Robust detection of GW chirps

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Outline

1. *GW chirps, an advanced detector perspective*
2. *Methodologies for chirp detection*
   - Viewpoint 1: Time-frequency heuristics
   - Viewpoint 2: Detection theory and optimality criterion
3. *Conciliate both viewpoints*
   - Chirplet chains, Phys. Rev. D73, 042003, 2006

GW chirps, an advanced detector perspective

- *(1st?) detection* plausible with "verification" sources
  go beyond: look for unexpected event, **explore!**
- consequence: our analysis pipelines should include algorithms which do not rely on precise model (like burst searches today)

GW chirps, an advanced detector perspective

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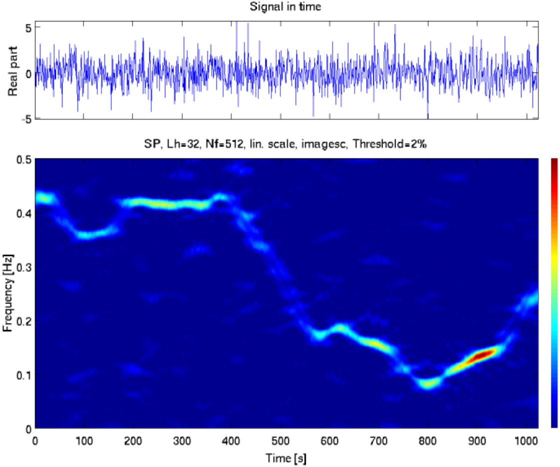
- GW = system "radiates away its asymmetries"
  if *orbiting* $\rightarrow$ quasi-periodic waves
  GW chirps: $s(t) \equiv A \cos(\phi(t) + \varphi_0)$, $\{A$, φ_0 unknown$\}$
  typical duration $T \sim$ few sec in detector band

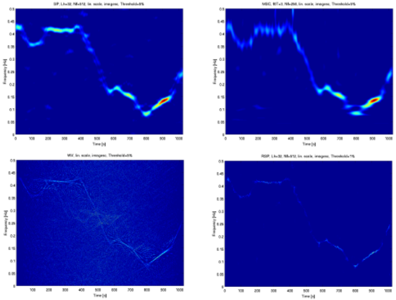
  $\phi(t)$ *is partially/totally unknown*
  restrict to realistic: impose $|\dot{f}(t)| \leq F'$ and $|\ddot{f}(t)| \leq F''$

Viewpoint 1: Chirps in the time-frequency plane (1)

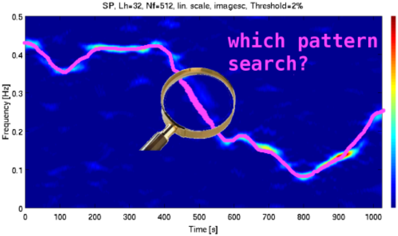


heuristic: chirp = "filiform" pattern in time-frequency plane

Viewpoint 1: Two degrees of freedom (2)



which TF representation?
spectrogram, wavelets, Wigner-Ville, Cohen, reassignment, etc.



which pattern search?
Hough, “crazy climbers”, “snakes”, road tracker in satellite images, etc.

Viewpoint 1: Multiple approaches. . . (3)

- Morvidone & Torrésani, *IJWMIP*, 2003
- Anderson & Balasubramanian, *Phys. Rev. D*, `grqc/9905023`
- Carmona, Hwang & Torrésani, *IEEE SP*, 1998
- Chassande-Mottin & Flandrin, *ACHA*, 1998
- Pinto et al., *Proc. of GWDAW*, 1997
- Innocent & Torrésani, *ACHA*, 1997

Viewpoint 2: Detection theory and chirps (1)

recall, chirp: $s(t) = A\cos(\phi(t) + \varphi_0)$ in white Gaussian noise
sampling : $s_k = s(kt_s)$ for $k = 0 \ldots N-1$

likelihood ratio: $\log(\lambda; A, \varphi_0, \phi) = \log \frac{\mathbb{P}(x_k|H_1)}{\mathbb{P}(x_k|H_0)}$

replace A and φ_0 by their max. likelihood estimates

$$\ell(x,\phi) = \log(\lambda_{\max}) \propto \left| \sum_{k=0}^{N-1} x_k \exp i\phi_k \right|^2 \lessgtr \eta$$

quadrature matched filtering

Viewpoint 2: When chirp phase is not known... (2)

