

On the gw emission from the collapse to a rotating bh

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(wishful) plan

- ❑ Source modelling at the AEI: **Cactus/Whisky**
- ❑ The things we can do with **Whisky**...
- ❑ 3D collapse to Kerr black holes:
 - excision/no-excision
 - uniformly/differentially rotating polytropes
- ❑ Conclusions

- SISSA: L. Baiotti, B. Giacomazzo, P. Montero, LR
- AEI/Soton/LSU: I. Hawke, F. Loeffler, E. Seidel, C. Ott
- Valencia: T. Font; Thessaloniki: N. Stergioulas
- Portsmouth/Austin: A. Nerozzi; Munich/LSU: B. Zink; Tuebingen: D. Kobras

Modeling gw sources with **Cactus** and **Whisky**...



Over the last 10 years the group at AEI (LSU) has been developing **Cactus**, a generic 3D, parallel computational toolkit for the solution of PDEs in general and of the Einstein eqs. in particular.

Over the last 3 years and within a European network, we have developed and tested **Whisky**: a 3D, parallel code for the solution of the relativistic hydrodynamics eqs on a generic curved spacetime.



Whisky is "coupled" to **Cactus** ie on a spacelike hypersurface:

$$\text{Cactus} \longrightarrow G_{\mu\nu} = 8\pi T_{\mu\nu} \longleftarrow \text{Whisky}$$

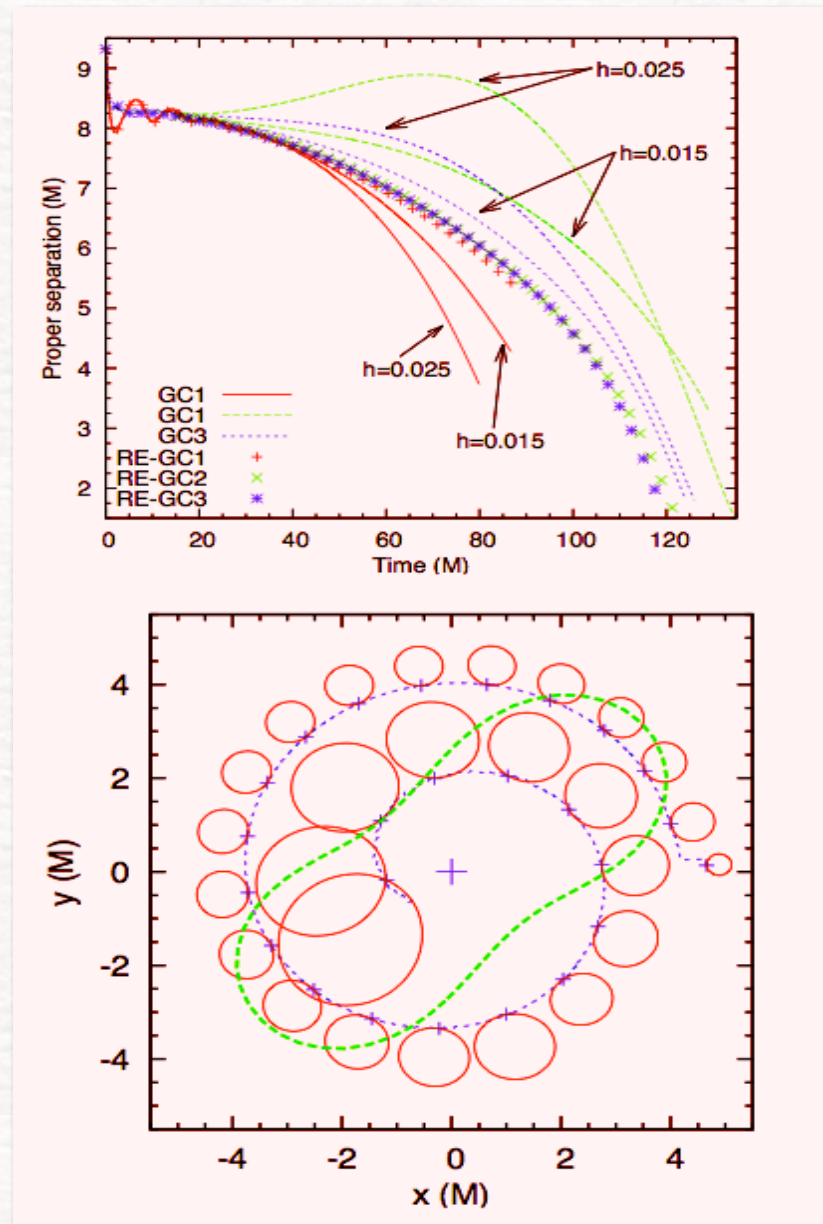
Vacuum sources: binary black holes

- We have recently carried out binary BH evolutions for one or more orbits.
- Convergent and consistent results under variation of gauges.

Diener, Pollney et al. PRL (2006)

- Use of both a “co-rotating” frame with dynamically adjusted gauges or free motion of punctures without excision.
- Extraction of waveforms comparable with those reported by NASA and UTB groups.
- Development of new code for solution of harmonic formulation of the Einstein eqs

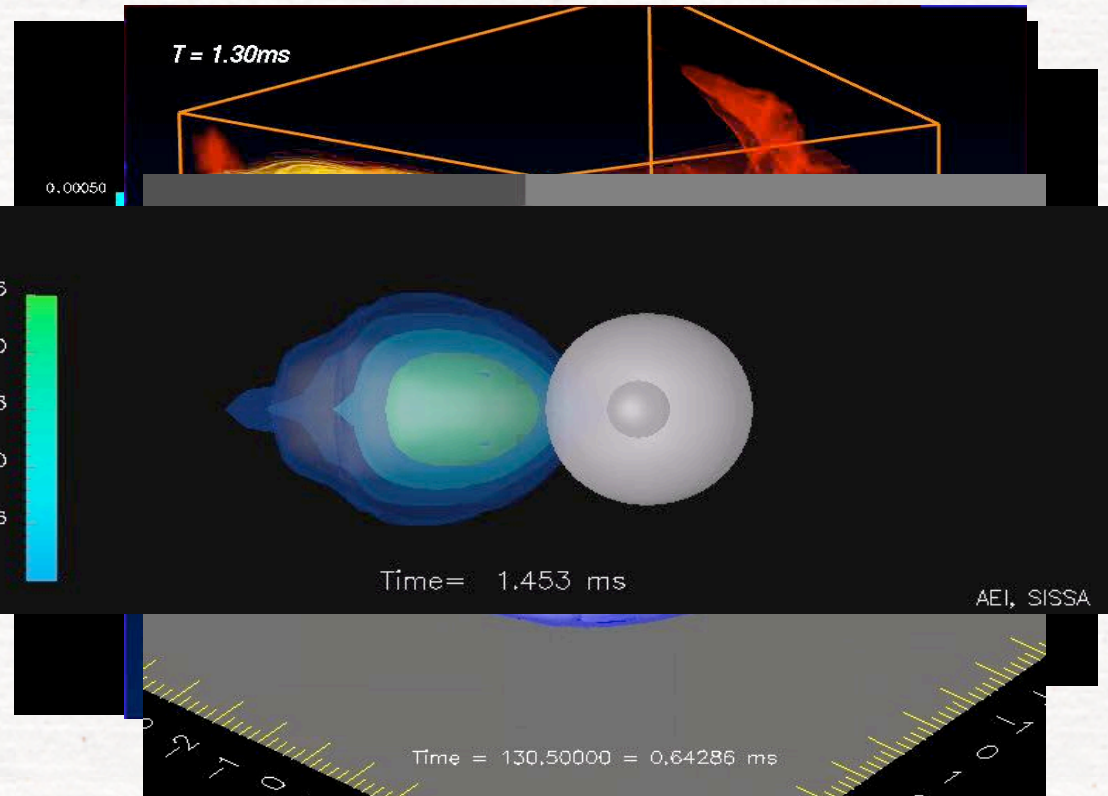
gwadw06 Elba



Non-vacuum sources: ...

