

Some Ideas on Coatingless all-reflective ITF

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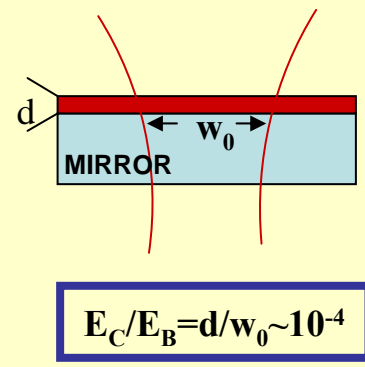
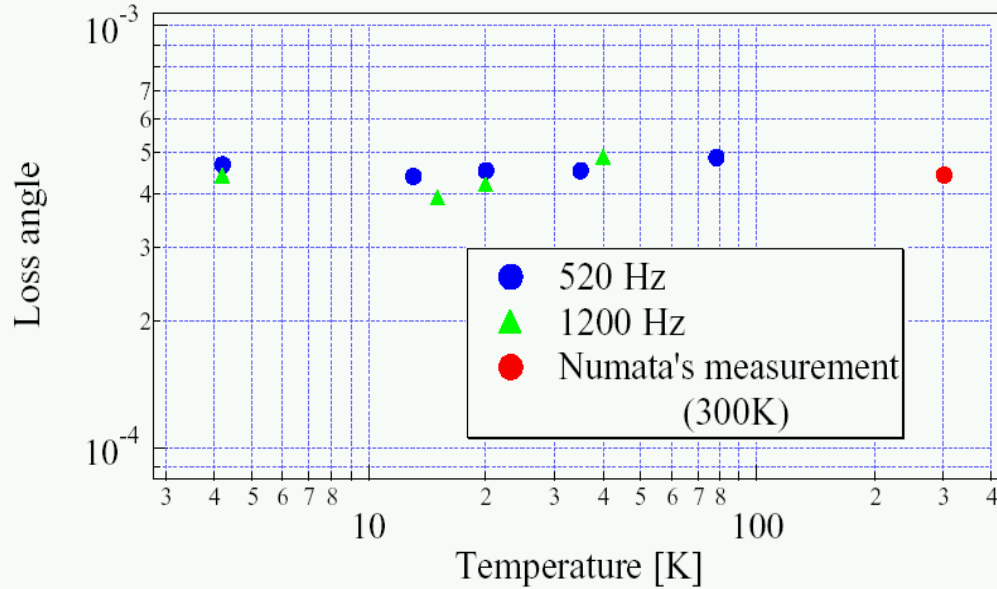
INFN- Pisa

(*) Work done in collaboration with G. Cella

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Why Total Reflecting Mirrors?

Experimental data from Yamamoto and Numata show a very high coating loss angle even at low temperature.



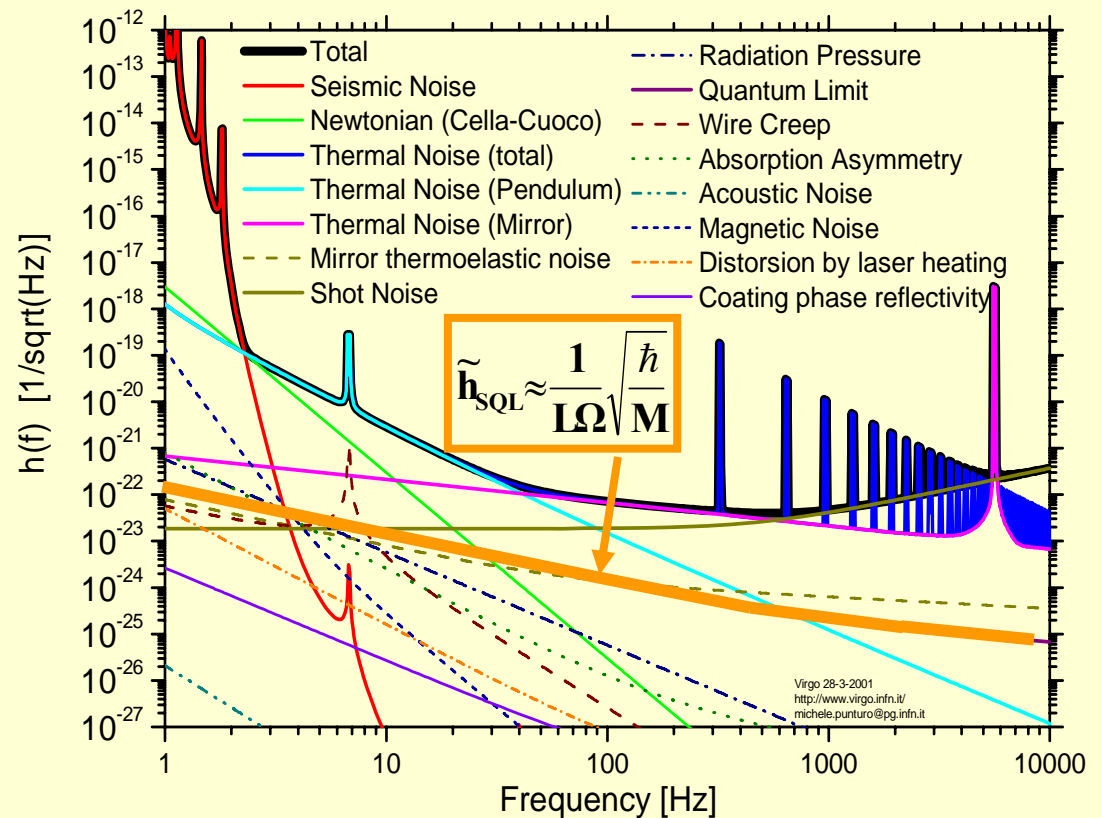
$$\tilde{\chi}_{Total}(\omega) = \frac{\sqrt{16k_B T (\phi_{Bulk} + \frac{E_C}{E_B} \phi_{Coating})}}{\omega^{1/2} E^{1/2} w_0^{1/2}}$$

$$\phi_{Total} = \phi_{Bulk} + \frac{E_C}{E_B} \phi_{Coating} \sim \frac{E_C}{E_B} \phi_{Coating} \sim 410^{-8}$$

$$Q = \frac{1}{\phi_{Total}} \sim 2.510^7$$

The Virgo design Thermal noise curve is evaluated with a $Q \sim 10^6$, and the maximum expected Q , due to coating losses, is $Q \sim 2.5 \cdot 10^7$. Standard Quantum Limit is about a factor 100 below Virgo TN i.e. equivalent to a $Q = 10^{10}$; consequently, for reaching SQL sensitivity Q should improve by $10^{10} / 2.5 \cdot 10^7 \sim 400$ - **For this reason it is interesting to explore Coating-less Mirrors**

This argument will become even more important if we want to go below SQL.



