Free-fall and the observation of low frequency gravitational waves with LISA

Bill Weber
Università di Trento

Caltech
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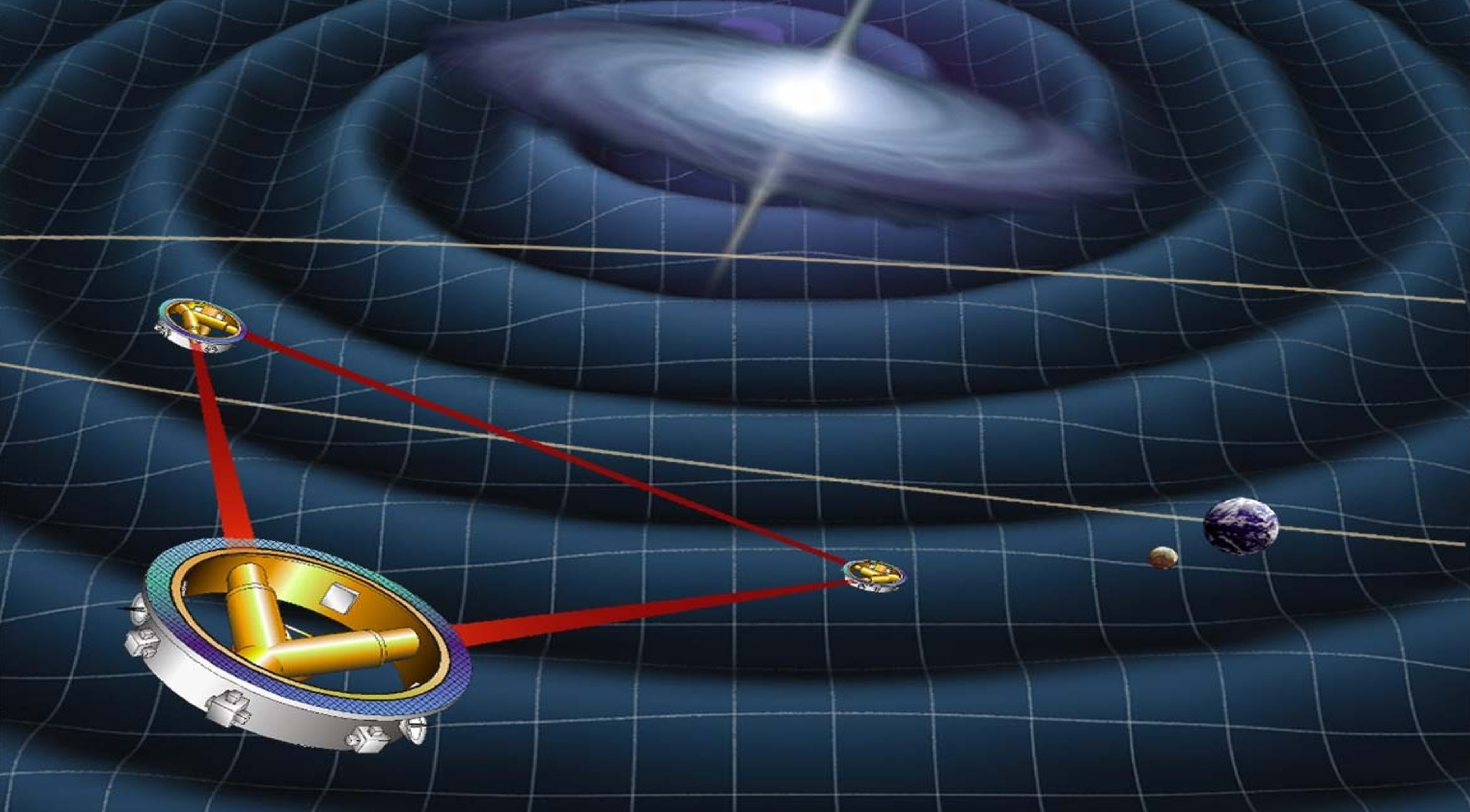
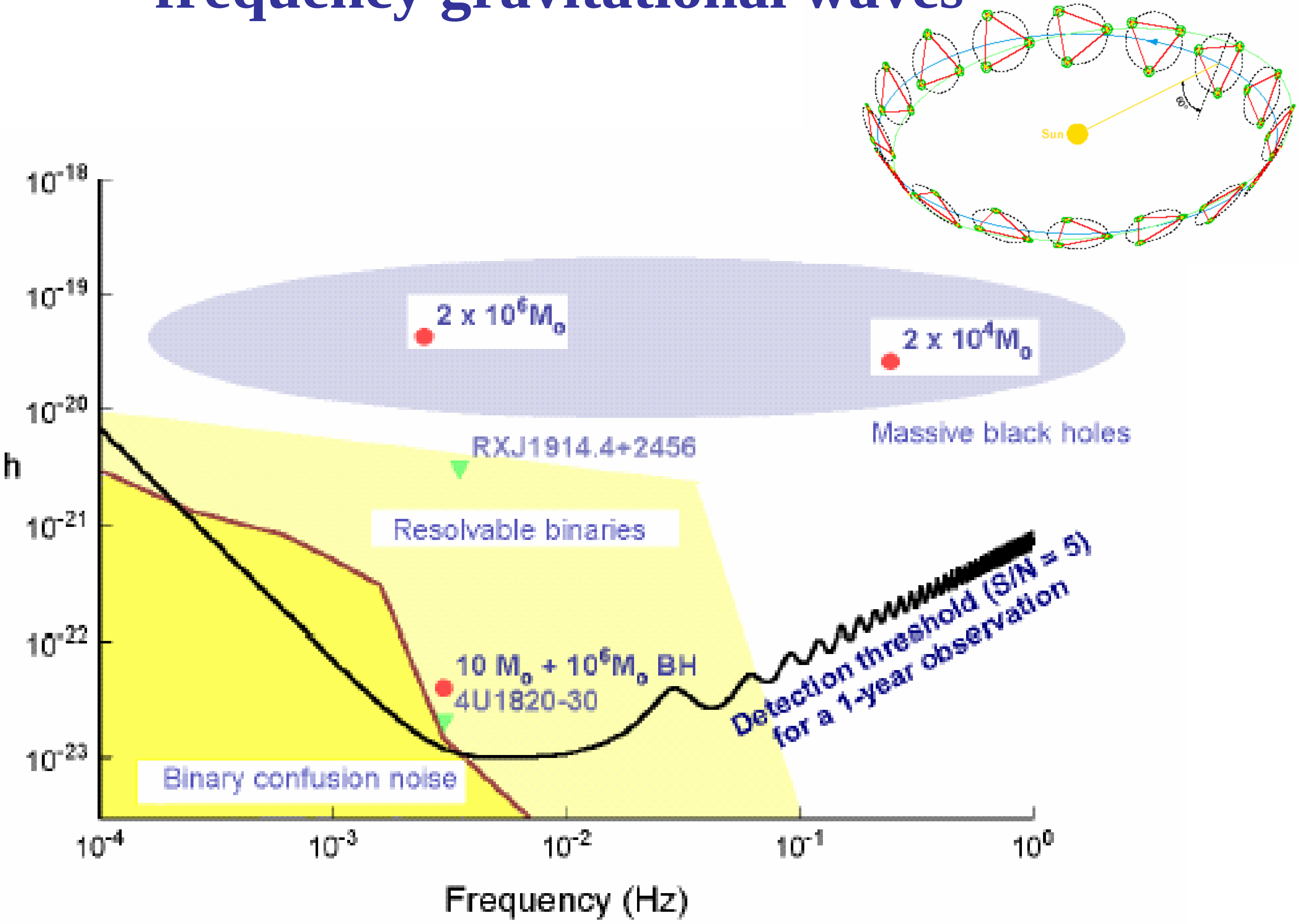
LISA



Laser Interferometer Space Antenna

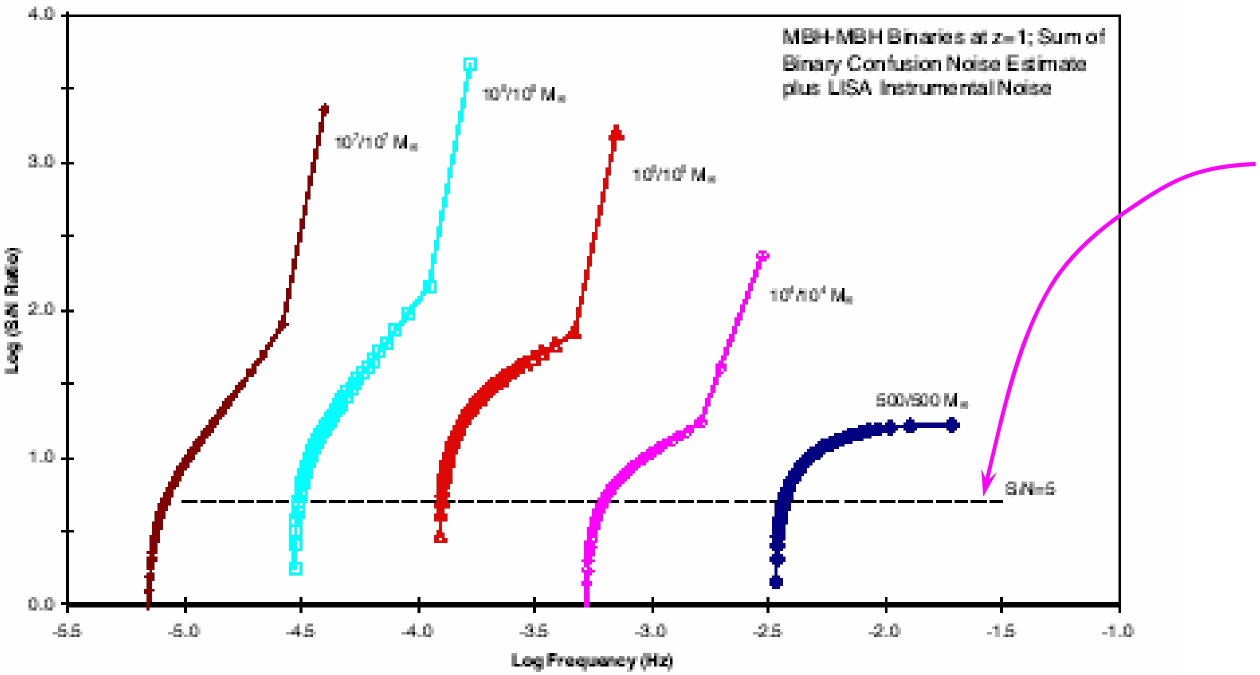
LISA: an orbiting observatory for low frequency gravitational waves



Sensitivity curve for 1 year integration and S/N=5

Purity of free-fall critical to LISA science

Example: massive black hole (MBH) mergers
Integrated SNR at 1 week intervals for year before merger



Assuming LISA goal:

$S_a^{1/2} < 3$ fm/s^2/Hz$^{1/2}$
at 0.1 mHz

- Factor 10 in acceleration noise →decreased observation time (year →weeks)
  - LISA sweeps out only 10 degrees rather than a full circle
  - Lose information on source location and thus source luminosity distance

How "guaranteed" is LISA's low frequency sensitivity and projected scientific return?

- What are the sources of force noise that can compromise purity of free-fall for LISA?

- What is the proposed “drag-free control” system that aims to minimize force noise?

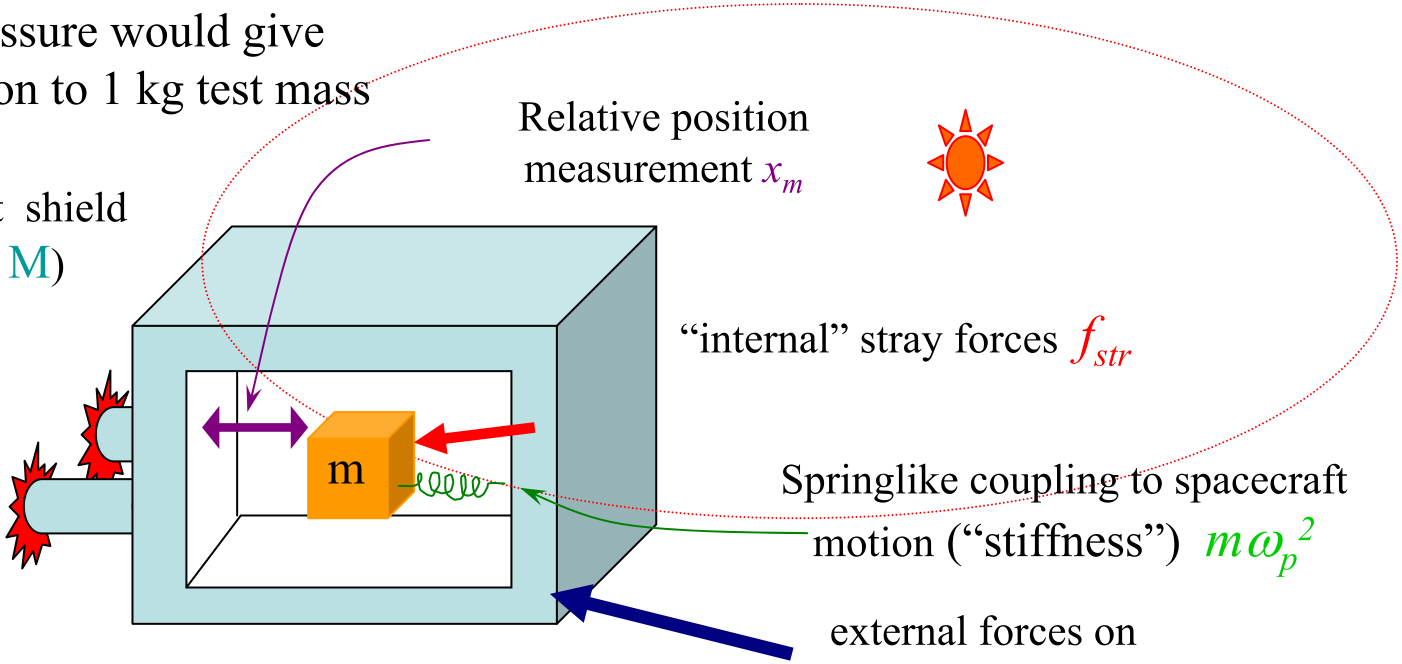
- What do we know and what can we learn quantitatively about these sources of force noise?

