A Modelled Cross-Correlation Search for Scorpius X-1

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Outline

1. Gravitational Waves from Low-Mass X-Ray Binaries
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   - Search Strategies
2. Cross-Correlation Method
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   - Parameter Space Metric
   - Sensitivity
3. Conclusions

Gravitational Waves from Low-Mass X-Ray Binaries

- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides "hot spot"; rotating non-axisymmetric NS emits gravitational waves
- Bildsten *ApJL* **501**, L89 (1998) suggested GW spindown may balance accretion spinup; GW strength can be estimated from X-ray flux
- Torque balance would give ≈ constant GW freq
- Signal at solar system modulated by binary orbit

Scorpius X-1

- 2nd brightest X-Ray source in the sky, after the Sun
- Favored model is $1.4 M_{\odot}$ NS + $0.42 M_{\odot}$ companion
  Steeghs & Casares *ApJ* **568**, 273 (2002)

Parameters (see LSC *PRD* **76**, 082001 (2007) for refs)

RA	α	$16^{\mathrm{h}}19^{\mathrm{m}}55^{\mathrm{s}}$
dec	δ	$-15^{\circ}38'25''$
orb period	P_{orb}	$(68023.84 \pm 0.08)\,\mathrm{s}$
ref time	$\widetilde{T}$	$(731163327 \pm 299)\,\mathrm{s}$
proj orb radius	a_p	$(1.44 \pm 0.18)\,\mathrm{s}$

GW Searches for Sco X-1

- Fully coherent $\mathcal{F}$-statistic search
  Jaranowski, Królak & Schutz *PRD* **58**, 063001 (1998)
  - ☞ w/6 hours of LIGO S2 data LSC *PRD* **76**, 082001 (2007)
- Directed stochastic ("radiometer") search
  Ballmer *CQG* **23**, S179 (2006)
  - ☞ w/LIGO S4 data LSC *PRD* **76**, 082003 (2007)
- Sideband search Messenger & Woan *CQG* **24**, S469 (2007)
- Modelled cross-correlation search

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

Basics of Cross-Correlation Method

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

- [BTW, other targets include SN1987A supernova remnant; see Chung, Melatos, Krishnan & JTW *MNRAS* **414**, 2650 (2011)]
- Divide data into segments of length T_{sft} & take "short Fourier transform" (SFT) $\tilde{x}_I(f)$
- Label segments w/indices I, J, etc
  - ☞ I & J can be same or different times or detectors
- Use CW signal model ($\mathcal{A}_+ = \frac{1+\cos^2\iota}{2}$; $\mathcal{A}_\times = \cos\iota$)

$$h(t) = h_0 \left[\mathcal{A}_+ \cos\Phi(\tau(t)) F_+ + \mathcal{A}_\times \sin\Phi(\tau(t)) F_\times\right]$$

to determine expected cross-correlation btwn SFTs I & J

$$E\left[\tilde{x}_I^*(f_{k_I})\,\tilde{x}_J(f_{k_J})\right] = \tilde{h}_I^*(f_{k_I})\,\tilde{h}_J(f_{k_J}) = h_0^2\,\tilde{\mathcal{G}}_{IJ}\,\delta_{T_{\text{sft}}}(f_{k_I} - f_I)\,\delta_{T_{\text{sft}}}(f_{k_J} - f_J)$$

Expected Cross-Correlation & Optimal Statistic

- Cross-correlation of signal w/intrinsic frequency f_0:

$$E\left[\tilde{x}_I^*(f_{k_I})\,\tilde{x}(f_{k_J})\right] = \tilde{h}_I^*(f_{k_I})\,\tilde{h}(f_{k_J}) = h_0^2\,\tilde{\mathcal{G}}_{IJ}\,\delta_{T_{\text{sft}}}(f_{k_I}-f_I)\,\delta_{T_{\text{sft}}}(f_{k_J}-f_J)$$

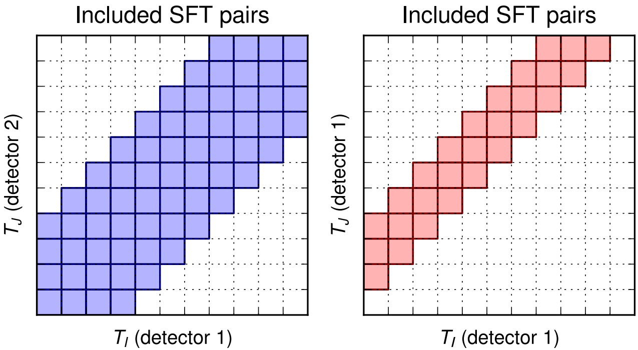
  - $\delta_{T_{\text{sft}}}(f-f') = \int_{-T_{\text{sft}}/2}^{T_{\text{sft}}/2} e^{i2\pi(f-f')t}\,dt$ so $\delta_{T_{\text{sft}}}(0) = T_{\text{sft}}$
  - f_I is signal freq @ time T_I Doppler shifted for detector I
- Label SFTs by $I, J, \ldots$ and pairs by $\alpha, \beta, \ldots$
- Construct $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{\tilde{k}_I})\tilde{x}_J(f_{\tilde{k}_J})}{(T_{\text{sft}})^2}$ (where $f_{\tilde{k}_I} \approx f_I$) so that