Whisky is intended as an “*astrophysics laboratory*” to study the dynamics of matter in highly-curved and dynamical spacetimes. Under investigation now:

- ❑ Neutron star oscillations: linear/nonlinear; magnetized/not (cf. *Giacomazzo*)
- ❑ Dynamical (barmode) instability
- ❑ Binary neutron stars
- ❑ Mixed Binary systems (cf. *Loeffler*)
- ❑ Rotating collapse to black holes



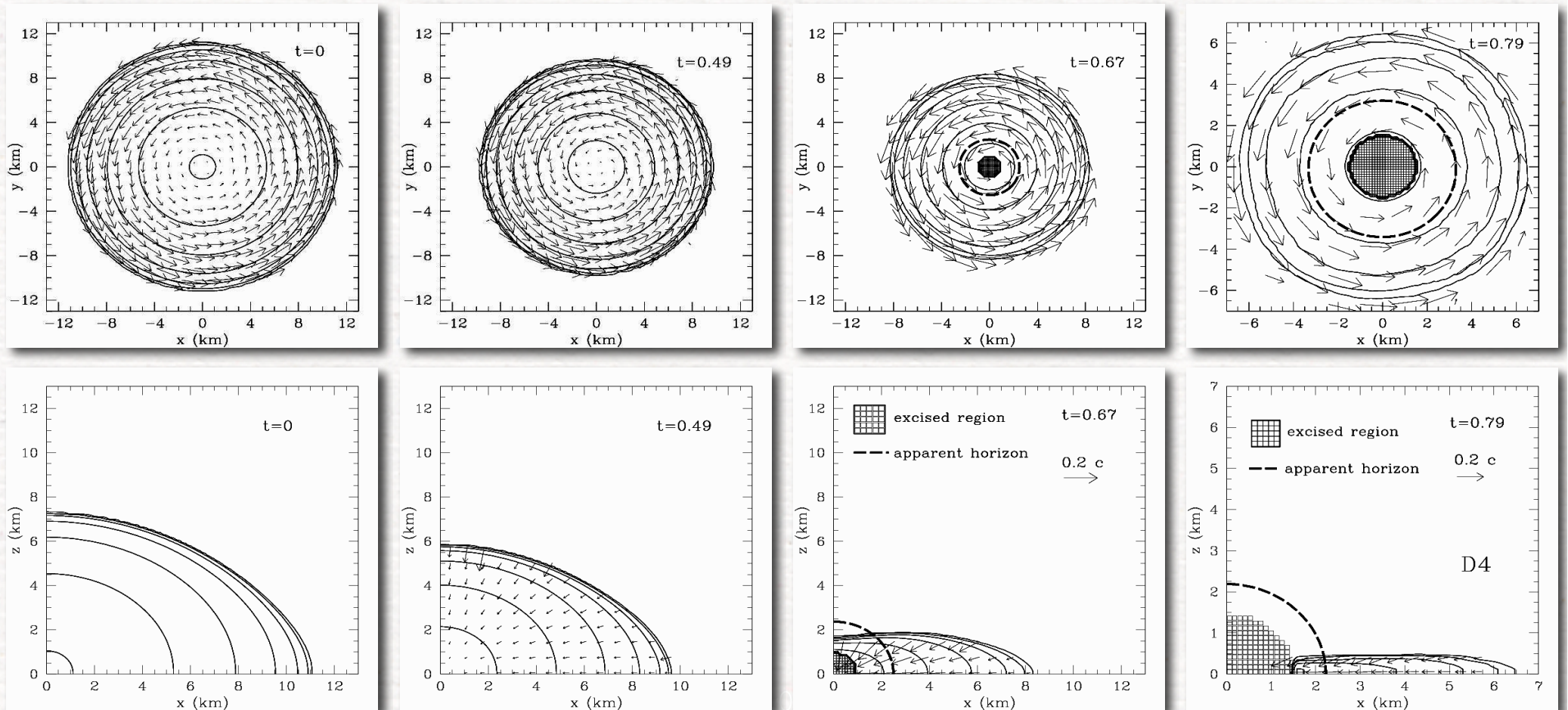
Baiotti et al PRD (2005)

Baiotti, Hawke, LR, Schnetter PRL (2005) gwadw06-Elba

Collapse with excision: rapidly rotating model

Baiotti, et al., PRD (2005)

Once an apparent horizon is found we **excise** a region inside it. Both the field and hydro eqs are solved inside the ah and out of the excision region



What about waveforms?...

This is the ultimate goal of all the collapse simulations. Progress in 3D simulations so far has been hindered by:

- use of uniform Cartesian grids (Stark & Piran in '85 were using non-uniform spherical polar coordinates in 2D)
- signal intrinsically **very small** ($\Delta M/M \sim 10^{-6}-10^{-7}$). This is to be compared with binary bhs where $\Delta M/M \sim 10^{-2}$