→ LISA Pathfinder in-flight test

→ torsion pendulum studies on the ground

Stray forces and drag-free control

- Solar radiation pressure would give $10\ \mathrm{nm/s^2}$ acceleration to 1 kg test mass



Relative position measurement x_m

Spacecraft shield
(mass M)

μNewton Thrusters
“Drag Free” loop
gain $M\omega_{DF}^2$

“internal” stray forces f_{str}

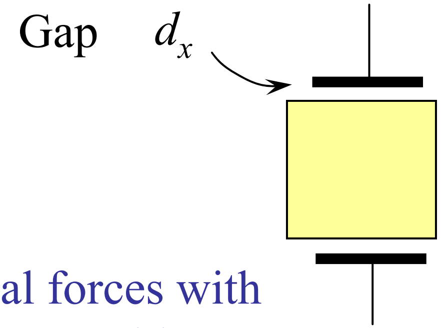
Springlike coupling to spacecraft motion (“stiffness”) $m\omega_p^2$

external forces on satellite F_{str}

Residual acceleration noise:

$$a_{res} = \frac{f_{str}}{m} + \omega_p^2\left(x_n + \frac{F_{str}}{M\omega_{DF}^2}\right)$$

Key LISA test mass acceleration noise sources



Springlike coupling to spacecraft:
sensor readout stiffness ($\omega_p^2 x_n \sim d$)
gravity gradients

External forces with
finite control loop
bandwidth

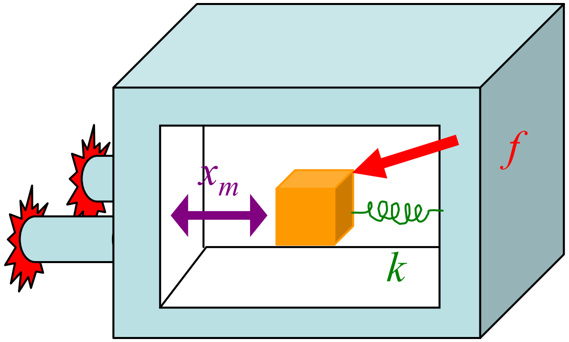
Residual acceleration noise:

$$a_{res} = \frac{f_{str}}{m} + \omega_p^2 \left(x_n + \frac{F_{str}}{M\omega_{DF}^2} \right)$$

gas damping
magnetic noise
readout back action ($\sim d^{-2}$)
DC electric fields + charge shot noise ($\sim d^{-1}$)
DC electric fields + dielectric noise ($\sim d^{-2}$)
thermal gradients
radiation pressure, radiometric effects

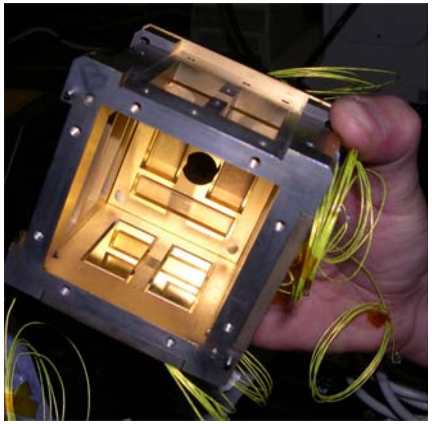
Sensor noise
Low frequency stability!

Gravitational Reference Sensor Design



- o ~ 1 nm/Hz$^{1/2}$ sensor noise floor
- o low force gradient (k ~ 100 nN/m)
- o low force noise ($S_f^{1/2}$~ fN/Hz$^{1/2}$)

- 40-50 mm cubic Au / Pt test mass (1-2 kg)



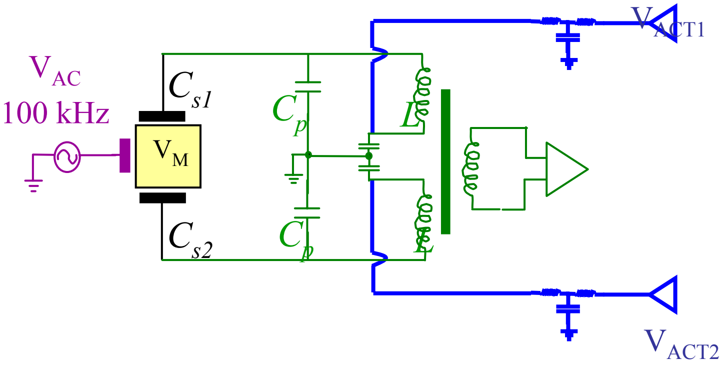
- 6 DOF "gap sensing" capacitive sensor
  - Contact free sensing bias injection
  - Resonant inductive bridge readout (100 kHz)

- Audio frequency electrostatic force actuation

  → avoid DC voltages

- Large gaps (2 – 4 mm)

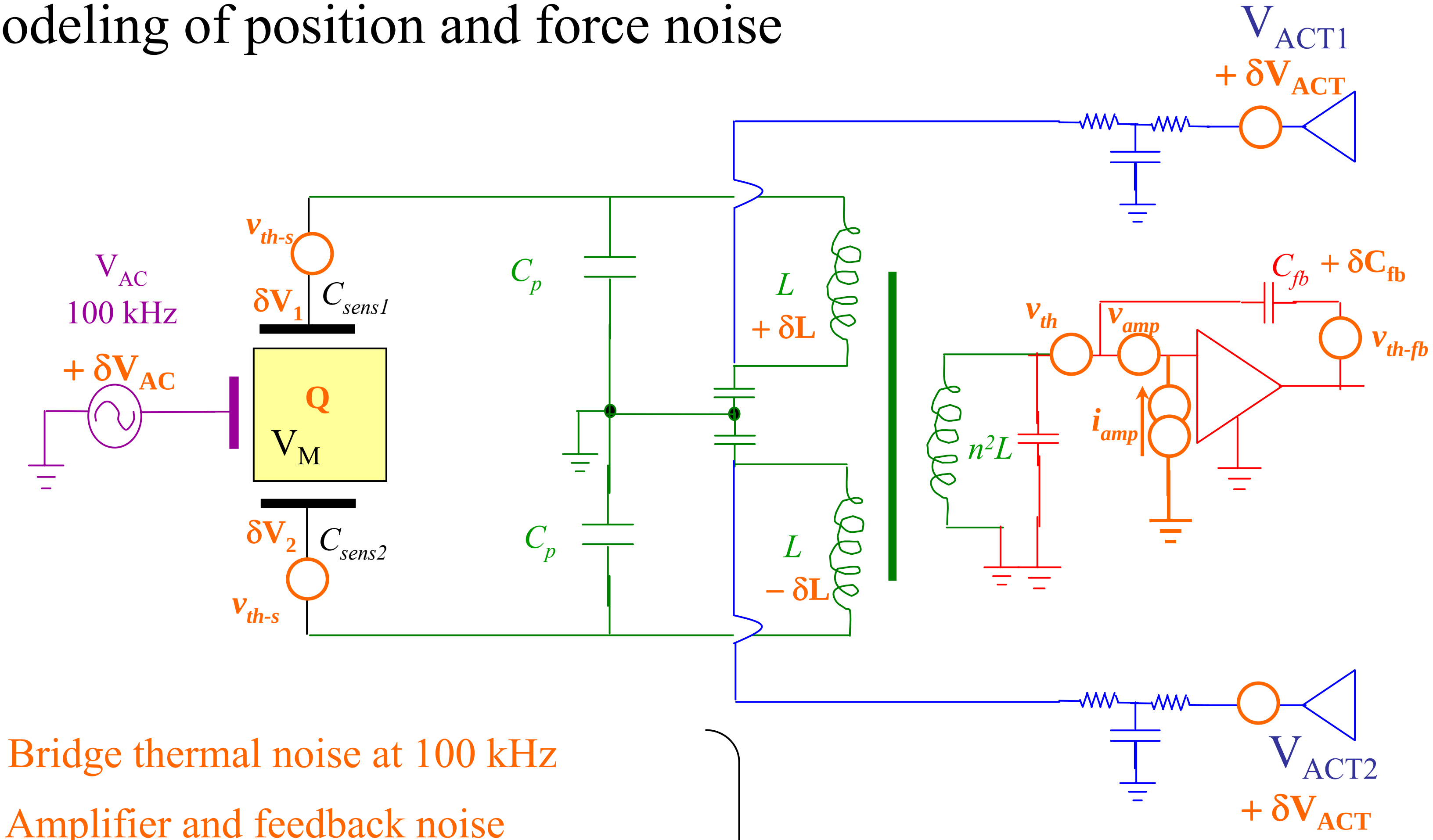
  → limit electrostatic disturbances



- High thermal conductivity metal / ceramic construction

  → limit thermal gradients

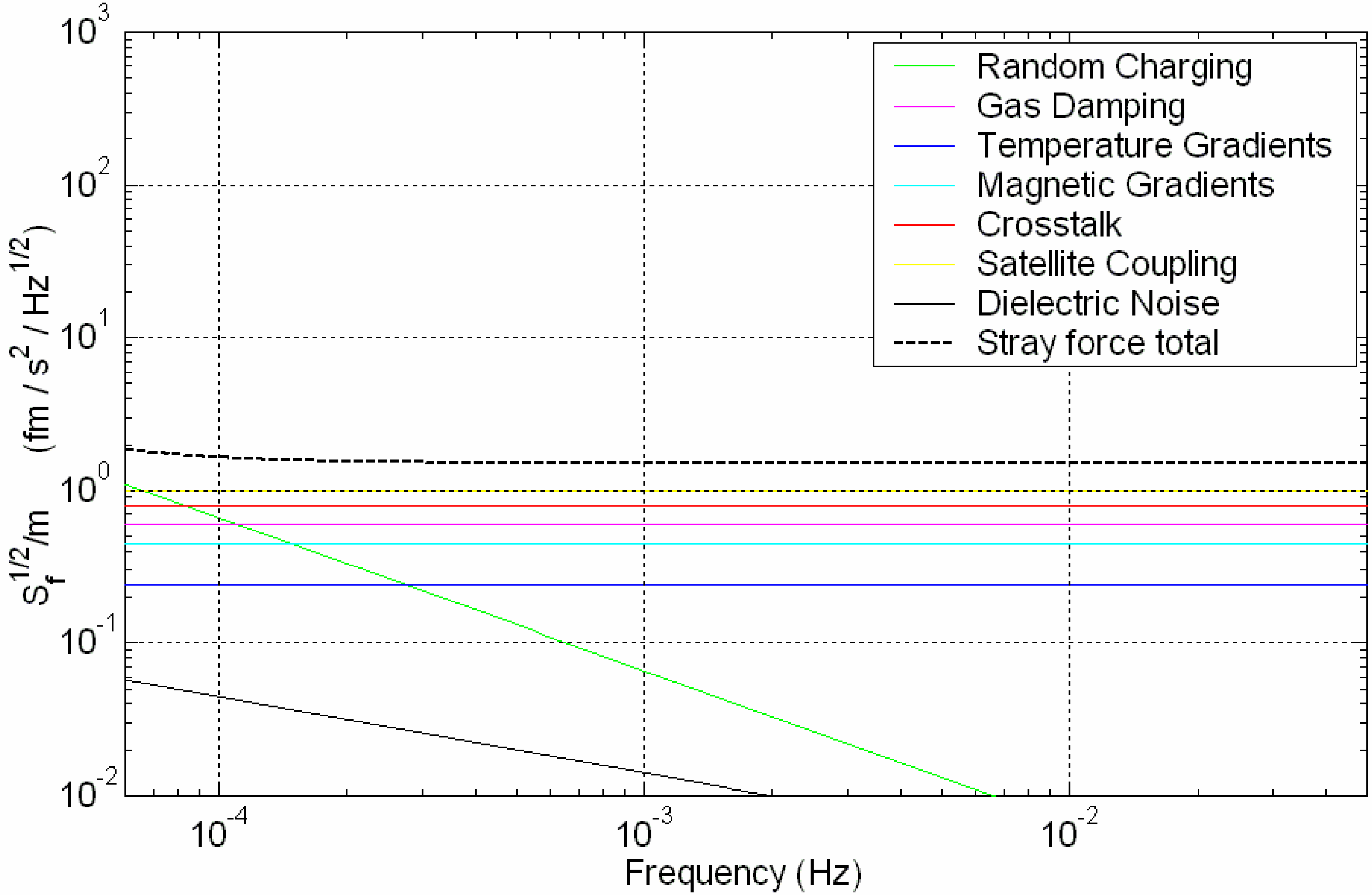
Capacitive sensing readout / actuation scheme: Modeling of position and force noise



- Bridge thermal noise at 100 kHz
- Amplifier and feedback noise
- Actuation noise at DC, 100 kHz, and f_{ACT}
- Voltage and component stability
- DC biases and test mass charging
- Low frequency thermal noise

Disturbances analyzed for readout noise, but also as force noise sources

Acceleration noise projections for LISA



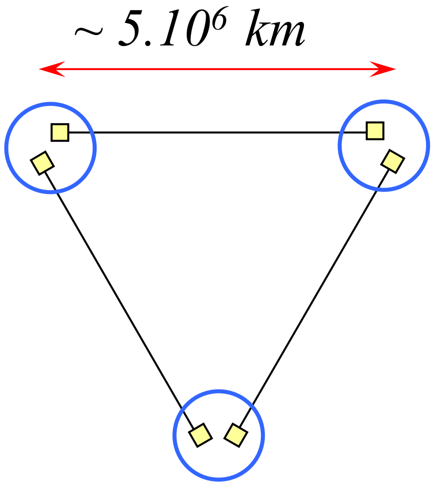
[Note: "worst case," assume performance at 0.1 mHz across whole band]

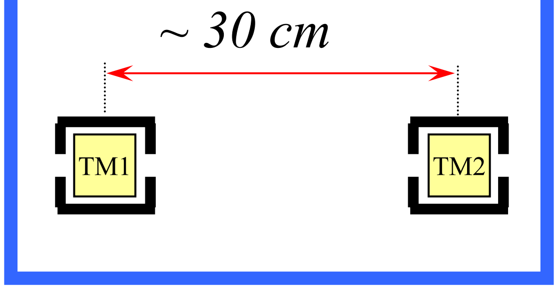
How do we verify these predictions for acceleration noise?

