LIGO-G970300-00-E

1 April 97
LIGO Science Mtg. Talk
A. Lazzarini

SCATTERED LIGHT AND ITS CONTROL IN THE LIGO BEAM TUBES*

• References

- Born & Wolfe (diffraction)
- Beckmann & Spizzichino (scatter from rough surfaces)
- Stover (optical scattering)
- R. Weiss - numerous LIGO internal reports dealing with application to B.T.
- K. Thorne / E. Flannigan - same as above
- J.Y. Vinet - VIRGO Analyses

• Outline

- noise equivalent strain
- dominant scattered light processes (LIGO BT)
  • offending paths - identification
  • paths & baffling
- time dependent phase shifts from vibrating surfaces
- Scattering & recombination from rough surfaces
  - BRDF
  - detailed balance
- combining effects (end-to-end)
- LIGO requirement
- LIGO design today - projected performance

* Similar issue for VE chambers, COS, LSC (P.D.)

Interferometer response to scattered light – noise equivalent strain

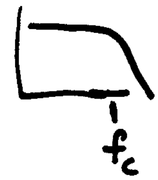
- GW

$$\bar{E}' = \bar{E}_0 + \delta\bar{E} = |E| e^{i\delta\phi_{GW}}$$

$$\frac{\delta E}{|E|} \sim i\delta\phi_{GW} \Rightarrow \boxed{\delta\phi_{GW} = \mathrm{Im}\left\{\frac{\delta\bar{E}}{|E|}\right\}} \quad ①$$

$$\boxed{\delta\phi_{GW} = \left|\frac{d\phi}{dh}\right| \delta h} \quad ②$$

↑ IFO response



$$\frac{d\phi}{dh}(f) = \frac{4\pi\tau_{ST}\, c}{\lambda\left[1 + (4\pi\tau_{ST} f)^2\right]^{1/2}}$$

τ_{ST} = Cavity storage time

$$\left|\frac{d\phi}{dh}\right| = \underbrace{2hLN_B}_{\text{D.C.}} \; \underbrace{\frac{1}{\left[1 + \left(\frac{2\pi N_B}{f_0} f\right)^2\right]^{1/2}}}_{\text{A.C.}}$$

$$\tau_{ST} = \underbrace{\frac{2L}{c}}_{1/FSR}\underbrace{\left[\frac{1}{1-R}\right]}_{1/\text{Losses}}$$

$$N_B = \frac{1}{2\pi}\,\frac{1}{1-R} = \text{"\# Bounces"}$$

$$\tau_{ST} = 2\pi N_B \tau_0 = \frac{1}{f_c} = \frac{2\pi N_B}{f_0}$$

①+②
$$\delta h = \frac{1}{\left|\frac{d\phi}{dh}\right|} \mathrm{Im}\left\{\frac{\delta E}{|E|}\right\}$$

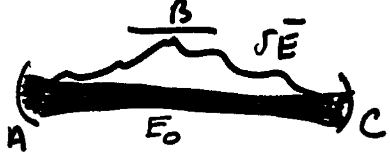
- Scattered light

– "EMI" : $\underbrace{\text{xmittr}}_{\text{imperfect optic}} \Rightarrow \underbrace{\text{path}}_{\text{moving VAC. envelope surfaces}} \Rightarrow \underbrace{\text{rcvr}}_{\text{imperfect optic}} \Rightarrow$ interference

– 3rd order process:

$$\frac{\delta P}{P_0} \sim \mathrm{Prob}(A|B)\,\mathrm{Prob}(B|C)\,\mathrm{Prob}(C|A) \qquad \text{"A"} = \text{main beam}$$

– Processes
  $A \to B$
  $C \to A$
  are time reversed pairs (antenna theorem, detailed balance)



– Scattered light also builds up resonantly by factor N_B

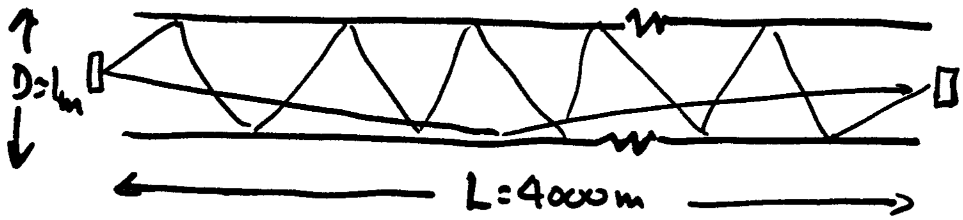
– determine $\delta\bar{E}^{\text{TOTAL}}_{\text{scatt}} \Rightarrow$
$$h_{\text{scatt}} = \frac{1}{\left|\frac{d\phi}{dh}\right|} \mathrm{Im}\left[\frac{\delta\bar{E}}{|E|}\right] \frac{1}{N_B}$$

$\sum_{\text{paths}} \delta\bar{E}_i \Leftarrow$ weighted by probability

noise equivalent strain

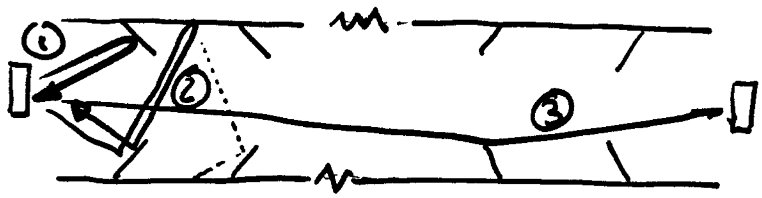
Scattered light in LIGO B.T.

- BT is a "light pipe" (lossy!)



- L/D ~ 4000 ⇒ lots of opportunity for scattering.

- identification of reflection paths is <u>not</u> analytical
  - need optical (rough) design
  - use numerical methods to identify multipath interfere
    - Monte Carlo
    - Launch rays with $P(\theta,\phi)$
    - follow rays through multiple reflections
    - keep track of cumulative probabilities / intensities
  - LIGO had "baseline" design analyzed by Breault Research Org. (BRO)
    - intracavity scatter (highest finesse) worst problem
    - recombination @ detector also a concern (cos?)



① Backscatter from baffles

② Reflect off baffles & Backscatter off BT wall

③ Diffract from baffle edges.

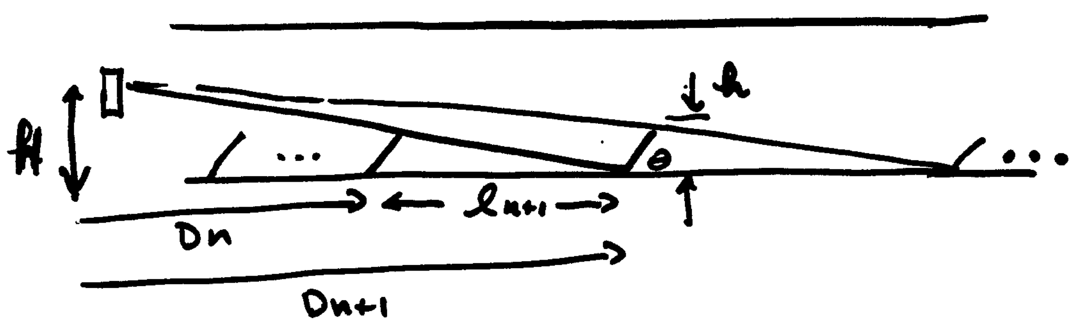
- use baffling to suppress direct viewing of BT wall by mirrors

- Paths ①, ②, ③ worst remaining offenders

- To estimate magnitude of effects need to formulate
  - scattering off (nearly perfect) mirrors
  - " BT walls / Baffles
  - Diffraction off baffle edges
  - recombination of scattered light into TEM_{00} beam

LIGO Baffle design -

- select spacing to completely mask BT surface from mirrors
- select height to keep ~~baffle~~ baffle count low while not impacting clear aperture
- incline baffles <u>away</u> from nearest mirror
  - avoid "retro angles"
  - direct light <u>away</u> from nearest mirror
- serrate edges to reduce (coherent) diffraction

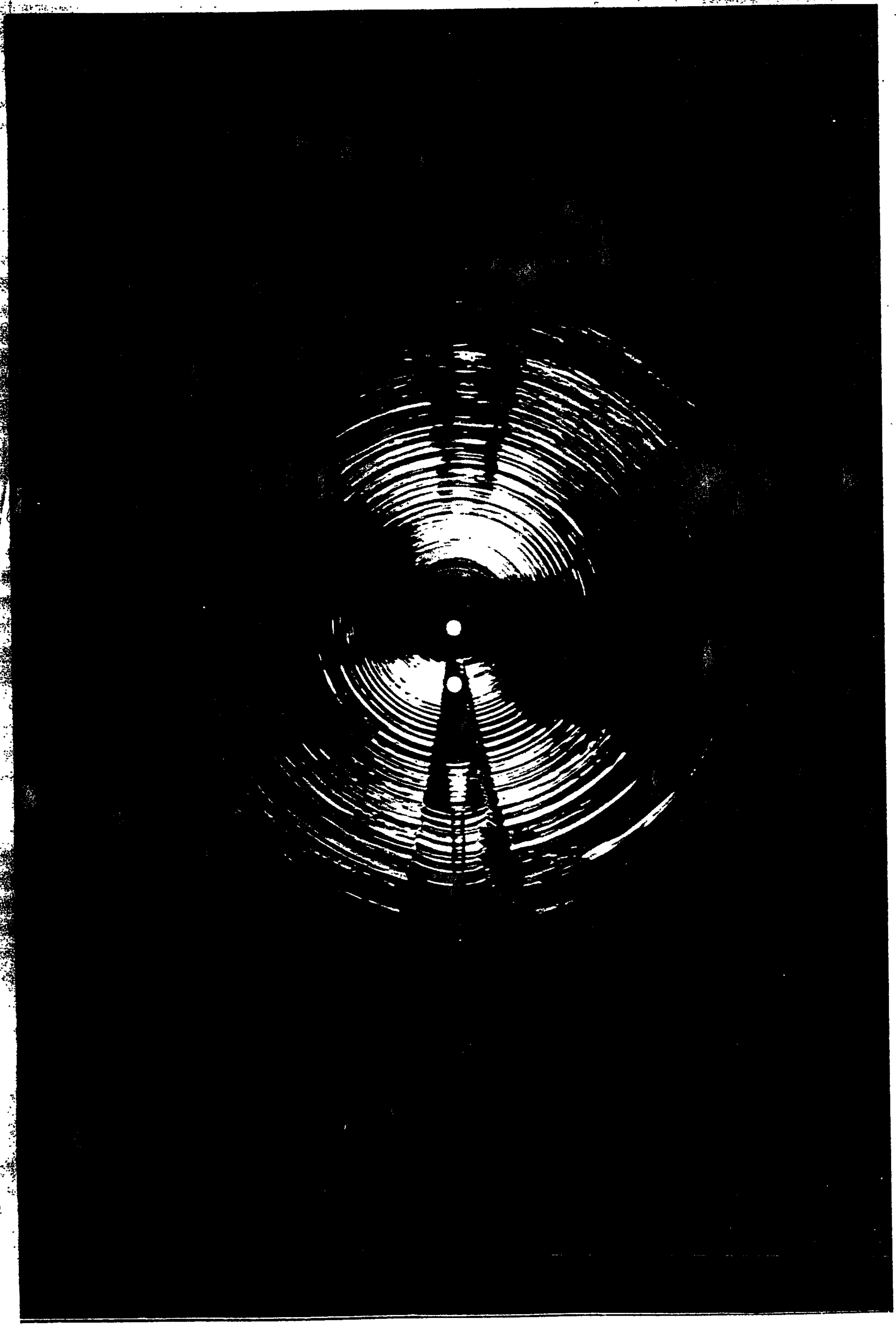


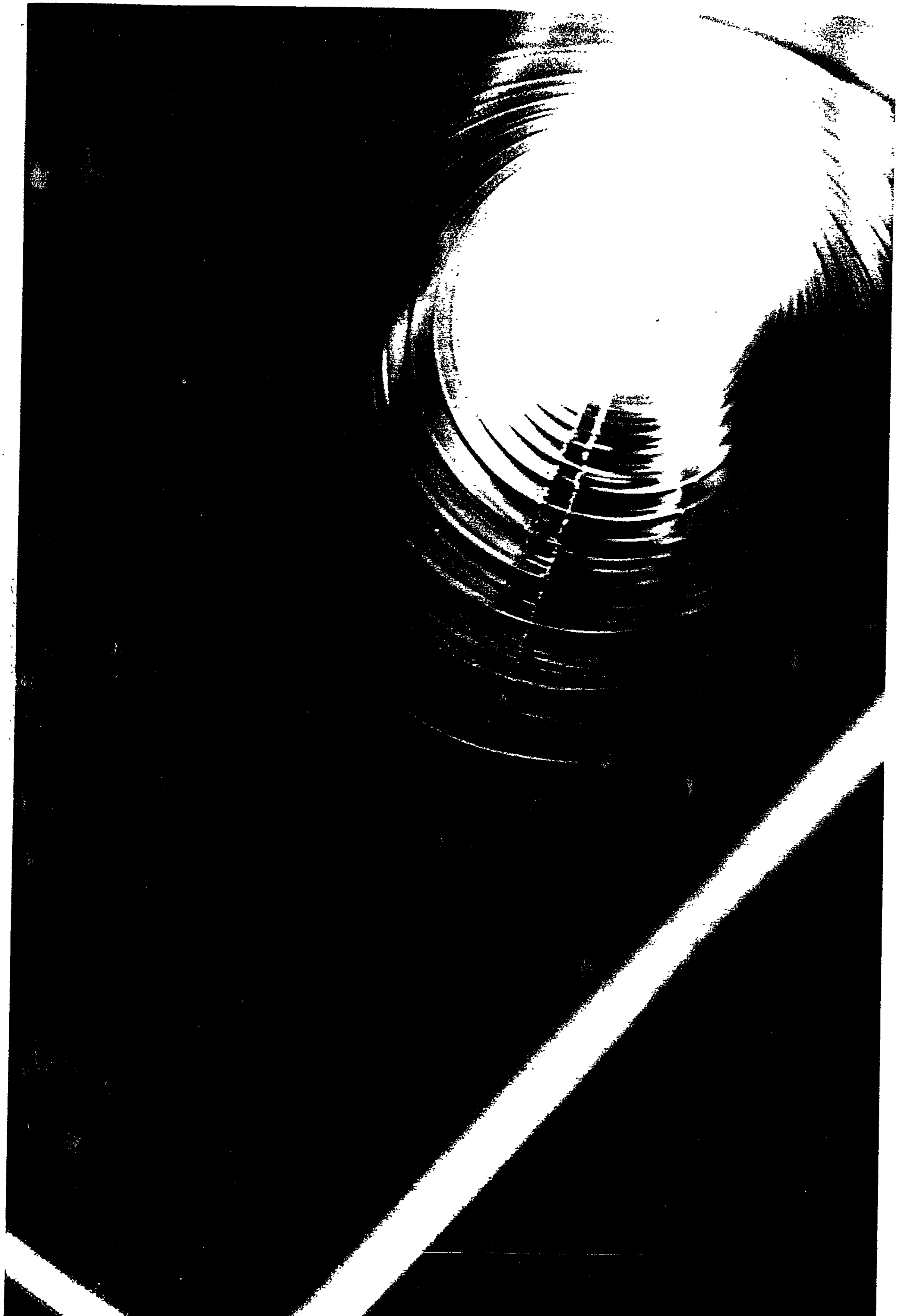
$$\frac{H}{D_{n+1}} = \frac{h}{l_{n+1}} \; ; \qquad D_{n+1} = D_n + l_{n+1}$$

$$\Rightarrow \quad \frac{l_{n+1}}{l_n} = \left[\frac{H}{H-h}\right] \equiv \beta > 1$$