Some History

Toraldo di Francia in 1965 proposes flat Roof Prism for creating a stable Radio Frequency cavity. Stability was successfully experimentally tested.

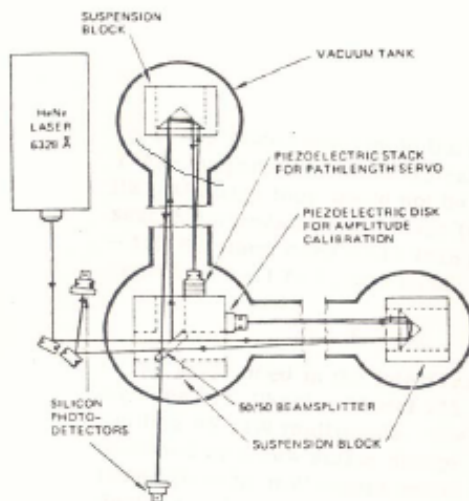
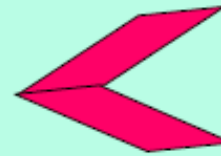
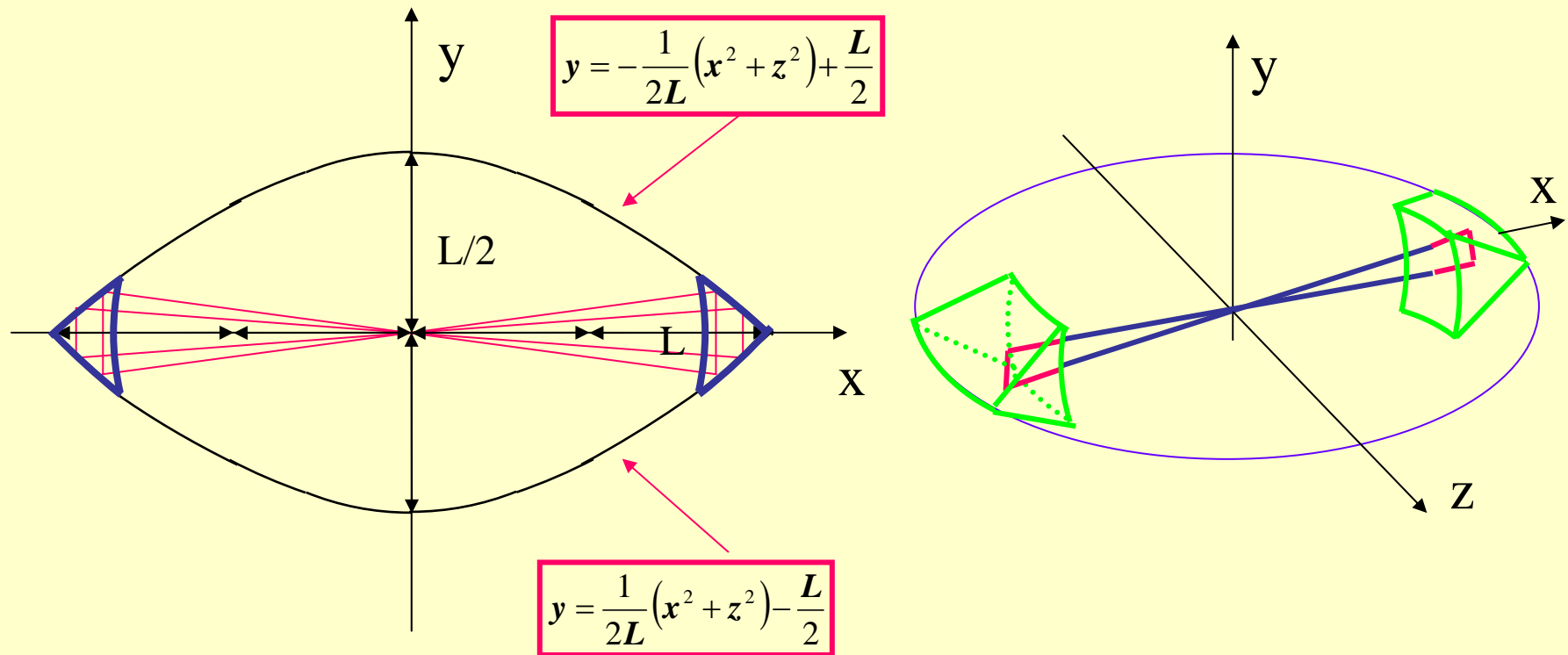


FIG. 7. Schematic of folded optical path.

In 1970 Robert Forward (Hughes Lab.) build the first Interferometer for GW detection. It was equipped with **Roof Prisms Mirrors. Arm length ~2m**

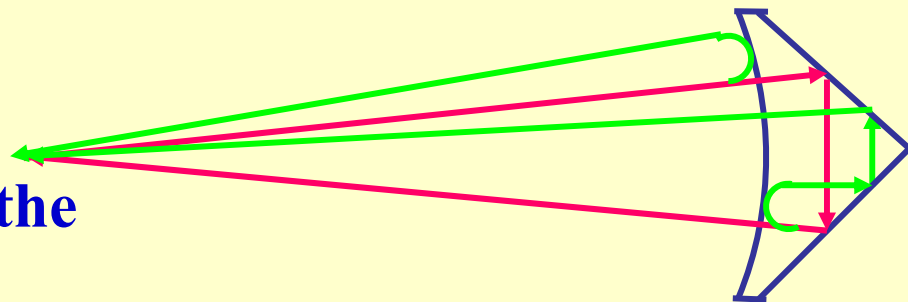
Braginsky et al. Recently Published a paper on the use of Roof Prism and Corner Cube mirrors in Fabry Perot cavities using Antireflective Coatings .

