# aLIGO Photon Calibrator Errors due to Elastic Deformation

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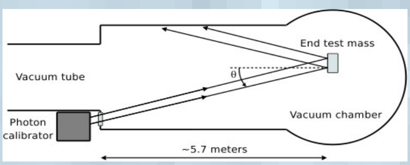
Calibration and actuation forces applied to the test masses of interferometric gravitational wave detectors produce both, local[3,4] and bulk[5,6] elastic deformations that can significantly alter the motion sensed by the interferometer, especially when the frequencies of the periodic forces is above *1kHz*. Our study show an increasing discrepancy between the actual sensed motion and the nominal free - mass motion to *1%* at about *4.3kHz* for an optimum two - beam Photon Calibrator configuration (2PCal). The discrepancy is weakly dependent on the position of the interferometer (ifo.) beam, but strongly dependent on the PCal location. For a ifo. and PCal deviation of *3mm* and *1mm*, respectively, we found a *1%* discrepancy at *3.5kHz*, and as much of *6%* at *5kHz*. If a three - beam (3PCal) or four - beam PCal (4PCal) configuration were used, the discrepancy could be reduced to *0.2%* for frequencies up to *5kHz*. This is a consequence of how a particular PCal configuration can effectively excite a normal mode. Nevertheless, a *1mm* deviation for 3PCal or 4PCal results in a discrepancy of *1%* at *3.5kHz*, as in 2PCal case, emphasizing the extra care that one has to have positioning the PCal beams.

## PCal

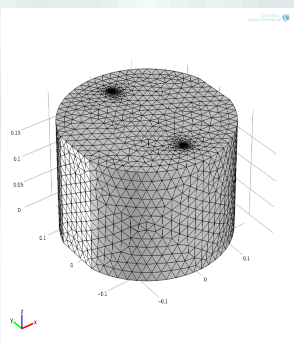
Photon Calibrator or PCal is a calibration technique that uses the radiation pressure of a power-modulated auxiliary laser to induce calibrated displacements of one of the End Test Masses (ETM) arm cavity mirrors.



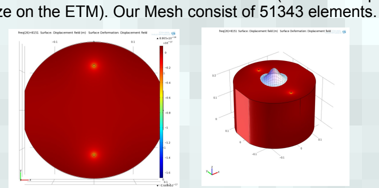

*Schematic diagram of initial LIGO Photon Calibrator with output beams reflecting from and ETM inside the vacuum envelope[3].*

## aETM and PCal COMSOL model

COMSOL is a Finite Element Modeling (FEM) software (for more information http://www.comsol.com/). The main idea behind FEM is to divide the system in many different elements that interact with each other through a "mesh" that connect those elements.

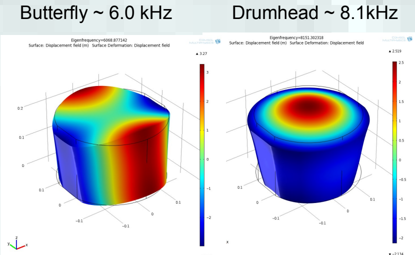


The advance LIGO End Test Mass (aETM) was modeled as a fused silica cylinder of radius R=170.1mm, height H=200mm (mean of 0.077° wedge), with two flats separated by 326.5mm. The 2PCal were modeled as two circular spots of radius 25mm, diametrically opposed at 111.6mm from the center of the aETM. The ifo. beam was modeled as a gaussian weighted measurement of the surface motion with 1/e[2] radius of 62mm (the aLIGO spot size on the ETM). Our Mesh consist of 51343 elements.