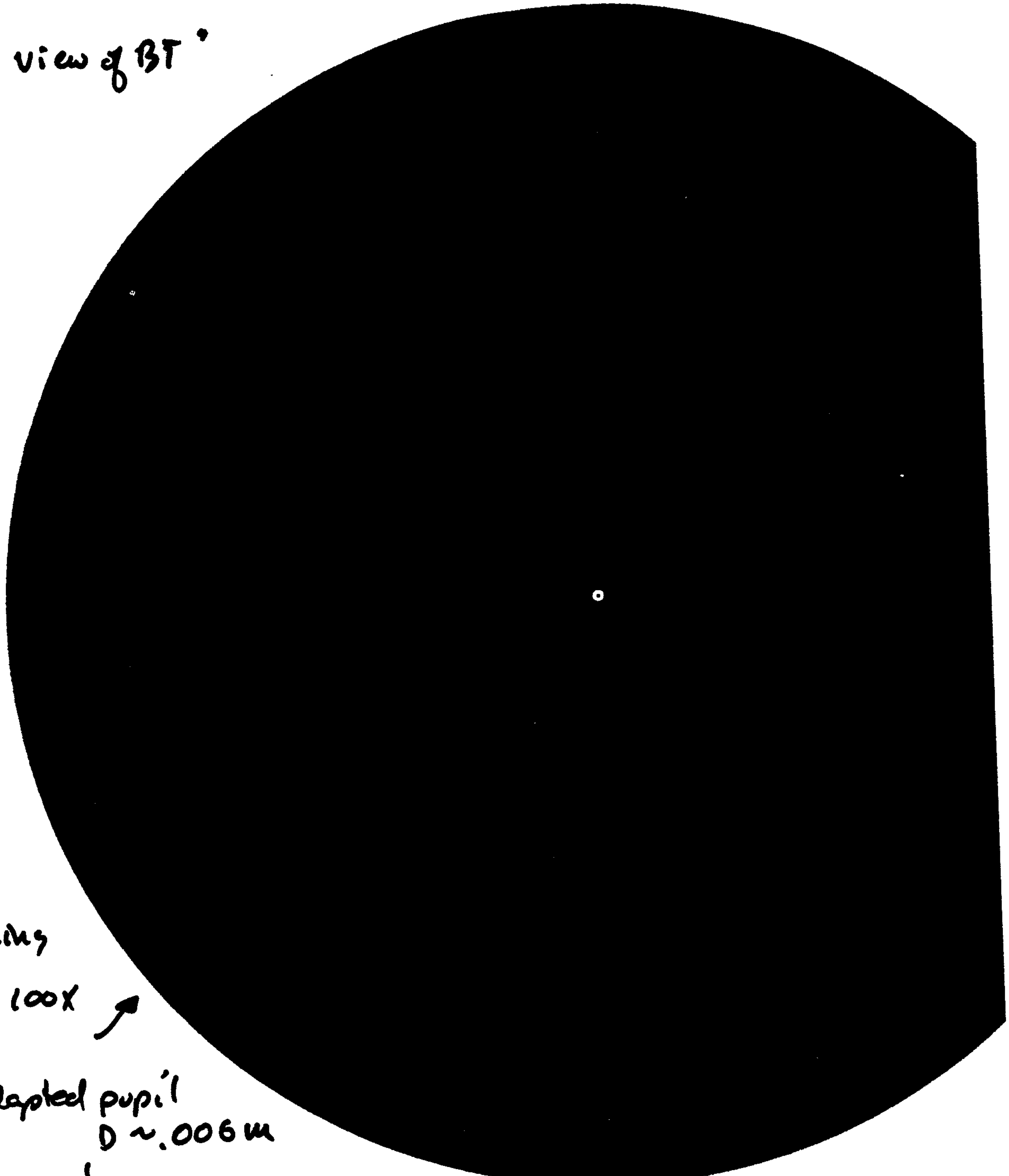
$$l_{n+1} = l_1 \beta^n$$

- Geometric progression
- $h = 9\,cm$
- $\theta = \cancel{35}^\circ$ ($\theta_i = 55^\circ$ for scattering processes)





"mirror's eye view of BT"



- End openning magnified 100X
- Dark adapted pupil $D \sim .006\,m$

$\Rightarrow \Delta\theta_{diff} = \frac{\lambda}{D} = 83\,\mu r$

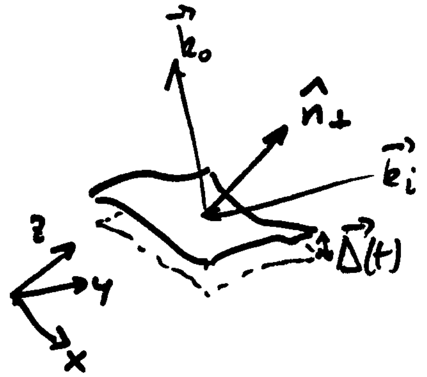
$\Rightarrow \Delta X_{min} = 0.3\,m \Rightarrow$ openning is a pointlike star

- Ambient sky brightness (day) $\sim 30\,W/m^2/sr$

$\Rightarrow 2\,\mu W/m^2$ @ $4000\,m \rightarrow$ very dark

$\Rightarrow m_v \sim +36$

I. Scattering from moving surface produces time-dependent phase shifts



Non specular: $\vec{k}_0, \vec{k}_i, \hat{n}_\perp$ not in same plane

surface: $z = f(x,y)$

motion: surface translates by $\vec{\Delta}(t)$

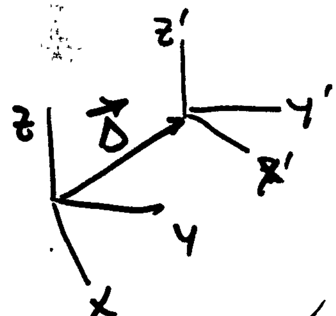
- Incident plane wave $\quad |\psi_i\rangle \sim e^{i\vec{k}_i\cdot\vec{r}}$

$$|\psi_i\rangle = e^{i[k_x x + k_y y + k_z f(x,y)]}$$

- Scattered wave $\quad |\psi_0\rangle = e^{i[k_x' x + k_y' y + k_z' f(x,y)]}$

Consider at $t=0$ $\quad \langle\psi_0|\psi_i\rangle_0 = \int dx dy\, e^{i(\Delta k_x x + \Delta k_y y + \Delta k_z f(x,y))}$

let $\quad \vec{\Delta}(t) = \{\xi(t), \eta(t), \zeta(t)\}$

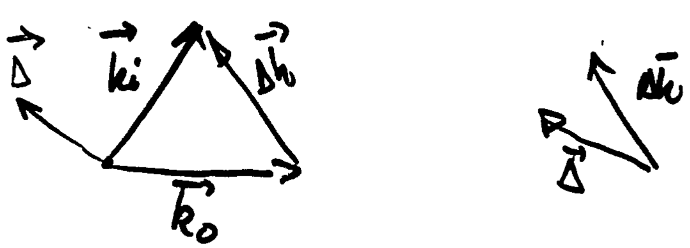


$$x = x' + \xi \qquad z' = f(x', y')$$
$$y = y' + \eta \qquad z = \zeta + f(x', y')$$
$$z = z' + \zeta$$

$$\langle\psi_0|\psi_i\rangle_t = \int dx' dy' \left[e^{i\vec{\Delta k}\cdot\vec{\Delta}} e^{i[\Delta k_x x' + \Delta k_y y' + \Delta k_z f(x',y')]} \right.$$

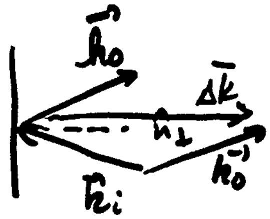
$$= \underbrace{e^{i\vec{\Delta k}\cdot\vec{\Delta}(t)}}_{\text{dynamic phase shift}} \underbrace{\langle\psi_0|\psi_i\rangle_0}_{\text{static form factor}}$$

$$\underline{\phi(t) = \vec{\Delta k}\cdot\vec{\Delta}(t)}$$



I Scatter by a moving surface

- Reflection

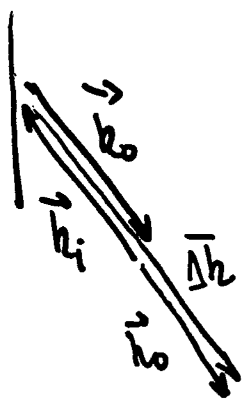


$$\Delta\bar{k} = -2(\bar{k}_i \cdot \hat{n}_\perp)\,\hat{n}_\perp$$

$$\delta\phi = -2\,(\vec{k}_i \cdot \hat{n}_\perp)(\vec{\Delta} \cdot \hat{n}_\perp)$$

- sensitive to motion $\perp$ to surface

- Backscatter

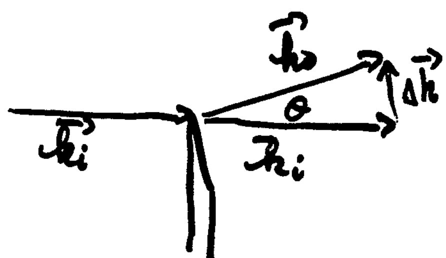


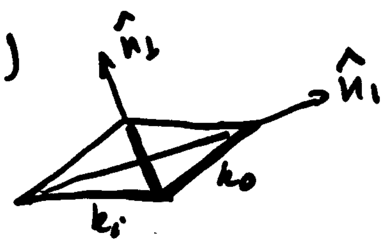
$$\Delta\vec{k} = -2\,\vec{k}_i$$

$$\delta\phi = -2\,\vec{k}_i \cdot \vec{\Delta}$$

- sensitive to motion along line of sight (L.O.S.)

- Small angle scatter (Diffraction)



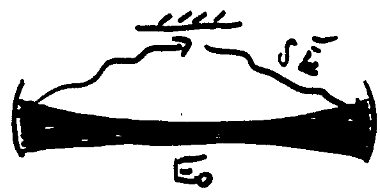


$$\Delta\vec{k} = k\theta\;\hat{n}_\perp$$

$$\delta\phi = k\theta\,\vec{\Delta}\cdot\hat{n}_\perp = k\theta\,\Delta_\perp$$

- sensitive to motion $\perp$ to L.O.S.

I Light Scattering phase noise



$$\frac{\delta \bar{E}}{E} = \cos\phi + i \sin\phi$$

- Whereas $\phi \lll 1$ for GW, cannot assume This for general light scatter
  - rough ($\gg\lambda$) surface
  - large amplitudes of motion (on resonance) are possible.

$$h^2 = \frac{1}{\left|\frac{d\phi}{dh}\right|^2} S_{\sin\phi}(f)$$

$\uparrow$ spectral density of $\sin\Phi(t)$

- Small (dynamic) motions

$$\phi \sim \Phi_0 + \delta\phi(t)$$

$\delta\phi(t)$ └ small excursion

Φ_0 └ random constant

$$\sin\phi = \cos\Phi_0\, \delta\phi(t) + \sin\Phi_0$$

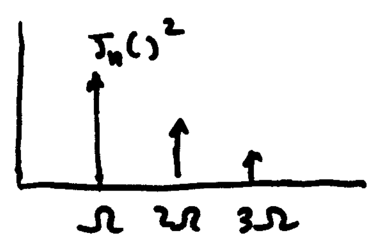
$$\langle \sin^2\phi \rangle = \frac{1}{2} \langle \delta\phi(t)^2 \rangle$$

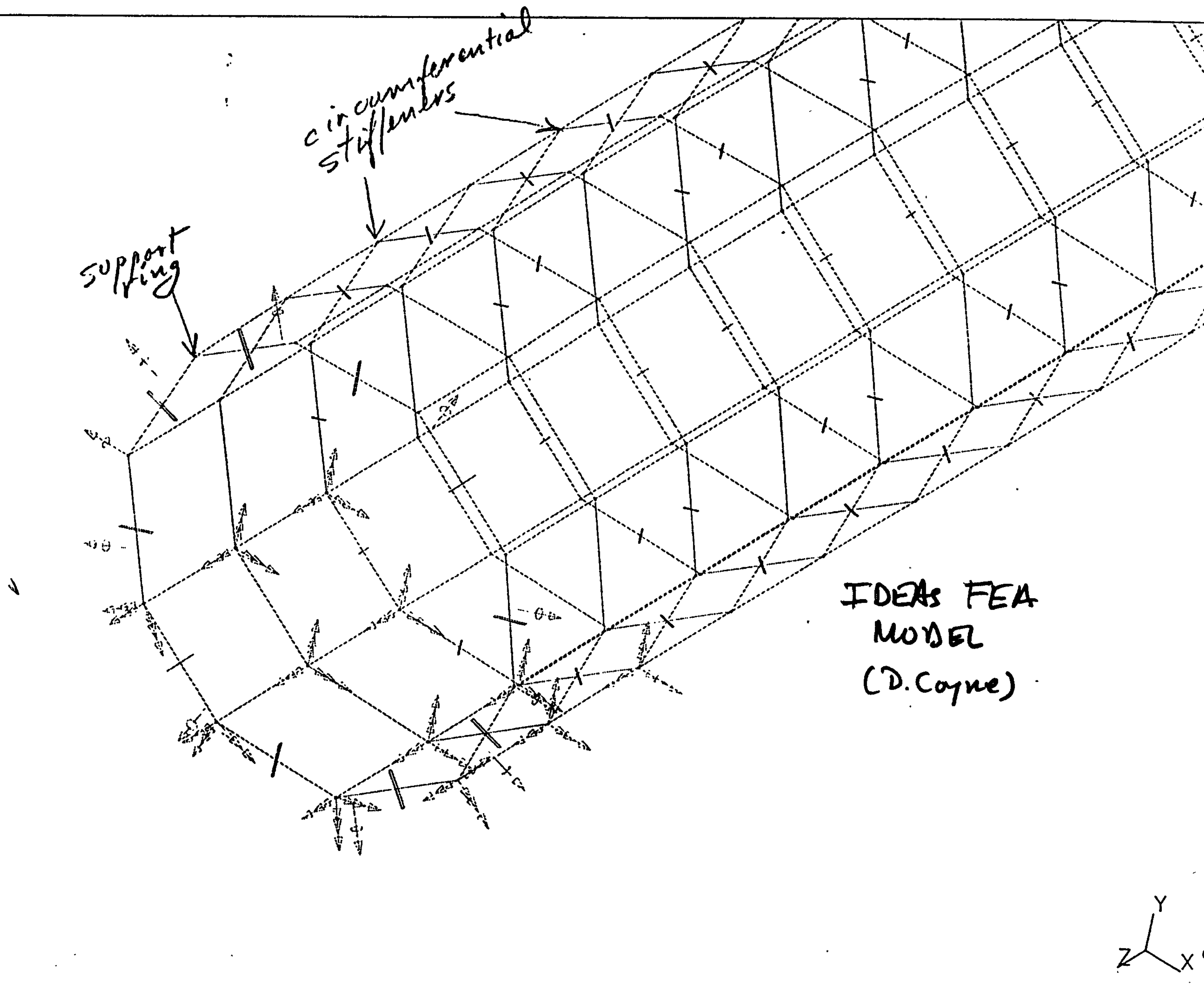
- large ($>\lambda$) motion $\quad x(t) = x_0 \cos\Omega t$

$$\sin\phi = \sin(\Phi_0 + k x_0 \epsilon \cos(\Omega t)) \leftarrow \text{PM}$$

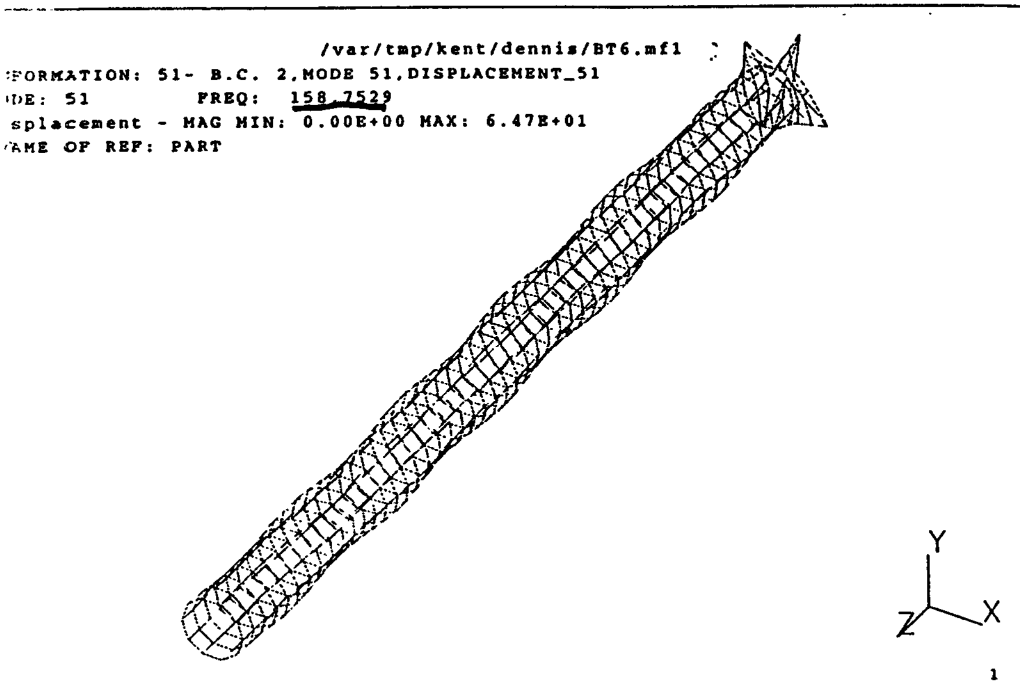
ϵ ← factor from orientation of motion & $\Delta\vec{k}$?