Rotation Paraboloids as exact reflectors for closed geometrical optical trajectories

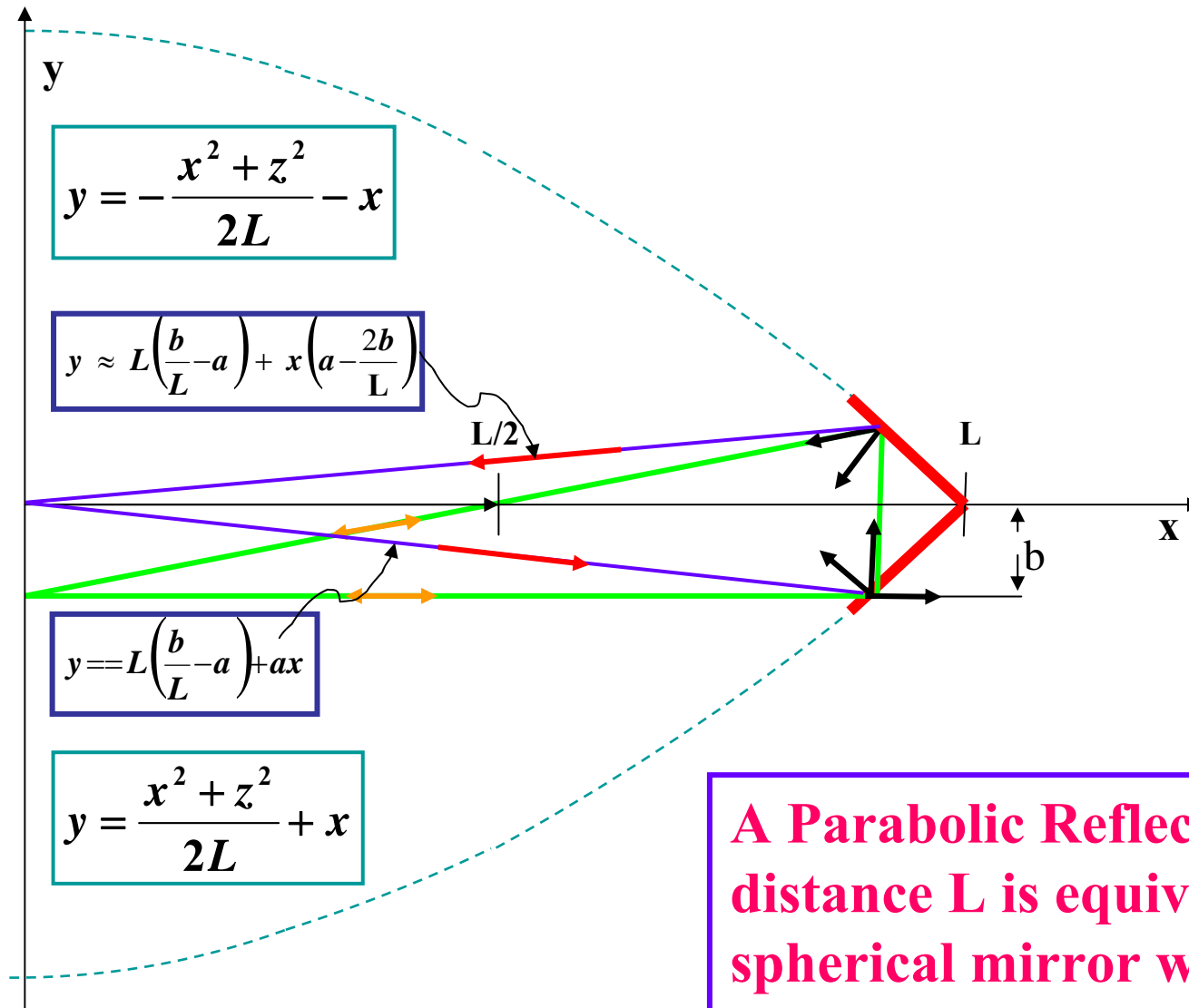


No Beam Losses

The spherical mirror surface is matched to the constant phase beam surface curvature