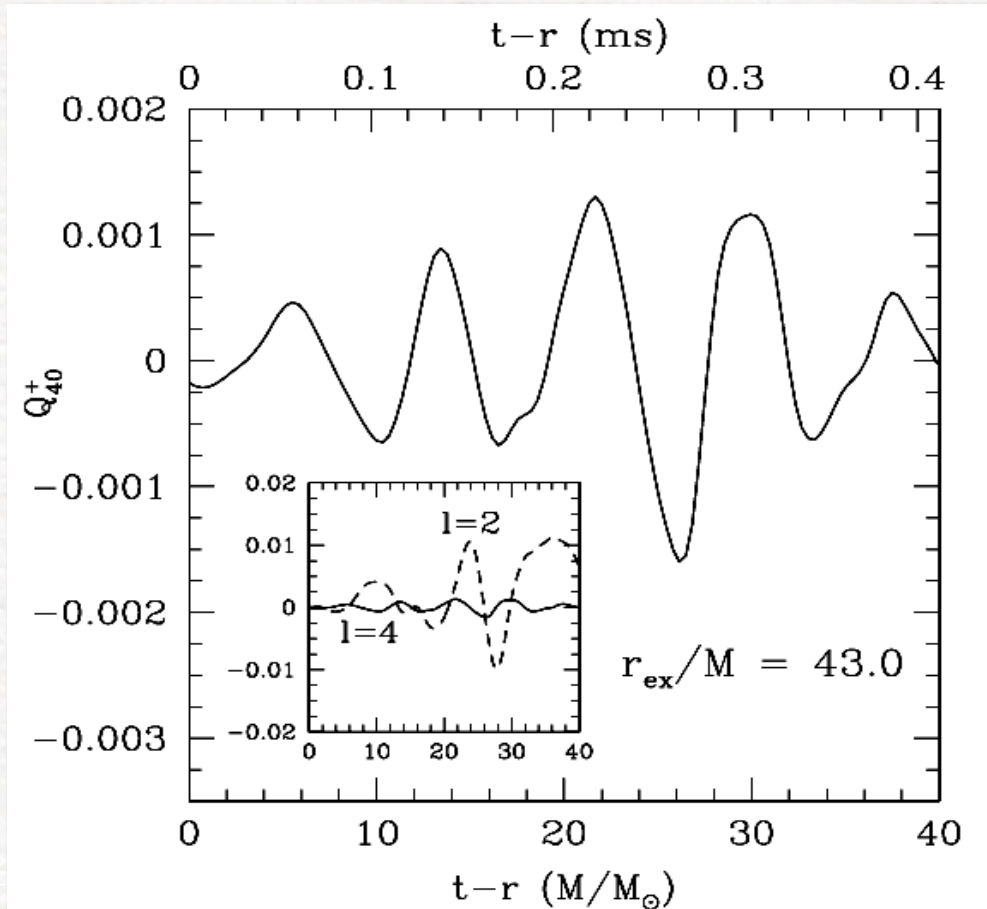
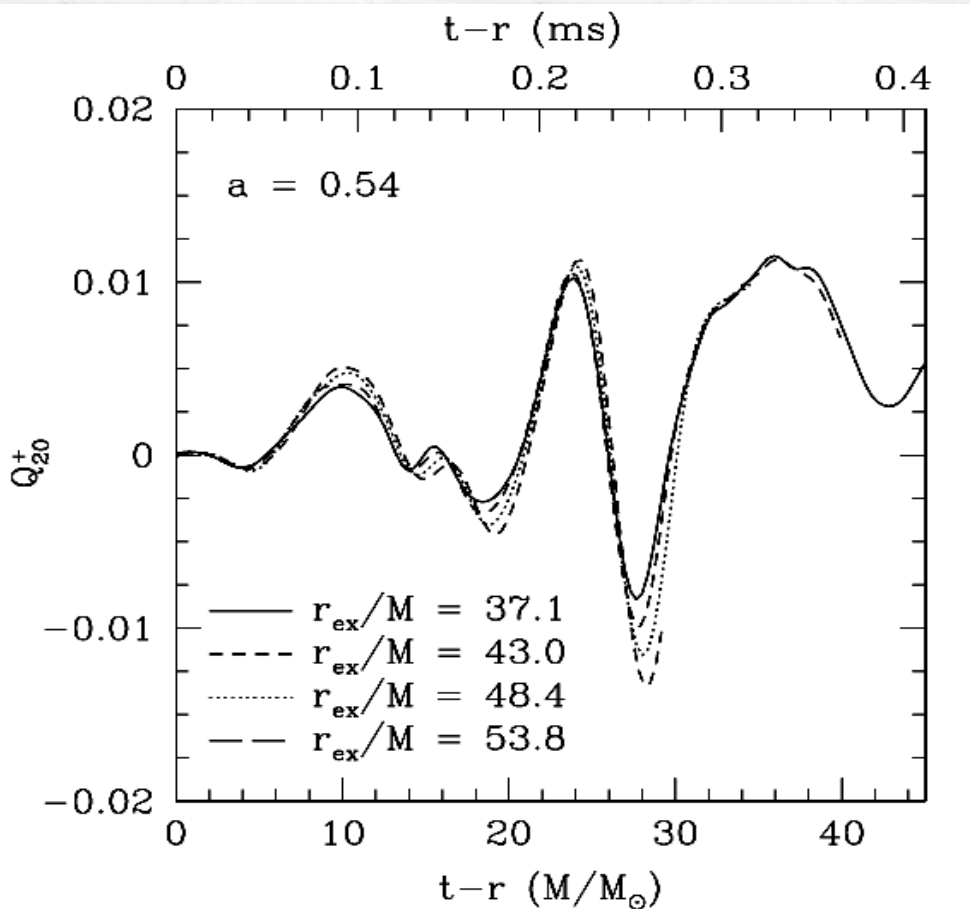
To avoid the limitations of uniform grids we have used FMR (Carpet) with 7 levels of refinements which are “switched-on” as the collapse proceeds

Waveforms with excision...

Baiotti, Hawke, LR, Schnetter PRL (2005)