$$E[\mathcal{Y}_\alpha] \approx h_0^2\tilde{\mathcal{G}}_\alpha \qquad \text{Var}[\mathcal{Y}_{IJ}] \approx \sigma_{IJ}^2 = S_I(f_0)S_J(f_0)/4(T_{\text{sft}})^2$$

- Optimally combine into $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$ w/$u_\alpha \propto \tilde{\mathcal{G}}_\alpha^*/\sigma_\alpha^2$ so $E[\rho] = h_0^2\sqrt{2\sum_\alpha |\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}$ & $\text{Var}[\rho] = 1$

Tuning the Cross-Correlation Search

- Computational considerations limit coherent integration time
- Can make tunable semi-coherent search by restricting which SFT pairs α are included in $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$
- E.g., only include pairs where $|T_I - T_J| \equiv |T_\alpha| \leq T_{\mathrm{max}}$



Metric for Cross-Correlation Search

- Consider dependence of ρ on parameters $\boldsymbol{\lambda} \equiv \{\lambda_i\}$
- Parameter space metric $g_{ij} = -\frac{1}{2} \frac{E[\rho_{,ij}]\big|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}_{\text{true}}}}{E[\rho^{\text{true}}]}$ from

$$\frac{E[\rho] - E[\rho^{\text{true}}]}{E[\rho^{\text{true}}]} = -g_{ij}(\Delta\lambda^i)(\Delta\lambda^j) + \mathcal{O}([\Delta\lambda]^3)$$

- Assume dominant contribution to $E[\rho_{,ij}]$ is from variation of $\Delta\Phi_{IJ} = \Phi_I - \Phi_J$; get phase metric

$$g_{ij} = \frac{1}{2} \frac{\sum_\alpha \Delta\Phi_{\alpha,i}\Delta\Phi_{\alpha,j}|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}{\sum_\beta |\tilde{\mathcal{G}}_\beta|^2/\sigma_\beta^2} \equiv \frac{1}{2}\left\langle \Delta\Phi_{\alpha,i}\Delta\Phi_{\alpha,j} \right\rangle_\alpha$$

- Note $\langle\ \rangle_\alpha$ is average over pairs weighted by $|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2$
- If you ignore that weighting factor you get back usual metric

$$\langle \Phi_{I,i}\Phi_{I,j}\rangle_I - \langle \Phi_{I,i}\rangle_I \langle \Phi_{J,j}\rangle_J$$

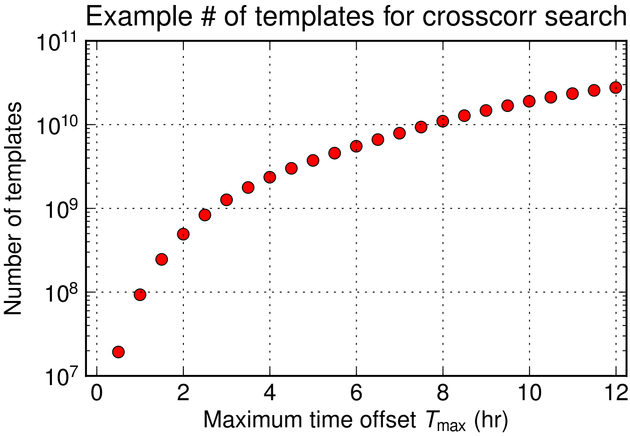
Approximate Phase Metric for LMXB

- $T_{IJ} = T_I - T_J \equiv T_\alpha$ is time offset btwn SFTs; T_α^{av} is avg time
- For each detector pair, avg over pairs is avg over T_α & T_α^{av}
- Assume average over T_α^{av} evenly samples orbital phase
- Metric in $\{f_0, a_p, \widetilde{T}\}$ space is

$$\mathbf{g} = \begin{pmatrix} 2\pi^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \langle T_\alpha^2 \rangle_{T_\alpha} + \begin{pmatrix} \pi^2 a_p^2 & \pi^2 f_0 a_p & 0 \\ \pi^2 f_0 a_p & \pi^2 f_0^2 & 0 \\ 0 & 0 & 4\pi^4 f_0^2 a_p^2 / P_{\text{orb}}^2 \end{pmatrix} \langle \sin^2[\pi T_\alpha / P_{\text{orb}}] \rangle_{T_\alpha}$$

