



A·S·P·E·N  
*Center for Physics*

**ASPEN WINTER CONFERENCE  
 ON  
 GRAVITATIONAL WAVES AND THEIR  
 DETECTION**

**JANUARY 14-20, 1996**

**DATA ANALYSIS  
 LIGO RESEARCH COMMUNITY  
 SPACE BASED DETECTORS-LISA**

**PROGRAM COMMITTEE/ASPEN ORGANIZING COMMITTEE**

Barry Barish ..... Caltech  
 Karsten Danzmann ..... Hanover  
 Sam Finn ..... Northwestern  
 Adalberto Giazotto ..... INFN Pisa  
 John Hall ..... JILA  
 William Hamilton ..... LSU  
 James Hough ..... Glasgow  
 Sydney Meshkov ..... Caltech  
 Charles Prescott ..... SLAC

Fred Raab ..... Caltech  
 Albrecht Ruediger ..... MPI Garching  
 Gary Sanders ..... Caltech  
 Peter Saulson ..... Syracuse  
 Bernard Schutz ..... MPI Potsdam  
 David Shoemaker ..... MIT  
 Kip Thorne ..... Caltech  
 Rainer Weiss ..... MIT  
 Stan Whitcomb ..... Caltech

**EDITOR: SYDNEY MESHKOV**

PROGRAM  
ASPEN WINTER CONFERENCE ON GRAVITATIONAL WAVES AND THEIR DETECTION

Aspen Center for Physics  
January 14 - 20, 1996

Monday AM: Interferometers and Bars  
January 15 Chair: R. Drever

8:00	S. Meshkov (Caltech) S. Mencimer (ACP)	Welcome
8:10	A. Ruediger (Garching)	Current Interferometer Projects
8:35	Discussion	
8:45	G. Pizzella (Rome)	Operation of Resonant Detectors
8:50	W. Johnson (LSU)	Status of Current Bar Experiments
9:10	Discussion	
9:20	Coffee Break	
9:35	S. Finn (Northwestern)	The Path to Data Analysis
10:00	Discussion	
10:10	D. Shoemaker (MIT)	LIGO Status Report
10:35	Discussion	

Monday PM: Interferometer Issues  
January 15 Chair: J. Centrella

4:30	R. Drever (Caltech)	Advanced Interferometers
4:55	Discussion	
5:05	M. Schrempf (Hannover)	GEO 600 Status Report
5:30	Discussion	
5:40	Coffee Break	
5:55	F. Raab (Caltech)	Time-Domain Behavior of the 40m Interferometer
6:20	Discussion	
6:30	R. DeSalvo (Pisa)	Virgo Experiment and Seismic Isolation
6:55	Discussion	
7:05	E. Mizuno (Kanagawa)	Data Acquisition and Analysis for TENKO100 and TAMA300
7:30	Discussion	
7:40	R. Byer (Stanford)	A Sagnac Interferometer for Gravitational Wave Detection
7:55	Discussion	

Tuesday AM Space Based Detectors-LISA I  
January 16 Chair: A. Ruediger

8:00	P. Bender (JILA)	Sources for LISA
8:25	Discussion	
8:35	H. Ward (Glasgow)	LISA Overview
9:00	Discussion	
9:10	Coffee Break	
9:25	H. Ward (Glasgow)	LISA Overview (continued)
9:50	Discussion	
10:00	D. Hils (JILA)	BH-SMBH Binaries
10:25	Discussion	

Tuesday PM      Space Based Detectors-LISA II  
January 16      Chair: R. Schilling

4:30      B. Willke(Hannover)      Lasers for LISA  
5:00      Discussion  
5:10      D. Robertson(Glasgow)      Spacecraft Pointing  
5:40      Discussion  
5:50      R. Stebbins(JILA)      LISA Error Budgets  
6:20      Discussion  
6:30      Coffee Break  
6:45      D. Robertson(Glasgow)      GEO 600 Developments  
7:15      Discussion  
7:25      D. Schnier(Hannover)      Power Recycling  
7:40      Discussion

Wednesday AM      LIGO Research Community(LRC)  
January 17      Chair: H. Ward

8:00      G. Sanders(Caltech)      The View From LIGO  
8:20      Discussion  
8:30      Meeting of LIGO Research Community and Discussion  
  
1:30      Meeting of LIGO Research Community(continued)

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~~Wednesday PM      Shot Noise; Detector Improvement  
January 17      Chair: D. Shoemaker~~

4:30      A. Ruediger(Garching)      History and Derivation of Shot Noise Equations  
5:00      Discussion  
5:10      J. Hall(JILA)      Dynamic Light-induced Mirror Birefringence  
5:40      Discussion

Wednesday Evening:      Public Lecture, Wheeler Opera House  
Chair: S. Meshkov

8:00      L. Krauss      The Physics of Star Trek  
            (Case-Western Reserve)

Thursday AM      Data Analysis I. Search for Compact Object Binaries  
January 18      Chair: S. Finn(Northwestern)

8:00      C. Will(Washington U)      Compact Binary Inspiral: A Post-Newtonian  
   Playground  
8:25      Discussion  
8:35      J. Wilson(LLNL)      Neutron Star Binaries Hydrodynamics  
9:00      Discussion  
9:10      A. Wiseman(Caltech)      Final Wave Forms for Binary Inspiral  
9:35      Discussion  
9:45      Coffee Break  
10:00      K. Thorne(Caltech)      The Final Merger of Compact Binaries:  
   Information Content, Waveform Computations,  
   and Data Analysis Changes  
  
10:25      Discussion  
10:35      R. Matzner(UT-Austin)      Grand Challenge Update  
11:00      Discussion

Thursday PM      Data Analysis II. Time-Frequency Techniques and Other Issues  
 January 18      Chair: C. Will

4:30	B. Owen(Caltech)	Searching for Coalescing Binaries: Templates, Strategies, Computing Requirements
4:55	Discussion	
5:05	J. Centrella(Drexel)	Coalescence Wave Forms and Rotational Instabilities
5:30	Discussion	
5:40	A. Krolak(Warsaw)	Estimating Parameters of GW Signals Using Wavelet Analysis
6:05	Discussion	
6:15	Coffee Break	
6:30	A. Vecchio(Cardiff)	Why Realistic Errors in Parameter Extraction Don't Follow Cramer-Rao Bounds
6:55	Discussion	
7:05	H. Yamamoto(Caltech)	LIGO Simulation Environment
7:30	Discussion	

Friday AM      Data Analysis III. Pulsar and Other Periodic Source Searches  
 January 19      Chair: F. Raab      Stochastic Background Searches

8:00	D. Nicholson(Cardiff)	Realistic Search of One Year's Data
8:30	Discussion	
8:40	G. Jones(Cardiff)	Results of a Practical Search of a Large Patch of Sky for a Pulsar-type Signal
9:10	Discussion	
9:20	Coffee Break	
9:30	B. Allen(Milwaukee)	Stochastic Background Searches
10:00	Discussion	
10:10	R. Fakir(UBC)	The GAP Concept
10:15	Discussion	

Friday PM      Data Analysis IV. Analysis of Existing and Future Data  
 January 19      Chair: N. Robertson

4:30	D. Nicholson(Cardiff)	Overview of Data Analysis for 100 hour Glasgow-Garching Run
5:00	Discussion	
5:10	R. Schilling(Garching)	Time Domain Analysis of 100hr Glasgow-Garching Run
5:40	Discussion	
5:50	Coffee Break	

6:05 S. Finn(Northwestern) Data Analysis for A Network of Interferometers  
6:35 Discussion  
6:45 B. Mours(LAPP-Annecy) Plans for Organizing Data Analysis for Virgo  
7:15 Discussion

Saturday AM Impulsive Signal Searches, Round Table  
January 20 Chair: K. Thorne

8:00 D. Nicholson(Cardiff) Nonlinear Adaptive Filters  
8:30 Discussion  
8:45 Coffee Break  
9:00 S. Finn, Round Table on Waveforms and Data Analysis  
W. Johnson for Stellar Core Collapse and Other Sources  
A. Krolak,  
B. Mours,  
D. Nicholson.  
D. Shoemaker,  
R. Stebbins  
K. Thorne  
11:00 Conference Adjourns

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January 15 Chair: R. Drever

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*Albrecht Rüdiger*

**1996 Aspen Winter Conference  
on  
Gravitational Waves and Their Detection**

**January 14-20, 1996**

Our thanks to those who made it possible:

Aspen Center for Physics

Syd Meshkov

Sally Mencimer

*Saint Peter*

**Three major topics selected for Conference :**

1. First meeting of LIGO Research Community
2. Signal processing in GW research
3. Laser-interferometric detectors in space (LISA)

**First meeting of LIGO Research Community**

**Historical event :**

Worldwide group of GW scientists

common platform for

intensive discussions

exchange of ideas

mutual assistance and consultation

**LIGO as condensation nucleus**

largest project

furthest developed

most heavily funded

but perhaps change the name ?

*what's in a name ?*

*A rose by any other name would smell as sweet*

**Expectation :**

become truly international,

mutually fertilizing entity

## LRC Executive Committee :

1. David Shoemaker
2. Sam Finn
3. Bill Hamilton
4. Harry Ward
5. Joan Centrella
6. Eric Gustafson
7. Bruce Allen

Chair: Sam Finn

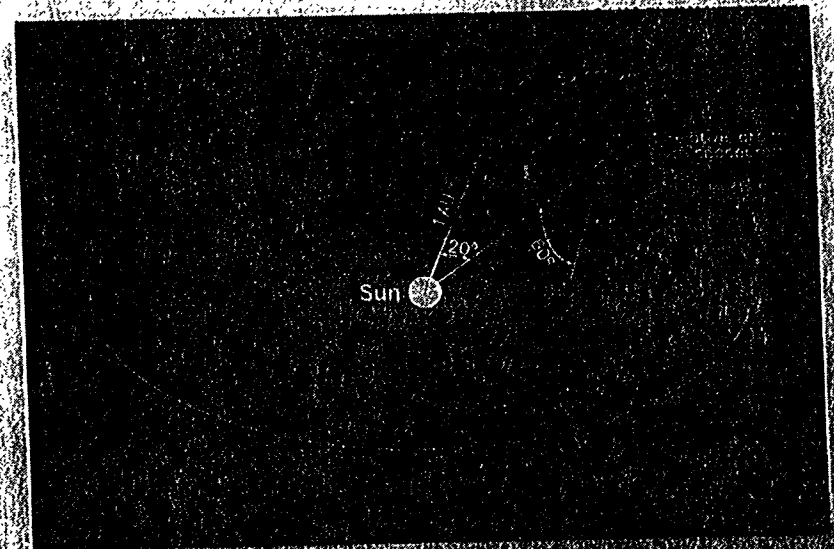
## Good blend of scientists active in

theory,  
astrophysics,  
experimentation of  
interferometric detectors,  
resonant mass detectors,  
signal processing.

# LISA

Laser Interferometer Space Antenna  
for the detection and observation of gravitational waves

A Cornerstone Project in  
ESA's long term space science programme  
"Horizon 2000 Plus"



Pre-Phase A Report

December 1995



## Signal Processing in GW Research

in case of laser interferometry:

in recent years (decades !) emphasis on  
 establishing feasibility  
 optimizing projects  
 getting funded

now with construction under way:

intensify studies of signal processing  
 optimize strategies  
 interaction between different detectors

**Sam Finn: overview on Signal Analysis, today**

deal with signal analysis both for  
 interferometer detectors  
 resonant mass detectors

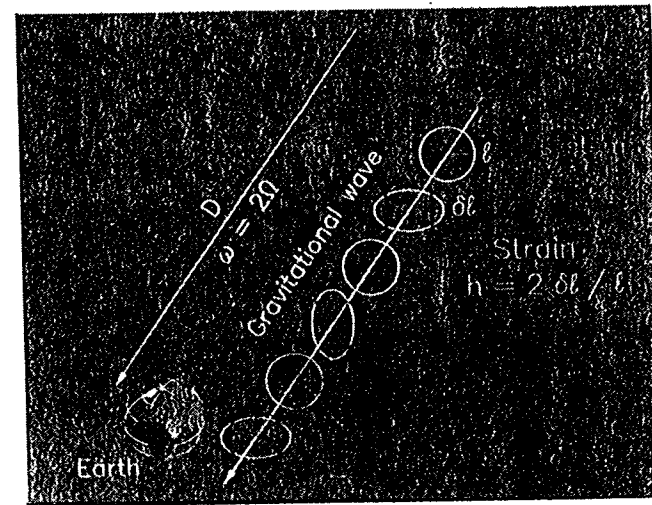
**Prior to that:**

lay foundations with review talks

Warren Johnson resonant mass detectors  
 Albrecht Rüdiger interferometer detectors

## Gravitational Waves

emitted at measurable strengths only when  
 cosmic masses undergo **strong** accelerations



**GW: transversal, quadrupole strain in space**

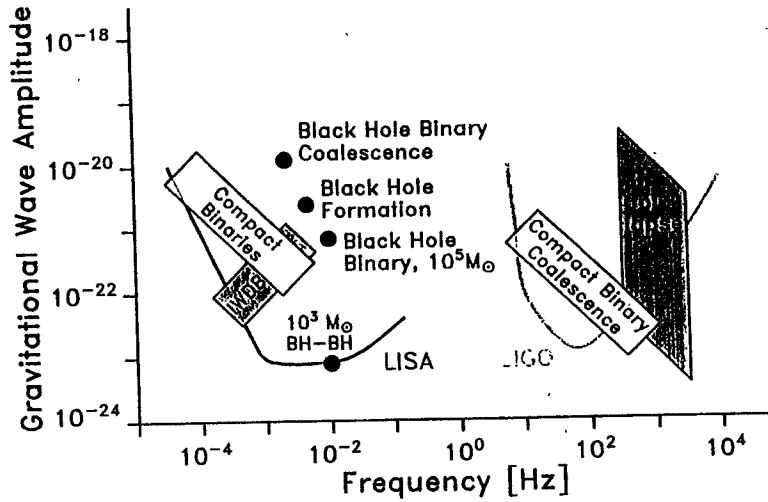
relative change in distances:  $\delta l / l$   
 expressed as  $h := 2\delta l / l$

**Michelson interferometer** is ideally suited:

measures changes in *path difference*:

$$h = \frac{\delta \mathcal{L}}{\mathcal{L}} = \frac{\delta(\mathcal{L}_2 - \mathcal{L}_1)}{\mathcal{L}}$$

# Sources of Gravitational Radiation



## vast frequency spectrum :

from kHz (supernovae, NS coalescence)  
 down to sub-mHz (supermassive black holes)

Observe two clearly separate regimes :

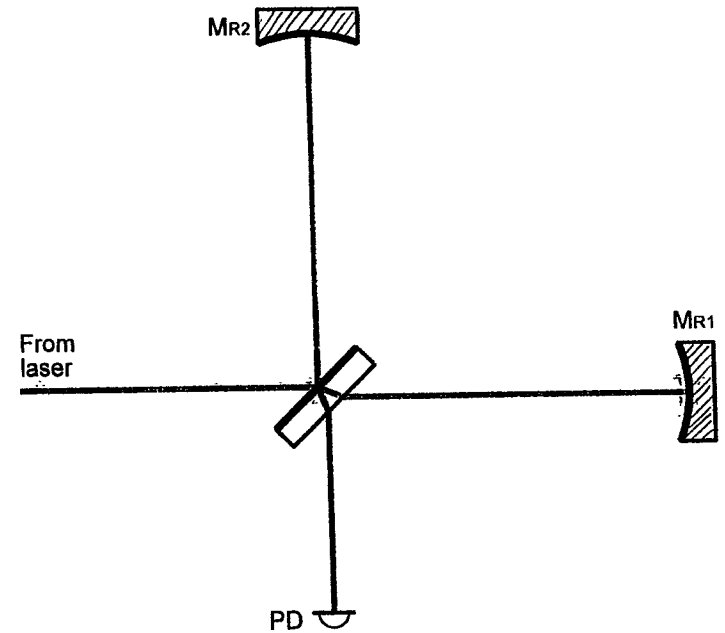
- no overlap; but ideal complementarity
- from 10 Hz to several kHz (*terrestrial*)
- from  $10^{-4}$  Hz to  $10^{-1}$  Hz (*space*)

Space project treated separately (LISA)

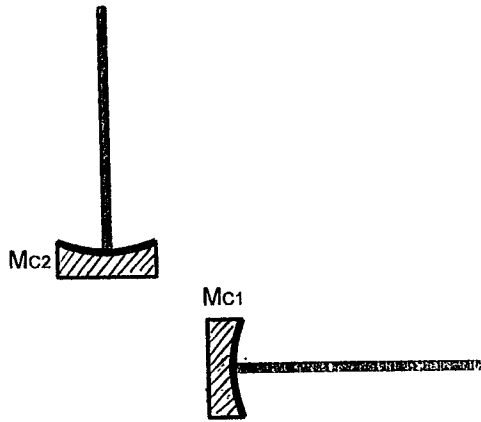
start out with :

ground-based interferometric detectors

Michelson Interferometer  
 Optical path  $L \rightarrow \frac{1}{2} \Delta_{GW}$  :



add near mirrors: DL or FP  
Shot noise limit reduces as  $P^{-1/2}$ .



add mirror  $Mp$ : "power recycling"



If  $L \ll \frac{1}{2} \Lambda$  (DL, few bounces)

Ms 

add mirror  $M_s$  for "signal recycling"  
(increases signal, at price of bandwidth)

If  $L \gg \frac{1}{2} \Lambda$  (FP, excellent mirrors, high  $f_{\text{aw}}$ ):  
same mirror  $M_s$  can avoid cancelling:  
"Resonant sideband extraction"

### Laser-Interferometric GW Detectors: Past - Present - Future

Country:	USA	USA	GER	GBR	FRA	ITA	JPN	JPN	AUS	AUS
Institute:	MIT	Caltech	MPQ	Glasgow	CNRS	INFN	ISAS	NAO	Perth	(all)
<b>Prototypes:</b>										
Start:	1972	1980	1975	1977	1983	1986	1986	1991	1991	
Laser:	Ar <sup>+</sup>	Ar <sup>+</sup>	Ar <sup>+</sup>	Ar <sup>+</sup>	(Ar <sup>+</sup> )	(Ar <sup>+</sup> )	Ar <sup>+</sup>	YAG	YAG	YAG
Arm length $l$ :	40 m		30 m	10 m	0.5 m		100 m	20 m	8 m	
Strain sensitivity $\tilde{h}$ [Hz <sup>-1/2</sup> ]:	1 · 10 <sup>-20</sup> 1995		11 · 10 <sup>-20</sup> 1986	6 · 10 <sup>-20</sup> 1992			20 · 10 <sup>-20</sup> 1994	10 <sup>-17</sup> 1994		
<b>Large Interferometric Detectors:</b>										
Planning (start):	1982	1984	1985	1986	1986	1986	1987	1994	1990	1994
Arm length $l$ :	4 km 2 km	4 km	600 m		3 km		300 m		400 m	
Site (State)	Hanford (WA)	Livingston (LA)	Hannover GER		Pisa ITA		Mitaka JPN		Perth AUS	
Cost (10 <sup>6</sup> US\$):	292		7		90		15		12	
Project name:	LIGO		GEO 600		VIRGO		TAMA 300		AIGO 400	

solicit your assistance in filling holes

# Development of prototypes

<b>Country:</b>	USA	USA	GER	GBR	FRA	ITA	JPN	JPN	AUS	AUS	USA
<b>Institute:</b>	MIT	Caltech	MPQ	Glasgow	CNRS	INFN	ISAS	NAO	Perth	(all)	Stanford

<b>Prototypes:</b>											
<b>Start:</b>	1972	1980	1975	1977	1983	1986	1986	1991	1991		1994
<b>Laser:</b>	Ar <sup>+</sup>	Ar <sup>+</sup>	Ar <sup>+</sup>	Ar <sup>+</sup>	(Ar <sup>+</sup> )	(Ar <sup>+</sup> )	Ar <sup>+</sup>	YAG	YAG	YAG	YAG
<b>Technique:</b>	DL	FP	DL	FP	FP	FP	DL	FP	FP	FP	FP
<b>Arm length <math>l</math>:</b>	(1.5 m) 5 m	40 m	30 m	10 m	0.5 m		100 m	20 m	8 m		10 m
<b>Strain sensitivity <math>\frac{1}{\hbar}</math> [Hz<sup>-1/2</sup>]:</b>	(4·10 <sup>-17</sup> ) (1980)	1·10 <sup>-20</sup> 1995	11·10 <sup>-20</sup> 1986	6·10 <sup>-20</sup> 1992			20·10 <sup>-20</sup> 1994	10 <sup>-17</sup> 1994			
<b>Special features</b>	pioneer, DL washing out phase sens. light power	Fabry-Perot leads in sensitivity	direct susp. mode cleaner power recycl. RSE	Fabry-Perot extern. mod. signal recycl. mode cleaner	extern. mod. power recycl. mode cleaner optical meas. simulations	extreme seism. isolation super-atten. optical meas. modeling	DL scheme longest prototype	first FP-recombin. 3m prototyp	seismic isol. cantilever springs		<i>Sagnac advanced technol.</i>

## Large Interferometric Detectors:

<b>Planning (start):</b>	1982	1984	1985	1986	1986	1986	1987	1994	1990	1994	1994
<b>Arm length <math>l</math>:</b>	4 km 2 km	4 km	600 m		3 km		300 m		400 m		400 m
<b>Project name:</b>	LIGO		GEO 600		VIRGO		TAMA 300		AIGO 400		

## Technology

Work on prototypes and for large-scale projects has spawned technological research in:

### Optics:

- substrate quality
- superpolishing
- supercoating
- modulation
- mode cleaning

revived interest in diffractive optics

### Lasers:

- Nd:YAG (MISER)
- efficiency
- laser diode pumping
- lifetime
- laser stabilization
- frequency
- power

### Mechanics:

- suspension techniques
- active seismic control
- cantilever springs
- high Q materials
- high Q bonding techniques

### Vacuum:

- tube construction
- low outgassing materials
- cleaning
- heat treatment

Country Institute	USA MIT	USA Caltech	FRA CNRS	ITA INFN	GER MPQ	GBR Glasgow	JPN ISAS	JPN NAO	AUS Perth	AUS (all)	USA Stanford
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**Prototypes:**

Start	1972	1980	1983	1986	1975	1977	1986	1991	1991		1994
Laser	Ar <sup>+</sup>	Ar <sup>+</sup>	(Ar <sup>+</sup> )	(Ar <sup>+</sup> )	Ar <sup>+</sup>	Ar <sup>+</sup>	Ar <sup>+</sup>	YAG	YAG	YAG	YAG
Arm length L	40 m		0.5 m		30 m	10 m	100 m	20 m	30 m		10 m
Strain sensitivity [1/√Hz]	1 · 10 <sup>-20</sup> 1995				11 · 10 <sup>-20</sup> 1986	6 · 10 <sup>-20</sup> 1992	20 · 10 <sup>-20</sup> 1994	10 <sup>-17</sup> 1994			

**Large Interferometric Detectors:**

Planning (start):	1982	1984	1986	1986	1985	1986	1987	1994	1990	1994	1994
Arm length L:	4 km 2 km	4 km	3 km		600 m		300 m		400 m		400 m
Site (State)	Hanford (WA)	Livingston (LA)	Pisa ITA		Hannover GER		Mitaka JPN		Perth AUS		Stanford (CA)
Cost (10 <sup>6</sup> US\$):	292		90		7		15		12		
Project name:	<b>LIGO</b>		<b>VIRGO</b>		<b>GEO 600</b>		<b>TAMA 300</b>		<b>AIGO 400</b>		
special features:	autonomous: 2 + 1 interf.		super-attenuator low frequencies		advanced optics tunable		"small is beautiful" <= low cost =>		ext'ble to 3 km suspension		

## Outlook

**often asked question:**

*when will interferometers be ready ?*

have to define what 'ready' means

construction finished

optics installed

first interference

first operation, at reduced level

full sensitivity reached

all finished for longer data-acquisition run

any answer to be taken with a grain of salt

**safest statement:**

**by turn of millennium**

*leaves one year contingency (2000, 2001 ?)*

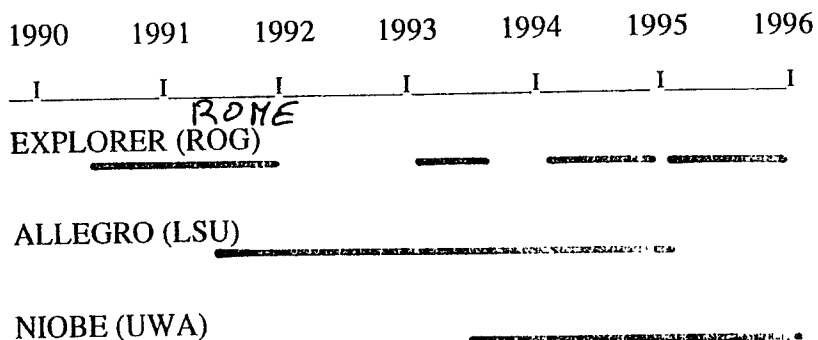
still much better off than LISA

scheduled for 2015 if all goes well!

AVAILABLE EXPERIMENTAL DATA

## Operation of Resonant Detectors

G. Pizzella  
 ROG collaboration (ROME)  
 I and II University of Rome  
 University of L'Aquila  
 National Institute for Nuclear Physics (INFN)  
 National Council of Research (CNR)

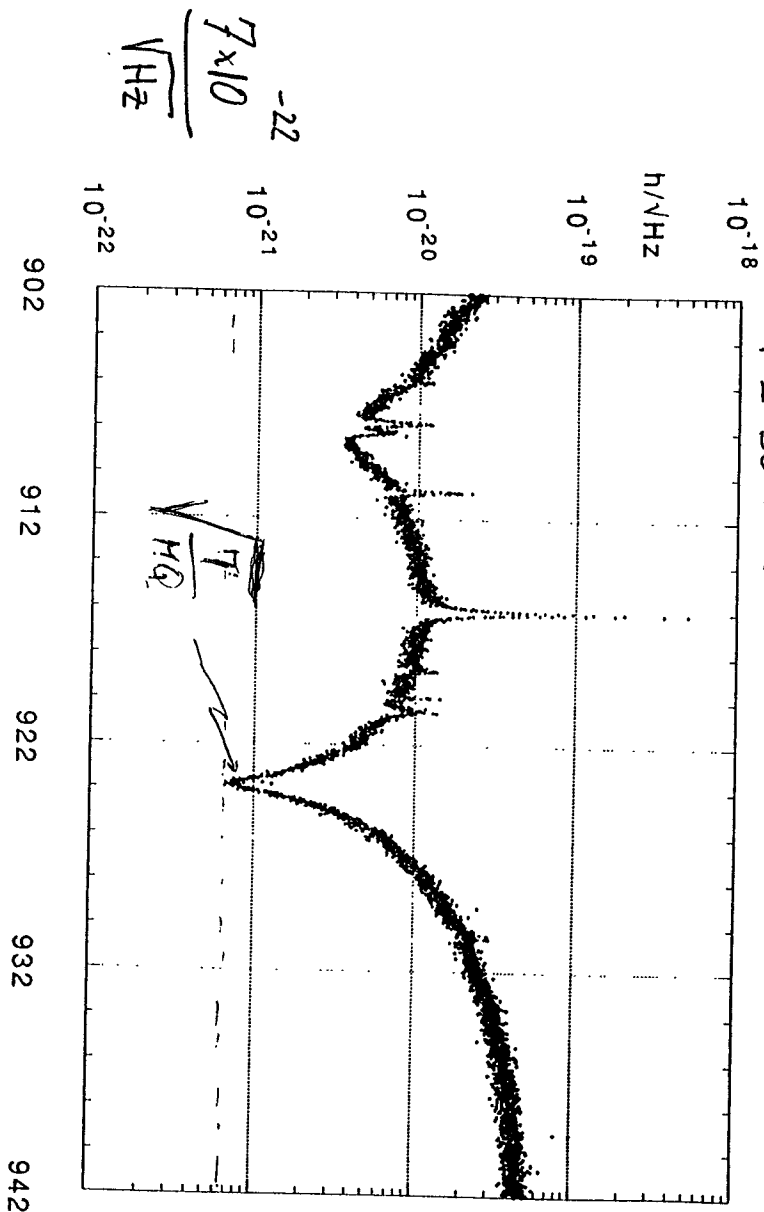


sensitivity for short bursts  $h \approx 6 \cdot 10^{-19}$   
 at the resonance  $\tilde{h} \approx \frac{10^{-21}}{\sqrt{\text{Hz}}}$

It is important to make available the raw data, in order to exploit the experiments with the best possible efficiency.

One should try to consider the various apparatuses as part of one single detector, each part giving its own contribution to the study of the still unknown phenomenon of gravitational waves.

This data exchange should be done soon



MA 7... December 1995  
 $m = 2300 \text{ kg}$   
 $Q \approx 5 \times 10^6$   
 $T \approx 30 \text{ mK}$

# STATUS OF BAR DETECTORS ↑ RESONANT CRYOGENIC

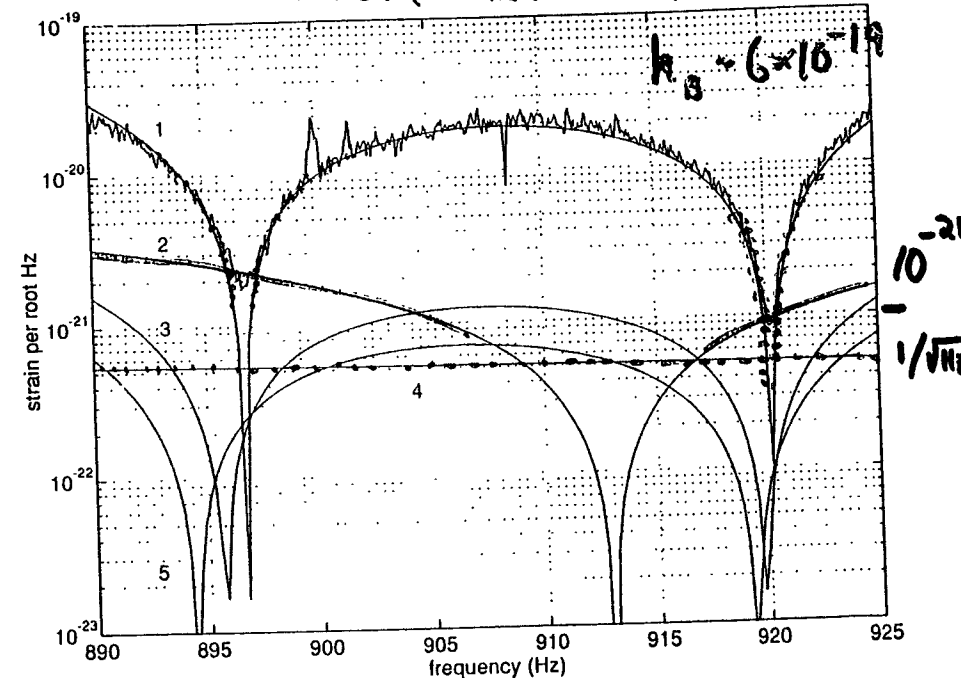
WARREN JOHNSON  
 LSU

- Observing Instruments
- Instruments in Progress
- Proposed Instruments

Search for events - Single Blind Search

## NOISE BUDGET - ALLEGRO '93 16

- RESONATOR THERMO-MECHANICAL FORCE ("BROWNIAN")
- ..... ANTENNA " "
- ..... AMPLIFIER (SQUID) NOISE



THERMO-MECHANICAL NOISE  $\propto \sqrt{\frac{T}{mQ}}$

AMP NOISE  $\propto \frac{\text{ADDITIVE NOISE}}{\text{COUPLING STRENGTH}}$

What can be done with cryogenic techniques



## Observing Instruments

- ( Stanford 4K - operational early '80's  
damaged '89 )
- Rome (EXPLORER) - operational late '80s  
- to present
- LSU (ALLEGRO) - operational '91 - '95  
spent '95 fixing cryogenics & vacuum  
1st try at improved sensitivity has failed  
will probably return to observing anyway.
- Australia - (NIobe) - operational '94 - present

## In Progress

- ( Stanford 50mK - discontinued '95 )
- Rome (NAUTILUS) - first & only mK a. line  
shows 50mK cryogenics are possible  
isolation & upconversion are problems  
operational late '95
- Legnaro (AURIGA) - younger sister to NAUTILUS  
cryostat done, antenna under construction

Omni-Directional Antennas (large, spherical)

International Efforts  
US collaboration - TIGA project  
Direction Finding-Spin of Graviton(s)

Technology - Very Recent Progress  
Antenna Fabrication- Explosive Bonding  
Mechanical Q  
Multi-stage Resonators  
Transducers  
    Inductive  
    Optical  
real SQUIDS  
Modeling  
Sources ?

4

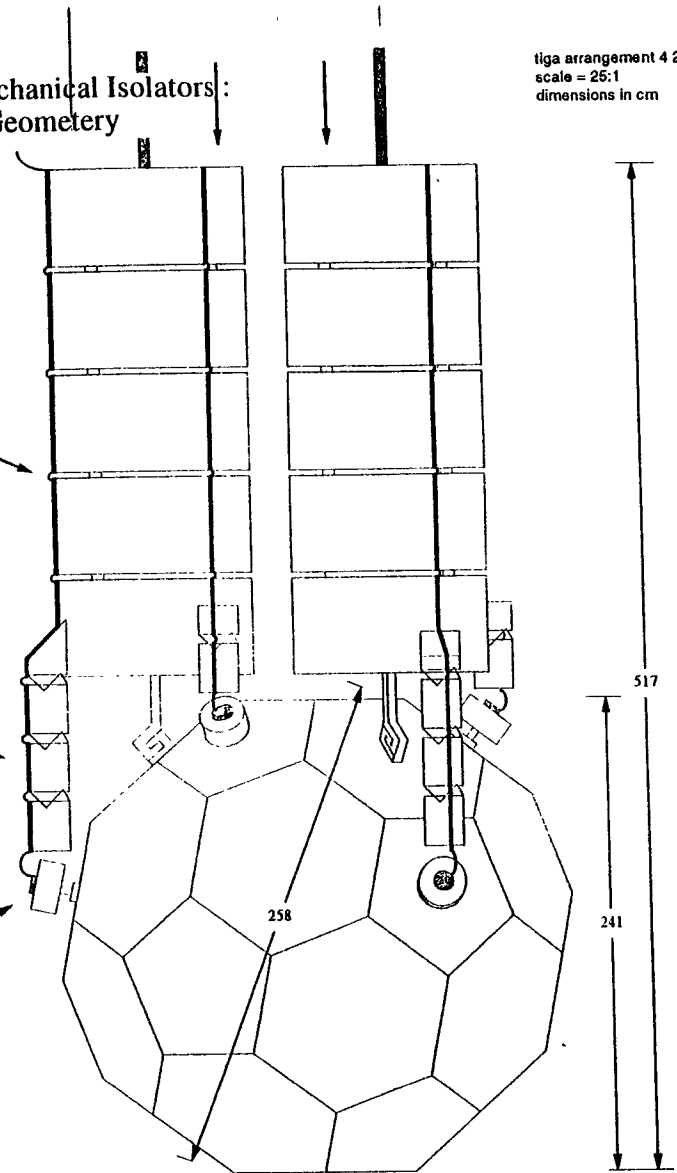
3 Multistage Mechanical Isolators:  
Tube and Disk Geometry

tiga arrangement 4 2  
scale = 25:1  
dimensions in cm

Wiring &  
Thermal Links

Mechanical  
Isolator

Resonator and  
Transducer



Side View

5

# Spherical Resonant GW Detector International Effort

## "OMEGA"

OMnidirectional Experiments with Gravitational Antennae

### US TIGA

- LSU
- Maryland
- Rochester

### Italy "NAIAO"

- Rome, "Tor Vergata"
- Rome
- L'Aquila
- Legnaro

### Netherlands GRAIL

- Leiden
- Amsterdam
- Twente
- Eindhoven

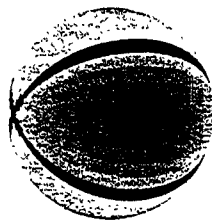
### Brazil GRAVITON

- INPE
- Sao Paolo
- Brasilia

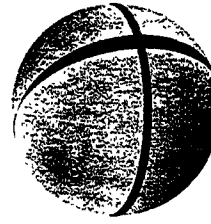
+ Barcelona = EUROMEGA  
+ Paris "Inst. d'Aphys"

A spherical detector can determine the SPIN(s) of the G-RAVITON(s)

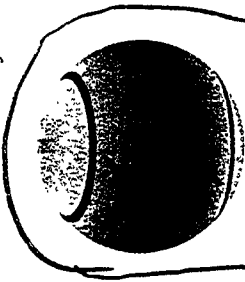
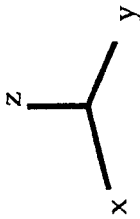
$\vec{k}$  = propagation direction



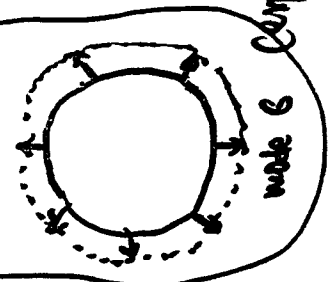
mode 2



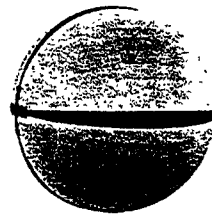
mode 4



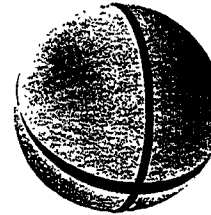
mode 5



mode 6 (Compression)



mode 1



mode 3

combinations are SPIN 0. TIGA detects modes 5 compact

N. Nagaiwa

Terminology

L

# World-wide Status Report

## US - TIGA (2.6 m (1 kHz) Al sphere @ 50 mK)

TIGA physics  
Cryogenic design - prelim. eng. drawings  
Transducers - inductive, optical  
Materials (Al alloys)

Presently separately funded efforts  
Submit joint R&D proposal 12/95  
Construction proposal 1997-8

## Italy

Materials (Al alloys, BeCu, CuAl)  
Ultracryogenics (10 mK)  
Coordinate EUROMEGA

Nautilus (100 mK 1.5 ton bar)  
EUROMEGA funding  
Construction proposal

## Netherlands (3 m 'sphere' - 660 Hz (100 tons) @ 10 mK)

Materials (Al alloys, BeCu, CuAl)  
Ultracryogenics (10 mK)

R&D funding - 1995  
2 m prototype - 1996-1998.5  
3 m sphere - operation in 2001

## Brazil (3 m 'sphere' < 100 mK)

Materials (Bronze alloys)

Complete feasibility studies in 1995  
Submit construction proposal by 1996

## US collaboration (Gravity Coop)

### (TIGA - Truncated Icosahedral Gravitational Antenna)

L.S.U.	- W. Hamilton, W. Johnson, K. Geng
Maryland	- H.J. Paik, T. Stevenson
	- J.P. Richard, Y. Pang
	- F. Wellstood, I. Jin
Rochester	- M. Bocko
Santa Clara	- W. Duffy

## Working Groups - systematically attacking the technology

Modeling - understand tensor and multimode behavior  
- guide component development

SQUIDS - design and construct devices that reach intrinsic noise

Inductive Transducer - increase  $Q_m$  and  $Q_e$   
- maintain coupling  
- integrate the advanced SQUIDS

Optical Transducer - develop alternate technology that is compatible with cryogenics and shot noise limited

Resonators and Antenna - increase bandwidth and coupling  
- increase  $Q_m$

## Cryogenics and Isolation

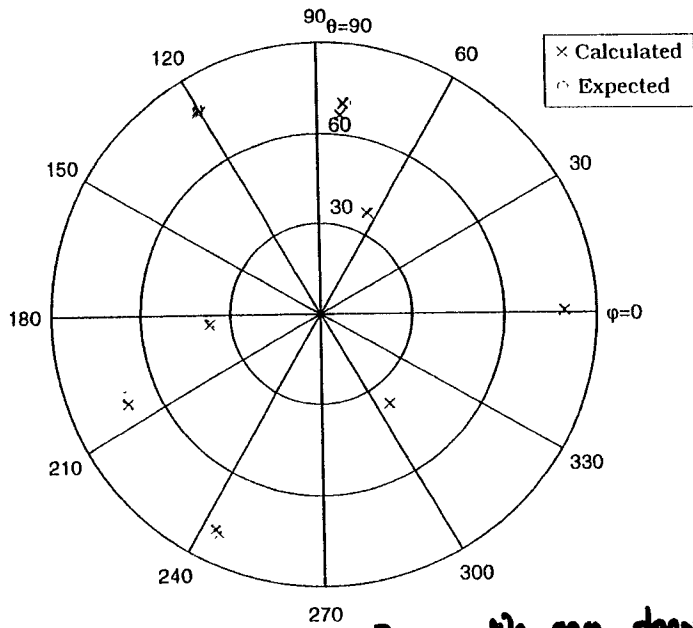
- operate at <60 mK with 40-50 tons
- engineered for antennadevelopment
- assure sufficient vibration isolation
- eliminate non-thermal noise

**SOURCES !**

# Impulse Location

S. Merkwita  
L.S.U.

10

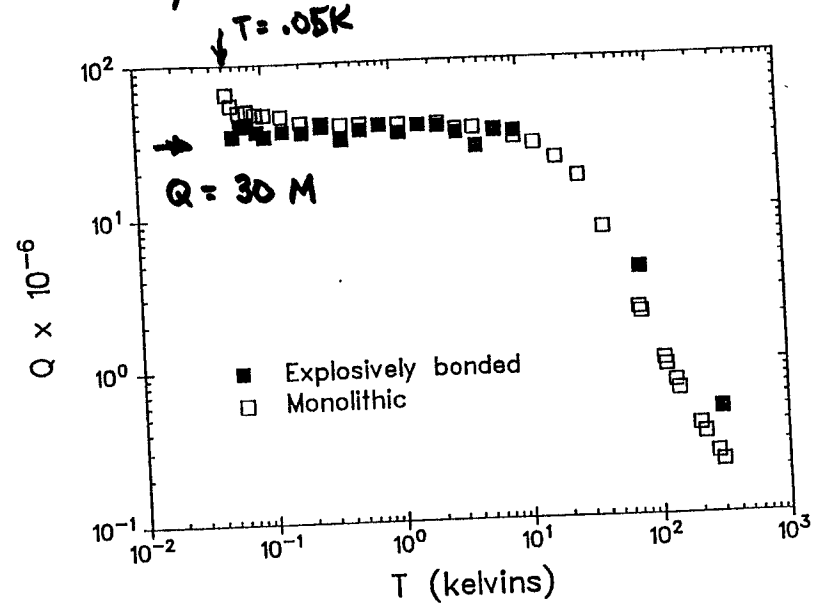


Proves We can deconvolve:  
→ modes → direction

Impulse Face	$\theta$ Calculated	$\theta$ Expected	$\phi$ Calculated	$\phi$ Expected
Pent 1	36.8°	37.3°	64.6°	60.0°
Pent 2	37.0°	37.3°	- 53.4°	- 60.0°
Pent 3	36.8°	37.3°	- 175.1°	180.0°
Pent 4	79.3°	79.1°	- 116.4°	- 120.0°
Pent 5	79.4°	79.1°	0.6°	0.0°
Pent 6	78.6°	79.1°	120.2°	120.0°
Top Hex 6	70.4°	70.5°	82.9°	82.2°
Bottom Hex 6	66.5°	70.5°	83.4°	82.2°
Top Hex 8	70.6°	70.5°	- 155.6°	- 157.7°

EXPLOSIVE Bonding - proposed by E. Coccia  
'unlimited' size! if rect. + made of plates 2" thick  
layered to 3 meters seems possible

11



Proves Material OK. (Coccia & Frossetti  
also)

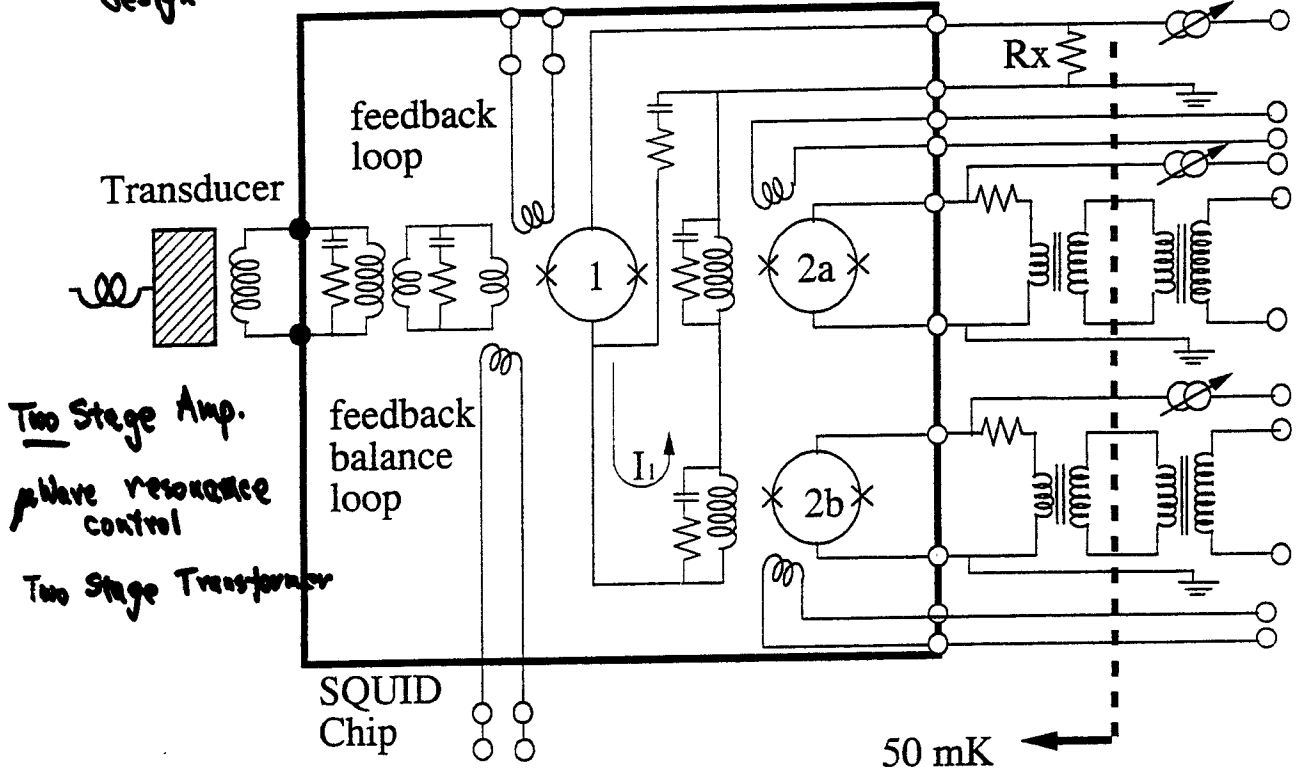
Preprint 12/95

Cryogenics, W. Duffy, Jr. and S. Dalal, Fig. 3

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# NEW SQUID Simplified Chip Diagram

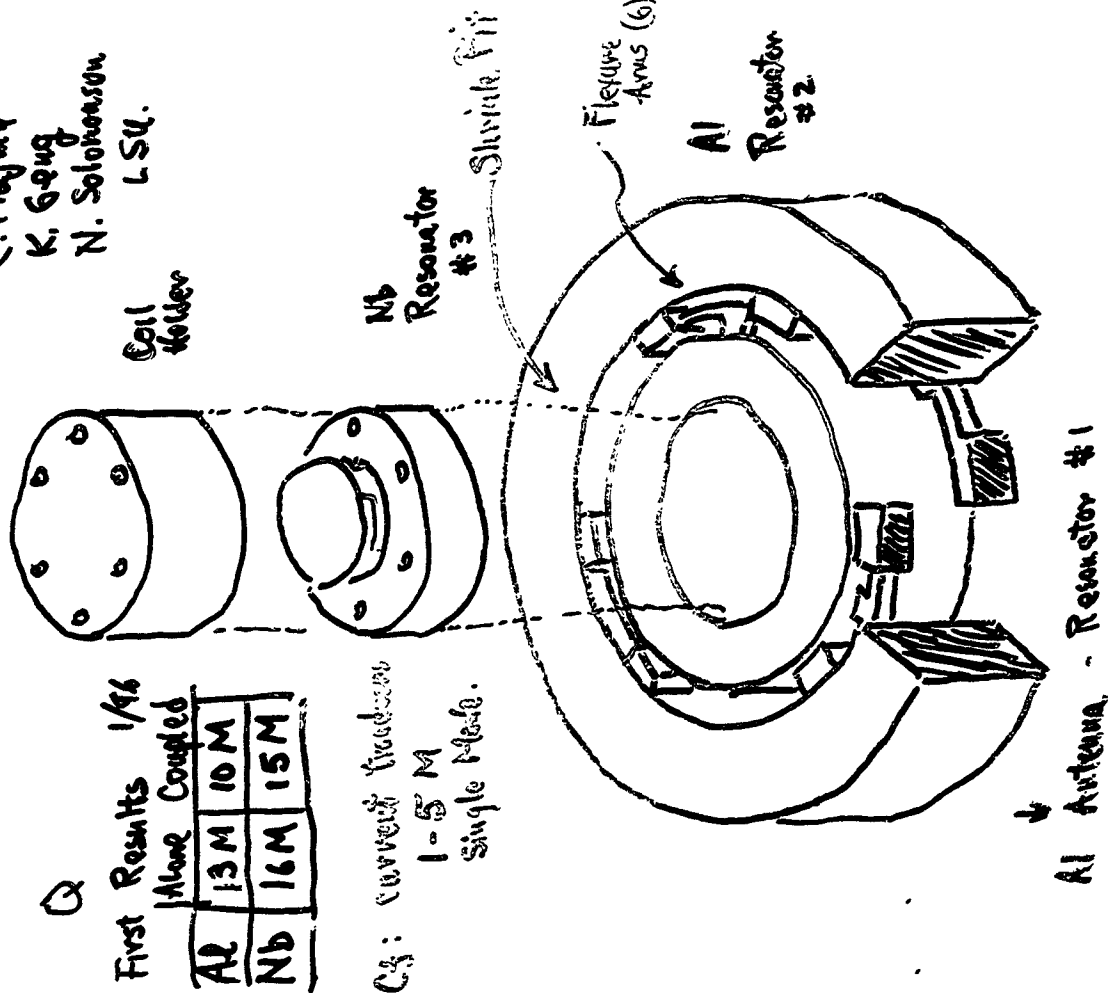
I. Jia, F. Wellsted  
U. Maryland



12

## TWO MODE TRANSDUCER Prototype

C. Prajma  
K. Geug  
N. Solomonson  
LSU.



Q

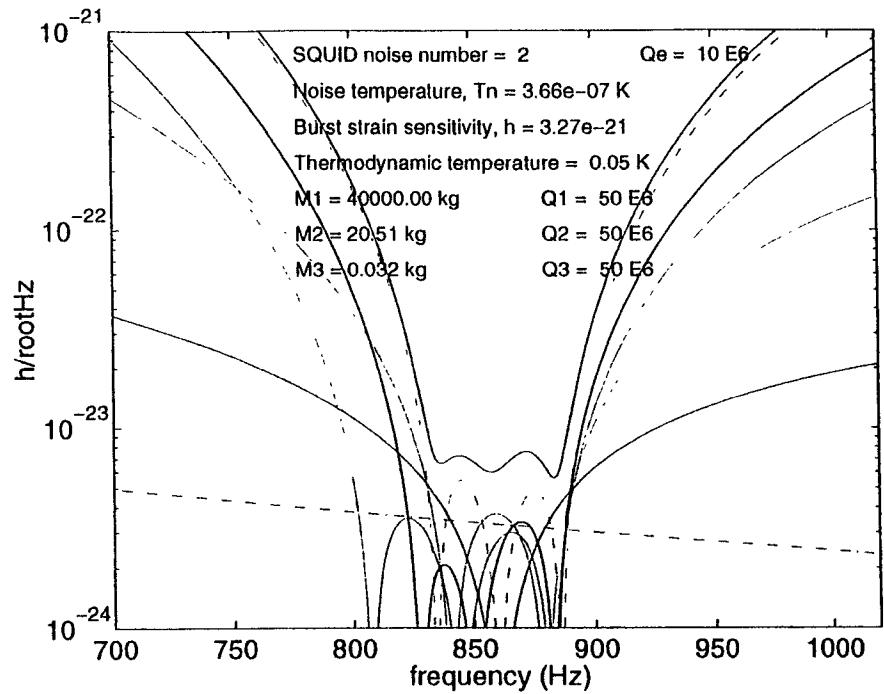
First Results 1/96

Mode Coupled	AL	Nb
13 M	10 M	15 M
16 M	15 M	

Cg: current transducer  
1-5 M  
Single Mode.

