
 - Infrastructure is in place to add fields, refine model
 - As modeled, ISI Acts don't meet interpreted requirements
- Other sinks (for QUAD): UIM & PUM magnets, Blade Spring ECD Magnets,...
- Other sources (for QUAD): ST2 Actuators, Giant ECD Magnets on ACB, ...
- Magnetic short might be worse for HAM Triples since optics are closer to ISI
- Magnetic coupling is hard to predict, because we've tried hard already to get rid of the easy stuff
- Should get rid of any loaded guns (e.g. unused ECD magnets on QUAD*) * Action items

Modeling Magnetic Coupling to Suspensions Part 3: The details of the "Simple" Model

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A "Simple" Model ST1 Loop Design: The Controller



K_{ISI}

- Remember, controller design is independent of blends
- Went for quick, simple, "vanilla"
 design -- not perfect
- 30Hz UGFs
- ~35 deg phase margin
- < 2 gain peaking</p>
- At least a factor of 10 suppression at 10 Hz

– Ignored high-frequency

Designed all DOFs, to see **total colocated drive** (sans damping loops) See attached plot collection for actually legible other DOF plots





κ_{1SI} A "Simple" Model ST1 Loop Design: Proof it's not Crazy



- Sensor noise contributionfrom SEI_sensor_noise.m(Thanks Brian!)
- HEPI Input Motion fromeLIGO HEPI Data, T0900312(Thanks Sam!)
- -Reasonable blend filters (Thanks Rich!)
- No feed forward
- Totally SISO Model (no THC)



*I*_{ctrl} A "Simple" Model The total current output



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Coils

A "Simple" Model The Coil Actuators [T/A]: "Simple" Dipole

Remember from slide 5, we calculated the field of a loop of wire, with an oscillating current:

$$B_{r} = \frac{2\mu_{0}A\cos\theta}{4\pi}I_{0}\left[\frac{1}{R^{3}}\cos[\omega(t-R/c)] - \frac{\omega}{R^{2}c}\sin[\omega(t-R/c)]\right]$$
$$B_{\theta} = \frac{\mu_{0}A\sin\theta}{4\pi}I_{0}\left[\left(\frac{1}{R^{3}} - \frac{\omega^{2}}{rc^{2}}\right)\cos[\omega(t-R/c)] - \frac{\omega}{R^{2}c}\sin[\omega(t-R/c)]\right]$$
$$E_{\phi} = \frac{\mu_{0}A\sin\theta}{4\pi}I_{0}\left[\frac{\omega^{2}}{Rc}\cos[\omega(t-R/c)] + \frac{\omega^{2}}{R}\sin[\omega(t-R/c)]\right]$$

But thankfully, we're in a non-relativistic regime, where

$$\frac{\omega R}{c} << \omega t \qquad \qquad \frac{\omega}{R^2 c} << \frac{1}{R^3} \qquad \qquad \frac{\omega^2}{R c^2} << \frac{1}{R^3} \qquad \qquad \frac{\omega}{R^2} << \frac{\omega^2}{R c}$$

So our equations reduce to

$$B_{r} = \frac{2\mu_{0}A\cos\theta}{4\pi}I_{0}\left[\frac{1}{R^{3}}\right] \qquad B_{\theta} = \frac{\mu_{0}A\sin\theta}{4\pi}I_{0}\left[\frac{1}{R^{3}}\right] \\ E_{\phi} = \frac{\mu_{0}A\sin\theta}{4\pi}I_{0}\left[\frac{\omega^{2}}{R}\right]$$

(where I've dropped terms involving $sin(\omega t)$ and $cos(\omega t)$ because we're already in the frequency domain)



Then multiply by N turns to get a solenoid dipole model

Coils A "Simple" Model The Coil Actuators: They're **not** a "Simple" Dipole

Again, there are two counter-wound coils to create a quadrapole Also, they're shielded in the "X" and "Y" direction





A "Simple" Model The Coil Actuators: Dimensions



Coils

A "Simple" Model The Coil Actuators: Measured Data vs. Radius

Back when we were worried about magnetic coupling to ISI sensors, Rich made some useful measurements of the stray fields from real actuators

But, he was looking for near field, stray magnetic fields and it was before the re-design of the large actuators (to increase the gap between magnets and coils – it shouldn't make that much of a difference)

It's a Start!!

His Data @ DC vs. Radius (as measured by a magnetometer)



I took "worst" case: Old Coarse Acts "X on X," and extrapolated as $1/R^3$ to 1m



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Coils

A "Simple" Model

The Coil Actuators [T/A]: Measured Data vs. Freq

Concerned about frequency dependence of the [T/A] generator constant, both Rich and Fabrice made independent measurements of the frequency response of coil drivers (See [old] SEII Log 1725 and T0900226 respectively)

Fabrice measured the force on the magnets, Rich measured the leakage field, both show a roll-off around 30 Hz.

For other work by Robert Schofield (not included, but consistent): T050087 and LHO i/eLOG May 11, 2008



B_{ctrl} A "Simple" Model Stray Fields [T]: Interpreting the Reqs.

Requirements:

-From **T010007**, 4.2.7 (Barton, Robertson, Fritschel, Shoemaker, Willems) "... We require that [technical noise] be ... 10% [the] amplitude of ... pendulum thermal noise. Sources include ... ambient magnetic field fluctuations at the magnetic actuators...."

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