- if ϕ is not known, proceed with same scheme: find phase which maximizes $\ell(x, \phi)$
- analytical maximization impossible $\rightarrow$ numerical bank of matched filters, *ex:* inspirals

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- griding must be sufficiently tight, how to be sure?
  we "receive" $x_k \hat{=} A \cos(\phi_k + \varphi_0)$ and we "search" with template ϕ_k^*

$$\text{distance:} \quad \Delta\ell(\phi, \phi^*) \equiv \frac{\ell(s, \phi) - \ell(s, \phi^*)}{\ell(s, \phi)}$$

the distance between two grid nodes should be small

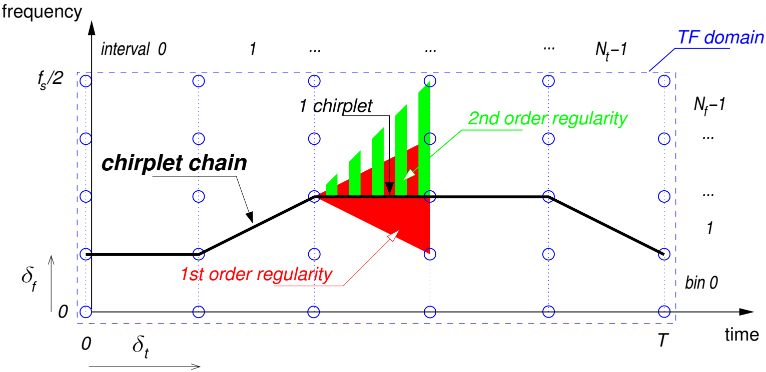
Conciliate viewpoints?

Does this method apply in general?

1. can we build a bank of matched filters for GW chirps?
2. with which templates?

Chirplet chains (CC), Phys. Rev. D73, 042003

CCs are piecewise linear chirps



free parameters: N_t, N_f, N'_r, N'_r

CCs form a tight template grid

if N_r' and N_r'' are large enough, for all smooth chirp ϕ, there exists a CC ϕ^* such that

$$\Delta\ell(\phi,\phi^*) \lesssim C\left[\frac{1}{2}\left(\frac{\sqrt{3F''T^3}}{N_t}\right)^2+\frac{1}{2}\left(\frac{2N}{N_f}\right)\right]^2$$

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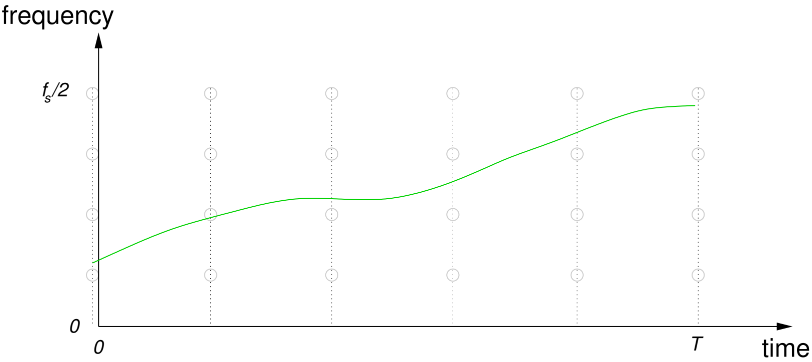

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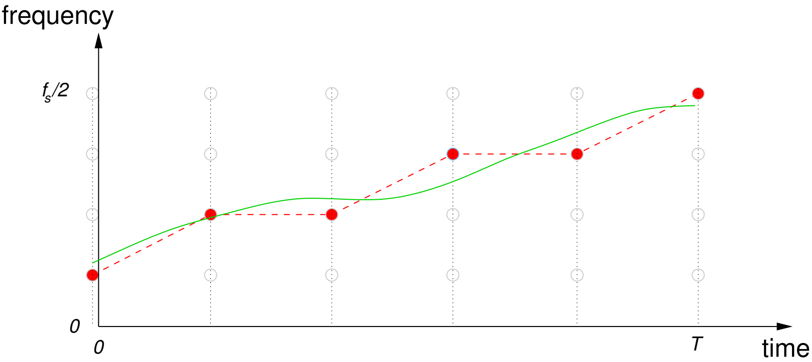

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CC grid is tight!