ESA / NASA LISA Pathfinder Mission

Launch 2008

Testing TM free-fall purity to within an order of magnitude of the LISA goals

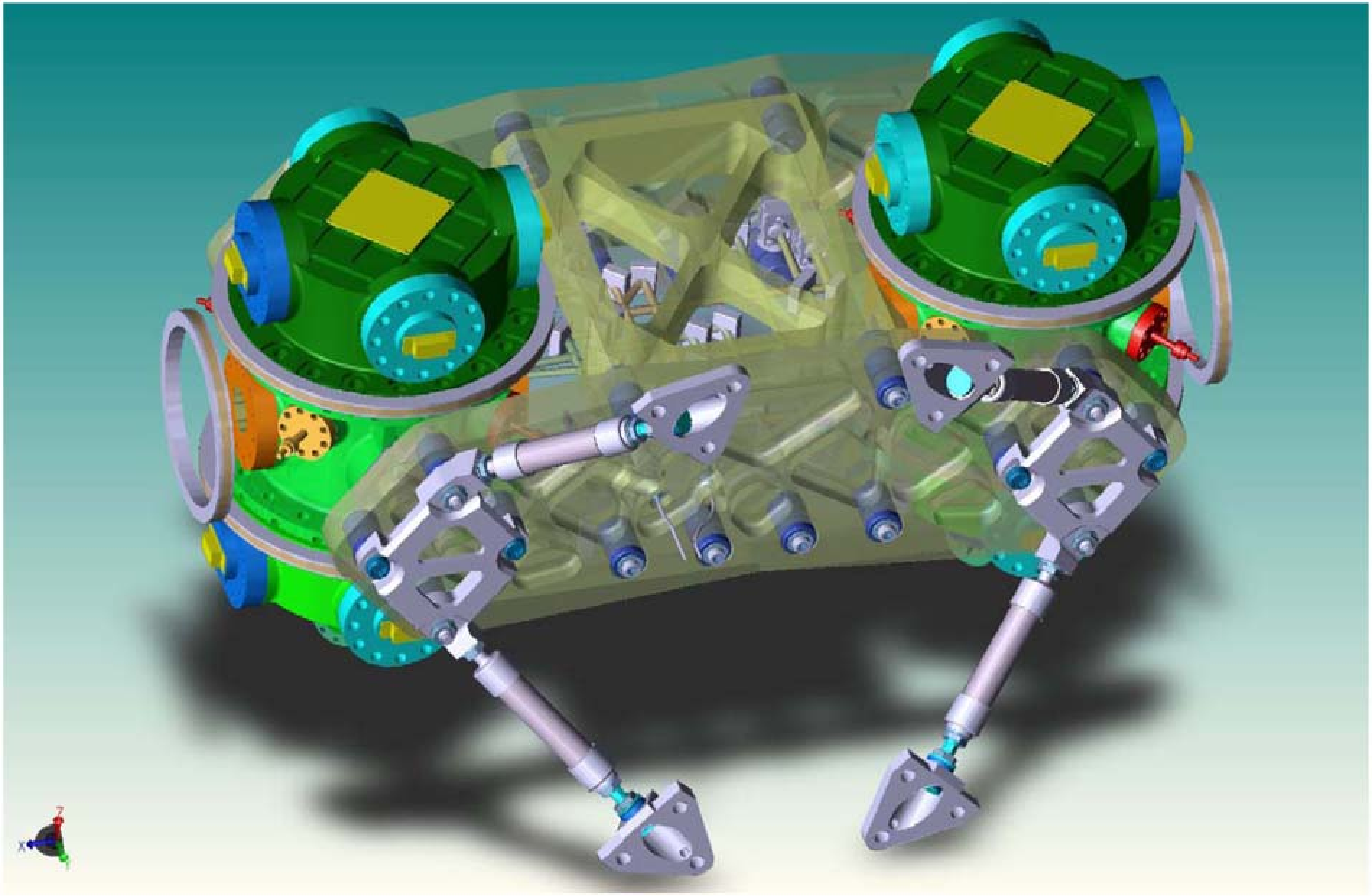




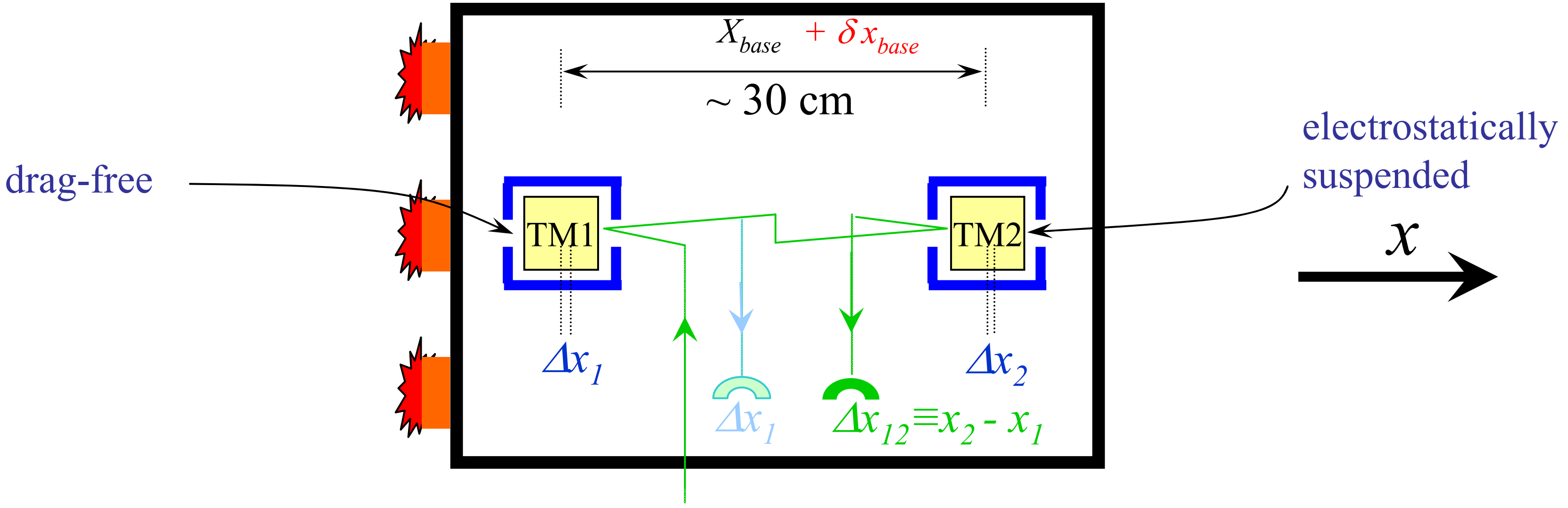
LISA: $a_{res} < 3\ 10^{-15}\ \mathrm{m/s^2/Hz^{1/2}}$
$f > 0.1\ \mathrm{mHz}$

LTP: $a_{res} < 30\ 10^{-15}\ \mathrm{m/s^2/Hz^{1/2}}$
$f > 1\ \mathrm{mHz}$

LISA Technology Package (LTP) aboard LISA Pathfinder



LTP Configuration, Dynamics, and Measured Quantities



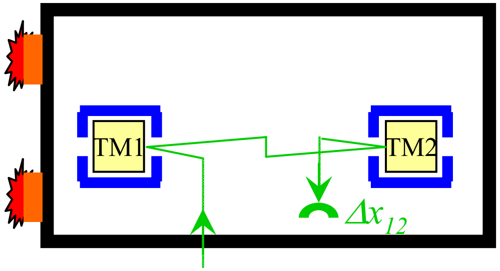
			Noise:
Capacitive position sensors	Relative displacements	$\Delta x_1, \Delta x_2$	x_{n1}, x_{n2}
Optical interferometer	Differential displacement	Δx_{12}	$x_{n,opt}$
	Relative displacement	Δx_1	

2 Masses, 1 measurement axis (x)

Control scheme:

- Satellite follows the "drag-free" TM1 with drag-free gain ω_{DF}^2
- TM2 electrostatically forced to follow TM1 (null Δx_{12}) with gain ω_{ES}^2
- Relative displacement Δx_{12} measured with interferometer to probe drag-free performance
- Note: $\omega_{ES}^2 , \omega_p^2 \ll \omega^2 \ll \omega_{DF}^2$

LTP Measurement of stray force noise f_{str}



$$\Delta x_{opt} = \frac{1}{\omega^2 - \left(\omega_{2p}^2 + \omega_{ES}^2\right)} \left\{ \frac{f_1 - f_2}{m} - \delta x \omega_{2p}^2 + x_{n,opt}\left(\omega^2 - \omega_{2p}^2\right) \right\}$$

$$S^{1/2}_{\Delta x_{opt}} > \frac{1}{\omega^2 - \left(\omega_{2p}^2 + \omega_{ES}^2\right)} \frac{\sqrt{2} S_f^{1/2}}{m}$$

- for $x_{n,opt} \sim .1\ \mathrm{nm/Hz}^{1/2}$, measure random differential force noise $S^{1/2}_{\Delta f}$ to $\sim 5\ \mathrm{fN/Hz}^{1/2}$

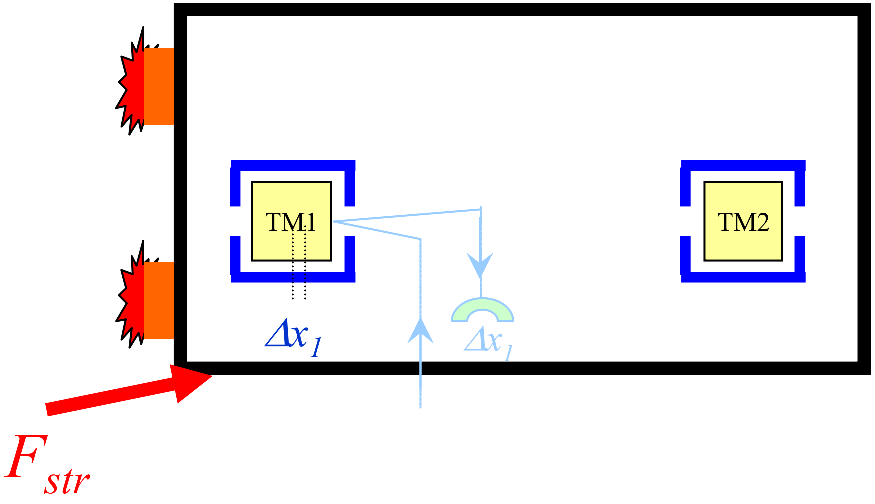
LISA

$$a_{res} = \frac{f_{str}}{m} + \omega_p^2 \left(x_n + \frac{F_{str}}{M \omega_{DF}^2} \right)$$

LTP Measurement of External Force and Sensor Noise

- Closed-loop: satellite control nulls the sensor 1 output to an accuracy limited by the finite gain control loop response to external forces

$$S^{1/2}_{\Delta x_{1s}} \approx \frac{S^{1/2}_{F_{str}}}{M\omega^{2}_{DF}}$$



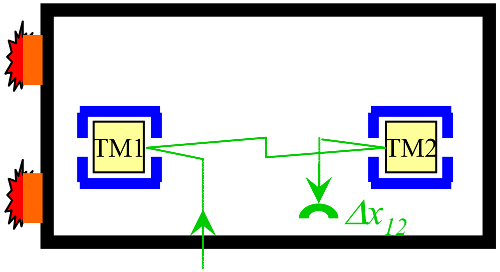
- optical interferometry measurement of TM1 with respect to satellite gives redundant, higher precision measurement of Δx_1 → measure sensor noise $S^{1/2}_{x_{1n}}$

LISA

$$a_{res} = \frac{f_{str}}{m} + \omega_p^2 \left(\enclose{circle}{x_n} + \enclose{circle}{\frac{F_{str}}{M\omega_{DF}^2}} \right)$$

Drag-free control setpoint modulation: stiffness measurement

- control satellite to TM1 to a modulated setpoint $x_0 \sin \omega t$
- control TM2 to follow TM1 (mode 3)
- “shake” satellite, observe differential motion



$$\Delta x_{opt} \approx x_0 \frac{\omega_{2p}^2 - \omega_{1p}^2}{\omega^2 - (\omega_{2p}^2 + \omega_{ES}^2)}$$

- acceleration noise limited differential stiffness resolution:

$$\Delta(\Delta\omega^2) \approx 10^{-8}/\mathrm{s}^2 \times \left(\frac{100\ \mathrm{nm}}{x_0}\right)\left(\frac{S_a^{1/2}}{30\ \mathrm{fm/s^2/Hz^{1/2}}}\right) \times \left(\frac{1\ \mathrm{h}}{T}\right)^{1/2}$$

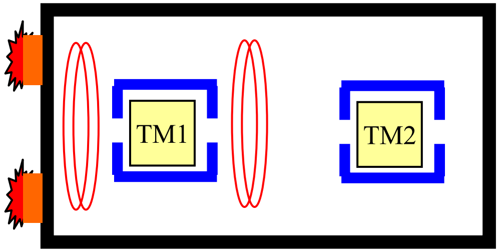
- Roughly 2% of LISA stiffness goal of 4 10^{-7} /s^2
- Other schemes allow 10-20% absolute stiffness measurement via sensor signal

LISA

$$a_{res} = \frac{f_{str}}{m} + \boxed{\omega_p^2}\left(x_n + \frac{F_{str}}{M\omega_{DF}^2}\right)$$

Coherent force measurements:

Magnetic field effects

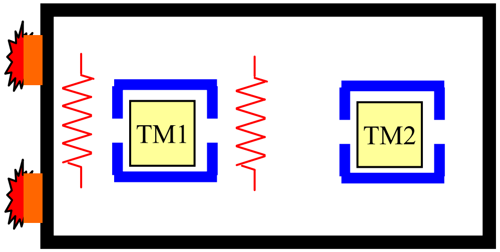


$$F_x \approx \frac{\partial}{\partial x}\left(\left(\vec{M} + \frac{\chi V \vec{B}}{\mu_0}\right).\vec{B}\right)$$

Measurement of coupling with magnetometer and field/gradient coils

Thermal gradient effects

- radiation pressure difference
- radiometric effects
- temperature dependent outgassing

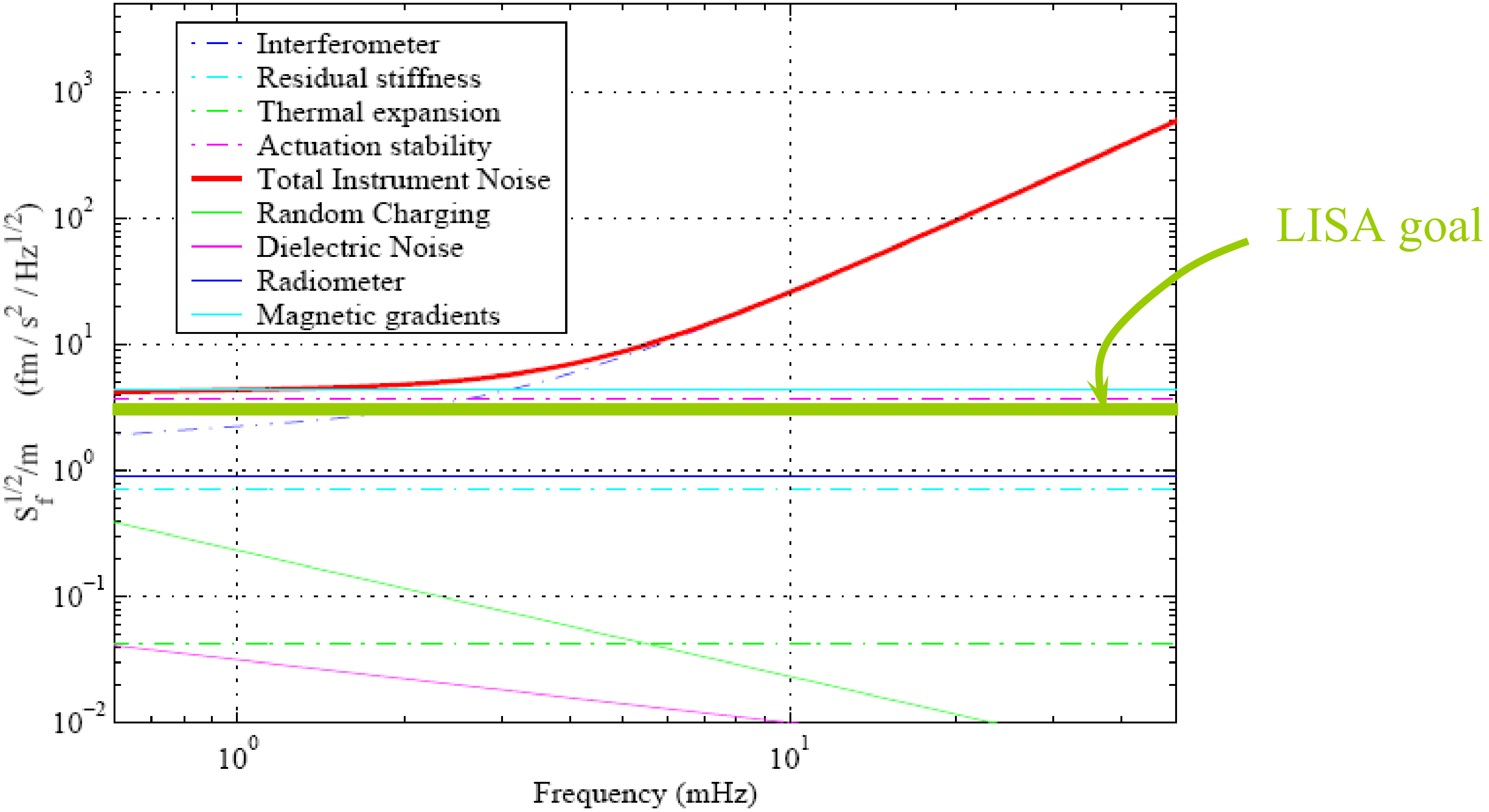


Measurement of coupling with thermometers and heaters

- Measurement of disturbance time series allows correlation analysis of noise sources, measurement of actual coupling parameter allows possible correction
- LTP is a true experiment, “debuggable”