# Calculated Strain Noise Advanced TIGA

Carlos Frajuca  
LSU



Total Noise	—————	Antenna Brownian	-----
SQUID White Noise	-----	Mode 2 Brownian	=====
SQUID Back Action	=====	Mode 3 Brownian	=====
Transducer Electrical loss	=====		

4

T=0.109 K, 1SQUID feedback loop

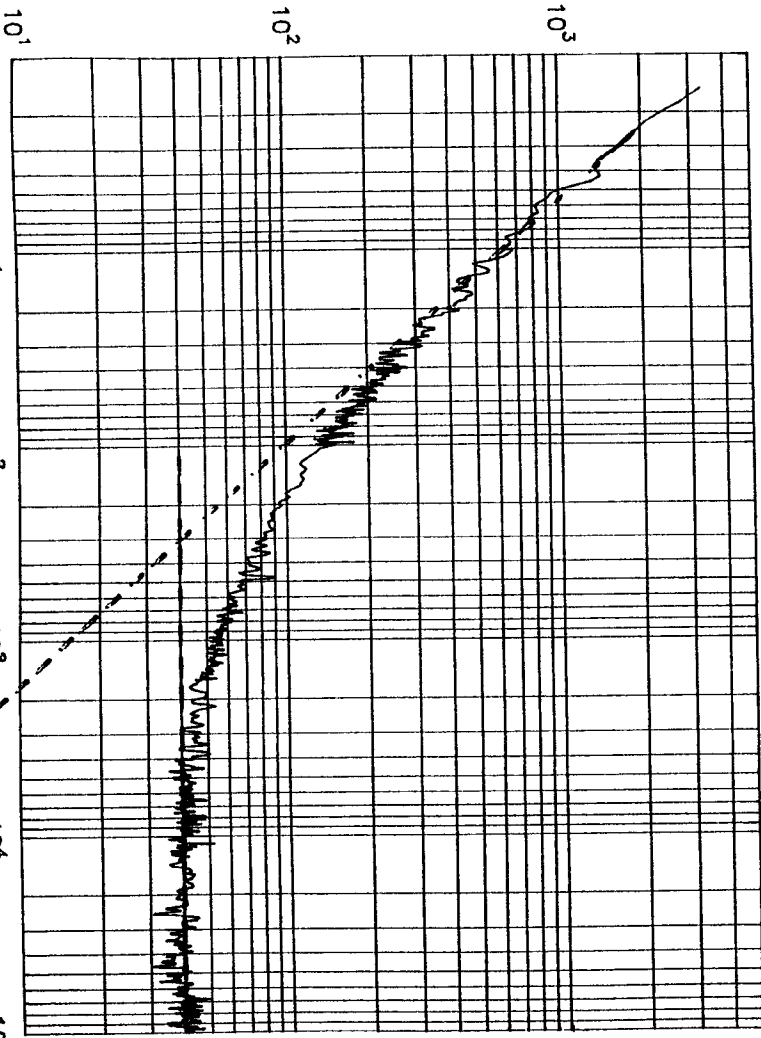
$E_v$  (hbar)

Jan, 2013

Frequency (Hz)

$E_v \sim 54$  K  
@ 1 kHz

10<sup>1</sup>  
10<sup>2</sup>  
10<sup>3</sup>  
10<sup>4</sup>  
10<sup>5</sup>



## Summary & Questions

- Nearly quantum limited 'spherical' detector arrays are being developed world-wide.

But

- What are the best candidate sources for such detectors?
- Which frequency range should we try to cover first? (600 Hz to 4 kHz)

'Guaranteed' sources may be necessary for U.S. funding

## Gravitational Astronomy - Search for Impulsive Events

Filter the raw data to get Event Candidates  
Eliminate bad data  
Coincidence Search using data from another detector

### Problems

Background is not stationary  
Event signatures are arguable  
Difficult to correctly evaluate accidental rate  
High confidence required for discovery of new physics

A Solution? - Single Blind Multiple Search W. Johnson  
E. Meucci

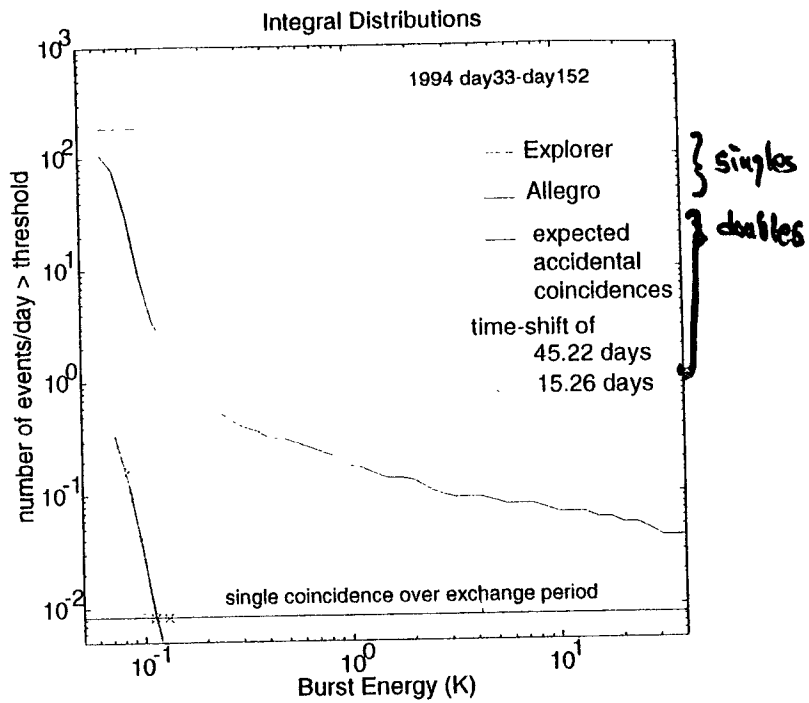
Search through N event lists from other detector  
Compare them to his own:  
N-1 are to evaluate REAL accidentals  
because real list, but time-shifted  
1 list is the real one

Blind, because searcher does not know which list is ~~which~~ real one.

Can the search distinguish the real from the shifted?

Is there any penalty for blind searches?





### Symmetric Single Blind Coincidence Search

#### Data Exchange

1. Generate a list of 1000 random time-shifts.  
Pick one ( $\equiv x$ ), whose value is undisclosed.
2. reported event times = (real event times +  $x$ ) mod  $L$ ,  
where  $L$  is the length of the exchange (120 days)
3. Replace  $x$  by its "inverse"  $L-x$  in the list of time-shifts.
4. Send the list of reported times  
and the list of 1000 time-shifts to other group(s).

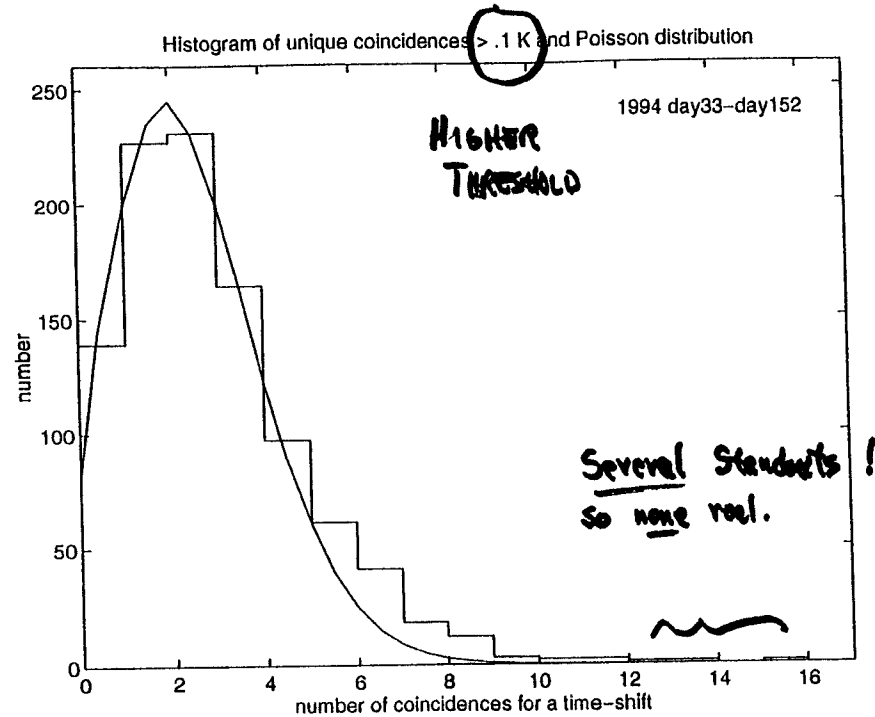
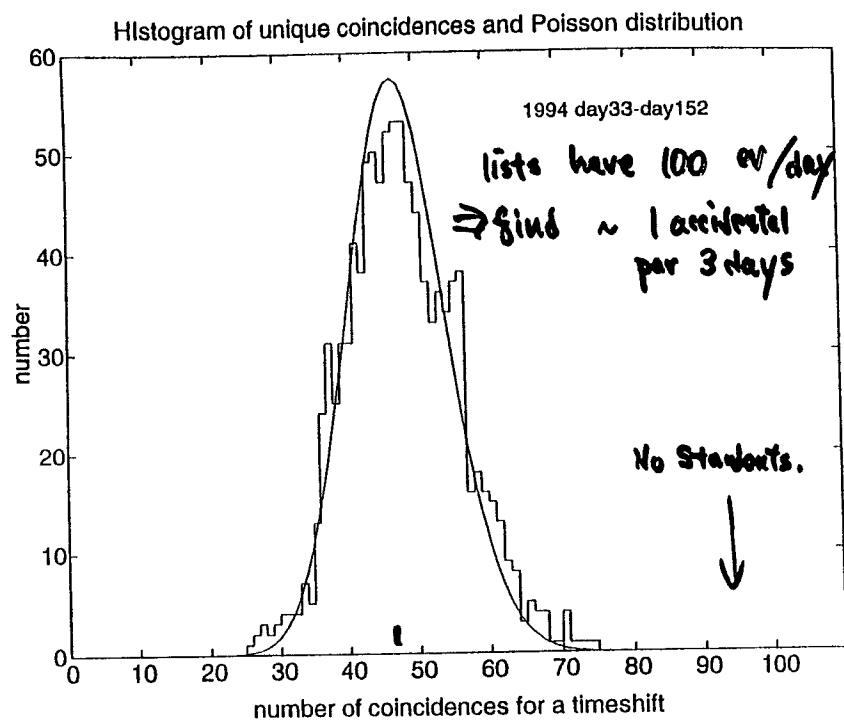
#### Coincidence Search

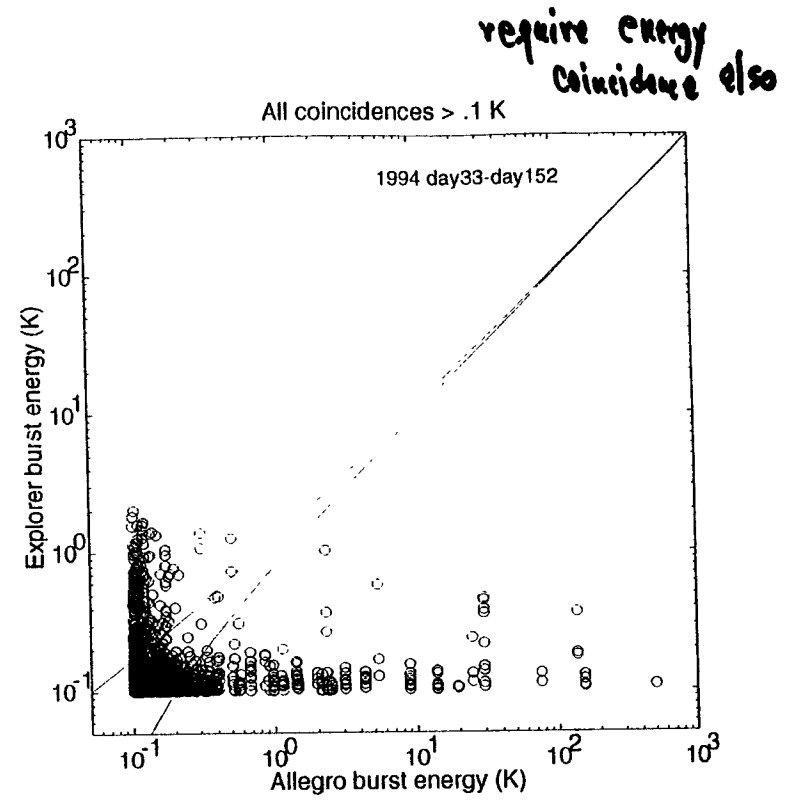
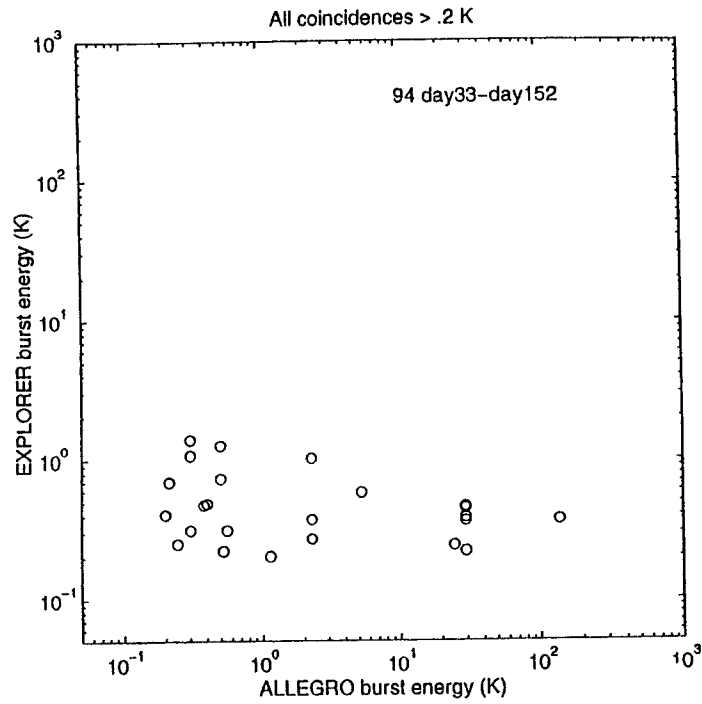
1. Use any reasonable selection criteria on both event lists.
2. Choose (first) time-shift from the received list ( $\equiv y$ ).
3. Candidate event times = (reported times +  $y$ ) mod  $L$
4. Find coincidences between candidate event times  
and own real event list.
5. Redo with next time-shift from list, until done.

( Real event times returned when  $y = L-x$  )

So 999 searches for (guaranteed) accidental coincidences,  
1 search for real coincidences

The Big Question : Does the real search stand out  
from the accidentals ?





Conclusion: >2 detectors essential unless?  
Use Blind Search with 3 detectors?

# The Path To Data Analysis

Sam Finn  
Northwestern Univ.

Data Analysis:

Finding a needle  
in a haystack.

Haystack.

Needle.

Method for separating  
needles from hay

Data Stream:

Signals

Statistics

Data Stream

Integrity

Continuity

Soundness

Resolution, duty cycle.

Calibration

Content

Detector response.

Noise

Intrinsic instrument noise

Extrinsic environmental noise

Gaussian vs. non-gaussian noise

Instrument/Environmental "vetos"

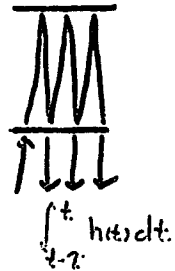
## Sources & Signals

Signal is not waveform

Stochastic vs. "deterministic" signals

Detector: Response

Detector: motion



Known vs. Unknown Sources

No unknown signals!

Signal Models

Particular sources

General physical principles

## Signal-model Accuracy

Accurate model where signal power overlaps detector bandwidth

Example: Binary Inspiral

$$\rho \approx 8 \left( \frac{r_0}{d_t} \right) \left( \frac{M}{1.2 M_\odot} \right)^{5/6} \text{ (1)}$$

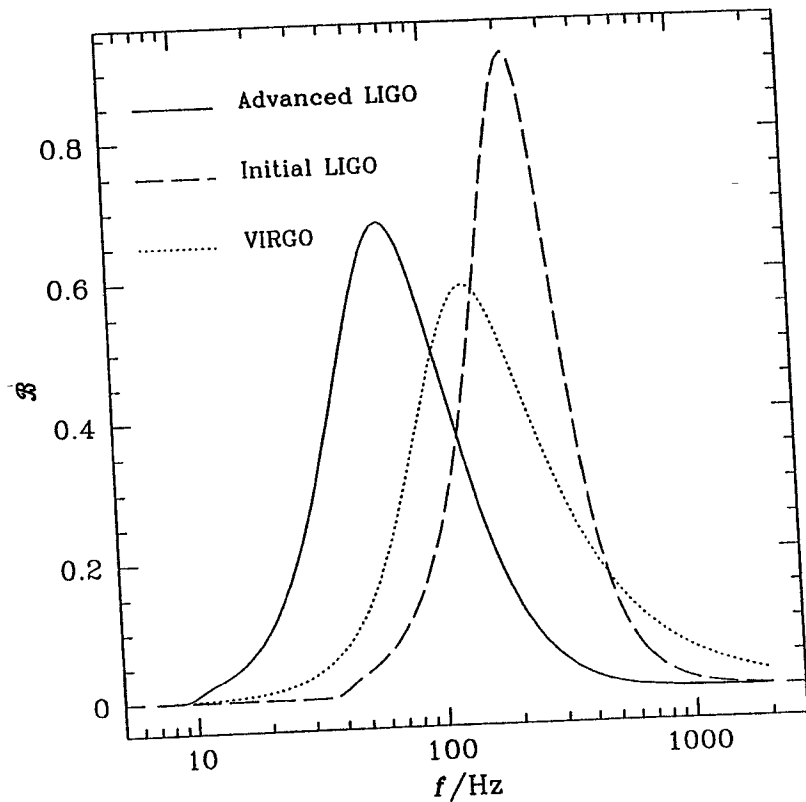
$$r_0^2 \equiv \frac{5 M_\odot^2}{12 \pi} \left( \frac{3}{20} \right)^{5/3} \chi_{7/3}$$

$$\chi_{7/3} \equiv \int_0^\infty \frac{df (\pi M_\odot)^2}{(\pi f M_\odot)^{7/3} S_n(f)}$$

$$\frac{dp}{df} \propto \frac{1}{f^{7/3} S_n(f)}$$

$$d\log(\rho) / d\log(f)$$

( $f_{\max}$  large)



## Statistics

What is signal? What is noise?

CWDBs & LISA

Supernovae & binary inspiral.

Quantifying confidence

Frequentist / Classical / Neyman-Pearson statistics

Bayesian statistics

Algorithms

## Detection † Frequentist Statistics

Pick a descriptive statistic

e.g., maximum likelihood SNR  $\rho$

Evaluate statistic distribution

Signal absent  $P(\rho)$

Signal present  $Q(\rho)$

Set Detection Threshold  $p$

If  $\rho$  in  $\{\rho: P(\rho) < p\}$  say signal present.

$\alpha \equiv \int_{\{\rho: P(\rho) < p\}} P(\rho) d\rho$  False alarm prob.

$\beta \equiv \int_{\{\rho: P(\rho) > p\}} Q(\rho) d\rho$  False dismissal prob.

Interpretation

Probability  $\leftrightarrow$  Frequency, not likelihood  
Long-run performance of statistic  $p$

## Measurement † Frequentist Statistics

Pick estimator  $e$  of source property

Average  $e$  over detector ensemble is source property  $E$

Define test statistic  $t \propto e - E$

$P(t)$  is distribution of  $t$ , is prob  $e$  differs from  $E$

$I(x)$  is shortest interval of  $t$  s.t.  $x = \int_{I(x)} P(t) dt$

$I(x)$  covers  $e - E$  with probability  $x$ .

Interpretation

Longrun behavior of  $e$   
Not probability that  $E$  takes particular value

Frequency interpretation of probability central

Relative frequency of repeatable measurements  
Not likelihood of proposition

## Detection, Measurement & Bayesian Statistics

Probability: Likelihood of proposition

Bayes Law:  $P(A|B)P(B) = P(A,B) = P(B|A)P(A)$

Signal described by param.  $\mu$   $\int$   
 $P(s|\mu) | g \rangle = \frac{P(g|s,\mu) P(\mu|s) P(s)}{P(g|0) P(0) + \int d\mu P(g|s,\mu) P(\mu|s)}$

detector output  $\Lambda$   
 $= \frac{\Lambda_{\mu}}{\Lambda + P(0)/P(1)}$

$$\Lambda_{\mu} = \frac{P(g|s,\mu) P(\mu|s)}{P(g|0)}$$

$$\Lambda = \int d\mu \Lambda_{\mu}$$

Prior probabilities:

$P(\mu|s)$  Expectation/prior knowledge regarding distribution of  $\mu$

$P(s), P(0)$  Expectation/prior knowledge regarding event rate

Interpretation

## Conclusions

Data Analysis: Searching for a needle in a haystack

Data Stream: Haystack

Integrity, Content:  
Characterizing non-gaussian noise

Signals: Needles

Detection response, & signal model accuracy  
Source event rate, distribution  
Signal modeling and unknown sources

Statistics: Distinguishing hay from needles

One persons signal is another's noise  
Bayesian & Frequentist statistics



# LIGO Status Report

---

David Shoemaker

LIGO Project

15 January 1996

## Organization of talk: From the outside in

- sites
- beam tubes
- buildings
- vacuum equipment
- detector, supporting research
- systems
- people
- plans

# LIGO Sites

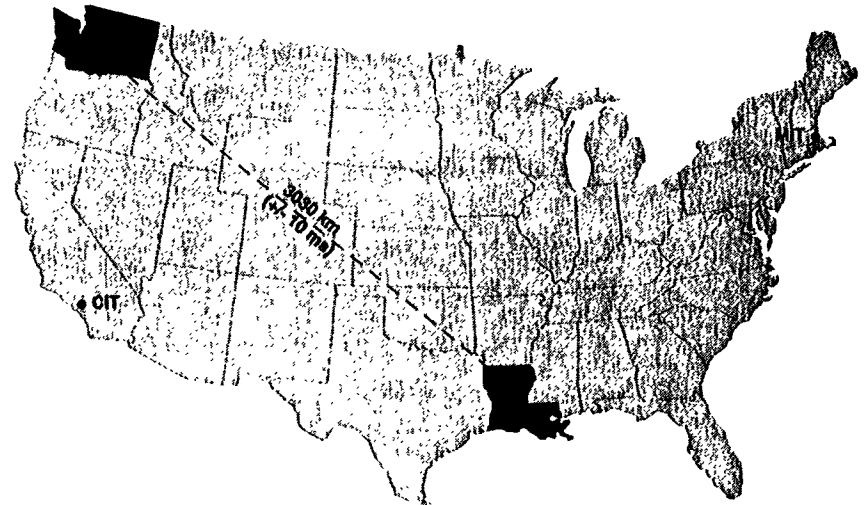
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## Hanford, WA

- 25 km from Richland, WA
- cleared, graded, and settling
- seismic noise survey completed

## Livingston, LA

- 50km from Baton Rouge, LA
- dedication 6 July 95
- cleared; rough grading started



## Beam Tube

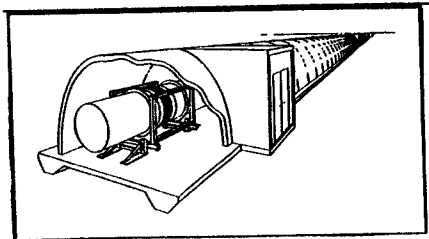
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### Beam Tube Chicago Bridge and Iron

- total of 16 km of 1.2 m diameter high vacuum
- tests of fabrication, cleaning, vacuum performed
- *baffle design chosen: black porcelain on SS*

### Beam Tube Enclosure RM Parsons

- supports and aligns Beam Tube, protects (from bullets!)
- final design, Oct 95



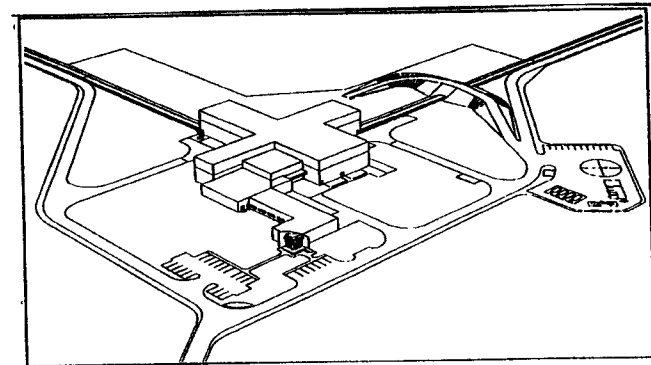
3 of 16

## Buildings

---

### Civil construction contract RM Parsons

- clean, quiet, temperature regulated environment
- work over requirements, realization from Dec 94 - Nov 95
- Preliminary Design Review in Nov 95



- WA: 2 full length or 1 full length, one half-length
- LA: 2 full length interferometers
- room to expand for additional interferometers

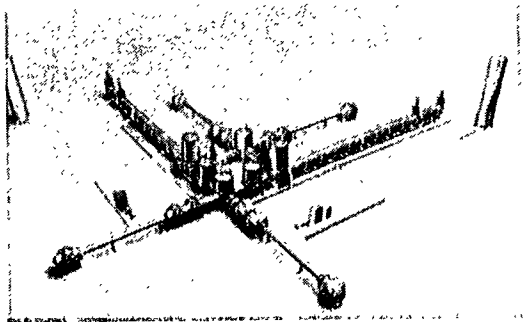
4 of 16

## Vacuum equipment

---

### Contractor selected

- Process Systems International
- provides flexible envelope for interferometer components
- allows isolation of elements, mechanical and electrical interface



- Preliminary Design Review in October, 95
- basic design of system, components well advanced

## Detector: Laser

---

### LIGO baseline was Argon Ion laser

- workhorse since 70's
- well established control technology
- dated technology, obsolete in 2000

### Solid State laser adopted as new baseline

- Nd:YAG or equivalent
- 1064 nm, near infrared wavelength

### Advantages

- allows adiabatic increase in power—no mirror changes
- relaxed mirror surface requirements (longer yardstick)
- some better optical components available (modulators...)
- common with other GW efforts, industry momentum

### Disadvantages

- reworking of (advanced) laser system, control design

### Nd:YAG development effort

- contract for commercial development (early stages)
- internal effort to transfer technology
- incorporation into high-sensitivity instruments (5m, 40m)

## Detector: Optics

### Suspended Mode Cleaner

- 12m model of LIGO system
- control tests (locking, manual alignment)
- performance tests (beam jitter, passing of modulation)

### LIGO Optics Pathfinder

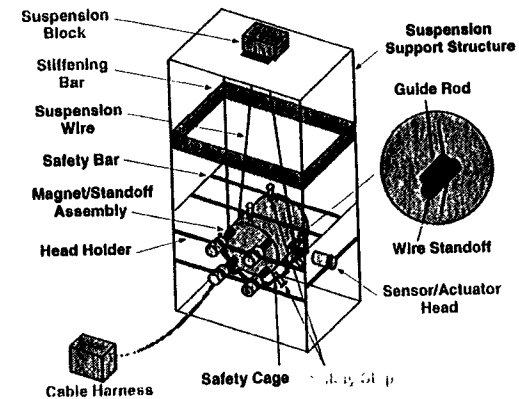
- to find path for industrial fabrication of LIGO optics
- substrates (Corning at present)
- polishing (several contractors)
- metrology (with help from NIST)
- coating (REO)

### Coating: real progress

- development of requirements (FFT and analytic modeling)
- development of implications for coating (modeling: 0.1% uniformity required over ~10cm diameter)
- development of masking techniques for coating
- comparison of transmission, AR coating reflectivity measurements
- looks feasible!

## Detector: Suspensions

### Refinement of single sling suspension



- techniques for maintaining high substrate ' $Q$ ' used
- refined versions of coil-magnet motor, LED-PD sensor
- version for 40m in preparation, soon to be installed
- designs for LIGO to follow

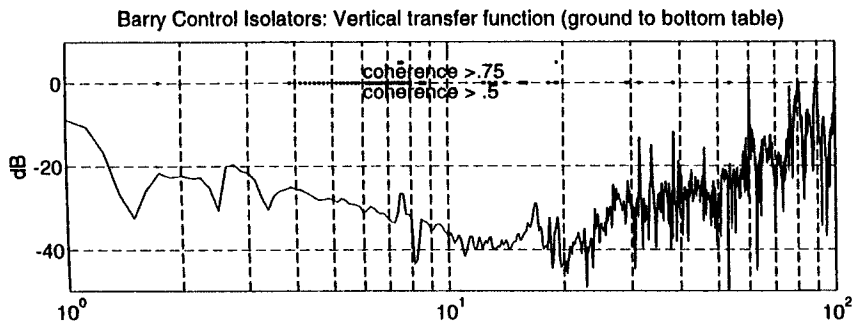
### Measurement of $Q$ for full-size mass underway

- electrostatic excitation
- study of sling, attachments
- no results yet

# Detector: Isolation

## Test of Commercial active vibration-isolation system

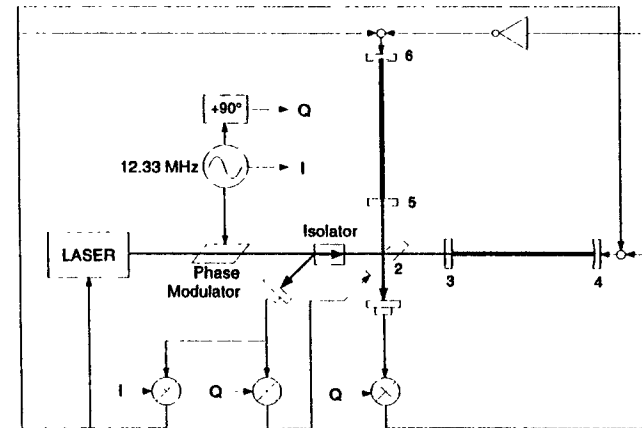
- 'beta' test of commercial system
  - > active servo system, measures ground noise, feedback
  - > six degrees of freedom
  - > at MIT, suppresses local non-stationary noise (trucks, subways...)
  - > for LIGO, can reduce controller dynamic range, up-conversion
- system functional
  - > required collaborative work between Barry, Inc. and LIGO staff
  - > significant reduction in noise: factor 100 in vertical, 2-30 Hz
  - > still short of prediction in horizontal, but
  - > makes qualitative difference in seismic environment



# Detector: Length sensing/control

## Recombined-Beam 40m Interferometer configuration

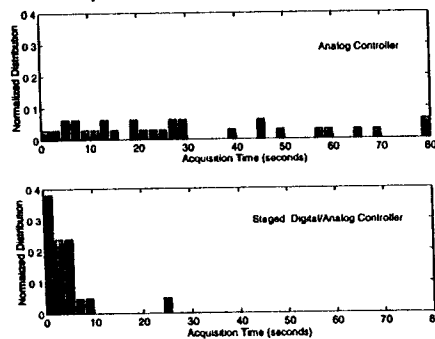
- Objective: give tests of models for dynamics of length control
  - > linear model (time constants similar to LIGO)
  - > acquisition (pendulum suspensions, correct D.O.F.)
- Requires optical configuration of LIGO
  - > recombination of beams from the two arms
  - > addition of recycling mirror
- Significant construction project finished
  - > modifications to in-vacuum components
  - > arms asymmetrized for modulation technique
  - > new modulation and control system



# Detector: Lock Acquisition

## Guided Lock Acquisition

- traditional brute-force method for acquiring 'lock' of cavity
  - > system moves very quickly through resonance - non-adiabatic
  - > very wide-bandwidth, high-force servo control
  - > short effective duty-cycle
- better idea: reduce test-mass velocity
  - > analyze signal output in time domain of unlocked system
  - > calculate relative velocity of test masses
  - > apply force X duration to 'brake' masses
  - > THEN acquire lock
- success
  - > considerably reduced locking time
  - > valuable information for locking strategies
  - > good technical experience for hardware/software



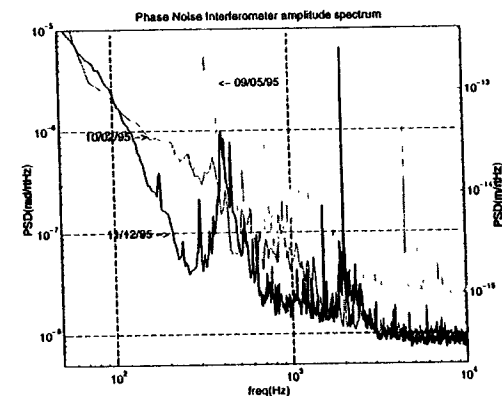
# Detector: Phase Noise sensitivity

## OBJECTIVES:

- demonstrate the initial LIGO phase sensitivity ( $\sim 10^{-10}$  rad/ $\sqrt{\text{Hz}}$ )
- develop the sensors, electronics, scattering control needed

## Activity in '95 dominated by construction

- re-commissioning of vacuum system
- fabrication of seismic isolation system
- design, production of optic suspensions
- replication of the Argon pre-stabilized laser
- control and monitor systems
- recycled Michelson now being commissioned



## Detector: And...

### Alignment research

- detailed design of wavefront sensing system
- construction of complete tabletop model
- fully digital servo and control system
- system requirements, subsystem specifications for LIGO

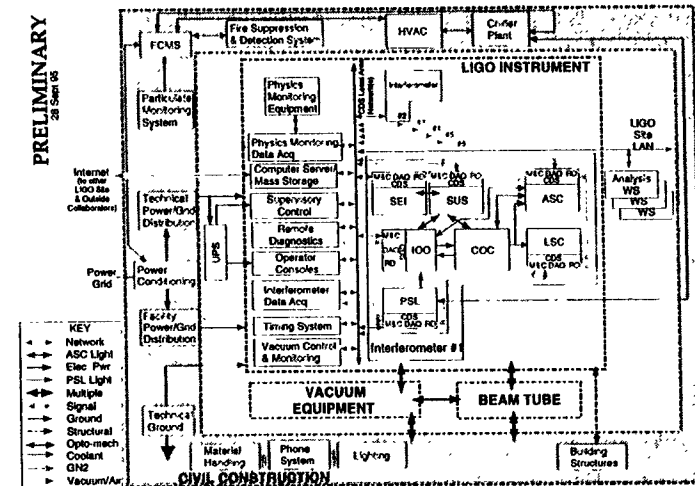
### Modeling (and joint with System Integration, or SysInt alone)

- FFT model: optimizing, automating, propagating
- also, dual-recycling FFT model, real optics (in process)
- dynamics of coupled cavities, means to generalize
- data analysis of 40m
- end-to-end noise modeling, housekeeping

...etc.

## System Integration

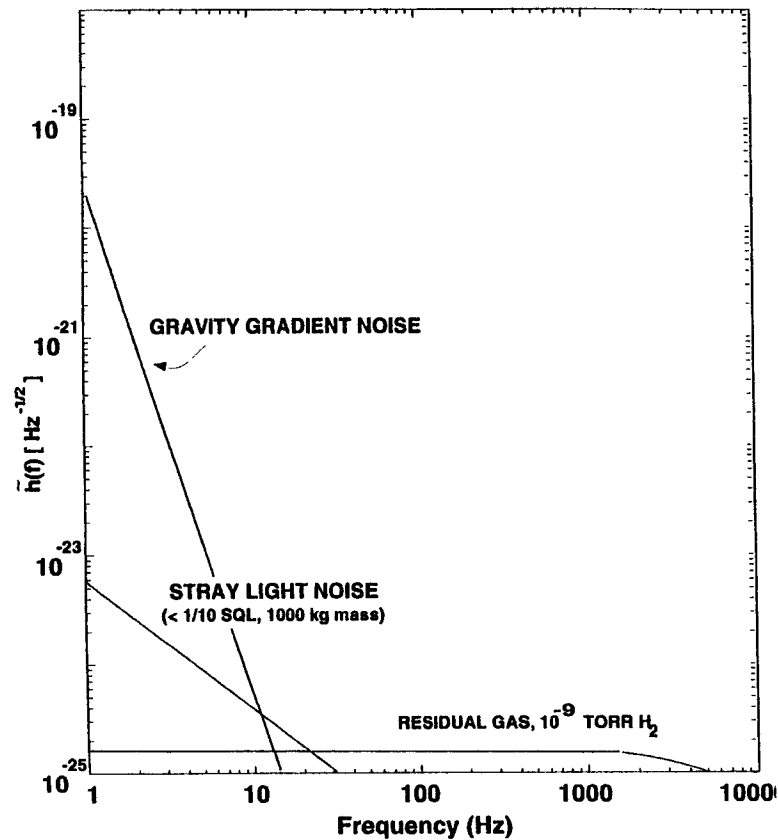
### Interfaces, top-level requirements, and everything else



### Interface Control Documents

- example: Vacuum Equipment/Beam Tube - Civil Construction

## System Integration: Facility Limits



## Structure of LIGO

### The heart of the project: People

- 85 people, 29 new in '95
- several more in '96
- Visitors (B. Allen, D. Gustafson, P. Saulson, K. Sliwa)

### Organizational

- Detector and R&D groups brought together (S. Whitcomb)
- Site directors named (F. Raab, M. Coles)
- LRC born and moved out (S. Finn)

### Milestones (WA, LA)

- Initiate Beam Tube Fabrication: imminent
- Initiate building construction: 6/96, 1/97
- Accept beam tube and cover: 3/98, 9/98
- Accept buildings, became equipment: 3/98, 9/98
- Initiate interferometer installation: 7/98, 1/99
- **Begin coincidence tests: 07/00**



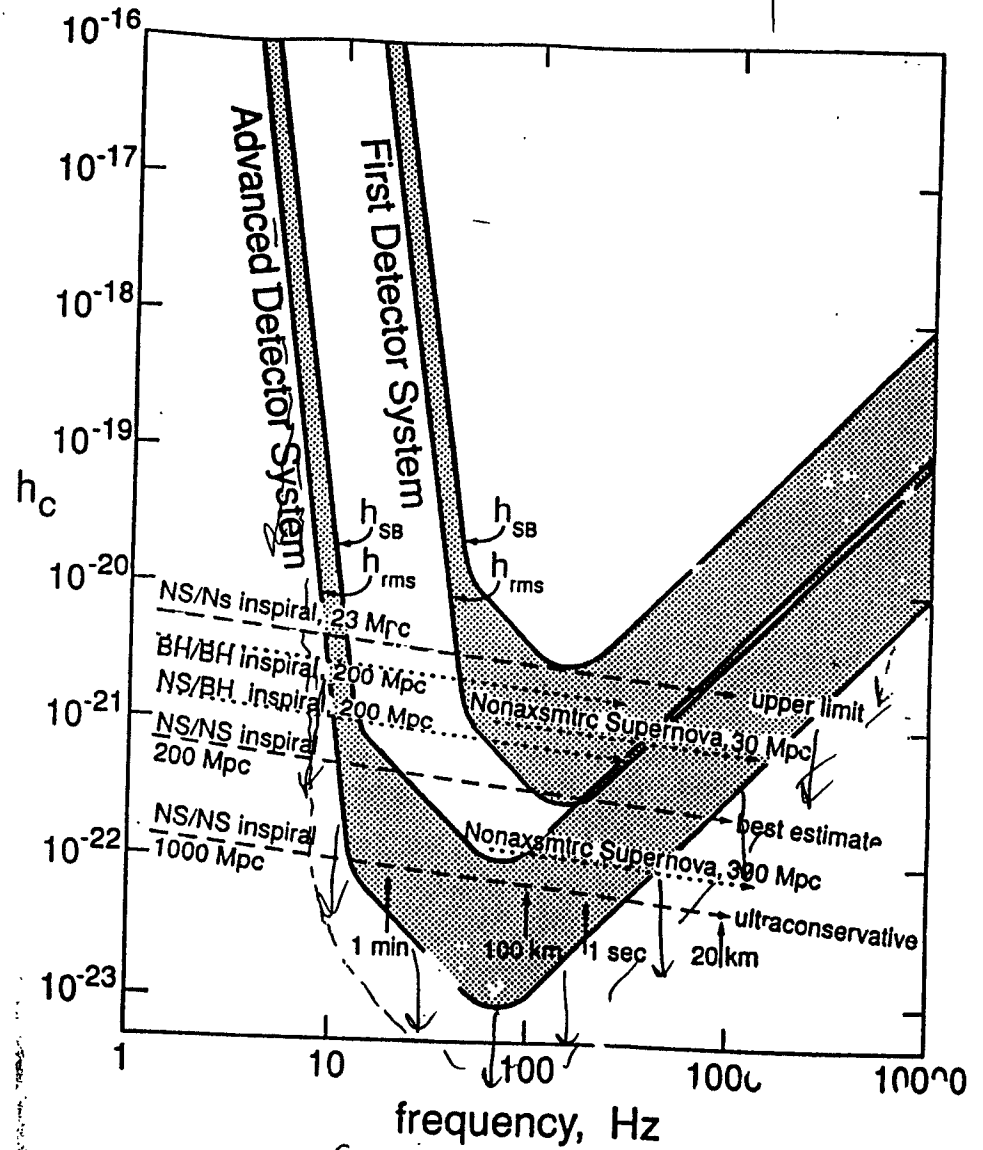
Sunday, 18th September 1989  
Chair: J. Campese

1:30	P. Meyer (Caltech)	Advanced Interferometry
2:00	Discussion	
2:30	M. Schmitt (Darmstadt)	500 MHz Gravitational
3:00	Discussion	
3:30	G. Lee (York)	Time-Delay Behaviour of a Laser Interferometer
4:00	Discussion	
4:30	R. Passivo (Pisa)	Virgo Experiment and Seismic Isolation
5:00	Discussion	
5:30	D. Mizuno (Kanagawa)	Data Acquisition and Analysis for GEO600 and TAMA400
6:00	Discussion	
6:30	R. Price (Stanford)	A Sagnac Interferometer for Gravitational Wave Detection
7:00	Discussion	

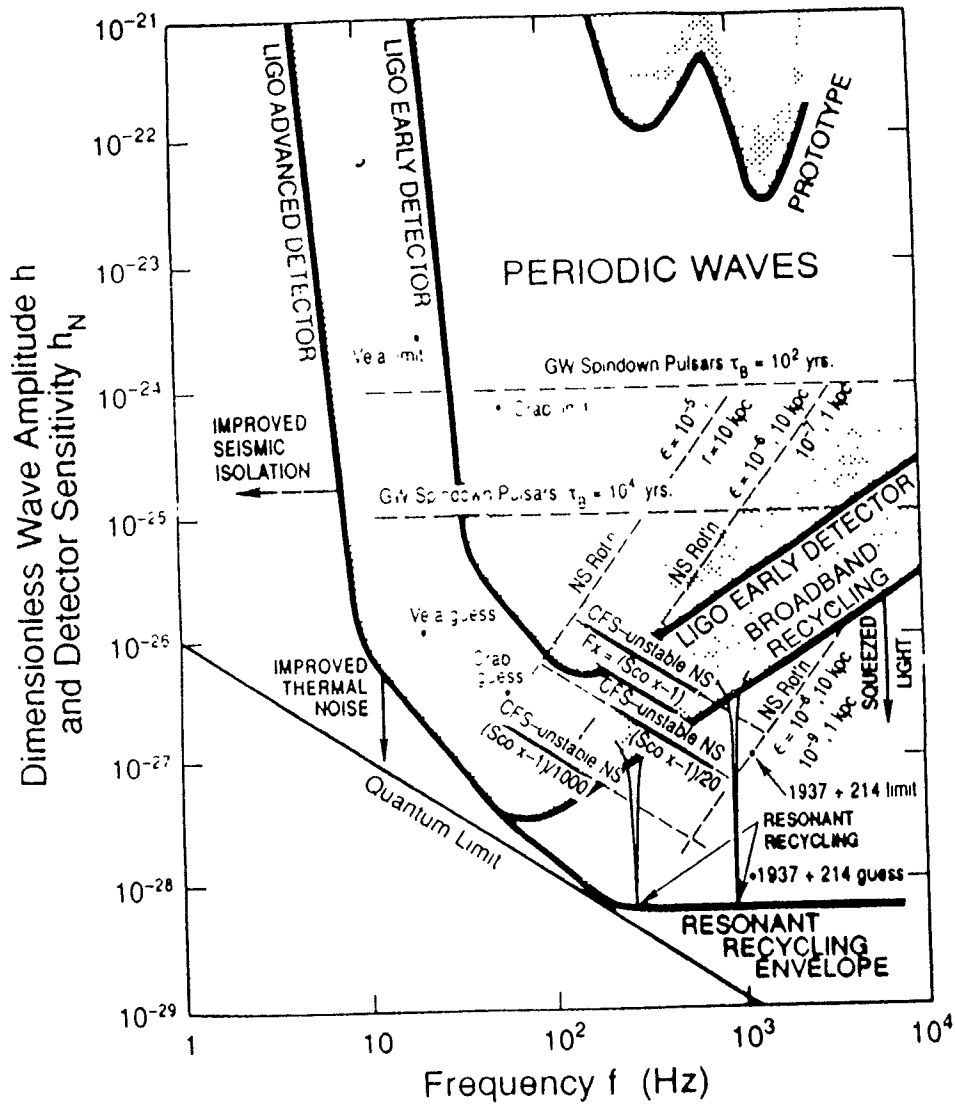
SOME CONCEPTS FOR ADVANCED INTERFEROMETERS.