...but too large to be searched exhaustively

we want $\max_{\text{all CCs}}\{\ell\}$

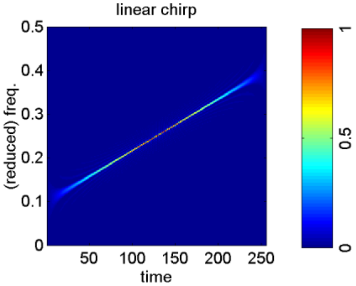
(combinat. count: number of CCs is exponential. growing with N_t)

best CC: near optimal search

maps to time-frequency: discrete Wigner-Ville

$$\text{Moyal:} \quad \ell = \frac{1}{2N} \sum_n \sum_m W_x(n,m) W_e(n,m)$$

approximation: W_e is almost Dirac $\approx \delta(m - m_n^{(cc)})$



path integral:
$\ell \approx \sum_n W_x(n, m_n^{(cc)})$

$\max\{\ell\}$ = **longest path** prob.

dynamic programming solves this in polynomial time

which TFR ? DWV, which pattern search? largest path int. + dyn. prog.

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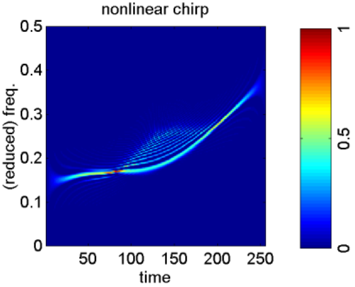

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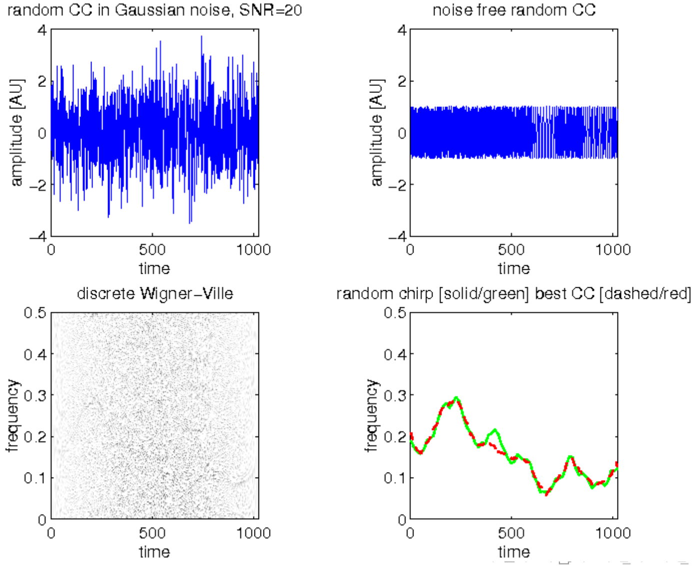
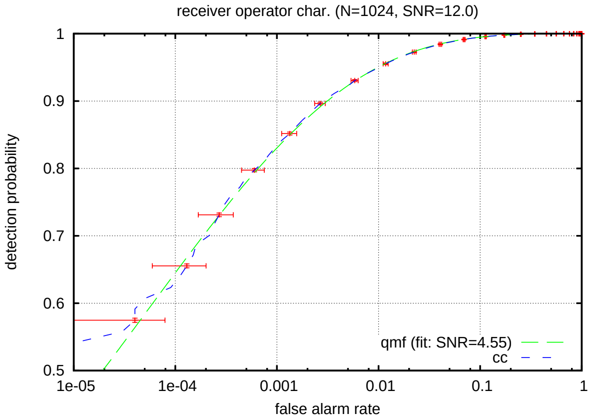
best CC: check



best CC: performance, ROCs

ROC: detection prob. *vs* false alarm



"clairvoyant" observer knows incident chirp *a priori*

the SNR of "clairvoyant" observer is set such that ROC fits the other.

reduction factor in the sight distance wrt "clairvoyant" ≈ 2.6

Future plans: extension to multiple GW antennas

extend CC search to quasi-coherent analysis of GW network data

open a post-doctoral position at APC (Paris) with support of VESF

APC is a new institute in Paris devoted to astroparticle physics including GWs

very soon, announcement on `http://www.apc.univ-paris7.fr`

to know more/apply, contact: `ecm@obs-nice.fr`