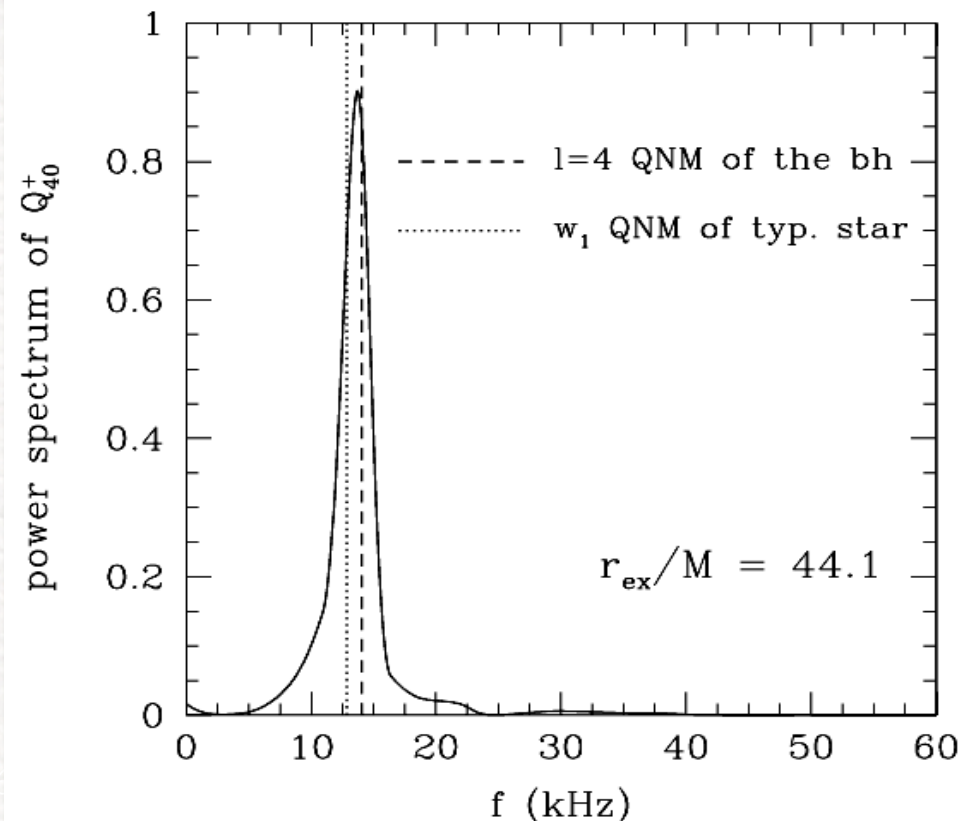
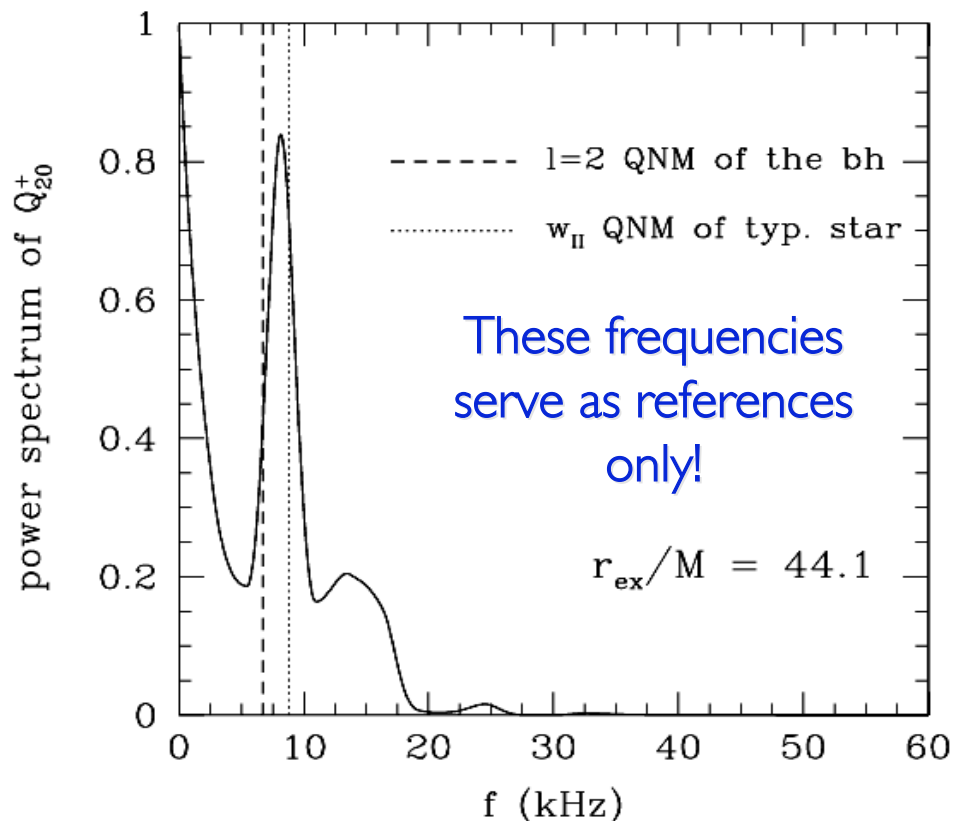
The overlapping of waveforms extracted by different detectors indicate the asymptotic wave nature.

Most of the signal is in the lowest multipole: $Q_{20} \gg Q_{40}$. As a result: $h_+ \gg h_x$ (this is an essentially axisymmetric spacetime)



Are these gravitational waves ?

- Convergence in the wave-zone guarantees that what is extracted behaves as a wave
- Is there consistency with the expected frequencies?



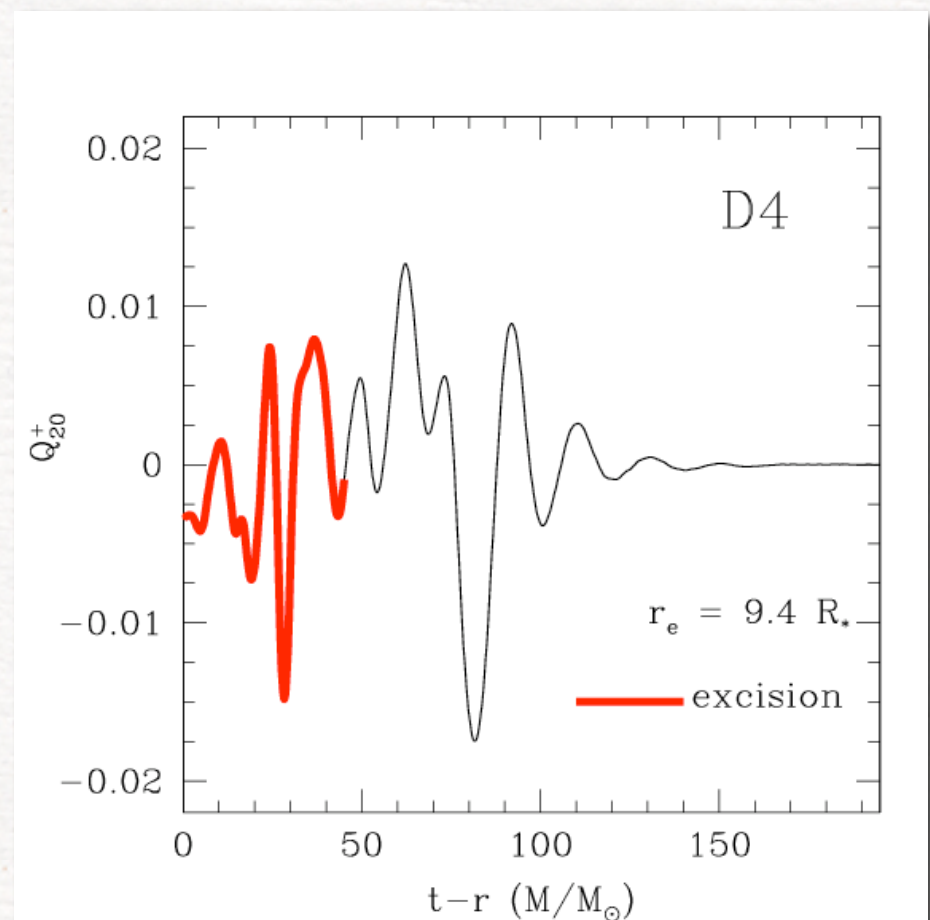
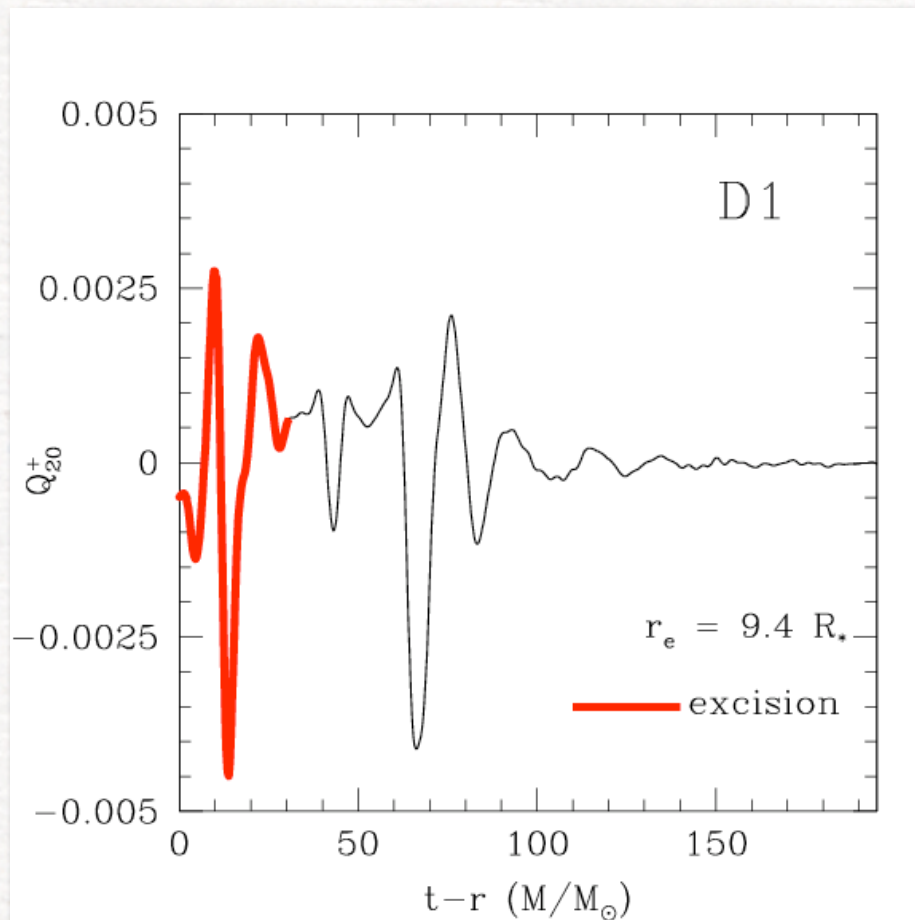
Challenging the excision paradigm