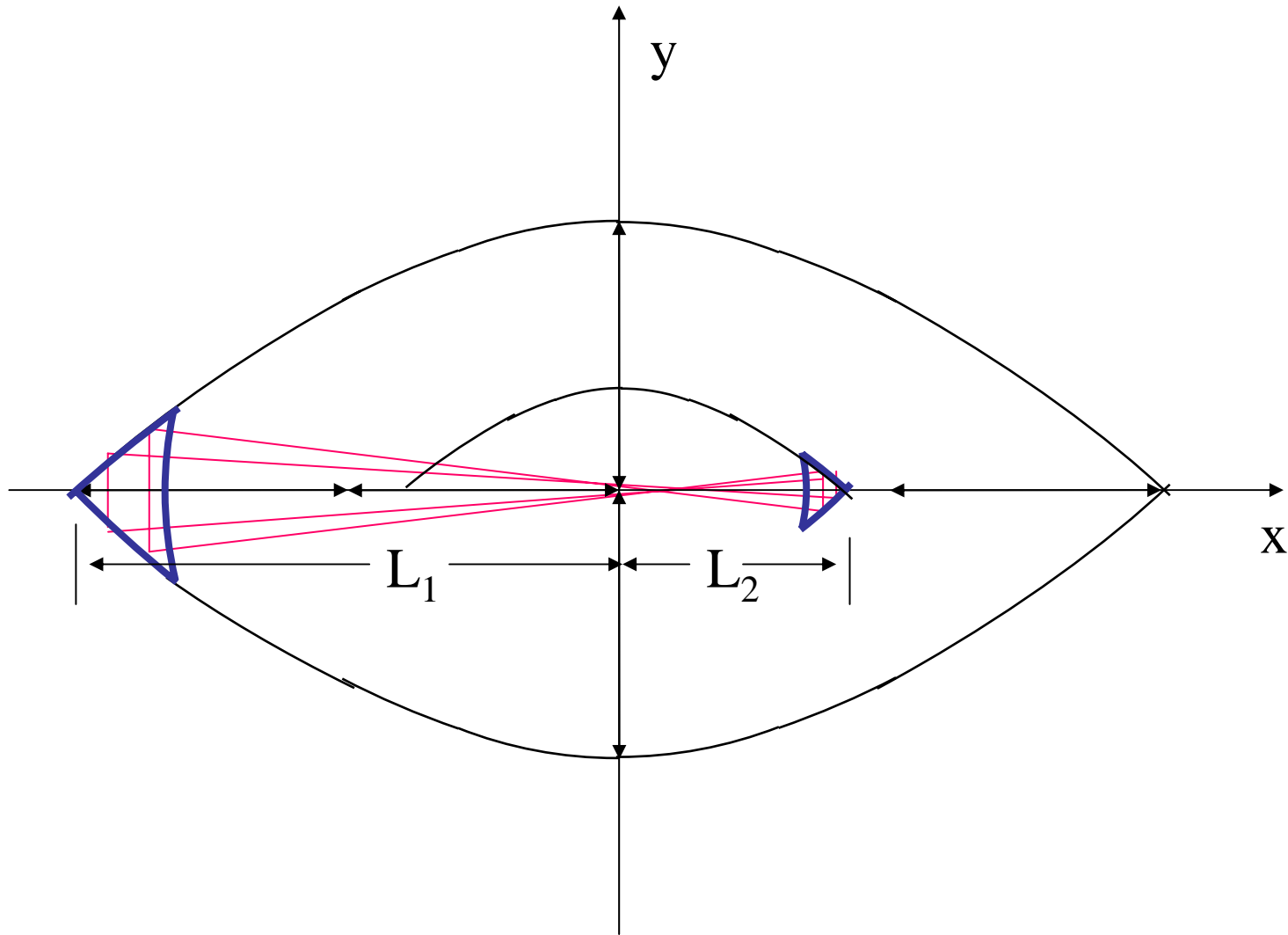


Rotation Parabolas as reflectors for closed geometrical optics trajectories



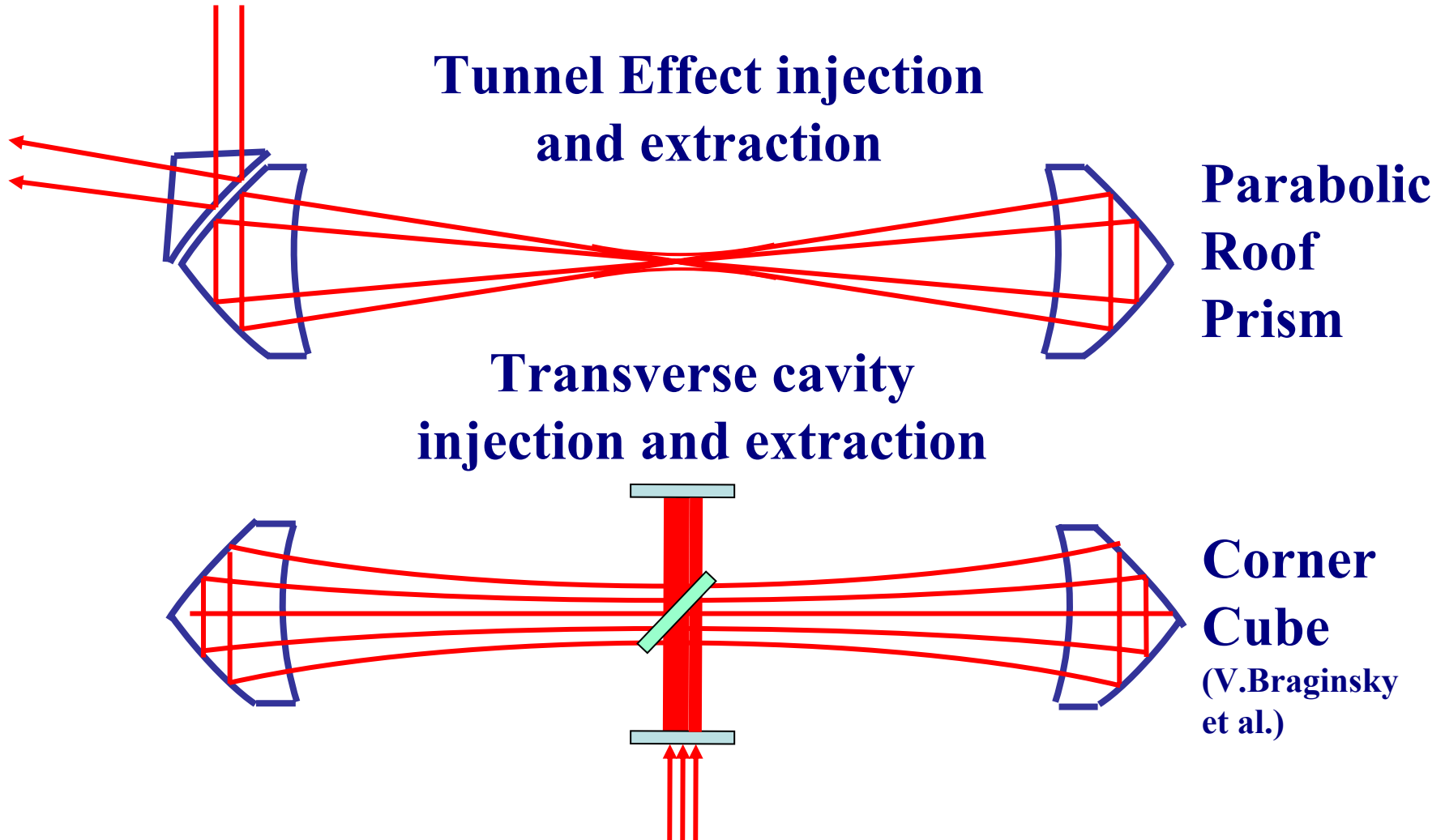
A Parabolic Reflector at distance L is equivalent to a spherical mirror with curvature radius $L/2$