LTP “instrument noise limit”

- resolution with which we can measure LISA force noise
- 5 fm/s^2/Hz$^{1/2}$ (within 2 of LISA goal at 1 mHz)
- limited by interferometer and actuation noise

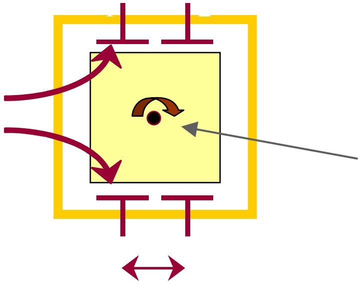


Torsion pendulum measurements of small forces originating in gravitational reference sensor

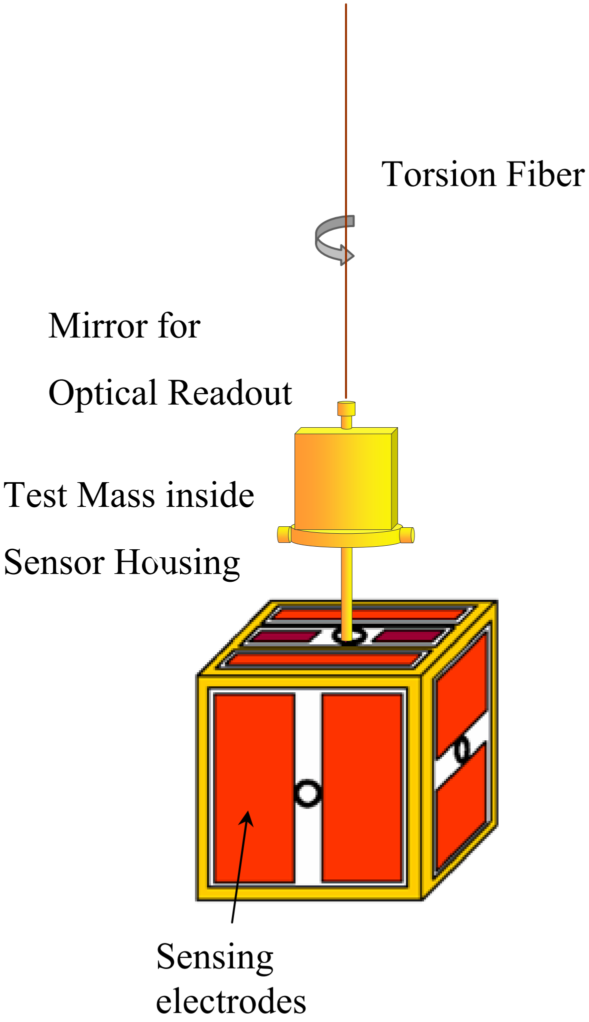
Light-weight test mass suspended as inertial member of a low frequency torsion pendulum, surrounded by sensor housing

Measure stray forces as deflections of pendulum angular rotation
to within 100x LISAgoal,
10x LTP goal

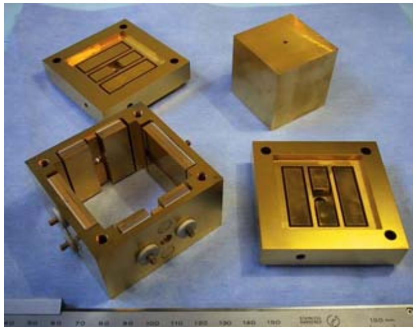
Precision coherent measurement of known disturbances



Pendulum suspension and axis of rotation

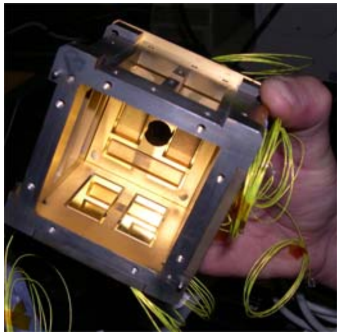


Results obtained with 2 different sensors



Trento prototype

LTP EM sensor



Design differences

Gaps: 2 mm → 4 mm

→ further reduction of short range electrostatic effects

Injection electrodes: z-axis → y and z axes

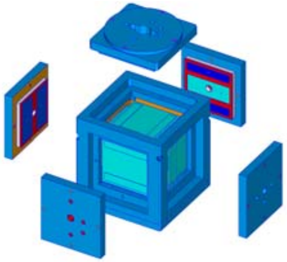
→ favors x axis

Electrode material: Au coated Mo → Au coated ceramic (shapal)

→ better machining tolerances

→ risk of exposed dielectric

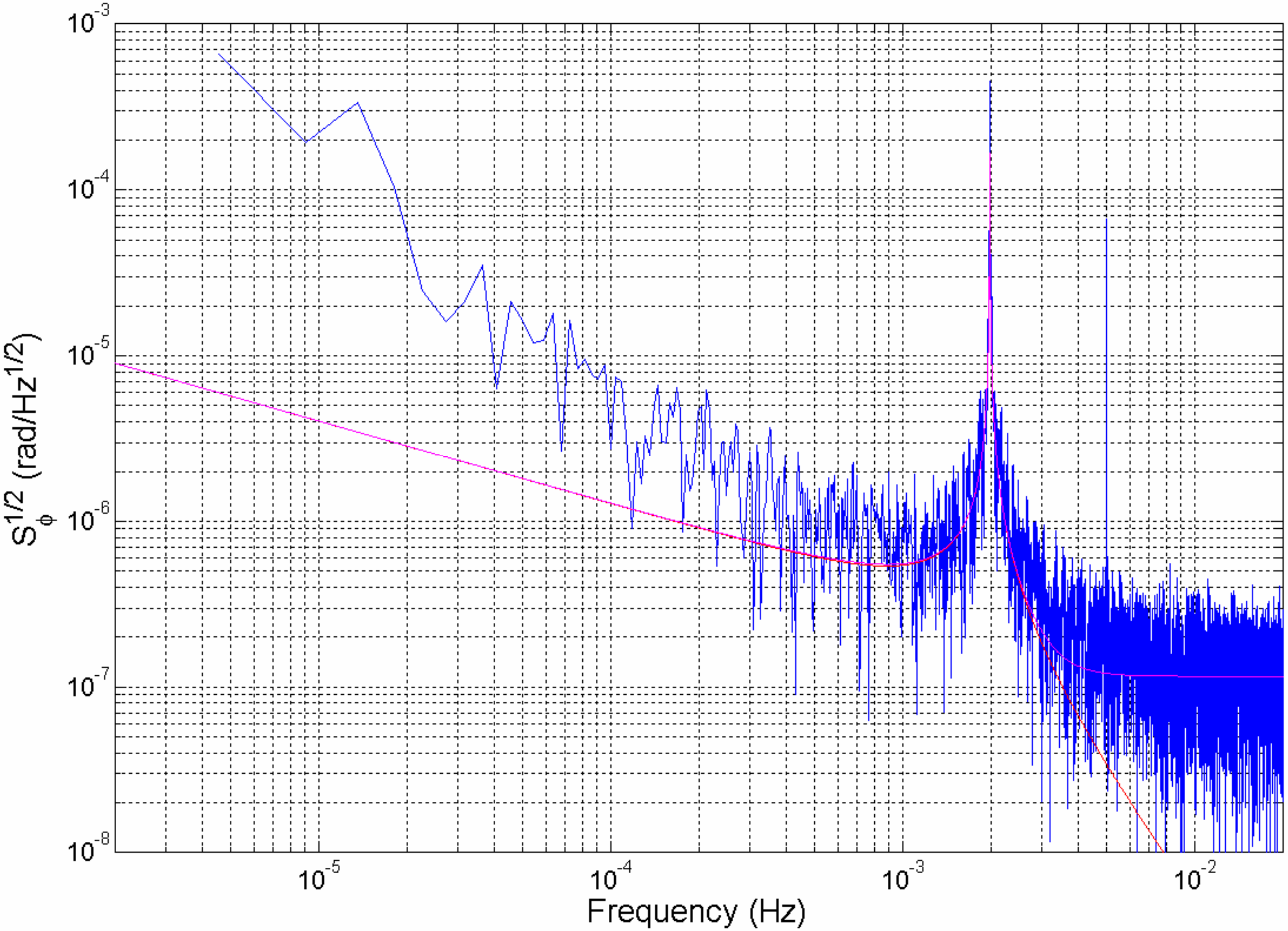
Construction techniques: HV glue / screws, pin contacts



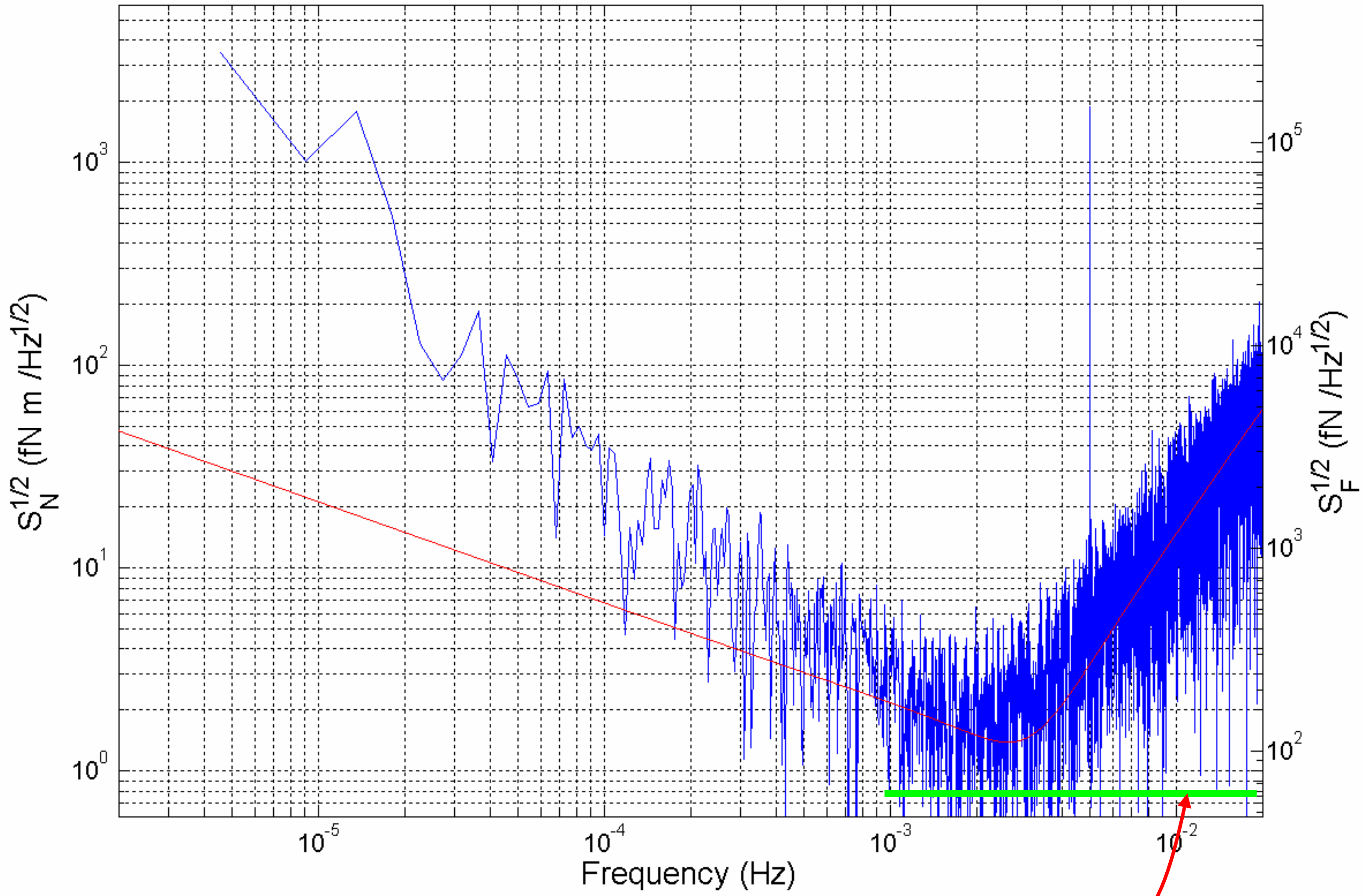
LTP Flight Model Sensor – Au coated sapphire electrodes

Force noise measurements: more stringent upper limits

Pendulum angular deflection noise measured over 3 days

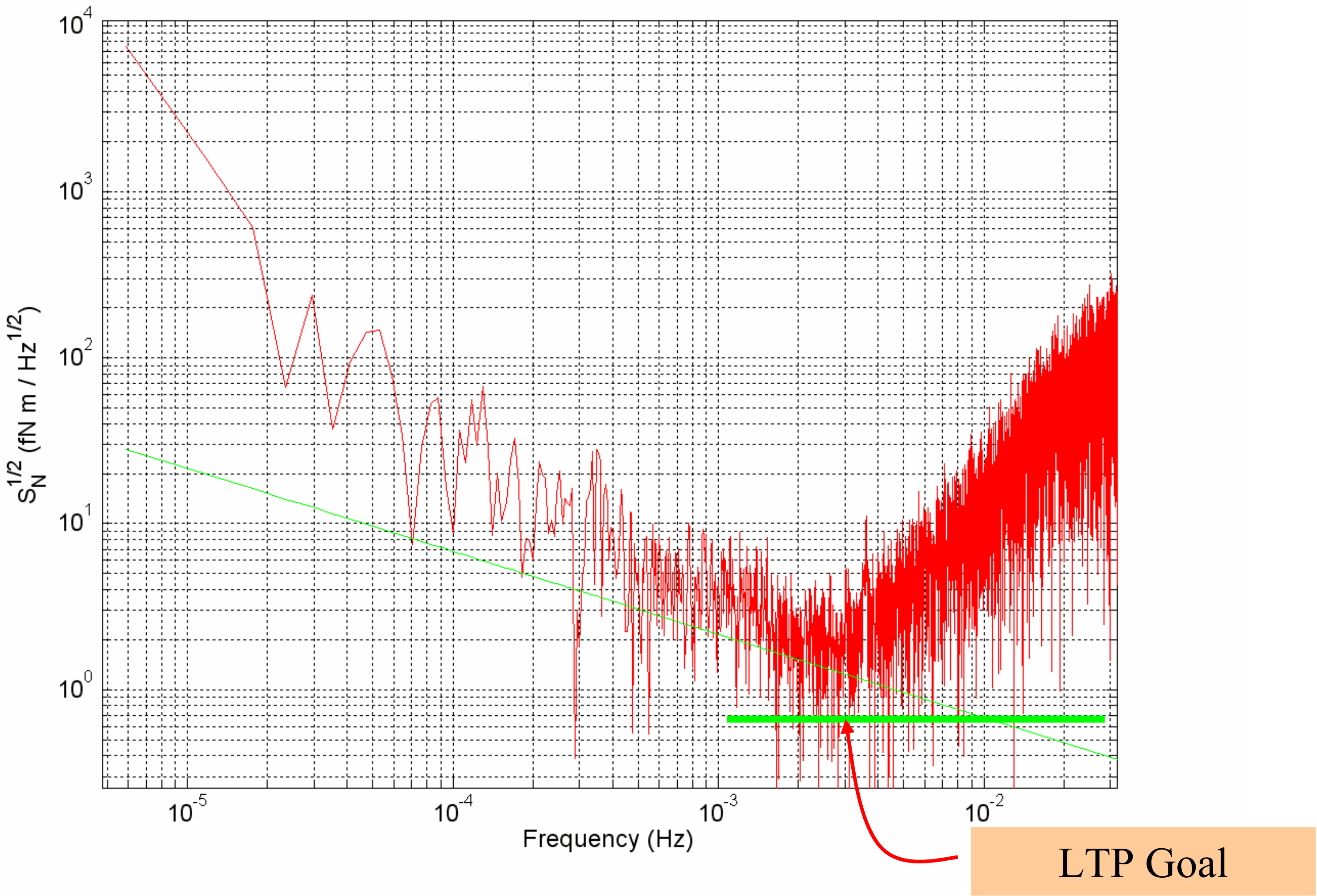


Force noise upper limits (old sensor)



