MELODY/MATLAB: OBJECT-ORIENTED MODEL IN GRAVITATIONAL-WAVE INTERFEROMETERS USING MATLAB

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MELODY/MATLAB OVERVIEW

- Goals and features
- Propagation model
- Object-level features
  - Interferometer configurations
  - Mirror physics: thermal loading, position, orientation
  - Automatic length locking
- Script-level features
  - Modulation schemes
  - Mirror parameters: thermal, position, orientation
  - Full interactive MATLAB functionality
- Milestones

ACKNOWLEDGMENTS

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- *MIT*
  - Daniel Sigg
  - Ryan Lawrence
- *Caltech*: Hiro Yamamoto

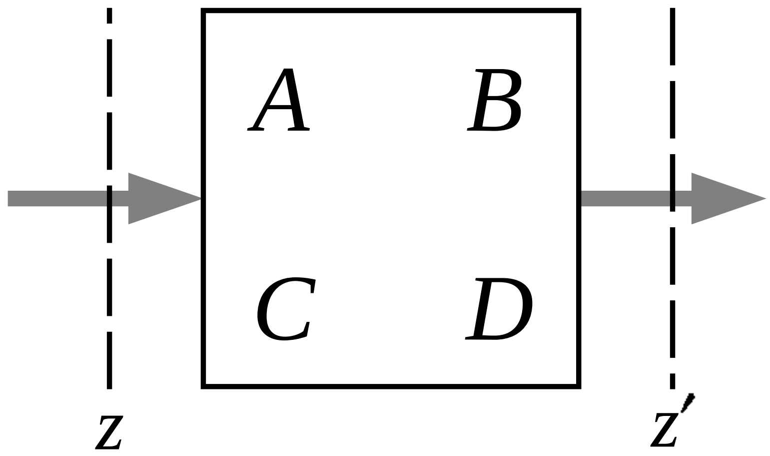
MELODY/MATLAB GOALS

- Provide an easily usable, flexible multiplatform framework for LIGO I/II/III calculations and simulations
- Allow users to write scripts to drive simulations tailored to their needs (post-processing, graphics, numerical analysis)
- Easily include physical effects in mirrors: thermal loading, translation, rotation
- Allow translation to a lower-level language for performance
- Provide a simple interface to industry-standard software for modeling control systems (SIMULINK)

MELODY/MATLAB FEATURES

- MATLAB classes for fields, mirrors, interferometers, and detectors; driven by user-written scripts
- Prebuilt LIGO I/II/III configurations
  - Power, signal, and dual recycling
  - Arbitrary modulation schemes
  - Automatic length locking for quick simulations
- Mirror physics
  - Thermal lensing due to bulk and coating absorption
  - Thermal distortions of the reflecting surface
  - 2D rotation and 3D translation

FORWARD PROPAGATION: HUYGENS-FRESNEL INTEGRAL



$$\mathbf{E}(\mathbf{r}, t) \equiv \mathrm{Re}\left\{\boldsymbol{\epsilon} E(\mathbf{r}) e^{i(kz-\omega t)}\right\}$$

$$\nabla_\perp^2 E(\mathbf{r}) + i2k\frac{\partial}{\partial z}E(\mathbf{r}) = 0$$

$$E(x,y,z) = \int_{\mathcal{A}_1} dx'\,dy'\,K(x,y;x',y')E(x',y',z') \equiv \hat{K}[E(x',y',z')]$$

$$K(x,y;x',y') =$$

$$\frac{1}{i\lambda B}\exp\left\{i\frac{\pi}{\lambda B}\left[A(x'^2+y'^2) - 2(x'x+y'y) + D(x^2+y^2)\right]\right\}$$

UNPERTURBED EIGENMODES

Forward and backward unperturbed eigenmodes:

$$\gamma_{mn} u_{mn}(x,y,0) = \int_{\mathcal{A}_1} dx'\, dy'\, K_0(x,y;x',y') u_{mn}(x',y',0)$$

$$\gamma^\dagger_{mn} u^\dagger_{mn}(x,y,0) = \int_{\mathcal{A}_1} dx'\, dy'\, K^\dagger_0(x,y;x',y') u^\dagger_{mn}(x',y',0)$$

Biorthogonality relation (Siegman), satisfied discretely:

$$\int_{\mathcal{A}_1} dx\, dy\, u^\dagger_{mn}(x,y,z) u_{m'n'}(x,y,z) = \delta_{mm'}\delta_{nn'}$$

Expand intracavity field:

$$E(x,y,z,t) = \sum_{mn} E_{mn}(t) u_{mn}(x,y,z)$$

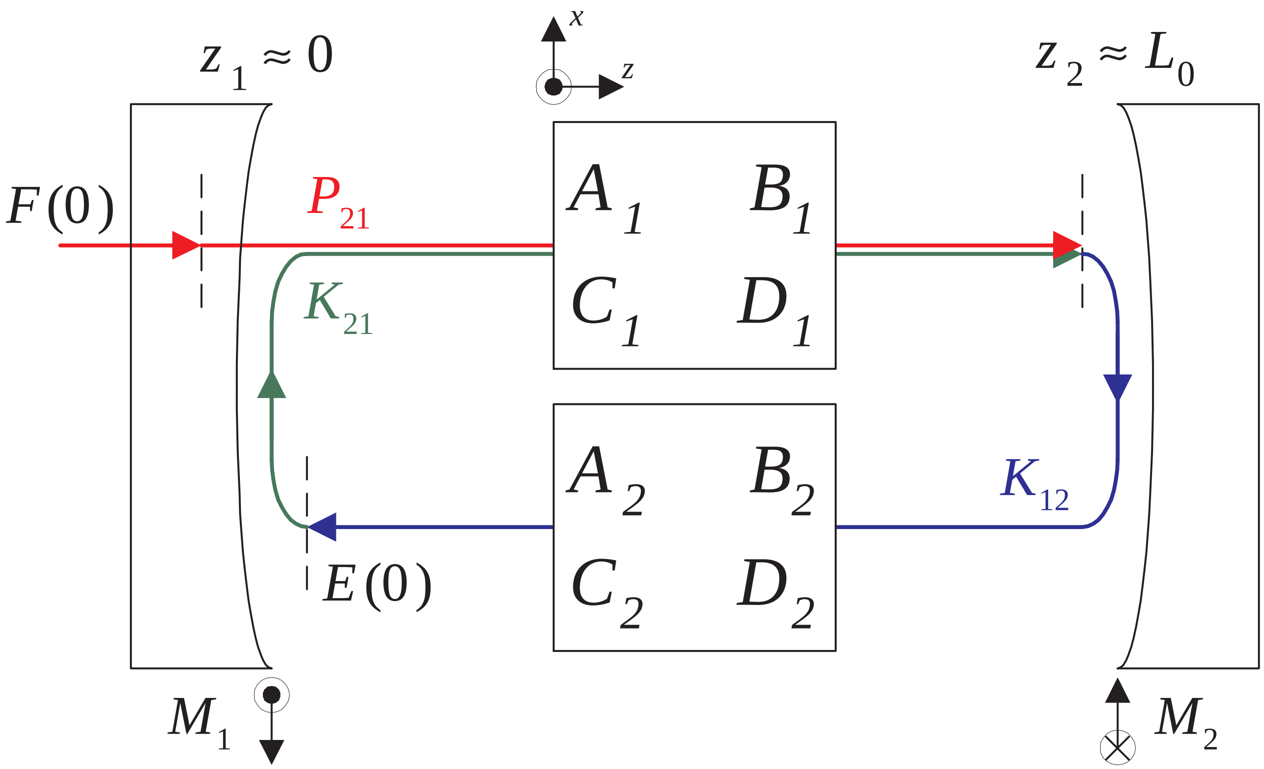
PROPAGATOR MATRIX ELEMENTS

Calculate $K_{mn;m'n'}(t)$ as the matrix element of the fully perturbed forward propagator (from reference plane $\mathcal{A}_1$ to reference plane $\mathcal{A}_2$) in the basis of the unperturbed eigenmodes:

$$K_{mn;m'n'}(t) = \int_{\mathcal{A}_2} dx\,dy \int_{\mathcal{A}_1} dx'\,dy' \times u^{\dagger}_{mn}(x,y)K(x,y;x',y';t)u_{m'n'}(x',y')$$

In general, we can compute $K_{mn;m'n'}(t)$ for each distinct propagation region in the basis of the unperturbed eigenmodes of the interferometer, and then construct a representation of the perturbed interferometer using simple matrix multiplication.

