Mapping Inspiral Sensitivity of Gravitational Wave Detectors

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LSC LIGO Scientific Collaboration

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NSF

Overview

1. Gravitational Waves
   - Fundamentals
   - Detection
   - GWs From Inspiral Sources
2. Simulation
   - Calculating $d_{50\%}$ by Monte Carlo method
3. Results
4. Summary

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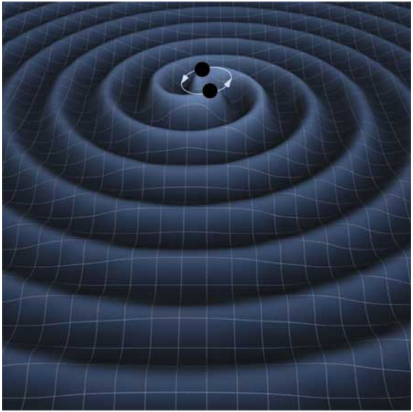
Overview

Gravitational wave basics

- "Ripples in space-time," caused by movement of massive objects; propagate at the speed of light.
- Similar to electromagnetic waves, which are caused by movement of charges.
- Negligible absorption and scattering by matter.



[Image: T. Carnahan (NASA GSFC)]

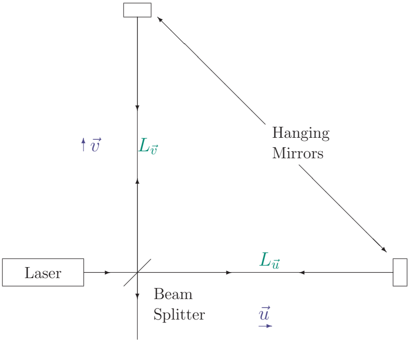
Propagating gravitational waves

- GW from single, distant source approximated as a plane wave, propagating along observer's line-of-sight.
- Wave represented by metric perturbation tensor

  $$\boldsymbol{h} = h_+ \boldsymbol{e}_+ + h_\times \boldsymbol{e}_\times ,$$

  in terms of polarization basis tensors $\boldsymbol{e}_+$ and $\boldsymbol{e}_\times$.

Interferometry



Laser interferometer measures strain h, given by

$$h = \frac{L_{\vec{u}} - L_{\vec{v}}}{L_0}$$

$$= h_+ F_+ + h_\times F_\times ,$$

where F_+ and $F_\times$ are functions of detector, source sky position, and source polarization angle ψ.

LIGO detectors

The Laser Interferometric Gravtational Wave Observatory

Two US detector sites:

Livingston, Louisiana

Hanford, Washington

[Images: Ligo Scientific Collaboration, www.ligo.org]

Other GW detectors

International detectors:

[Image: www.ego-gw.it]

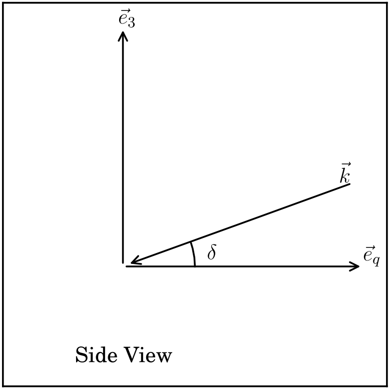
Virgo, Cascina, Italy

[Image: www.geo600.org]

GEO600, Sarstedt, Germany

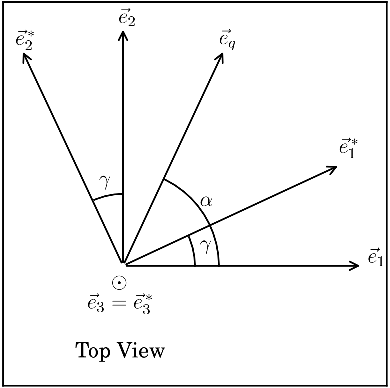
Equatorial Coordinates: Earth-Fixed and Inertial

- Earth-fixed, latitude λ, longitude β, correspond to $\vec{e}_1^*, \vec{e}_2^*, \vec{e}_3^*$ (Cartesian, rotates with Earth).
- Intertial declination δ, right ascension α, correspond to $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ (Stationary).