Reflective cavity with Asymmetric Arms



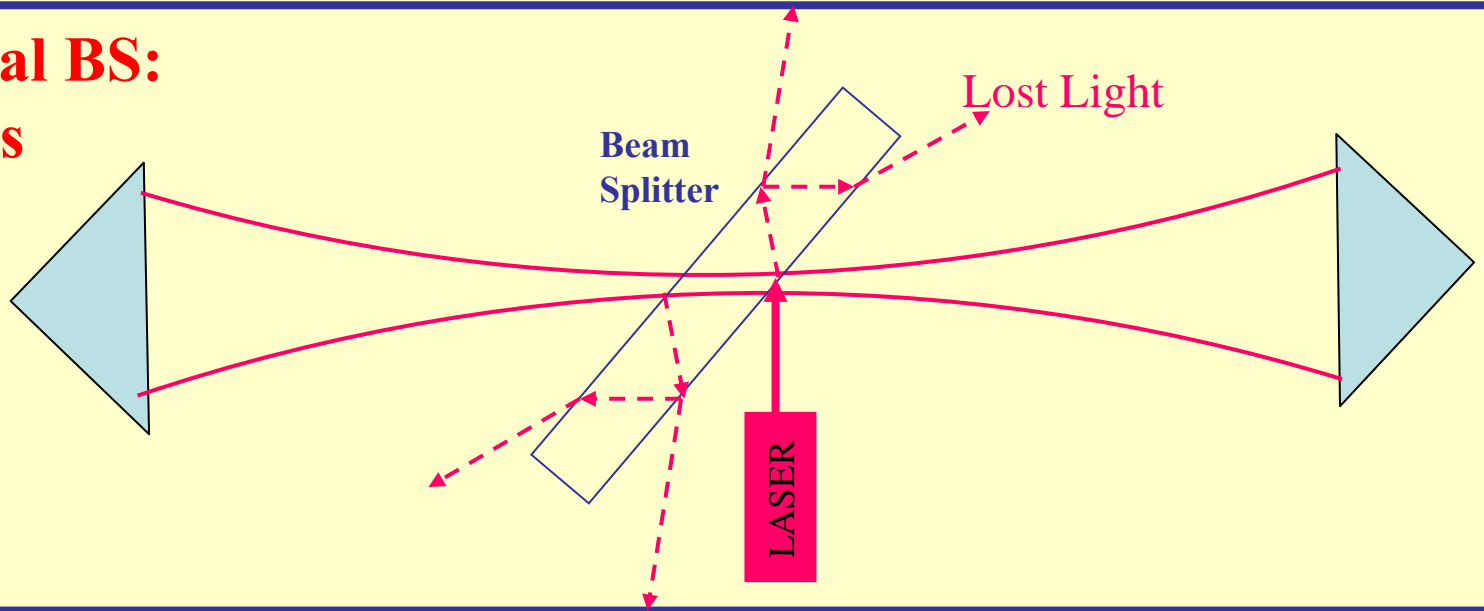
Cavity Beam Injection

One of the main problems we faced was to find the way to inject a beam in the All Reflective Cavity.