FABRY-PEROT INTERFEROMETER



$$A_2 = D_1 = 1 \qquad B_2 = B_1 = L_0$$

$$C_2 = C_1 = 0 \qquad D_2 = A_1 = 1$$

FPI OBJECT UPDATE PROCEDURE

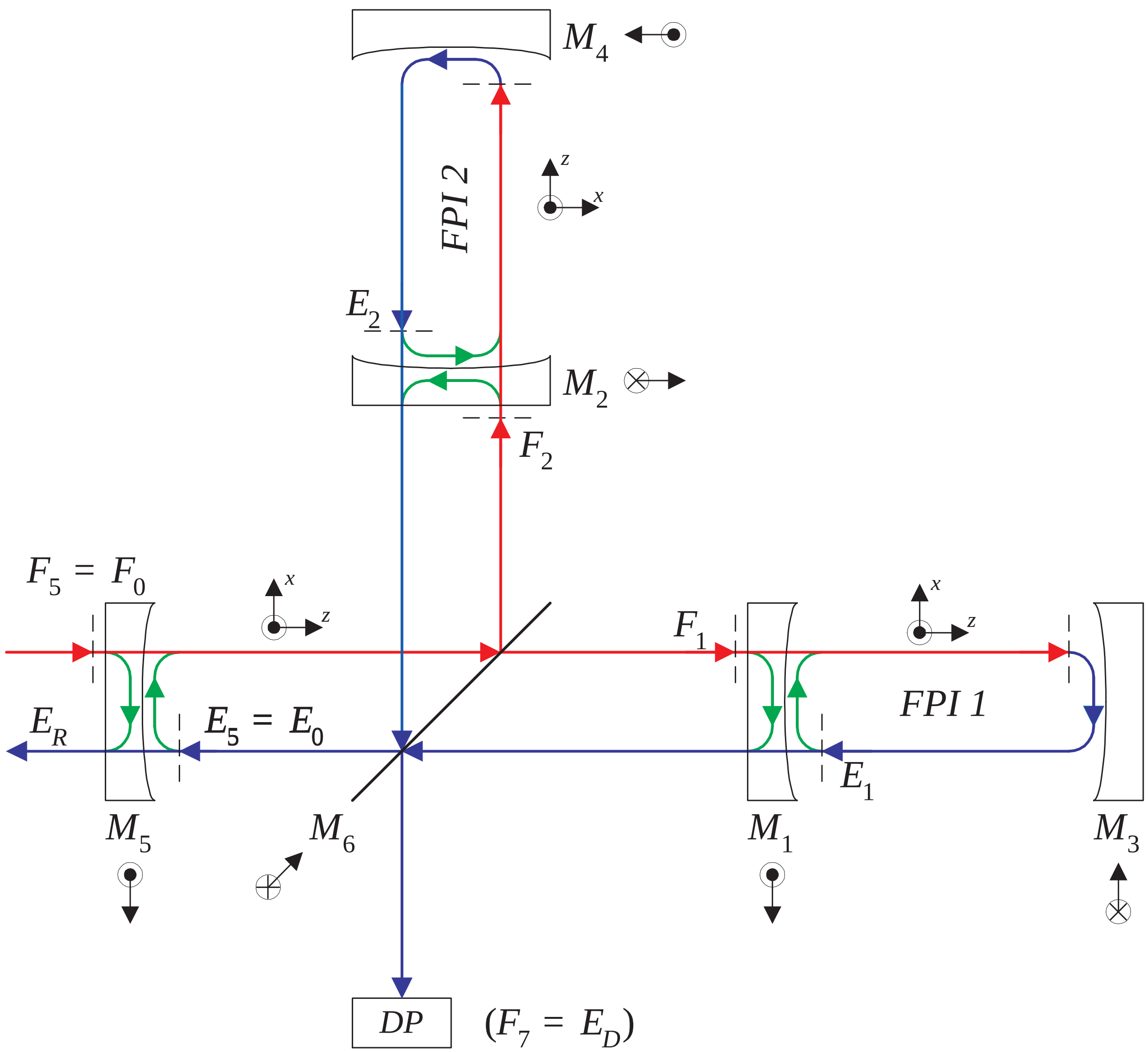
```
% Get the total field propagating away from the
% vacuum-coating interface of m_1, and then
% propagate that field to the vacuum-coating
% interface of m_2. This is the new 'front
% field' of m_2.
e_1_r = get_field(m_1, 'front');
e_2 = fp.gouy_prop * e_1_r * fp.kz_prop;
set_field(m_2, e_2, 'front');

% Get the total field propagating away from the
% vacuum-coating interface of m_2, and then
% propagate that field to the vacuum-coating
% interface of m_1. This is the new 'front
% field' of m_1.
e_2_r = get_field(m_2, 'front');
e_1 = fp.gouy_prop * e_2_r * fp.kz_prop;
set_field(m_1, e_1, 'front');
```

MODEL OF IFO COUPLING

- Choose a *primary* FPI as a reference cavity, defining the “fundamental” unperturbed eigenmodes
- Assume that the reference laser is very nearly mode-matched to the primary FPI
- Propagation around the recycling cavity and through the secondary input mirror directly couples the eigenmodes of the secondary FPI to those of the primary FPI
- Mode-mismatch matrix operators can be included if desired

IFO COUPLING SCHEMATIC



THERMAL LENSING

- Hello-Vinet model of substrate thermal lensing due to both substrate and coating absorption: complete

- Temperature distribution due to bulk absorption approximated by

$$T(r) = -\frac{\alpha_P P}{4\pi k_T}\left[\gamma + \ln\left(\frac{2r^2}{w^2}\right) + E_1\left(\frac{2r^2}{w^2}\right)\right]$$