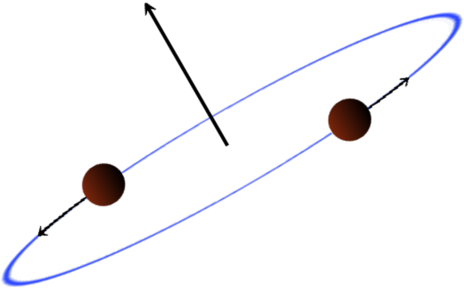


Equatorial Coordinates: Earth-Fixed and Inertial

- Greenwich sidereal time (GST, γ) measures angle between meridian at Greenwich, England ($\vec{e_1}^*$), and vernal equinox ($\vec{e}_1$).
- Local hour angle (LHA) measures angle from source meridian ($\vec{e}_q$) to observer meridian (Not shown in figure).



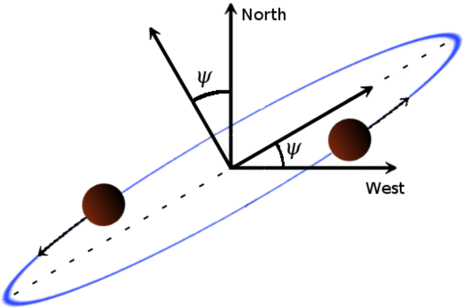
Binary Systems



Circular orbit projected onto plane of the sky

- Two compact, massive objects (black holes, neutron stars) orbit one another.
- System radiates energy as gravitational waves, objects spiral inwards (inspiral).
- Orbital frequency increases as system loses energy.

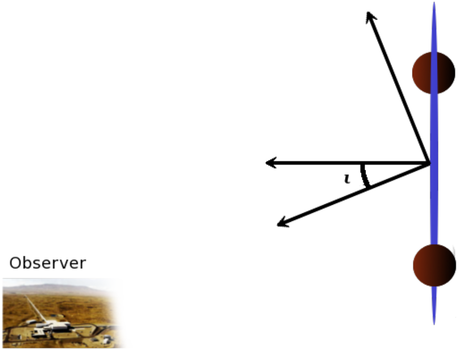
Orbital plane orientation



Orientation defined by two angles:

1. Polarization angle ψ
2. Inclination angle ι

Orbital plane orientation



Orientation defined by two angles:

1. Polarization angle ψ
2. Inclination angle ι

Effective distance

Definition

Effective distance: Distance at which an optimally located and oriented source would be seen with the same signal as a given source in a given detector.

$$d_{\text{eff}} = \frac{d}{\sqrt{F_+(\alpha,\delta,\psi)^2 \frac{(1+\cos^2\iota)^2}{4} + F_\times(\alpha,\delta,\psi)^2\cos^2\iota}}$$

- Quantity $\frac{d}{d_{\text{eff}}} = \sqrt{F_+(\alpha,\delta,\psi)^2 \frac{(1+\cos^2\iota)^2}{4} + F_\times(\alpha,\delta,\psi)^2\cos^2\iota}$ describes detector's *relative* sensitivity in a given direction to given binary orientation.

Threshold distance and chance of detection

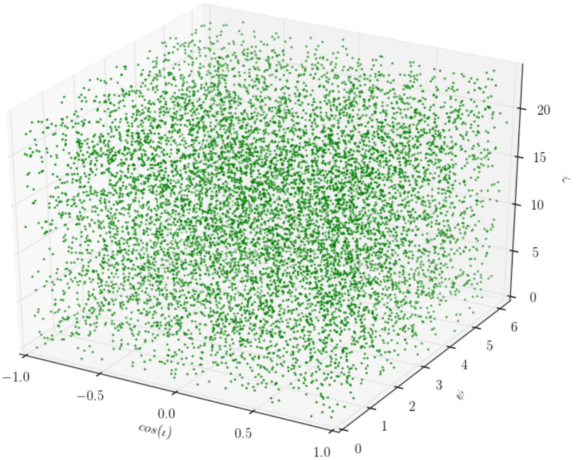
Definition

$d_{x\%}$: The distance at which, for a given detector, the chance of detecting a binary source with random polarization, inclination, and sidereal time is $x\%$.