Baiotti, LR, PRL (2006)

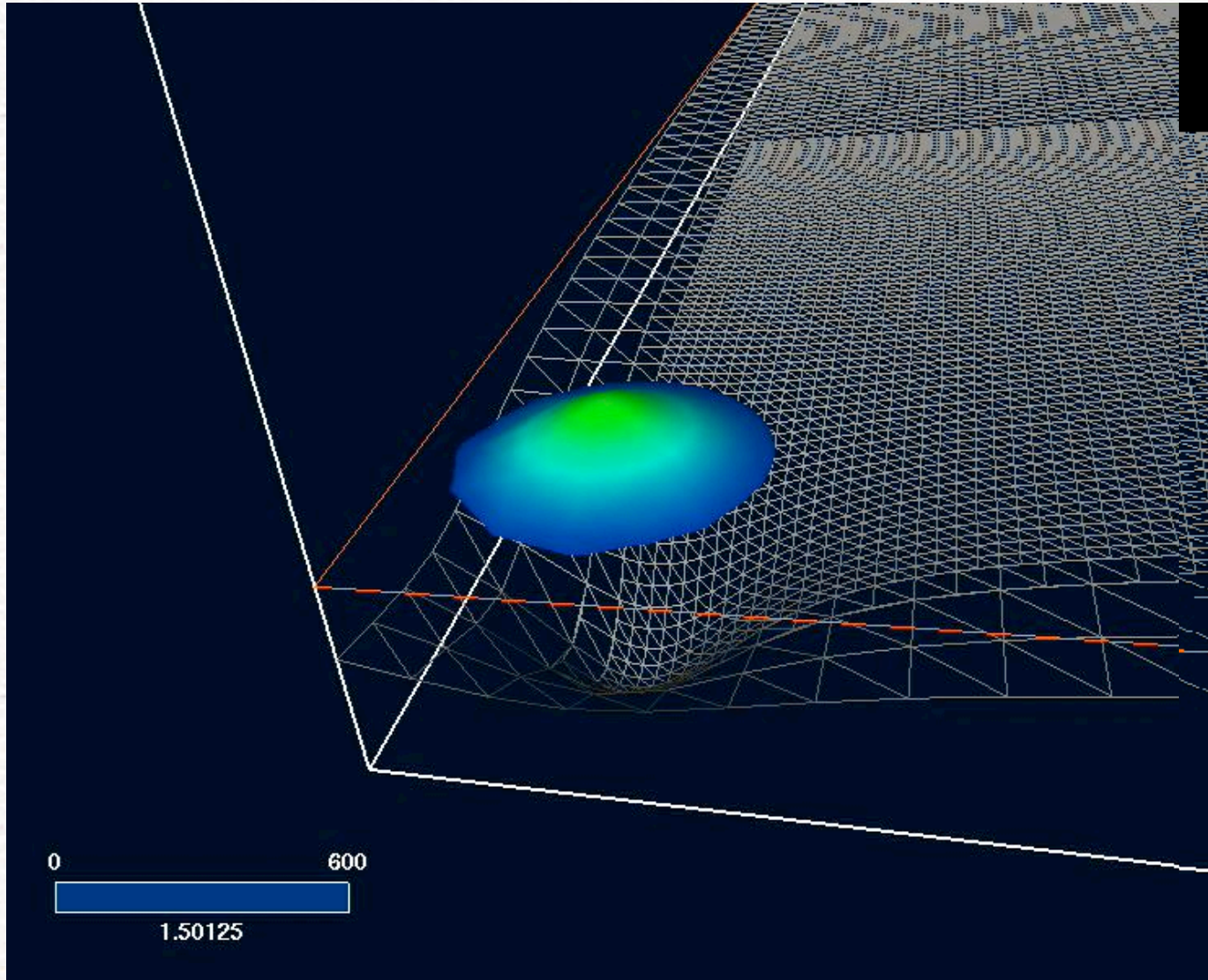
- All of the results presented so far refer to runs in which excision is made once the apparent horizon is found
- A good idea at low resolutions where it can increase the lifetime of the simulations, hence the length of the waveforms
- We have recently challenged this paradigm and avoided excision once an apparent horizon is found
- Note that the collapse leads to *generic spacetime singularities* with no relation to concept of *punctures* as done in binary bh simulations

Challenging the excision paradigm

In practice, the *only* changes amount to *avoiding excision* and to introducing a numerical *dissipation* in terms of 4th-order spatial differential operator for any field variable which is evolved in time



Challenging the excision paradigm



Collapse of model D1

Very similar is also the behaviour for the rapidly rotating model D4

Overall the simulation is carried out for $\sim 900 M$; excision would crash the code at $\sim 110 M$

Energy losses and detectability

The amplitudes can be used to calculate the energy lost to gws

$$\frac{dE}{dt} = \frac{1}{32\pi} \sum_{lm} \left(\left| \frac{dQ_{lm}^+}{dt} \right|^2 + |Q_{lm}^x|^2 \right) \longrightarrow \Delta E \sim 3 \times 10^{-6 \div -7} M/M_{\odot}$$