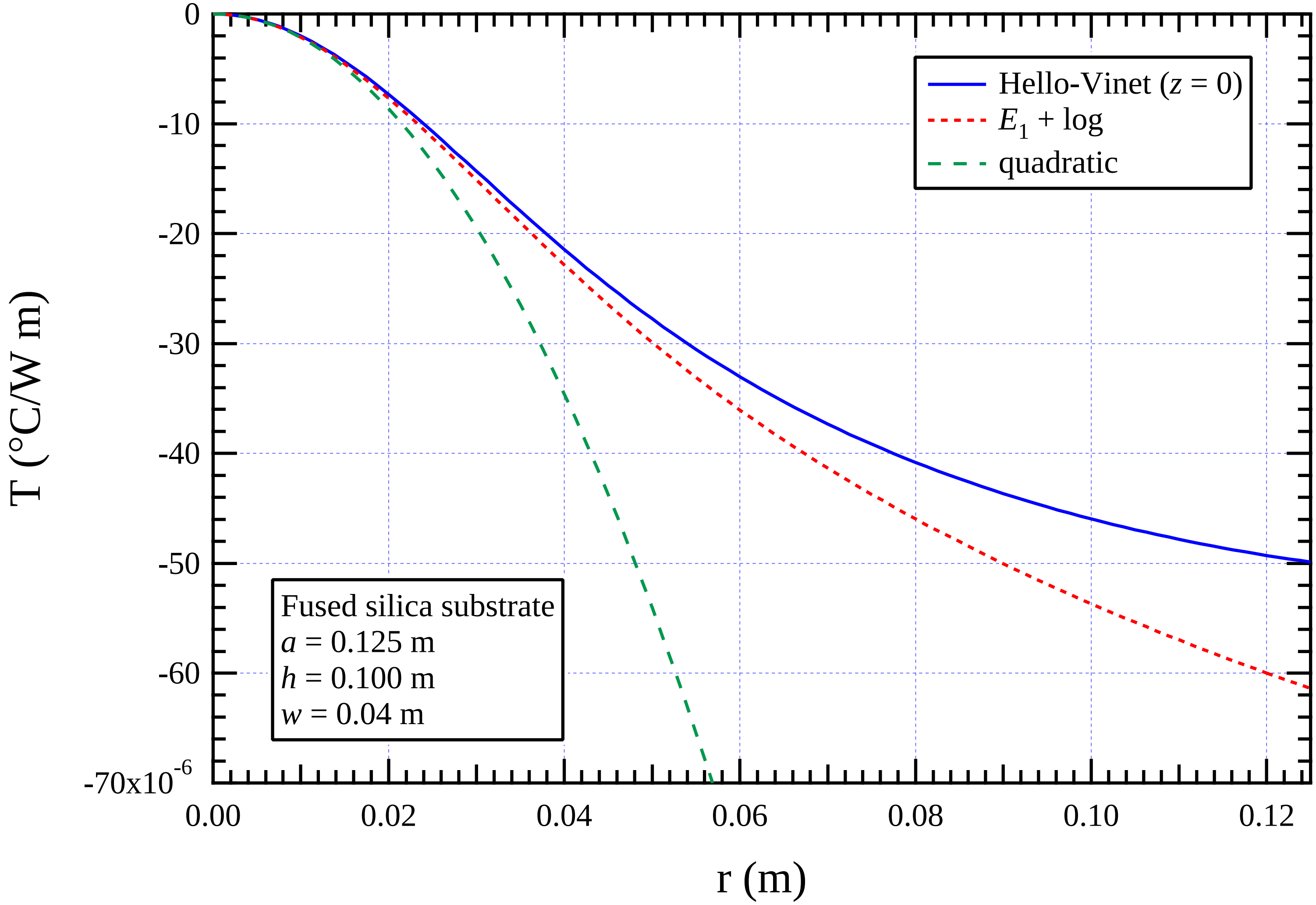
- Near $r = 0$, bulk absorption similar to a thin lens with focal length

$$f = \frac{\pi w^2}{\alpha_P h P}\frac{\kappa_T}{dn/dT}$$

- Numerical implementation of astigmatic thermal loading in beam-splitter almost complete (Hermite-Gauss basis)

- Coating deformation next (Hello-Vinet approximation)

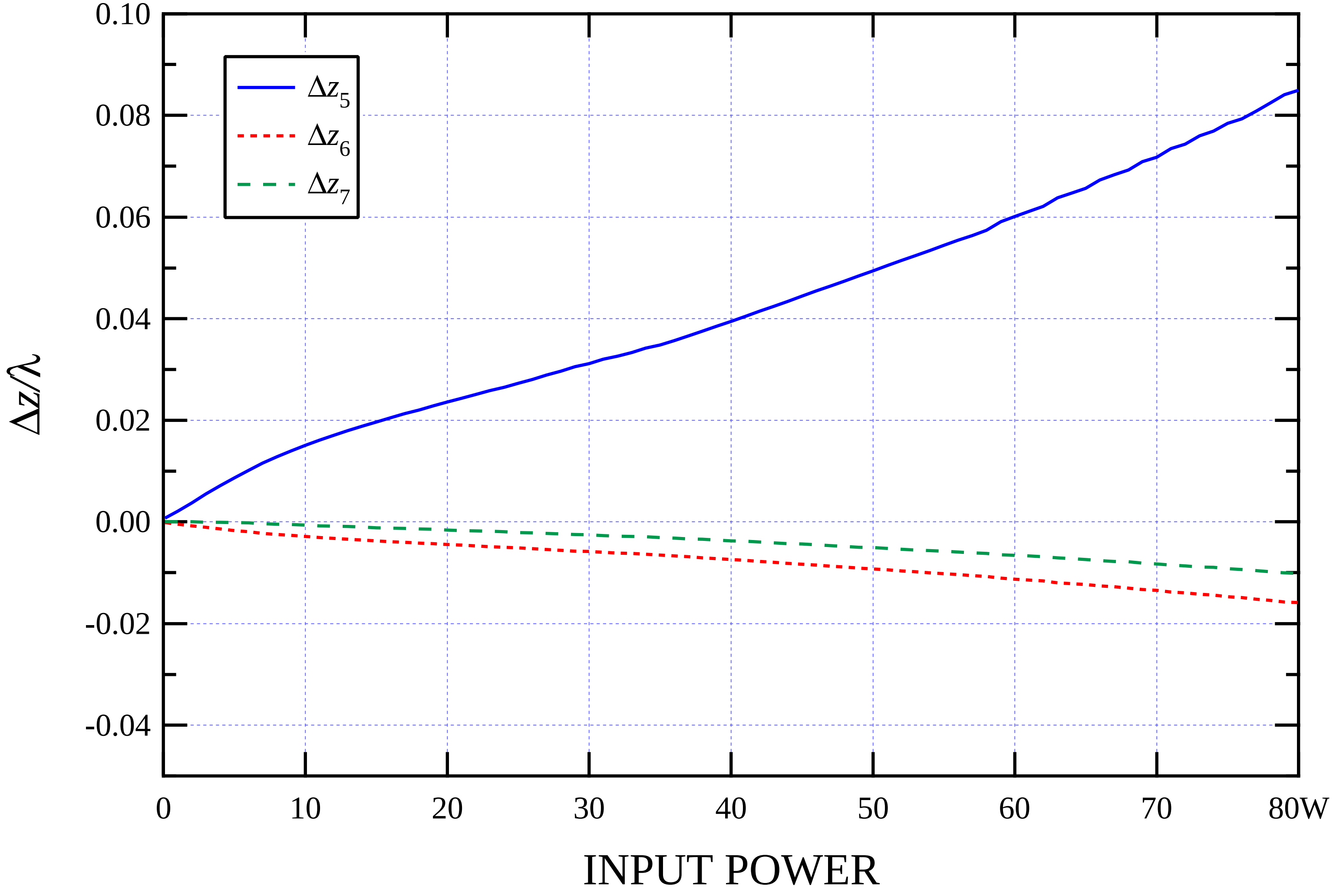
TEMPERATURE (SUBSTRATE ABSORPTION)



AUTOMATIC LENGTH LOCKING

- LIGO control systems will adjust mirror z-positions to compensate for thermal and angular perturbations; for quick calculations, we simulate these numerically without explicit external servo models using a *three-stage autolocker*.
- *Dark Port* stage adjusts the position of the beamsplitter so that the amplitude of the carrier TEM_{00} mode is minimized at the dark port.
- *Power Recycling* stage adjusts the position of the power recycling mirror to maximize carrier TEM_{00} enhancement.
- *Signal Recycling* stage adjusts the position of the signal recycling mirror to maximize resonant sideband TEM_{00} enhancement.