LTP Goal (most pessimistic torque – force conversion, 10.25 mm)

Force noise upper limits (new sensor)



Excess noise observed below 1 mHz → rises more steeply than thermal noise
Observed with both sensors → likely pendulum (not sensor) related

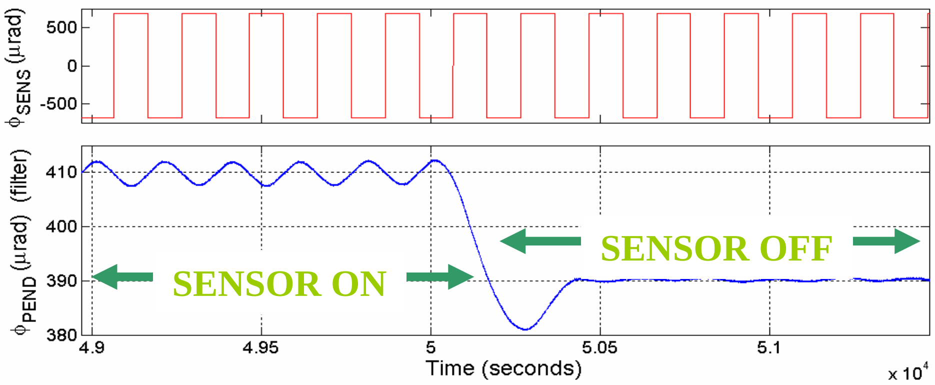
→ Currently under investigation!

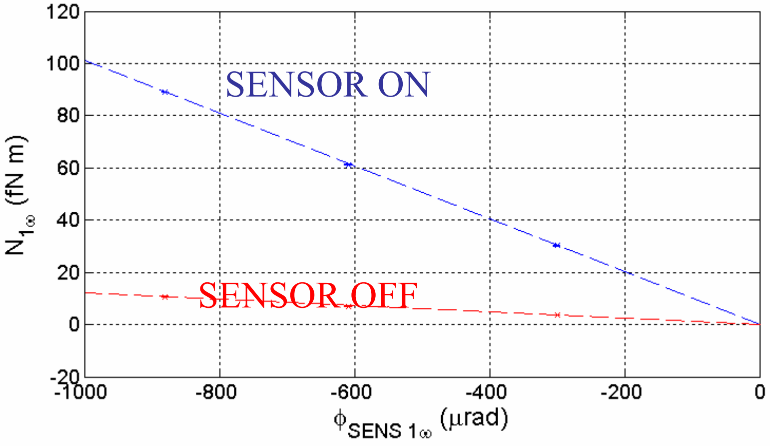
Noise source characterization
Stiffness: coupling to spacecraft motion

Move sensor (or spacecraft), measure force (or torque)

- Coherent torque excited by square wave oscillation of sensor rotation angle

- Search for all sources of stiffness, with and without sensing bias





Results: $\Gamma = \Gamma_{SENS} + \Gamma_0$

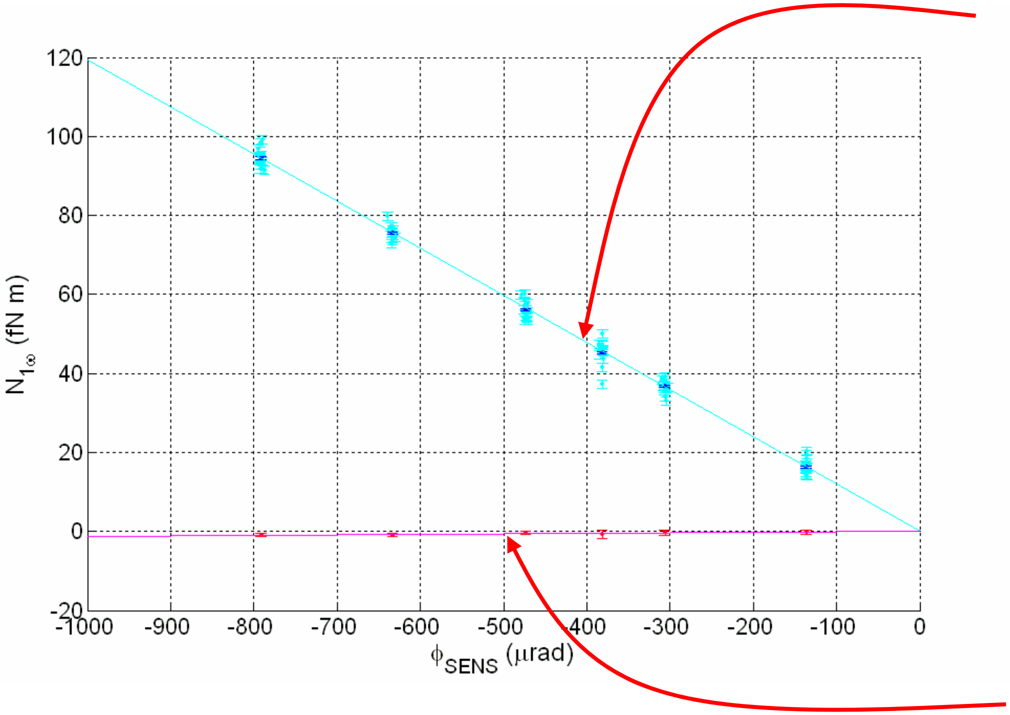
Γ_{SENS} = - 89.2 ± .5 pN m / rad

consistent with expected sensor bias stiffnes

Γ_0 = - 12.0 ± .3 pN m / rad

extra stiffness ... could be explained by 115 mV RMS patch voltages

Stiffness with 4-mm gap sensor



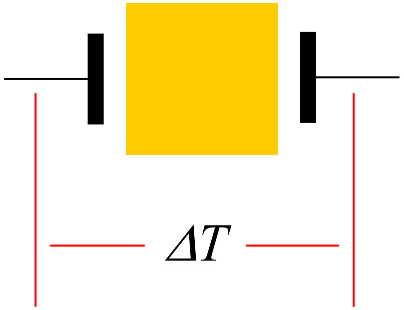
Sensor ON
electrostatic stiffness
roughly as modelled

Sensor OFF stiffness
essentially zero
→ “extra” stiffness
not observed

→ With 4 mm gap sensor, unmodelled force gradients are not likely to be an issue for LISA

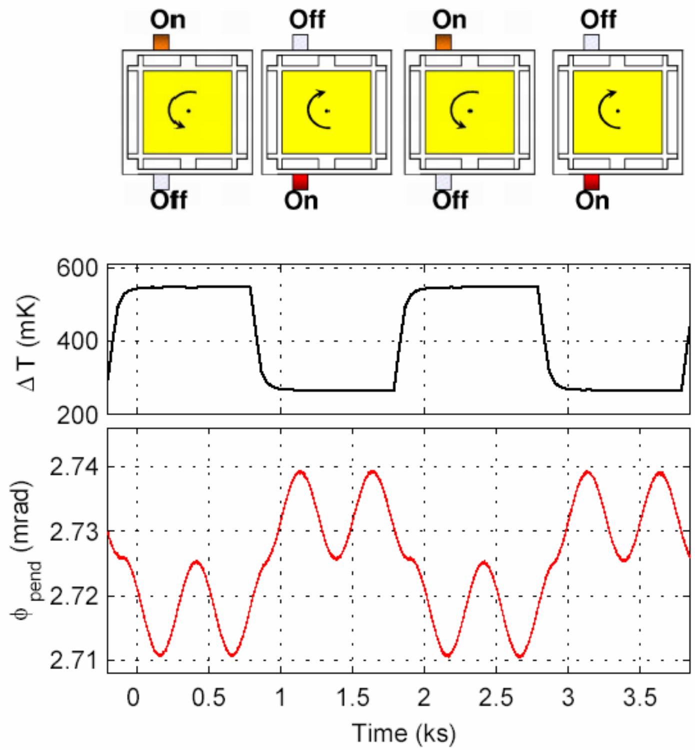
Noise source characterization

Thermal gradient measurement



(Noisy) temperature gradient converts to (noisy) force:

- radiation pressure
- radiometric effect
- temperature dependent outgassing (???)

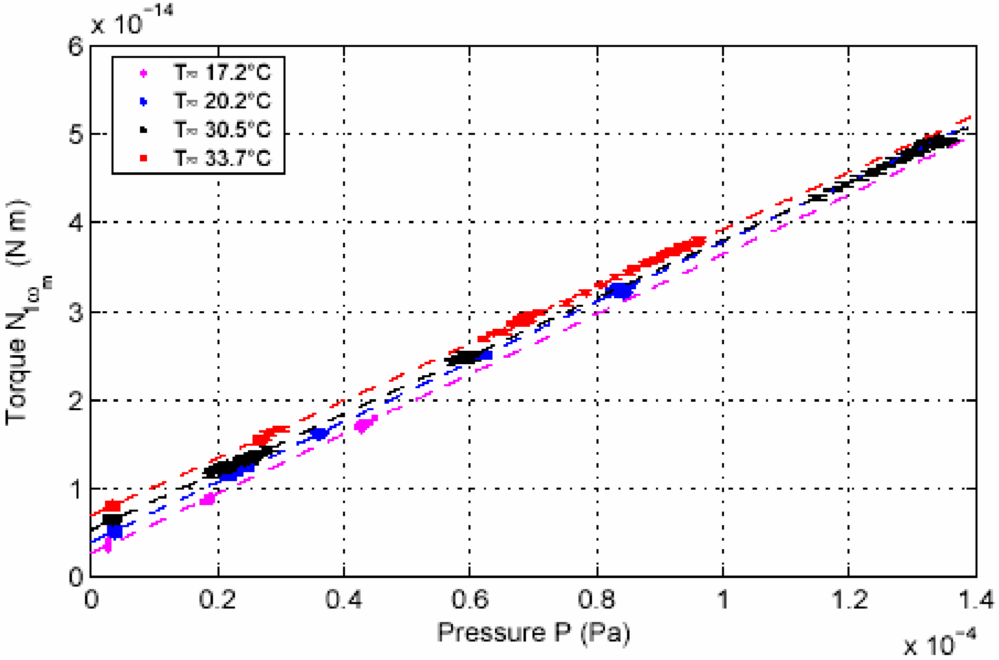


In the lab (and on LTP)

apply ΔT → measure force (torque)

Thermal gradient measurement: pressure dependence

- radiometric effect as expected $N \propto {}^{p}\!/_{T}$
- *N(p=0)* increases with temperature as expected



- measured torque is consistent with radiometric+radiation pressure effects (factor ≈ 2 uncertainty in effective *ΔT*)

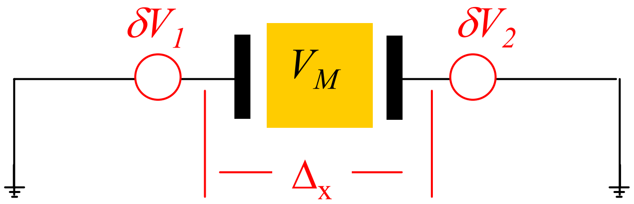
$$\frac{N(p=0)}{\left(\dfrac{\partial N}{\partial p}\right)}$$

Largely independent of *ΔT*, geometry

Measured value ≈ 1 10^{-7} mBar
Theoretical ≈ 1.5 10^{-7} mBar

- we actually see too small a torque coefficient
- radiation pressure effect probably overestimated (not infinite plates)
- **any temperature dependent outgassing effect is too small to hurt LISA**

Noise source: DC biases



$$k \equiv -\frac{\partial F}{\partial x} = -\frac{1}{2}\sum \frac{\partial^2 C_i}{\partial x^2}(\delta V_i)^2$$

Electrostatic stiffness

$$S_F^{1/2} = \frac{S_Q^{1/2}}{C_T}\sum \frac{\partial C_i}{\partial x}\delta V_i = \frac{\sqrt{2e^2\lambda_{EFF}}}{\omega C_T}\left|\frac{\partial C}{\partial x}\right|\Delta_x$$

Random charge noise mixing with DC bias (Δ_x)

$$S_F^{1/2} = \frac{\langle Q \rangle}{C_T}\left|\frac{\partial C}{\partial x}\right| S_{\Delta_x}^{1/2}$$