Smaller efficiency than calculated by Stark & Piran (i.e. $\Delta E \sim 1.5 \times 10^{-4 \div -5} M/M_{\odot}$), but consistent with estimates from core collapse

$$\frac{S}{N} \equiv \frac{h_c}{h_{rms}(f_c)} = \frac{h_c}{\sqrt{f_c S_h(f_c)}}$$

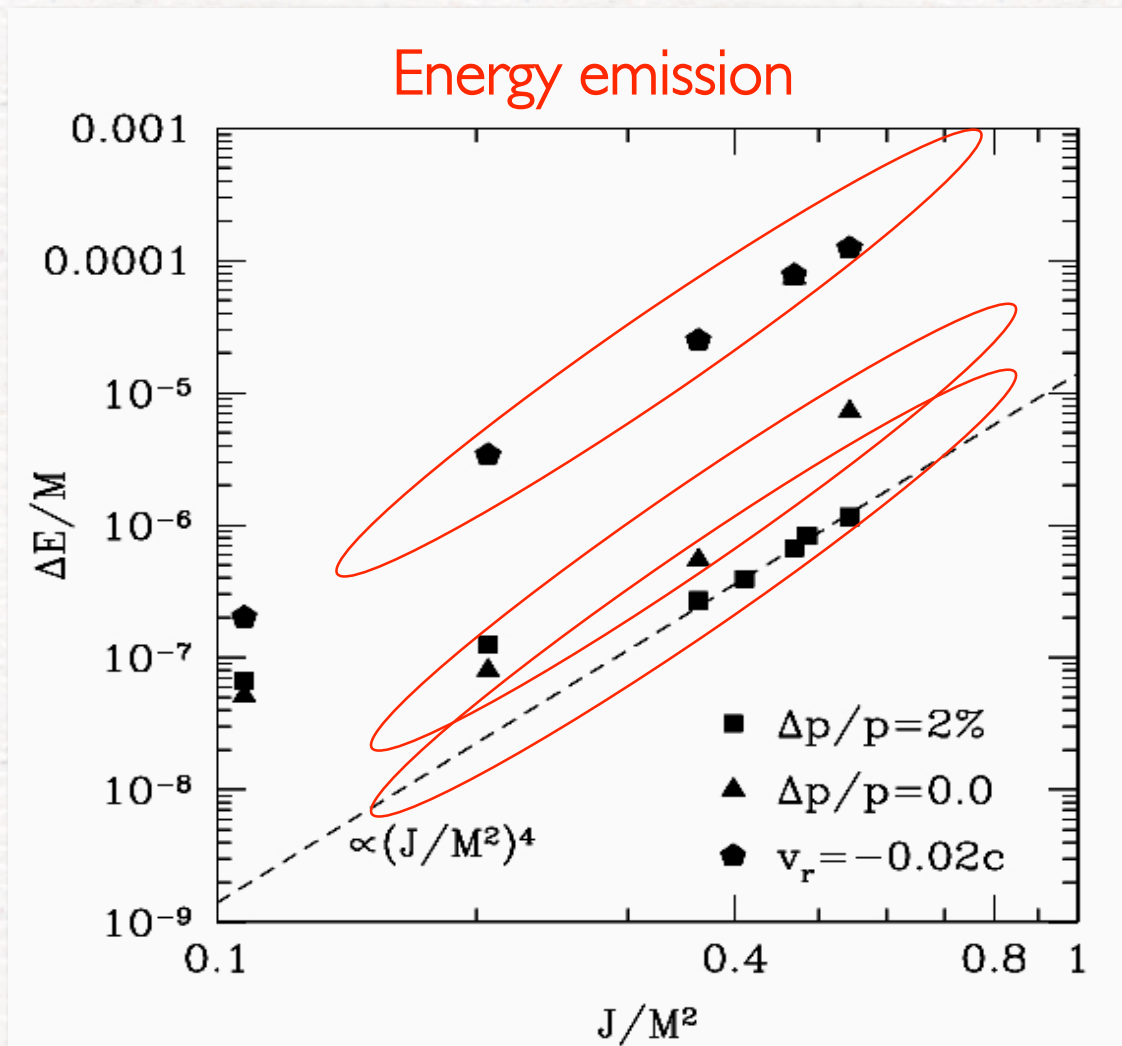
Slow	Rapid	detector
0.2	2	Virgo/Ligo
1	9	Advanced
3	28	Dual

Building our understanding: ie the first steps towards gw astronomy

With the basic picture clear, we are now looking at how the properties of waveforms depend (at times sensitively) on a number of factors:

- *perturbation type and amplitude*,
- *rotation rate*, Baiotti, LR, Hawke, Schnetter (in preparation)
- *velocity distribution (ie differential rotation)*
- *EOS* Giacomazzo, LR, Stergioulas (in preparation)

Energy emission from uniformly rotating stars



- The energy emitted scales like $\sim (J/M^2)^4$ but only for sufficiently large rotation rates

- Even small (i.e. $\sim 2\%$) pressure perturbations modify the collapse and reduce the energy emission

- Additional radial velocities (2%) can boost the collapse and enhance the emission

Because of the overall axisymmetry, the gw emission from uniformly rotating stars is efficient only at very large rotation rates: $S/N > 1$ at 10 kpc.

Differentially rotating polytropes

Giacomazzo, LR, stergioulas (in preparation)

- The **efficiency** in the gw emission can be increased if large deviations from axisymmetry develop via **dynamical instabilities**
- Differential rotation is expected in a star produced as a result of core-collapse or in the merger of a binary system of NSs
- Differential rotation can easily yield stars with $J/M^2 < 1$ (ie **sub-Kerr**) as well as stars with $J/M^2 > 1$ (ie **supra-Kerr**)
- A supra-Kerr star **cannot** collapse promptly to a Kerr bh; something must intervene to remove angular momentum, eg. **non-axisymmetric instabilities**

Collapse of sub-Kerr models

It is not too difficult to construct differentially rotating models with a law

$$\Omega_c - \Omega = \left(\frac{r_e}{\hat{A}} \right)^2 \left[\frac{(\Omega - \omega) r^2 \sin^2 \vartheta e^{-2\rho}}{1 - (\Omega - \omega)^2 r^2 \sin^2 \vartheta e^{-2\rho}} \right] \quad \text{which have } J/M^2 < 1 \text{ (sub-Kerr) and are close to the stability limit}$$