DUAL-RECYCLED LIGO IFO AUTOLOCK OUTPUT



SCRIPT-LEVEL FEATURES

- Input powers, modulation frequencies and depths
- All mirror parameters (e.g., thermal constants, orientation and micro-position)
- All interferometer cavity lengths
- Signal recycling
- Iteration and solution methods
- Graphics, object storage(!), post-processing
- Full interactive MATLAB functionality

TWO-PHASE THERMAL/TEMPORAL SIMULATIONS

Characteristic time: $t_c = \rho C a^2 / k_T \approx 5$ h for fused silica ($a = 0.125$ m)

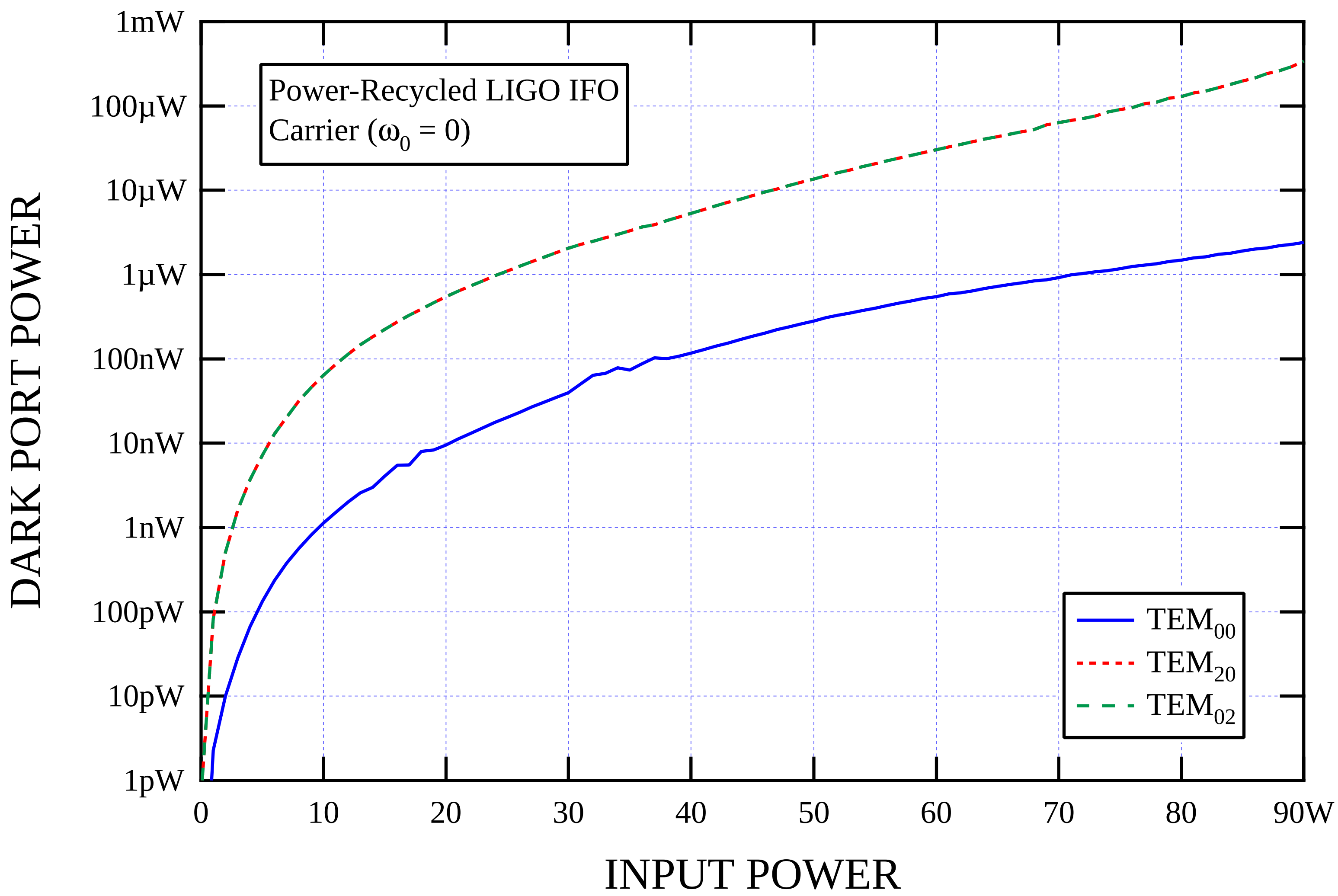
THERMAL (Script-driven)

1. Run thermal relaxation code, including power-dependent optimizations (e.g., modulation depths, SRM reflectivity)
2. SAVE ligo object after stability is reached for each power level

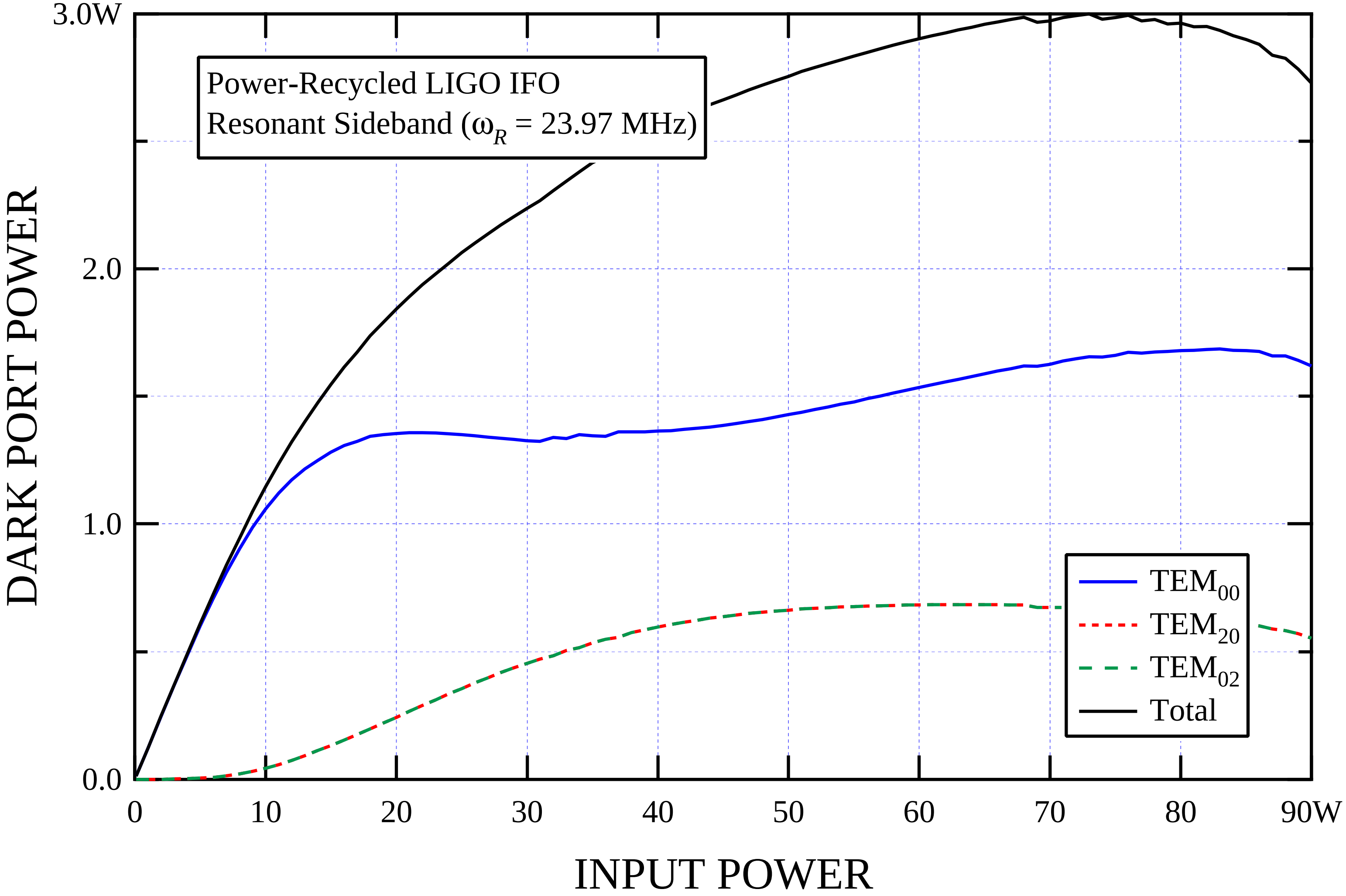
TEMPORAL (Script-driven, SIMULINK)