(ASPEN 15/1/96)

R.W.P. DREVER. (CALTECH)

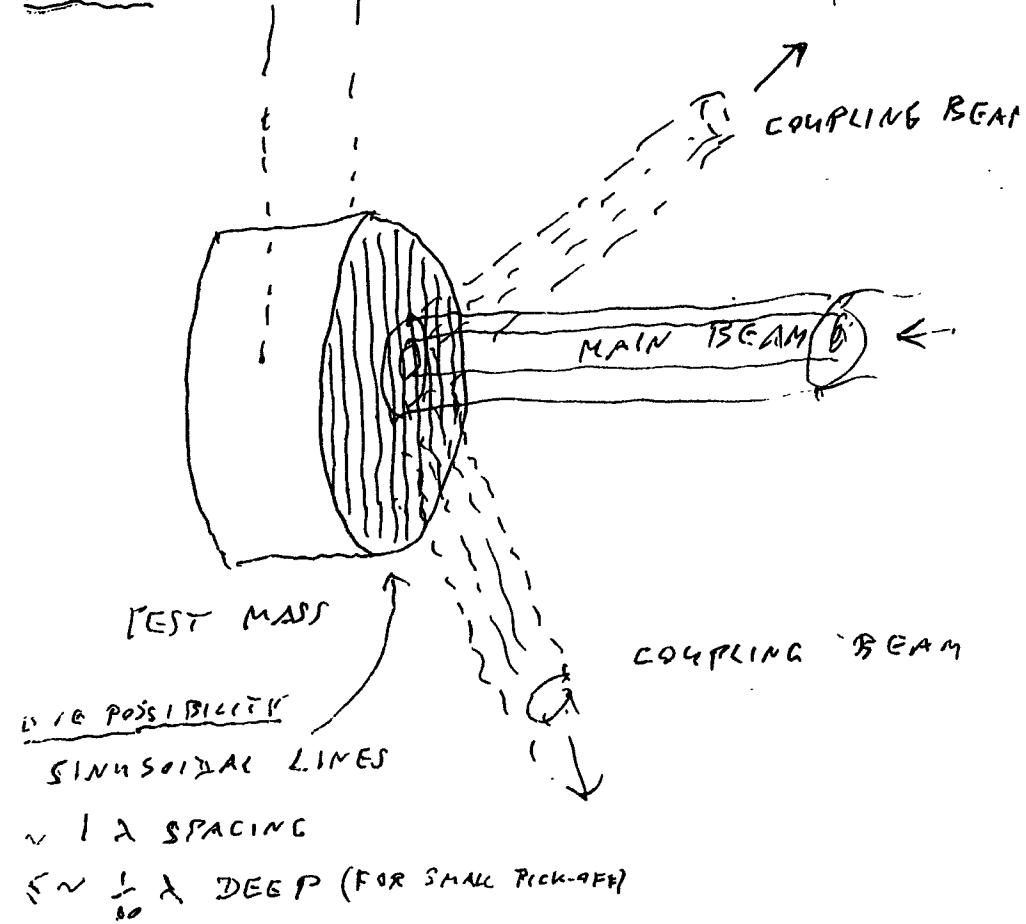


(PLOT OF K. THORNE,  
WITH ADDITIONS)



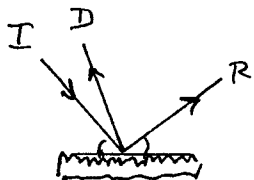
DIFFRACTION  
GRATING

ONE EXAMPLE 2-OUTPUT CASE

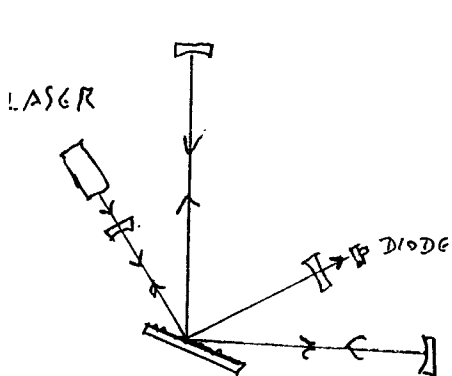


2. 5/1/96

HOLOGRAPHIC (DIFFRACTION) BEAMSPLITTER - SINGLE OUTPUT  
(OBLIQUE INCIDENCE)



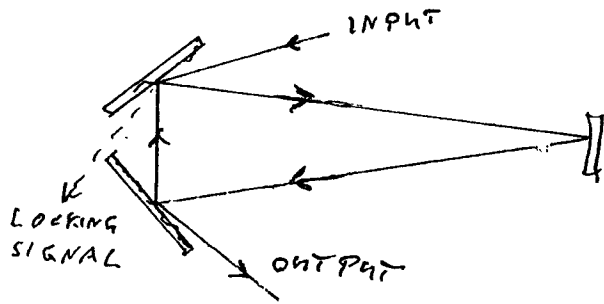
DUAL RECYCLING INTERFEROMETER



(50% GRATING)  
(I.E. 50% OF BEAM  
TO BE DIFFRACTED)

MODE CLEANING CAVITY

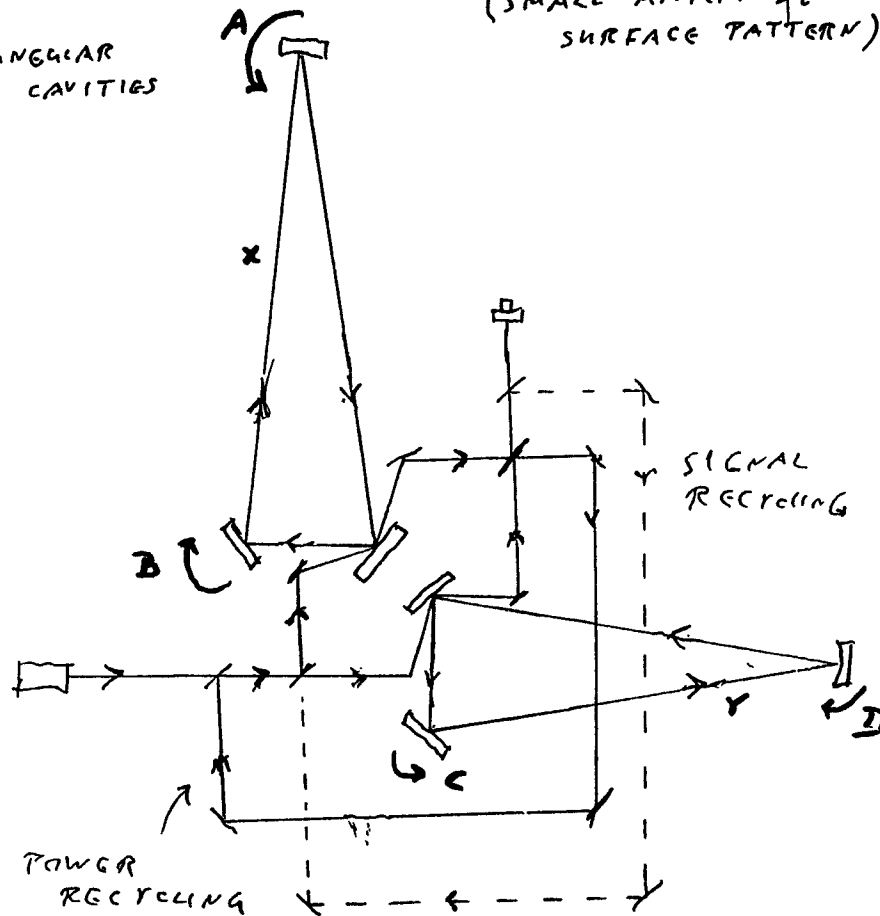
(SMALL COUPLING OK)



"HOLOGRAPHIC" INTERFEROMETERS

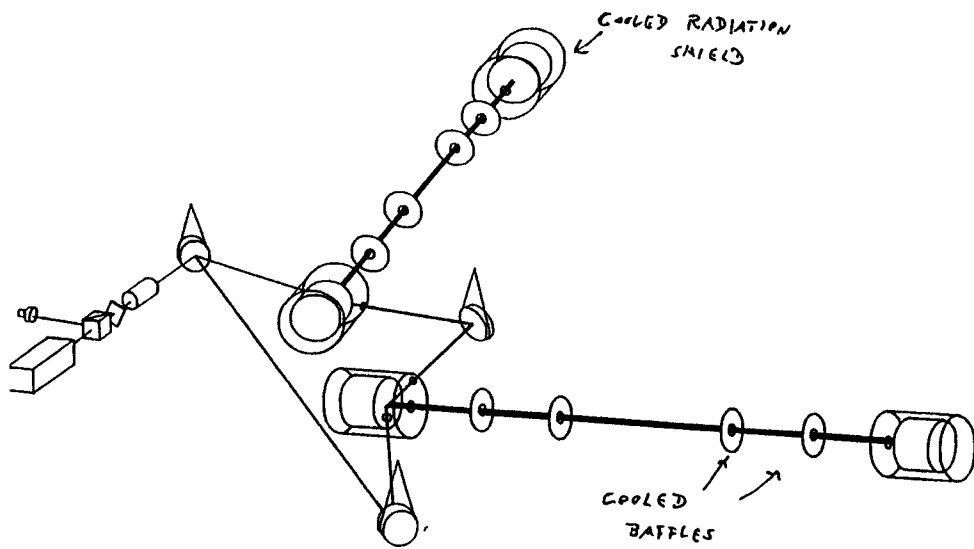
(SMALL-AMPLITUDE  
SURFACE PATTERN)

TRIANGULAR  
ARM CAVITIES



(NOTE: CAN ROTATE A, B, C, D TO ELIMINATE X, Y  
AND GET 2 FOLDED LINEAR CAVITIES)

INTERFEROMETER WITH TEST MASSES AT LOW TEMPERATURE



7/23/84B  
B. Dreyer

MAGNETIC LEVITATION GEOMETRIES  
BEING DEVELOPED. (CONCEPTS ONLY)

①

SIMPLE. PERIODS NOT THE LONGEST.



②



ADJUSTABLE FOR LARGE RANGE  
OF LOADS. LONG PERIODS POSSIBLE  
ROTATION CAUSES INSTABILITY PROBLEM.

(- 3 LEGS USED TO CONTROL THIS IN  
TEST EXPERIMENTS)

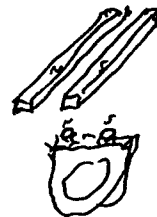
③



VERY LONG PERIOD POSSIBLE  
IN ONE DIRECTION

(PERIODS > 10 SECONDS FOUND EASILY OBTAINED  
IN PRELIMINARY EXPERIMENTS)

④



QUADRUPOLE - LESS SENSITIVE  
TO OUTSIDE MAG. FIELD.

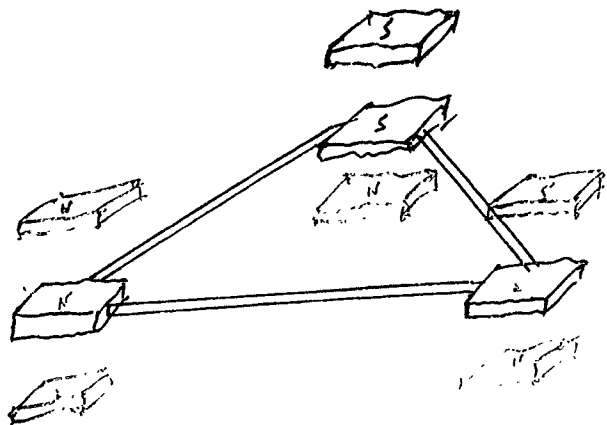
CAN EXTEND TO

4, 6, 8 ..... POLE  
SYSTEM.

12/1/96

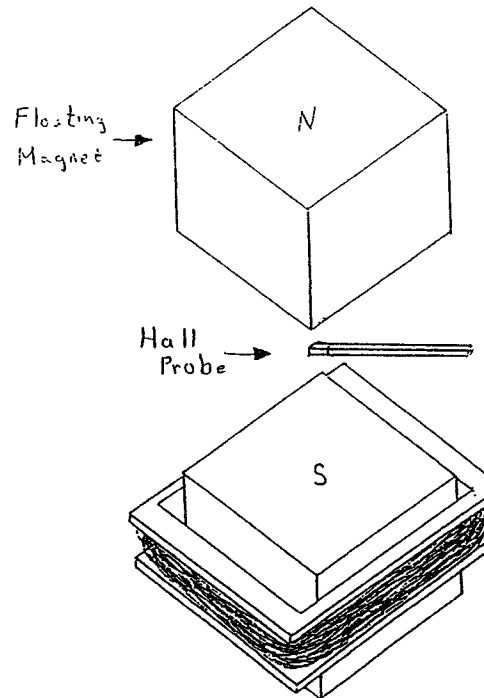
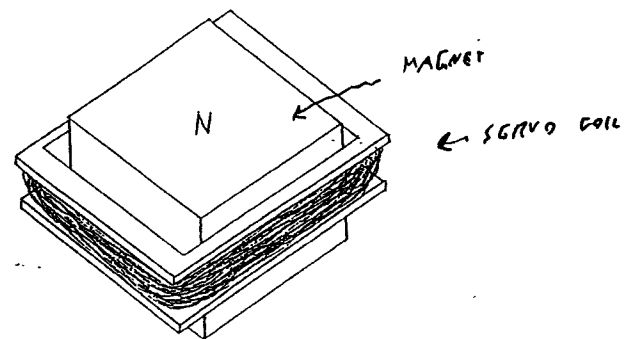
3-LEGGED TEST RIG FOR MAGNETIC ISOLATION

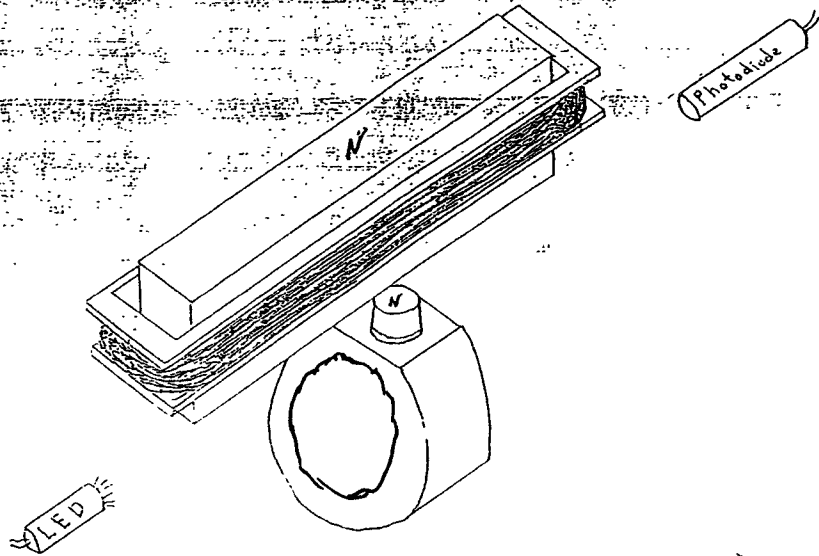
STAGE (CONCEPT ONLY)



(FOR SEISMIC ISOLATION)  
(PRIMARY)

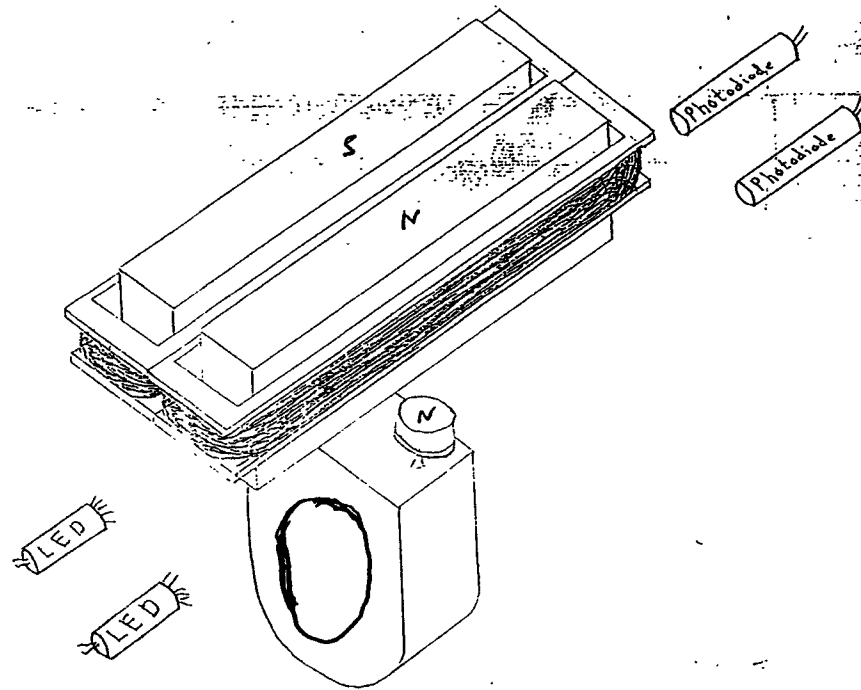
ONE UNIT OF 3-LEGGED TEST RIG.





EXPERIMENTAL TEST MASS SUSPENSION

15/1/96  
RD.

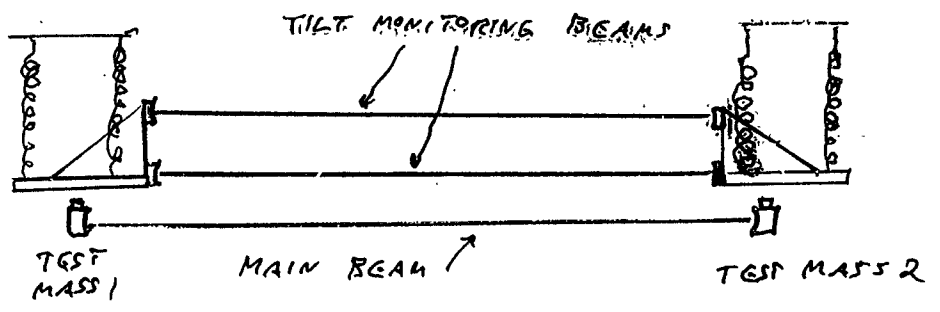


(CAN EXTEND TO MORE POLES)

19. 15/1/96  
RD.

21.

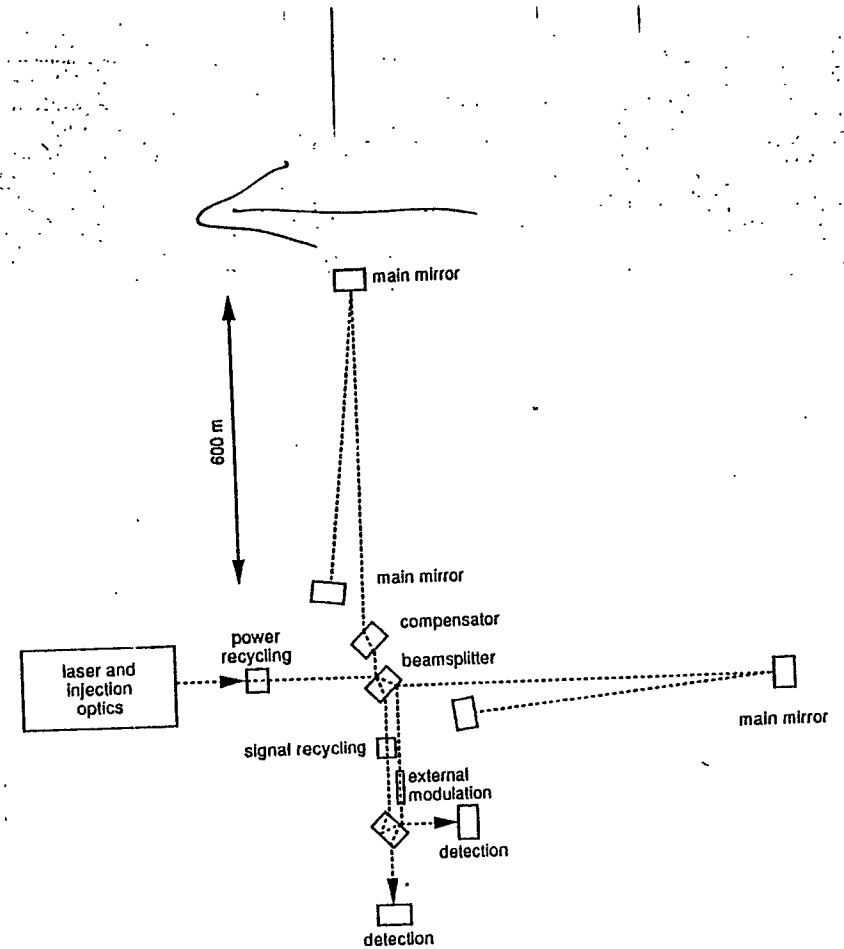
TILT-COUPLED LOW FREQUENCY SUSPENSION





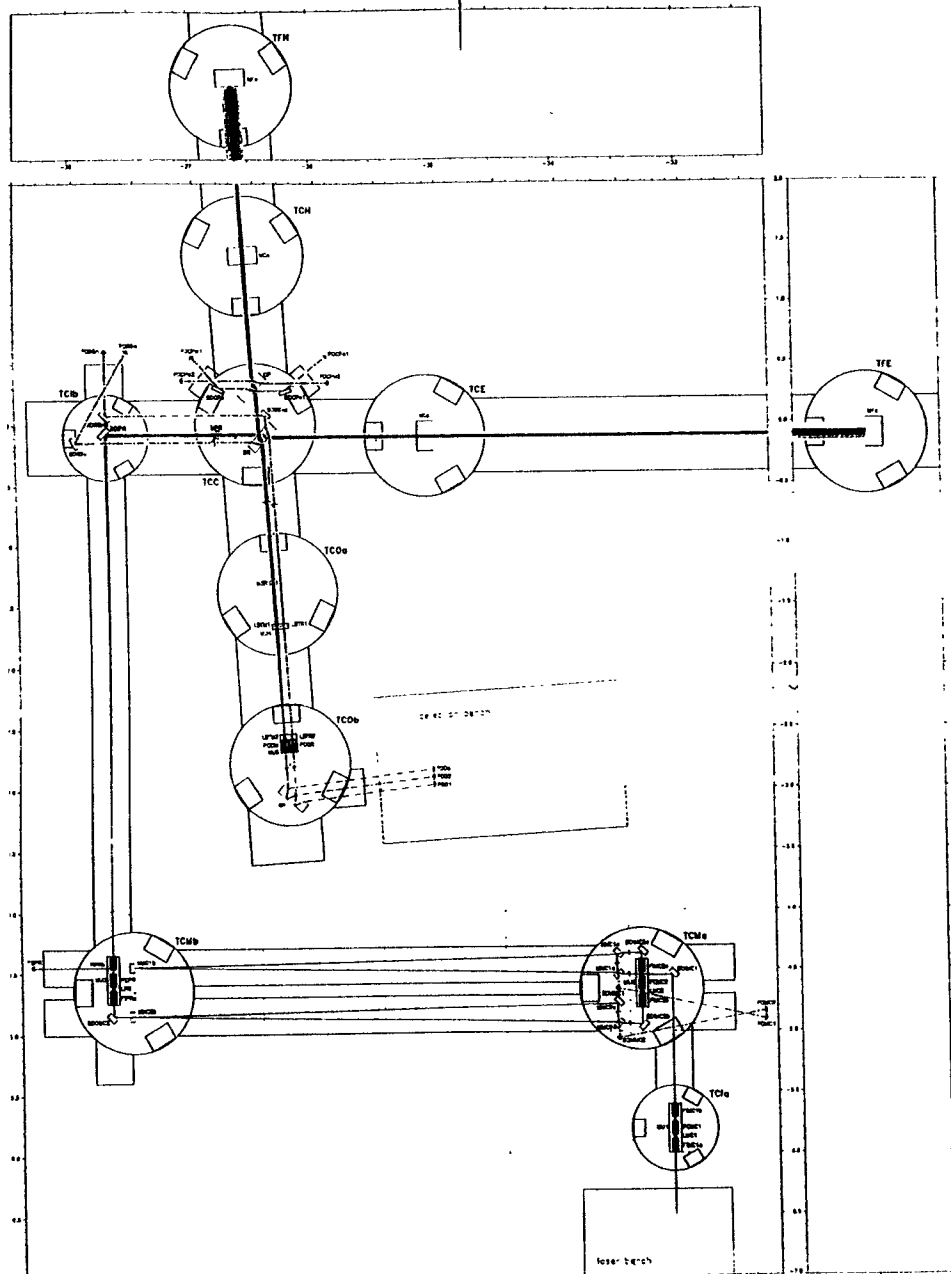
# GEO 600 Status Report

Mathias Schrempel  
and the GEO 600 Team



layout schematics

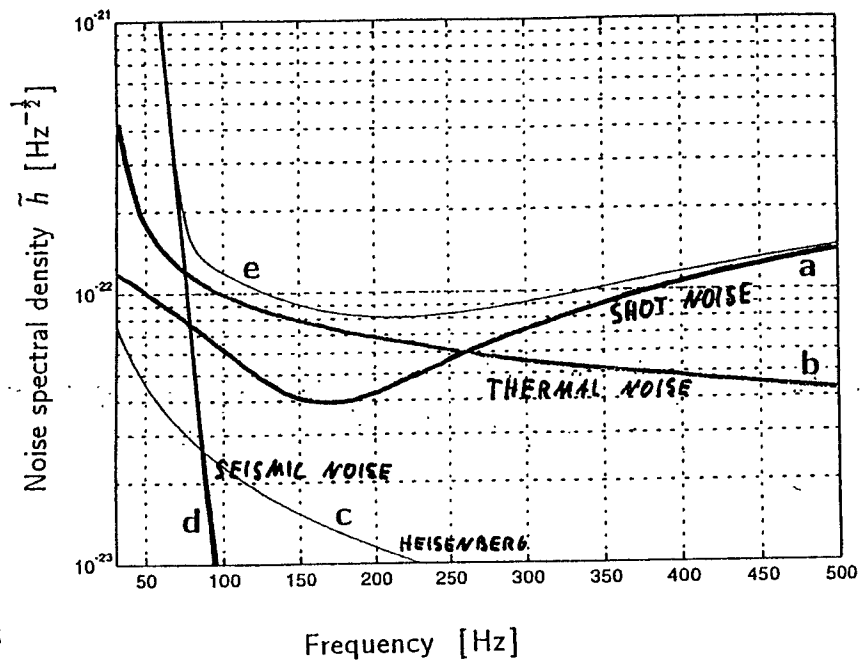
# GEO 600 optical layout



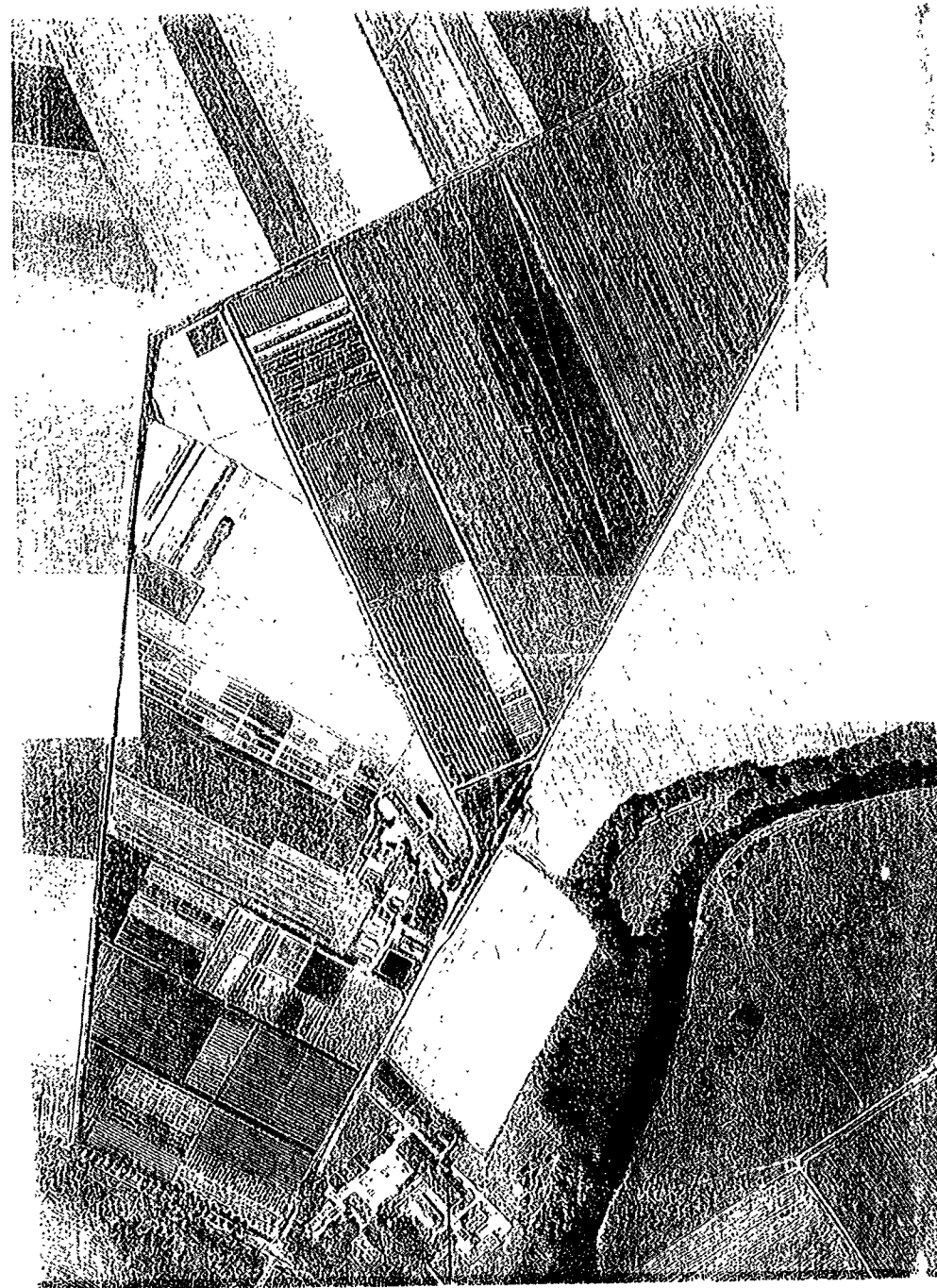
# GEO 600

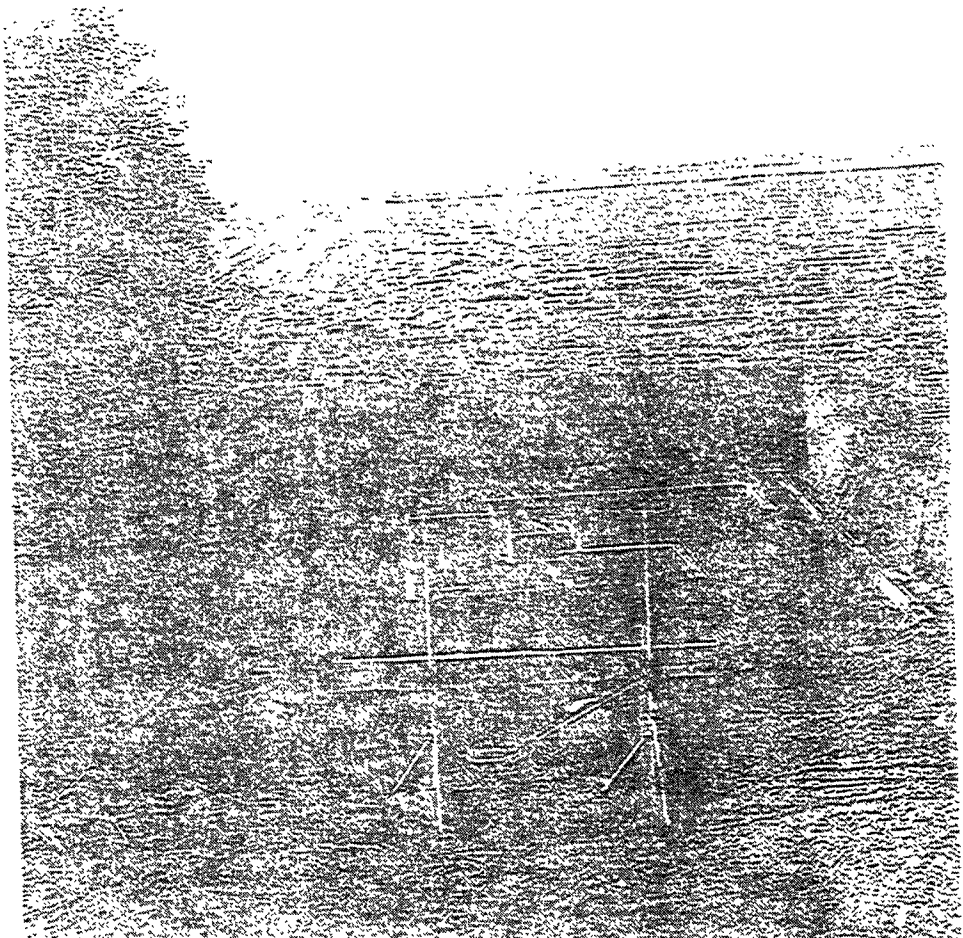
arm length	2 × 600 m, NNW and ENE
laser	diode pumped Nd <sup>3+</sup> YAG Laser, 10W Master - Slave system
test masses	25 cm dia., 15 cm thick, OH <sup>-</sup> -free fused silica, absorption ≤ 1ppm/cm
coatings	ionsputtered, absorption < 1ppm
power recycling	up to 10kW circulating power
signal recycling	150 to 1500 MHz bandwidth
tunability	0 to 1kHz
seismic isolation	encapsulated 4-layer stacks, active suspension optional
suspension	double pendulums, monolithic lower stage
vacuum system	60cm dia. corrugated beam tubes, hydrocarbon-free <5×10 <sup>-8</sup> mbar for H <sub>2</sub> , <5×10 <sup>-9</sup> mbar for other gases
sensitivity	depending on bandwidth h~10 <sup>-22</sup> to 4×10 <sup>-23</sup> /√Hz

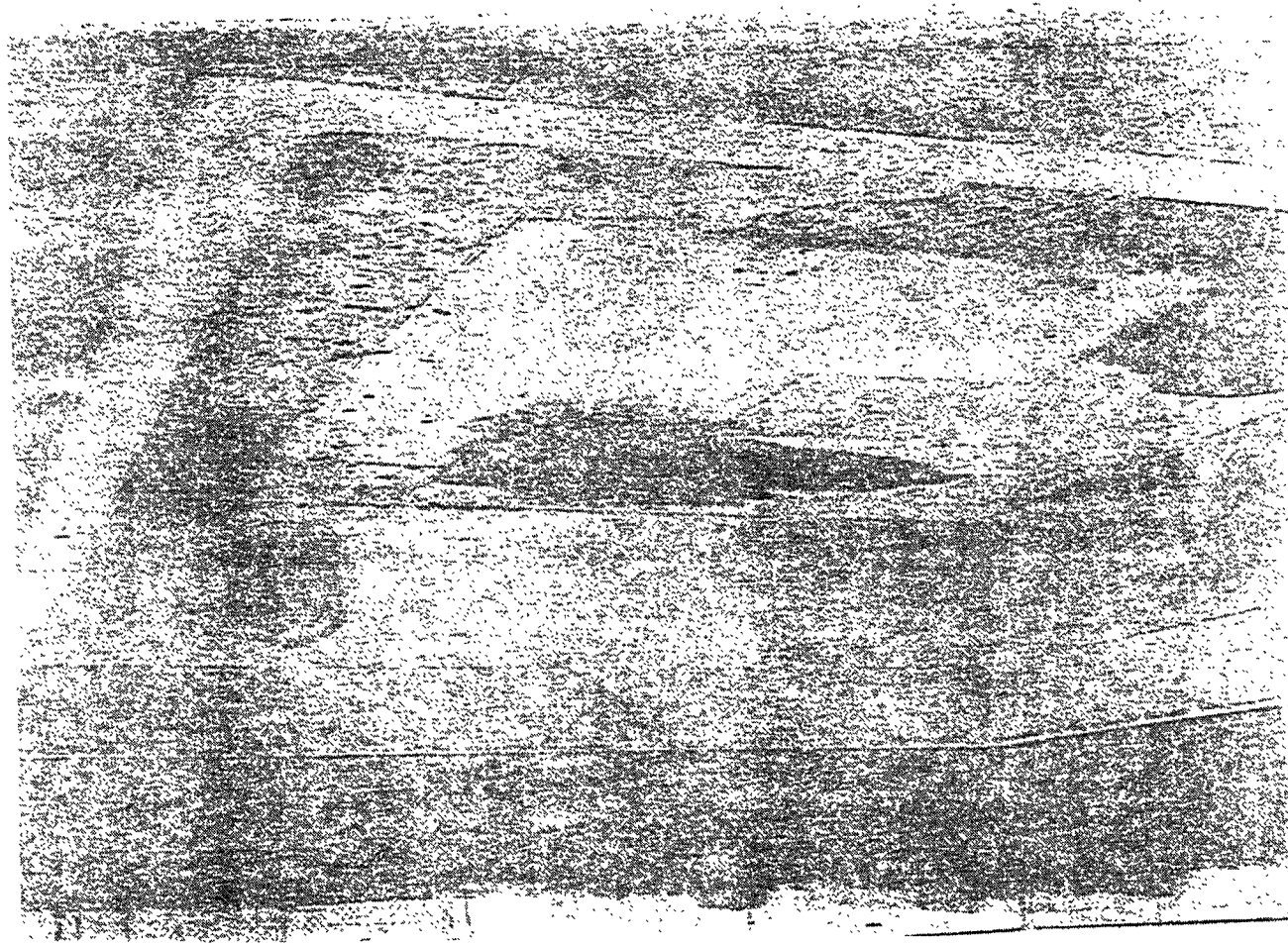
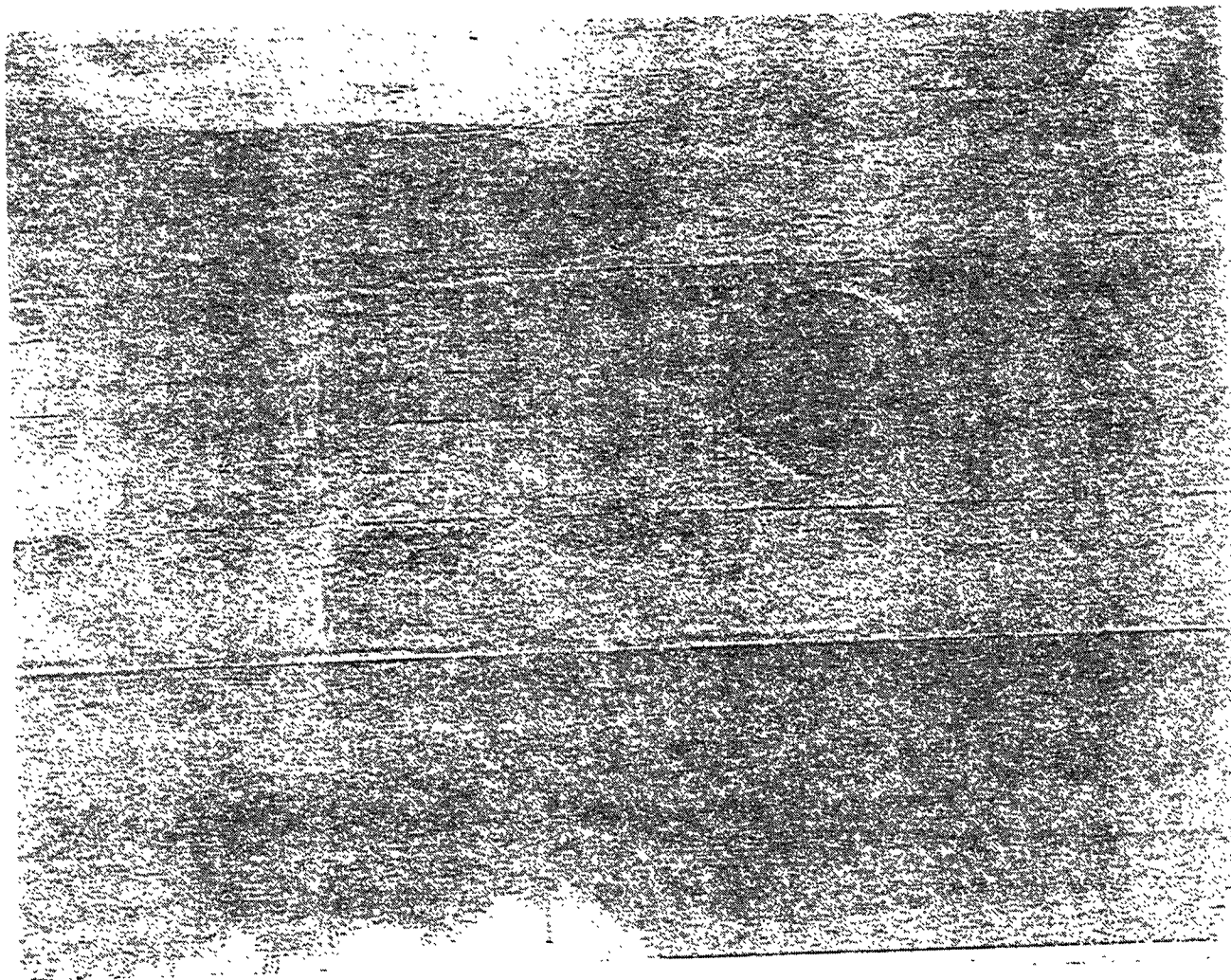
# Sensitivity Limits

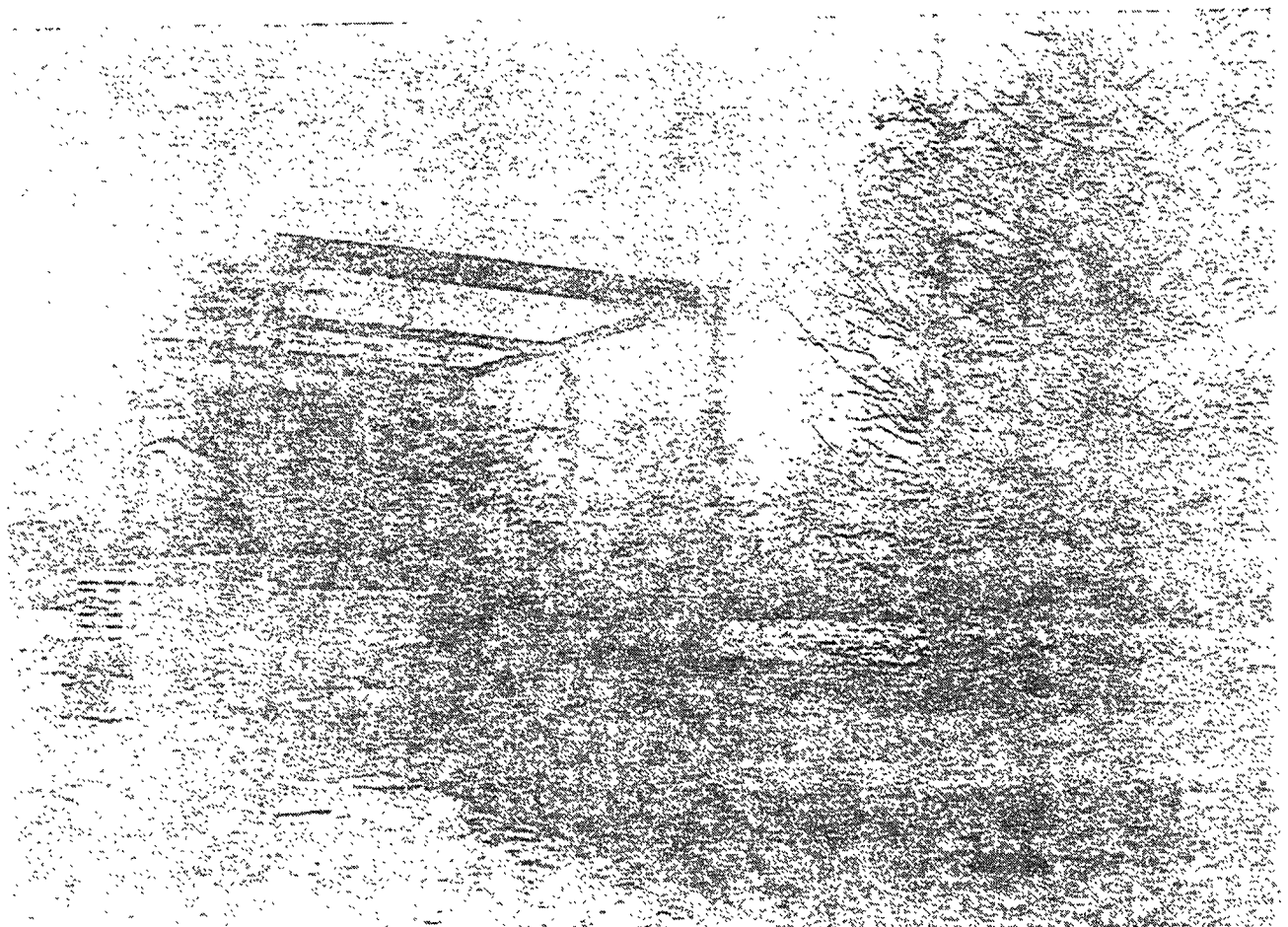
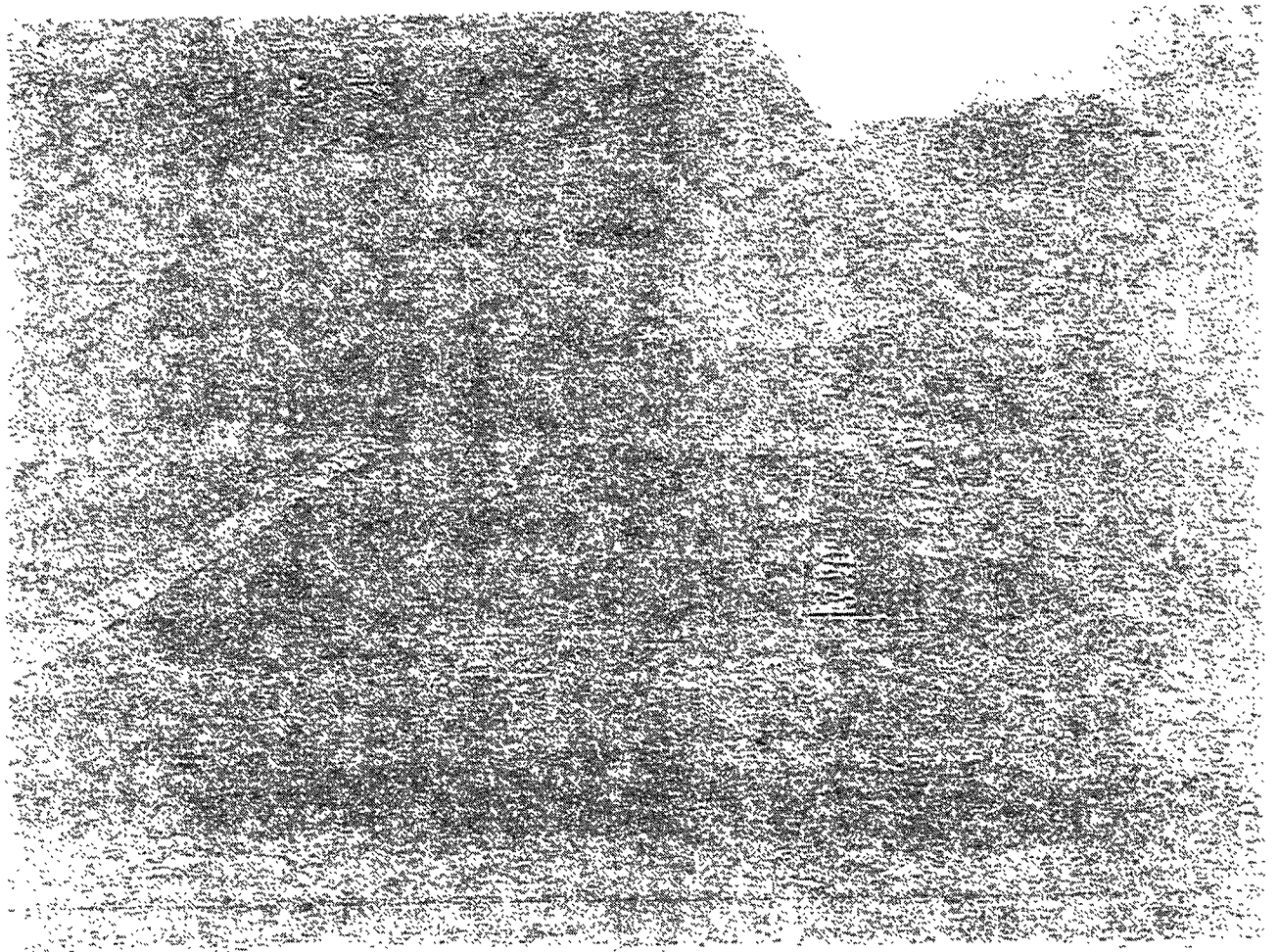