Noisy average “DC” bias ($S_{\Delta x}$) mixing with mean charge

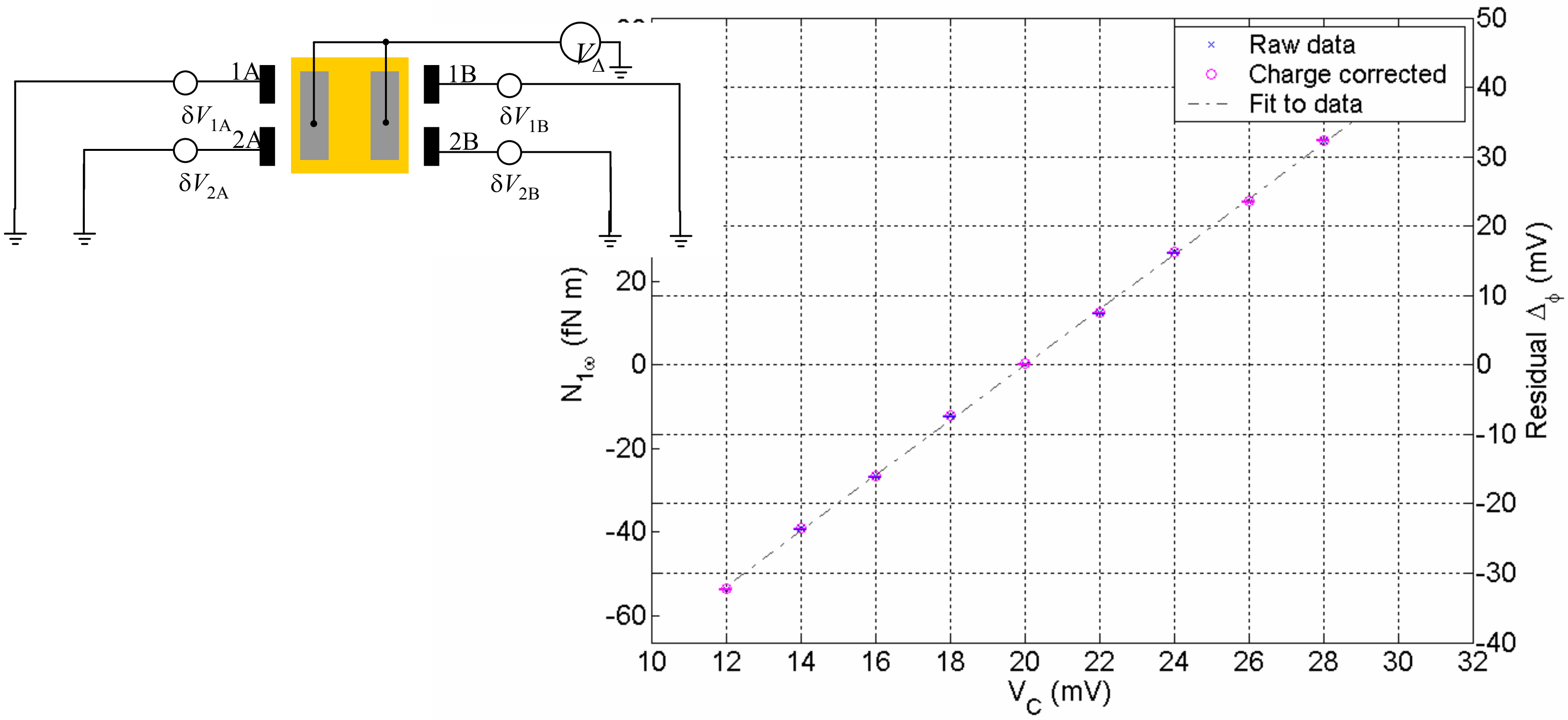
$$S_F^{1/2} = \sqrt{\sum \left|\frac{\partial C_i}{\partial x}\right|^2 \delta V_i^2 S_{\delta V_i}}$$

Noisy “DC” biases interacting with themselves

Individual noise source characterization
DC Bias: measurement and compensation

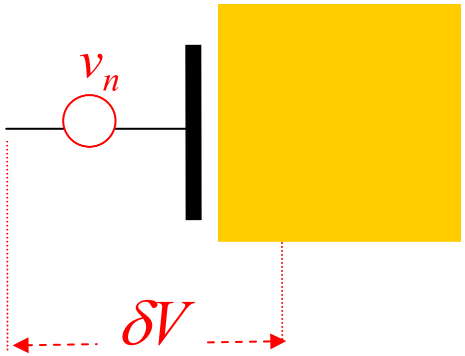
Average DC bias difference couples to charge shot noise

- Apply “charge”, measure force (extract ΔV)
- Compensate ΔV



- DC biases of order 10’s of mV would be a relevant noise source
- Sub-mV compensation demonstrated with torsion pendulum, possible in flight
- Random charging should not be problematic under normal conditions

Noise source: in-band voltage noise mixing with DC bias



$$F \approx -\frac{C}{d}\delta V \;\; v_{\mathrm{n}}$$

Voltage noise: v_{n}

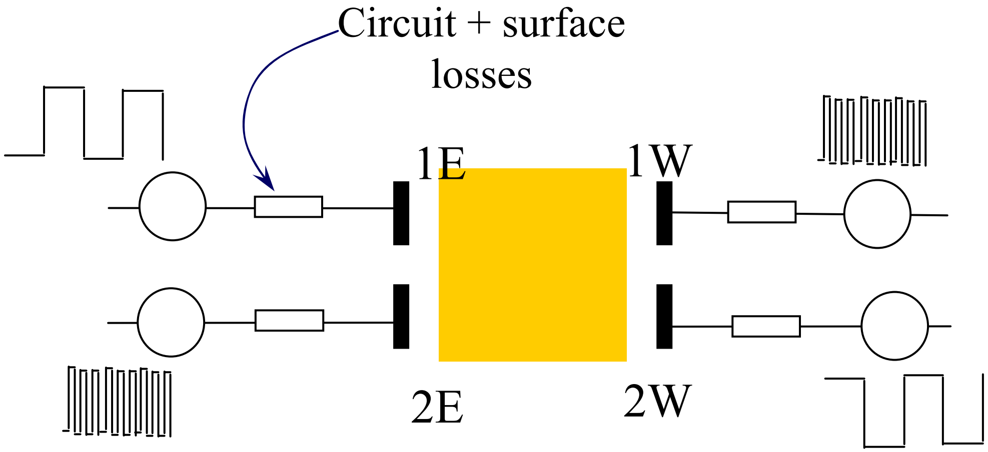
- Actuation amplifier noise (electronics)
- Thermal voltage fluctuations (δ)
- Drifting (not Brownian) DC bias $S_{\delta V}^{1/2}$

DC voltage difference: δV

- Residual unbalanced patch effects
- Test mass charge

LISA requires $v_{\mathrm{n}} \approx 20\ \mu\mathrm{V/Hz}^{1/2}$

Measurement of dielectric losses: new direct measurement technique



Force (torque) quadratic in voltage

$$F \propto V^2$$

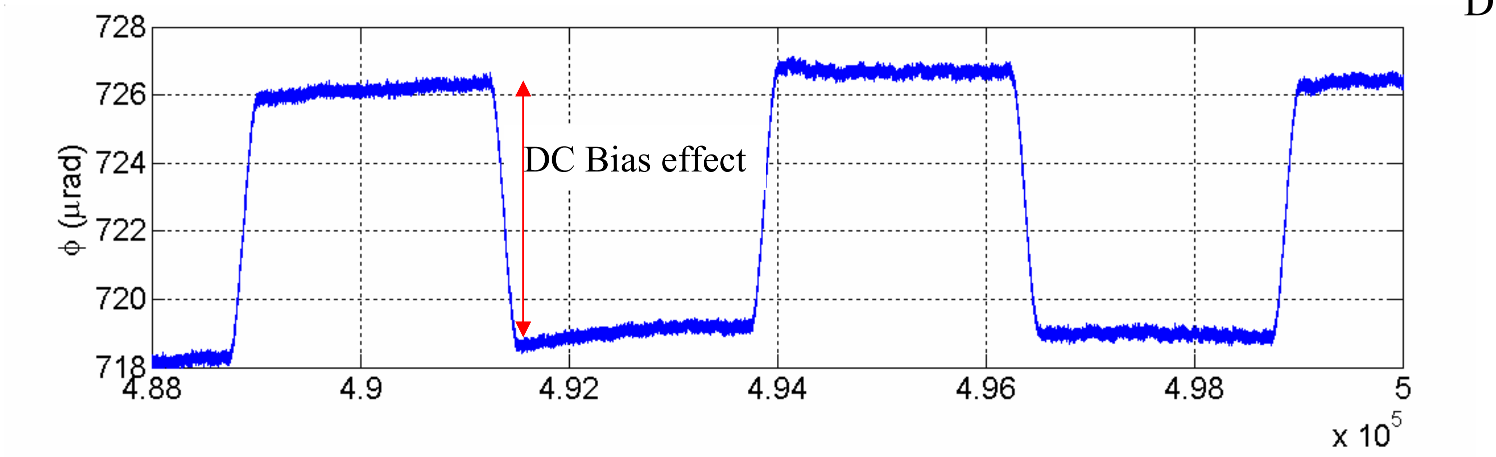
perfect square wave voltage produces only DC force (torque)

Electrode voltage:

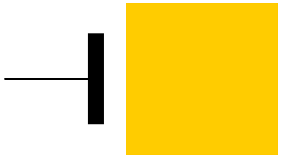
Force:

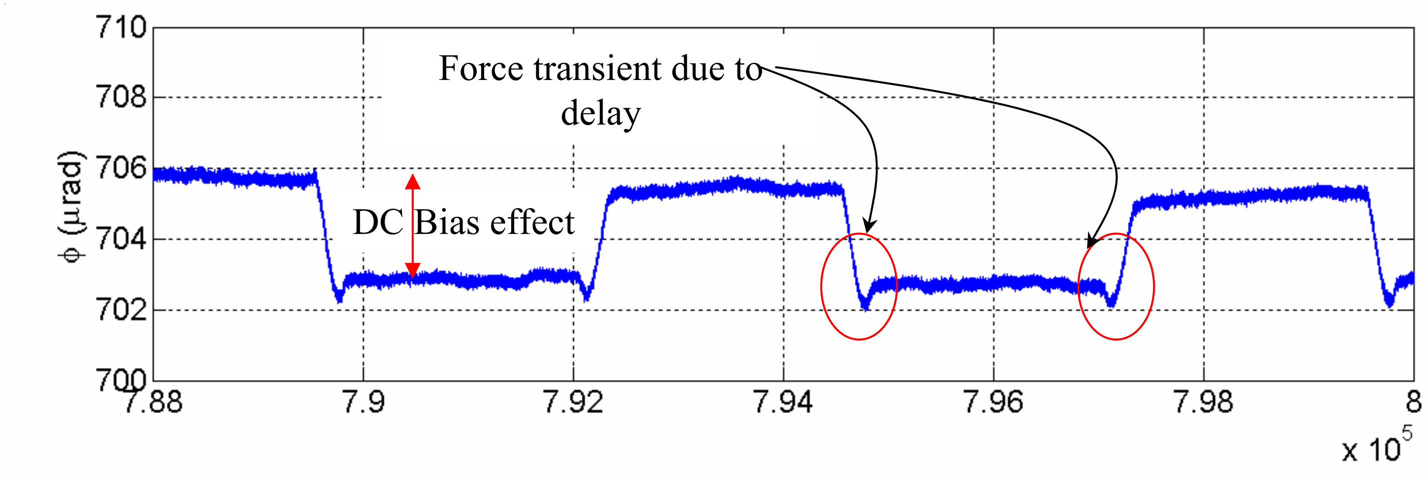
Measurement of dielectric losses: new direct measurement technique

Application of perfect square wave yields constant force
Any lossy element creates delays and thus force transients

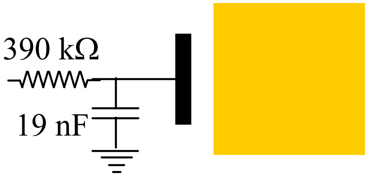


Direct application
(f = .4 mHz)

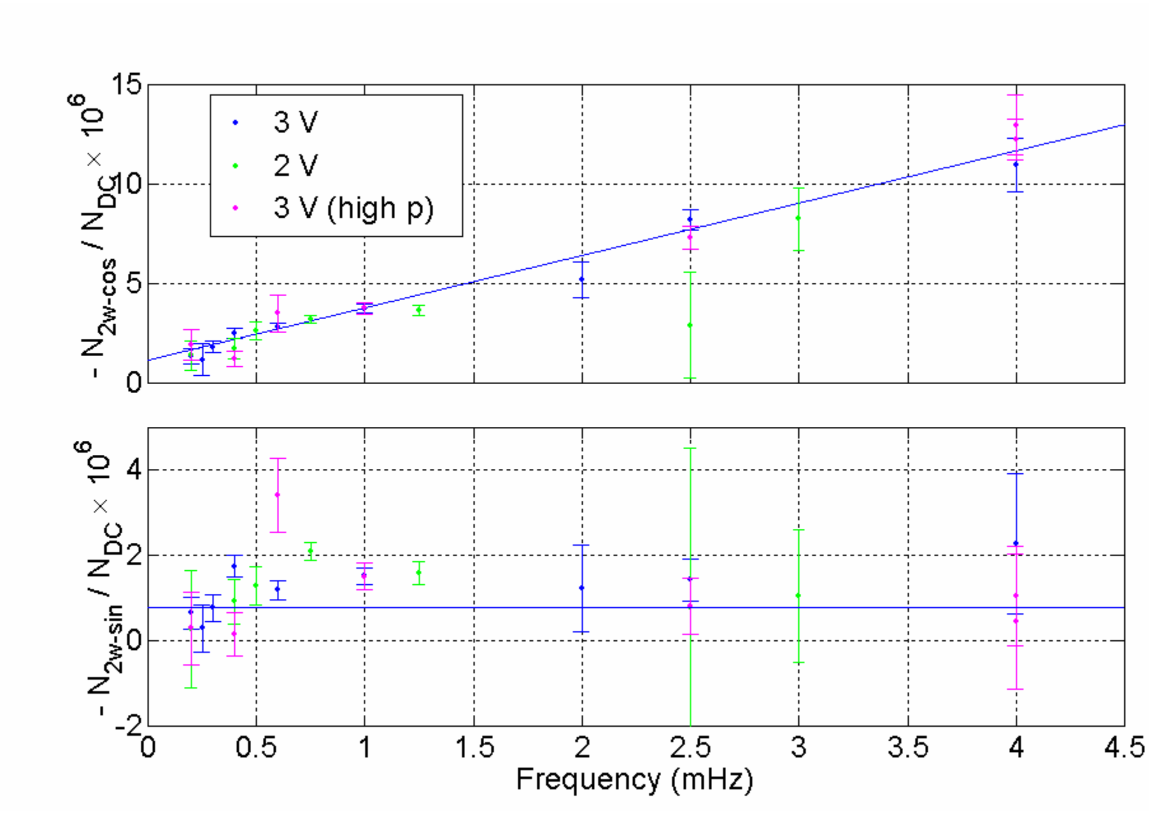




Application through an ohmic delay
($\tau \approx 7$ ms, $\delta \approx 2\ 10^{-5}$)