1. LOAD ligo object for a specified input power
2. Perturb mirrors and simulate temporal response

EXAMPLE: CARRIER (PR)



EXAMPLE: RESONANT SIDEBAND (PR)



MELODY/MATLAB MILESTONES

- **Simple object-oriented architecture in MATLAB**
- **Flexible modulation and resonance schemes**
- **Hello-Vinet mirror thermal lens**
- **Autolock routines for LIGO I/II/III**, *GEO*
- *Astigmatic beamsplitter thermal lens*
- Mirror misalignment operators
- Demodulation detector class
- SIMULINK interface

IMPLEMENTATION

- Use object-oriented programming in MATLAB: primitive classes, encapsulation, function/operator overloading, and inheritance
- Define classes for mirrors, Fabry-Perot interferometers, electric fields (Hermite-Gauss, RF-modulated), and detectors ($\{x, y\}$ geometry)
- Enclose classes representing simpler entities (mirrors, beamsplitters, laser fields) in classes representing interferometers
- Design simple class interfaces allowing calculations and simulations to be driven by MATLAB scripts
- Automatic translation to C++ available if performance is an issue

SUBSET OF CLASSES

`laser_field` Stores all spatial components for all operating sidebands, and the frequencies of those sidebands.

`mirror` Maintains all perturbation matrices (e.g., thermal and angular); encapsulates mirror parameters, two laser_field objects, detectors.

`beamsplitter` Special case of mirror for 45° beamsplitter; uses numerical temperature distribution.

`detector` Demodulation detector array; almost complete.

`fpi` Fabry-Perot Interferometer

`ligo` LIGO I/II/III Interferometers

LASER_FIELD OBJECT DATA

```
basis: Hermite-Gauss

[f\_0, f\_1, -f\_1, f\_2, -f\_2] = [0,  2.3971e+001, -2.3971e+001, 3.5956e+001, -3.5956e+001]

TEM_00   -7.36e-01 -2.45e-02i  -7.48e-02 -5.45e-04i  -7.48e-02 -4.79e-04i  -5.35e-02 -2.04e-03i -5.35e-02 -7.46e-04i
TEM_10             0                     0                     0                     0                    0
TEM_01             0                     0                     0                     0                    0
TEM_20   -1.64e-04 +1.86e-04i   8.15e-03 -9.54e-03i   7.54e-03 -9.98e-03i  -6.69e-07 +6.32e-06i -9.64e-08 +6.47e-06i
TEM_11             0                     0                     0                     0                    0
TEM_02   -1.64e-04 +1.86e-04i   8.15e-03 -9.54e-03i   7.54e-03 -9.98e-03i  -6.69e-07 +6.32e-06i -9.64e-08 +6.47e-06i
```

- laser_field class consists of data fields (*members*) and routines which operate on those fields

- Routines fall into two broad categories:

  procedures which alter the internal state of the object but do not return results (e.g., object update procedures)

  functions which return results but do not alter the internal state of the object (e.g., overloaded arithmetic operators)

SIDEBAND REPRESENTATION

Define the propagation vector

$$k = k_0 + \Delta k_q,$$

where $\Delta k_q / k_0 \ll 1$, $\omega_0 \equiv k_0 c$, and $\Delta\omega_q \equiv \Delta k_q c$. Write the time-dependent length as

$$L(t) \equiv L_0 + \Delta L(t),$$

where $2k_0 L_0 - \varphi_{00} = 2N\pi$ and $\Delta L(t) \approx \lambda = 2\pi / k_0$. Then

$$\begin{aligned} e^{i\left[2kL(t)-\varphi_{00}\right]} &= e^{i\left(2k_0L_0-\varphi_{00}\right)} e^{i\left[2k_0\Delta L(t)\right]} e^{i\left(2\Delta\omega_q L_0/c\right)} e^{i\left[2\Delta k_q \Delta L(t)\right]} \\ &= e^{i\left[2k_0\Delta L(t)+\Delta\omega_q\tau_0\right]} \end{aligned}$$

Include $\Delta L(t)$ in mirror class; implement $\Delta\omega_q \tau_0$ as a diagonal propagation matrix.

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```
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% vacuum-coating interface of m_1, and then
% propagate that field to the vacuum-coating
% interface of m_2. This is the new 'front
% field' of m_2.
e_1_r = get_field(m_1, 'front');
e_2 = fp.gouy_prop * e_1_r * fp.kz_prop;
set_field(m_2, e_2, 'front');

% Get the total field propagating away from the
% vacuum-coating interface of m_2, and then
% propagate that field to the vacuum-coating
% interface of m_1. This is the new 'front
% field' of m_1.
e_2_r = get_field(m_2, 'front');
e_1 = fp.gouy_prop * e_2_r * fp.kz_prop;
set_field(m_1, e_1, 'front');
```
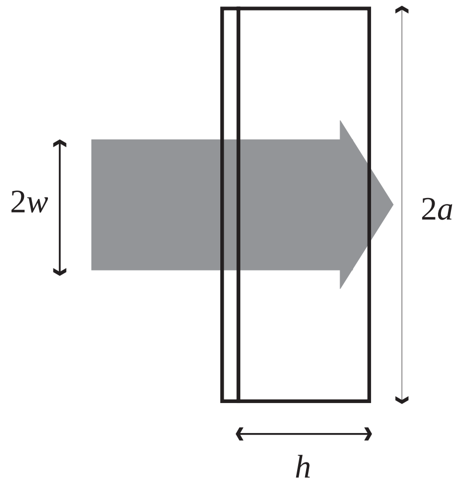

LASER_FIELD MTIMES FUNCTION

```
function e_3 = mtimes(e_1, e_2)
%
...
%
if isa(e_1, 'laser_field') & ~isa(e_2, 'laser_field')
% Initialize the structure e_3 with the same basis and sidebands
% as e_1, and multiply (matrix, using *) the elements of the
% matrix e_2 by the components of e_1.
   e_3.basis = e_1.basis;
   e_3.sideband = e_1.sideband;
   e_3.component = e_1.component*e_2;
elseif ~isa(e_1, 'laser_field') & isa(e_2, 'laser_field')
% Initialize the structure e_3 with the same basis and sidebands
% as e_2, and multiply (matrix, using *) the components of
% e_2 by the elements of the matrix e_1.
   e_3.basis = e_2.basis;
   e_3.sideband = e_2.sideband;
   e_3.component = e_1*e_2.component;
else
   error('Matrix multiplication of two laser_field objects is not allowed.');
end