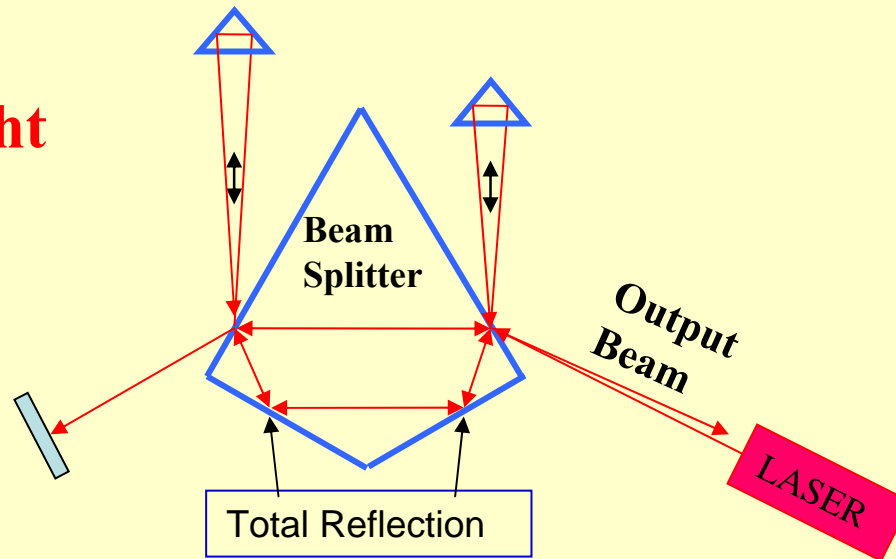


Example of Power Injection in a Parabolic Coatingless all-Reflective Cavity

**Conventional BS:
Light Losses**



**Closed trajectories
inside BS : No Light
Losses**

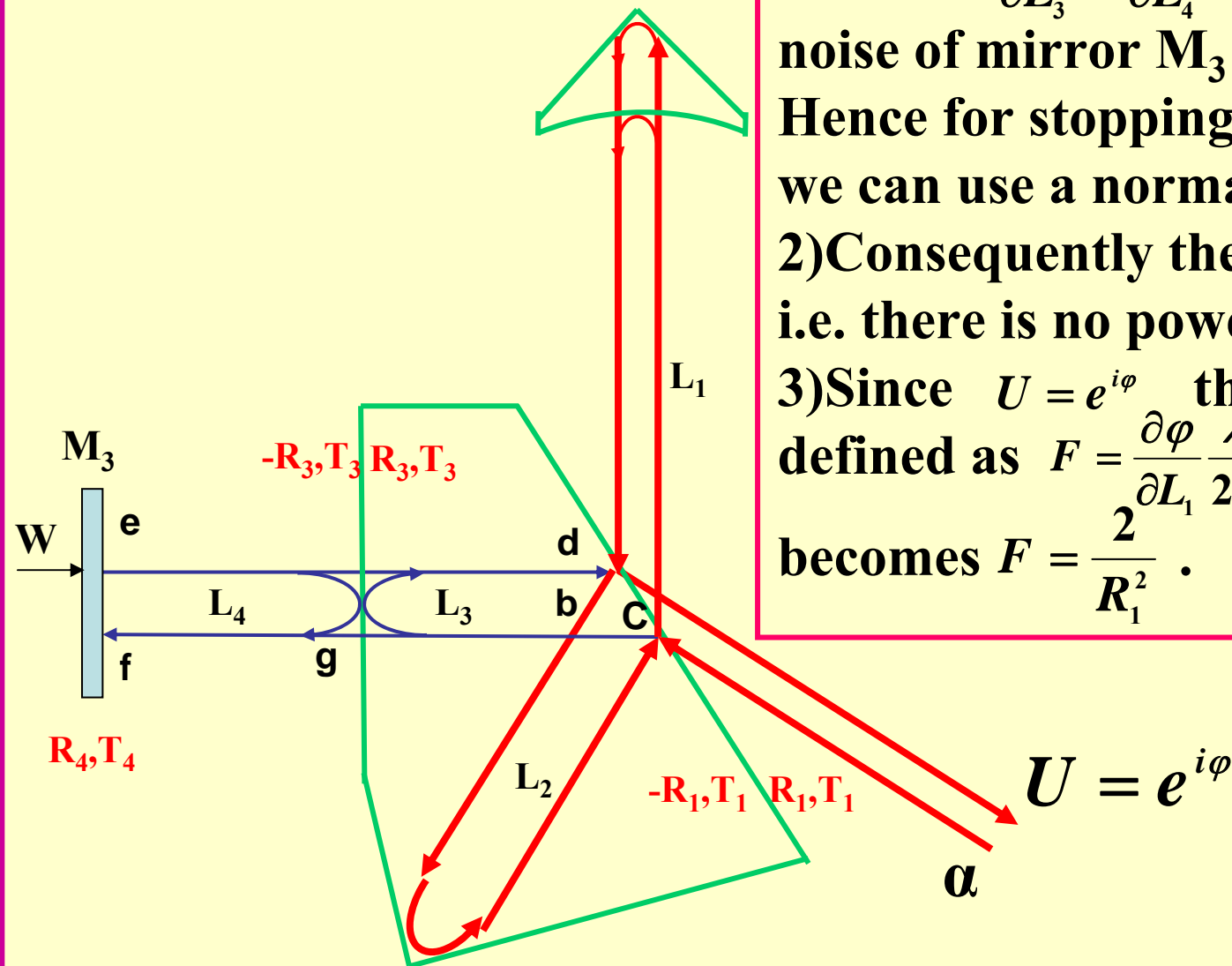


Some Peculiar Properties-1

If channel L3-L4 is antiresonating

- 1) Then $\frac{\partial U}{\partial L_3} = \frac{\partial U}{\partial L_4} = 0$ i.e. Thermal noise of mirror M_3 does not affect U . Hence for stopping input of vacuum we can use a normally coated mirror
- 2) Consequently the amplitude $C=0$. i.e. there is no power on M_3

- 3) Since $U = e^{i\varphi}$ the Finesse F , defined as $F = \frac{\partial \varphi}{\partial L_1} \frac{\lambda}{2\pi} \Rightarrow -\frac{i}{U} \frac{\partial U}{\partial L_1} \frac{\lambda}{2\pi}$, becomes $F = \frac{2}{R_1^2}$.