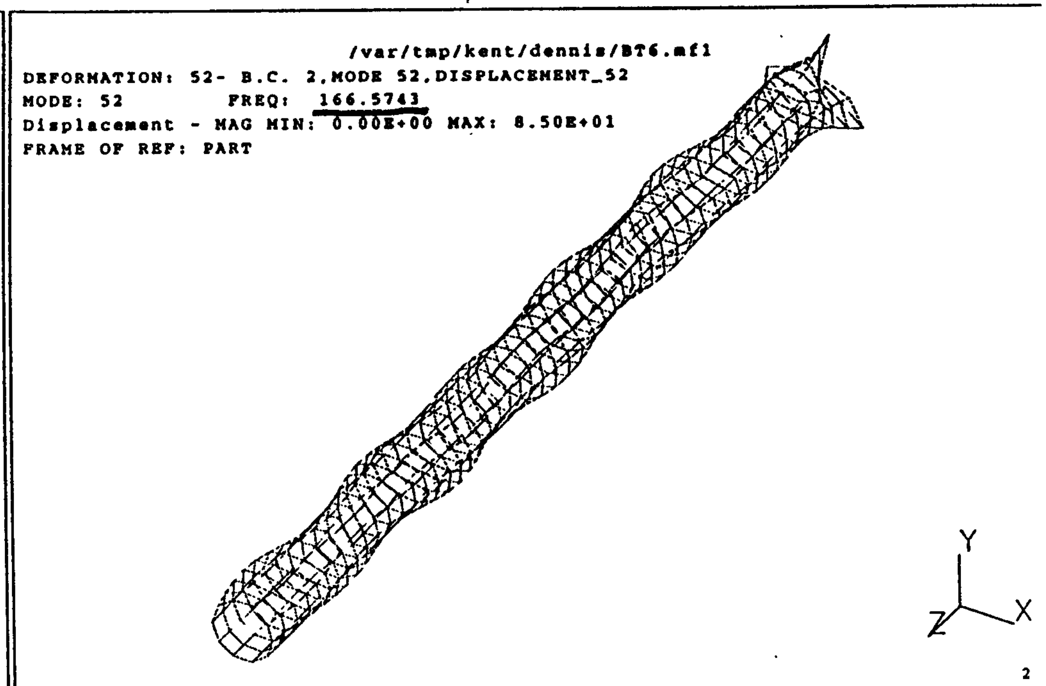
$$\sin\phi = \sum_n J_n[k x_0 \epsilon]\, e^{i n\Omega t + \alpha_n} \leftarrow \text{up conversion}$$

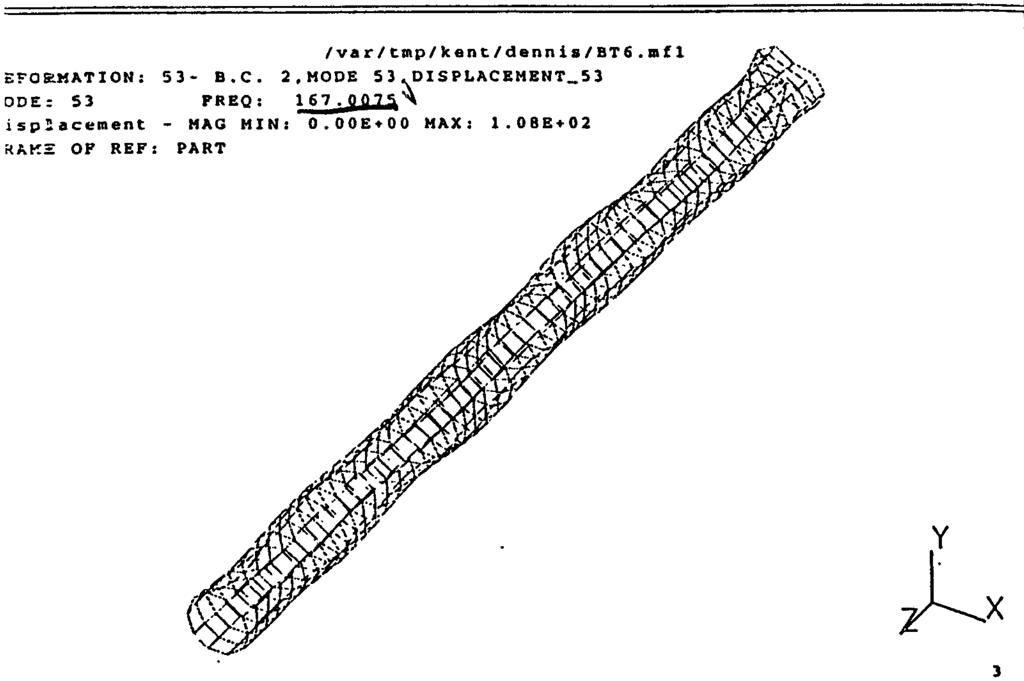


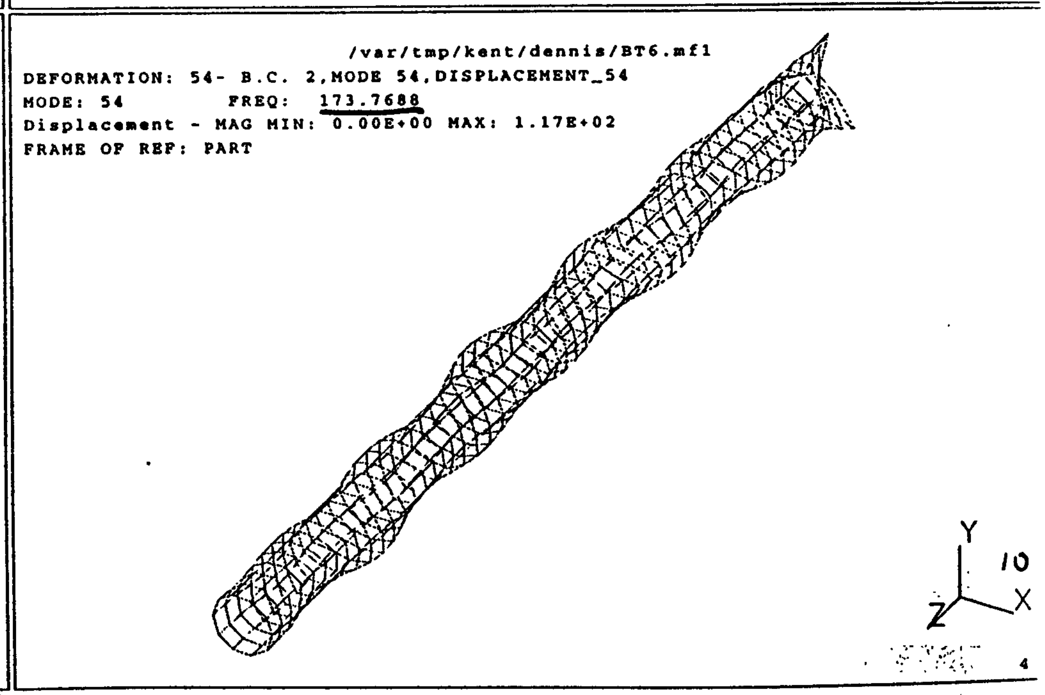

circumferential stiffeners
support ring
IDEAS FEA MODEL
(D. Coyne)
Y
Z
X

BT FEA NORMAL MODES









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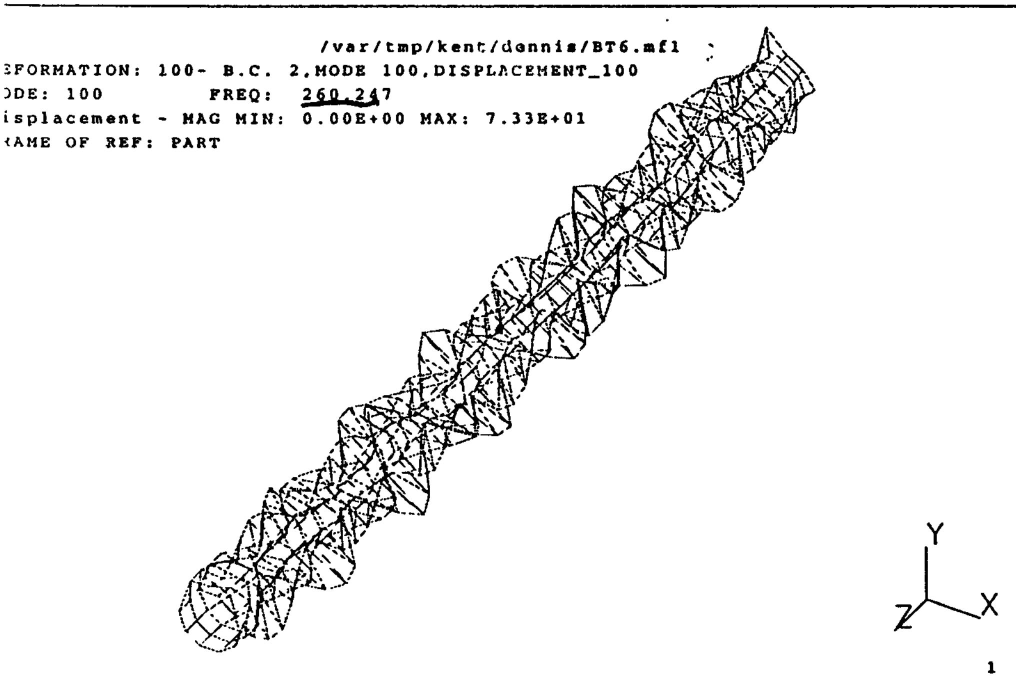
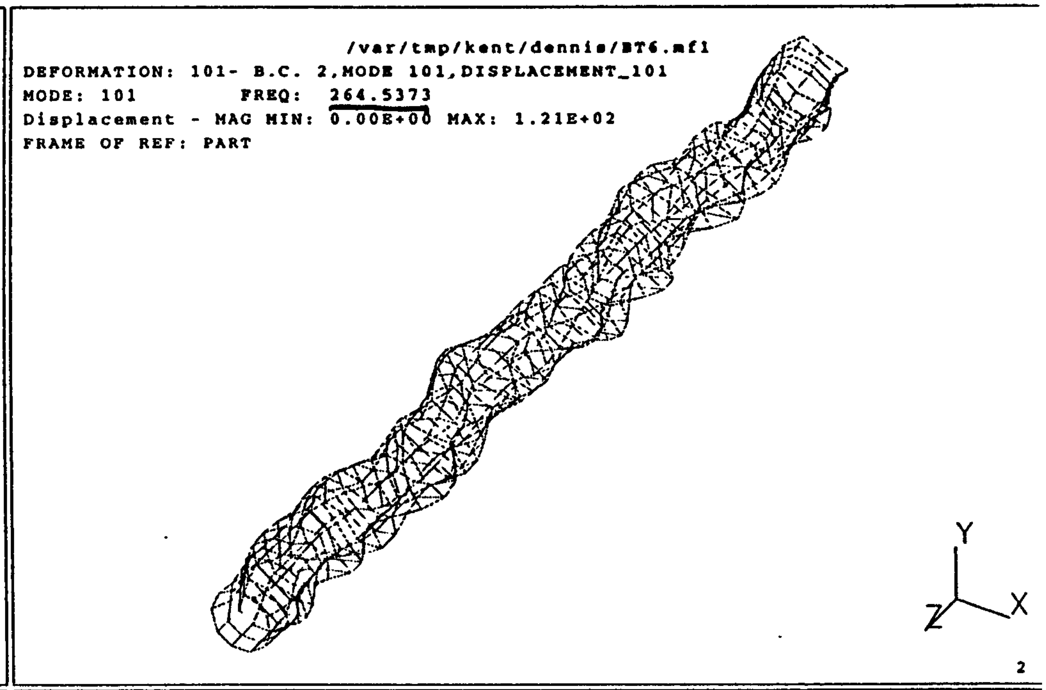
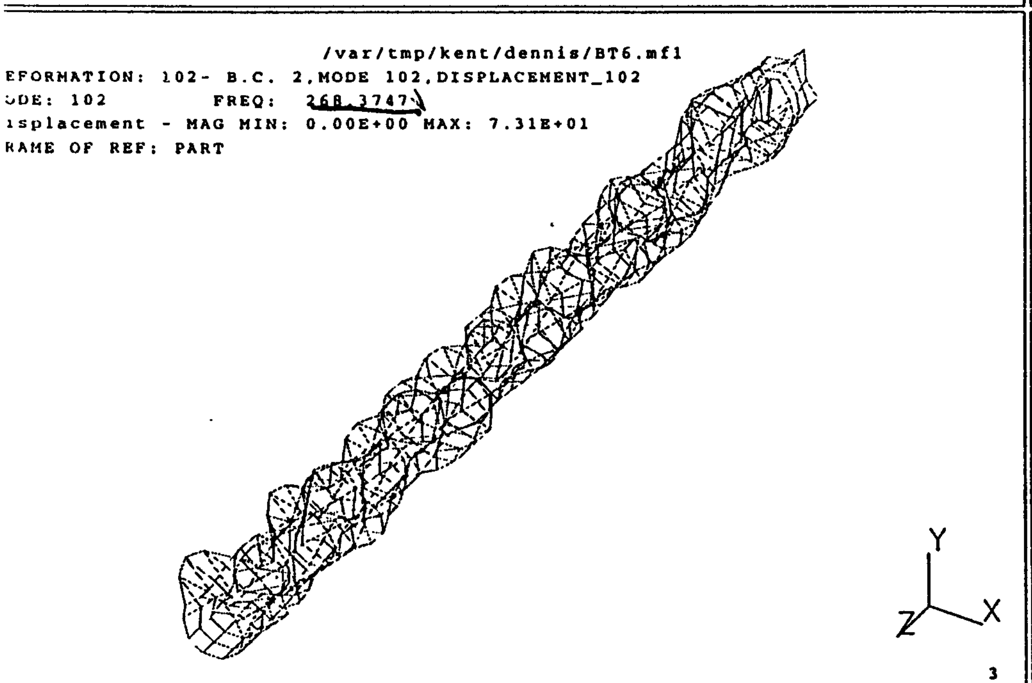
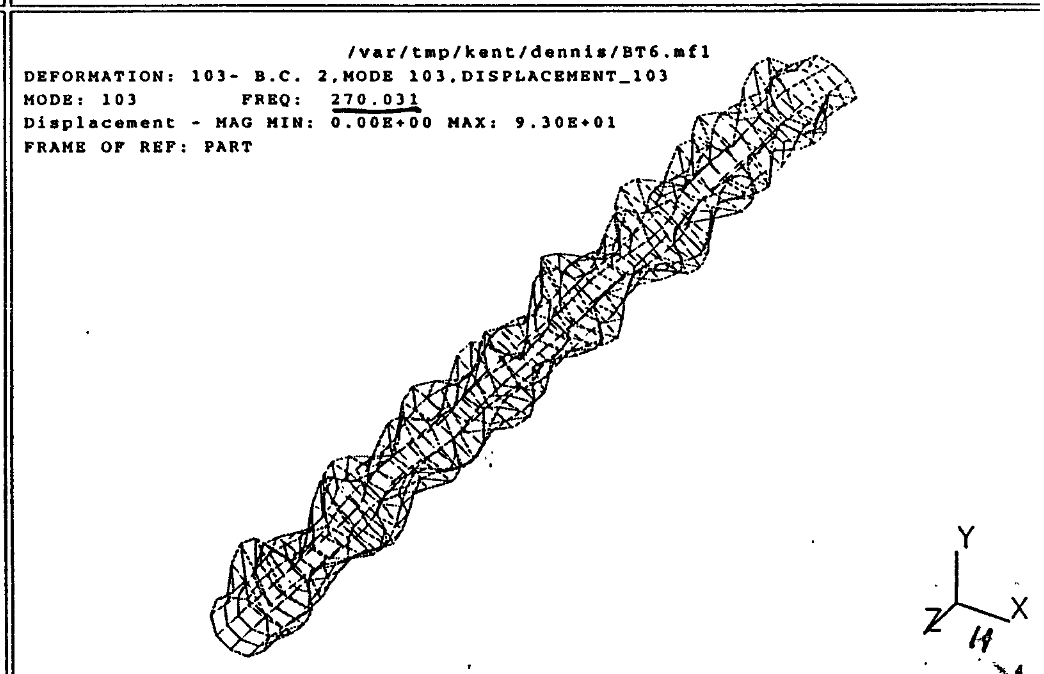
BT FEA NORMAL MODES