Dielectric Loss Angle Measurement Results

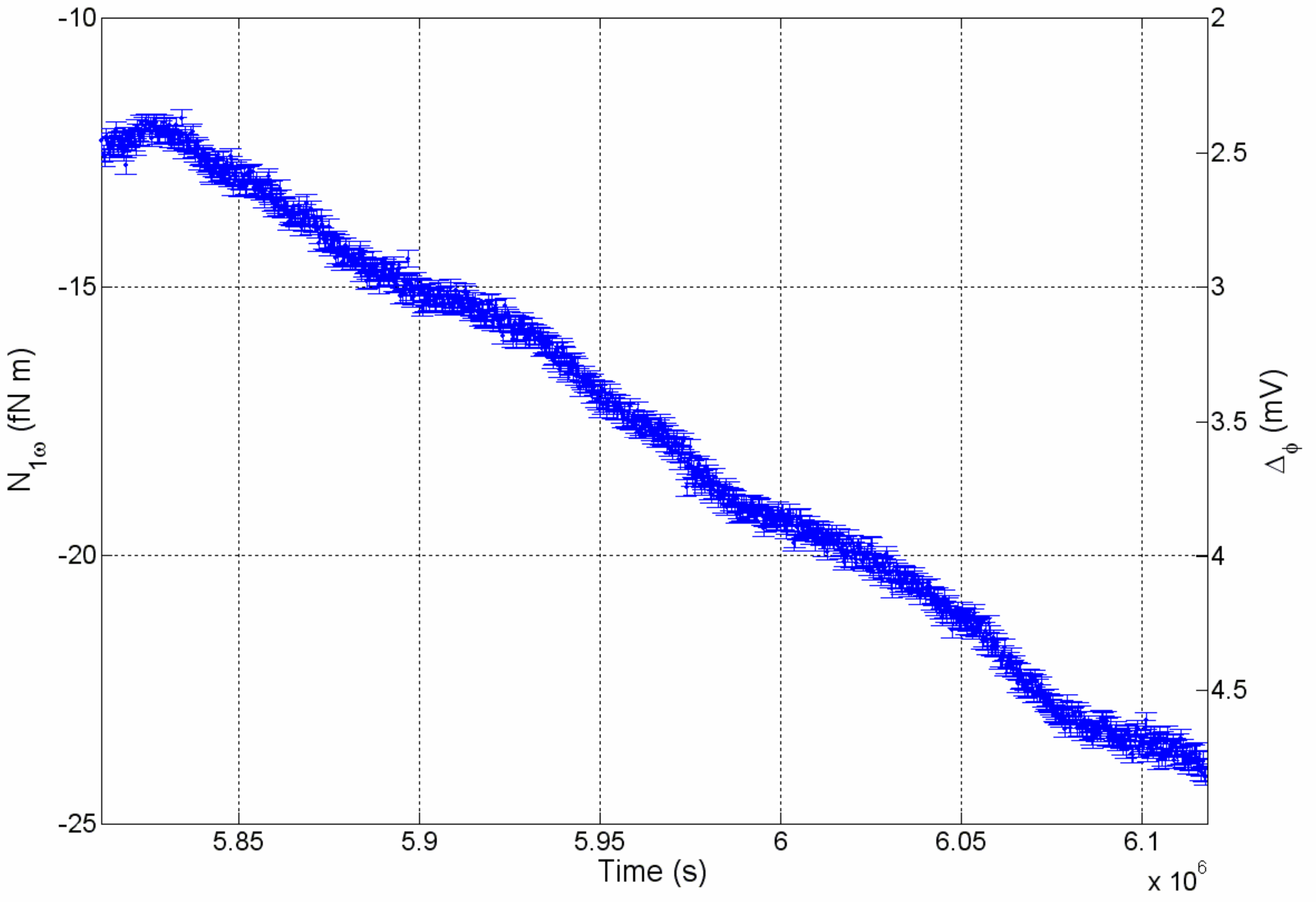


- 2ω cosine torque frequency dependence → ohmic delay time $\tau \approx 0.3$ ms (agrees with calculated value)
- 2ω sine + cosine intercept values → $\delta \approx 10^{-6}$ (likely not a problem for LISA!!)

Electrodes 2W/1E	Averaged sine data		Linear fitted cosine data		
	δ ($/10^{-6}$)	χ^2	**τ (ms)**	δ ($/10^{-6}$)	χ^2
3 V (p ≈ 5.e-8 mBar)	.79 ± .07	1.8	.33 ± .02	1.06 ± .16	.86
2 V (p ≈ 5.e-8 mBar)	1.08 ± .09	1.36	.23 ± .05	1.48 ± .31	1.27
3 V (p ≈ 4.e-5 mBar)	.73 ± .14	2.25	.36 ± .03	.60 ± .27	1.27

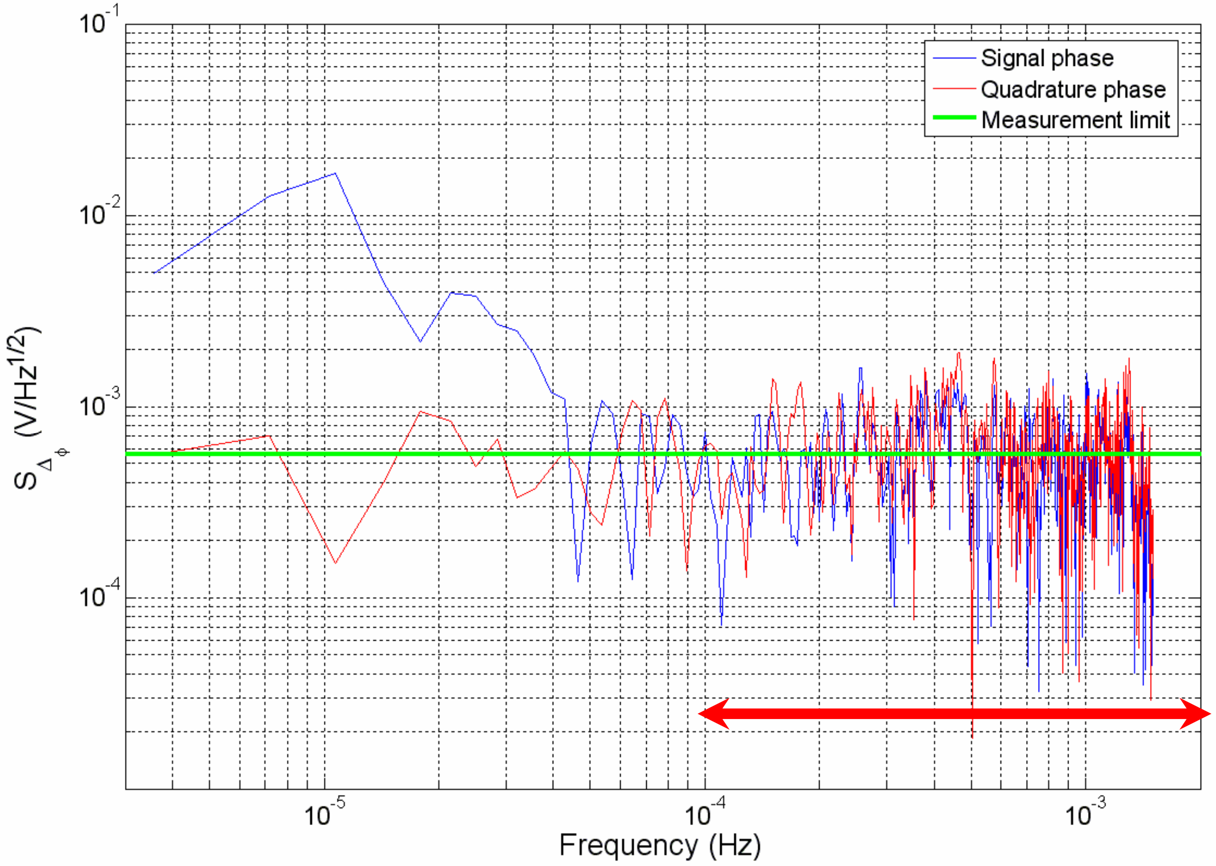
DC Bias measurements: stability

4 day measurement of residual DC balance stability after compensation



- Observe long term drifts in the DC bias imbalance of mV over several days

DC Bias measurements: stability



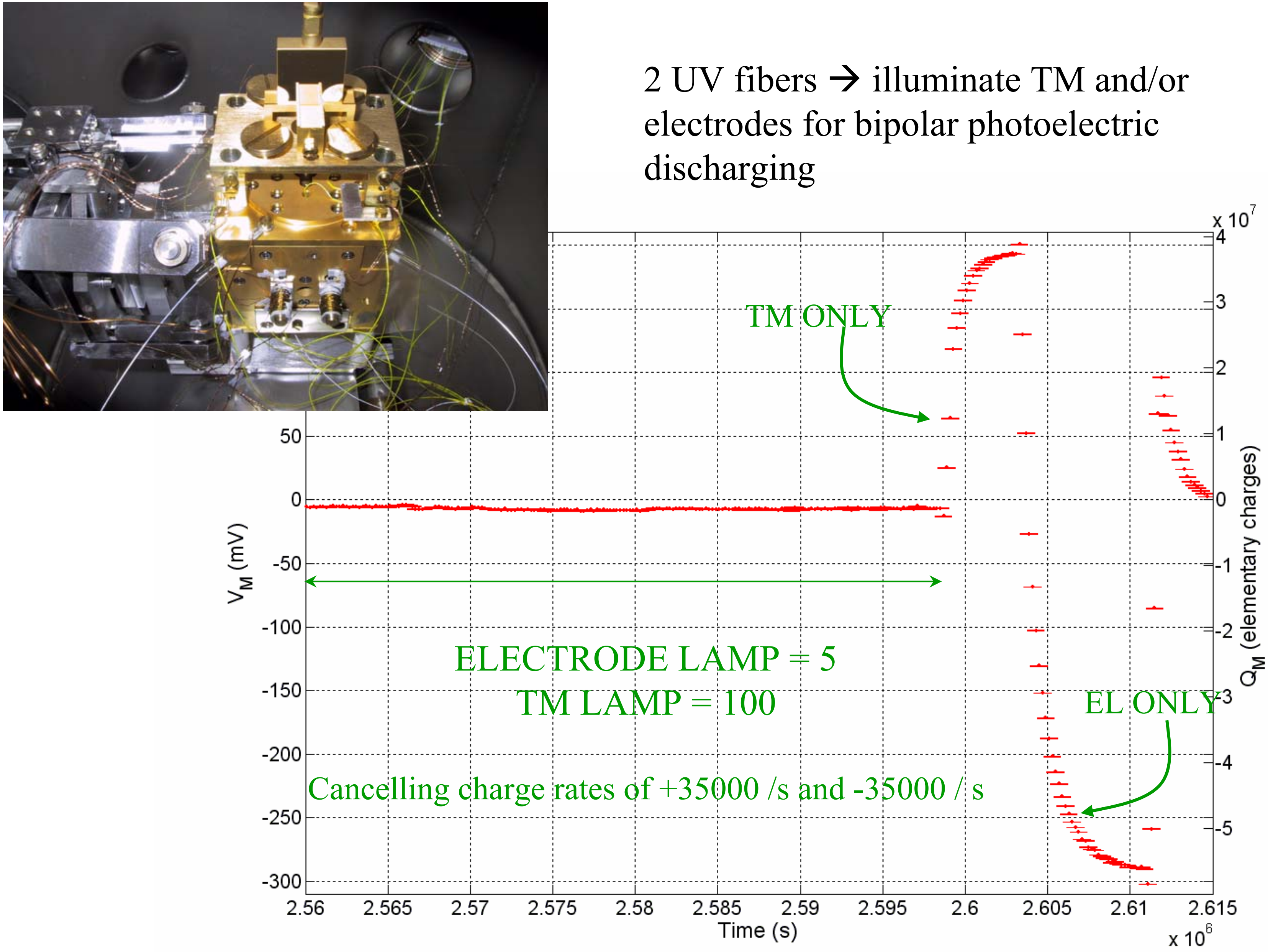
LISA goal

- Limited by pendulum force noise measurement resolution above 50 μHz
- excess noise (drifting) below 50 μHz

- current measurement resolution not sufficient to guarantee LISA performance!

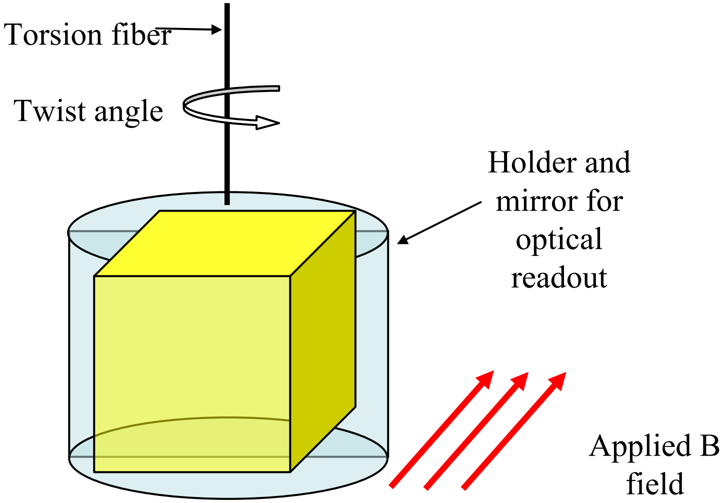
Continuous charging with UV light

2 UV fibers → illuminate TM and/or electrodes for bipolar photoelectric discharging



Magnetic testing of full Au – Pt test mass

- Measuring LISA TM magnetic properties (residual moment and susceptibility) with a torsion pendulum



$$\vec{m}(t) = \vec{m}_0 + \frac{\chi V}{\mu_0}\vec{B}(t)$$

$$\left|\vec{N}(t)\right| = \left|\vec{m}(t) \wedge \vec{B}(t)\right|$$

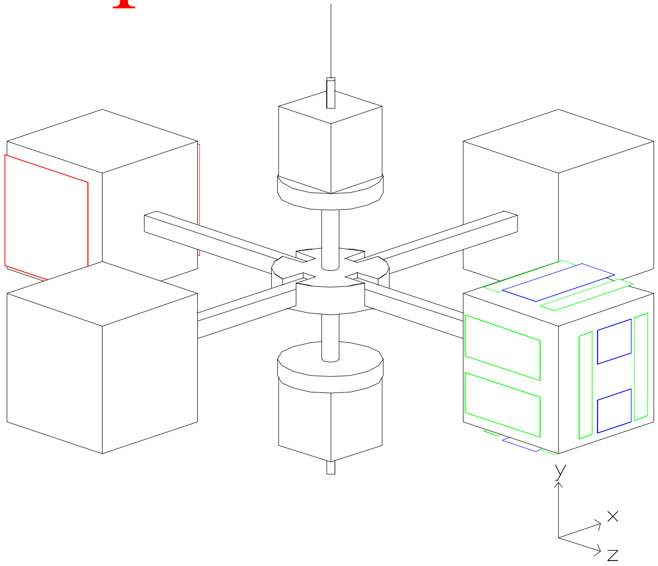
- Measure moment detection with pendulum deflection in homogeneous field

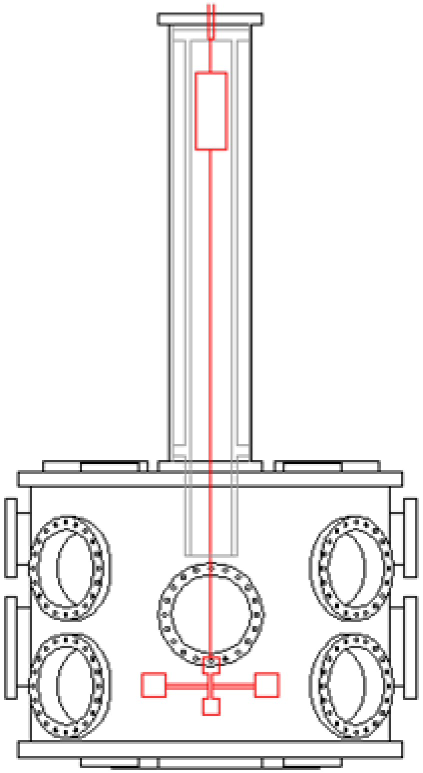
$$\boxed{\left|\vec{m}_0\right| \leq 10^{-8}\,\mathrm{A}\cdot\mathrm{m}^2}$$