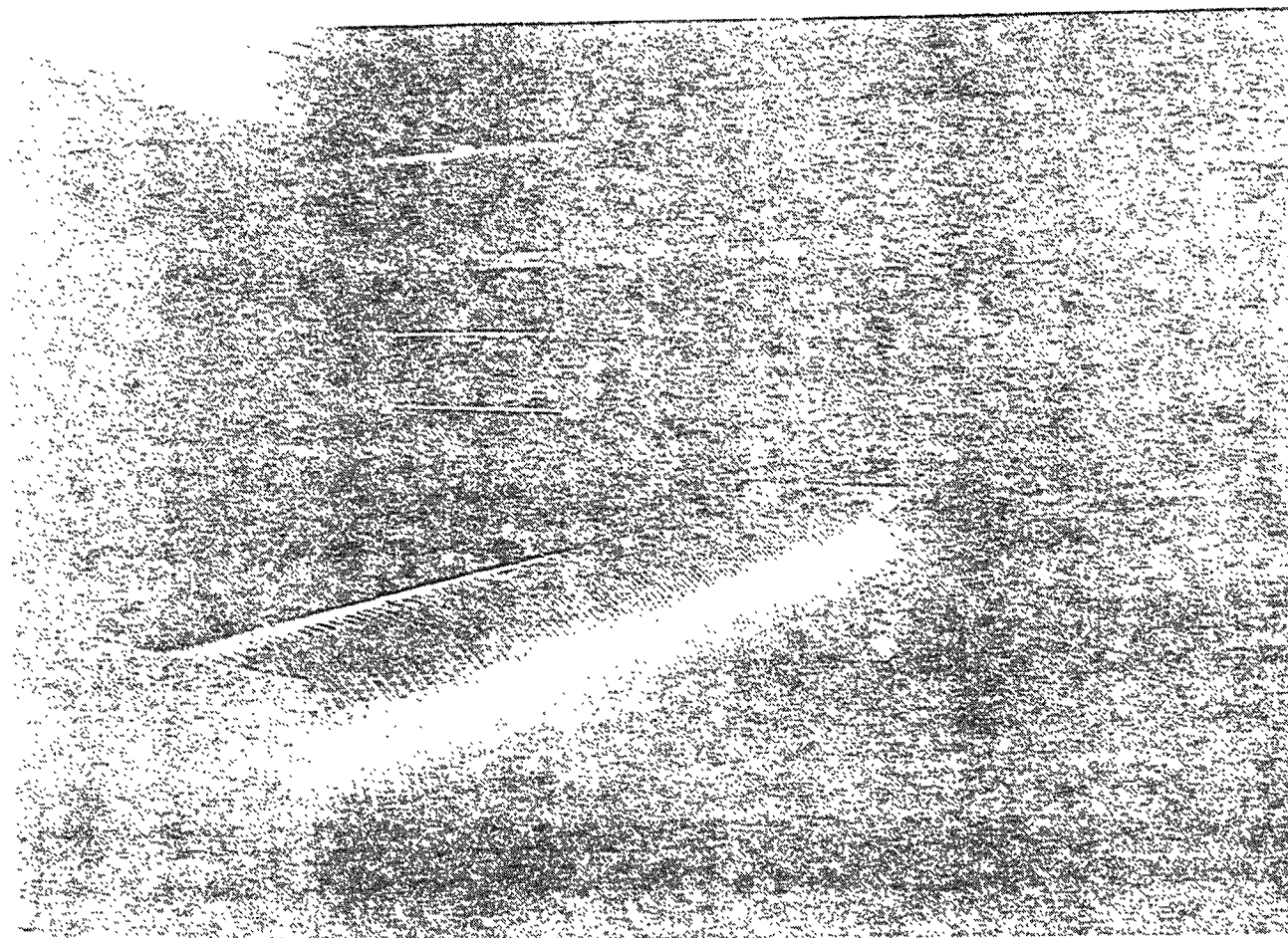
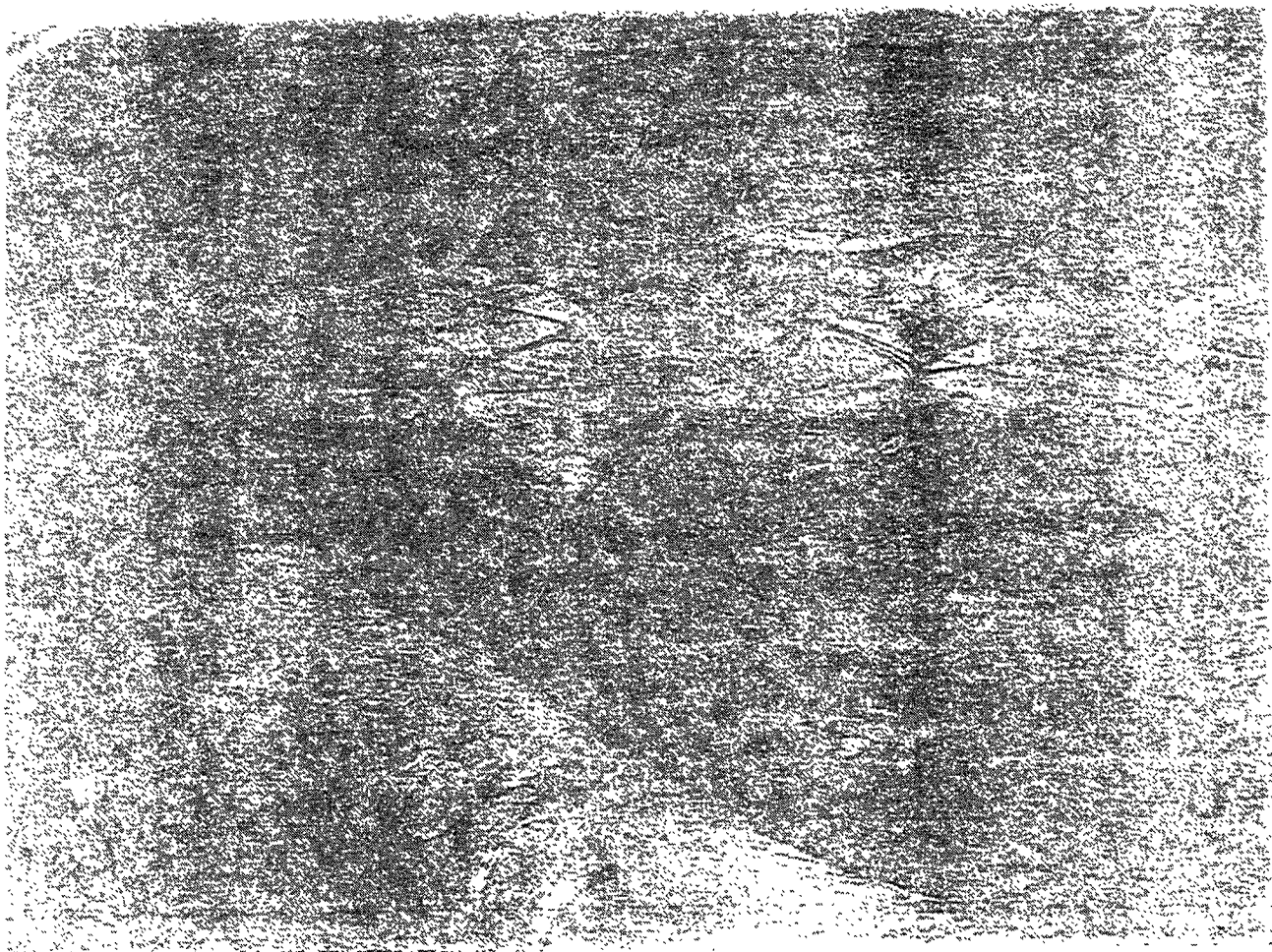
- 10kW internal power, 20ppm mirrors
- Q:  $5 \times 10^6$  internal,  $10^8$  pendulum
- 4-layer stack, double pendulums



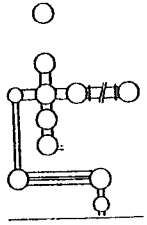




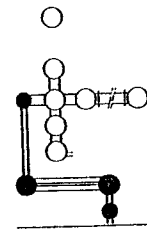




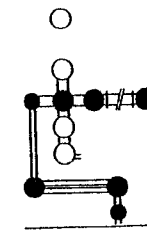
# Time Table



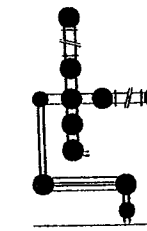
Vacuum System 6 '96



Modecleaners,  
Laser Bench 6 '97

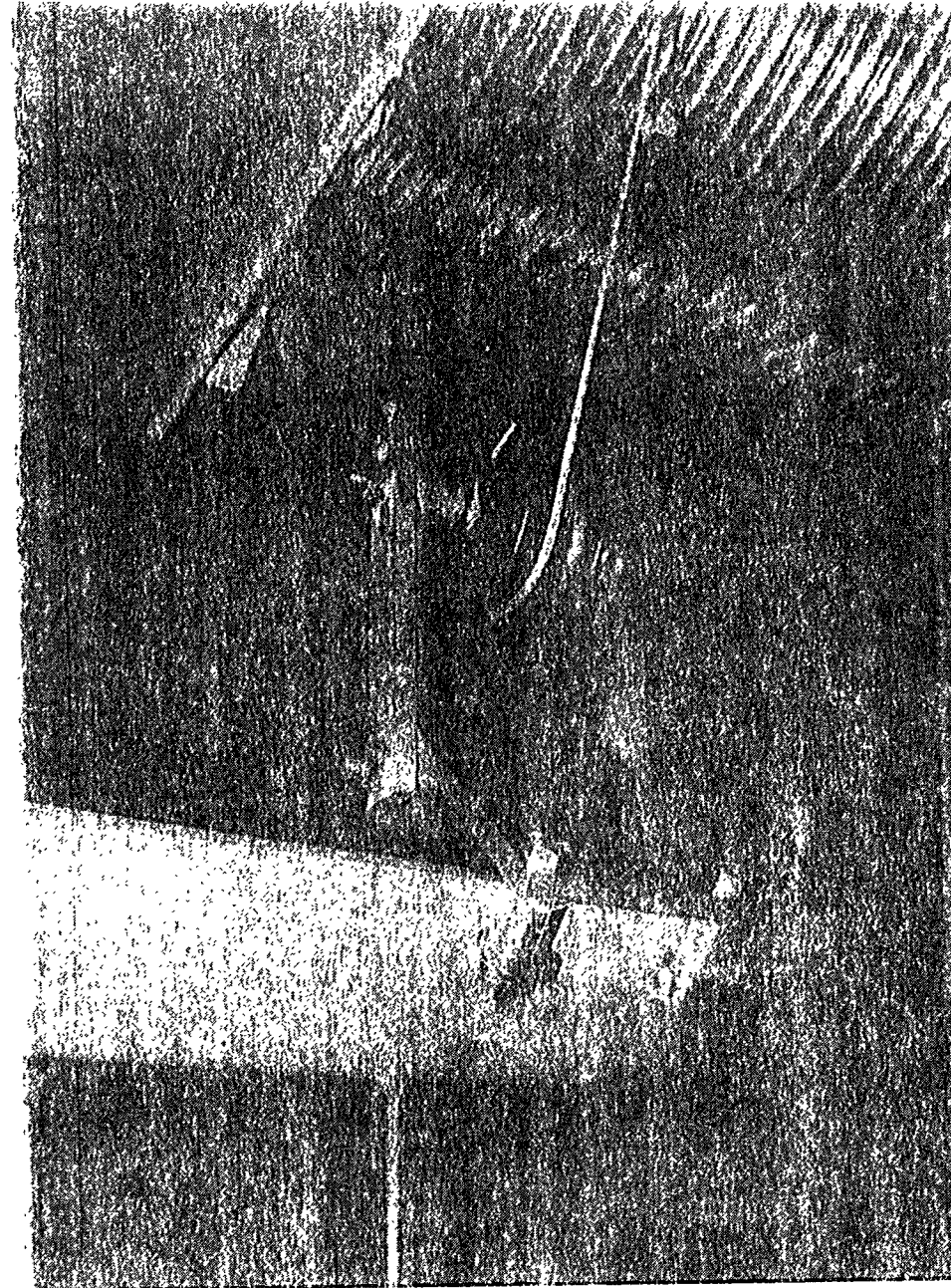


1200m Cavity 6 '98



Michelson 6 '99

Final Optics 2000





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# Time-Domain Behavior of the 40-Meter Interferometer

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Fred Raab  
January 15, 1996

A Report on Preliminary Studies and Future Plans for Work with Collaborators:

- Torrey Lyons
- Aaron Gillespie (now at NBS, Gaithersburg)
- Kent Blackburn
- Andy Kuhnert
- James Mason

Goal: Institute Regular Time-Domain Operation and Analysis.

1 of 5

LIGO-G960000-00-M

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## Non-Stationary Noise in a Laser Interferometer

---

- Most source detectability arguments are based on smooth stationary gaussian noise spectra with  $\Delta f \approx f$ ; is this reasonable?
- Excess (non-gaussian) noise must be vetoed through environmental monitoring and coincidence techniques.
- Coincidence strategies often assume uniform rates for non-gaussian events (i.e. stationary). For LIGO:

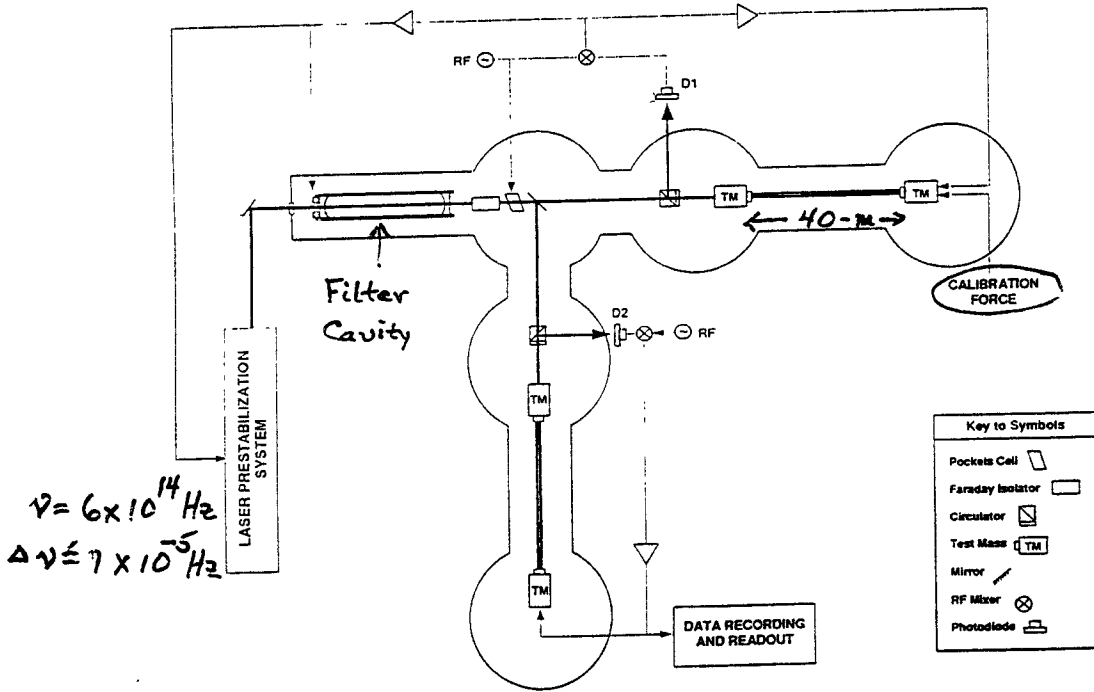
$$R_{TRIPLE} \approx \tau_{12}\tau_{13}R_1R_2R_3$$

where  $\tau_{ij}$  refer to coincidence window widths and  $R_k$  refer to the post-veto singles rates for interferometers 1, 2 and 3.

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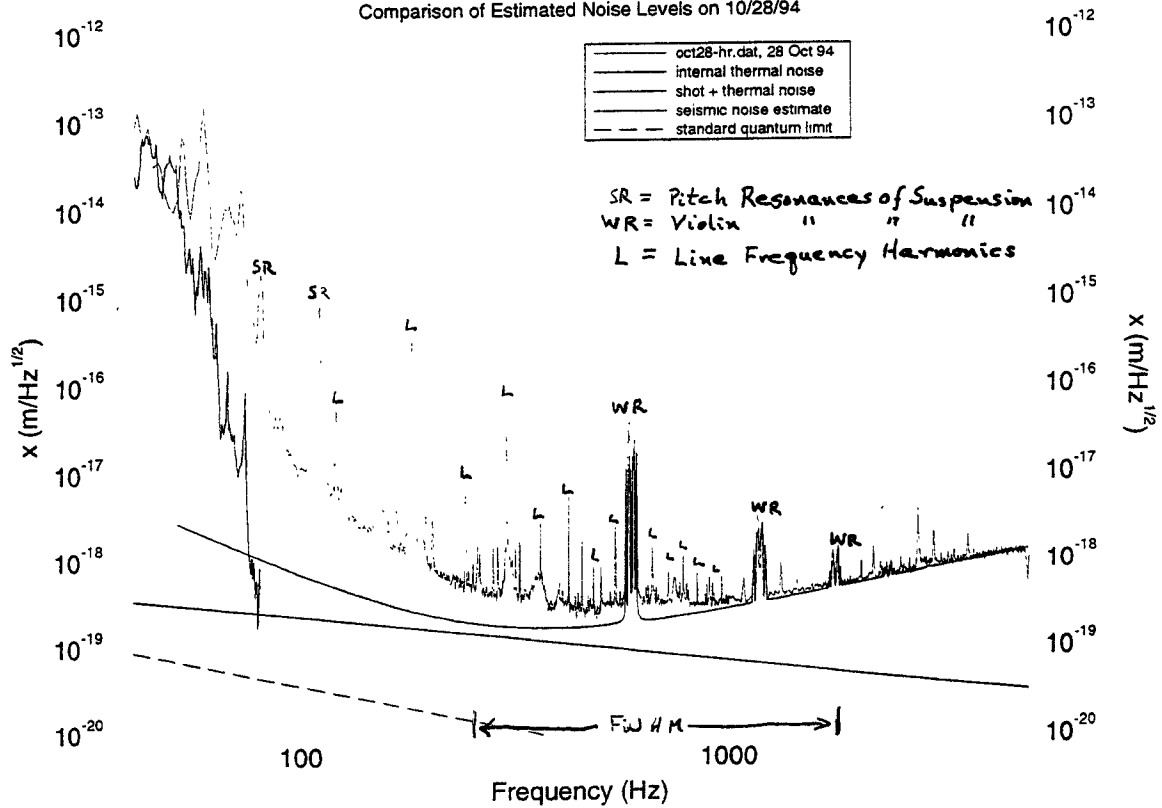
LIGO-G960000-00-M

# Mark II 40-Meter Interferometer



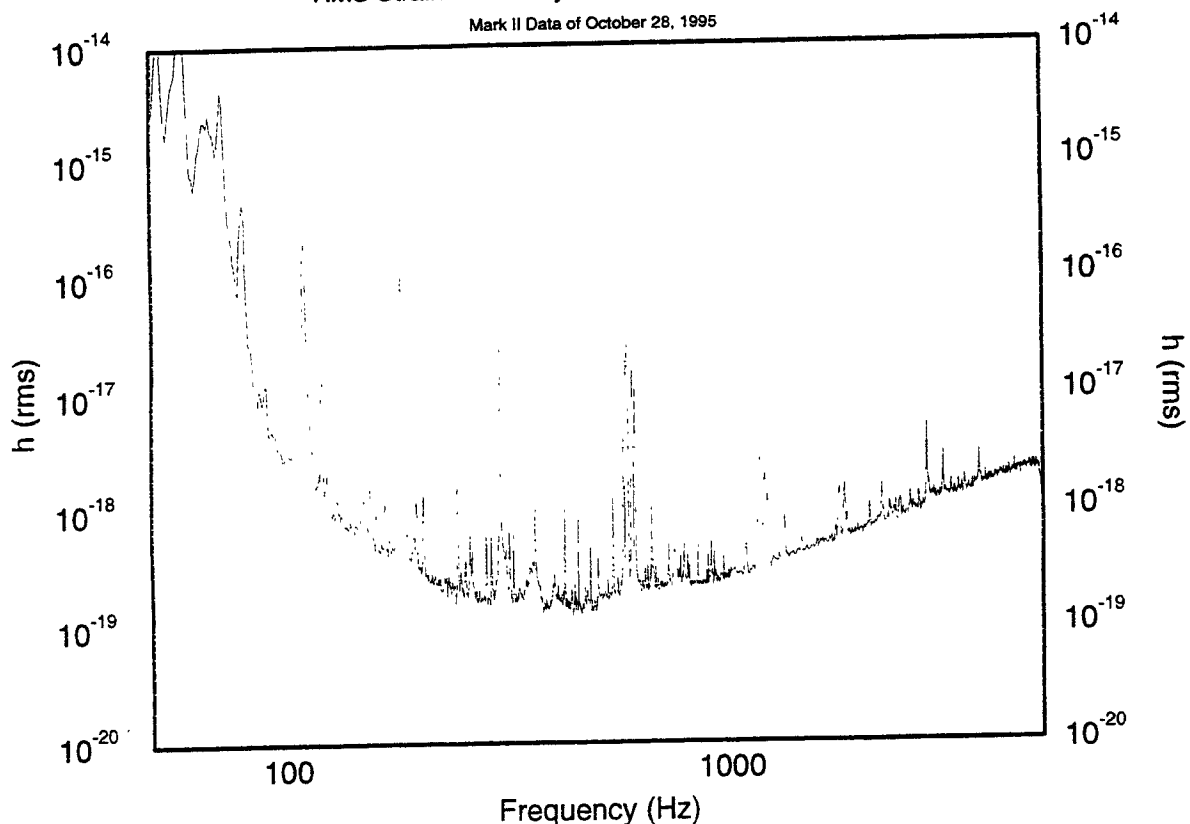
## 40 m Displacement Sensitivity

Comparison of Estimated Noise Levels on 10/28/94



## RMS Strain Sensitivity of 40-Meter Interferometer

Mark II Data of October 28, 1995



Nov 10 1995 17:31

-fjr/fort/mk2/hrms\_oct28.xvgr

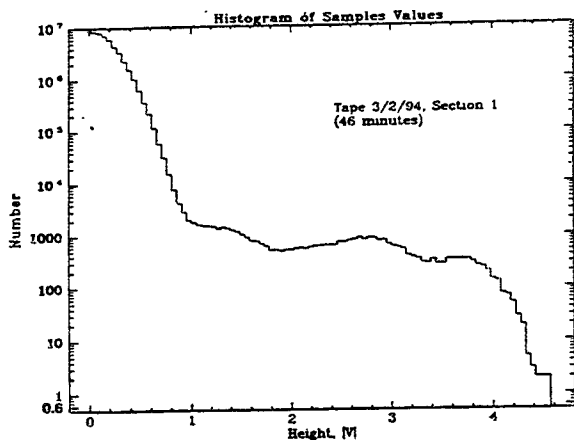
## How Can Experimental Data From a Noisy Interferometer Be Characterized?

- Sample histograms from a wide-band channel are not very informative (at least not to us).
- Correlating data with templates: great for finding the expected, but surprises could get away!
- Machine artifacts need identification and characterization, but may not trigger sharply even in extensive template sets.
- Some tool kit for identifying “events” and characterizing is needed.

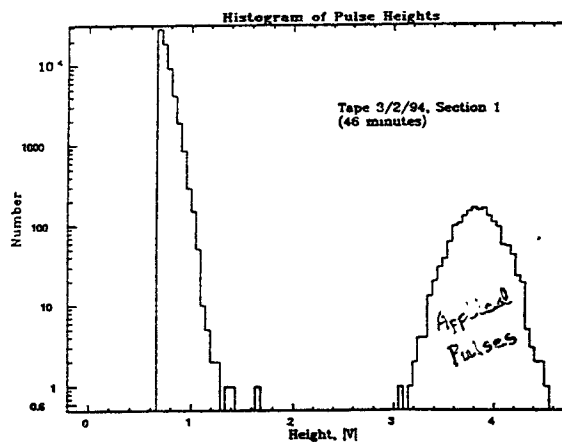
# Utility of a Simple Event Finder Algorithm

- Threshold for event turn-on, variable dead time filter

## Sample Histogram



## Event Histogram

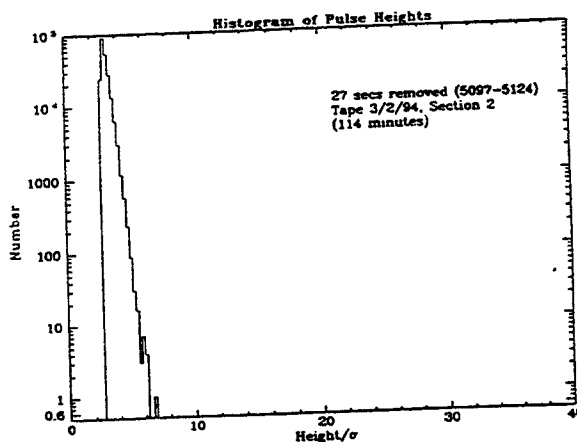
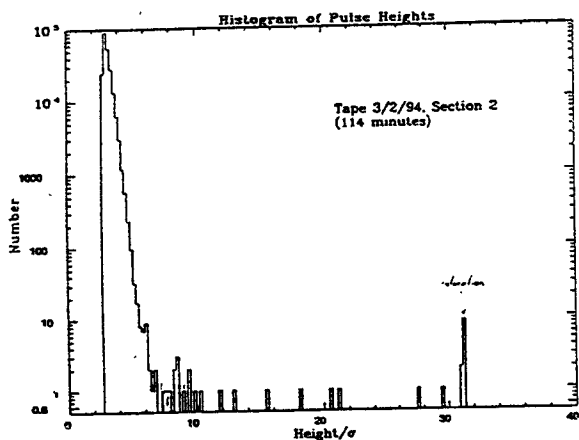


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LIGO-G960000-00-M

# An Example of Non-Stationary Non-Gaussian Events

- Event histogram for this locked section is almost purely gaussian, except for a “minute” interval:

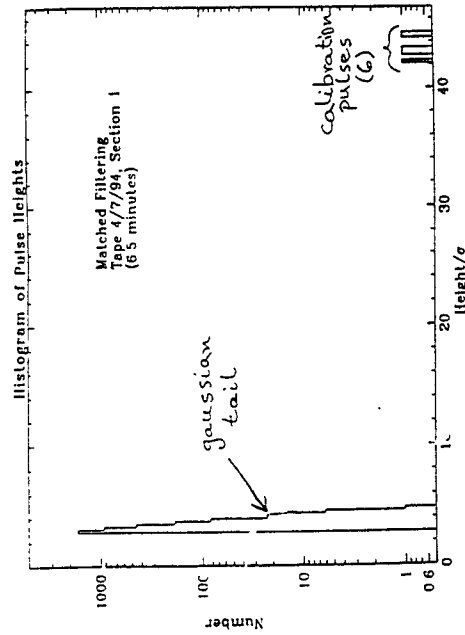
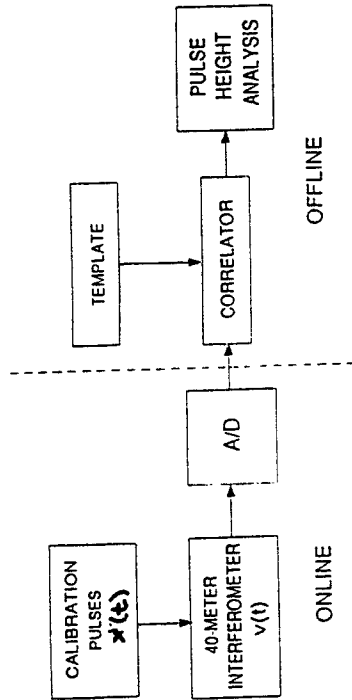


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LIGO-G960000-00-M

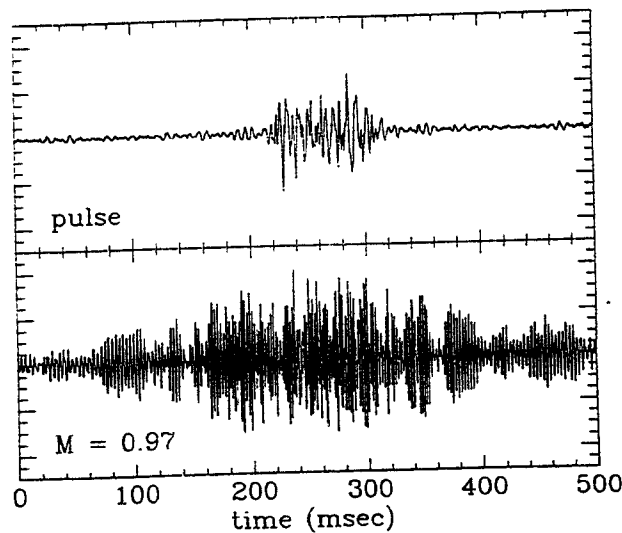
# Pulse Sensitivity

(preliminary data)



## An Example of a Non-Gaussian Event Feeding Through a Template

- Coalescing binary filter output is typical over large range of filters



# Findings From Preliminary Look at 40-Meter IFO Time-Domain Data

---

- Observed non-gaussian events are non-stationary
- Events vary in character: clicks, “the scraper”, “the howler”, “the whistler”, etc.
- Causes which have been identified:
  - ››higher-order transverse optical modes
  - ››maladies in test-mass damping servos
  - ››connector noise
  - ››edge effects on photodiodes
- Promising news: event rates are sometimes very low!
- To keep rates always low will require comprehensive data on machine status

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## Future Plans

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- Commence program of regular data runs with rapid analysis turn-around time:
  - ››Increase running time with split between R&D and DATA shifts
  - ››Limited manpower requires automated hardware and modest but efficient analysis
  - ››Constant changes required by R&D/Detector programs means sensitivity will often be less than optimal
- Institute low-bandwidth, high-channel environmental/status monitoring system
  - ››Use housekeeping statistics for cross-correlation studies of non-stationary noise
  - ››Learn about drift and aging effects in machine operation

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# THE VIRGO PROJECT

INFN - CNRS .

Experiment funded by INFN Italy (55%) and CNRS France (45%)

Presented by **Riccardo DeSalvo**

INFN Sez. Pisa  
via Livornese 1291  
56010 S. Piero a Grado

tel. ++ 39 50 880 349  
fax ++ 39 50 880 350

## The Virgo Collaboration

INFN	CNRS
LNF-Frascati	LAPP-Annecy
Napoli	IPN-Lyon
Perugia	LAL-Orsay
Pisa	ESPCI-Paris
Roma 1	

### Responsibility Subdivision

Infrastructures	
Central Building	Pisa
Tunnels	Pisa
Equipments	Pisa
Clean Areas	LAPP-Annecy
Infrastructures for mirrors	IPN-Lyon
Vacuum	
Tube	Pisa
Tube	LAL-Orsay
Baffles	LAL-Orsay
Towers	LAPP-Annecy
Pumping	Pisa

<b>Interferometer</b>	
<b>Optics</b>	<b>LAL-Orsay</b>
<b>Laser-Injection Bench</b>	<b>LAL-Orsay</b>
<b>Detection Bench</b>	<b>LAPP-Annecy</b>
<b>Mirrors</b>	<b>IPN-Lyon</b>
<b>Alignement</b>	<b>LNF-Frascati</b>
<b>Calibration</b>	<b>LAPP-Annecy</b>
<b>Suspensions</b>	
<b>Seismic Isolation</b>	<b>Pisa</b>
<b>Marionetta</b>	<b>Pisa</b>
<b>Wires and Clamps</b>	<b>Perugia</b>
<b>Local Electronics</b>	<b>Pisa</b>
<b>Electronics and Software</b>	
<b>Global Control</b>	<b>LAL-Orsay</b>
<b>Networks and cabling</b>	<b>LAL-Orsay</b>
<b>Data acquisition</b>	<b>LAPP-Annecy</b>
<b>Data archive</b>	<b>Napoli</b>
<b>Simulation</b>	<b>LAPP-Annecy</b>

- 1 ) **Generalities on the experiment**  
very very fast
- 2 ) **Civil engineering status**  
fast
- 3 ) **Super attenuators technical development**



## Interferometric detection of gravitational waves

Gravitational waves are **quadrupolar waves**

a gravitational wave of dimensionless **amplitude h**  
propagating along the **Z axis**  
with **polarization** along the **X and Y axis**  
induces a change  $\Delta L_x$  of an arbitrary length  $L_x$  ( $L_y$ )

$$\Delta L_x = h L_x \quad (\Delta L_y = h L_y)$$

typical **h** values are  $10^{-21}$

Typical GW signal **mirror displacement**

$$\Delta X = h 3 \text{ Km} = 3 \cdot 10^{-18} \text{ m}$$

Typical GW signal **phase shift**

$$\Delta \Phi = 4 \pi \Delta X f_{FP} / \lambda = 4 \pi \cdot 3 \cdot 10^{-18} \cdot 50 / 10^{-6} = 15 \cdot 10^{-10} \text{ radians}$$

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# THE VIRGO PROJECT

INFN - CNRS .

**Broad Band Gravitational Wave Interferometric Antenna**

**10 Hz to 10 KHz sensitivity**

**Main characteristics**

**3 + 3 Km vacuum Michelson Spectrometer**

**Arms made with  $f=50$  Fabry Perot cavities**

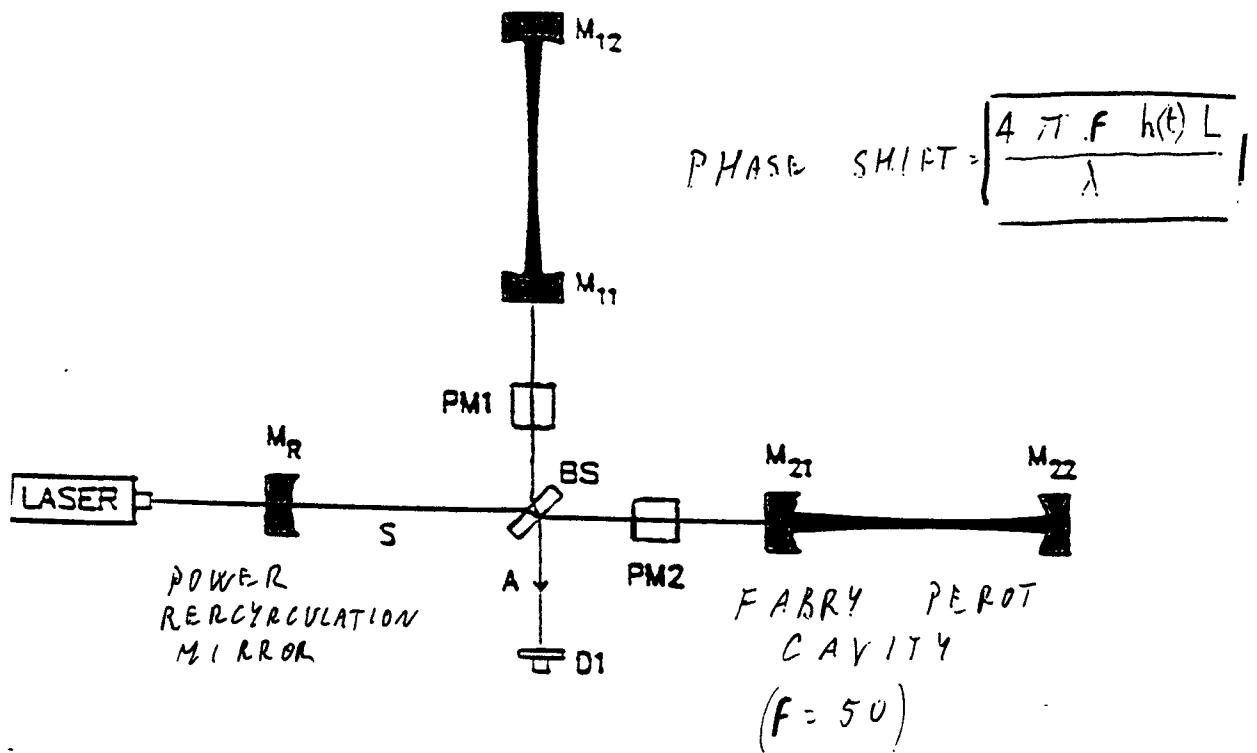
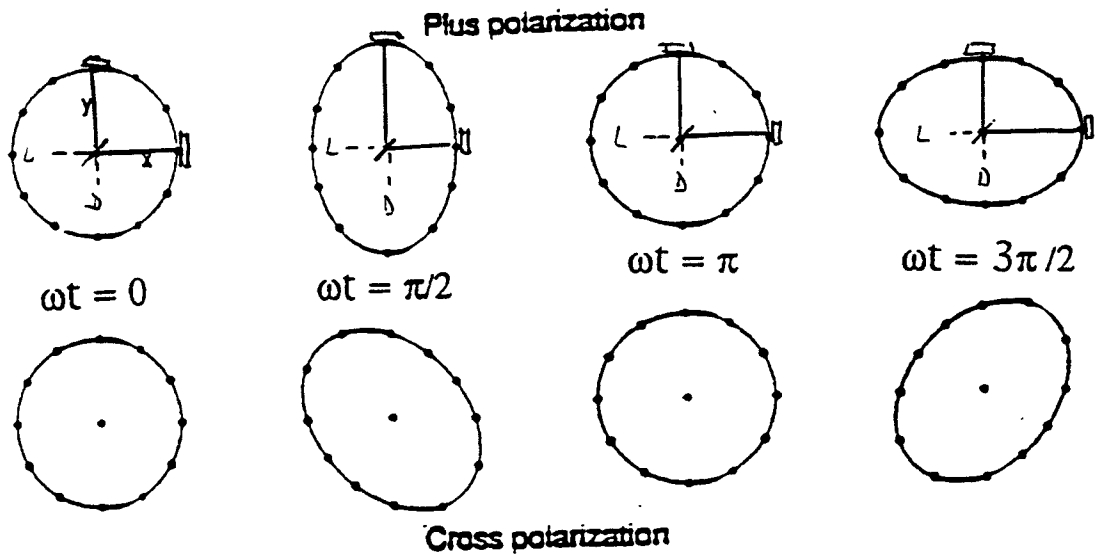
**1  $\mu\text{m}$  wavelength laser light**

**1 KW laser power at the beam splitter**

**Very high degree of seismic isolation at low frequency (> 5 Hz)**

# EFFECTS OF A GRAVITATIONAL WAVE ON A SUSPENDED INTERFEROMETER

PHASE SHIFT  $\frac{4\pi h(t)L}{\lambda}$



# Signal / Noise Ratio

For detection the signal must be higher than noise

## Dominating noise sources

Seismic noise 0-5 Hz attenuated  
(0-50 Hz unattenuated)

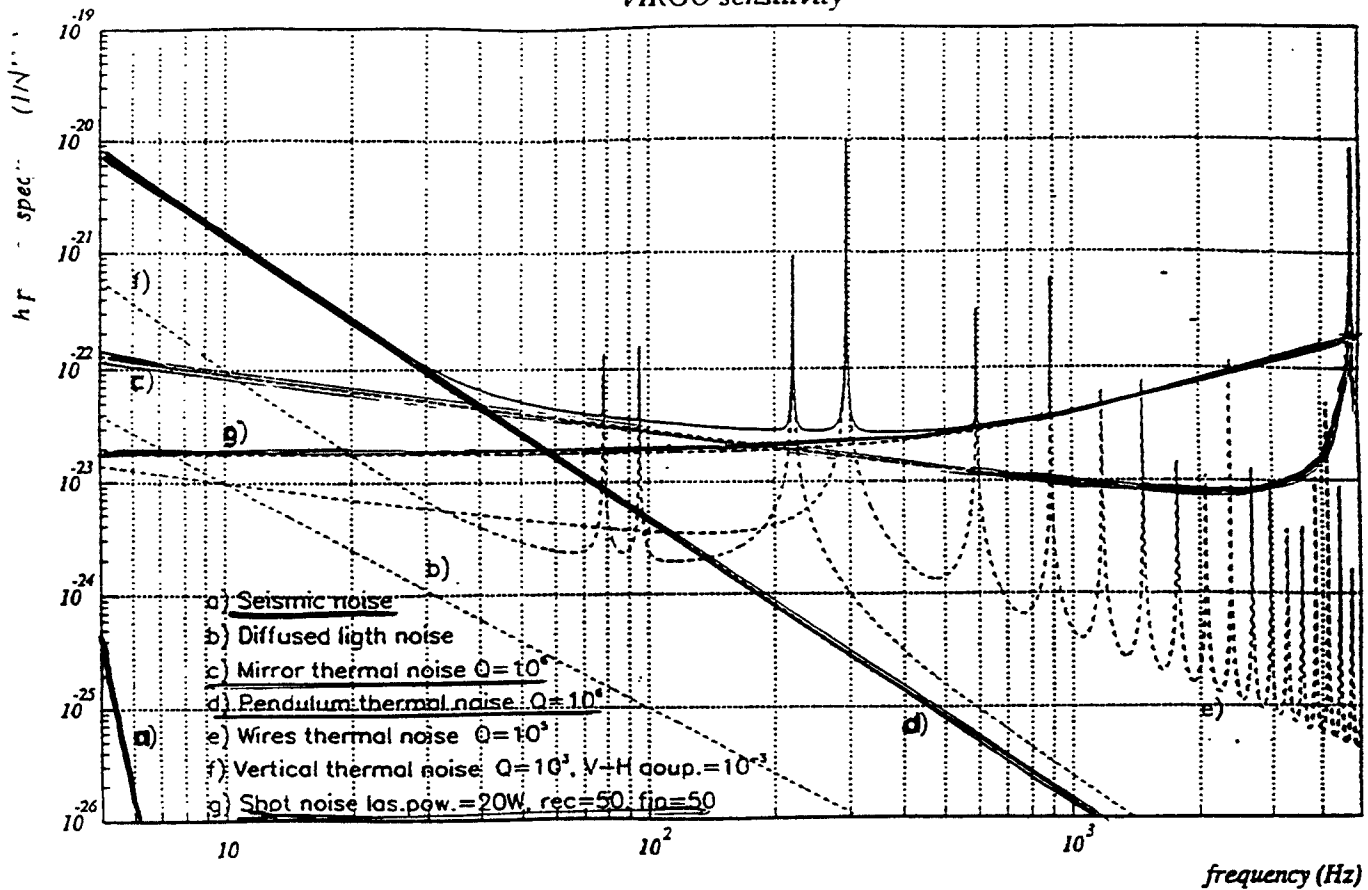
Suspension thermal noise 4-40 Hz high Q suspensions  
(50-200 Hz low Q suspensions)

Mirror thermal modes 40-200 Hz

Shot noise > 200 Hz  
(effective power enhanced by F.P. cavities  
and power recycling)

VIRGO sensitivity

Parte Prima



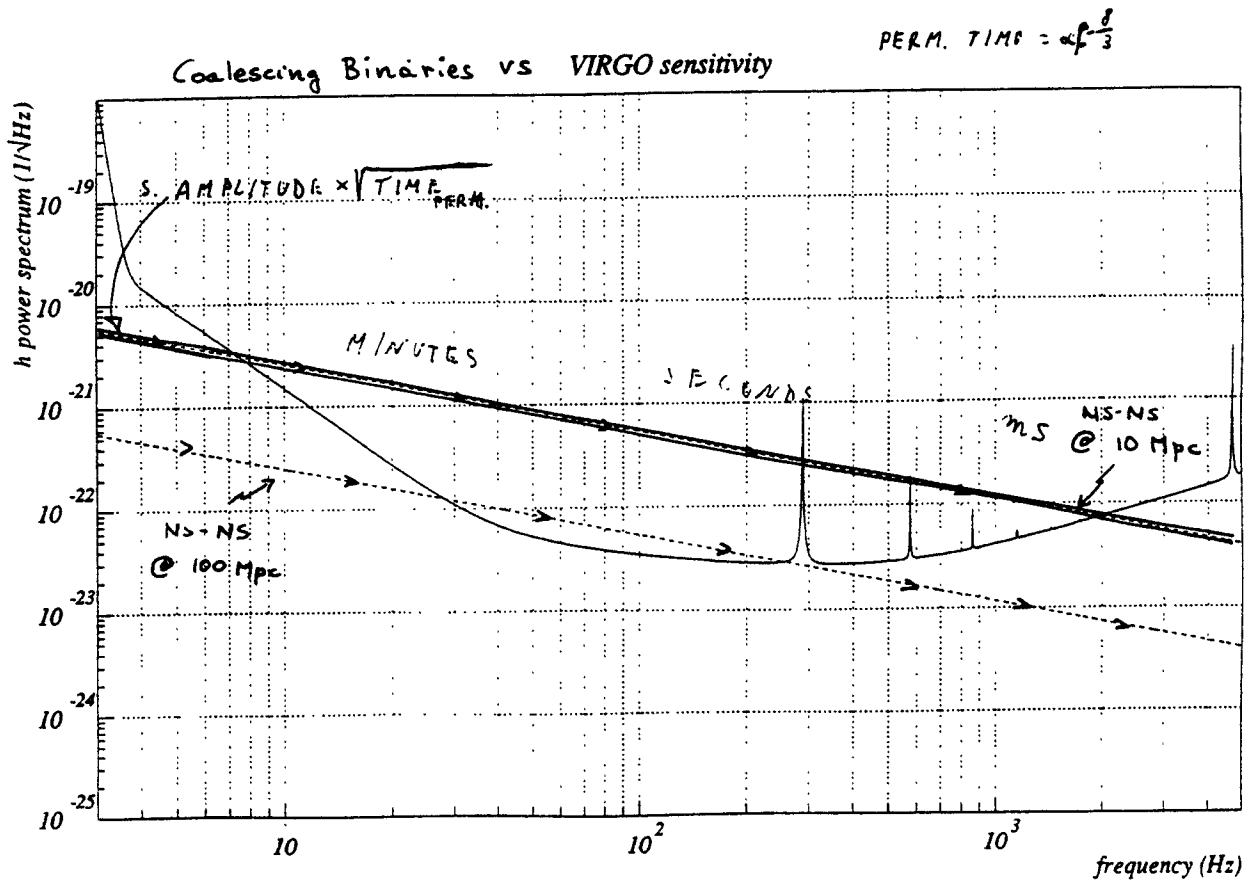
# Other Noise Sources

Laser Frequency fluctuations ( $10^{-6} \text{ Hz}^{-1/2}$ )

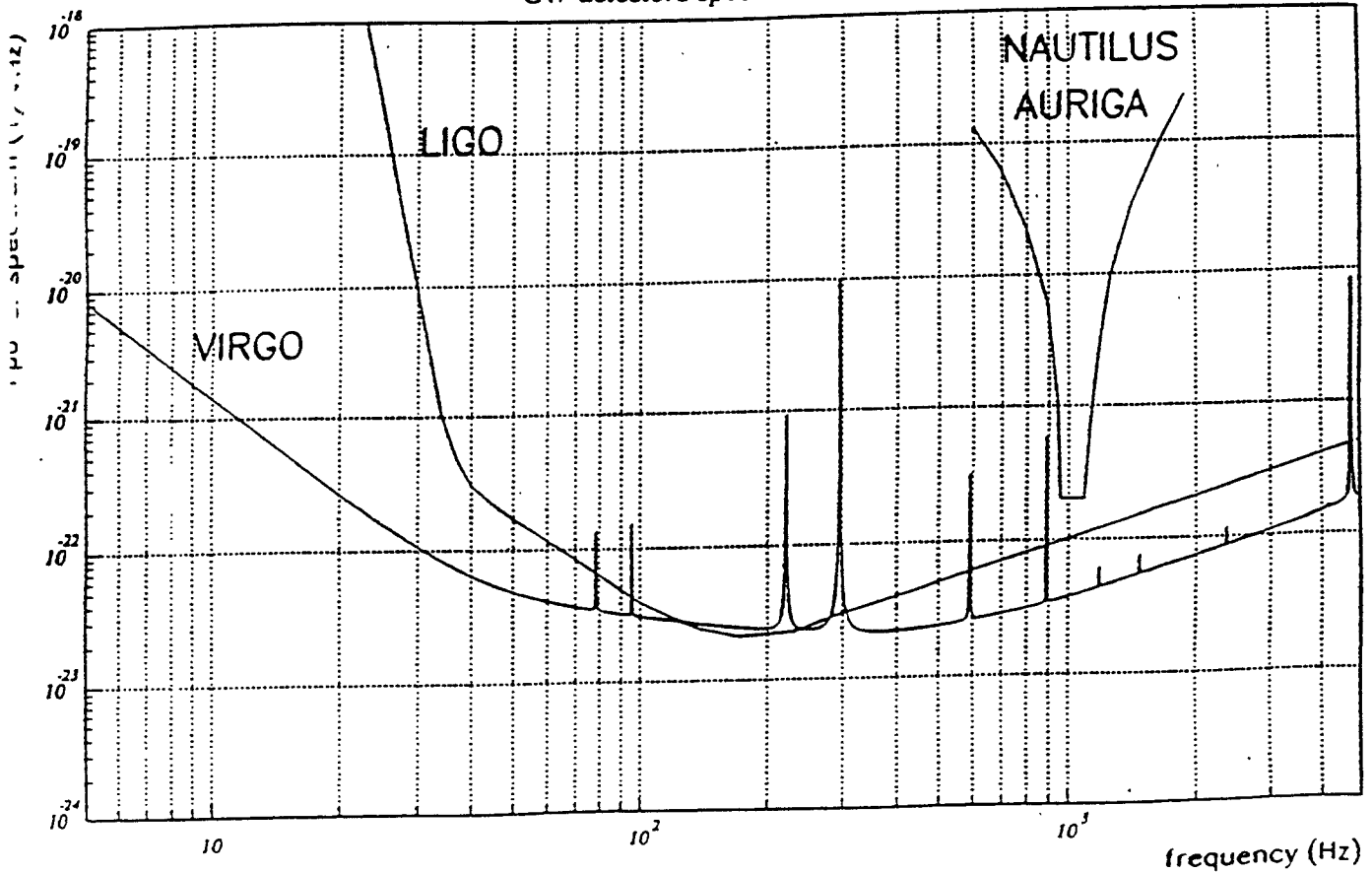
Laser Power fluctuations ( $4 \cdot 10^{-10} \text{ Hz}^{-1/2}$ )