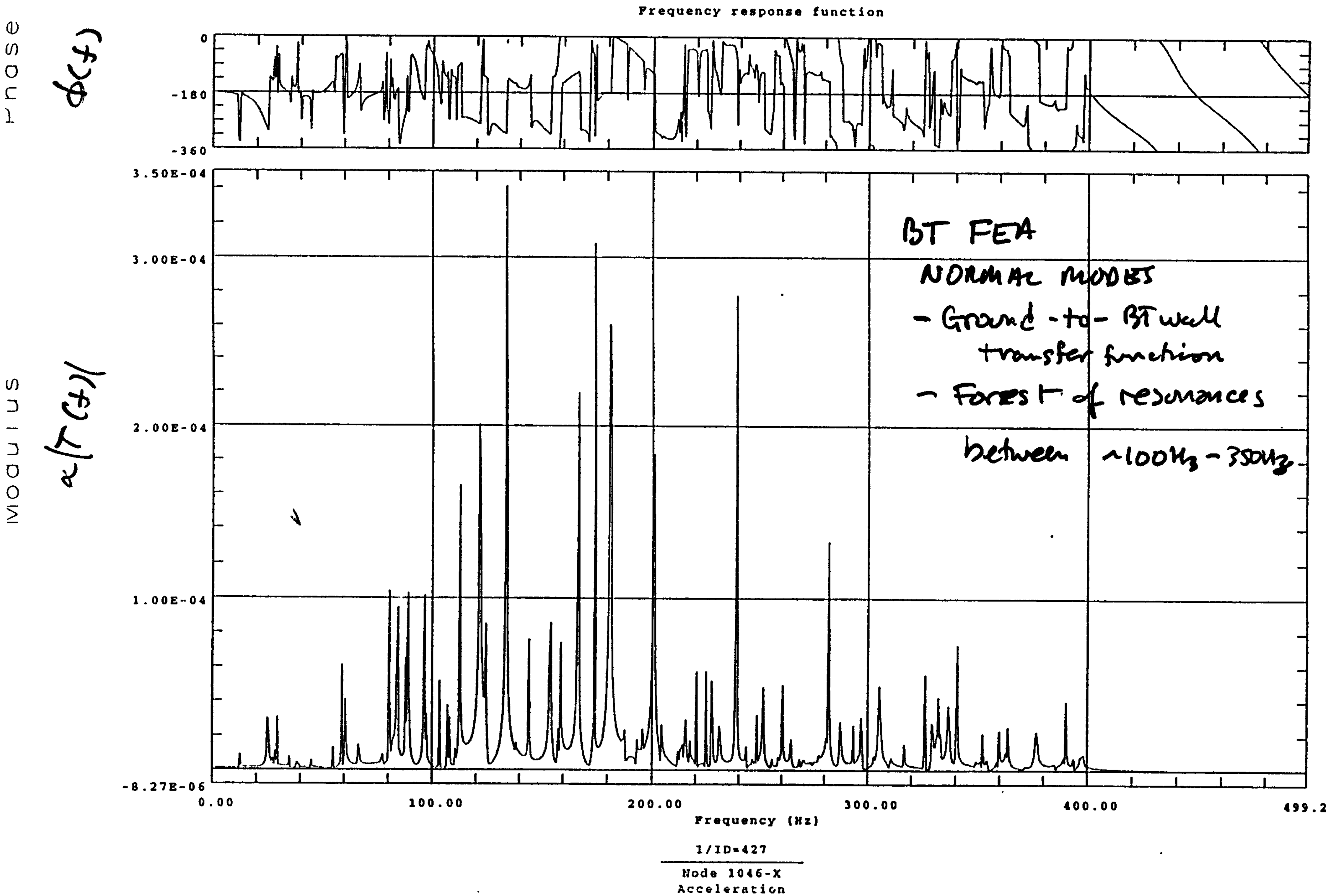




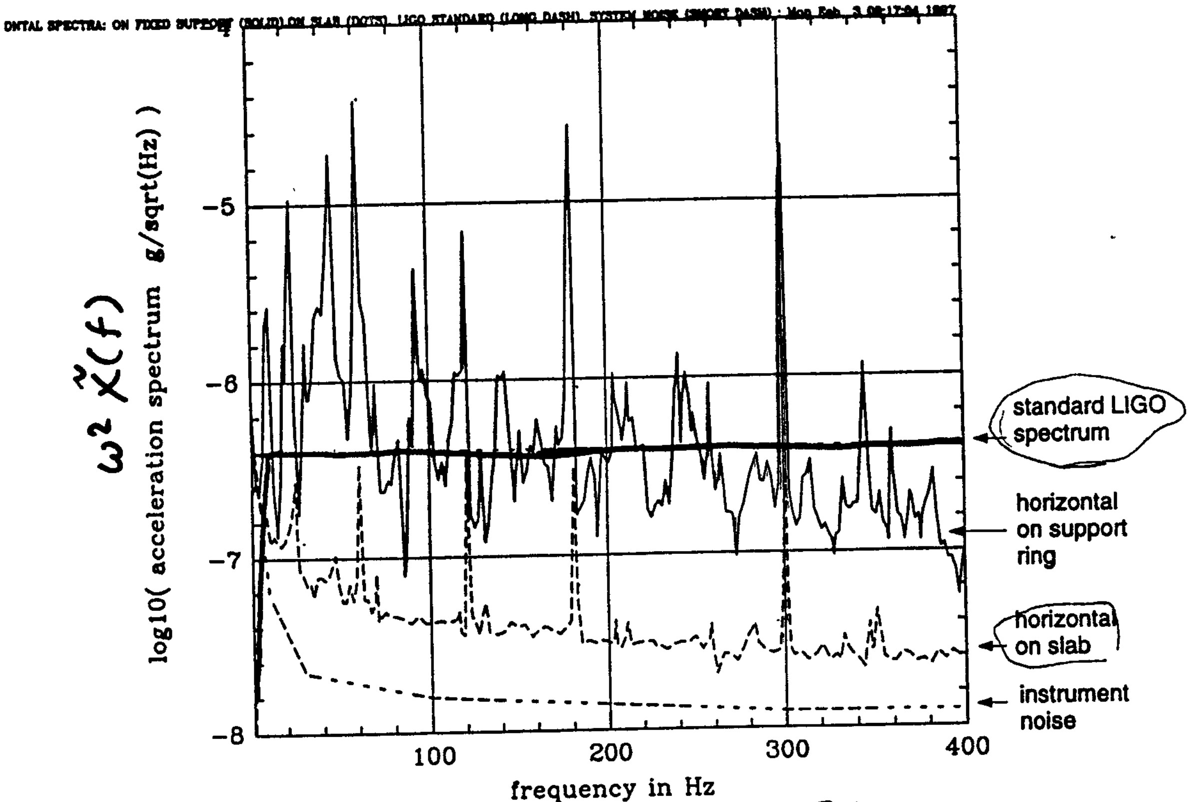




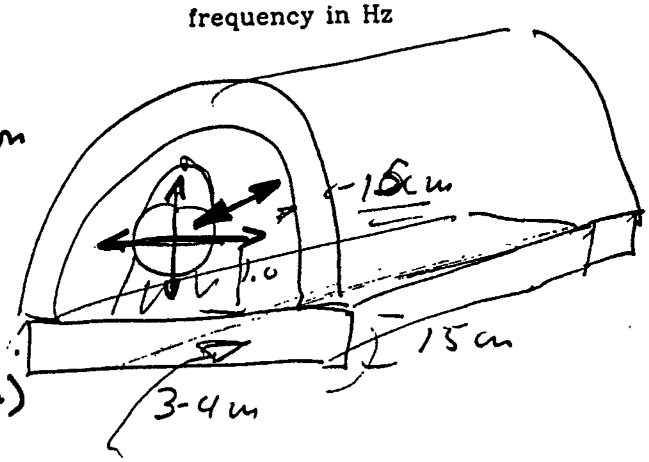
S6 → A5x


Frequency response function
Phase
$\phi(f)$
0
-180
-360
Modulus
$\propto |T(f)|$
3.50E-04
3.00E-04
2.00E-04
1.00E-04
-8.27E-06
0.00
100.00
200.00
300.00
400.00
499.25
Frequency (Hz)
BT FEA
NORMAL MODES
- Ground-to-BT wall transfer function
- Forest of resonances between ~100Hz - 350Hz
1/ID=427
Node 1046-X
Acceleration

LIGO X arm BT wall motion Field Measurements
(Weiss)



- lots of BT motion
  - still under construction
  - * - not coherent w/ ground
  - acoustically driven
  - without Bakeout insulation (15cm)
  - will attenuate

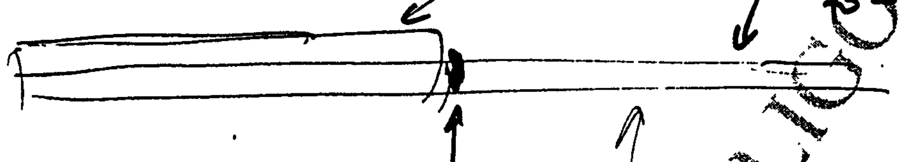


- Line harmonics due to mech. vib of xformer cores on slab (GB&I)

Figure 2 Horizontal acceleration spectrum

covered

exposed to elements



sandbags to provide impedance mis match

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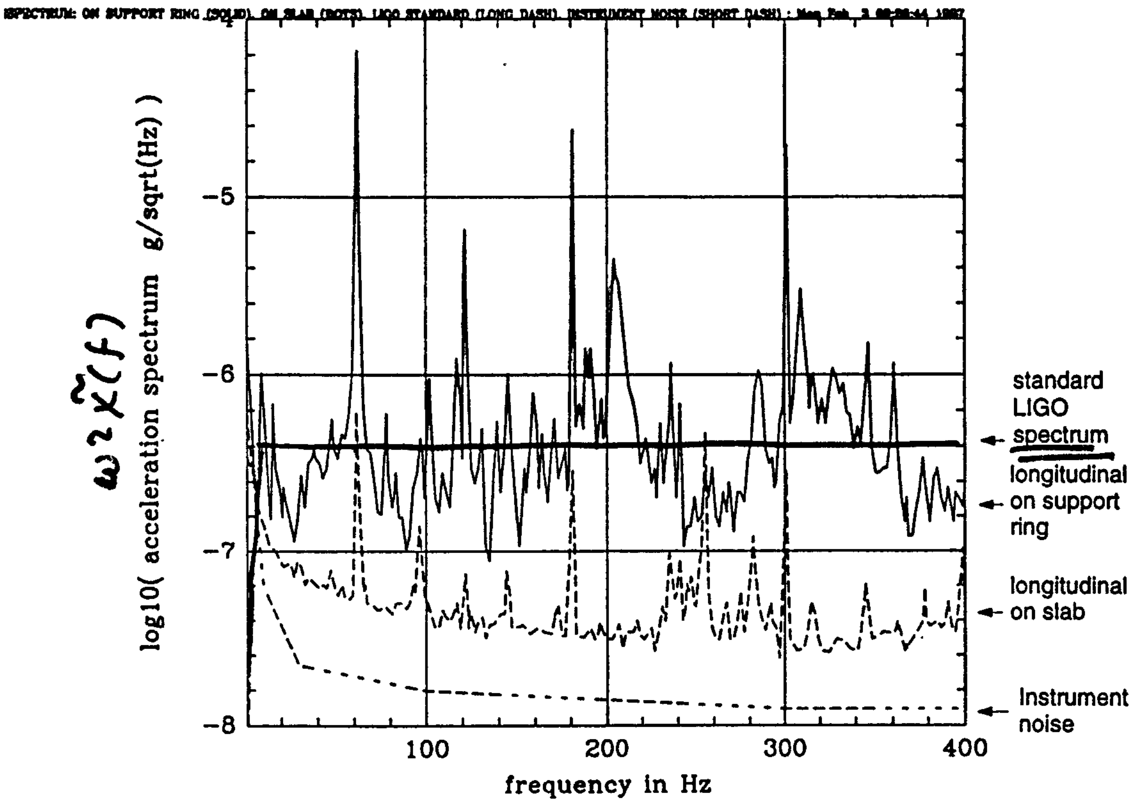
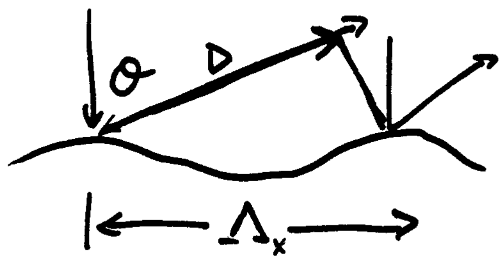


Figure 1 Longitudinal acceleration spectrum.