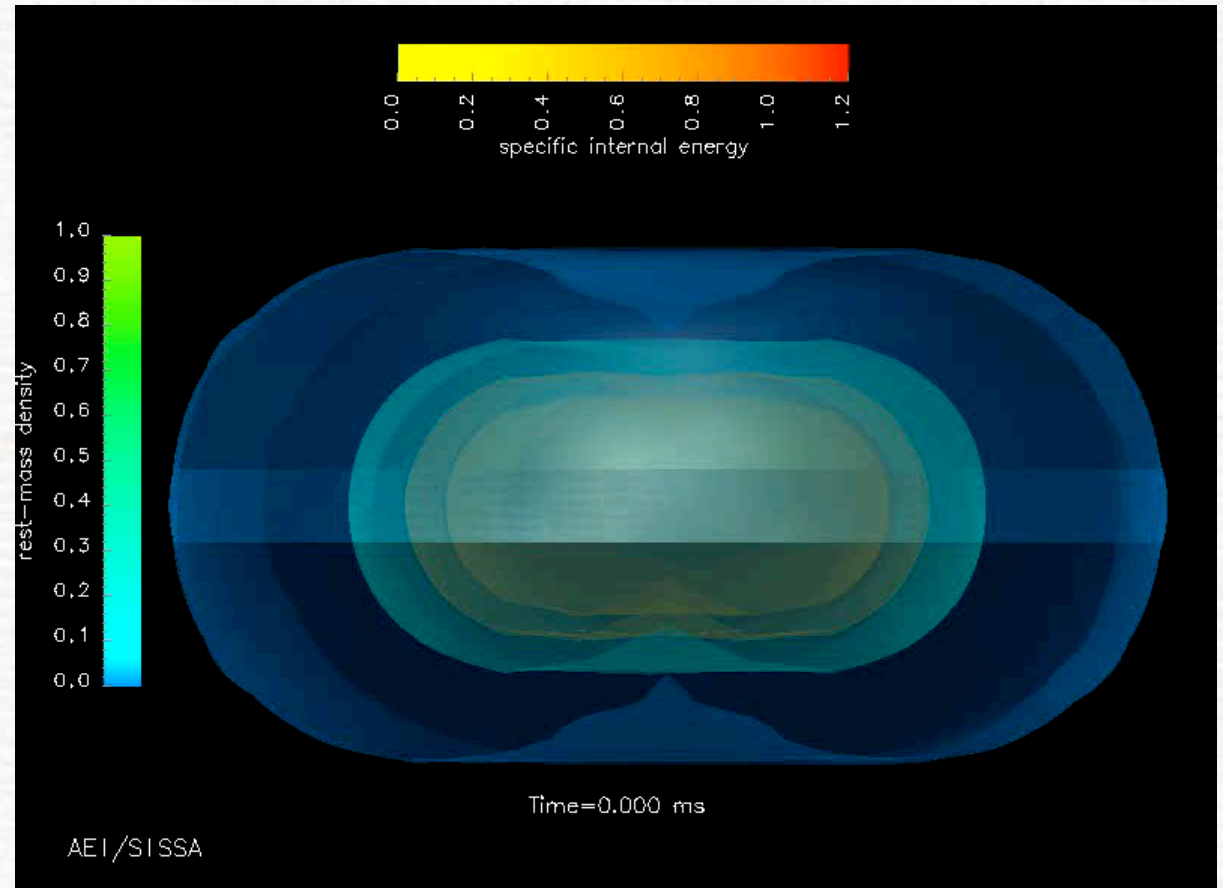
A $\sim 2\%$ pressure perturbation sufficient to trigger the collapse.

As an example: A10

$$M = 1.812 M_{\odot}; J/M^2 = 0.477$$

$$T/|W| = 0.06; \Gamma = 2$$

Also in this case, the collapse is essentially axisymmetric but contributions from higher multipoles are larger



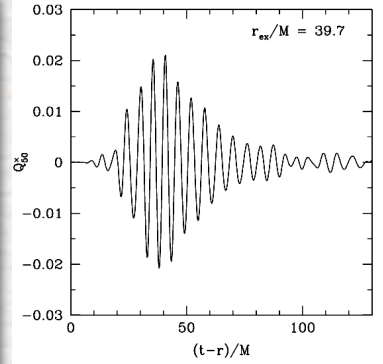
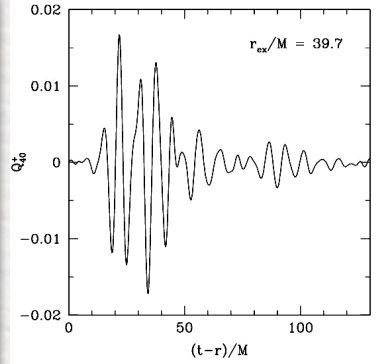
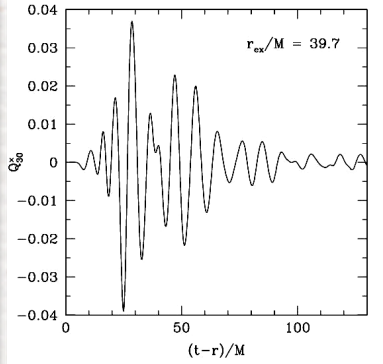
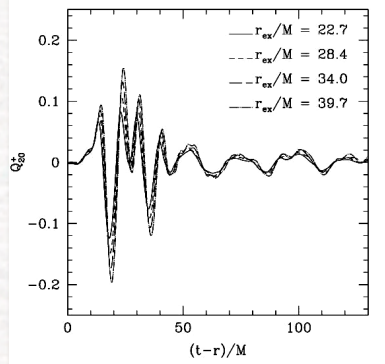
Building a database...

(l, m): 2,0

3,0

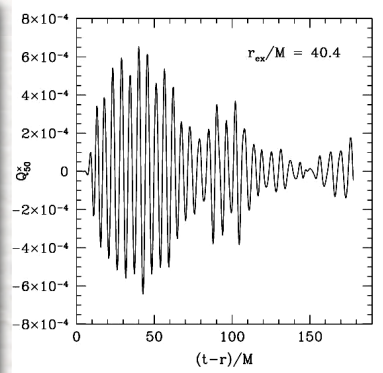
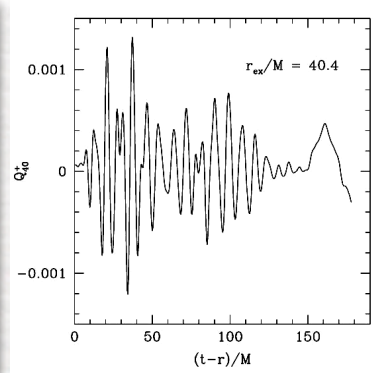
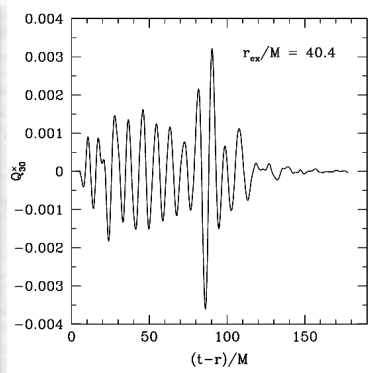
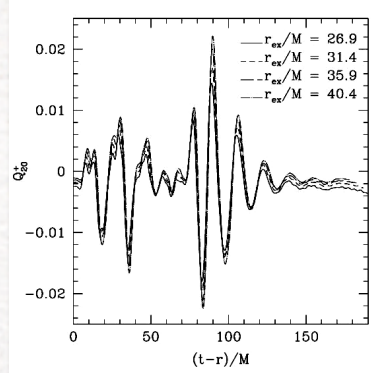
4,0