## aETM Normal modes

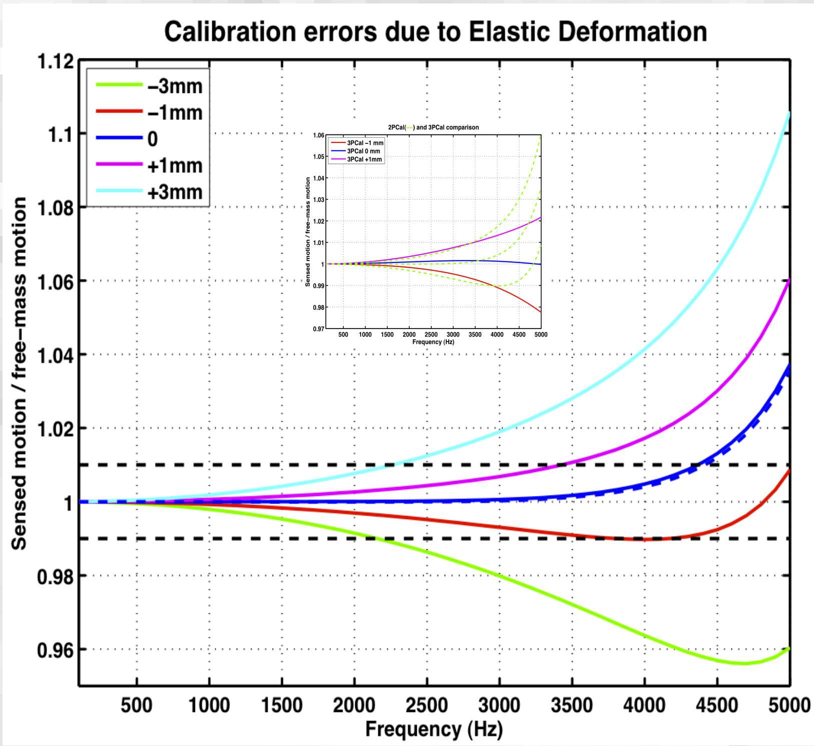
Butterfly ~ 6.0 kHz Drumhead ~ 8.1kHz



At first, the positions of he PCal spots were chosen to minimize the excitation of the Drumhead mode. For an ideal cylinder and a perfectly centered ifo. beam, the Butterfly deformation averages to zero. However, the flats on the aETMs break the azimuthal symmetry. The FEM analysis indicates that with the flats the optimum radius to position the PCal beams increases by 0.3 mm.

### Comparison of normal frequencies

| Measurements[7] | COMSOL |
|---|---|
| 8149 Hz | 8151 Hz |
| 10411 Hz | 10427 Hz |
| 12970 Hz | 12965 Hz |
| 15040 Hz | 15023 Hz |



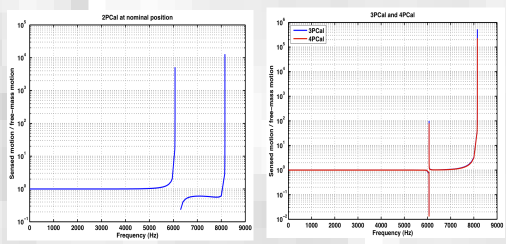

*Ratio of motion sensed by the interferometer to the ideal free-mass motion vs. the frequency of the applied forces for 2PCal with the beams located at 111.6 mm from the center. The interferometer beam is displaced by 3 mm (solid lines) from the center and centered (dashed line). The modeled ratios for optimally located and ±1mm off set 3PCal are also shown.*

The inserted image in the main plot, 2PCal and 3PCal comparison, show the different behavior for this configurations below 5kHz. Even though 3PCal have a preferred response at nominal position, you can see that this configuration is worthless unless you keep the position of the PCal within an error less than ±1m.
It is worth to mention that below 5kHz the difference between 3PCal and 4PCal is less than 0.05%, and the discrepancy between ifo. 3mm off set and centered for 3PCal and 4PCal is less than 0.001% for that range of frequencies.
For those reasons and due to the added complexity, issues with blockage by baffles, and since the discrepancy for 2PCal can be kept less than 1% below 3.5kHz (with an off set lower than 3mm for ifo. and 1mm for PCal), it is planned to utilize two - beams PCal configuration for aLIGO.