II. Scatter from a rough surface – BRDF

BRDF = Bidirectional Reflectance Distribution Function

- Angular distribution of scattered light"
- Surface features modify wavefront → far field diffraction ⇓ scatter
- Related to spectral density of surface errors through Bragg condition

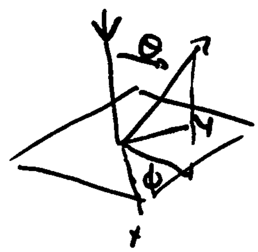


$$\Delta_n = n\lambda = \Lambda_x \sin\theta \qquad n \to 1 \text{ (small angle scatter)}$$

$$\lambda = \Lambda_x \theta \to \nu_x \lambda = \theta_x$$

spatial frequency ν_x → maps into scattering angle θ.

2D:



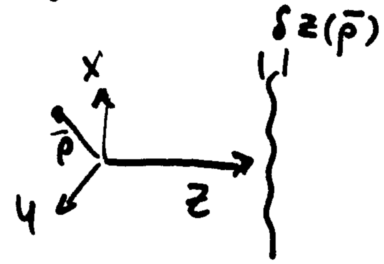
$$\lambda = \Lambda_x \sin\theta\cos\phi$$
$$\lambda = \Lambda_y \sin\theta\sin\phi$$

$$\nu_y = \frac{1}{\lambda}\sin\theta\sin\phi$$
$$\nu_x = \frac{1}{\lambda}\sin\theta\cos\phi$$

$$d\nu_x d\nu_y = \frac{1}{\lambda^2} J[\nu_x, \nu_y, \theta, \phi]\, d\theta\, d\phi$$

$$d\nu_x d\nu_y = \frac{1}{\lambda^2} d\Omega$$

II irregular surface Scatter from a surface



$\longleftarrow \psi_0 = \sqrt{I_0}\, e^{ikz}$

$\longrightarrow \psi' = \sqrt{I_0}\, e^{ikz'}\, \underbrace{e^{2ik\delta z(\bar{\rho})}}$ excess phase perturbation upon reflection $\lll 1$

$\langle \delta z \rangle = 0 \qquad \langle \delta z^2 \rangle = \sigma^2$

$\langle \delta z(\bar{\rho}_1)\, \delta z(\bar{\rho}_2) \rangle = \sigma^2\, \underbrace{C}(\bar{\rho}_1, \bar{\rho}_2)$ spatial correlation fn. (normalized)

Consider F.T. of ψ' @ $z=0$

$$\psi(\nu_x, \nu_y) = \sqrt{I_0} \int dA\; e^{i2\pi\nu_x x}\, e^{i2\pi\nu_y y}\, e^{2ik\delta z(\bar{\rho})}$$

$$|\psi(\nu_x,\nu_y)|^2 = I_0 \int dA\, dA'\; e^{i2\pi\nu_x(x-x')}\, e^{i2\pi\nu_y(y-y')}\, \underbrace{e^{2ik[\delta z(x,y) - \delta z(x',y')]}}$$

$$e^{i\varepsilon} = 1 - \frac{\varepsilon^2}{2} + i\varepsilon + \dots \mathcal{O}(\varepsilon^3)$$

$$|\psi(\nu_x,\nu_y)|^2 = I_0 \int dA\, dA'\; e^{i2\pi\nu_x(x-x')}\, e^{i2\pi\nu_y(y-y')} \times$$
$$\times \left\{ 1 + 2ik[\delta z(x,y) - \delta z(x',y')] - 2k^2[\delta z^2_{(x,y)} + \delta z^2_{(x',y')} - 2\delta z(x,y)\, \delta z(x',y')] \right.$$

statistical avg:

$$\langle |\psi(\nu_x,\nu_y)|^2 \rangle = \overbrace{\underbrace{I_0 \pi a^2}_{P_0} [1 - 4k^2\sigma^2]\, \delta(\nu_x)\, \delta(\nu_y)}^{\text{specular}}$$
$$+ \overbrace{\underbrace{I_0 \pi a^2}_{P_0} 4k^2\sigma^2\, \tilde{C}(\nu_x,\nu_y)}^{\text{diffuse}}$$

$$\int \langle |\psi(\nu_x \nu_y)|^2 \rangle\, d\nu_x d\nu_y = P_0$$

Consider scattered (diffuse part):

$$\int \langle |\psi(\nu_x \nu_y)|^2 \rangle_{scatt}\, d\nu_x d\nu_y = P_0\, \underbrace{4k^2\sigma^2}$$ surface irregularity leads to scatter

$$P_0\, 4k^2\sigma^2 = \int \underbrace{\langle |\psi|^2 \rangle \frac{1}{\lambda^2}\, d\Omega}$$

$$\frac{dP_{sc}}{d\Omega} = \underbrace{\frac{P_0}{\lambda^2}[4k^2\sigma^2]}\, \overbrace{\tilde{C}}^{\text{spectral density of surface errors}}(\nu_x,\nu_y) \qquad \underline{\underline{W/sr}}$$

$1/\lambda^4$ dependence $\rightarrow$ Rayleigh scat. i.e.

II

Scatter from a surface

$$\underbrace{\frac{1}{P_0}\frac{dP}{d\Omega}}_{\text{scattering probability}} = \frac{4k^2}{\lambda^2}\underbrace{\left[\sigma^2 \tilde{C}(\nu_x,\nu_y)\right]}_{m^2/[m^{-1}m^{-1}]} \quad sr^{-1}$$

- measurable in lab for rough surfaces (BT, Baffles ....)

$$\left[\frac{1}{P_0}\frac{dP}{d\Omega}\right] = \frac{16\pi^2}{\lambda^2}\left[\underbrace{\frac{\sigma^2}{\lambda^2}\tilde{C}(\nu_x,\nu_y)}_{\tilde{S}_{2D}(\vec{k}):\ \frac{\text{waves}^2(@\lambda)}{m^{-1}m^{-1}}}\right] \Leftarrow \text{BRDF}(\Omega)$$

Predict

measure (i.e. PeThfinder)

difficult to measure for good surfaces [RT's being built]

easy to calculate

- Frequently measure 1D $S_1(K_x)$:

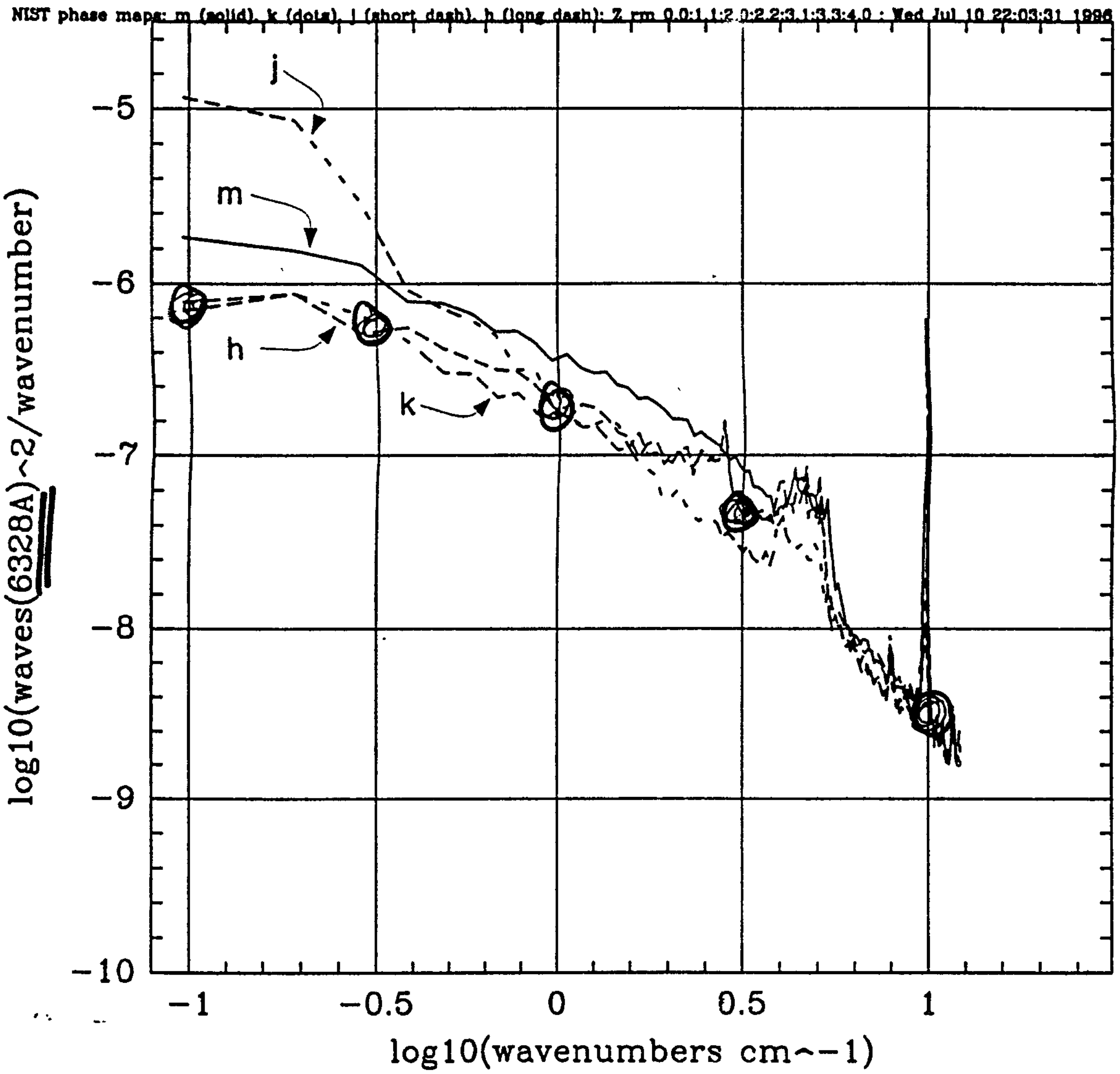
$$S_1(K_x) = 2\int S_2(k_x,k_y)\left[dK_y/2\pi\right]$$

- in general cannot invert to get S_2 (needed for BRDF)
  - ↳ measurement
    - ↳ too noisy
- simple analytical model (fit) to data

$$S_1 = \frac{A}{\left[1+[Bf_x]^2\right]^{c/2}} \qquad \text{fit } A, B, C$$

Then $$S_2 = \frac{A\,B\,\Gamma(\frac{c+1}{2})}{2\sqrt{\pi}\,\Gamma(c/2)} \cdot \frac{1}{\left(1+[Bf]^2\right)^{\left[\frac{c+1}{2}\right]}}$$

$\frac{1}{1\,cm} = 1\,cm^{-1}$ $\frac{1}{.01\,m} = 100\,m^{-1}$



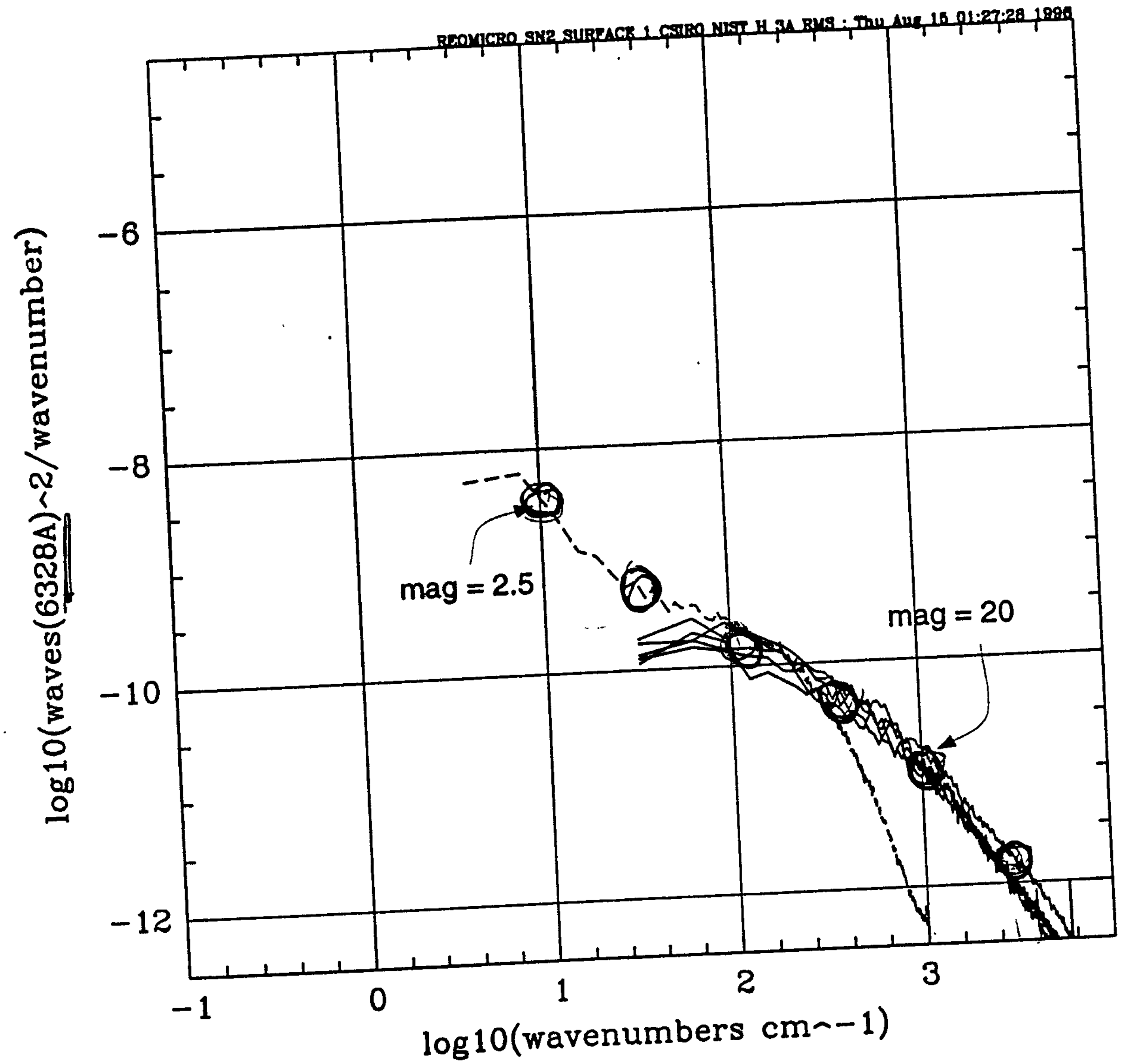
All spectra derived from phase maps with Z(0,0),Z(1,1),Z(2,0),Z(2,2),Z(3,1) Z(3,3),Z(4,0) removed

h = long dash = CSIRO surface 2 #2
j = short dash = HDOS serial04 side 2
k = dots = CSIRO surface 2 #6
m = solid = GO