Some Peculiar Properties-2

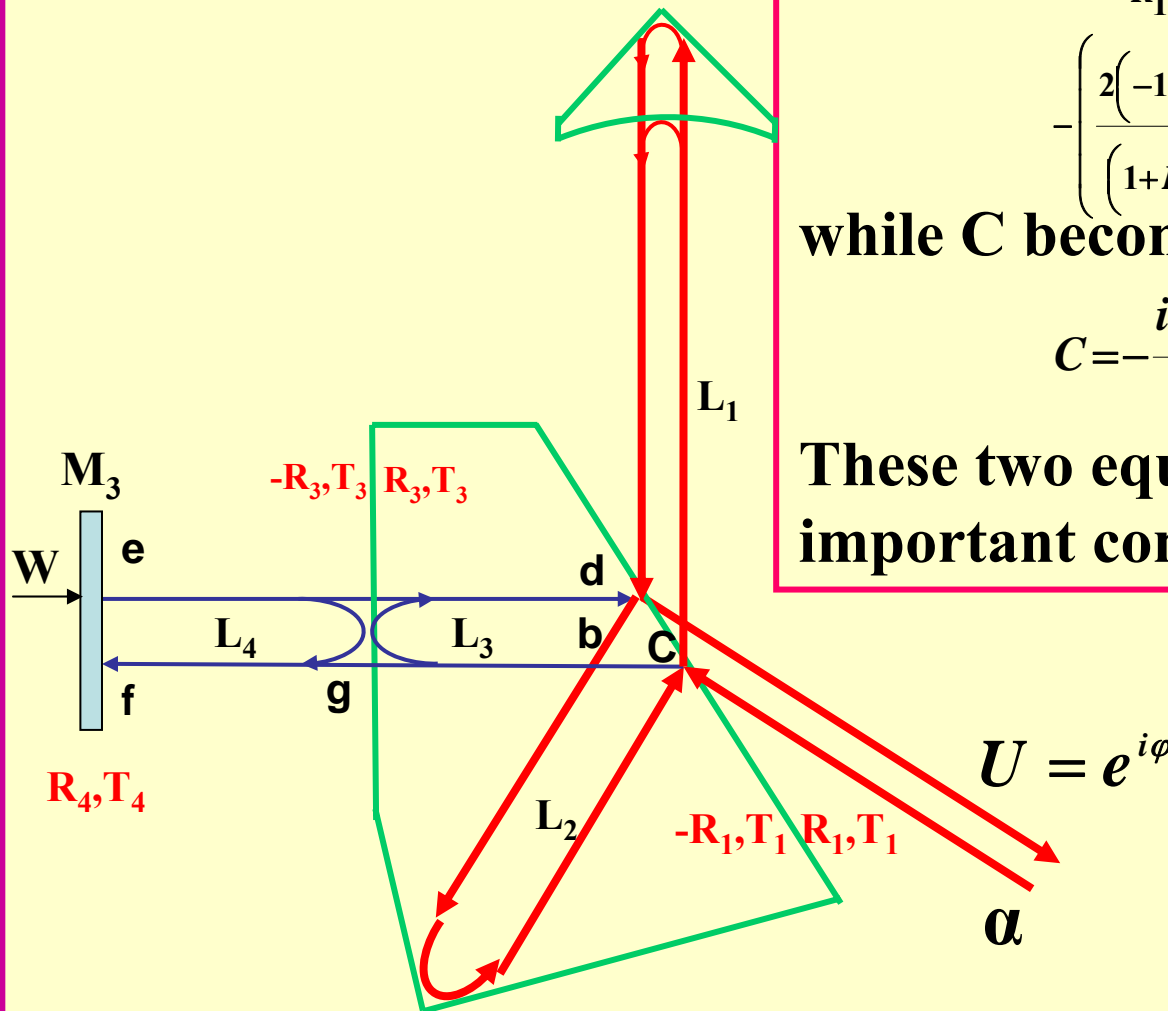
If channel L_3 - L_4 is out of antiresonance with a small phase offset χ then

$$\delta\phi = \frac{2}{R_1^2} \left(1 + \frac{1-R_1^2}{1+R_1^2} \chi^2 + O(\chi^4) \dots \right) \frac{\omega_0}{c} \delta L_1 - \left[\frac{2(-1+R_1^2)}{(1+R_1^2)^2} \chi^2 + O(\chi^4) \dots \right] \frac{\omega_0}{c} \delta L_4$$

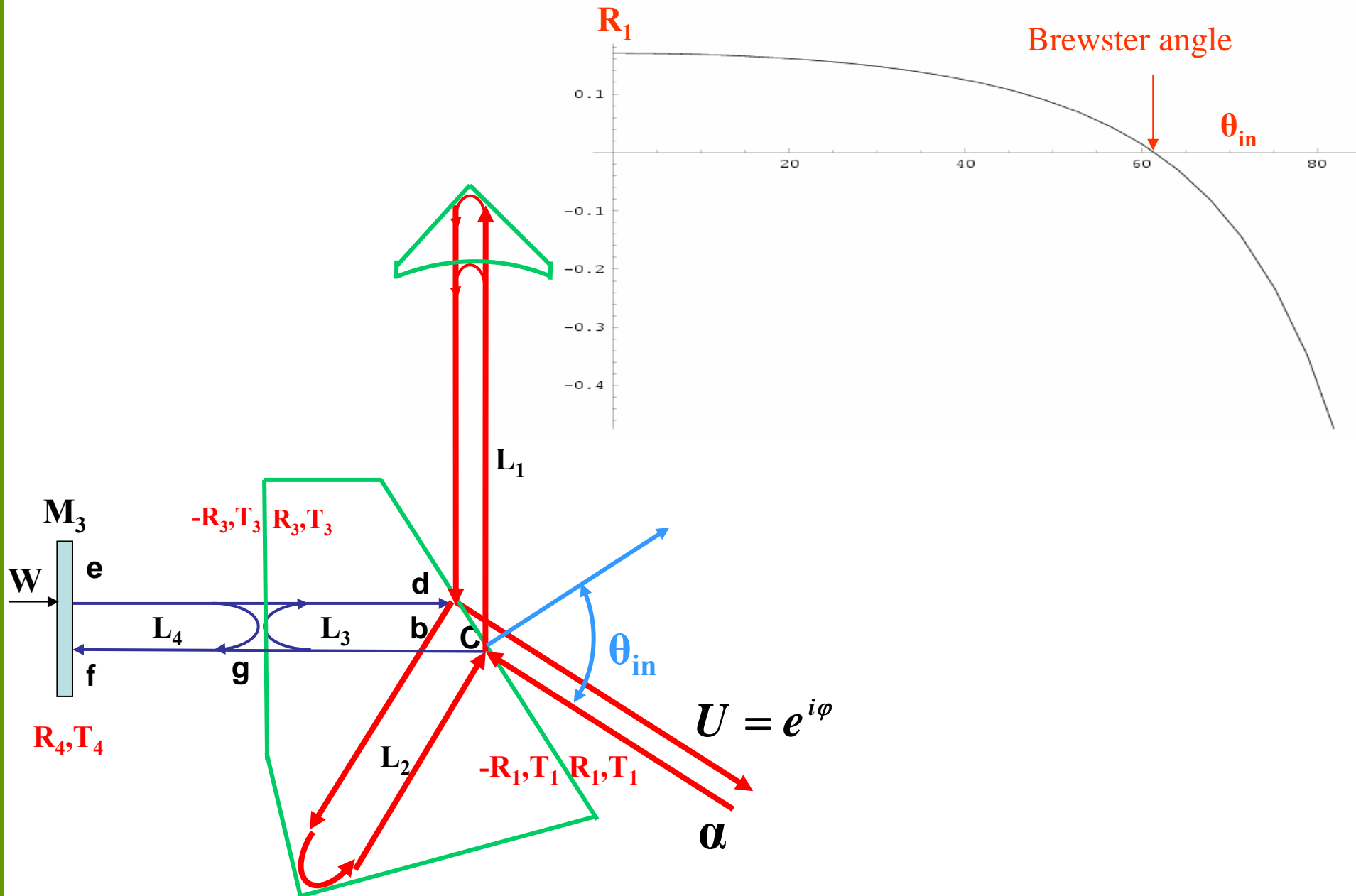
while C becomes

$$C = -\frac{i\sqrt{1-R_1^2}}{1+R_1^2} \chi + O(\chi^3) \dots$$

These two equations seem to have important consequences.