## Calibration Errors due to Elastic Deformation

LIGO's calibration takes the ETM as a solid rigid body; for frequencies highers than the pendulum resonant frequency its motion is modeled as a free – mas motion. The main plot show the ratio between the motion sensed by the ifo. and the free mass motion in the case where the ifo. is off set by 3mm. Each curve represent a different position of the 2PCal beams, nominal (111.6mm) or zero off set, ±3mm and ±1mm off set.
The dashed black lines shows the discrepancy of 1% between the free mass motion and the sensed motion:
- *2PCal off set by -3mm ~ 2.1kHz.*
- *2PCal off set by +1mm ~ 3.5kHz.*
- *2PCal nominal position ~ 4.3kHz*

The discrepancy between ifo. off set by 1mm and ifo. centered is less than 0.01%, for frequencies below 5kHz. As you can see from the plot, there is a little difference between ifo. centered (dashed blue line) and ifo. 3mm off set (solid blue line), for 2PCal at nominal radius, which imply the importance of the PCal positions.
From the main plot you can see as well, that about 4kHz all 2PCal curves tend to go up; this is an effect of the Butterfly mode. The two plots below show how different the 2PCal, on one hand, and 3PCal and 4PCal behaves for frequencies below 9kHz; from them, it is ease to compare the magnitude for the Butterfly mode (around 6.0kHz) and the Drumhead mode (around 8.1kHz) for the three PCal configurations.



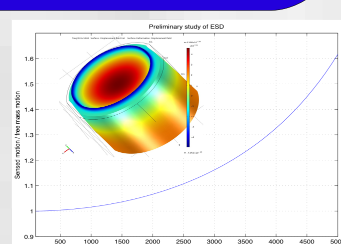

*Ratio of motion sensed by ifo. and ideal free-mass motion for 2PCal, 3PCal and 4PCal below 9kHz.*

## CONCLUSIONS

- Optimum location of the PCal beams is crucial to minimize the excitation of the *Drumhead* mode.
- The flats on the advance LIGO End Test Mass break the azimuthal symmetry so the *Butterfly* mode affects the calibration even for a centered ifo. beam, but the impact is much smaller than for the *Drumhead* mode.
- If the PCal beams can be maintained within less than 1 mm of their optimum locations, three-beams and four-beam PCal configurations give much smaller errors than the two-beam configuration, even for large ifo. beam mis-centering. However, if the PCal beam location tolerances are on the order of 1 mm, then two-, three- and four-beam configurations yield similar calibration errors due to the excitation of the *Drumhead* mode.
- For a two-beam PCal configuration, the discrepancy between the ideal free-mass motion and the motion sensed by the interferometer is less than 1% for frequencies below 3.5kHz with PCal beam positioning tolerances of 1 mm and an interferometer beam offset of less than 3mm. The discrepancy grows up to 6% at 5kHz.

## Future work:

- Addition to the model of the fused silica ears and fibers at the flat parts of the aETM.
- Investigation of the Electrostatic Drives (ESD) actuation. Preliminary studies show that driving the ESD produces a discrepancy of about 10% at 2.5kHz and as much as 60% at 5kHz. This effect is a consequence of the ESD position; they are located at radius position that efficiently excites the Drumhead mode.




3) S. Hild, et al.: *"Photon Pressure Induced Test Mass Deformation in Gravitational - Wave Detectors"*. Class. Quantum Grav. 24 5681 - 5688 (2007).
4) E Goetz, et al. (LIGO p080118): *"Precise calibration of LIGO test mass actuators using photon radiation pressure"*.
5) Phil Willems: LIGO Technical Document T080190-00-R.
6) Mahmuda Afrin Badhan, et. al., LIGO Technical Document LIGO-T0900401-v1.
7) John Miller, et al.: "Damping parametric instabilities in future gravitational wave detectors by means of electrostatic actuators".