VG 10 One d fft NIST phasemaps of flat surfaces

$S_1(K_x)$

Interferometer Phase Map Data



FILES:
211,212,213,214,215,216

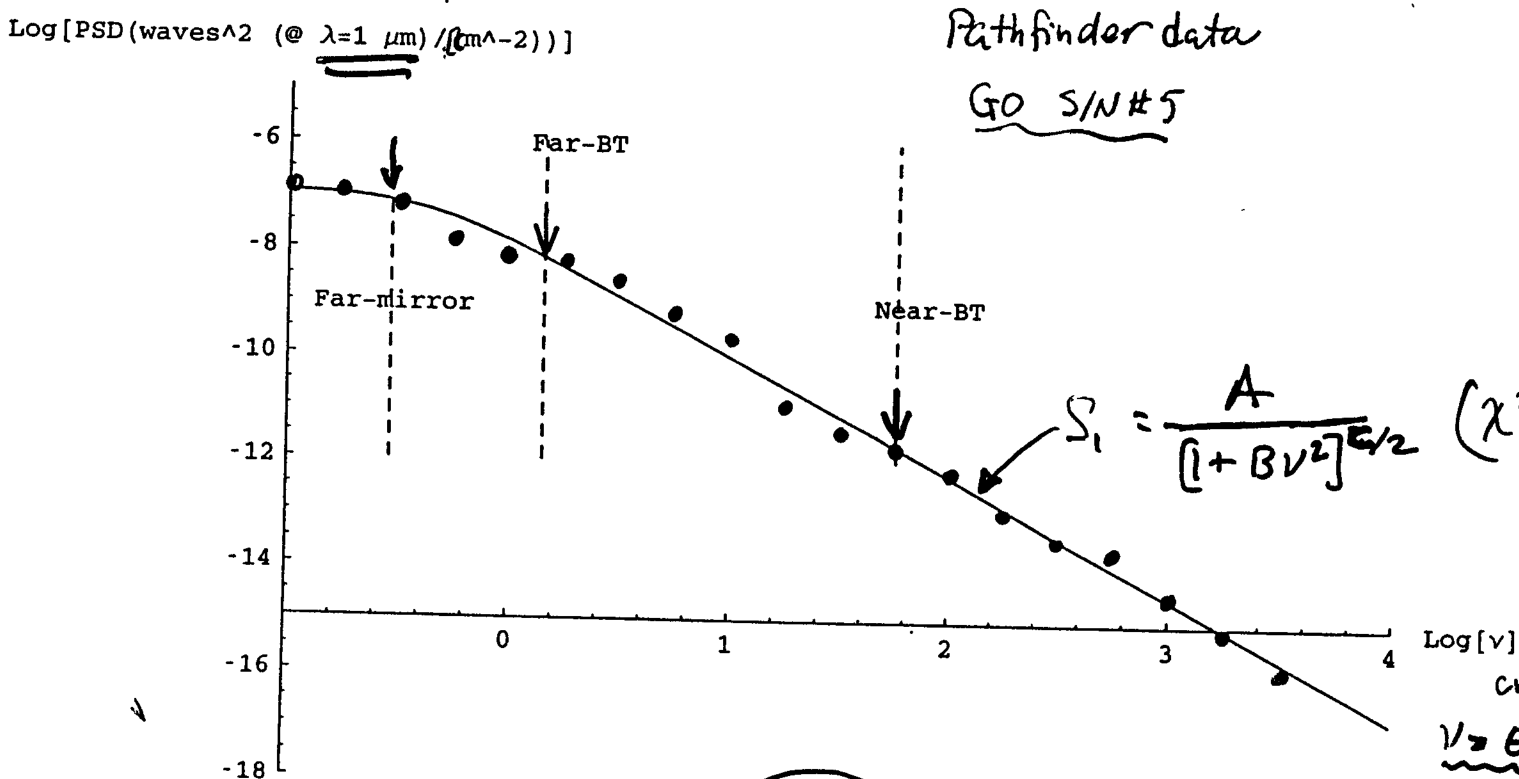
sn2 surface 2 CSIRO NIST H 3A rms

VG 16 Reo Micro

Profilometer Data

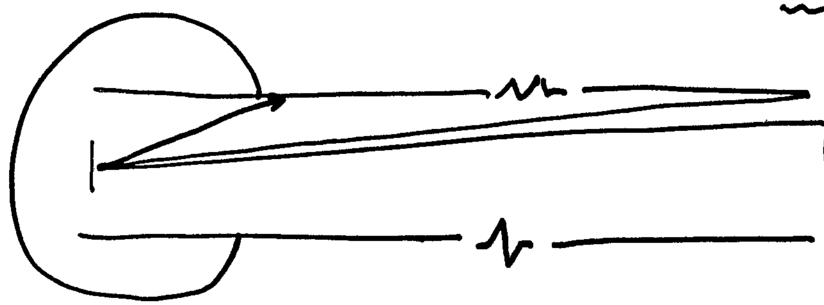
Pathfinder data

GO S/N #5

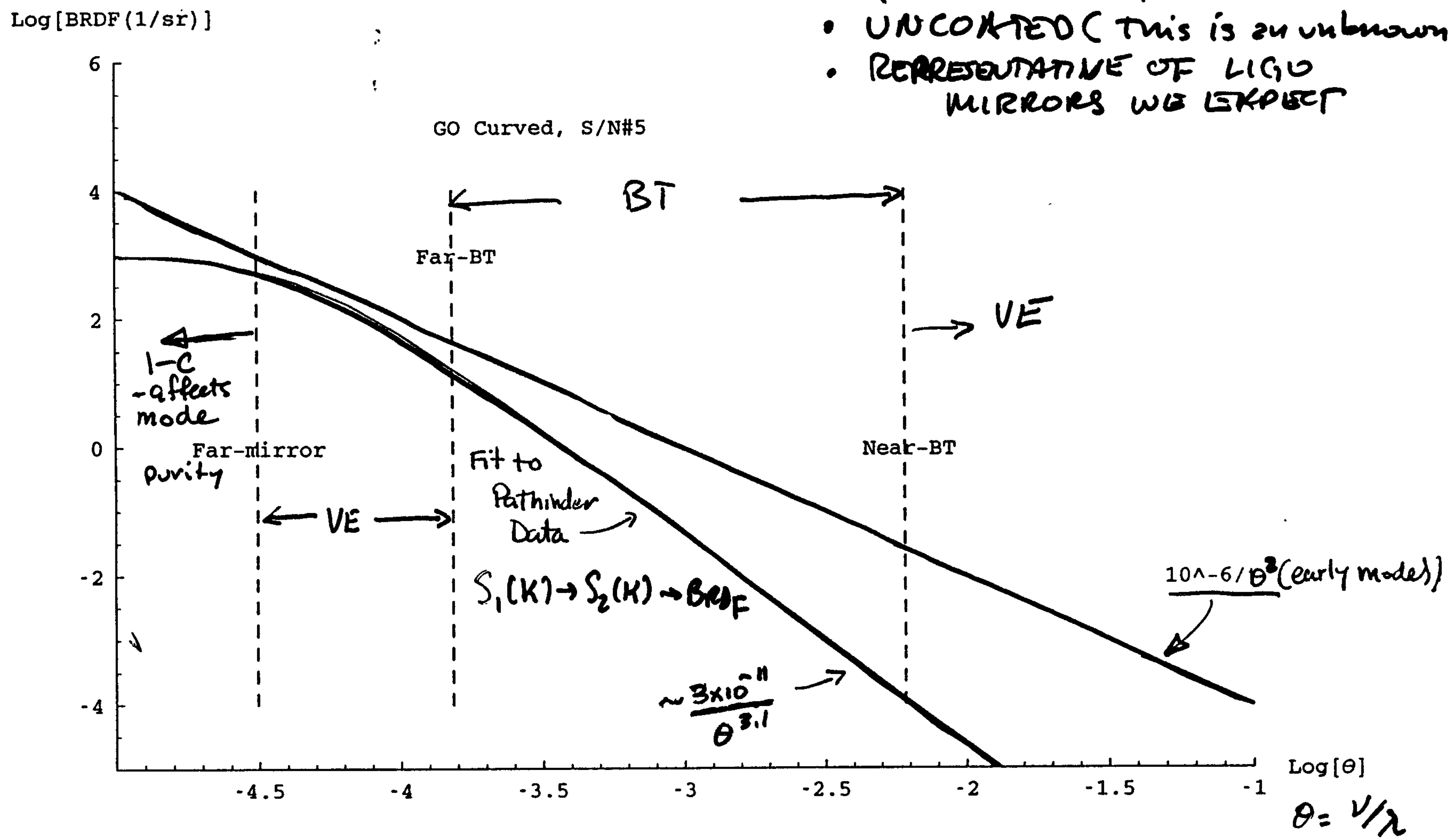


$$S_1 = \frac{A}{[1+B\nu^2]^{C/2}} \quad (\chi^2 \text{ fit})$$

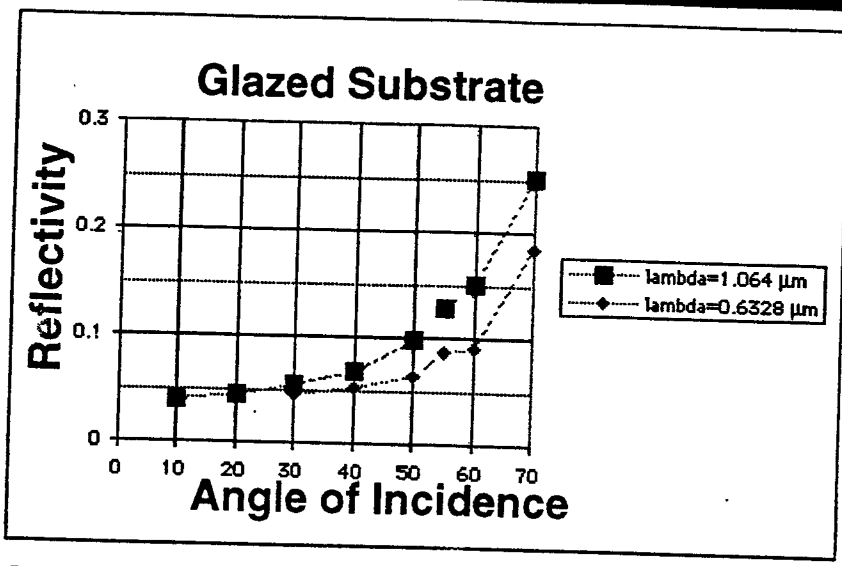
$\nu = \theta\lambda$

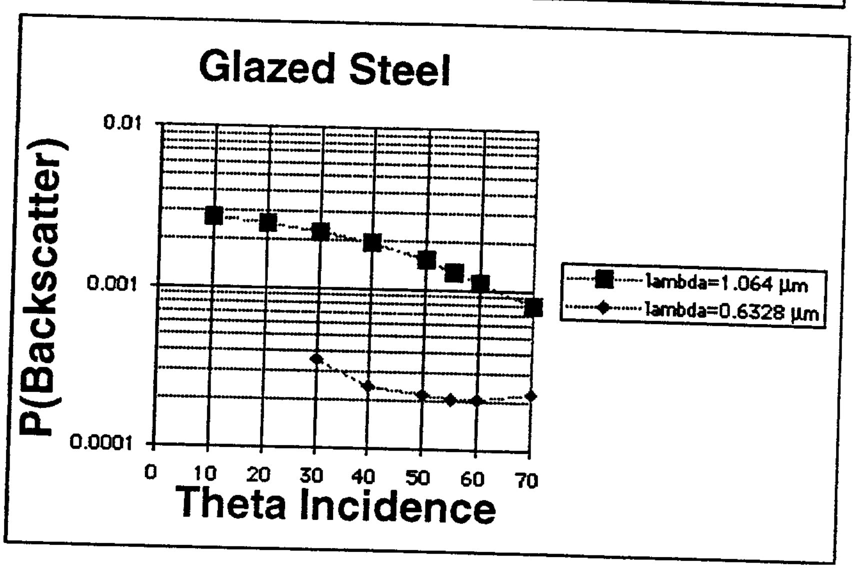


- GOOD QUALITY
- UNCOATED (This is an unknown !)
- REPRESENTATIVE OF LIGO MIRRORS WE EXPECT