Vacuum fluctuations (req.  $10^{-8}$  Torr, obt.  $10^{-9}$  Torr)

Seismic noise on light diffused in beam pipe walls (baffles)



GW detectors spectral sensitivity



1 19

Status of the  project

**Central building project adjudicated**

**Decree of public utility emitted**

**Currently mine clearing the land**

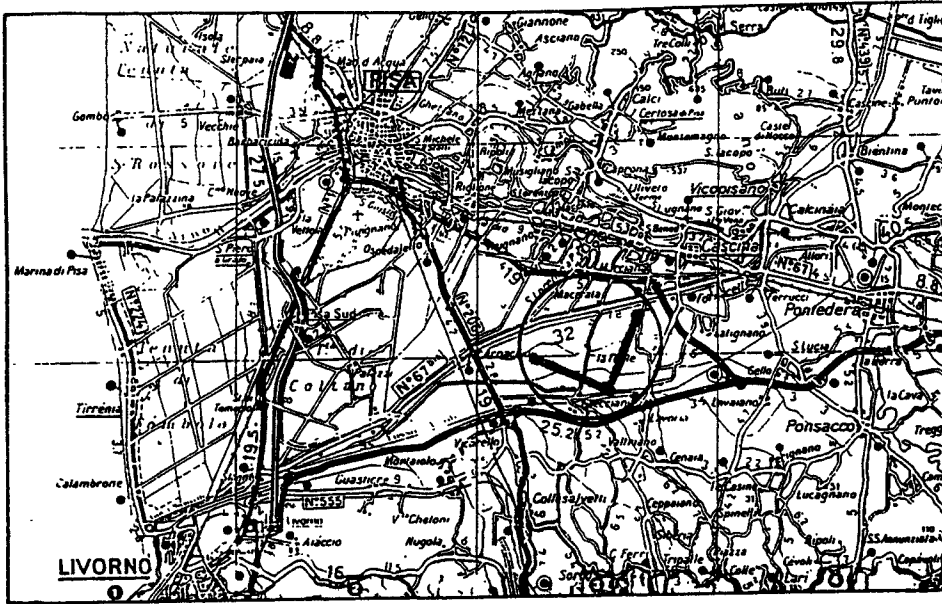
**Construction beginning shortly (One year construction time)**

**Other buildings to follow**

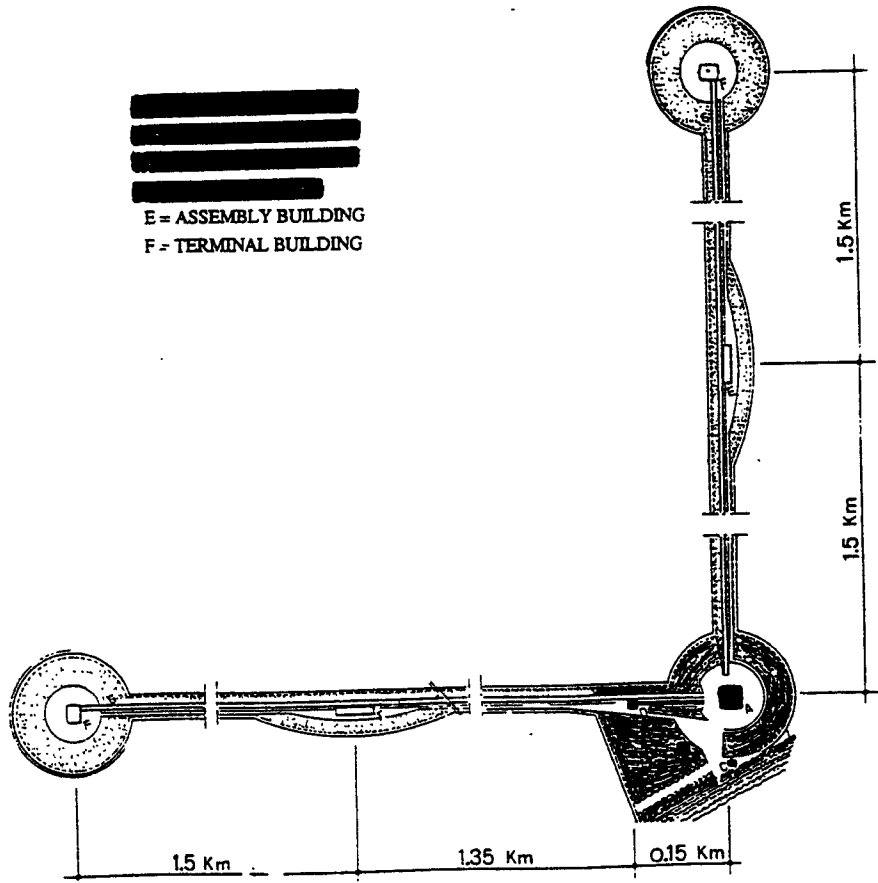
**Mechanics and tubes**

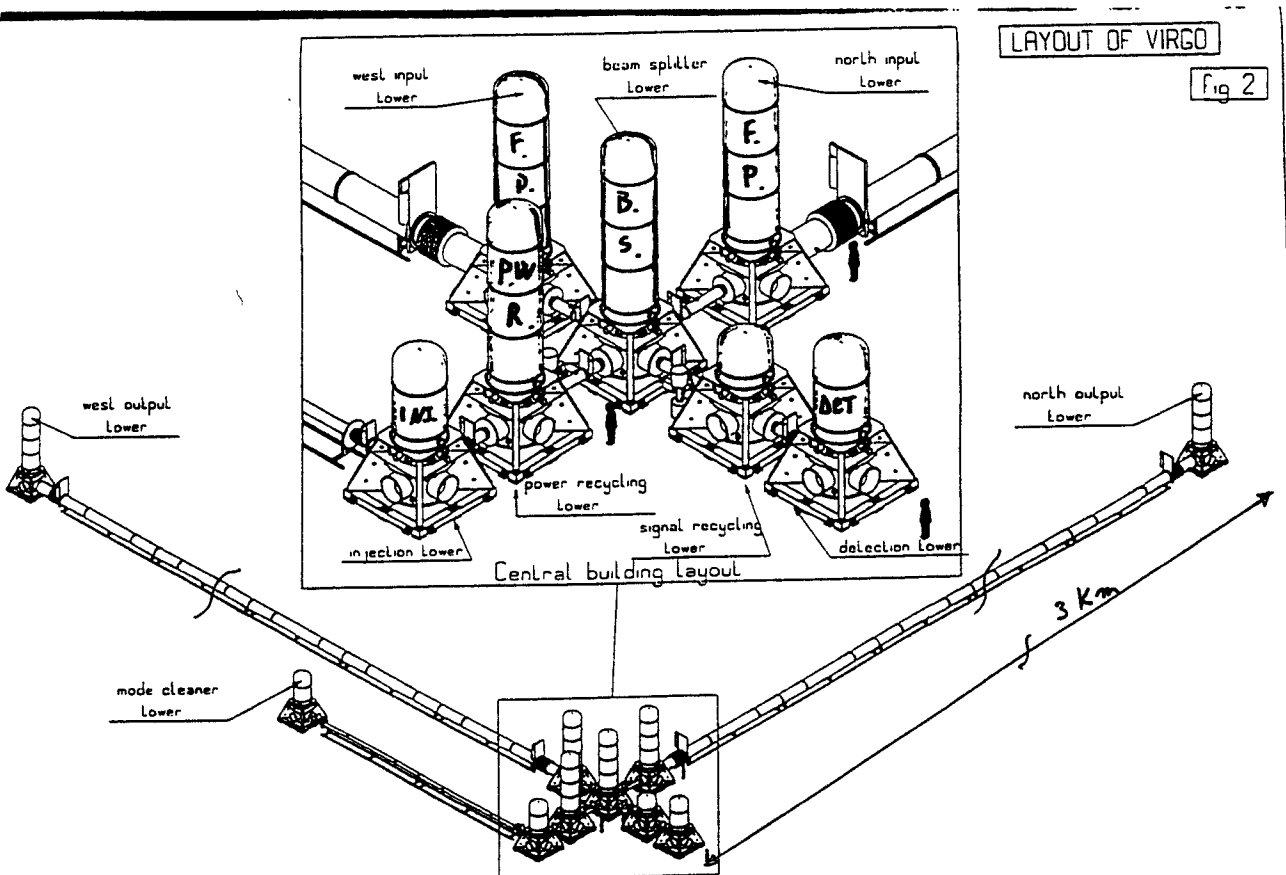
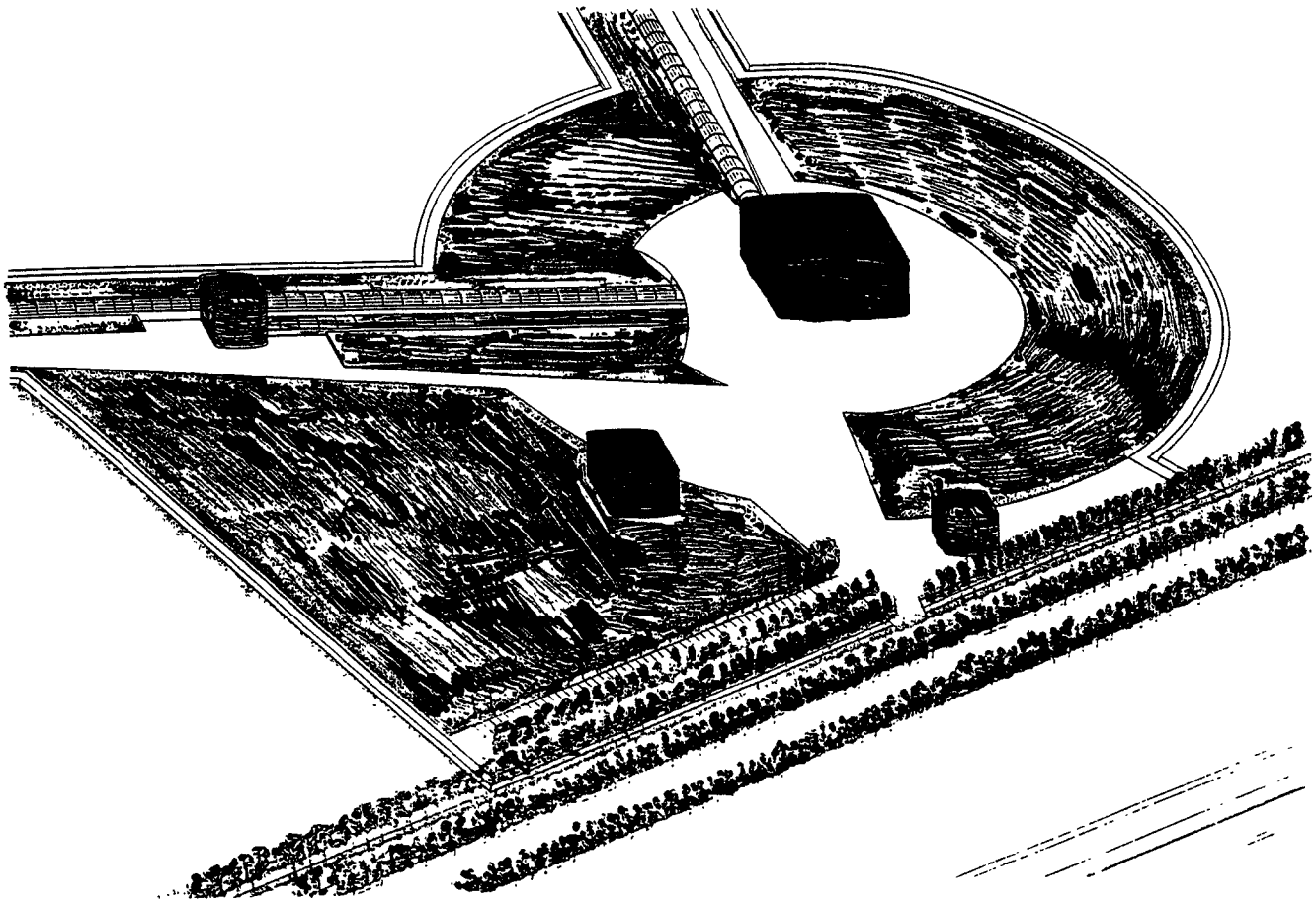
**Finalise R&D in some components**

**Bidding process in remaining components**

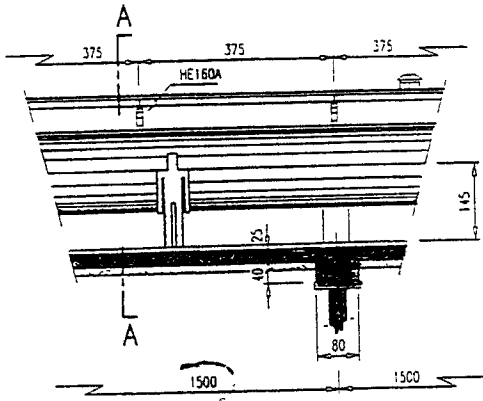


Sept. 1993 - De Carolis

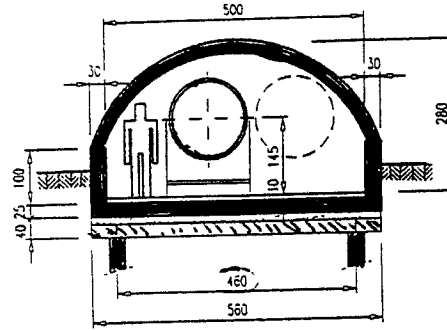




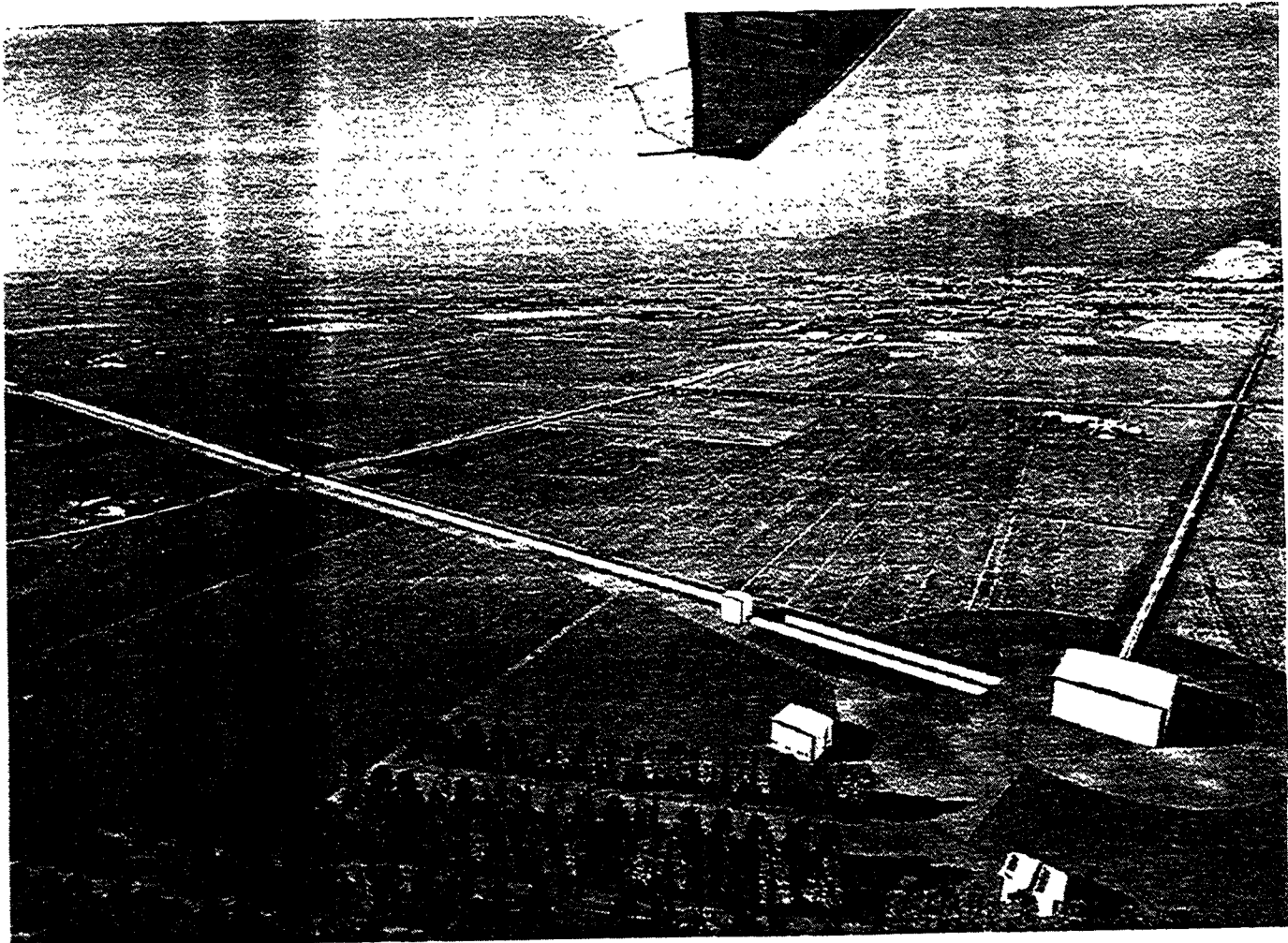
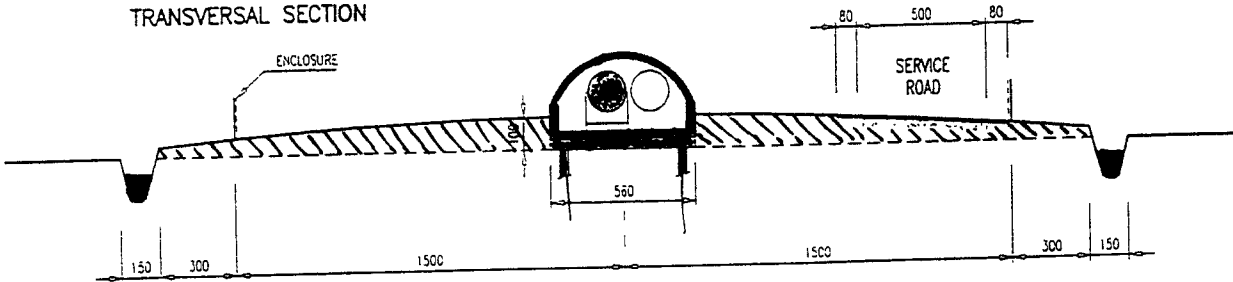
LONGITUDINAL SECTION



SECTION A-A



TRANSVERSAL SECTION





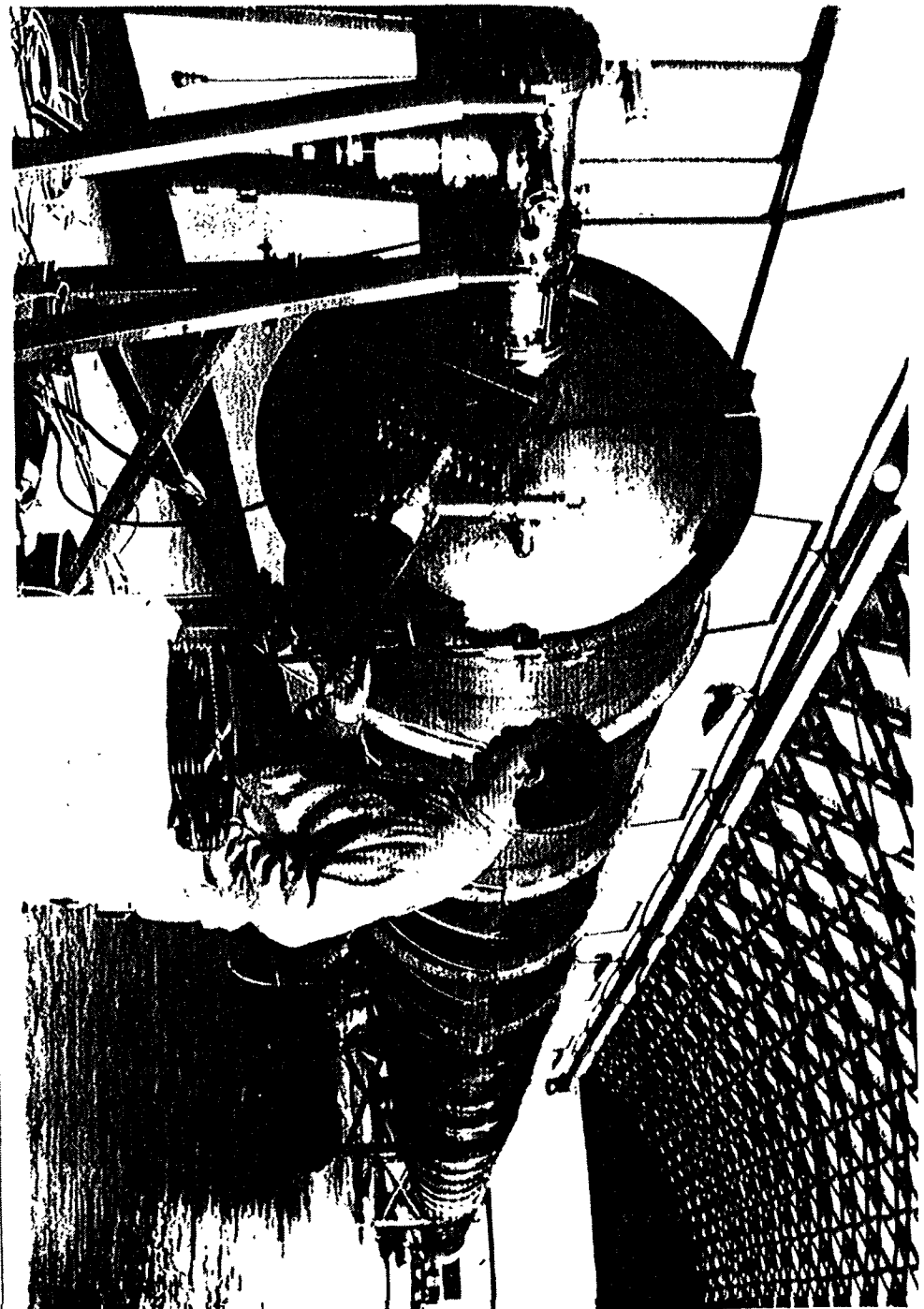
## ARM TUBES OF THE ANTENNA

### I. GENERAL SPECIFICATIONS

The general sketch of the antenna, limited to tubes and towers is presented in Fig.x.

The main parameters of the two 3km long vacuum tubes are summarized below:

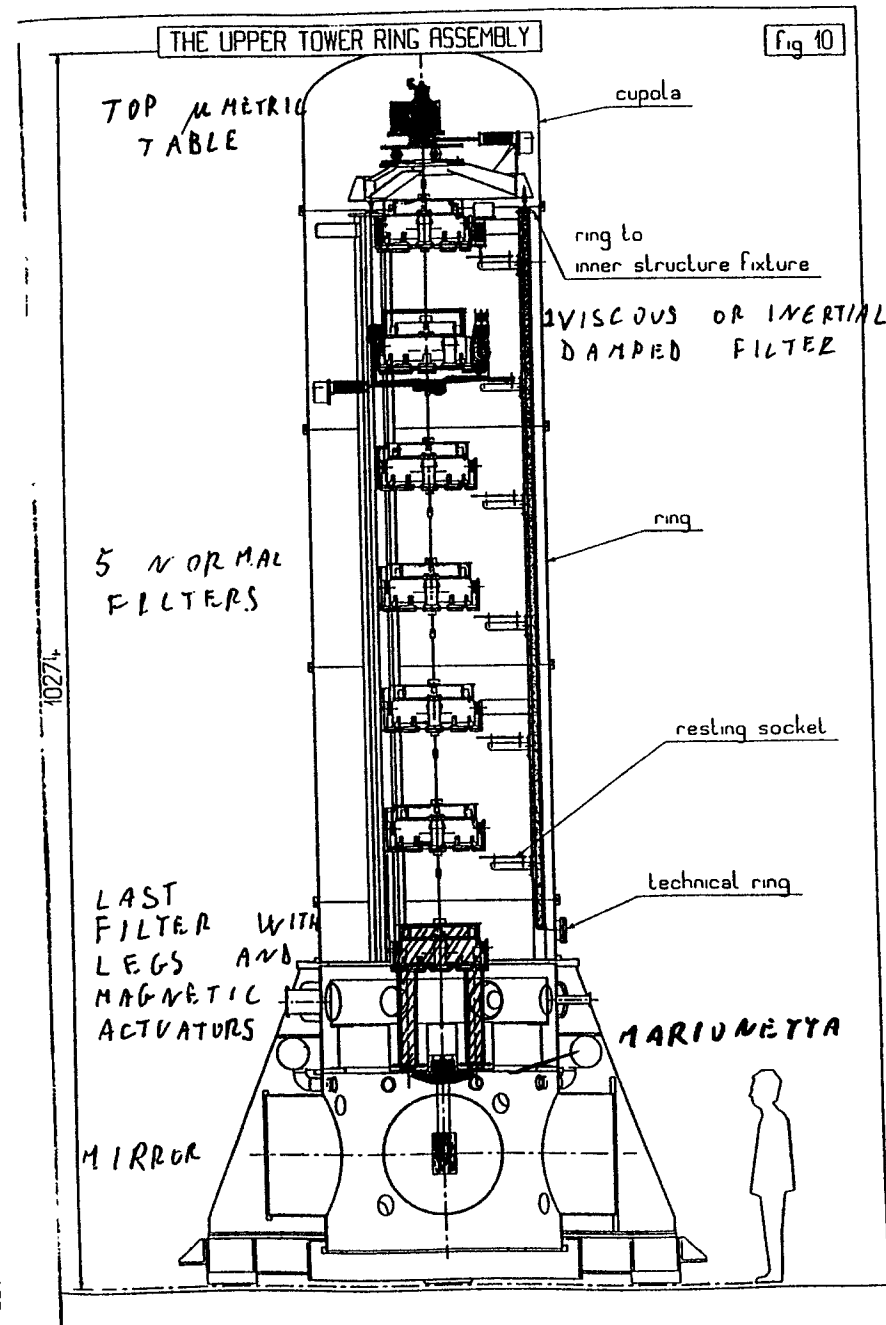
Inner diameter	1200 mm
External pressure	1 bar (3 for the calculation)
Length of each arm	3000 m $\pm$ 30m
Residual pressure	10 <sup>-9</sup> mbar for H2 10 <sup>-10</sup> mbar for other gas
Outgassing rate	< 10 <sup>-12</sup> mbar l/s cm <sup>2</sup> for H <sub>2</sub>
Temperature of reference	20°C
Temperature range	-20°C, +150°C
Working temperature range	-5°C, +40°C
Lifetime	20 years
Gate valves at end of arms	1200mm
Bake-out temperature	150°C (maximum power 1MW)
Distance between pumping groups	300 m
Distance between floor and tube axis	$\geq$ 1300m
Straightness: the tube axis has to be contained within a 50mm diameter	
Presence of baffles in the tube	
No elastomere gaskets for the permanent seals	
Minimum amplification of the seismic noise for any frequency	



## Super Attenuator Tower components

Top table movement  
 7 attenuation filters.  
 (2nd filter actively cooled)  
 Steering marionetta  
 Suspended mirror and recoil mass

10 m tall tower  
 2 m diameter  
 1.1 tons suspended mass  
 10<sup>-6</sup> Torr vacuum



# Top table movement

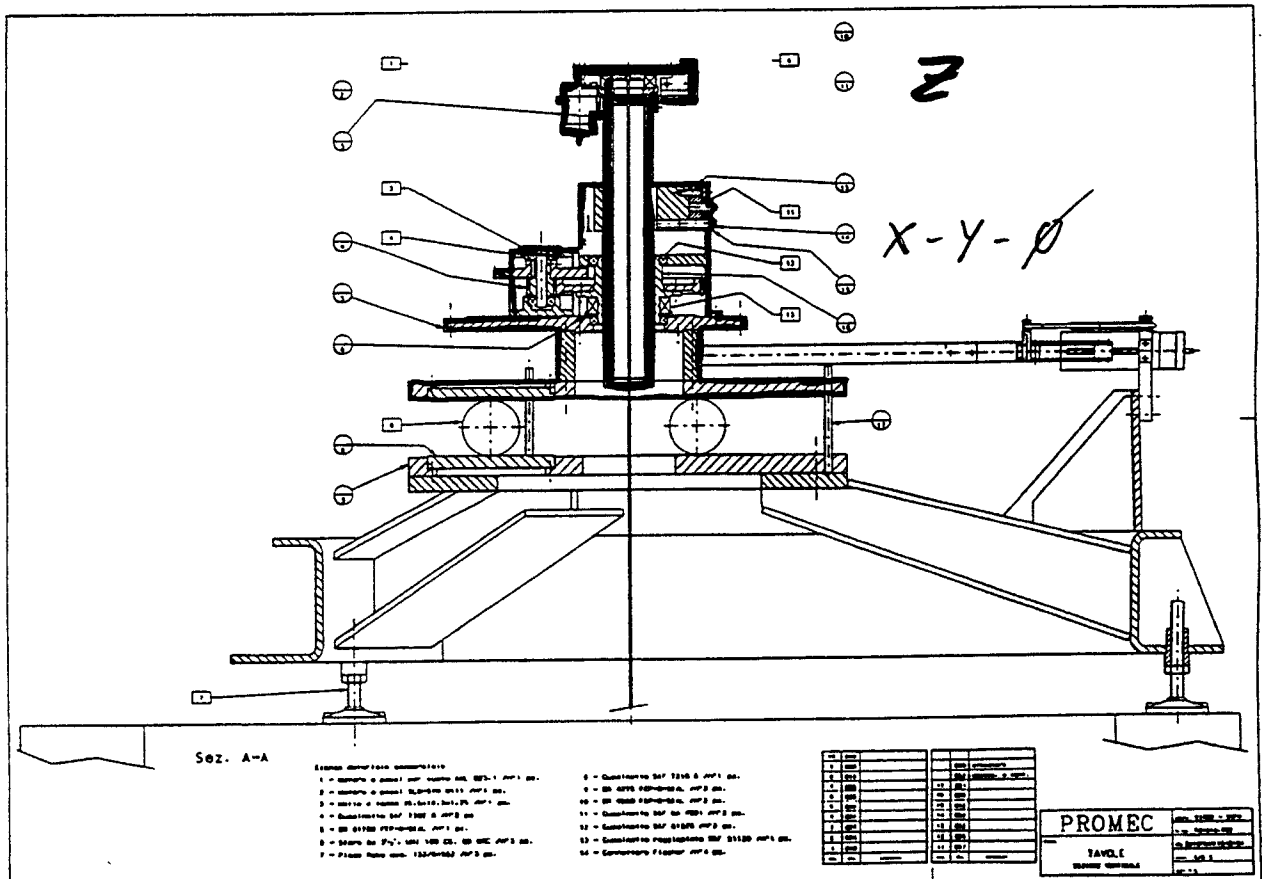
Main issues

Cold welds in movements under stress

1.1 tons load under vacuum

Movements in 3 dimensions + 1 rotation

smoother than seismic vibrations ( $1 \mu\text{m}$ )



## **FILTER 2 DAMPING**

**VISCOUS OR INERTIAL DAMPING IS NECESSARY  
AT SOME STAGE OF THE ATTENUATION CHAIN  
IN ORDER TO DAMP THE MOTION OF NORMAL MODES  
WITHIN THE LOCKING SYSTEM DYNAMIC RANGE**

**VISCOUS DAMPING WITH RESPECT WITH THE EXTERNAL  
STRUCTURE**

**OK On 6 Degrees of Freedom**

**INERTIAL DAMPING  
CAN PROVIDE ALSO SEISMIC NOISE SUPPRESSION**

**IN DEVELOPMENT**

---

### **Inertial damping of filter 2**

Principle of attenuation:

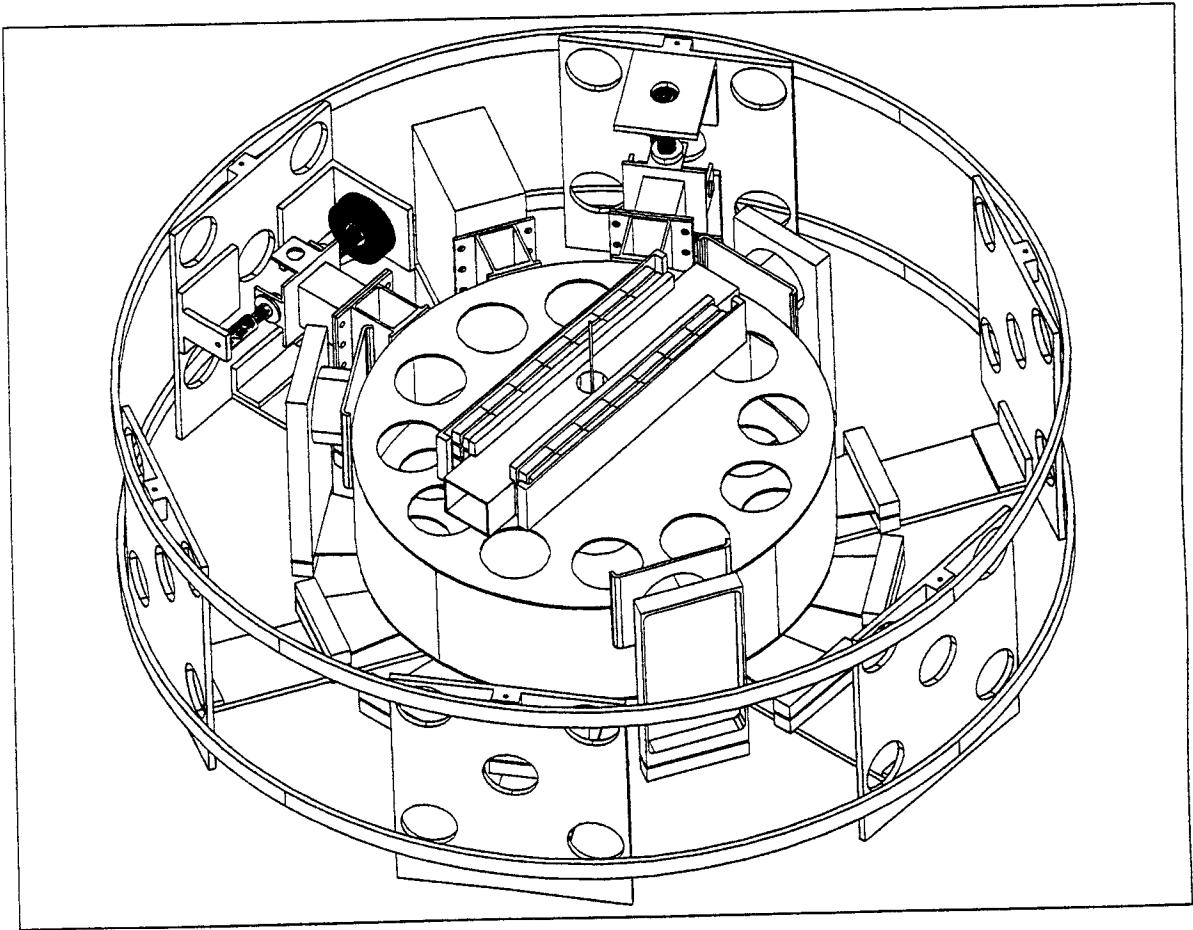
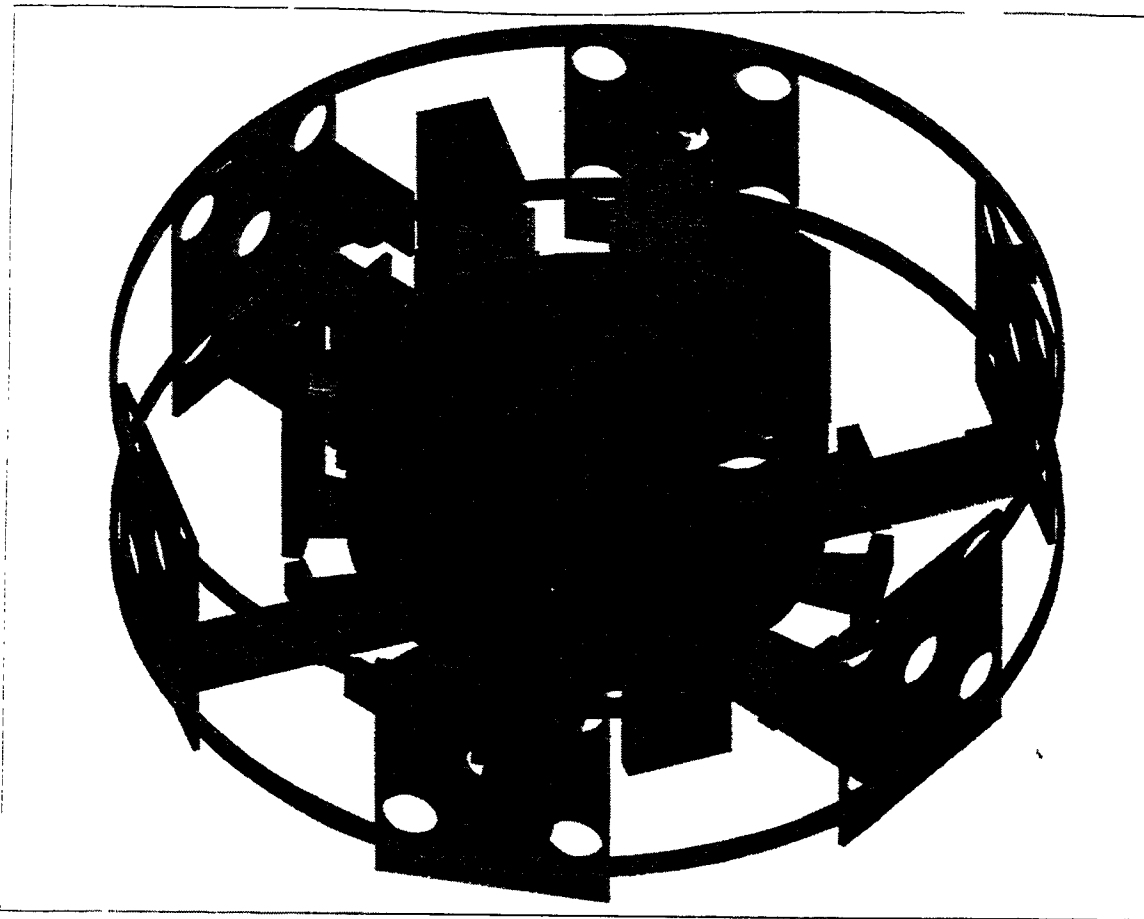
**Acceleration detected with six low frequency**

**linear and torsional accelerometer**

**Acceleration neutralised with a feedback of magnetic forces**

**Damping of main oscillation resonances**

Very complex mechanics and electronics



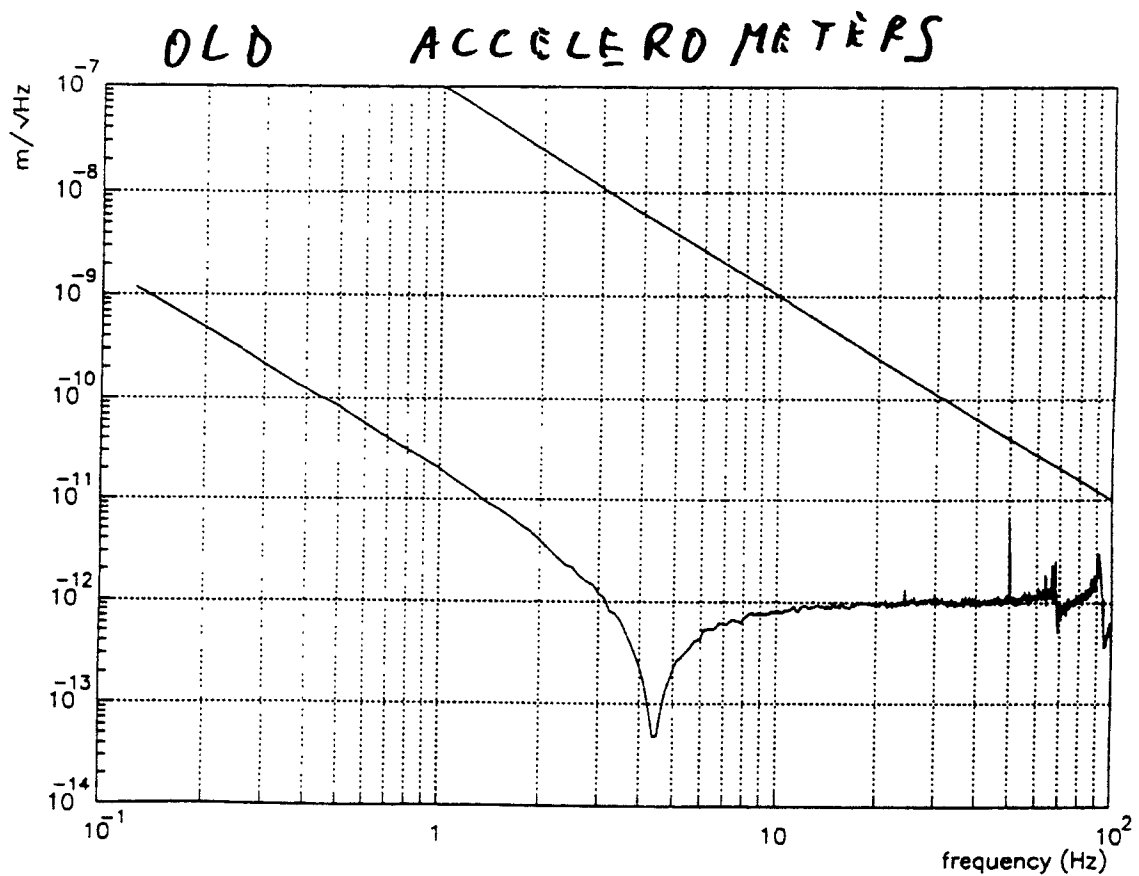
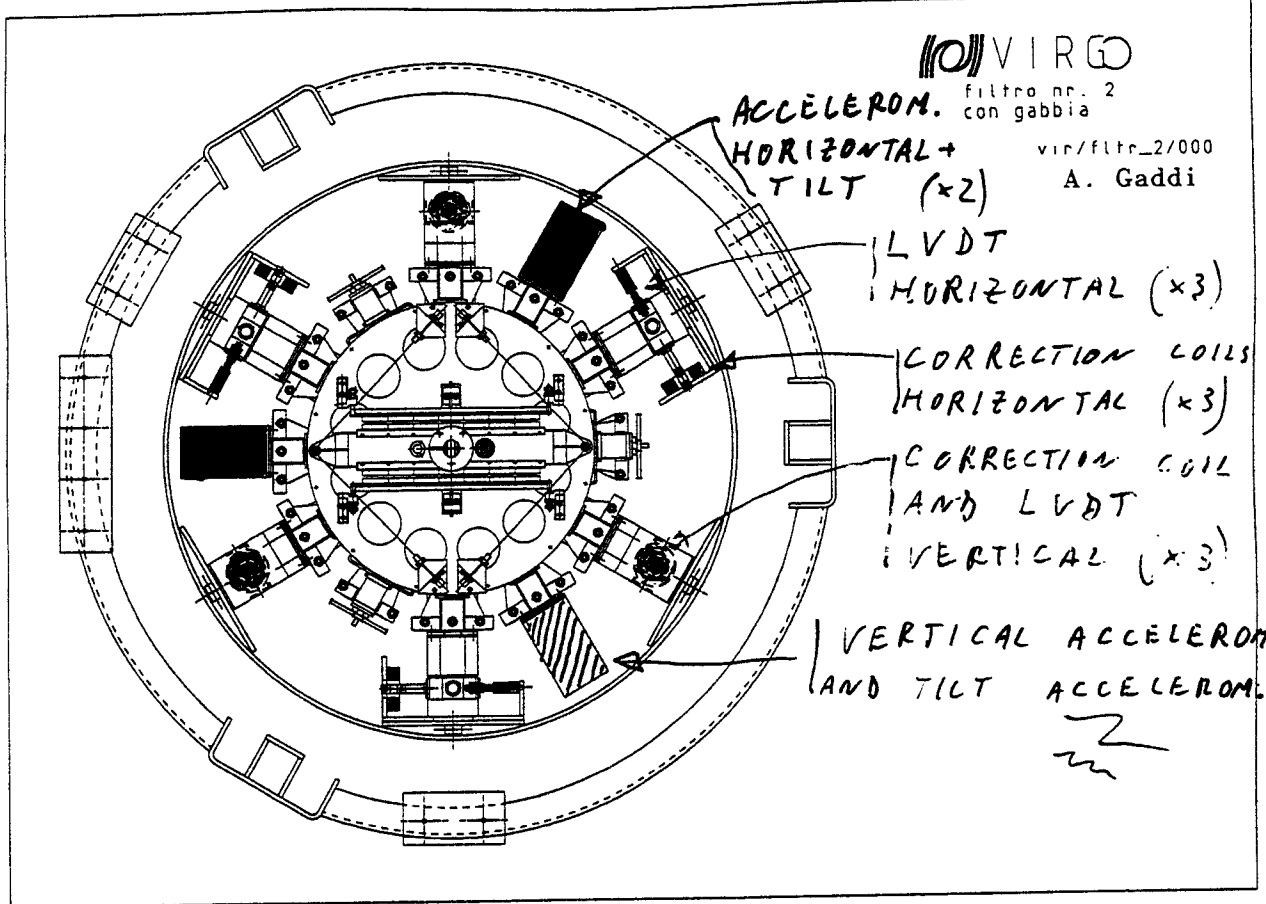


FIG. 5. Displacement spectral sensitivity function (compared with the behavior of a typical spectrum of seism:  $x_{seism}(f) = 10^{-7}/f^2 \text{ m}/\sqrt{Hz}$ ).