- Since $\langle T_\alpha^2 \rangle_{T_\alpha} \gg a_p^2 \langle \sin^2[\pi T_\alpha / P_{\text{orb}}] \rangle_{T_\alpha}$ (recall $a_p \approx 1.4\,\text{s}$), metric approximately diagonal

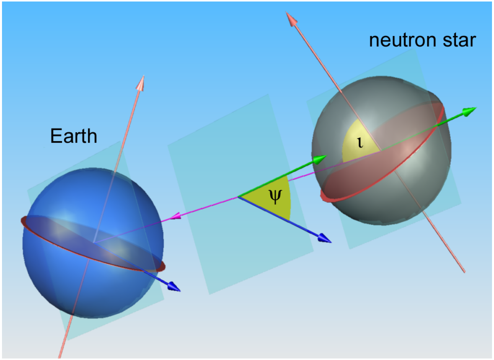
Ballpark Estimate of Template Count



For illustrative purposes, to show dependence on T_{max};
Don't read too much into absolute numbers

Sensitivity Estimates

- Sensitivity of search is $h_0 = \left(\frac{\mathcal{S}^2}{\sum_\alpha |\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}\right)^2$
- $\tilde{\mathcal{G}}_\alpha$ depends on (unknown) spin orientation angles ι & ψ; standard approach is to average value of $\tilde{\mathcal{G}}_\alpha$ over $\cos\iota$ & ψ



Sensitivity Estimates

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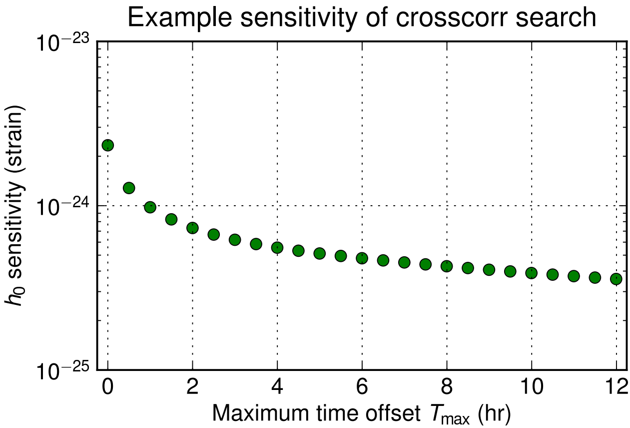
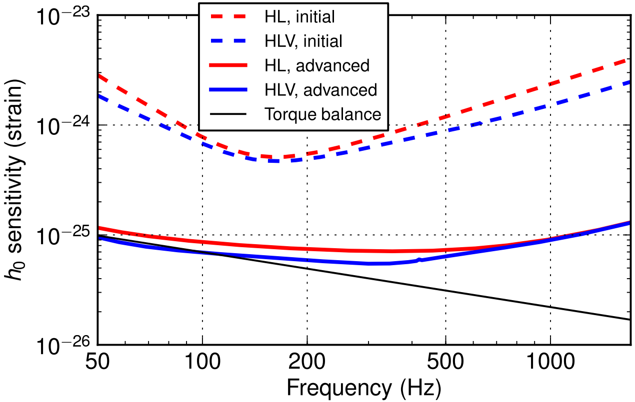
- ψ effect small after average over sidereal time
  ι effect means actually $E[\rho] \approx h_0^2 \frac{\mathcal{A}_+^2+\mathcal{A}_\times^2}{2}\sqrt{2\sum_\alpha |\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}$
  (recall $\mathcal{A}_+ = \frac{1+\cos^2\iota}{2}$ & $\mathcal{A}_\times = \cos\iota$)
- Net effect is to change statistical factor $\mathcal{S}$;
  for 10% false-alarm & -dismissal,
  h_0 sensitivity is a factor of 1.4 worse

Dependence of Sensitivity on T_{max}



For illustrative purposes, to show dependence on T_{max}; note $T_{\text{max}} = 0$ measurement $\sim$ stochastic radiometer

Preliminary Sensitivity Estimates



Assumes 10% false-alarm &-dismissal, 1yr @ design, $T_{\text{max}} = 6\,\text{hr}$

Summary

- Cross-correlation method adapted for CW signals
- Inclusion of signal model & Doppler effects
  allows correlation of non-simultaneous data
- Promising target is the low-mass X-ray binary Scorpius X-1
- For Sco X-1, must search over freq & orbital params
- Advanced detector era sensitivity
  should reach torque balance prediction