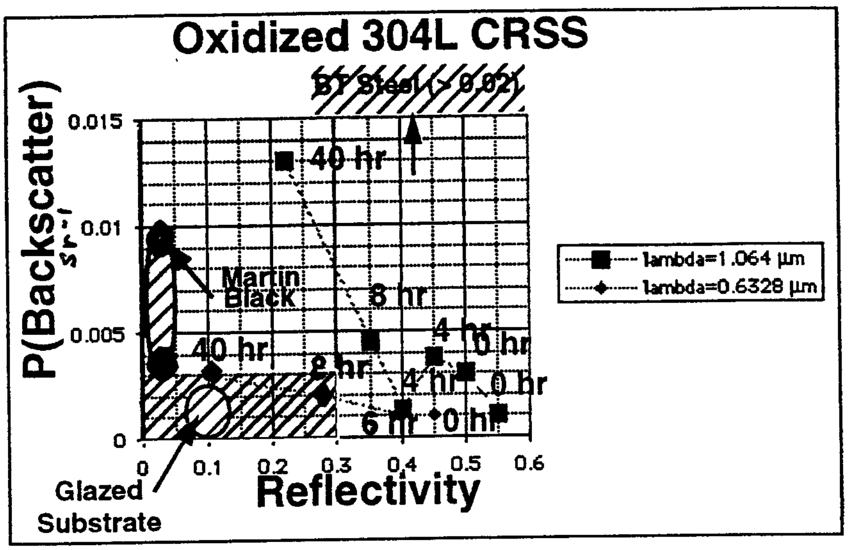


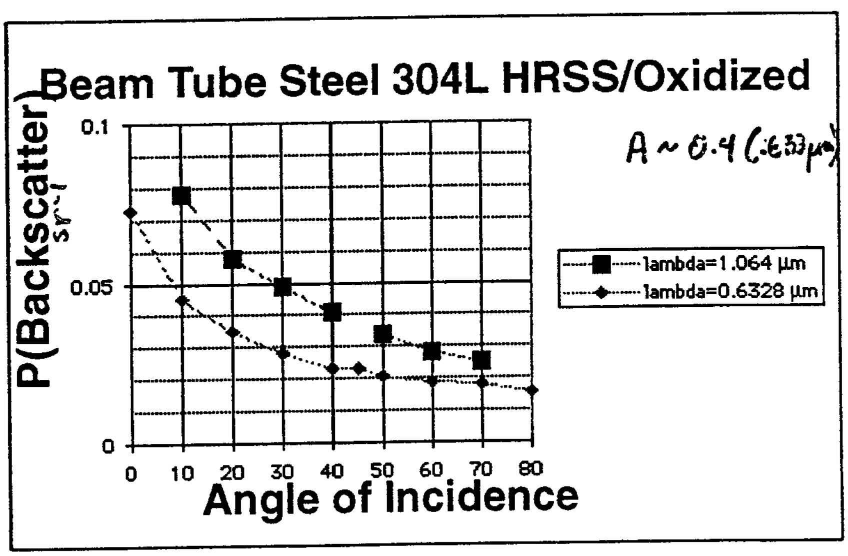
Beam Tube Baffle Materials



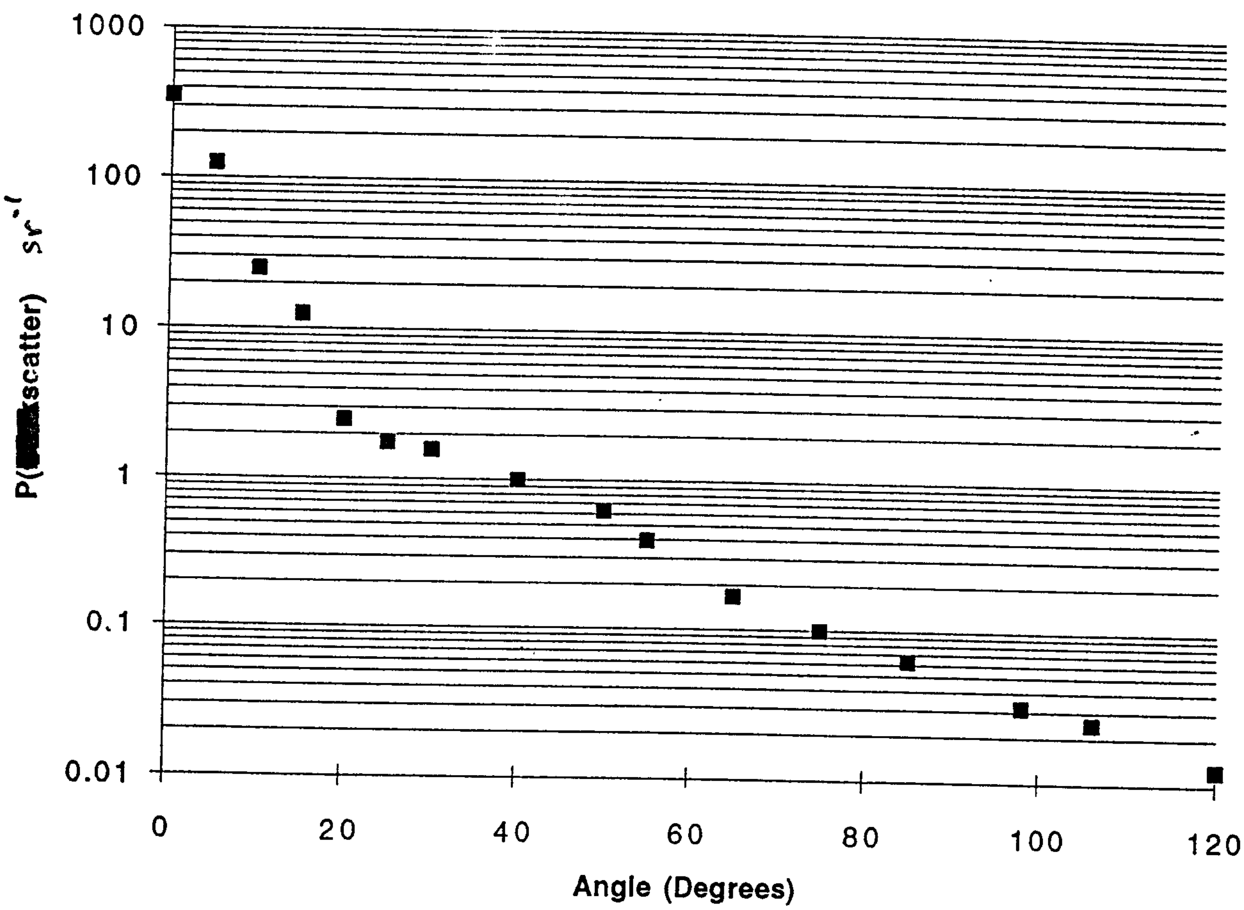


Beam Tube Baffle Materials

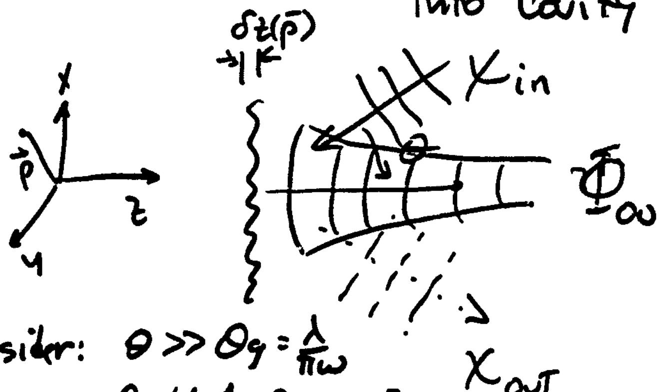




Beam Tube Steel Forward Scattering



III. Recombination of scattered light into Cavity mode on rough surface



χ_{in}, χ_{out} are scattered plane waves
- scatter into $\Psi_{00} \Rightarrow$ couples into cavity

Consider: $\theta \gg \theta_g = \frac{\lambda}{\pi w}$
$\theta \ll 1$ [forward]

- Coupling into mode proportional to $|\langle$mode overlap$\rangle|^2$

$\delta P = |\Gamma|^2$

$$|\chi_{in}\rangle_{z=0} = \sqrt{I_0}\, e^{i\bar k_i \cdot \bar r} \quad [W/m^2]^{1/2}$$

$$|\chi_{out}\rangle_{z=0} = |\chi_{in}\rangle e^{2ik\delta z(\bar\rho)} \quad [W/m^2]^{1/2}$$

$$= \sqrt{I_0}\, e^{ik_x x} e^{ik_y y} e^{ik2\delta z(\bar\rho)}$$

$$|\Phi_{00}\rangle_{z=0} = \underbrace{\sqrt{\frac{2}{\pi w^2}}\, e^{-\frac{(x^2+y^2)}{w^2}}}_{\phi_{00}} \quad [1/m^2]^{1/2}$$

$$\Gamma = \langle \Phi_{00} | \chi_{OUT} \rangle = \sqrt{I_0} \int dA\ \phi_{00}(x,y) e^{ik_x x} e^{ik_y y} e^{2ik\delta z(x,y)} \quad [W^{1/2}]$$

- expand $e^{2ik\delta z}$ as before ; statistical avg of Γ^2

$$\langle|\Gamma|^2\rangle = I_0 \int dA\,dA'\ \phi_{00}(x,y)\phi^*_{00}(x',y') e^{ik_x(x-x')} e^{ik_y(y-y')} \times$$

$$\times \{ 1 - 2k^2 [\delta z^2(x,y) + \delta z^2(x',y') - 2\delta z(x,y)\delta z(x',y') \} \quad [W]$$

$$\langle|\Gamma|^2\rangle = [I_0 2\pi w^2][1 - 4k^2\sigma^2] \, |\tilde\Phi_{00}(k_x,k_y)|^2 \xrightarrow{} 0 \text{ for } \{k_x,k_y\} \neq 0 \leftarrow \text{direct coupling}$$

$$+ I_0 4k^2\sigma^2 \int dA\,dA'\ \Phi_{00}(x,y)\Phi_{00}(x',y') e^{ik_x(x-x')} e^{ik_y(y-y')} \tilde C(x-x', y-y')$$

$$\tilde\Phi_{00}(k_x,k_y) = \sqrt{2\pi w^2}\, e^{-\frac{(k_x^2+k_y^2)}{4} w^2}$$

$k_x^2 + k_y^2 = k^2 \sin^2\theta \sim k^2\theta^2$

$$|\phi_{00}|^2 = 2\pi w^2\, e^{-\frac{k^2\theta^2 w^2}{2}}$$

$$\frac{w^2 k^2}{2} = \frac{2\pi^2 w^2}{\lambda^2} = \frac{2}{\theta_g^2}$$

$$= 2\pi w^2\, e^{-2\theta^2/\theta_g^2} \to 0 \text{ for } \theta \gg \theta_g$$

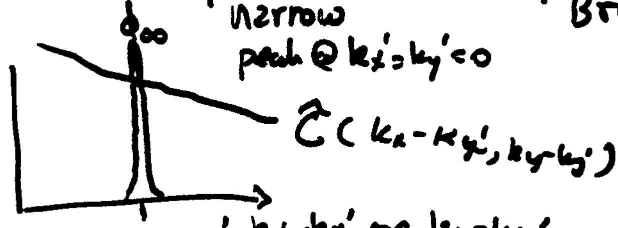
θ_g = Beam divergence

III Recombination at a surface

$$\langle|\Gamma|^2\rangle = I_0 4k^2\sigma^2 \int dA\,dA'\ \phi_{00}(x,y)\,\phi_{00}^*(x',y')\ C(x-x',y-y') \times e^{ik_x(x-x')}e^{ik_y(y-y')}$$

- let $\lambda = x-x'$; $\mu = y-y'$
- invoke convolution Thm & FFT

$$\langle|\Gamma|^2\rangle = I_0\ 4k^2\sigma^2 \int \frac{dk_x'\,dk_y'}{4\pi^2}\ \underset{\uparrow\ \text{narrow peak @ } k_x'=k_y'=0}{|\phi_{00}(k_x',k_y')|^2}\ \underset{\uparrow\ \text{Broad}}{\tilde{C}(k_x-k_x',k_y-k_y')}\quad [W]$$



$$\langle|\Gamma|^2\rangle = \underset{\substack{\downarrow \\ \text{irradiance on surface } (W/m^2)}}{I_0}\ 4k^2\sigma^2 \overbrace{\underbrace{\tilde{C}(k_x,k_y)}_{1/m^{-2}}}^{\text{surface roughness}} \cdot\ 1 \quad [W]$$

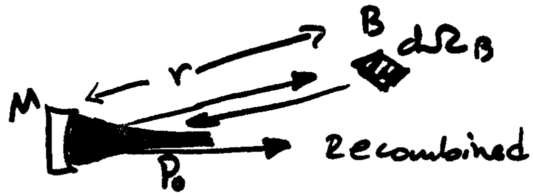
↳ recombined power

From before: $4k^2\sigma^2\,\tilde{C}(k_x,k_y) = \lambda^2\left[\underbrace{\frac{1}{P}\frac{dP}{d\Omega}}_{\text{BRDF}}\right]$

$$\boxed{\langle|\Gamma|^2\rangle = I_0\,\lambda^2\ \mathrm{BRDF}(\theta,\phi)}$$

IV Put it all TOGETHER

- Backscatter



δP at B: $\delta P_B = P_0 \cdot BRDF_M \cdot d\Omega_B$ Power incident @ patch $\delta\Omega_B$

Irradiance returned to mirror M:

$$\frac{\delta P}{\delta A} \text{ at } M = \frac{d\delta P_M}{dA} = \frac{1}{r^2}\left[\frac{dP_b^{scatt}}{d\Omega}\right] = \frac{1}{r^2}\,\delta P_B \cdot BRDF_B$$

$$\frac{d\delta P_M}{dA} = P_0 \frac{1}{r^2}\; BRDF_M \cdot BRDF_B \cdot d\Omega_B$$

Power recombined into beam TEM00:

$$\delta P_{00} = \left[\frac{d\,\delta P_M}{dA}\right] \cdot \lambda^2 \cdot BRDF_M \qquad [\text{Watts}]$$

$$\boxed{\frac{\delta P_{00}}{P_{00}} = d\Omega_B \frac{\lambda^2}{r^2}\left[BRDF_M\right]^2 BRDF_B}$$ ⟸ fractional power contribution per path $d\Omega$

- combining many paths → integration of contribution to $\tilde{h}(f)^2$ weighted by $\frac{\delta P}{P}$ ⟹ $\delta h^2 = \frac{\delta P}{P}\frac{1}{|d\phi/dh|^2}\,\delta\phi^2$
- back scatter is incoherent – many paths with differences of many wavelengths.
  - rough surfaces
  - different distances, angles
  - different times

$$\delta\tilde{h}(f)^2 = \frac{1}{\left(\frac{1}{N_B}\left|\frac{d\phi}{dh}\right|\right)^2}\,\frac{\lambda^2}{r^2}\left(BRDF_M\right)^2\left(BRDF_B\right)\tilde{S}^2_{\sin\phi}(f)\;\delta\Omega$$

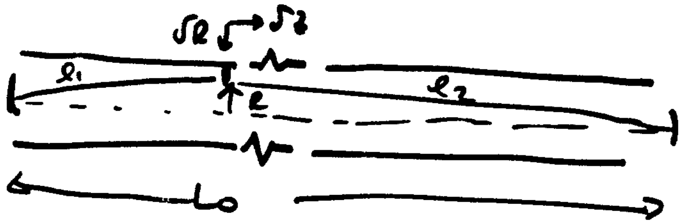
$$\delta\tilde{h}(f,\Omega)^2 \simeq \frac{1}{2}\left[\frac{\lambda^2}{4\pi r L}\right]^2 \left(BRDF_M\right)^2\left(BRDF_B\right)\left\{4k^2\,\xi^2_{BT_{11}}(f)\right\} d\Omega_B$$

$BRDF_M$: calculate from measurement; $BRDF_B$: measure; $\xi_{BT_{11}}$: measure

$$\boxed{h \sim BRDF_M \cdot \sqrt{BRDF_B} \cdot \xi_{11}}$$

V PUT IT ALL TOGETHER

- DIFFRACTION (FORWARD SCATTERING)



- diffraction must be dealt with as a <u>coherent</u> process – add amplitudes:

$$\tilde{h} = \Sigma \frac{\delta\psi}{\psi} \delta h$$

- Fresnel zones in mid tube ($L_0/2$ = 2 km) are macroscopic:

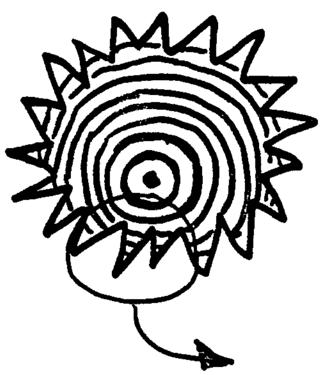
1st zone: (axial) $\lambda = \delta \ell(\delta z, \delta R) = \ell_1(z+\delta z, R+\delta R) + \ell_2(z-\delta z, R+\delta R) - [\ell_1(z,R)+\ell_2(z,R)]$

$$\Rightarrow \Delta z = \left[\frac{\lambda L_0^3}{8R^2}\right]^{1/2} = \underline{172\text{ m}} \quad 4\text{ km IFO (!)}$$
$$= \underline{61\text{ m}} \quad 2\text{ km IFO (!)}$$

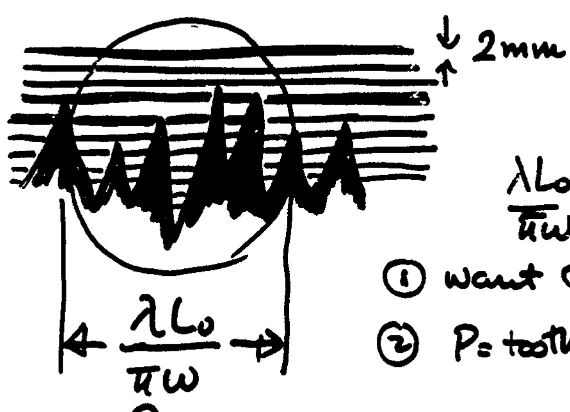
Groups of baffles may contribute coherently

(radial) $\Rightarrow \Delta R = \frac{\lambda L_0}{4R_0} =$ 2 mm 4 km IFO
1 mm 2 km IFO

- Want serrated baffles to reduce coherence effects.