In Fig. 1 a block diagram of the electronic circuit is shown. The suspended central coil of the LVDT is driven at

(proportional integrative derivative) network, whose

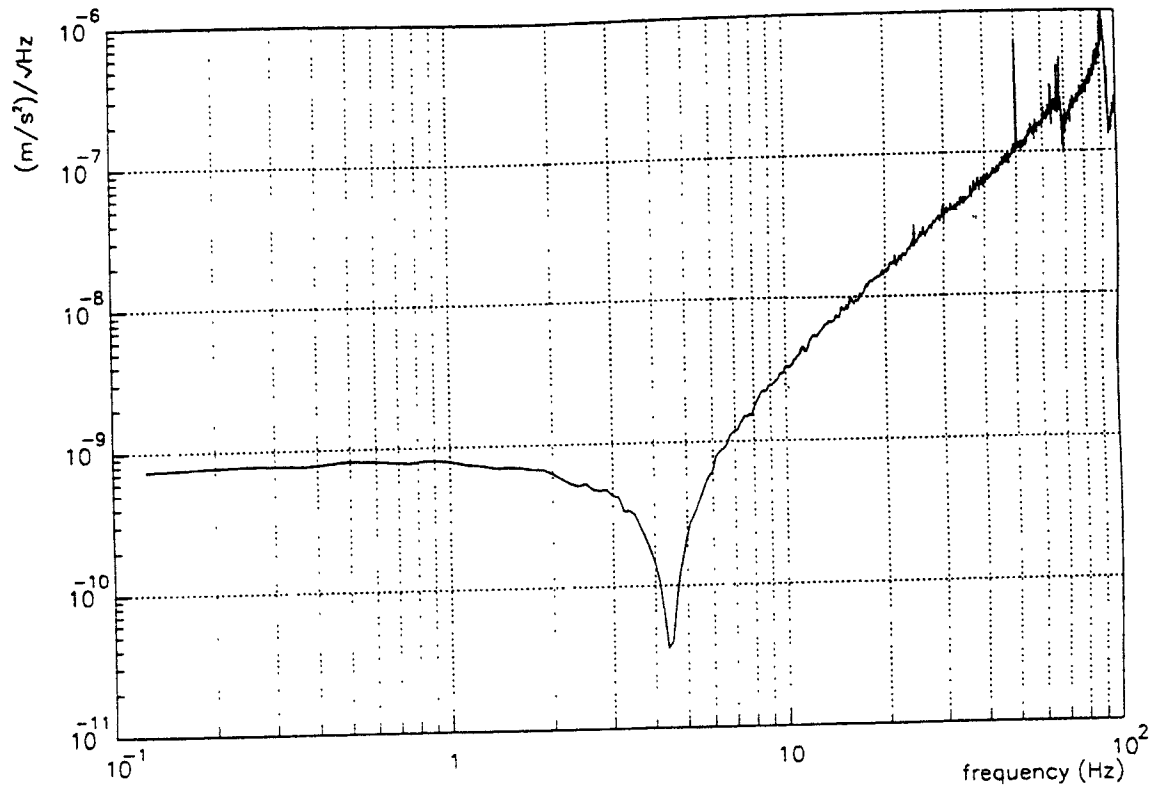
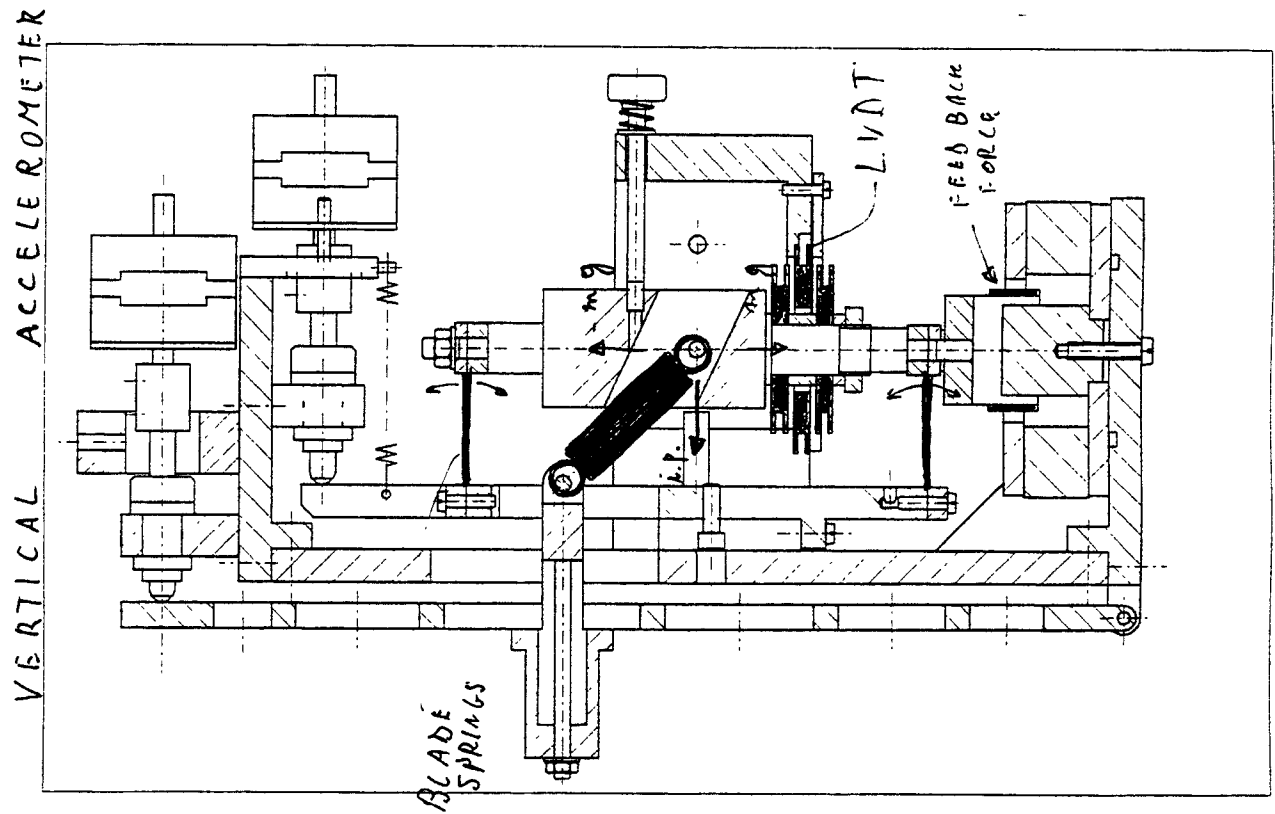
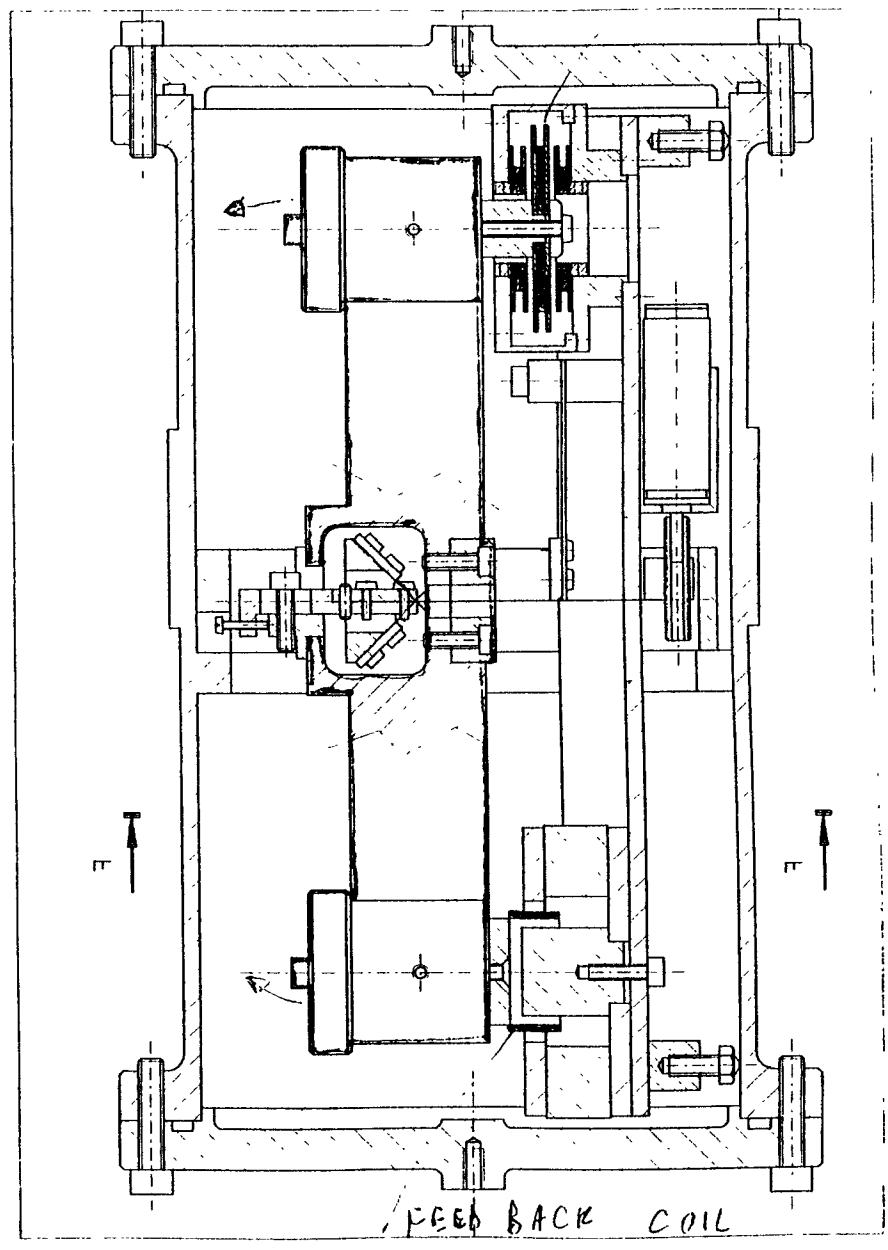
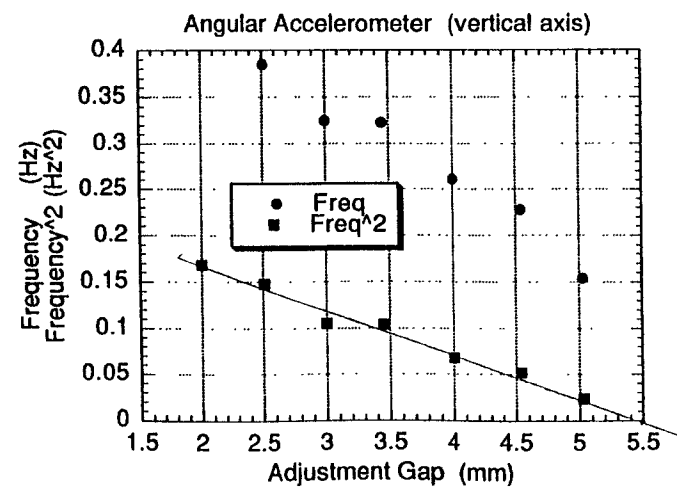
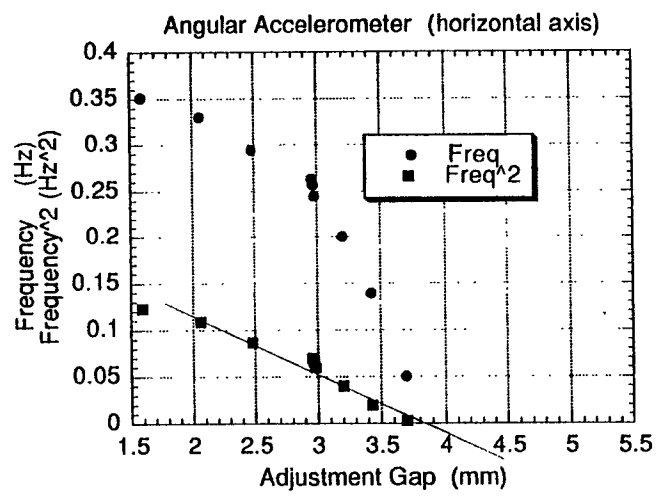


FIG. 4. Acceleration spectral sensitivity function.



LVA7





# Marionetta positioning of mirror and reference mass

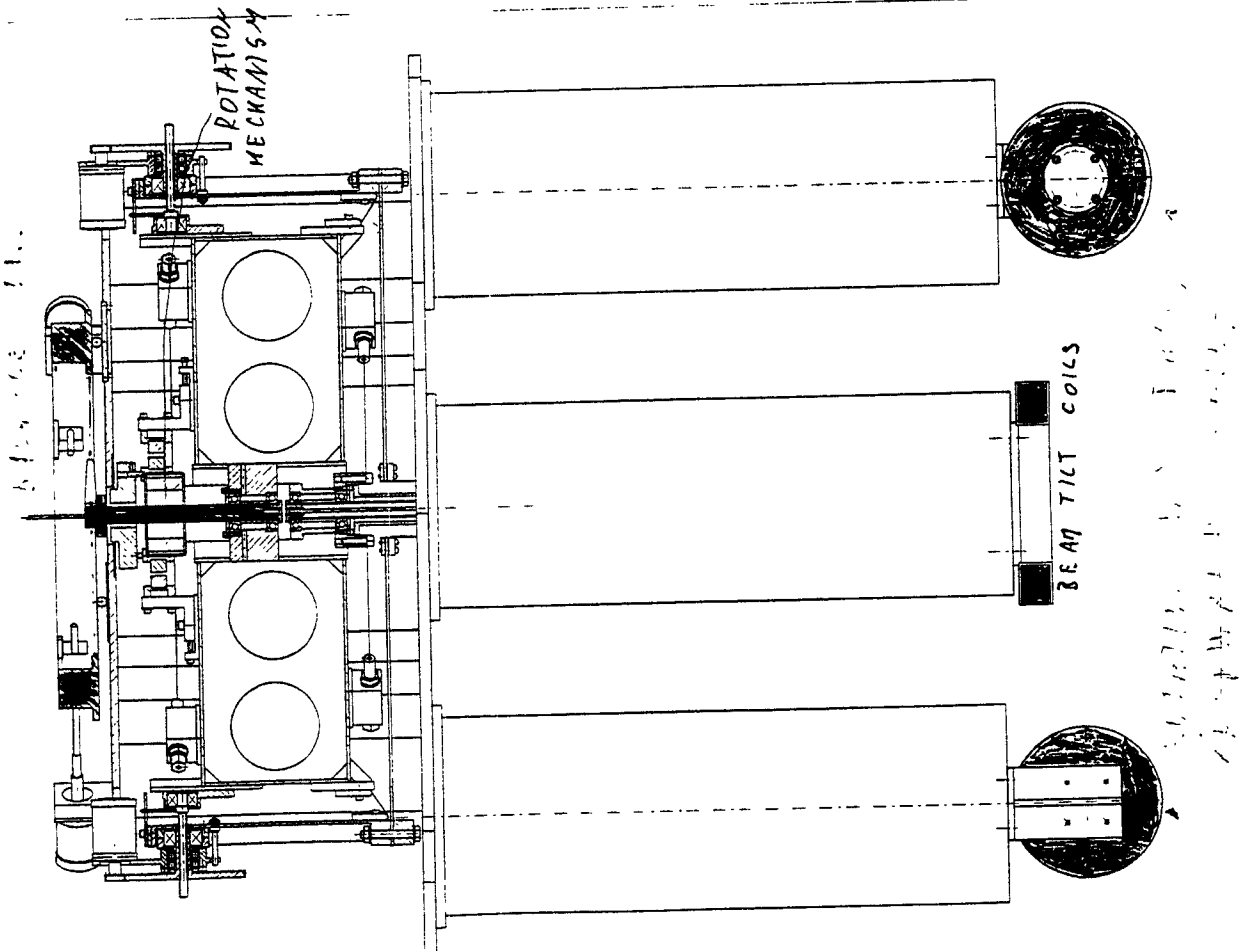
The mirror and its reference mass are  
suspended from a cross shaped structure (marionetta).

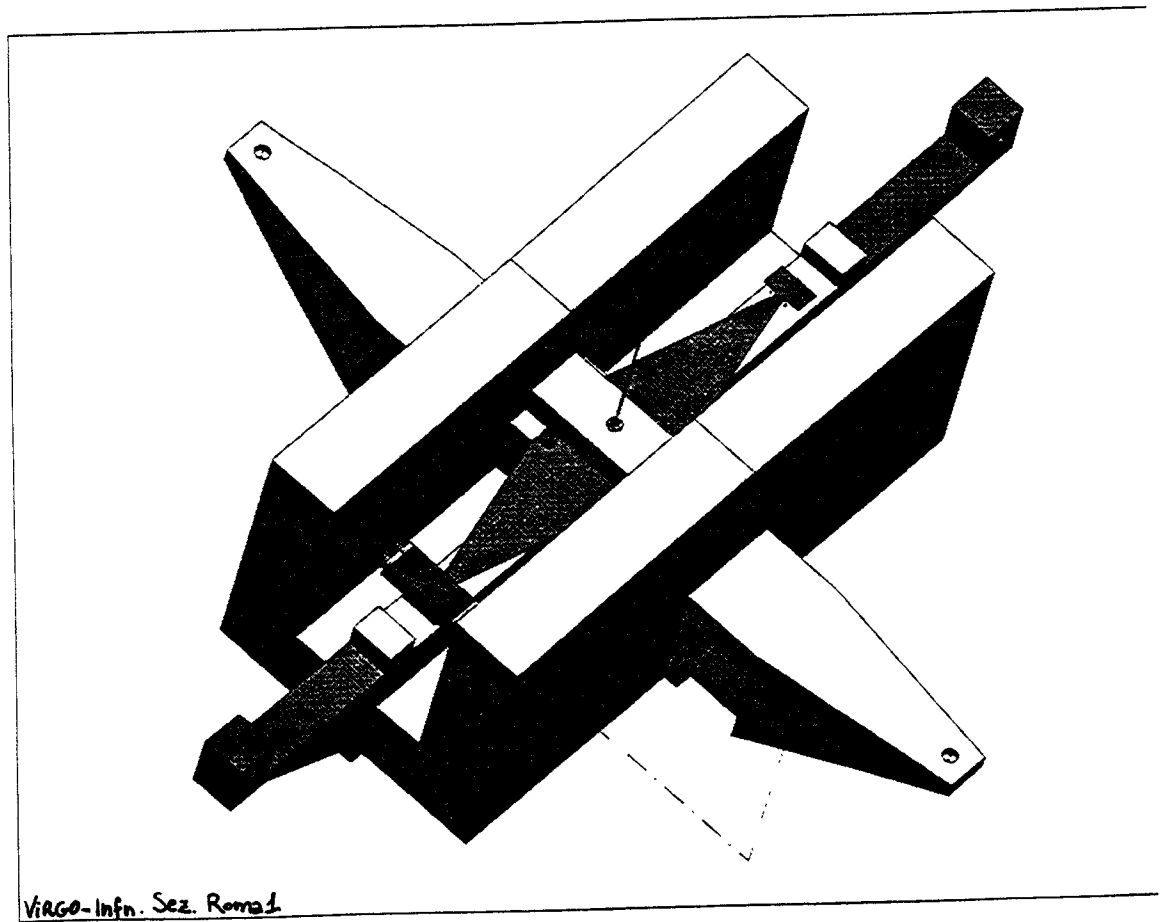
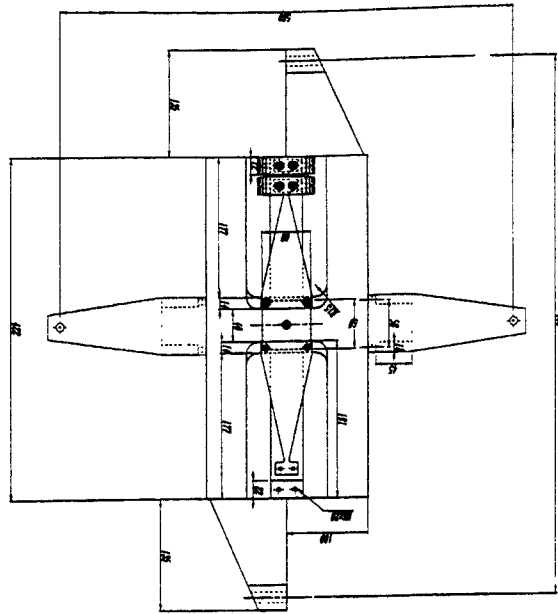
The marionetta is suspended from the last attenuator

The marionetta is actuated through magnetic coils

mounted on the last attenuator and pushing on the cross arms  
rotation, x-y tilt and back/forward movements are possible

Total movement 2-3 microns





VIRGO-Inf. Sez. Romad.

## Mirror positioning

The mirror and its reference mass are  
independently suspended from the marionetta

The reference mass is a cup with mass equal to the mirror mass

The reference mass encases the mirror and  
houses four coils acting on tiny magnets on the mirror

Mirror tilt around the z and y axis and x movements are possible

x positioning better than  $10^{-18} \text{ m/Hz}^{-1/2}$

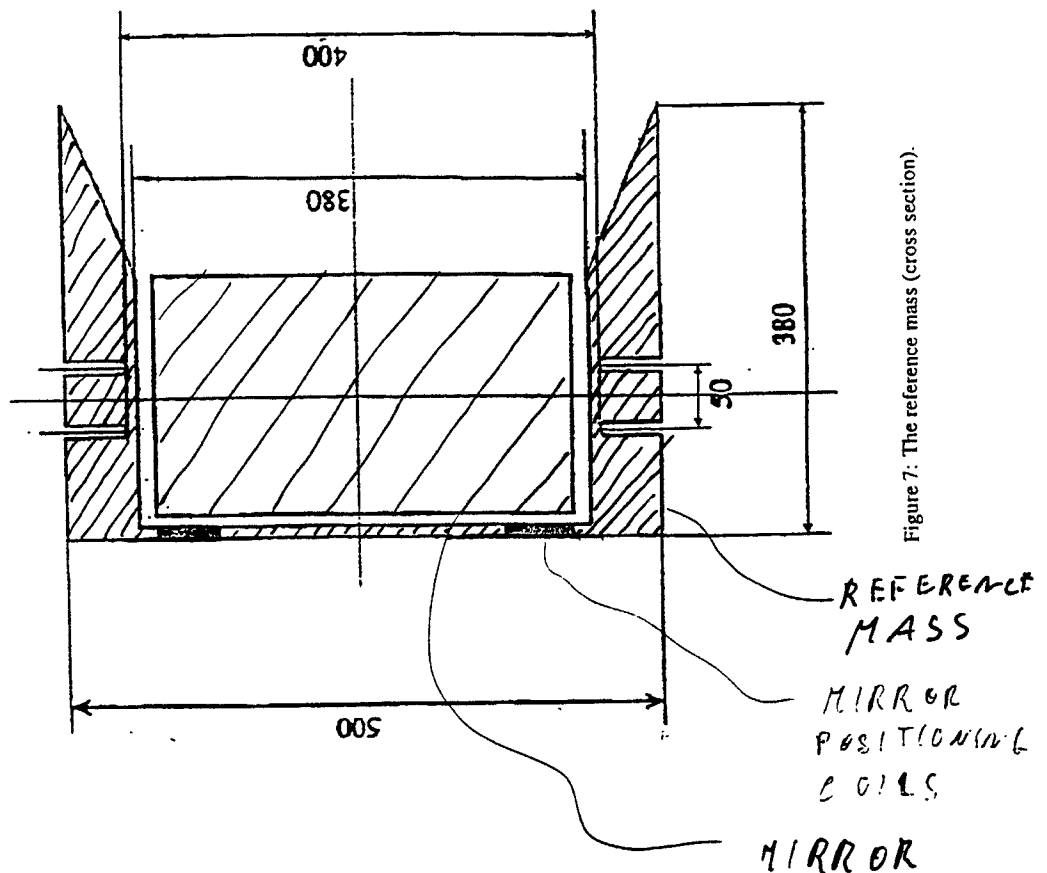
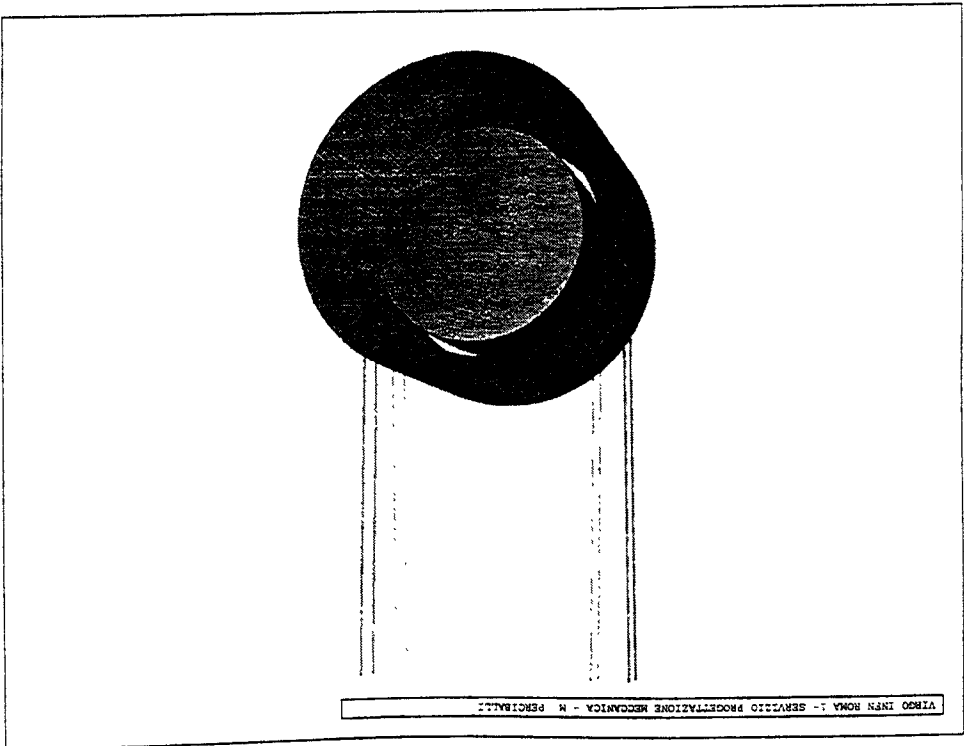
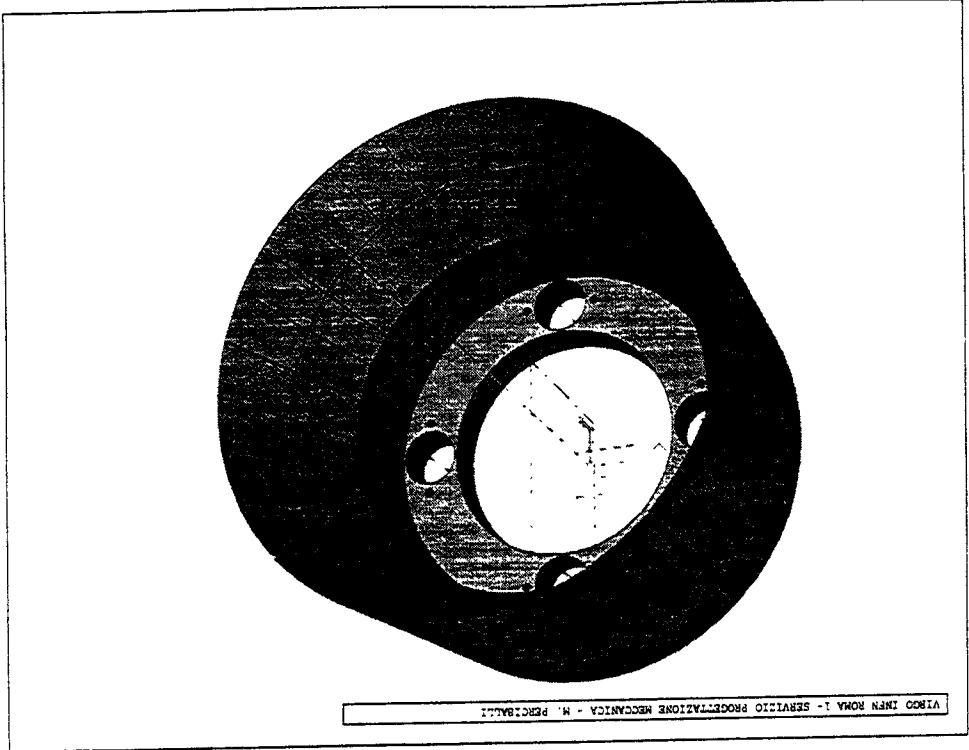


Figure 7: The reference mass (cross section).



# Super Attenuation filters

## Principle of attenuation

Each super attenuator filter is a

**5 + 1 Degrees of Freedom pendulum**

with resonant frequency below the Hertz

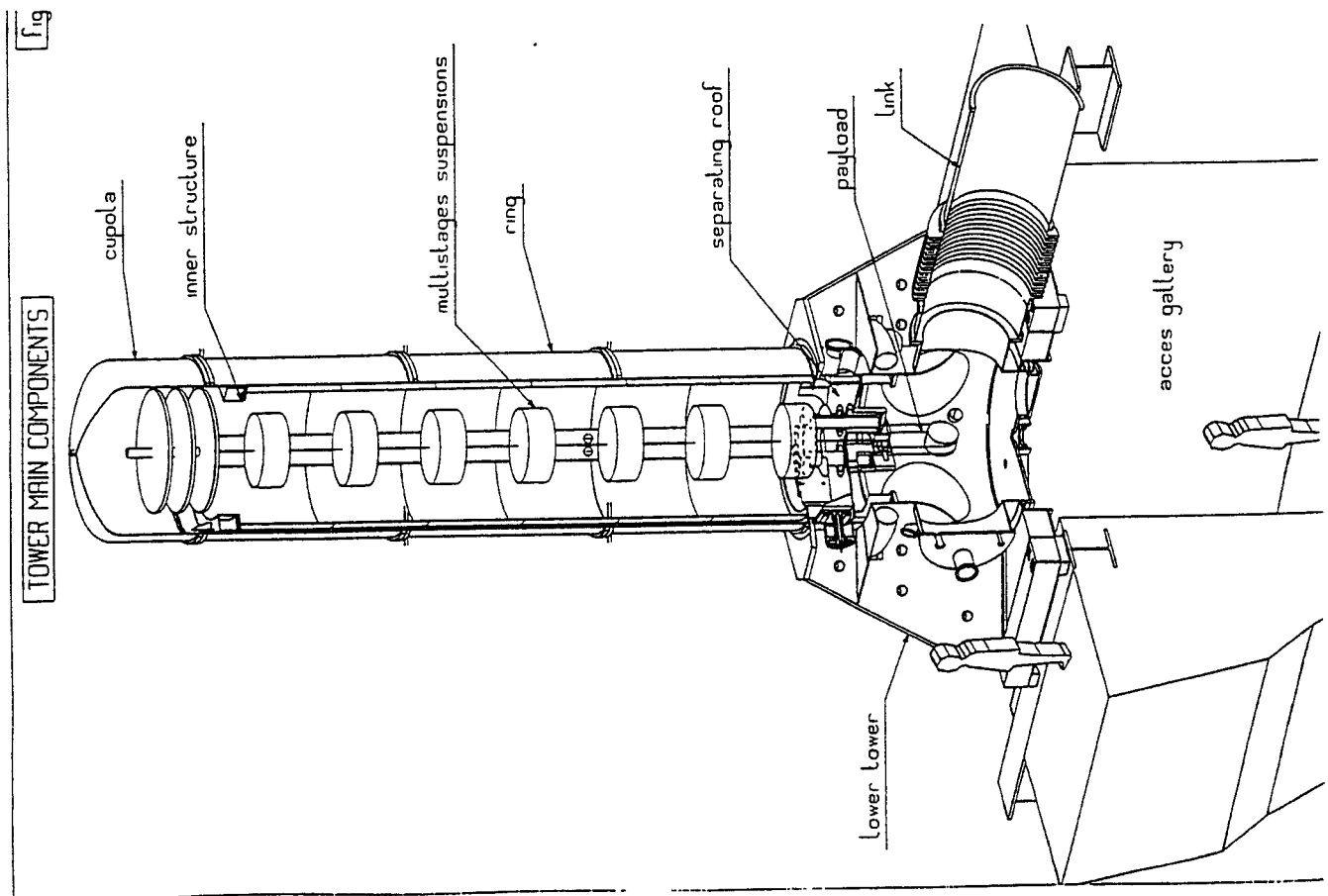
The center of mass of a pendulum which suspension point is shaken at a frequency higher than its resonant frequency will move with amplitude proportional to the suspension point vibration amplitude times an attenuation factor  $A_f = f_0^2/f^2$ .

At 10 Hz  $A_f = 10^{-2}$  per stage

With seven stages  $A_f = 10^{-14}$

If seismic noise is  $10^{-6}/f^2 \implies @ 10 \text{ Hz } 10^{-8} \text{ m/Hz}^{-1/2}$

Mirror will vibrate less than  $10^{-18} \text{ m/Hz}^{-1/2} @ 10 \text{ Hz}$  as required



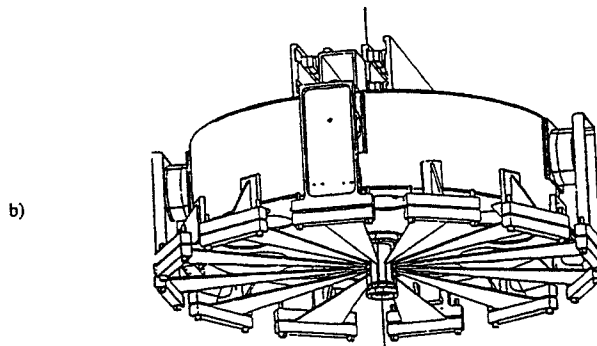
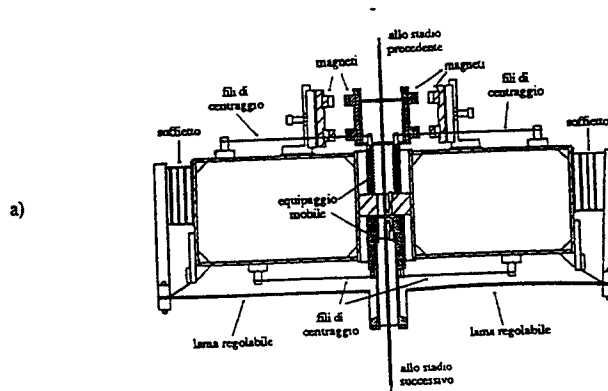
# Super Attenuation filters

## Characteristics

5 + 1 Degrees of Freedom pendulum below 1 Hz

- x - y pendulum OK
- torsion pendulum OK
- x-y tilt pendulum OK
- z pendulum a real MESS

very complex mechanics



## Super Attenuation filters

### Mechanical design and construction

Prototypes under test

work on

final engineering

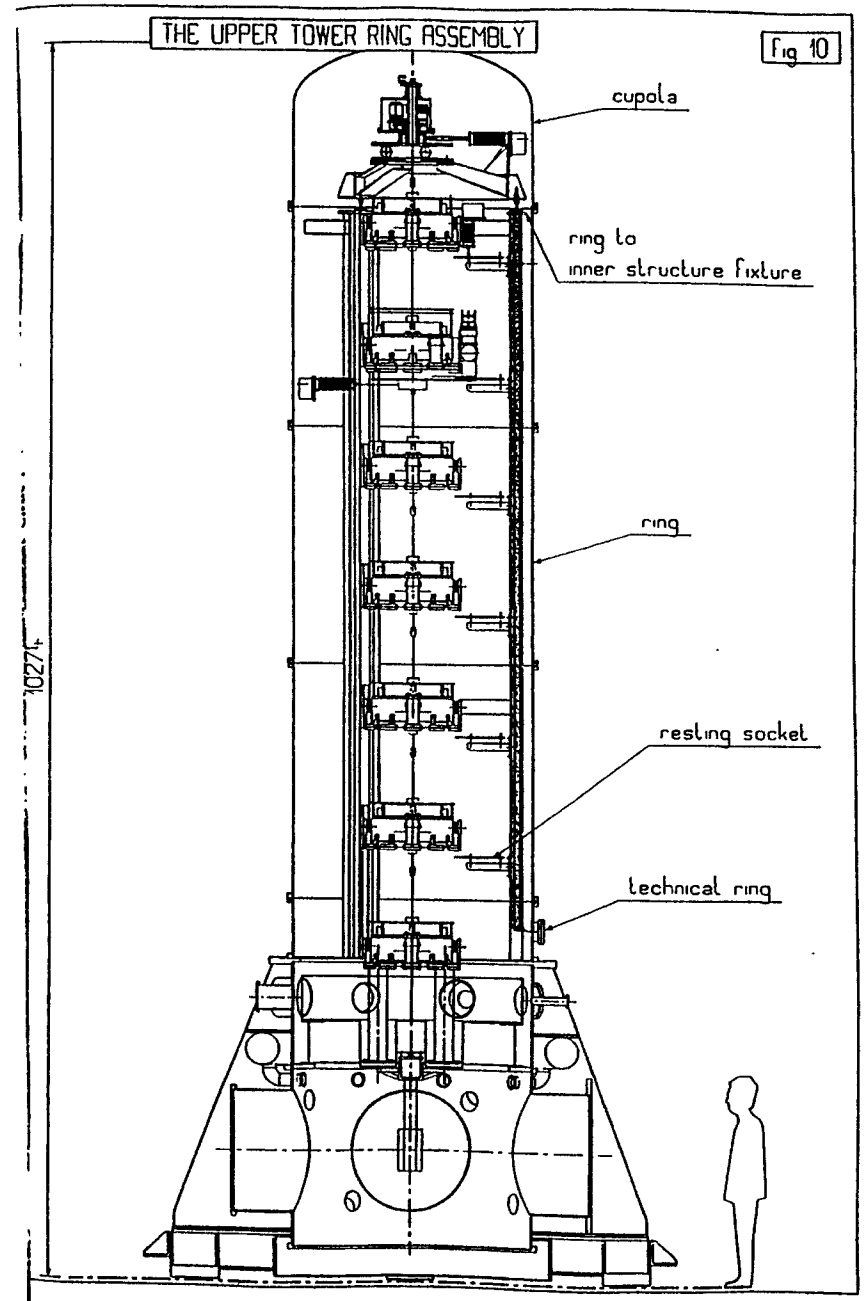
vacuum proofing

controls

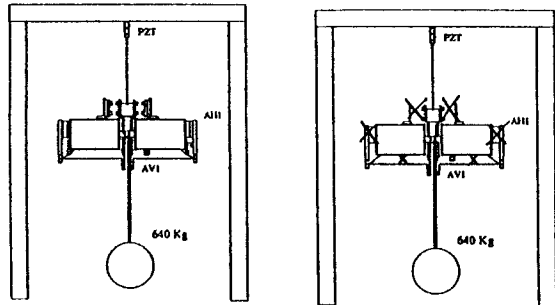
characterisation

heat dissipation

connectivity



**Phase 1: Measurements on the bare filter (concluded)**



- 1.a) ==> Vertical Transfer Function
- 1.b) ==> Horizontal Transfer Functions
- 1.c) ==> Quality factor of the fundamental vertical resonance
- 1.d) ==> Anharmonic behaviors of the blades system
- 1.e) ==> Dependence of the fundamental frequency on the offset position
- 1.f) ==> Creep of the blades !!!! ==> the only bad surprise

**Super Attenuation filters**

**Status of characterisation**

Separate measurement of attenuation characteristics of

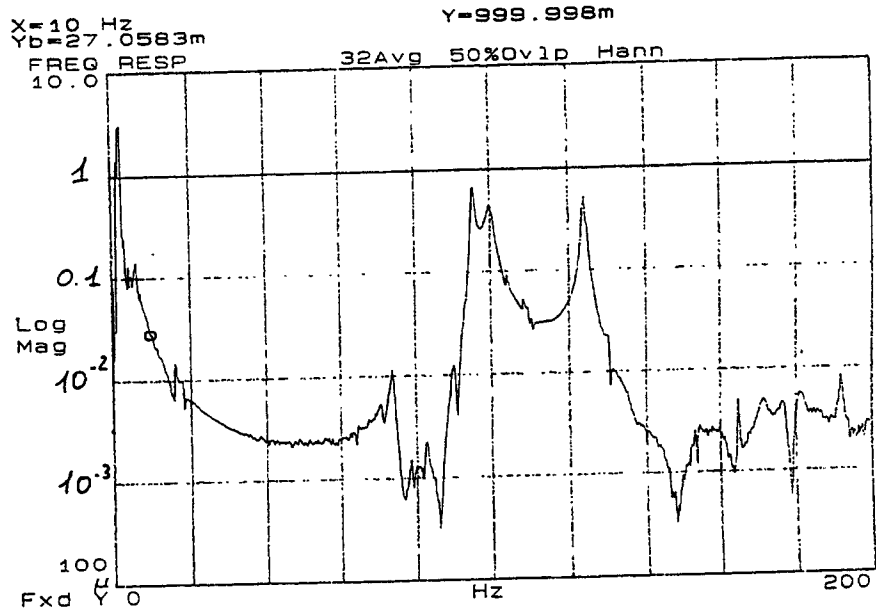
**Bare filter**

With **Resonant frequency reduction Anti Spring System**

With **tuning mechanisms**

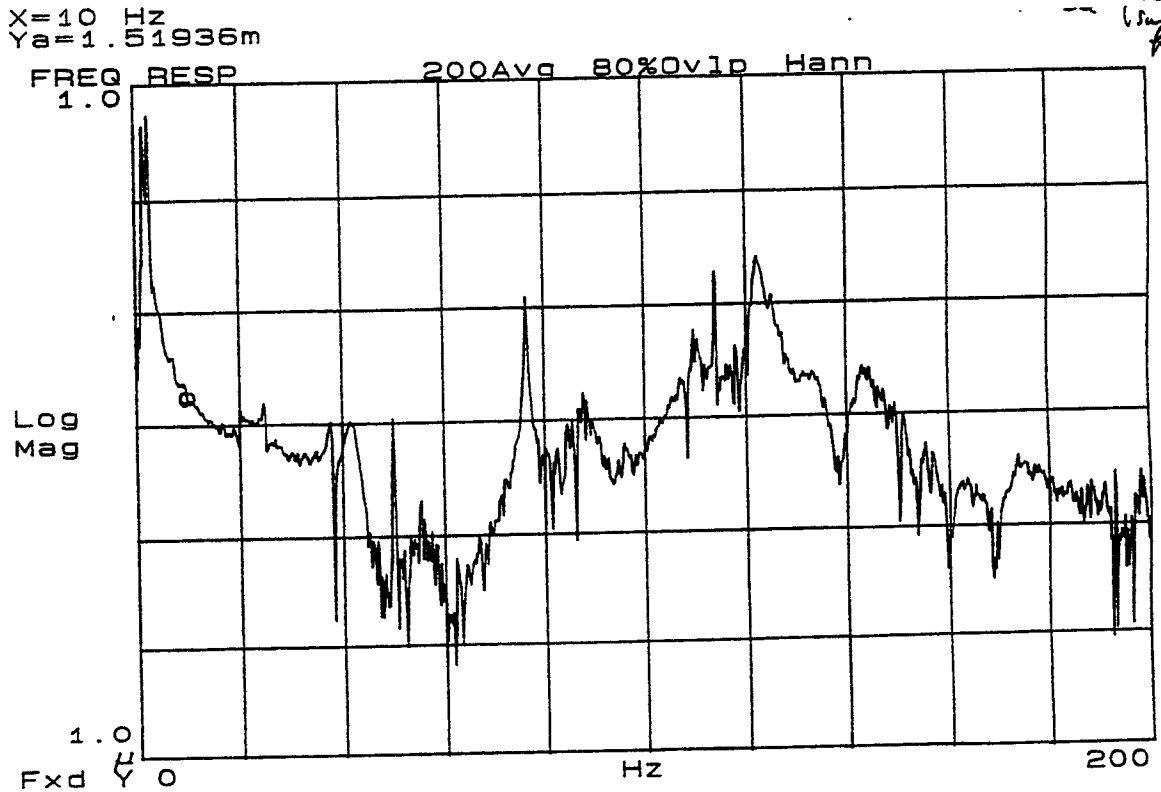
With **internal resonances dampeners**





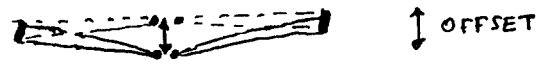
VERTICAL TRANSFER FUNCTION

HORIZONTAL TRANSFER FUNCTION OF THE BARE FILTER

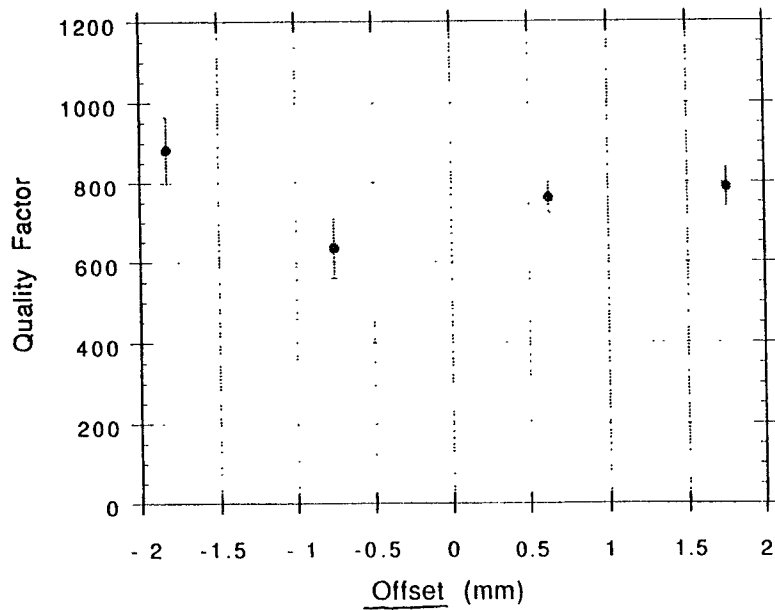


# QUALITY FACTOR vs.

OFFSET OF  
BLADES WORKING POSITION

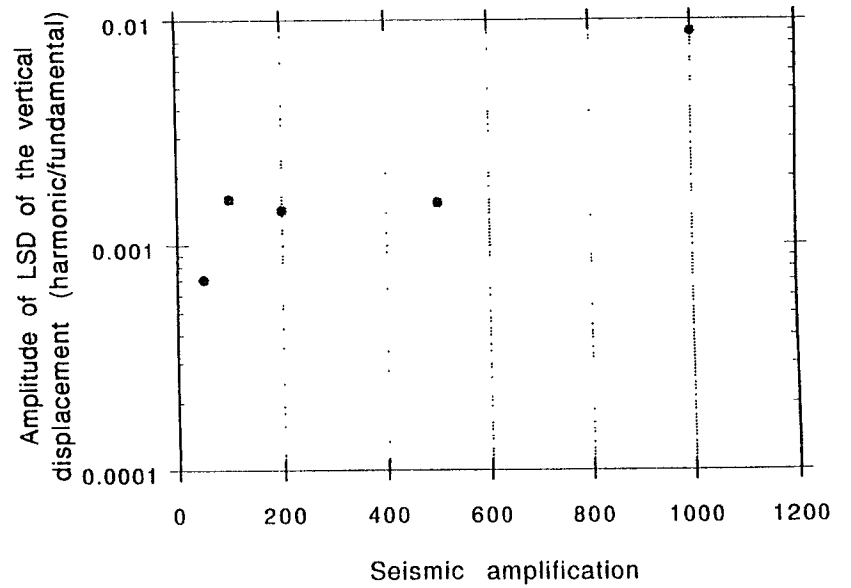
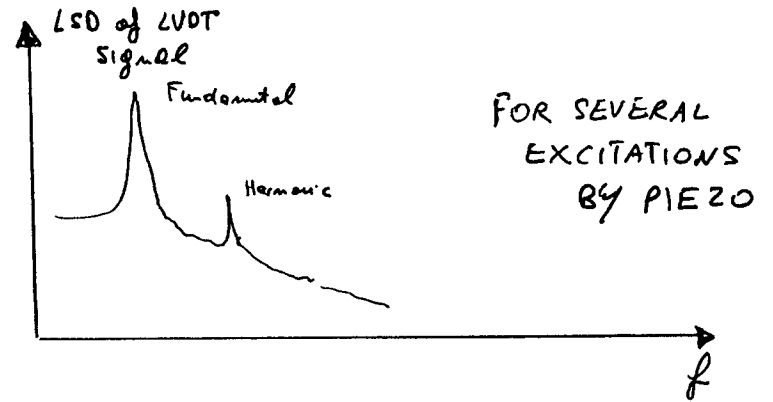


Q vs offset (In air)



NO RUBBING EFFECT EVEN IF  
THE SYSTEM WORKS OUT  
OF ITS STANDARD POSITION

8



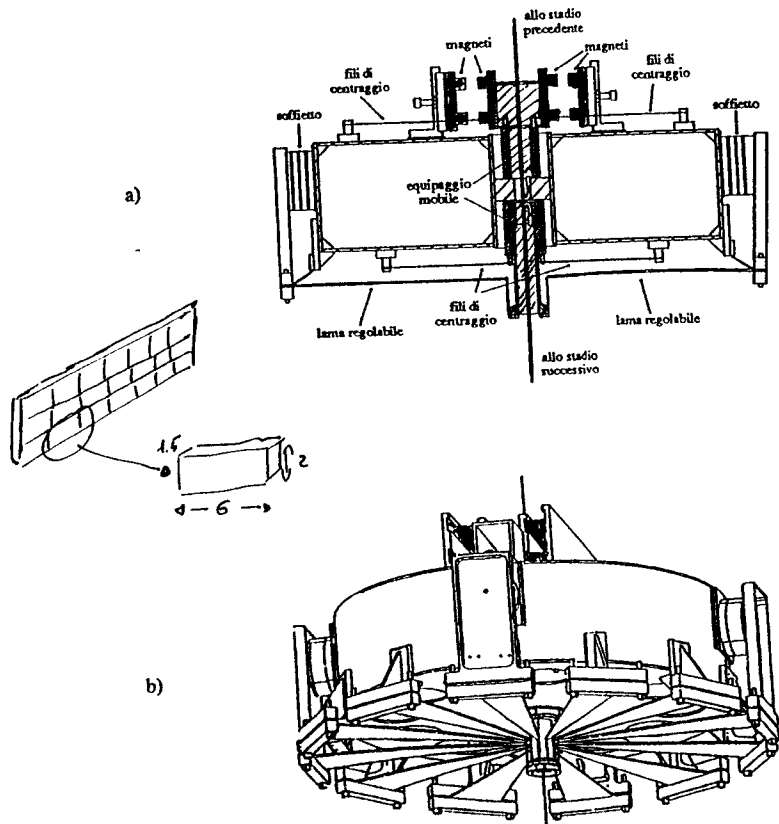
OFFSET = + 1.1 mm

NO UP CONVERSION!