Calculating $d_{50\%}$ by Monte Carlo method

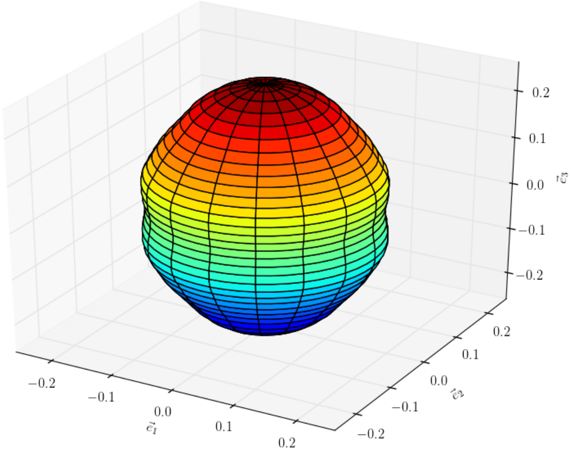
- Population of 1×10^6 “sources” generated, each with random polarization, inclination, and sidereal time.
- Values of each parameter sampled from uniform distribution on the appropriate interval to allow for full range of possible values, e.g. sidereal time sampled from $[0, 24)$
- d/d_{eff} calculated for each source
- "Values of d/d_{eff} placed inhistogram to find median $d_{50\%}$"

Calculating $d_{50\%}$ by Monte Carlo method

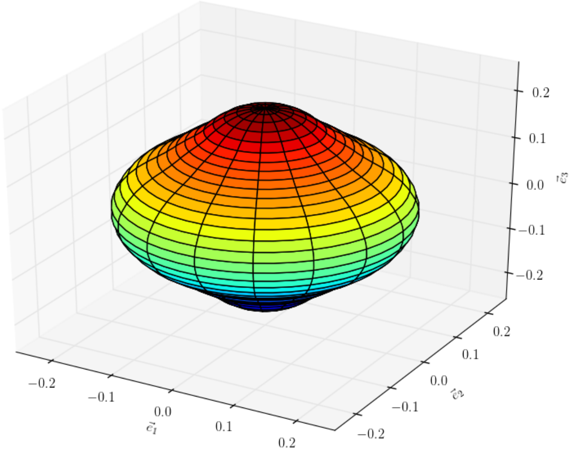


Distribution of 1×10^4 points in $(cos(\iota), \psi, \gamma)$ space

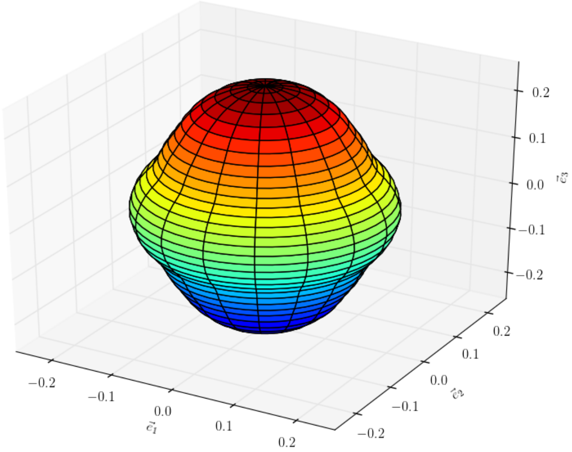
$d_{50\%}$, LIGO Hanford



$d_{50\%}$, LIGO Livingston



$d_{50\%}$, Virgo



Summary and Outlook

Summary

- GW signal seen at detector depends on location, orientation of binary.
- $d_{50\%}$ Calculated for LIGO and Virgo detectors by Monte Carlo method.

Outlook

- Optimize Monte Carlo implementation to allow for faster, simpler calculation of $d_{x\%}$ for different values of x.
- Calculate and plot surfaces corresponding to different values of x.