- Serrations cut across fresnel zones



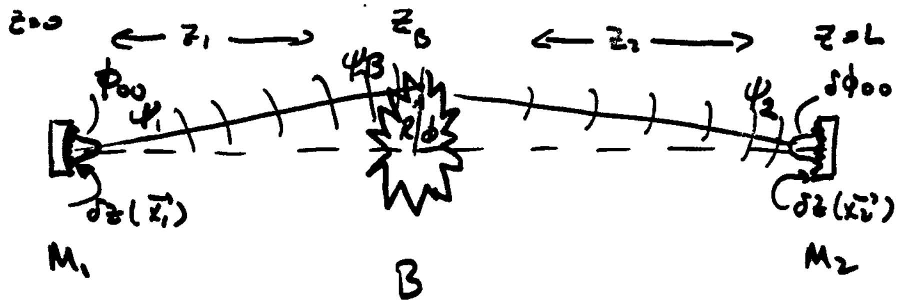
Resolution element of a LIGO optic

$\frac{\lambda L_0}{\pi w} \simeq 2$ cm @ 2 km

① want $\sigma_{PV} \gtrsim 2$ mm

② P = tooth period $\lesssim \frac{2\text{ cm}}{2} \sim 1$ cm

V ∘ DIFFRACTION – COHERENT CONTRIBUTION



① Surface errors $\delta z(x_1, y_1)$ on M_1 scatter light out of cavity mode Φ_{00} towards baffle B

$$\Psi_1(z=0) = \sqrt{P}\,\Phi_{00}(x_1, y_1)\, e^{2ik\,\delta z(x_1,y_1)}$$

② Propagation of ψ_1 from $z=0$ to $z=z_B$ results in incident wave at baffle, ψ_B:

Fresnel Kirchhoff scalar diffraction $\Rightarrow$

$$\psi_B = \frac{-ik}{2\pi}\,\frac{e^{ikz_1}}{z_1}\int dA_1\, \Psi_1(x_1,y_1)\, e^{i\frac{k}{2z_1}\left((x_1-x_B)^2+(y_1-y_B)^2\right)}$$

③ Baffle diffracts light towards M_2. Light incident on M_2 is obtained by another application of Fresnel-Kirchhoff integral:

$$\psi_2 = \frac{-ik}{2\pi}\,\frac{e^{ikz_2}}{z_2}\int dA_B\, \psi_B(x_B,y_B)\, e^{i\frac{k}{2z_2}\left[(x_2-x_B)^2+(y_2-y_B)^2\right]}$$

$\Rightarrow$ we are interested in fluctuating part due to baffle motion.

$$R(\phi,t) = R_0 + \xi(\phi,t)$$

$$\delta\psi_2 = \psi(R_0+\xi) - \psi(R_0) \Rightarrow \int \underbrace{r\,dr\,d\phi}_{\text{entire aperture}} \rightarrow R_0 \underbrace{\int \xi(\phi)\,d\phi}_{\text{(small) area which changes with time}}$$

$$\delta\psi_2 = \frac{-ik}{2\pi}\,\frac{e^{ikz_2}}{z_2}\int R_0\,\xi\, d\phi\, \psi_B(x_B,y_B)\, e^{ik/2z_2\left[(x_2-x_B)^2+(y_2-y_B)^2\right]}$$

④ Surface errors at M_2 rescatter light into cavity mode, $\delta\Phi_{00}$

$$\delta\Phi_{00} = \int dA_2\, \delta\psi_2(x_2,y_2)\, e^{2ik\,\delta z(x_2,y_2)}\, \Phi_{00}(x_2,y_2)$$

V. DIFFRACTION

$$\frac{\delta\phi_{00}}{\sqrt{I_0}} = \frac{k^4}{\pi^2}\frac{e^{ikL}}{z(L-z)}\left[\int R_0 \xi \, d\phi \cdot \left\{\int dA_1 \, \delta z(x_1,y_1)\,\phi_{00}(x_1,y_1)\, e^{i\frac{k}{2z}((x_1-x_B)^2+(y_1-y_B)^2)}\right\}\right.$$

$$\left. + \underbrace{\left\{\int dA_2 \, \delta z(x_2,y_2)\,\phi_{00}(x_2,y_2)\, e^{i\frac{k}{2z}((x_2-x_B)^2+(y_2-y_B)^2)}\right\}}_{\Gamma = \sqrt{BRDF(\theta)}\, e^{i\gamma_1(\vec{x}_B)}}\right]$$

$$\frac{\delta\phi_{00}}{\sqrt{I_0}} = \frac{k^4}{\pi^2}\frac{e^{ikL}}{z(L-z)} BRDF(\theta) \int \underbrace{R_0 \xi \, e^{i\gamma_1(\vec{x}_B)} e^{i\gamma_2(\vec{x}_B)}}_{\text{Contains details:}} d\phi$$

Contains details:
- mirror surfaces
- baffle edges

$$h(f) = \frac{1}{2kL}\sum_{\text{mirrors}}^{m}\sum_{\text{baffles}}^{n}\frac{\delta\phi_{00}(m,n)}{\sqrt{I_0}}$$

$$\boxed{h(f) = \frac{1}{2kL}\frac{k^4}{\pi^2}\sum_{\text{mirrors}}^{m}\sum_{\text{baffles}}^{n}\left\{\frac{BRDF(\theta_n)}{z_n(L-z_n)}\int R\,\xi_n\, e^{i\gamma_{m_1}(\vec{R}_n)} e^{i\gamma_{m_2}(\vec{R}_n)}\right\}}$$

$$h(f) \sim \underset{\substack{\uparrow \\ \text{Mirror} \\ \text{Roughness}}}{BRDF_M} \cdot \underset{\substack{\uparrow \\ \text{baffle (radial)} \\ \text{motion}}}{\xi_{\perp}^{\text{baffle}}}$$

- Independent of baffle optical (backscatter, reflective) properties

VI. What does LIGO Need?

Considerations:

- baffles cannot (easily) be replaced
  $\Rightarrow$ need to plan for advanced instruments

- Limiting noise sources will be:

(1) - gravity gradients (Rayleigh waves, atmosphere)
$f < 10\,Hz$

(2) - Internal thermal noise

- $h \sim \sqrt{\frac{8kT}{m(2\pi f L)^2}} \sim 4\times10^{-24}\left[\frac{10}{f}\right]\ 1/\sqrt{Hz}$
  $\underline{m = 1000\,kg}$

- m = 1000 kg deemed likely to be heaviest T.M. conceivable - ...
- $f \lesssim 200\,Hz$

(3) - Residual gas noise $\sim 2\times10^{-25}\ 1/\sqrt{Hz}$
$(10^{-9}$ torr, $H_2)$ $\quad f \lesssim 10^4\,Hz$

$\therefore$ Design baffles to be contributing <u>less</u> than (2) in mid-range

$\therefore$ want 10x safety margin $\rightarrow$ calculational uncertainties

Above is for stationary noise $\Rightarrow$ Baffles cannot contribute significantly to non-Gaussian noise (< 1/hr singles rate)

$\Rightarrow$ Baffles must <u>not shed</u> particles

VII · What (do we estimate) we have (today):

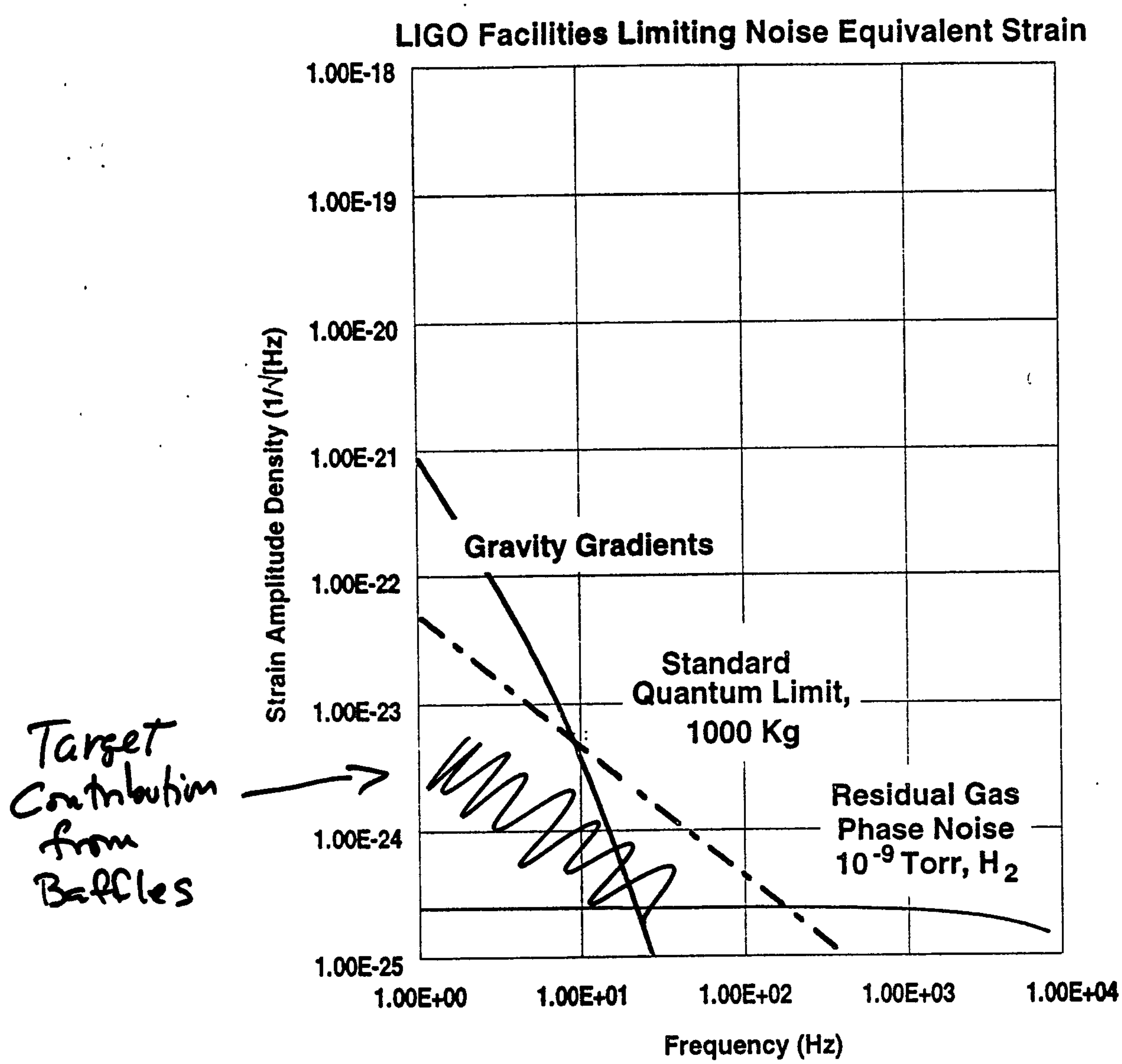
- Baseline goal (1995)
  - glazed material (assumed non shedding) $\sim 10^{-3}/sr$
  - L.S. spectrum for seismic, amplified up to 30x by FEA T(f) prediction for ground→BT motions.
  - mirrors no better than AXAF test flat
    $BRDF \sim 10^{-6}/\theta^2$

- Revised estimate (1997)
  - oxidized 304 SS (Bright Anneal) baffles (non shedding) $\sim 4.8 \times 10^{-3}$
  - Measured BT wall motion for partially completed X arm @ Hanford
    - Initial analysis missed biggest excitation mechanism: acoustics (wind)
      - motions up to 10x greater than 1995 model can occur at high f ($\gtrsim$ 200 Hz)
      - lower f motion consistent with expected values
  - Deduced BRDF for LIGO mirrors from Pathfinder experimental results
    $BRDF \sim 10^{-11}/\theta^{3.1}$
    - ~ 20X less scatter than 1995 values

- Net change is ± neutral:
  $\sqrt{BRDF_B}$ ↑ 2-3
  X_{BT} ↑ 10X
  $BRDF_M$ ↓ 20X

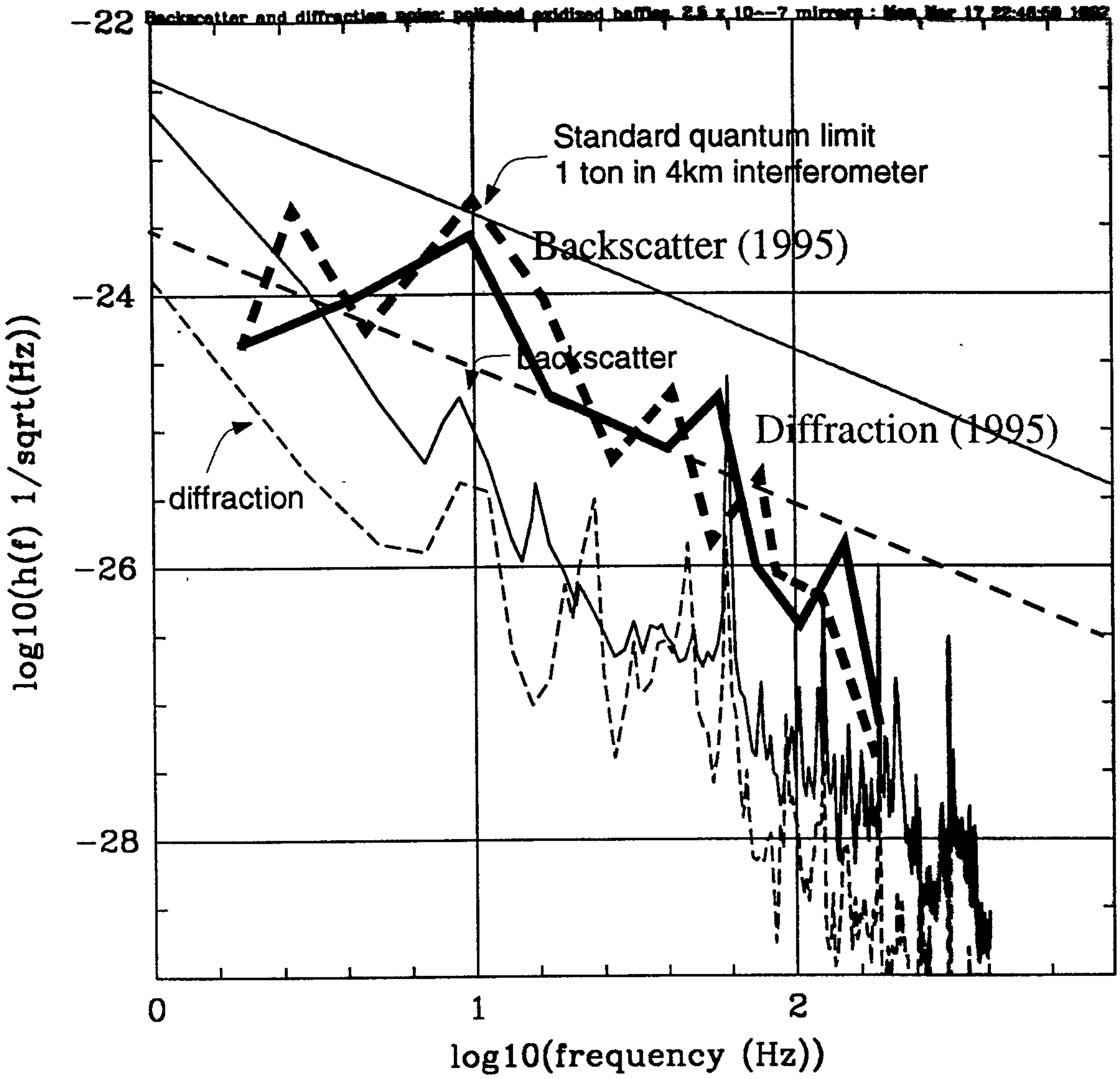
goal for the facilities is to not increase the naturally occurring environmental perturbations, such as mechanical vibrations, acoustic noise, electromagnetic fields and gravitational gradients, by more than a factor of two in the gravitational wave detection band.

Figure 3-4: Limiting interferometer performance attributable to the facilities



The ultimate LIGO vacuum levels are derived from the need to maintain optical phase noise due to fluctuations in the residual gas column density in the beam tubes and vacuum chambers at a level at or below an equivalent strain noise of 2×10^{-25} $Hz^{-1/2}$. This is expected to be the limiting noise source at the highest frequencies (100 Hz < f < 1 kHz).

The clear aperture of the beam tubes and vacuum chambers is in part determined by the requirement to maintain optical phase noise produced by scattered light to an acceptable level. The stray light requirement must be satisfied between 0.5 to 1.1 micron wavelengths, which is the range of



Baffle backscatter BRDF = 4.8 x 10^{-3} sr^{-1} Polished oxidized baffles

Mirror scattering BRDF = $\frac{2.5\times10^{-7}}{\theta^2}$ Super polished GO mirrors

Longitudinal spectrum 1/30/97 used for backscatter modulation
Horizontal spectrum 1/30/97 used for diffraction modulation