## PHASE 2: Antisprings

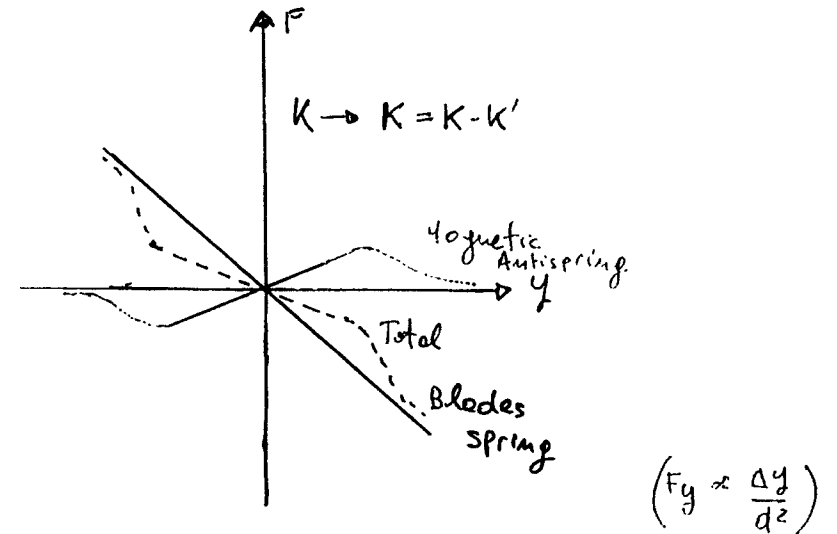
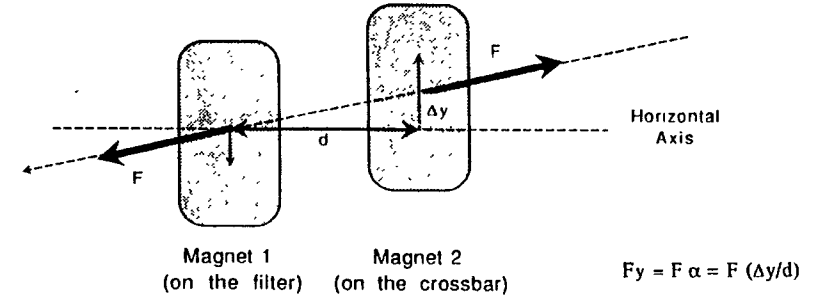
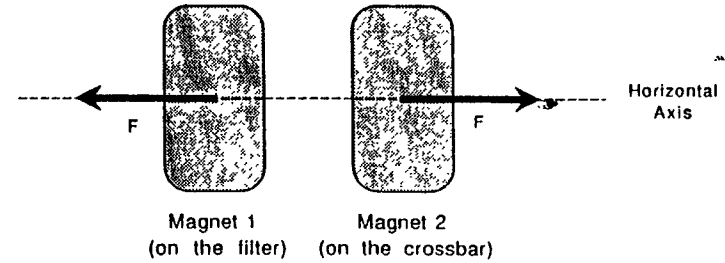
The following antisprings system has been mounted on the filter

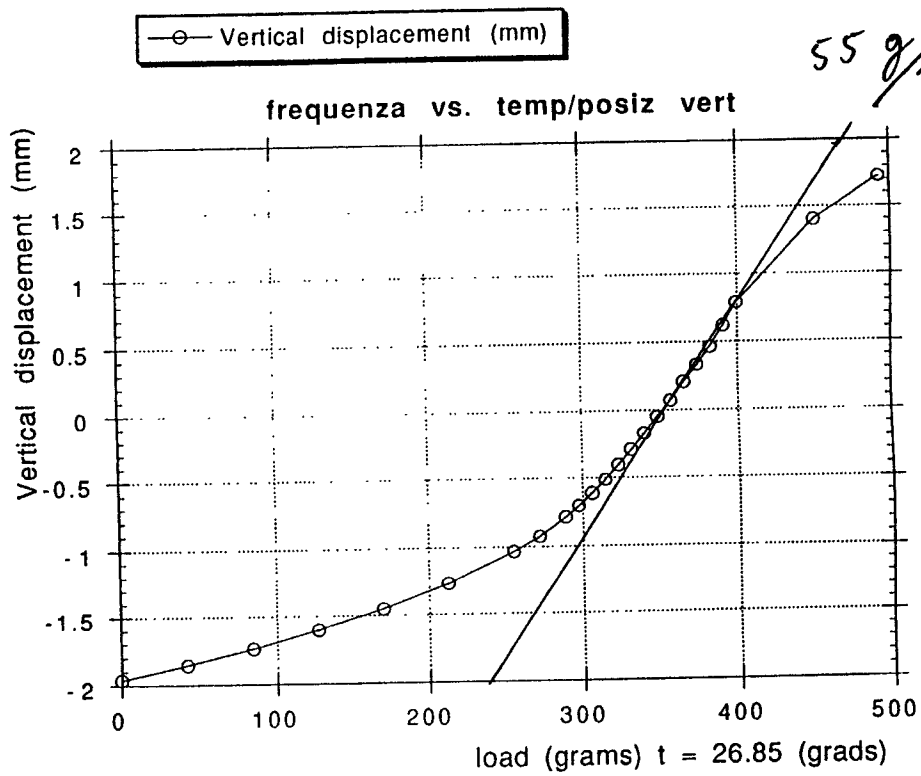
3 lines of 7 magnets (6 x 2 x 1.5 cm) on each of the four supports in a repulsive configuration.



## WORKING PRINCIPLE

### REPULSIVE CONFIGURATION





### GOAL

TO MAKE THE VERTICAL FUNDAMENTAL RESONANCE FREQUENCY LOWER THAN THE HORIZONTAL ONE (Pendulum = 0.5 Hz) FOR EACH STAGE.

### MEASUREMENT

- 1) Vertical transfer function
- 2) Vertical resonance frequency as a function of the system positioning (distance and offset)
- 3) Quality factors
- 4) Dependence on temperature of the system

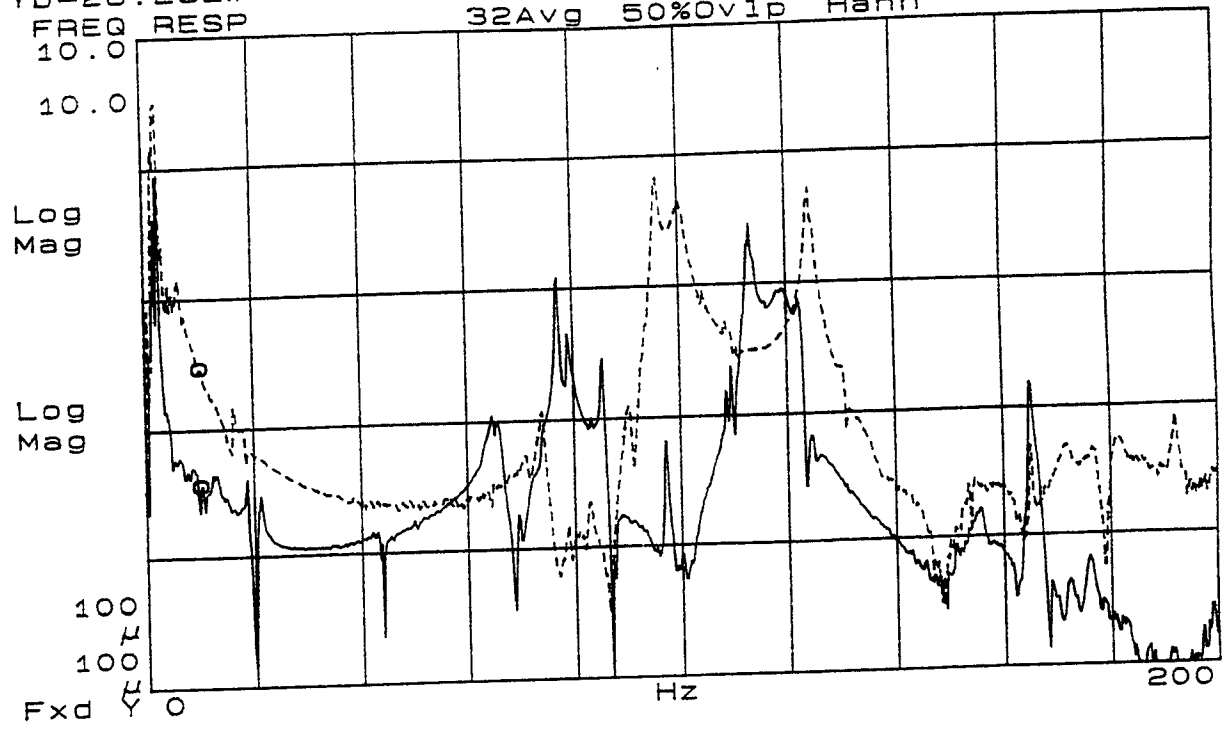
*In this way a complete characterisation of the system can be attained*

# MEASURED BY ACCELEROMETERS

VERTICAL TRANSFER FUNCTIONS  $f_0 \approx 0.35 \text{ Hz}$

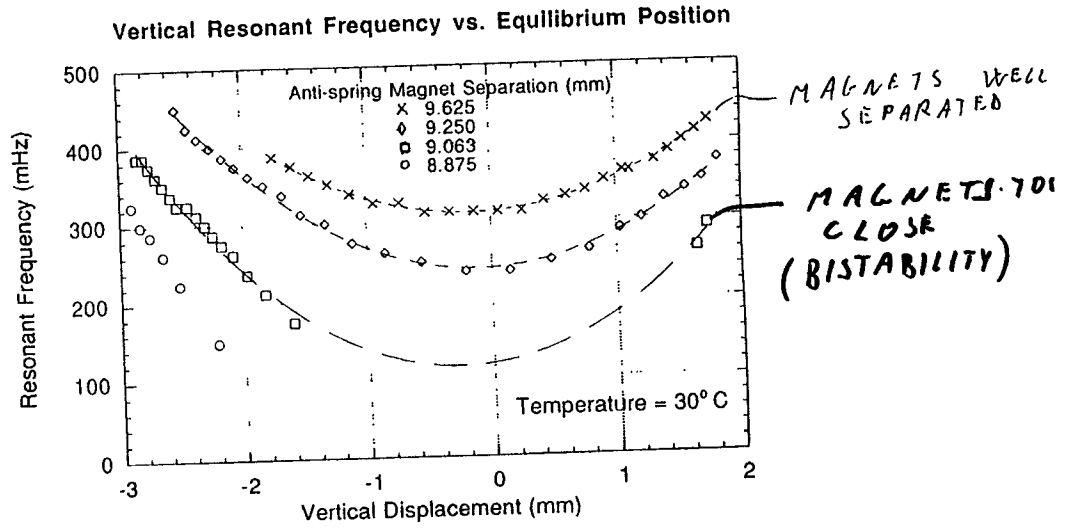
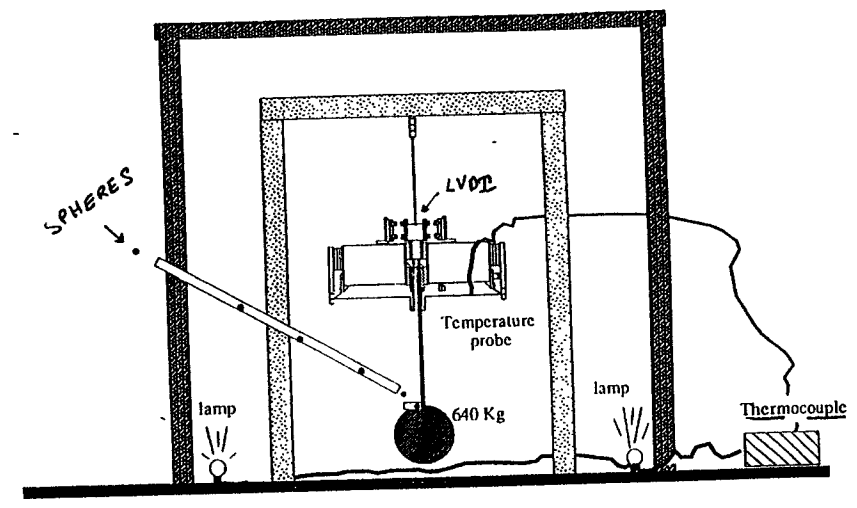
X=10.000000  
 Y=10.000000  
 FREQ RESP L  
 YD=28.28283  
 FREQ RESP L  
 10.0

400Avg 80%Ovlp Hann        WITH A.S.  
 32Avg 50%Ovlp Hann        WITHOUT A.S.



Measurement of  
 Vertical Resonance Frequency  
 of Super Attenuator filters

EXPERIMENTAL SET-UP



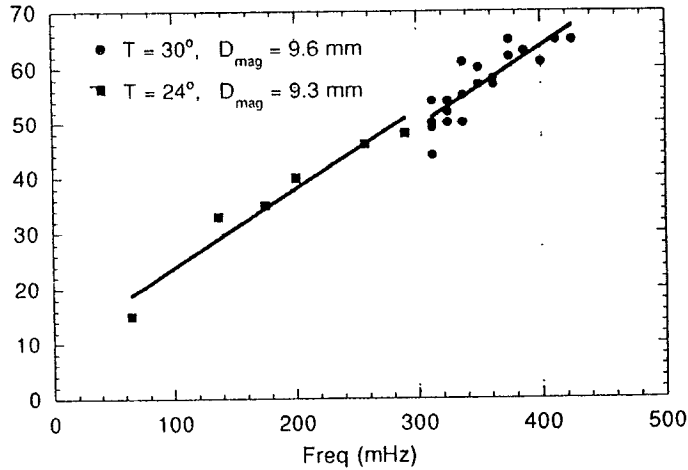
THE GOAL IS EASILY OBTAINED  
(Large tolerances)

→ CHANGE OF THE RELATIVE VERTICAL POSITION BETWEEN THE MAGNETS

→ SHOCK ⇒  $y$  LVDT SIGNAL ⇒  $\underline{Q}, \underline{\omega}$

MEASUREMENT OF  $\omega_0$  (or  $k$ ) AND  $Q$   
 AS A FUNCTION OF  $\Delta y$ .  
 (Can be done for several horizontal distances and for several temperatures)

Amplitude Lifetime vs. Vertical Resonant Frequency

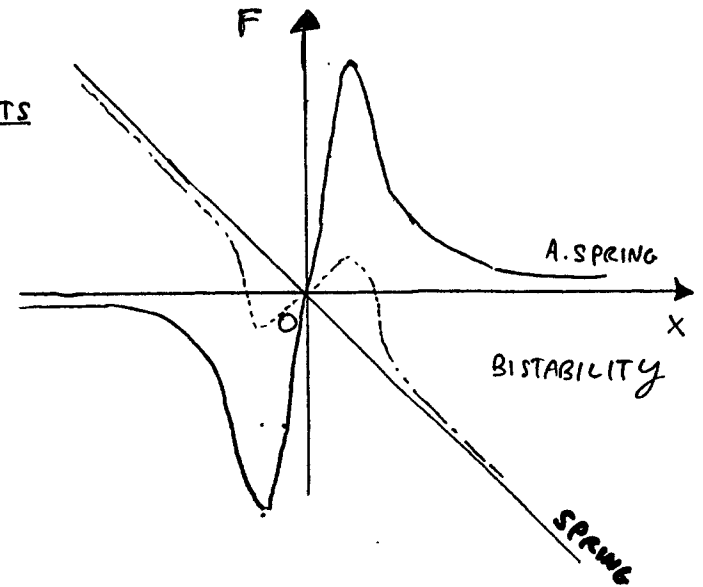


$$Q = \omega_0 \tau / 2$$

$$Q \propto \frac{\omega^2}{\omega_0^2} Q_0$$

⇒ INTERNAL FRICTION DOMINATES.

TOO CLOSE MAGNETS



THE FORCE INDUCED BY THE MAGNETS DEPENDS ON TEMPERATURE

44

$$K_{TOT} = K_{SPRING} - K_{ASPRING} \approx 1.000 \text{ N}/\mu$$

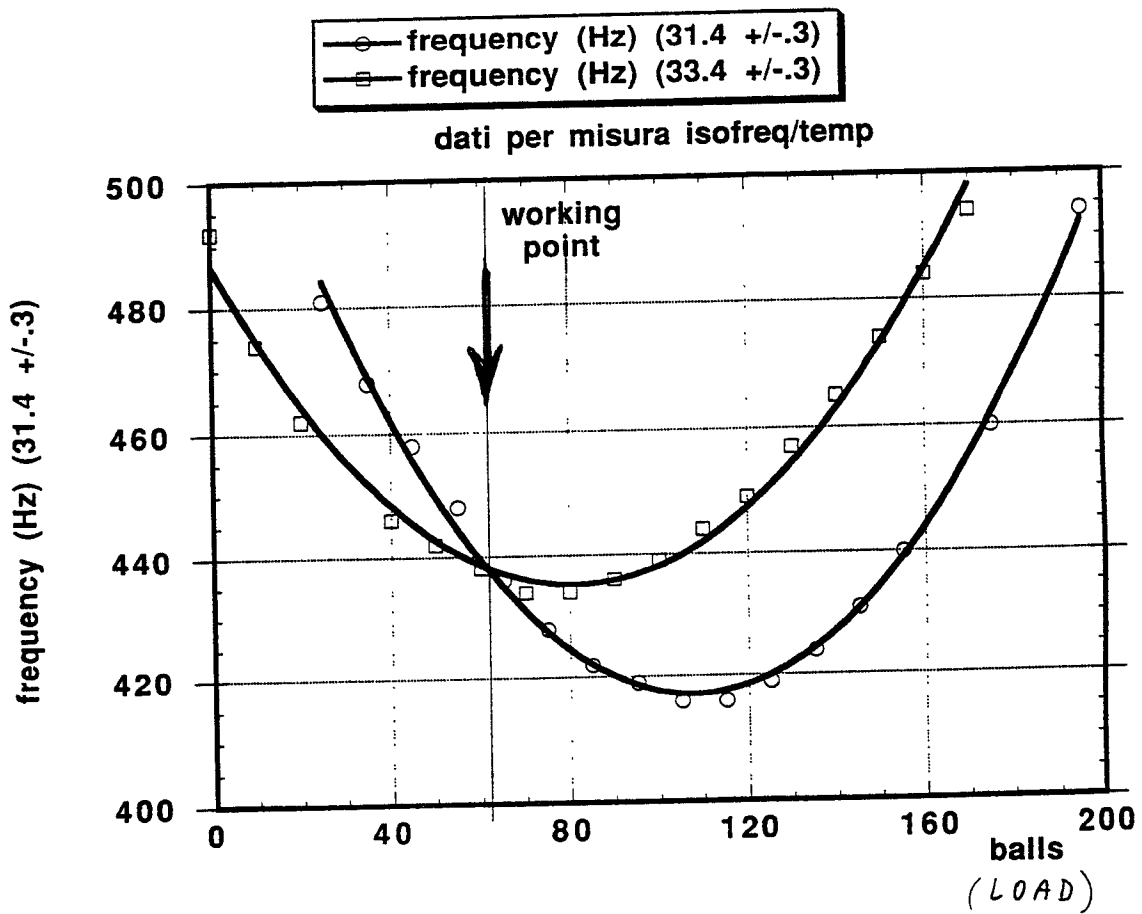
$$(30.000 \text{ N}/\mu) \quad (29.000 \text{ N}/\mu)$$

A SMALL VARIATION IN  $K_{AS}$  CAN BE A LARGE VARIATION IN  $K_{TOT}$

# Super Attenuation filters

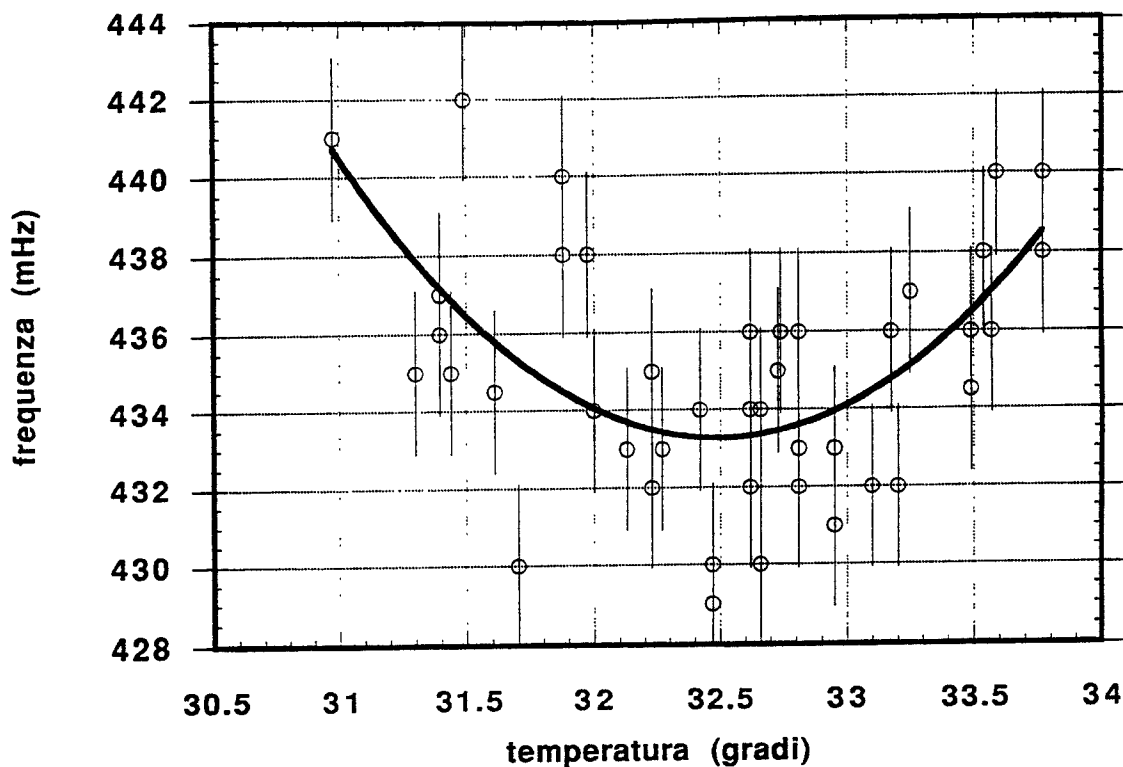
Working point determination and

Thermal stability



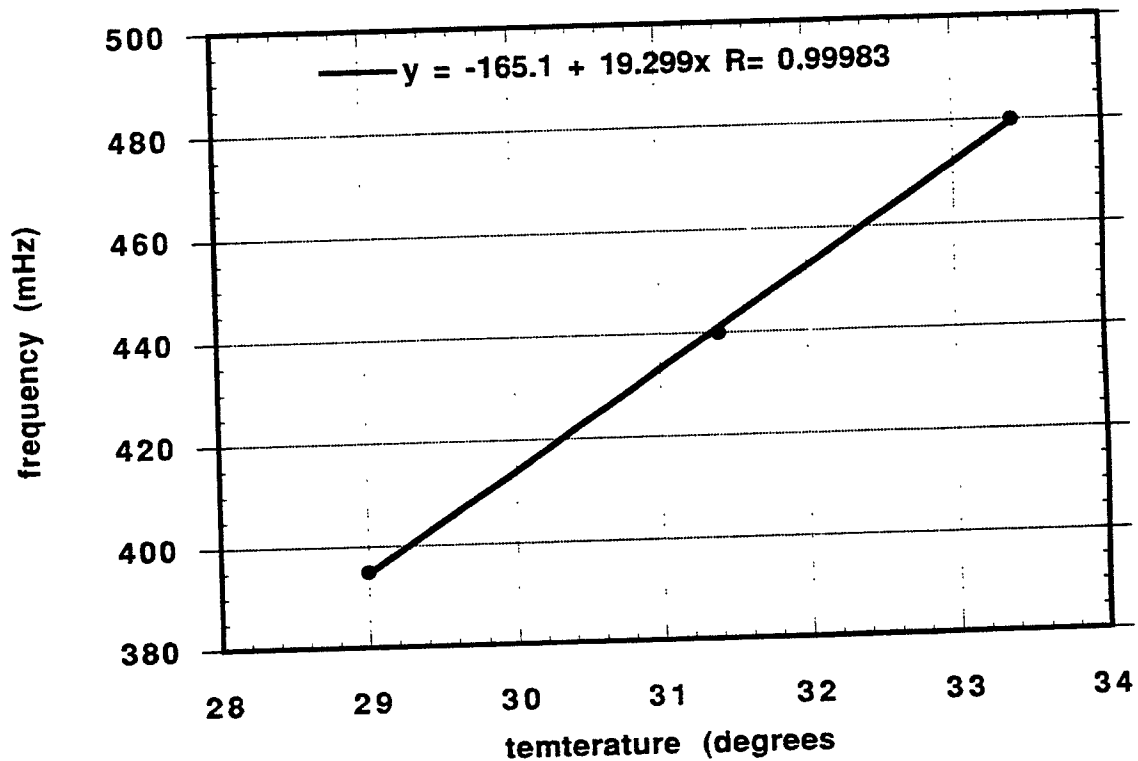


thermal stability  
 $f/dy * dy/dt - df/dt$



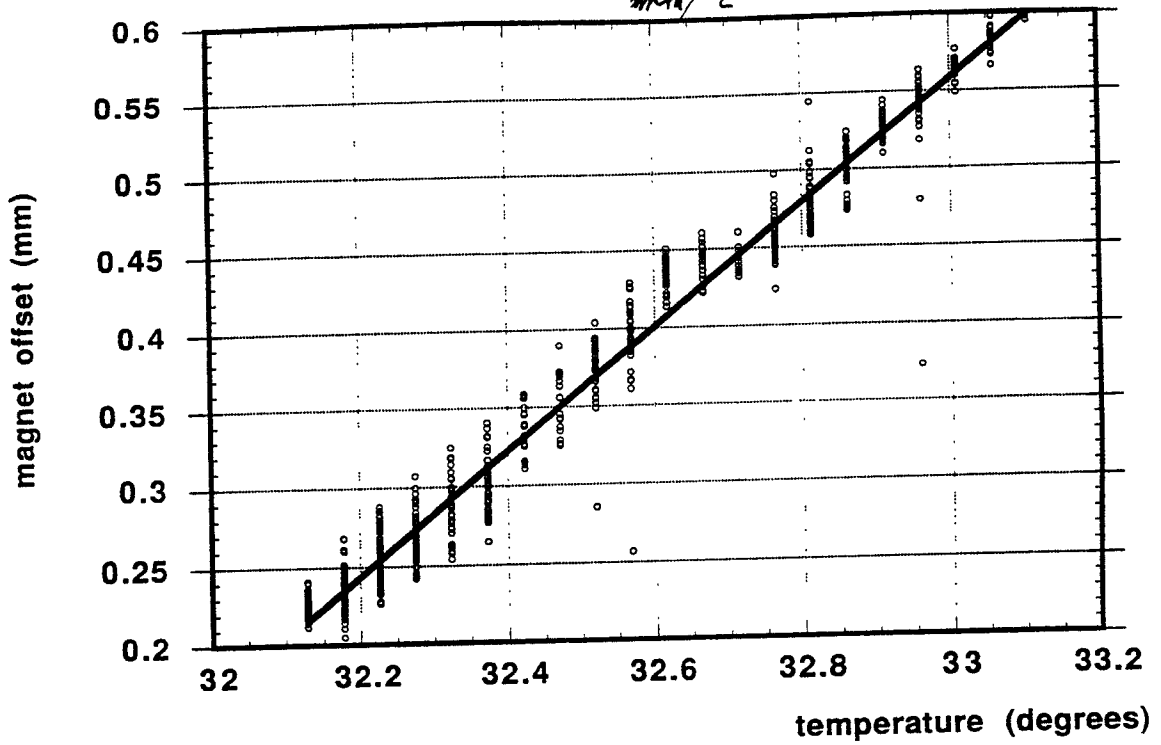
—●— frequency (mHz)

Data 39



THERMAL DRIFT  
FOR F FIXED

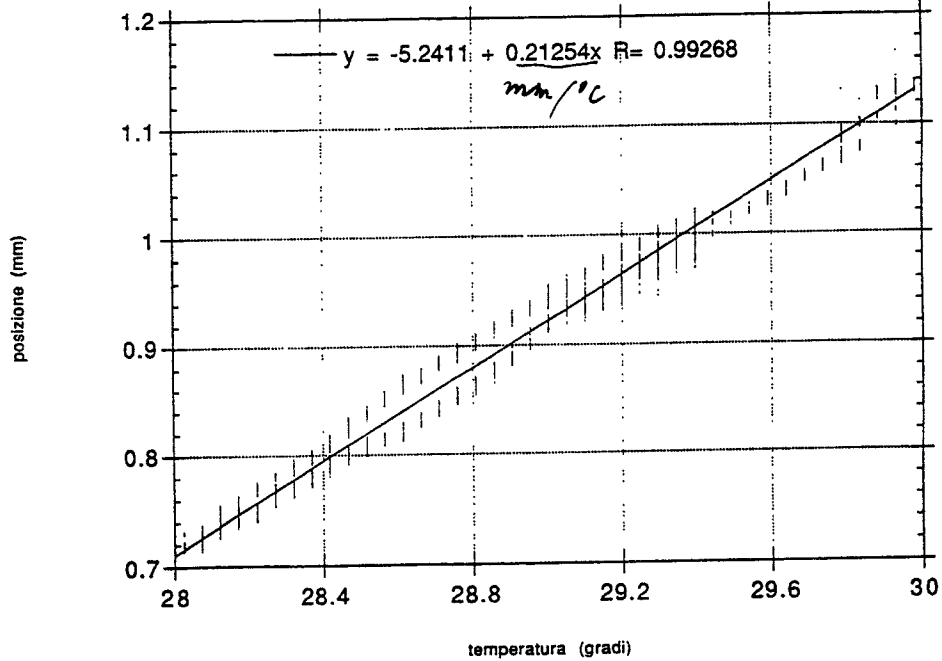
at  $df/dT = -df/dh * dh/dT$   
—  $y = -12.429 + 0.39356x$   $R = 0.98361$   
*mm/°C*



THERMAL DRIFT  
FOR F MINIMIZED

— posizione (mm)

fluttuazioni punto di lavoro



# Super Attenuation filters

## Creep

Observed abnormally high creep

Discrete process with random steps at random intervals

creep steps introduce internal nanoseisms

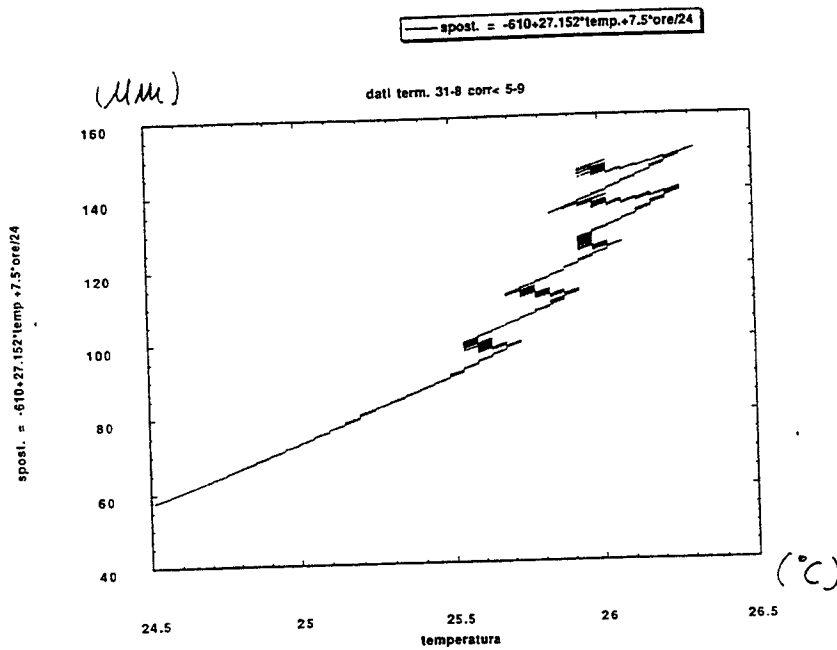
comparable to attenuated seismic activity

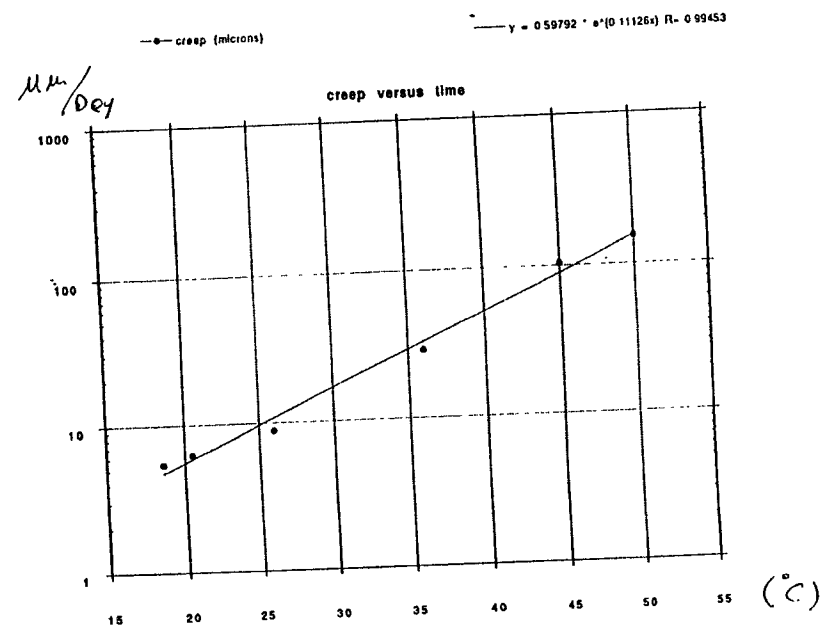
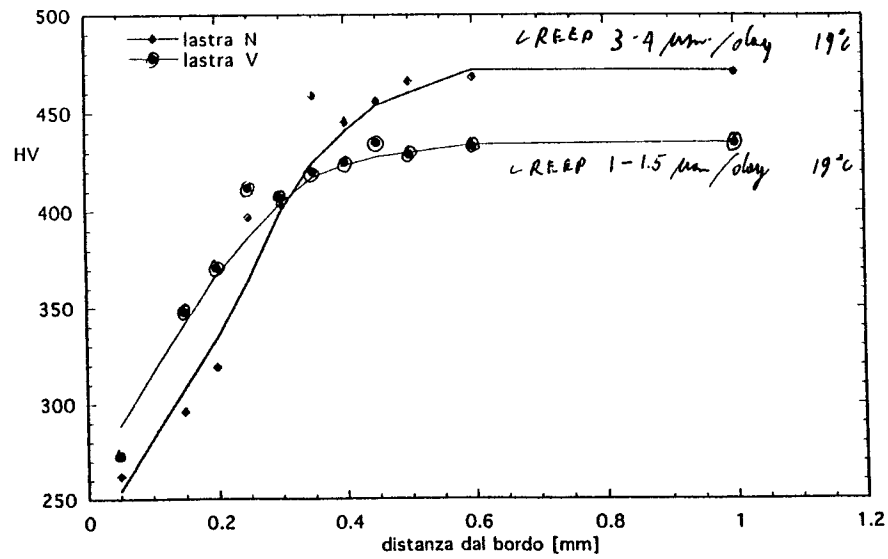
Must reduce creep induced noise below  $10^{-18}$  m/Hz<sup>-1/2</sup>

which requires creep activity less than  $10^{-4}$   $\mu\text{m}/\text{day}$  (not measurable)

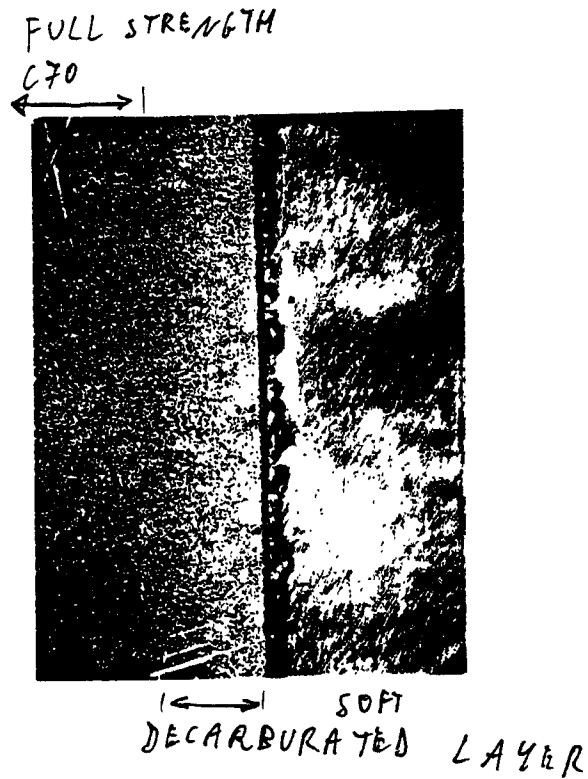
14

DISPLACEMENT VS.  
TEMPERATURE





→ CREEP (μm/day) VS. TEMPERATURE



## Super Attenuation filters

### Creep

Creep of 1  $\mu\text{m}/\text{day}$  is measurable

Creep step amplitude independent from temperature

dependent on crystalline structure

Creep event frequency  $f_{\text{creep}} = e^{-\Delta E/KT}$

Measured creep activity reduction by a factor of 10 every 20° cooling

Require less than 1  $\mu\text{m}/\text{day}$  creep activity 80° above operational temperature

Test superattenuators at 80°C and operation at -10°C

(Good for outgassing)



**INTERNATIONAL CONFERENCE ON GRAVITATIONAL WAVES:  
SOURCES AND DETECTORS**

**Cascina (Pisa), Italy    March 19-23, 1996**

**VIRGO KICKOFF MEETING**

**APPLICATIONS AVAILABLE**

**PLEASE ENQUIRE AND APPLY    A.S.A.P**

January 15, 1996 Aspen, USA  
 1996 Aspen Winter Physics Conference

● Present Status of the Tokyo 300m FP Interferometer (TAMA300)

M. Fujimoto

National Astronomical Observatory (NAO), Japan

● Data Acquisition and Analysis for TENKO100 and TAMA300

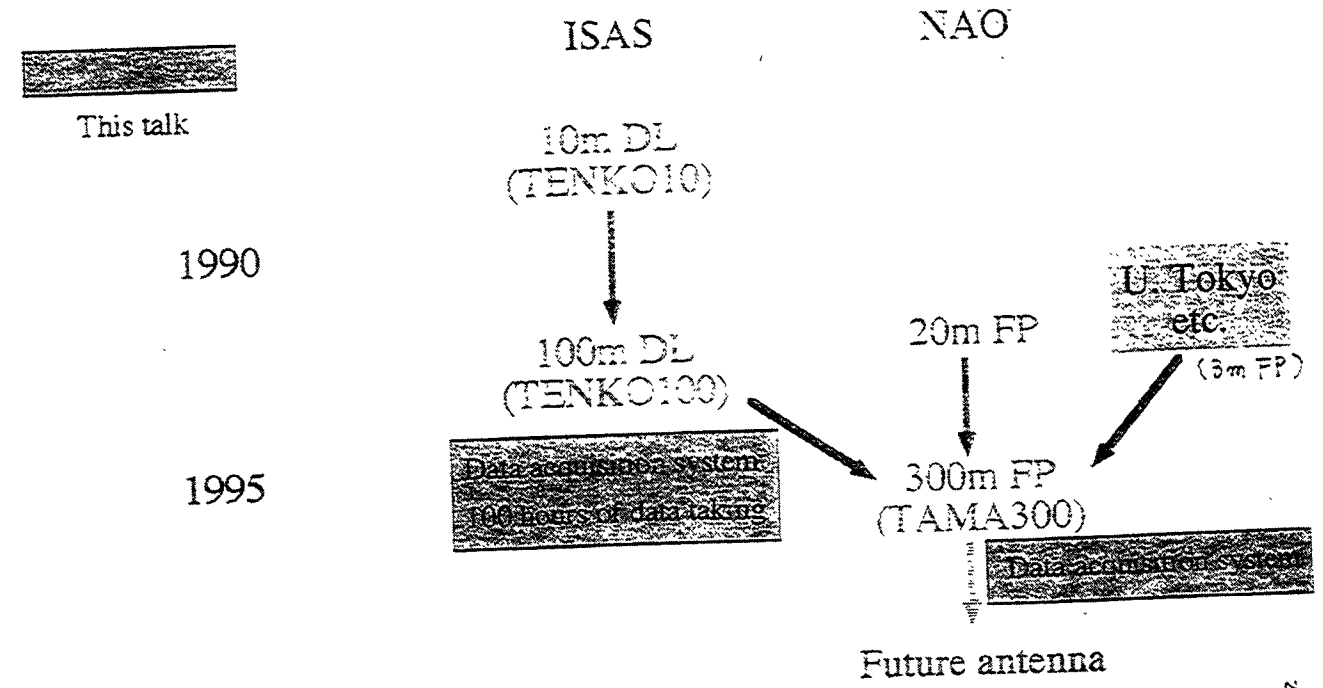
● Ei-ichi Mizuno

The Institute of Space and Astronautical Science (ISAS), Japan

6/4

● Technology  
 ● Researchers  
 → transfer

Laser interferometer GW detectors in Japan  
 (Arm length > 10m)



6/4

## 300-m laser interferometer fundamental parameters

Project Director

Yoshihide KOZAI

National Astronomical Observatory (NAO)

Institute for Cosmic Ray Research (ICRR)

National Laboratory for High Energy Physics (KEK)

Institute of Space and Astronautical Science (ISAS)

University of Tokyo

University of Electro-Communications

Kyoto University

target sensitivity

$$h=3 \times 10^{-21}$$

@300Hz

arm-length

300m

detector type

recombined Fabry-Perot  
type with power recycling

cavity finesse

520

laser

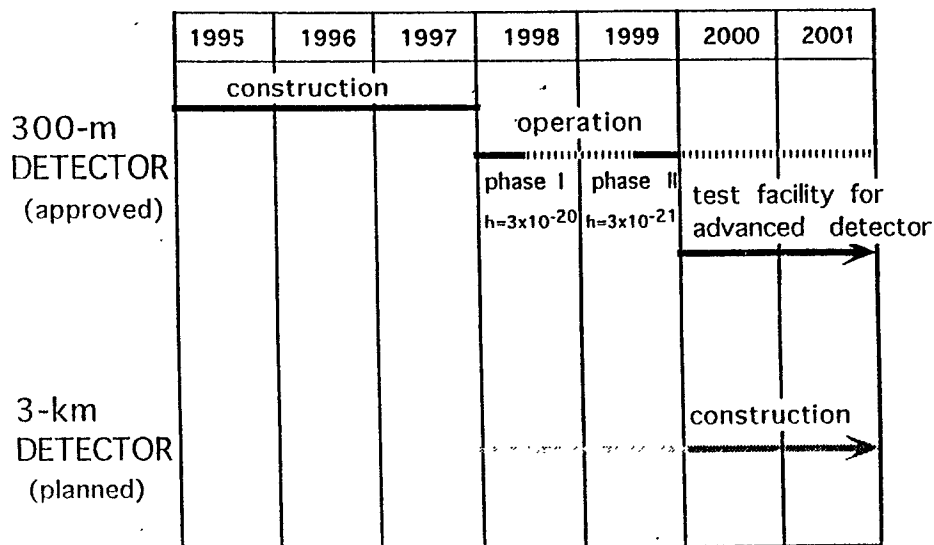
injection locked LD pumped Nd:YAG  
output power 10W  
wavelength 1064nm  
recycling gain 10

vacuum

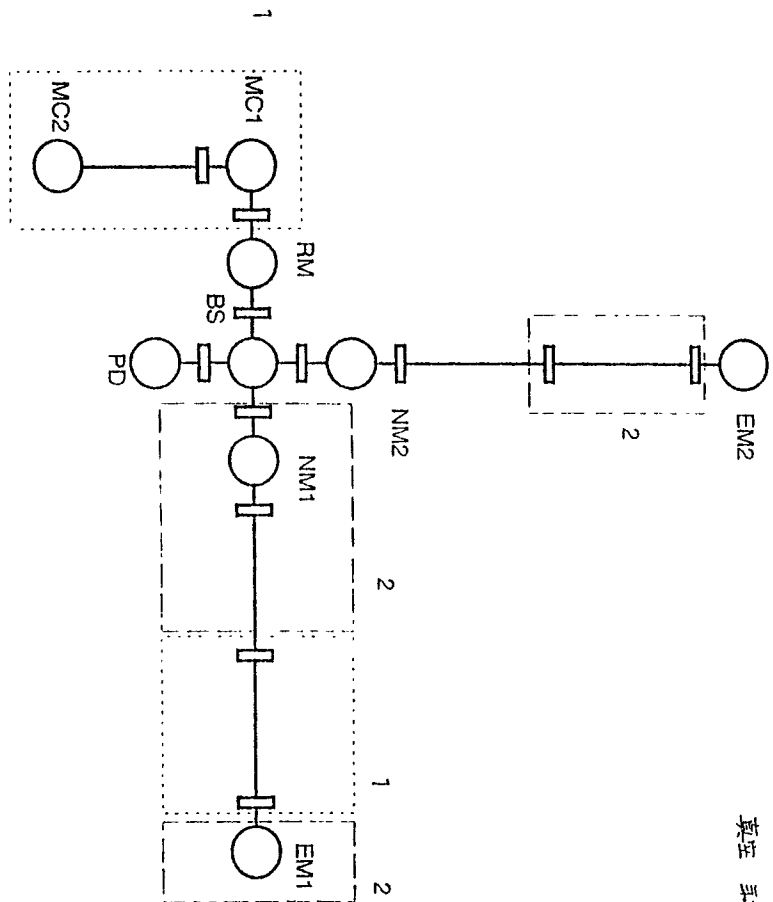
$10^{-6}$ Pa



### SCHEDULE OF THE LASER INTERFEROMETER GRAVITATIONAL WAVE DETECTORS



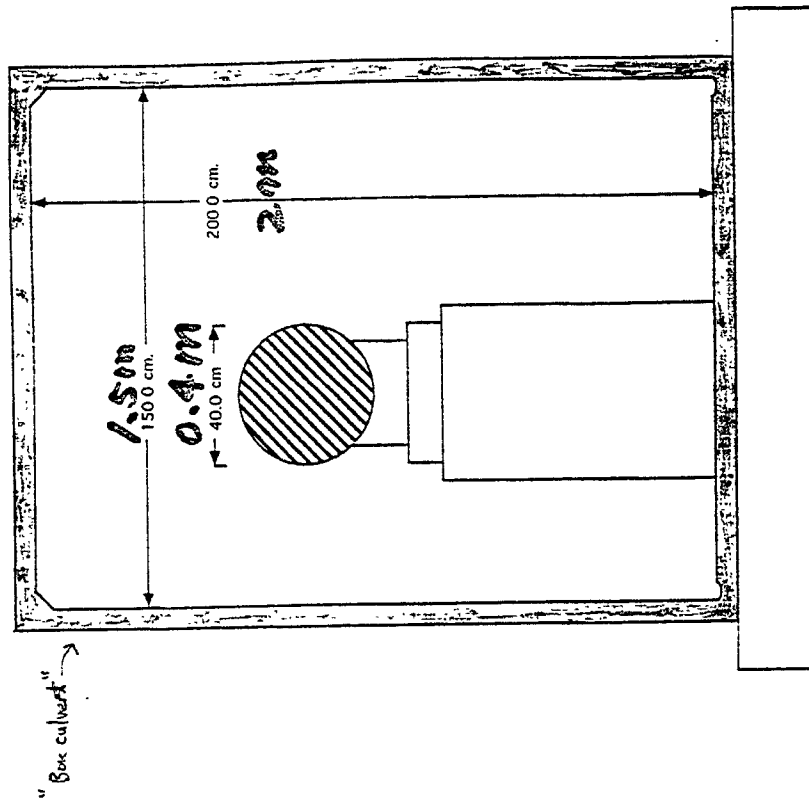
Building network telephone  
Nov. '96



真空系設備図

- 1. 1995 年度
- 2. 1996 年度
- 3. 1997 年度

GROUND LEVEL



## TAMA300 data management (d300)

### Tasks

#### Data acquisition

High rate (600 kbytes/sec), 300 channels, A week of online data storage

#### Realtime monitoring of slow data

CRT, printers, through network

#### Realtime data reduction, triggering and analysis

Strain sensitivity (h), analysis triggering by h

Template matching, Data reduction (by channels and frequency bands)

#### Remote control of instruments

Vacuum system, Laser system, through EPICS

#### Feedback over 300m

End room -> Near mirror signal feedback through fiber optics

#### Database management

User interface, data format, internet services

## TAMA300 data amount

### High speed (20kHz)

Interferometer signal  
 Low frequency (<700Hz)  
 High frequency (>500Hz)  
 Feedback  
 Main mirror (X2)  
 Beam splitter  
 Recycling mirror  
 Mode cleaner  
 Laser  
 Intensity  
 Frequency (X2)  
 Realtime analysis  
 Strain sensitivity (h)  
 Various filters (X3)

$$20\text{kHz} * 14\text{Ch.} * 16\text{bits}$$

$$= 560 \text{ kbytes/sec}$$

### Medium speed (200Hz)

Mirror angles, positions  
 Beam positions  
 Servo status  
 Housekeeping  
 Ground motion  
 Accoustic noise  
 Powerline voltage  
 Magnetic field  
 etc.

$$200\text{Hz} * 100\text{Ch.} * 16\text{bits}$$

$$= 40 \text{ kbytes/sec}$$

### Low speed (2Hz)

Temperature  
 Atmospheric pressure  
 Laser status  
 Vacuum pressure  
 Vacuum instruments status  
 GPS monitor  
 etc.

$$2\text{Hz} * 200\text{Ch.} * 16\text{bits}$$

$$= 0.8 \text{ kbytes/sec}$$

Total: 600 kbytes/sec

61/6

## Policy

Networks-based system and interface

Fundamental network: TCP/IP + Ethernet  
 User interface: Unix + X-window + EPICS

Separation of several different kind of data by networks

High speed data: Reflective memory - VXI  
 Low speed data: VME - Ethernet - EPICS  
 Vacuum monitoring: GPIB - Ethernet - EPICS  
 Analog data transfer: TTL data transfer through fiber optics

Interface with other observations (other GW antennas,  $\gamma$ -ray, infrared, etc.)