% Create a new laser_field object from the struct e_3.
e_3 = class(e_3, 'laser_field');
```

HELLO-VINET THERMAL LENS MODEL



Reference: P. Hello and J.-Y. Vinet, J. Phys. France 51, 1267 (1990)

Coating absorption:

$$T_c(r,z) = \frac{a_c a}{k_T} \sum_{m=0}^{\infty} p_m \left[A_m \cosh\left(\zeta_m \frac{z}{a}\right) + B_m \sinh\left(\zeta_m \frac{z}{a}\right)\right] J_0\left(\zeta_m \frac{r}{a}\right)$$

Substrate absorption:

$$T_s(r,z) = \frac{\alpha_P a^2}{k_T} \sum_{m=0}^{\infty} \frac{p_m}{\zeta_m^2} \left[1 - 2\tau A_m \cosh\left(\zeta_m \frac{z}{a}\right)\right] J_0\left(\zeta_m \frac{r}{a}\right)$$

HELLO-VINET THERMAL CONSTANTS

ζ_m: Roots of the equation

$$\zeta J_1(\zeta) - \tau J_0(\zeta) = 0$$

Since $\tau \equiv 4\epsilon T^3 a/k_T = 0.27734$,

$$\zeta_m \approxeq (m + 1/4)\,\pi, \quad m \in \{0, 1, 2, \dots\}$$

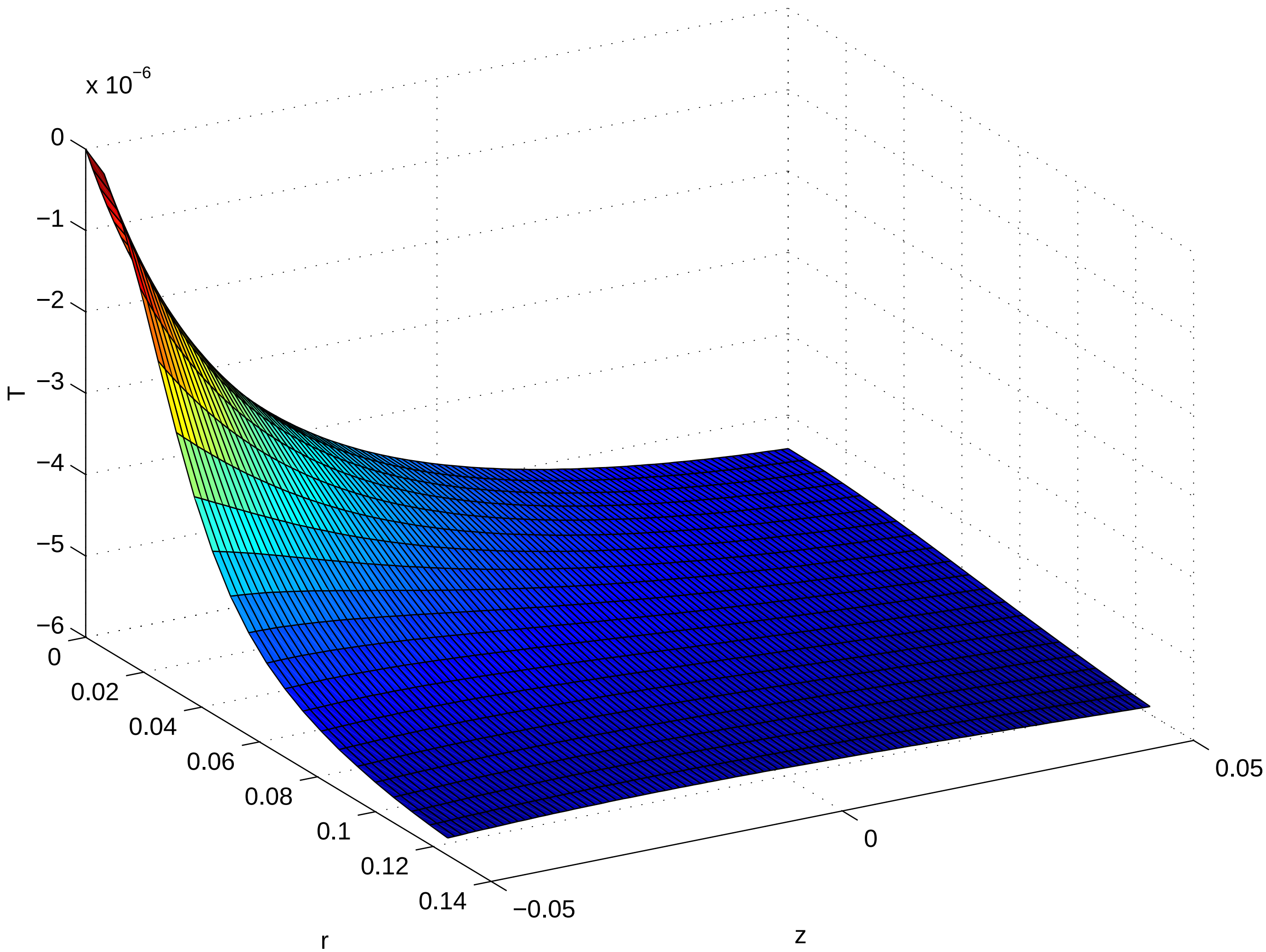
p_m: Normalized expansion coefficients