- Measurement of susceptility (c) requires non-zero second derivative of B (2*f* signal, analysis in progress)

$$\boxed{\left|\chi\right| \leq 10^{-5}}$$

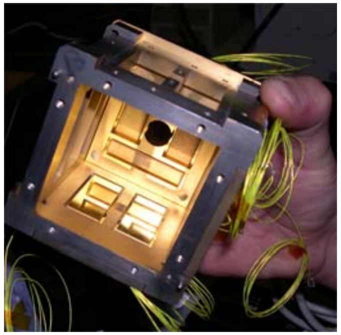
Development of Four-mass torsion pendulum





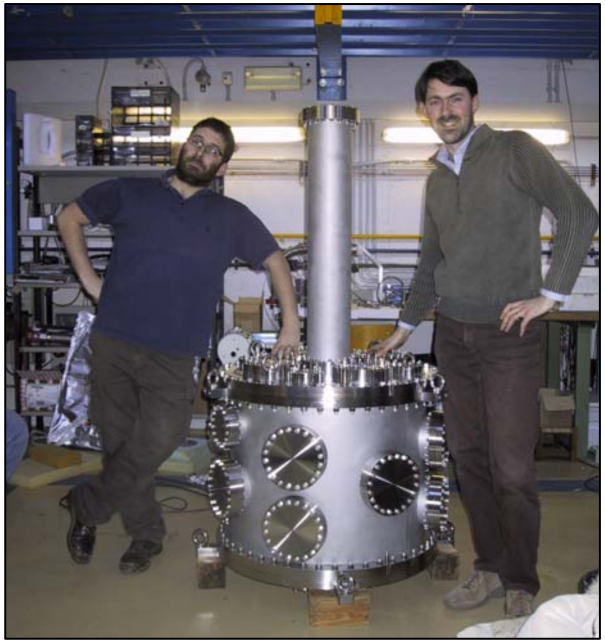
- in LTP / LISA, force matters (not torque!)
- Direct sensitivity to net forces (F_x rather than just N_ϕ) not achievable with 1-mass pendulum design

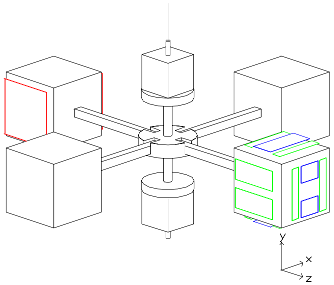
  - thermal outgassing, DC electrostatic problems could arise at central edges of the electrodes
  - translational stiffness qualitatively different from rotation stiffness with current electrode design



Four-mass pendulum: facility construction

Go big(*) or stay home!





Torque signal $\propto R$

(*) How big?

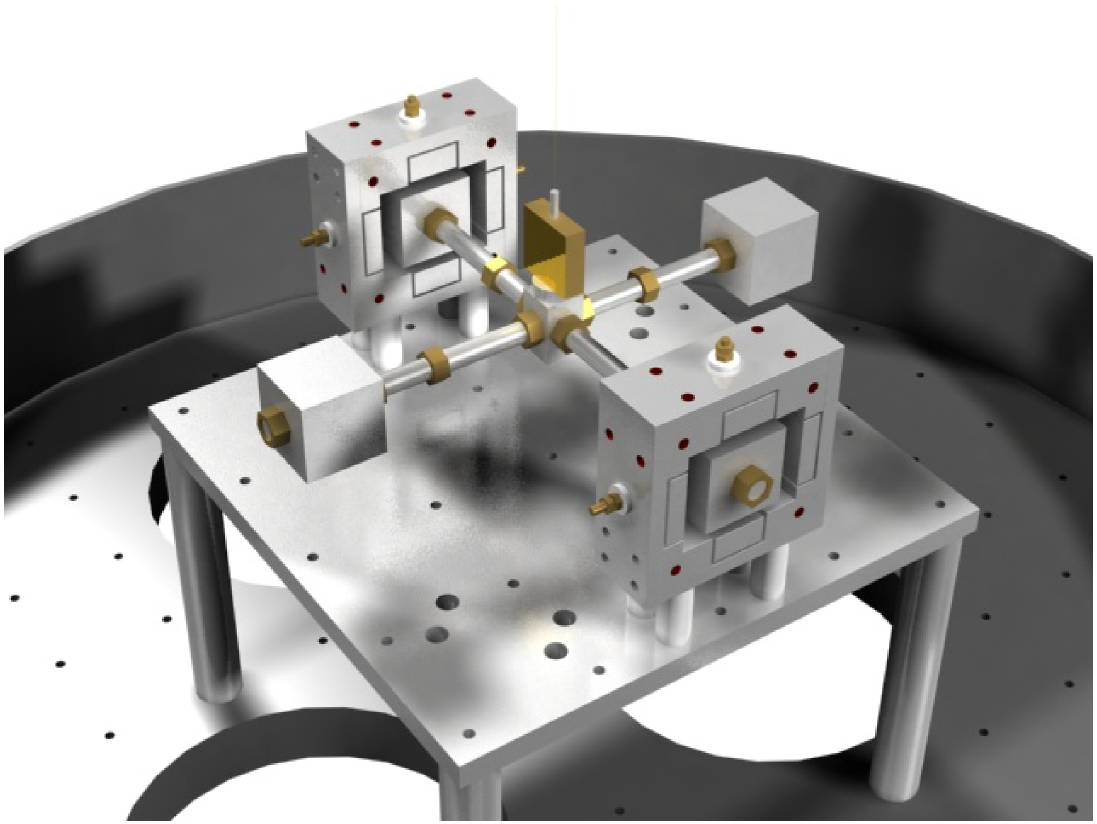
Gravitational gradient noise $\propto R^2$ (Quadrupole imperfection)

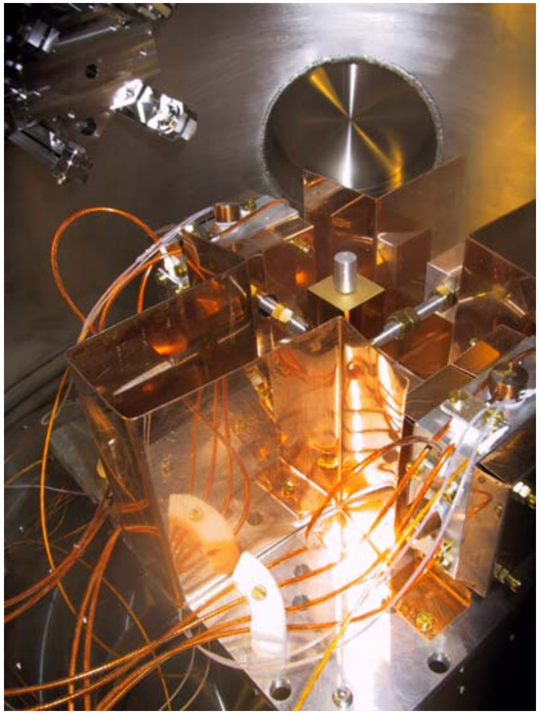
$\propto R^4$ (nominal hexadecapole)

→ First inertial member has arm length R = 10 cm

→ Gravitational gradient measurements underway

Four-mass pendulum:
initial testing with prototype inertial member

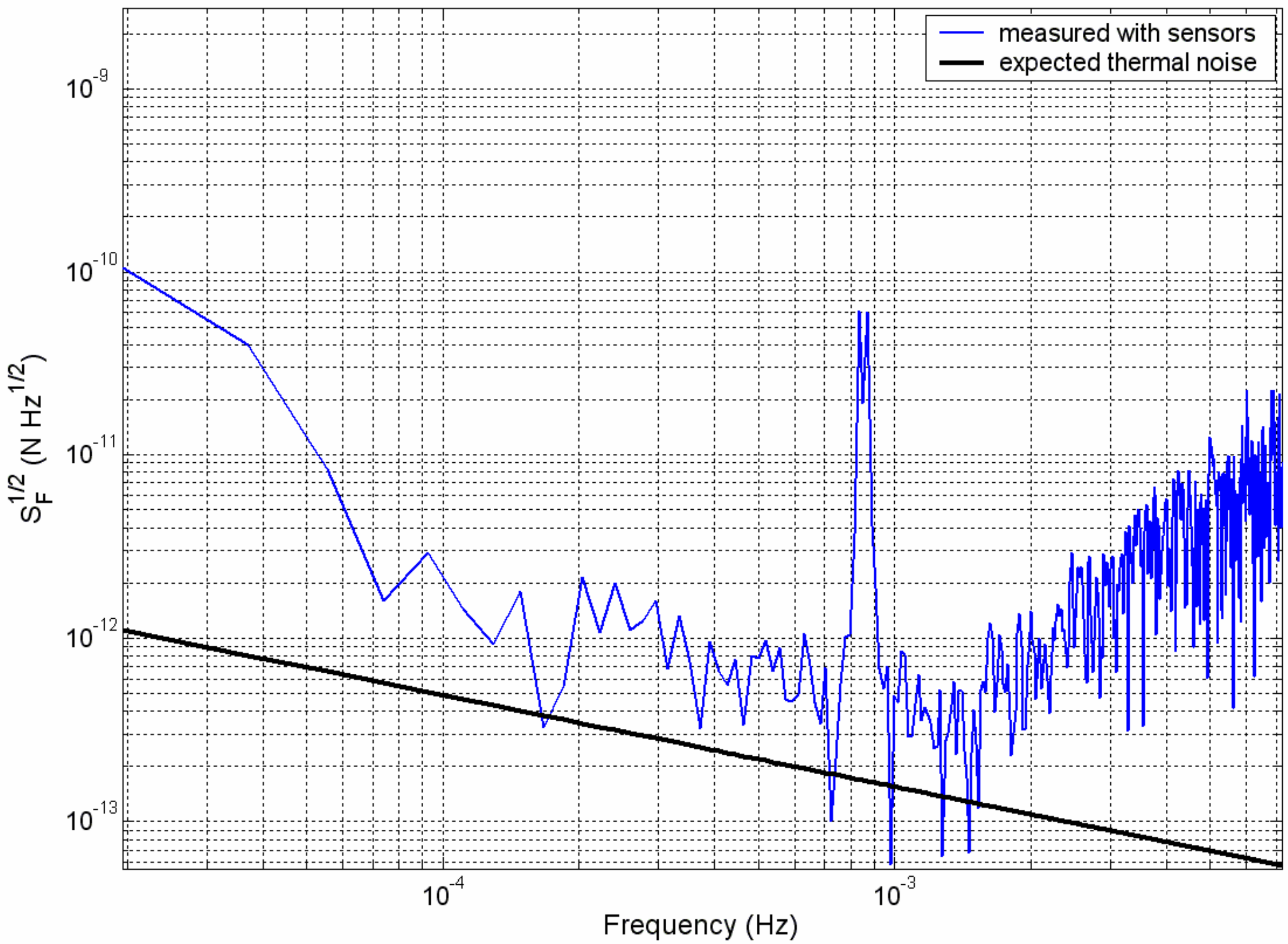




- “blank” measurement to measure pendulum noise in absence of sensor

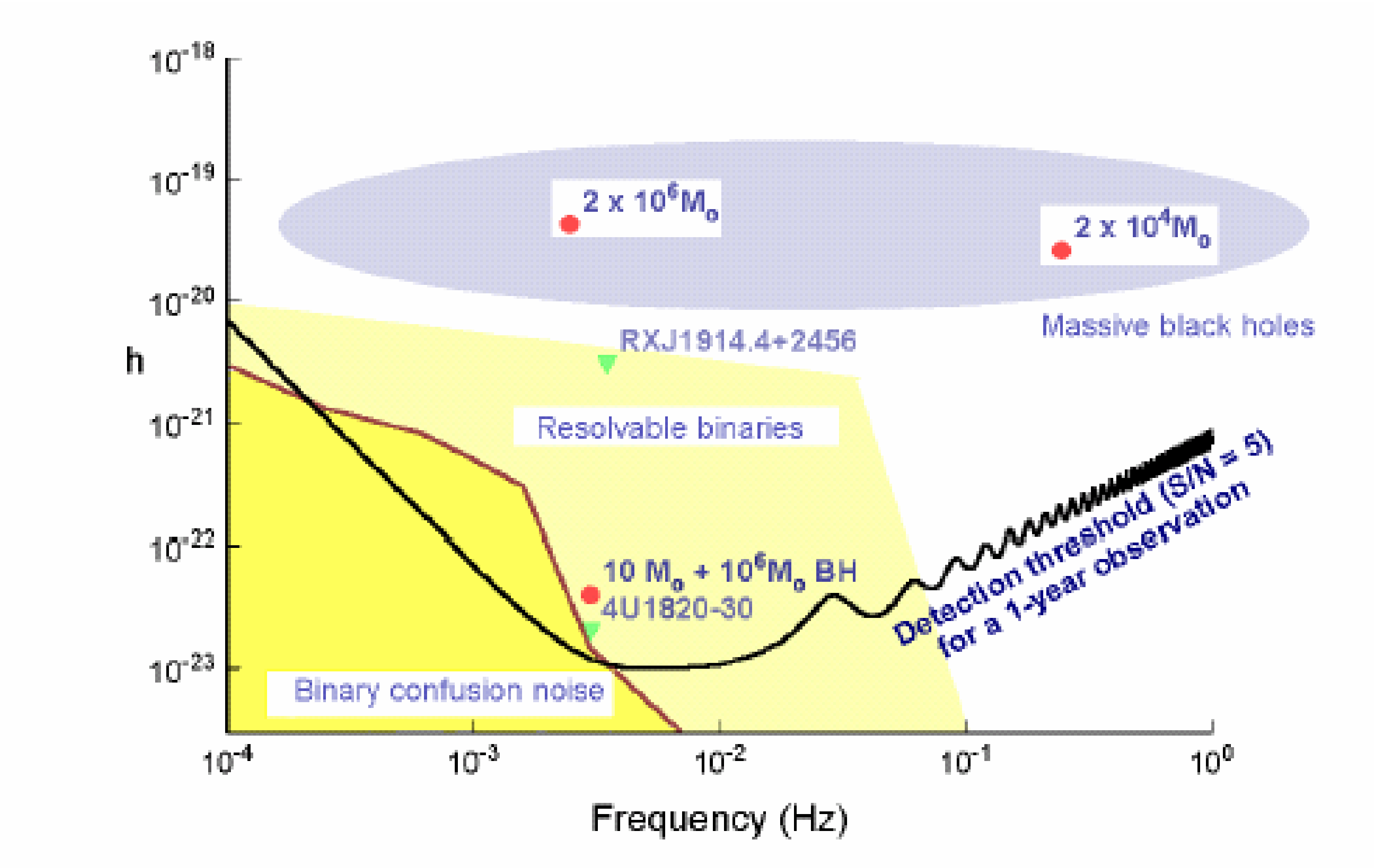
→ thermal noise, twist/tilt, temperature sensitivity, gravity gradient noise

Preliminary data 4 mass pendulum



→ Pendulum ready to make relevant direct force measurements for LISA

LISA low frequency sensitivity goal requires test masses to be in perfect free-fall to within 3 fm/s^2/Hz$^{1/2}$



Trento physicists* contemplate free fall and free food while celebrating the PhD of Doctor Ludovico Carbone

[* minus Antonella Cavalleri, plus Tim Sumner]

Trento LTP / LISA Group

Michele Armano
Ludovico Carbone
Antonella Cavalleri
Giacomo Ciani
Rita Dolesi

Mauro Hueller
David Tombolato
Stefano Vitale
Bill Weber