5,0

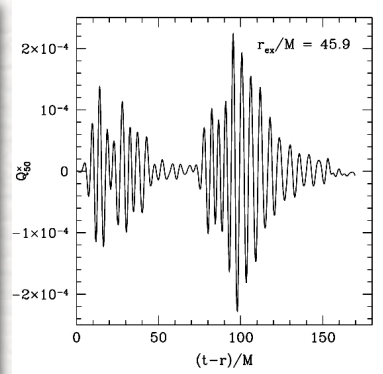
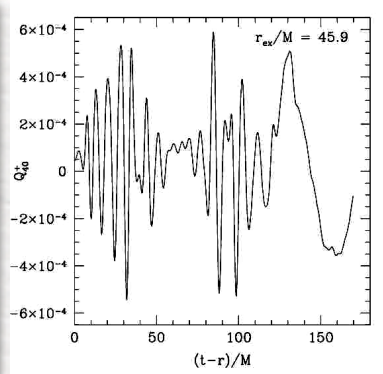
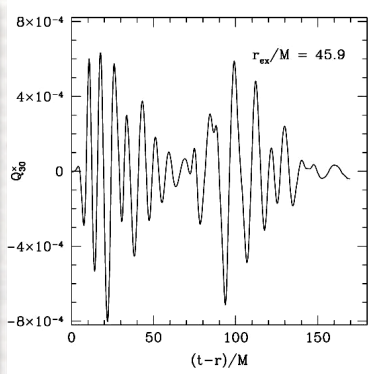
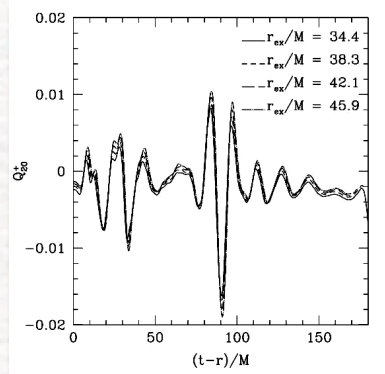


Degree of differential rotation

$A=0.6$



$A=1.0$



$A=1.4$

Collapse of a supra-Kerr model

Equally simple is to construct supra-Kerr models.

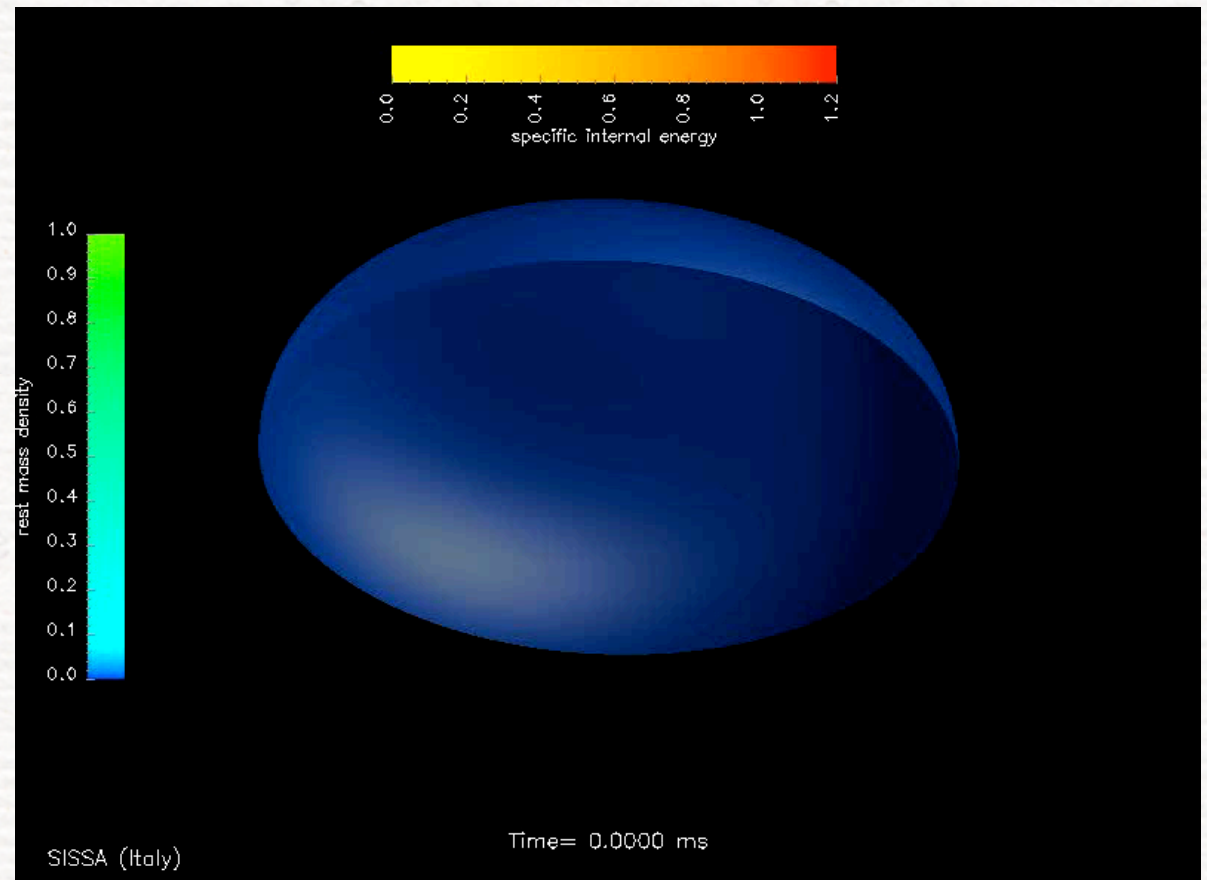
As an example B1:

$$M = 1.909 M_{\odot}; r_p/r_e = 0.390$$

$$J/M^2 = 1.09; |T/W| = 0.215$$

$$\Gamma = 2$$

This is the same model evolved by Duez et al. 03. Note the collapse was triggered with a pressure depletion of 99%



Animation by B. Giacomazzo

Modulo numerical (Cartesian) “imprints”, this seems an excellent source of gravitational radiation. *Unfortunately, this is rather unrealistic...*

Conclusions

- Collapse from uniformly rotating stars is less efficient than previously expected with $S/N \sim 1$; several factors (rotation, EOS, etc) can increase this
- All differentially-rotating models that are **unstable** have $J/M^2 \leq 1$; collapse's dynamics/efficiency is similar to the one for uniformly rotating models
- All differentially-rotating models $J/M^2 > 1$ are **stable**; unlikely to produce non-axisymmetric instabilities without huge pressure reductions; not yet excluded.
- A suitable combination of good gauge conditions and dissipation remove the need for excision for generic singularities, with dramatic improvements on the performance of the codes.
- **Whisky** has proven to be an excellent tool to study the collapse to a Kerr bh providing the most accurate simulations and the first waveforms in 3D. Equally exciting results are also coming in other studies in which **Whisky** is used (cf. Loeffler, Giacomazzo's talks)