$$u_{00}^2 = \sum_{m=0}^{\infty} p_m J_0\left(\zeta_m \frac{r}{a}\right)$$

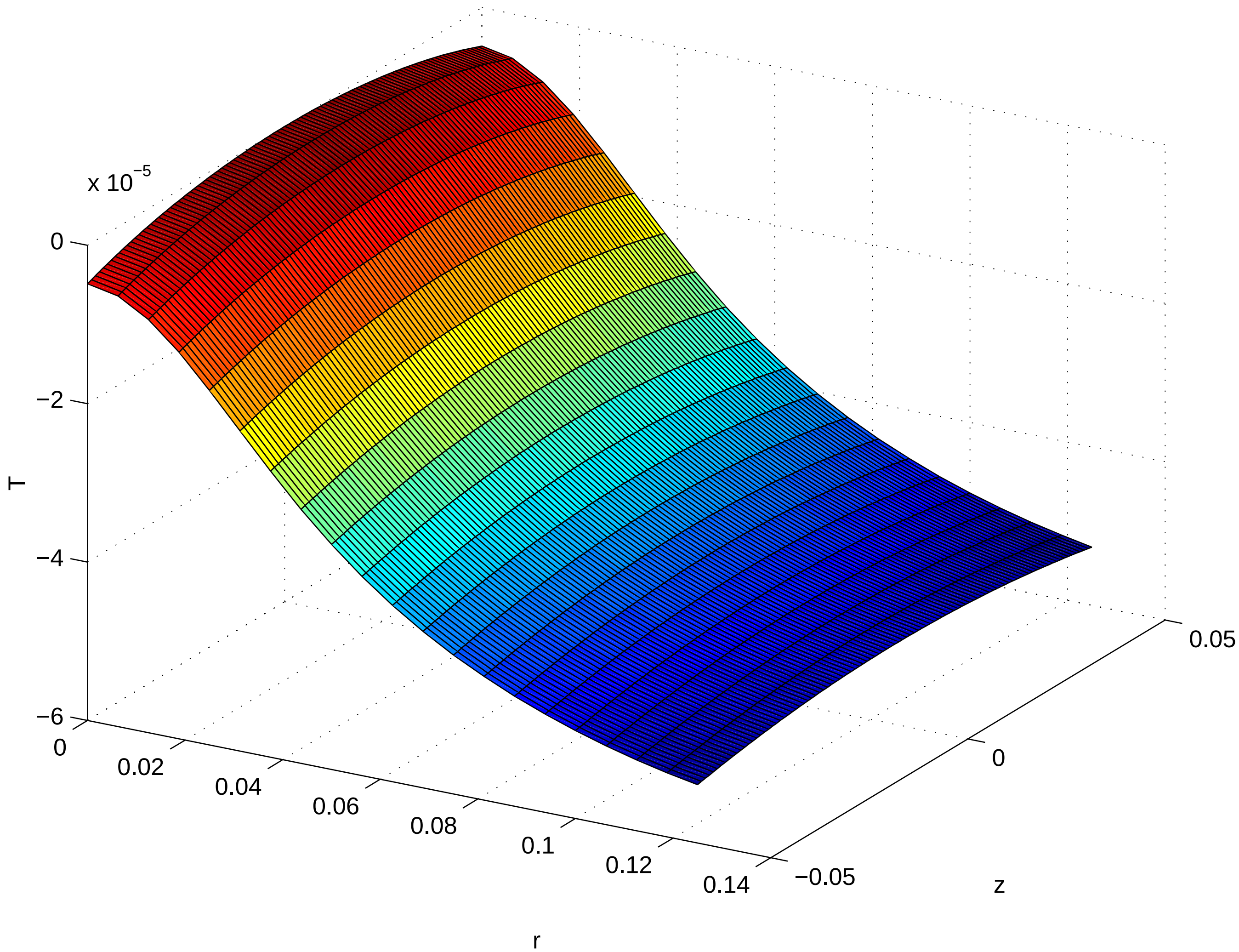
Since $(w/a)^2 \ll 1$,

$$p_m \approxeq \frac{P}{\pi a^2} \frac{\zeta_m^2}{\left(\zeta_m^2 + \tau^2\right) J_0^2(\zeta_m)} e^{-(\zeta_m w/a)^2/8}$$

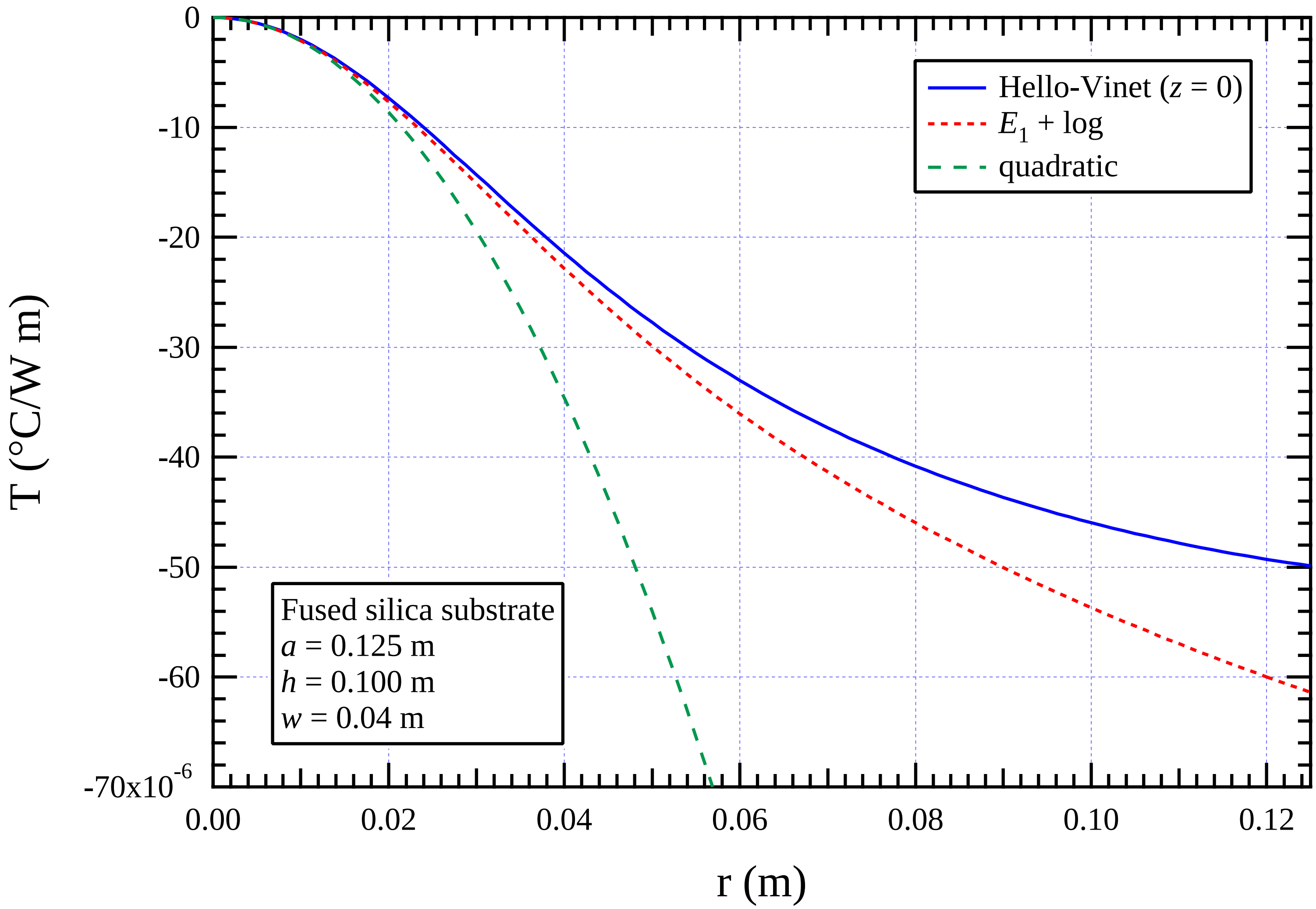
$T(r, z)$ FROM COATING ABSORPTION



$T(r, z)$ FROM SUBSTRATE ABSORPTION



APPROXIMATE $T(r,0)$ (SUBSTRATE ABSORPTION)



THERMAL LENS OPERATOR

The propagation phase perturbation due to the OPD is

$$\phi(r) = \frac{2\pi}{\lambda_0} \frac{dn}{dT} \int_{-h/2}^{h/2} dz\, T(r,z)$$

where $T(r,z)$ is the *linear* sum of contributions from heating due to absorption in both coatings (HR and AR) and the substrate.

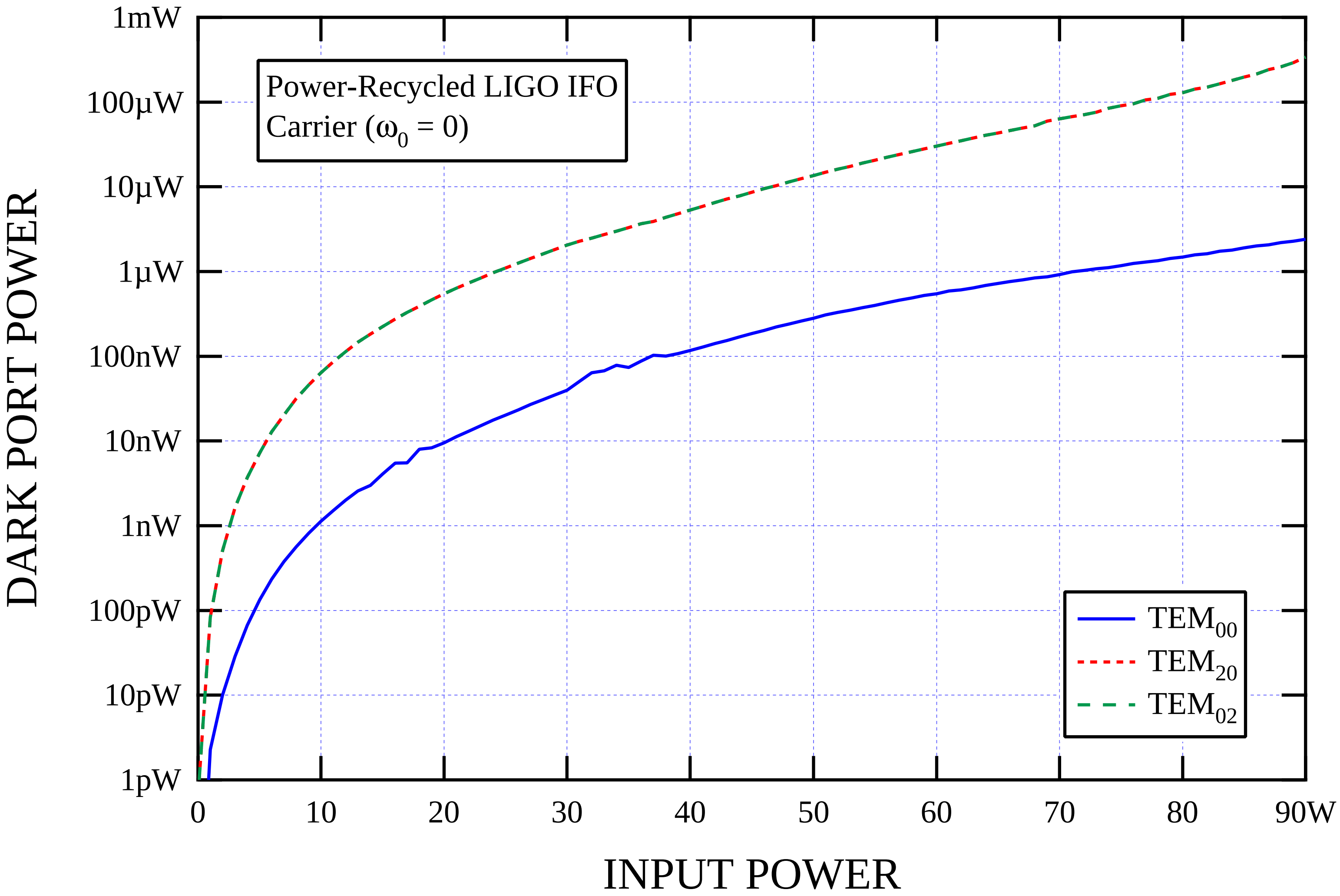
Matrix elements of the thermal lens operator:

$$\Phi_{m'n';mn} = \iint_{-\infty}^{\infty} dx\,dy\, u^{\dagger}_{m'n'}(x,y)\, u_{mn}(x,y)\, e^{i\phi(r)}$$

$$\cong 1 - i \iint_{-\infty}^{\infty} dx\,dy\, u^{\dagger}_{m'n'}(x,y)\, u_{mn}(x,y)\, \phi(r)$$