Silica Reflectivity



DISCUSSION

By considering that $R_1^2 \ll 1$, by stopping expansion to χ^2 terms and by putting in evidence $F = \frac{2}{R_1^2}$, we obtain:

$$\delta\varphi = \frac{2}{R_1^2} \left[\left(1 + \frac{1-R_1^2}{1+R_1^2} \chi^2 \right) \frac{\omega_0}{c} \delta L_1 - R_1^2 \frac{(-1+R_1^2)}{(1+R_1^2)^2} \chi^2 \frac{\omega_0}{c} \delta L_4 \right]$$

This equation shows that we may change the finesse by the factor $\Delta F = 1 + \chi^2$ and still the phase shift due to thermal noises of M_3 in L_4 is depressed by the factor $\frac{R_1^2 \chi^2}{1 + \chi^2} \ll 1$ because $R_1^2 \ll 1$.

Then it seems that in this cavity system we may change finesse, and consequently storage time, without affecting :

1) Thermal noise from channel L_3 - L_4 , which for $\chi^2=0$ is zero.

2) Cavity geometry.

The independence of geometry from finesse seems to be a very important feature of this system.

Thermal Noise

With this optical configuration it is evident that every photon traverses a large amount of material; here the problem is transferred from normal mirror reflective coating bad loss angle to **thermorefractive noise (TRN)** of coatingless cavity. TRN spectral density is:

$$S_{\phi}(\omega) = \frac{16\pi L_{\text{bulk}} k_B T^2 \kappa}{\lambda^2 \rho^2 C^2 w_0^4 \omega} \beta^2$$

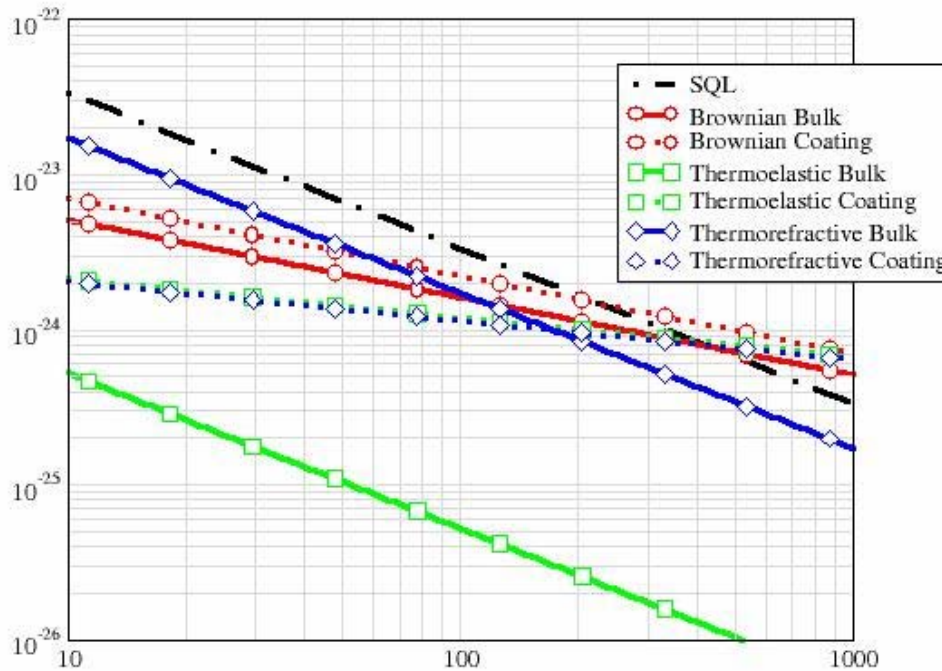
Where: L_{bulk} is the thickness of traversed material, w_0 the beam waist, ρ the bulk density, κ the bulk thermal conductivity coefficient, λ the wavelength, C the bulk thermal capacity and β is:

$$\beta = \frac{\partial n}{\partial T} = \frac{(n^2 - 1)(n^2 + 1)}{6n} \left[\frac{1}{\alpha_p} \frac{\partial \alpha_p}{\partial T} - 3\alpha \left(1 + \frac{\rho}{\alpha_p} \frac{\partial \alpha_p}{\partial \rho} \right) \right]$$

Where α_p is the bulk polarizability, n the bulk refractive index and α is the bulk linear thermal expansion coefficient.

We tried to find data on β at different temperature and we only found 300K data for Silica, which has large C and small κ .

Silica 300K



Data for Sapphire give bad results since C is small and κ is very large compared with Silica

Cristaline Quartz could be a good candidate for going to low temperature but we could not find any data.

Conclusions

This coatingless optical configuration has some interesting features:

- 1) Since all the surfaces are optically matched, antireflective coatings are not needed.**
- 2) Power can be injected in the cavity in a straightforward way; use of tunnel effect injection or transversal cavity are unnecessary.**
- 3) The optical scheme can be made with variable finesse. This property can also be applied also to “normal” mirror configuration.**

It is evident that the draw back is the Thermo Refractive noise; an investigation for selecting cristaline bulks with low thermal conductivity and large thermal capacity is instrumental.