Since $\phi(r) \propto r^2$, TEM_{00} is coupled to both TEM_{20} and TEM_{02}.

EXAMPLE: CARRIER (PR)



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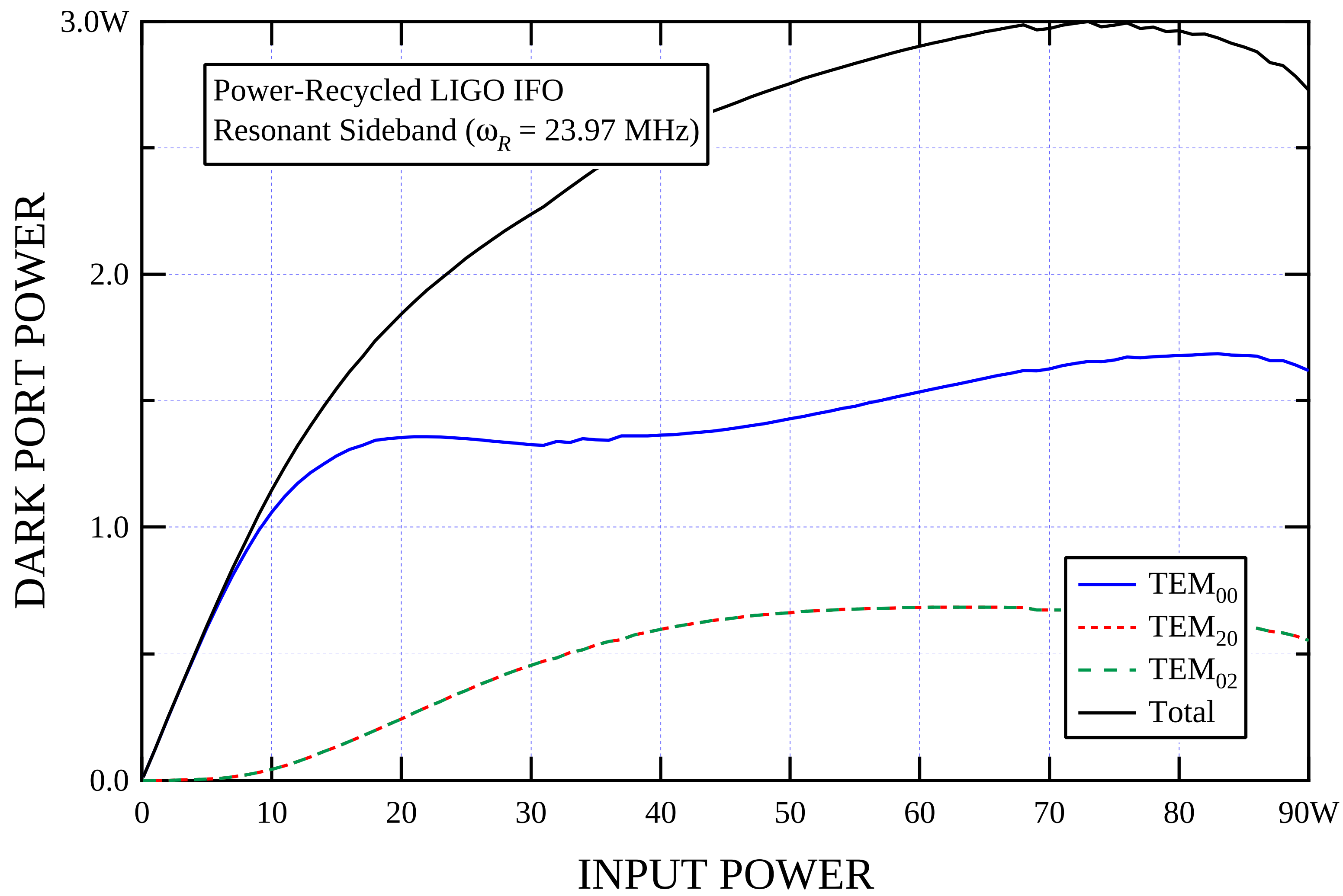

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MATLAB/OOP REFERENCES

- Duane Hanselman and Bruce Littlefield, **Mastering MATLAB 5: A Comprehensive Tutorial and Reference** (Prentice-Hall, 1998); ISBN 0-13-858366-8
- Bertrand Meyer, **Object-Oriented Software Construction**, Second Edition (Prentice-Hall, 1997); ISBN 0-13-629155-4
- Paul F. Dubois, **Object Technology for Scientific Computing: Object-Oriented Numerical Software in Eiffel and C** (Prentice-Hall, 1997); ISBN 0-13-257808-X
- Jean-Marc Jézéquel, **Object-Oriented Software Engineering with Eiffel** (Addison-Wesley, 1996); ISBN 0-201-63381-7

Note 1, LIGO, 03/18/99 08:12:47 AM
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