Time keeping: GPS / ntp  
 Establish an international standard data format  
 WWW information services

Collaboration with other groups (LIGO/VIRGO/GEO/etc.)

1/01

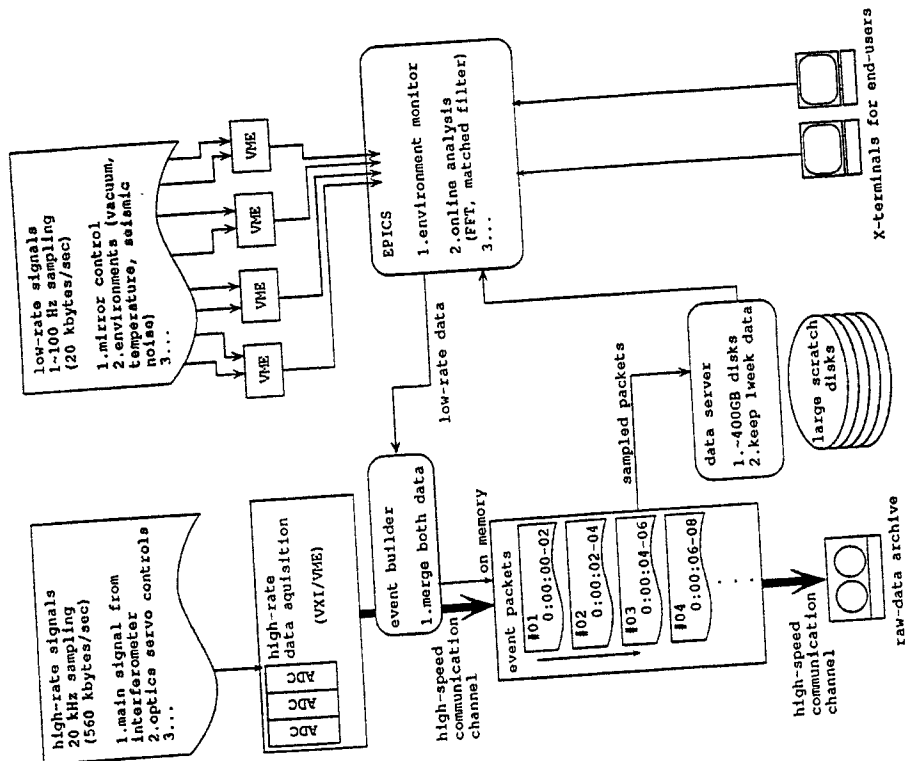


Figure 1: Schematic view of the data flows between the high-rate data-acquisition, the EPICS, the event builder, the data server, and the raw-data archiver.

## TENKO100 long-term operation

### Objectives:

- Study the reason why interferometer unlocks
- Correlations between various kind of signals
- Analysis of non-stationary noise, with long time scales
- Establishment of the data acquisition system and the realtime monitoring system
- Preparation of the TAMA300 data acquisition system

### Period:

(1) July 24 - August 10, 1995

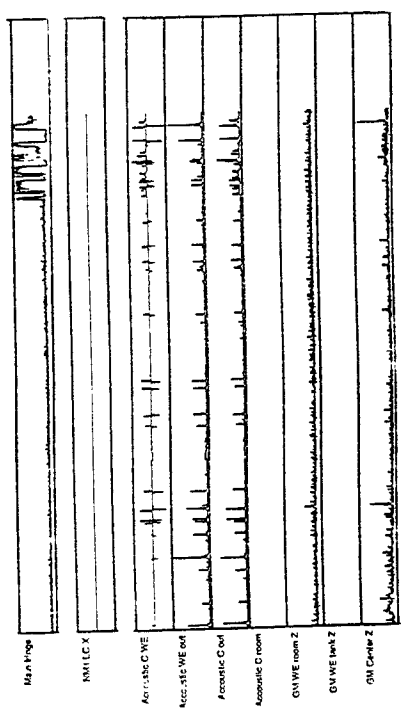
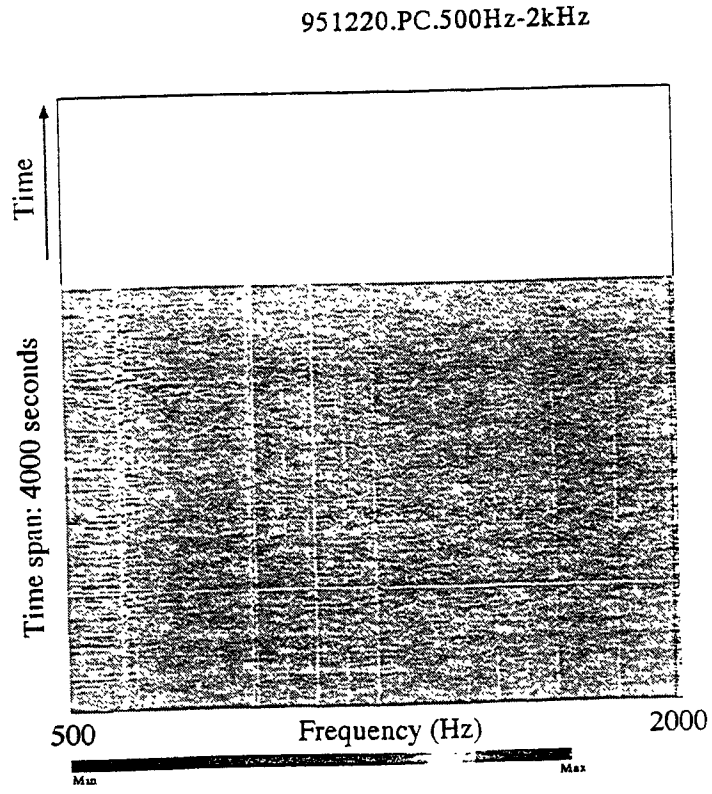
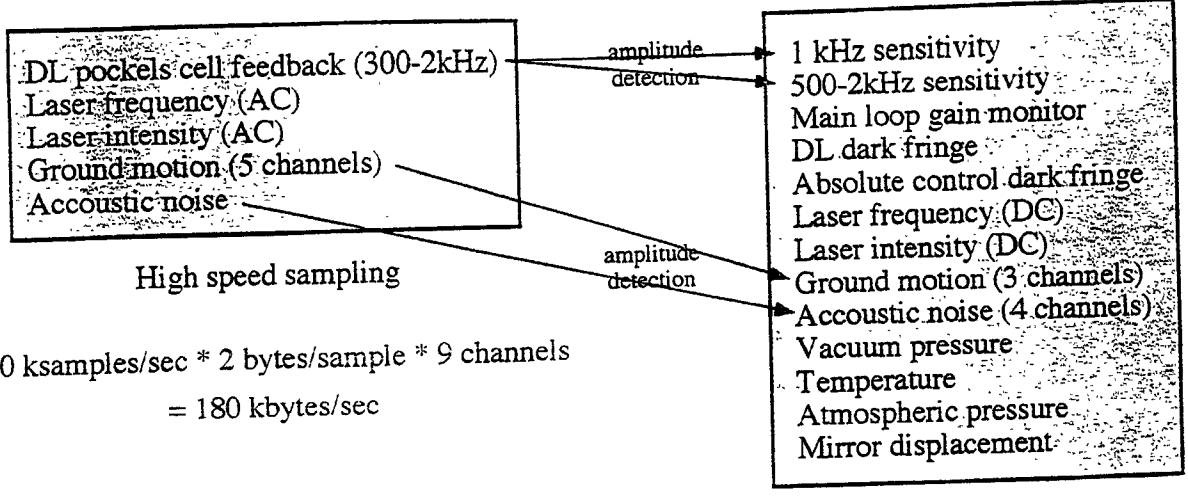
Operation: 100 hours, Data recording: 48 hours

(2) December 4 - 25, 1995

Operation: unknown, Data recording: 30 hours

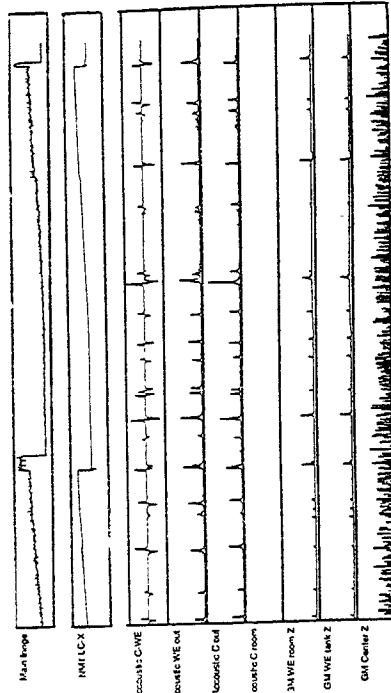
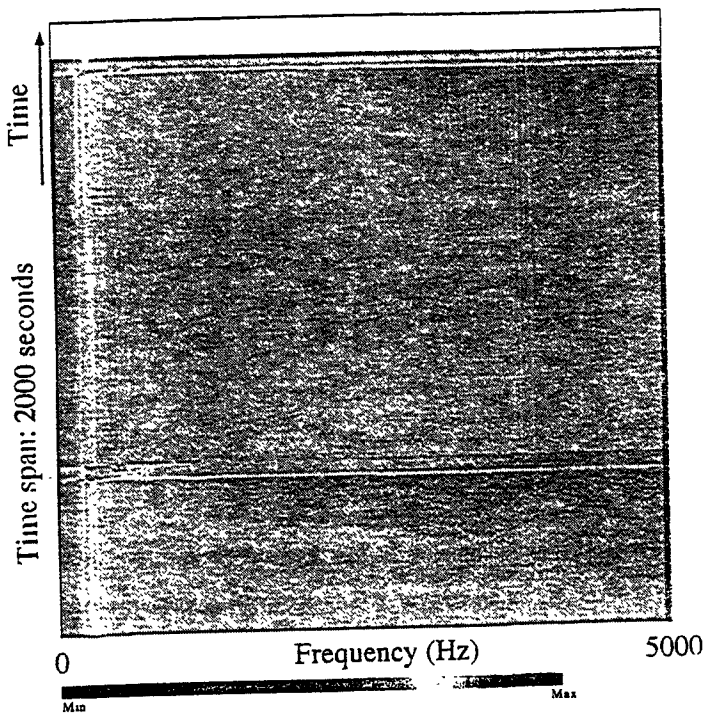
Improvement of the system

# TENKO100 recorded data

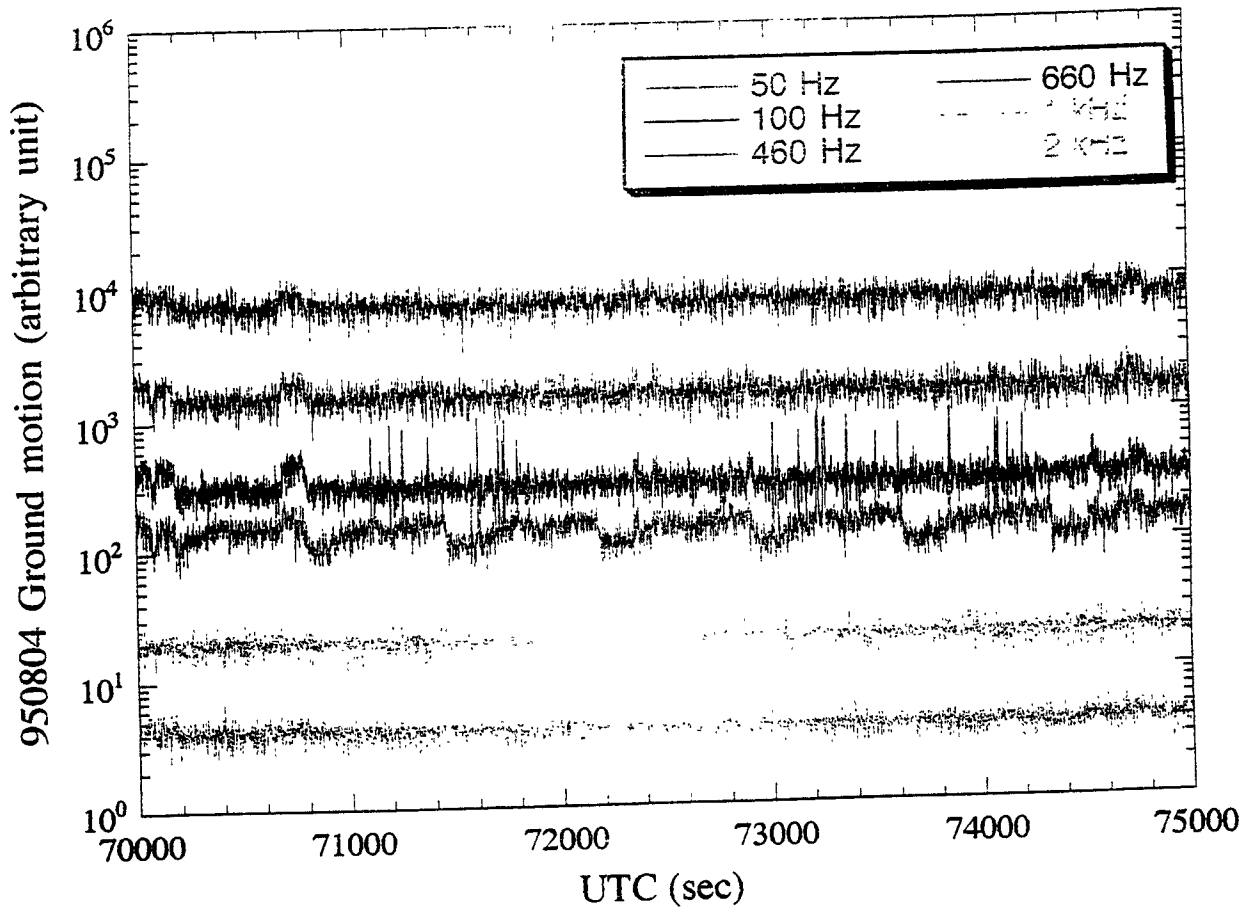


13/19

14/19



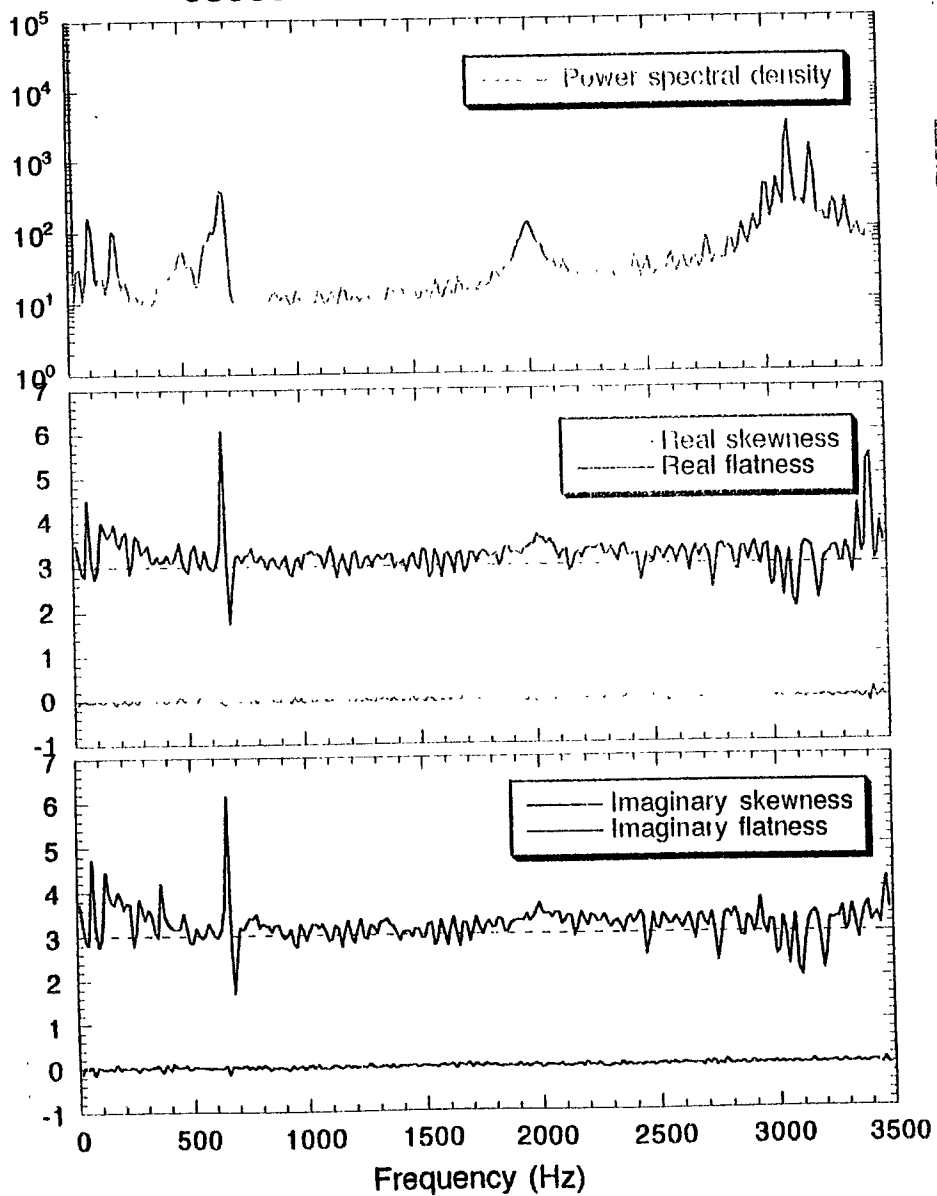
6/5/91



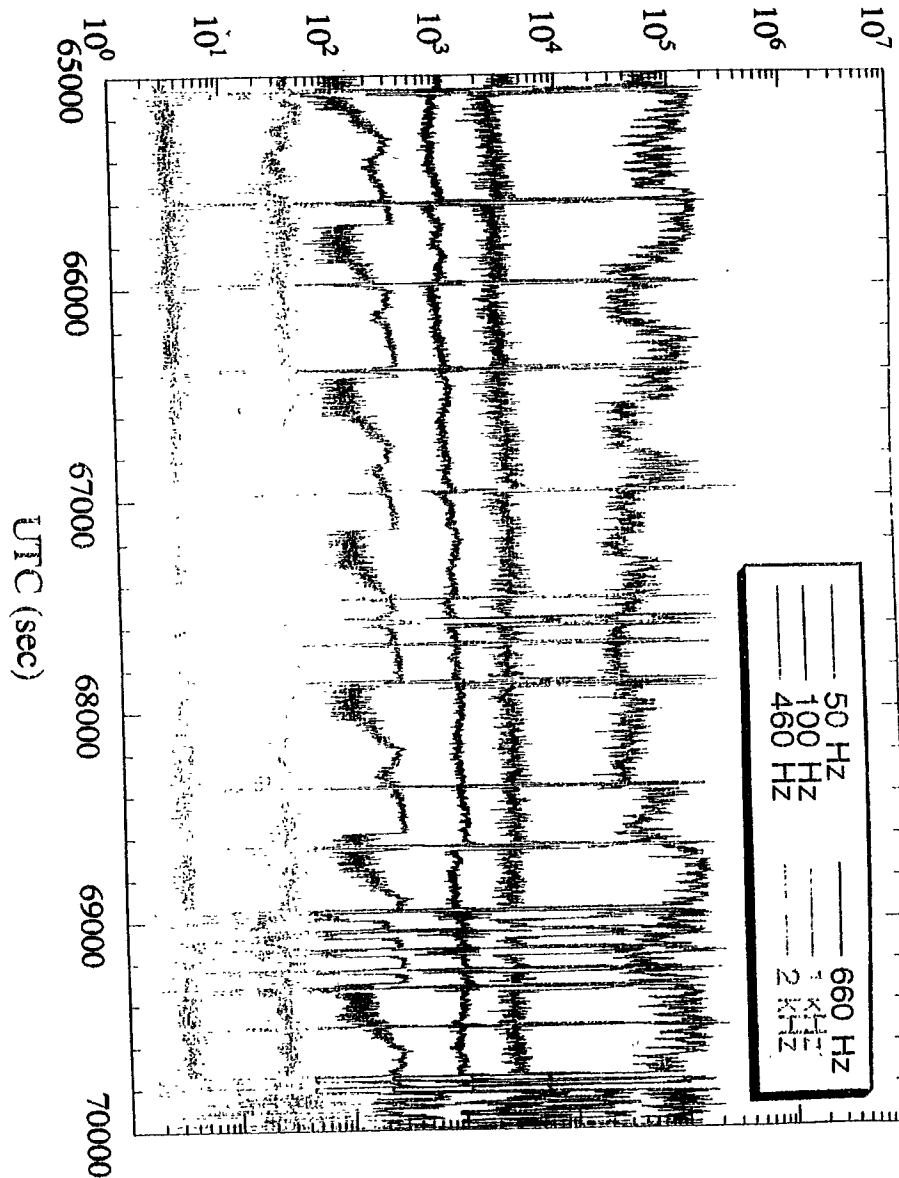
6/9/91

950804 20:18:00 - 20:29:40 UTC

18/19



950804 Sensitivity (arbitrary unit)



6/01

## What has become clear from the long-term operations?

19/9

### ○ Interferometer signal and the unlocking of the system

Sensitivity varies by factor of 10 (max) from day to day  
(especially below 600Hz)

Slow motion of the DL mirror changes the dark fringe condition

Dark fringe condition is highly correlated with the unlocking

48Hz ground motion, due to an air conditioner, has a time scale of 10 minutes, and it also makes the system unlock.

Car passing the street nearby, also makes the system unlock

### ○ Data acquisition system and analysis

Daily variation of the analog signal was difficult to handle with a small dynamic range (12 bit). Several days of data were lost. Choice of filter is important.

Data compression at the analog stage simplified the data handling

### ○ Future

Realtime data compression

Realtime data analysis

A flag showing the validity of data



Revised manuscript submitted to *Physical Review Letters* on November 27, 1995

Edward L. Ginzton Laboratory  
Stanford University  
Stanford, CA

# A Sagnac Interferometer for Gravitational Wave Detection

Ke-Xun Sun, M. M. Fejer, Eric Gustafson and Robert L. Byer

Edward L. Ginzton Laboratory, Stanford University, Stanford, CA 94305-4085

## Abstract

We have investigated a zero-area Sagnac interferometer as a broadband gravitational wave detector. Frequency response measurements of a laboratory-scale interferometer are in excellent agreement with theory. The measured contrast ratio, 0.996, is insensitive to induced birefringence, laser-frequency variation, arm-reflectance imbalance, and DC mirror displacement. A near shot-noise-limited phase sensitivity of  $3 \times 10^{-9}$  rad/ $\sqrt{\text{Hz}}$  was measured at the interferometer's maximum sensitivity frequency, 90.9 MHz.

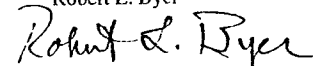
PACS numbers: 04.80.Nn, 07.60.Ly, 42.25.Hz, 42.62.-b

POST DEADLINE  
PAPER  
MONDAY, JAN 45, 1996  
ASPEN WORKSHOP


## Stanford Advanced Gravitational-Wave Laser Interferometer Program

### GALILEO

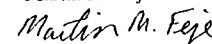
Principal Investigator  
Program Director of Galileo  
Robert L. Byer



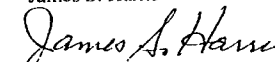
Co Investigators  
Daniel B. DeBra



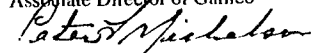
Martin M. Fejer



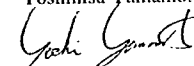
James S. Harris



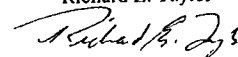
Peter F. Michelson  
Associate Director of Galileo



Yoshihisa Yamamoto



Richard E. Taylor



October 19, 1995

# Stanford Advanced Gravitational-Wave Laser Interferometer Program

p 3

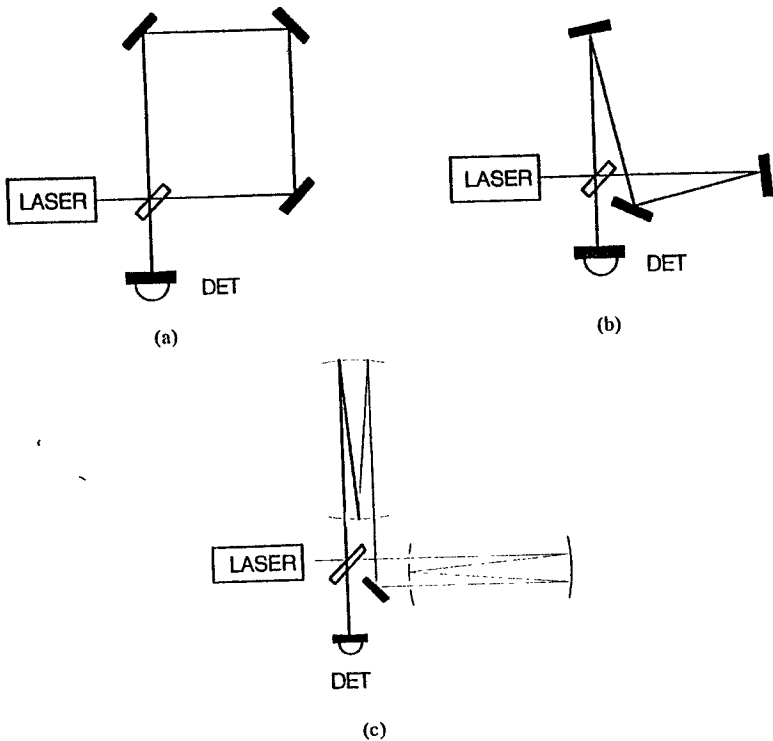


Figure C19 (a) Sagnac interferometer for rotational sensing (b) Sagnac interferometer for displacement sensing. Rotational sensitivity is suppressed by area cancellation. (c) Two-round trip Delay line Sagnac interferometer for gravitational wave detection.

## I. The Sagnac Common Path Interferometer

The Sagnac interferometer, invented in 1910 for rotation sensing, is a common-path interferometer in which waves traveling in the opposite direction experience identical optical paths. In the original version of this interferometer, and the version most widely used today for laser gyro rotation sensing, the Sagnac interferometer is an open ring that encompasses an area  $A$ . The rotation sensitivity is increased as the product  $NA$  where  $N$  is the number of times that the laser beam traverses the perimeter of the enclosed area. Fiber optics is used today to increase  $N$  and yet maintain the rotation sensitivity in a

## Stanford Advanced Gravitational Wave Interferometry Program

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Richard E. Taylor

**Principal Investigator**  
Robert L. Byer  
Ginzton Laboratory  
Stanford University  
94305-4085  
tel (415) 723 0226  
fax (415) 723 2666

## The Null Area Sagnac Interferometer for Gravitational Wave Detection

The concept of the common-path, reciprocal-path interferometer

Calculated Response Function for a LIGO scale interferometer

Comparison with the Michelson Interferometer

Laboratory experimental measurements

Response function

Contrast Ratio

Phase sensitivity measurements

near quantum limited phase sensitivity at 90 MHz

Signal extraction and detector local oscillator

$$|\Delta\phi| = \frac{4f_l h_g}{f_g} \sin^2(\pi\tau_s f_g), \quad \text{Sagnac interferometer.} \quad (2)$$

The absolute magnitude of the differential phase for a Michelson interferometer which uses delay lines rather than Fabry-Perots in the arms can be calculated similarly [2],

$$|\Delta\phi_m| = \frac{2f_l h_g}{f_g} |\sin(\pi\tau_s f_g)|, \quad \text{Michelson interferometer.} \quad (3)$$

For both the Sagnac and the Michelson interferometers, the single arm storage time  $\tau_s$  is defined as  $\tau_s = \frac{2NL}{c}$ , where  $L$  is their arm length,  $N$  is the number of bounces on one end mirror of the delay line, and  $c$  is the speed of light.

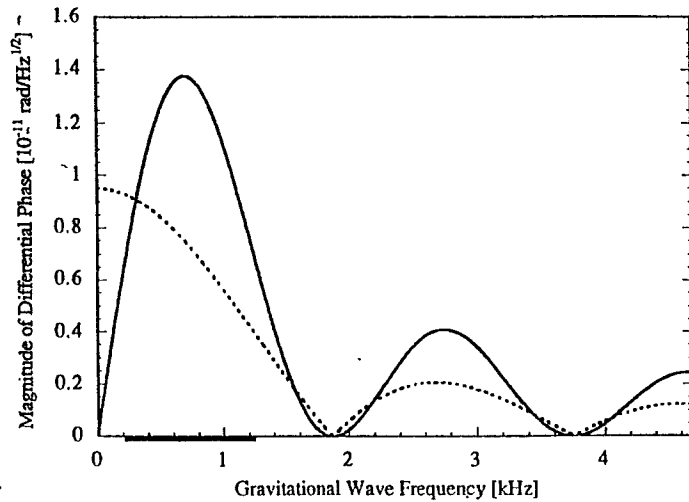


Figure 1(c) shows the response function for 4-km 20 bounce ( $N = 20$ ) LIGO-scale interferometers. The Michelson interferometer has its peak response at DC and sub-peaks at frequencies determined by the transcendental equation  $x = \tan x$ , where  $x = \pi\tau_s f_g$ . The Sagnac interferometer has its peak and sub-peak responses at frequencies determined by  $2x = \tan x$ . The first and highest peak occurs at  $f_{g,max} = 0.37/\tau_s = 0.74/\tau_{loop}$ , where  $\tau_{loop}$  is the loop time of the Sagnac interferometer. Setting  $\tau_s$  allows the Sagnac peak frequency to be tuned to the gravitational wave band of interest, and for this LIGO example  $f_{g,max} = 690$  Hz. The first leaf of Sagnac interferometer responses has a 3-dB bandwidth of  $0.54/\tau_s$ , which for the LIGO example extends from 220 to 1250 Hz.

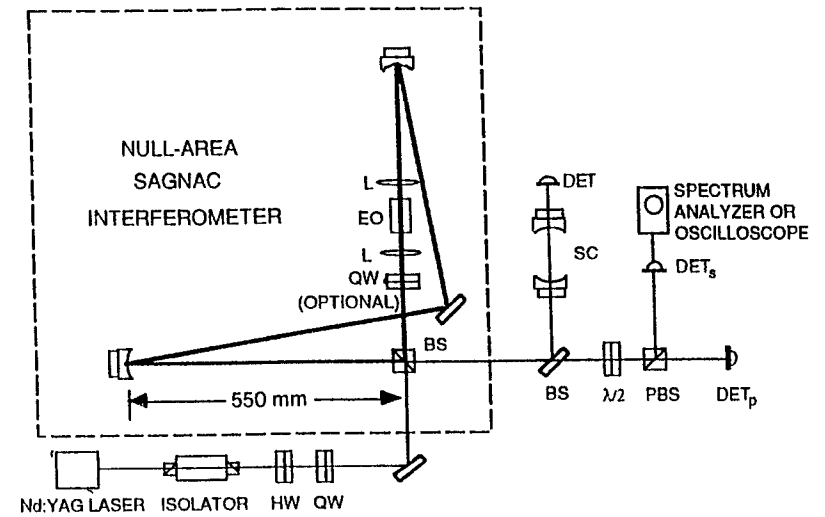
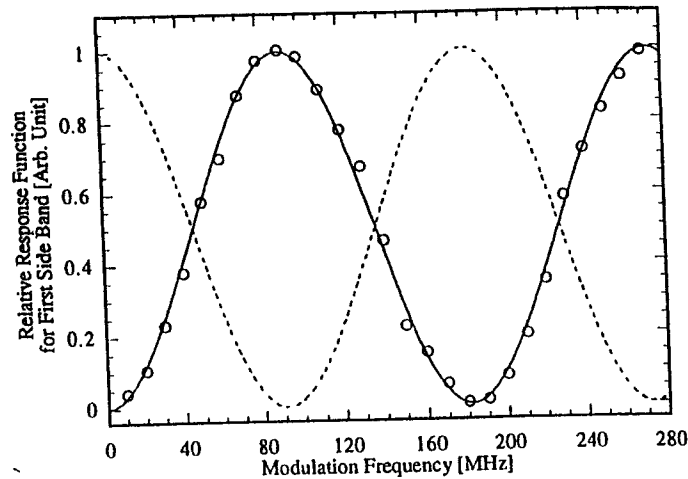


Figure 2 (a)



The frequency response of a Sagnac interferometer to a spatially lumped phase modulation is derived from Eq.(1) by replacing the integrand with  $\delta$ -functions, and expanding the field in a series of Bessel functions. With a 50/50 beam splitter, the first-side-band response function normalized to the input power for is given by

$$T_{1,s}(f_{\text{mod}}) = \frac{I_1}{I_{\text{in}}} = J_1^2(m) \sin^2[\pi f_{\text{mod}}(\tau_a - \tau_b)] \quad (4)$$

The peak response is at the modulation frequency

$$f_{\text{max}} = \frac{2}{|\tau_a - \tau_b|} \quad (5)$$

For small modulation ( $m \ll 1$ ), Eq. (5)

reduces to

$$T_{1,s}(f_{\text{mod}}) \approx \frac{1}{4} m^2 \sin^2[\pi f_{\text{mod}}(\tau_a - \tau_b)] \quad \text{Sagnac interferometer.} \quad (6)$$

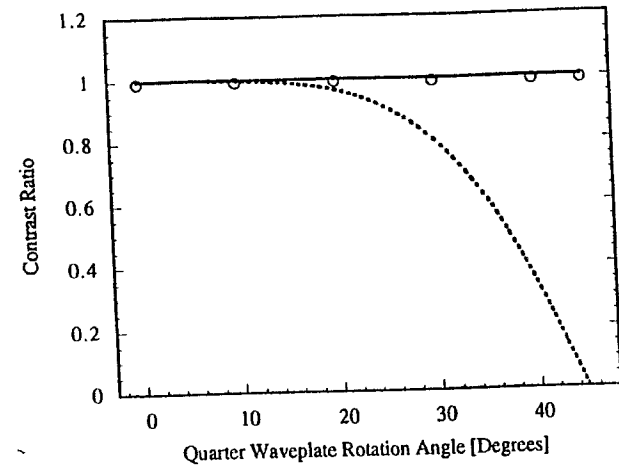
$$T_{1,M}(f_{\text{mod}}) = \frac{1}{4} m^2 \cos^2[\pi f_{\text{mod}} \tau_M] \quad \text{Michelson.} \quad (8)$$

If  $\tau_M = |\tau_a - \tau_b|/2$ , this result is phase shifted by  $90^\circ$  with respect to the result in Eq.(6).

p7

Contrast Ratio = 0.998  
30dB below bright port!

p8



The shot-noise-limited phase sensitivity of a Sagnac interferometer assuming a homodyne detection scheme at the dark port is given by

$$|\Delta\phi| = \sqrt{\frac{2(I_{LO} + I_{\text{min}})2\pi\hbar f_l}{I_{LO} I_{\text{in}} \eta C^2}} \quad (9)$$

where  $I_{LO}$  is the local oscillator power,  $I_{\text{min}}$  is the leakage power from the dark port,  $\hbar$  is the reduced Planck constant,  $I_{\text{in}}$  is the interferometer input power,  $\eta$  is the quantum efficiency of the photodetector, and  $C$  is the contrast ratio defined as

$$C = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \quad (10)$$

where  $I_{\text{max}}$  is the maximum power out of the bright port. For less-than-unity contrast ratio, the power incident onto the detector is given by  $I_{LO} + I_{\text{min}}$ . A near-unity contrast ratio is desirable in an interferometer for two reasons; first, the phase sensitivity reaches the quantum limit only if the contrast ratio approaches unity; second, the photo detector saturates if illuminated with too much leakage power,  $I_{\text{min}}$ . Therefore, we paid particular attention to the measurement of the contrast ratio and the dark-port leakage power of the Sagnac interferometer.

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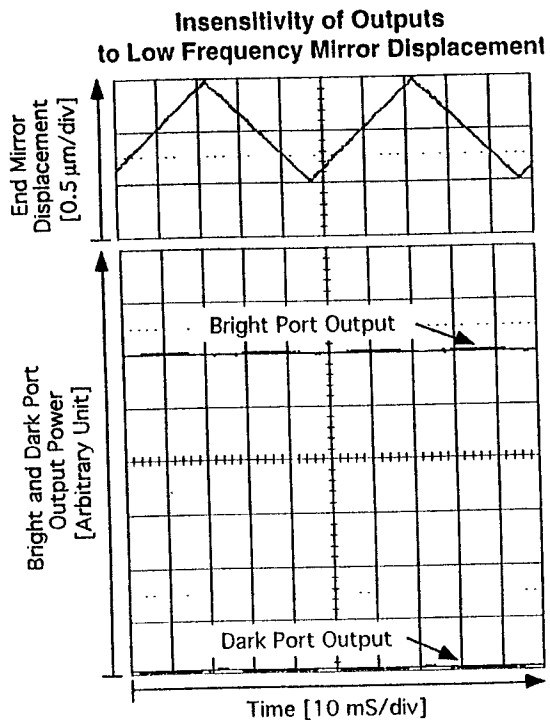


Figure C24 The outputs from the dark and bright ports when the mirror PZT was driven over 1  $\mu\text{m}$  distance, a full wavelength of YAG laser. Both of the outputs stayed nearly constant during the scan, for which a Michelson interferometer would produce two periods of interference fringes. The small ripple was due to PZT tilt. The PZT was driven at 20 Hz, 400 V peak to peak.

We undertook to investigate the contrast ratio as a function of depolarization of the optical beam within the interferometer. This study simulates the impact of thermal induced birefringence in a optical element in a under high power illumination expected for an advanced interferometer. For this experiment we inserted a quarter wave plate into the Sagnac interferometer and measured the dark port power as a function of the angle of the axis of the wave plate. Figure C25 shows the dark fringe contrast ratio vs quarter wave plate orientation angle and also shows the expected contrast ratio for a Michelson interferometer. As is well known, the Michelson interferometer has no fringes for

210 #

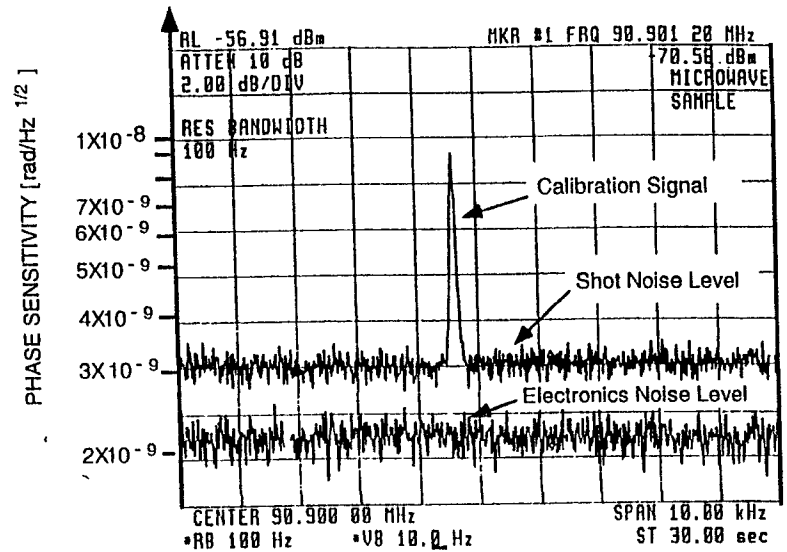


Figure 4

### Stanford Advanced Gravitational-Wave Laser Interferometer Program

conductivity but which are absorbing at 1064 nm. As a final demonstration of the potential for these interferometers, and to attempt to determine limits to the performance of such a system we will make a suspended mass interferometer using a simple design and measure the phase noise as a function of the laser power.

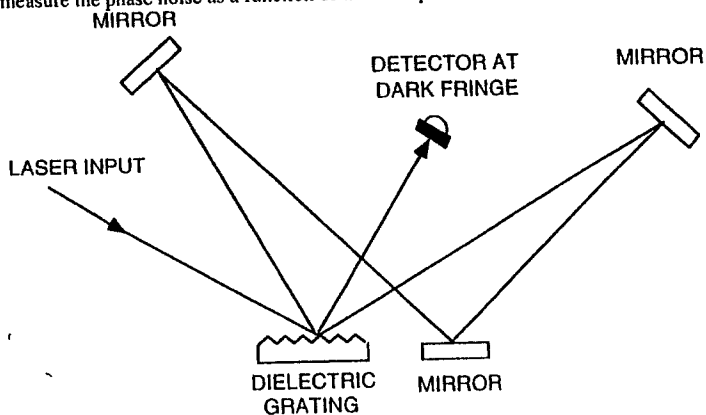


Figure C29. An all reflective Stanford Sagnac Interferometer using a grating beam splitter.

We have consulted with the team at LLNL regarding the design and the fabrication of high reflecting gratings on dielectric substrates. (Boyd 95) The LLNL group has indicated that they wish to cooperate to assist the LIGO effort. Further, they have demonstrated the capability to fabricate gratings using lithographic techniques with dimensions that exceed one meter. Thus this approach is appropriate to consider for an advanced LIGO interferometer. Combined with one dimensional face cooling technology, an all reflective element interferometer can be designed to accommodate kilowatts of laser power with minimum optical distortion and no induced birefringence.

#### iii. Reflective Beamsplitter

#### b. Thermal Noise and Materials Effects

#### i. Thermal Noise and Q in Test Masses and Suspensions

Thermal problems are of two types. The first type results from thermally-induced figure distortions in the optical components that produce spatial variations in the phase fronts, and also thermally-induced stresses that produce spatial variations in the polarizations of the optical beams which reduce the fringe contrast. The second type of problem is thermal noise in the test masses and test mass suspensions, the spectral densities of which

### Stanford Advanced Gravitational Wave Interferometry Program

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**Principal Investigator**  
Robert L. Byer  
Ginzton Laboratory  
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94305-4085  
tel (415) 723 0226  
fax (415) 723 2666

### Summary

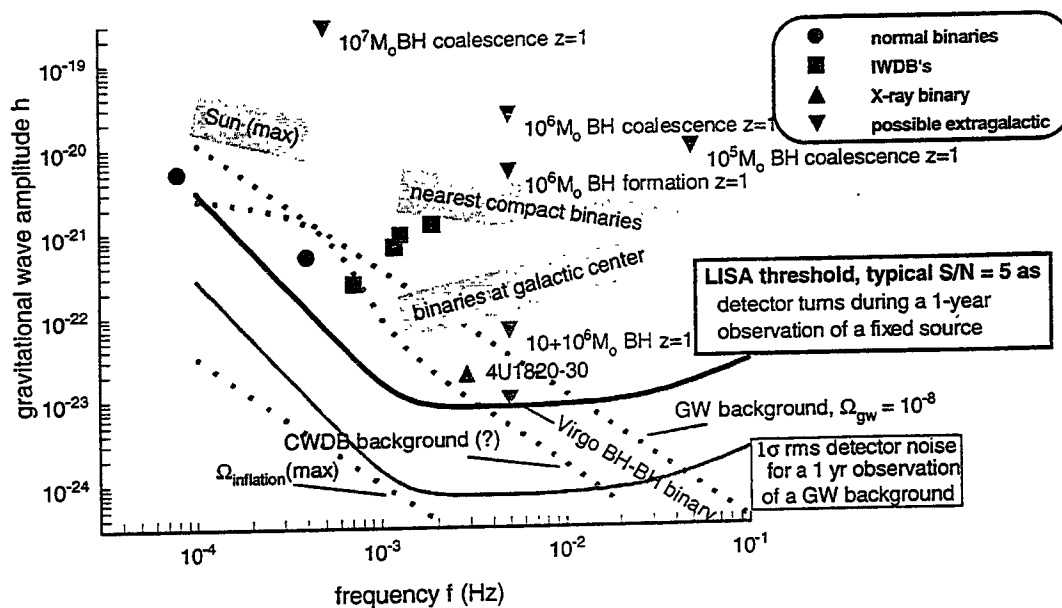
Next steps in the laboratory

The 10 meter Interferometer

Tuesday AM      Space Based Detectors-LISA I  
January 16      Chair: A. Ruediger

8:00	P. Bender (JILA)	Sources for LISA
8:25	Discussion	
8:35	H. Ward (Glasgow)	LISA Overview
9:00	Discussion	
9:10	Coffee Break	
9:25	H. Ward (Glasgow)	LISA Overview (continued)
9:50	Discussion	
10:00	D. Hils (JILA)	BH-SMBH Binaries
10:25	Discussion	

## LISA Sensitivity



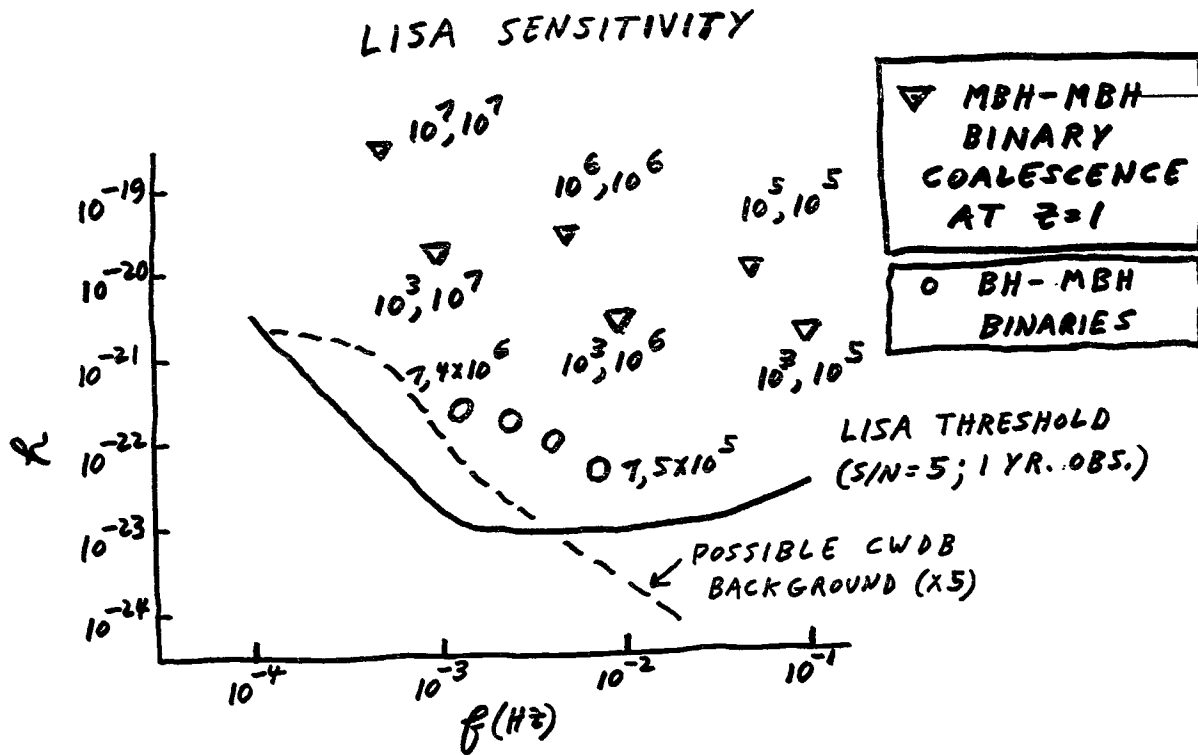
## Extragalactic Astronomy

**Objective:** To detect and study gravitational wave signals from sources involving massive black holes (MBHs) with masses of  $10^3$  to  $10^8 M_{\odot}$ .

- 5 or 10  $M_{\odot}$  black holes (BHs) or compact stars orbiting  $10^5$  to  $10^7 M_{\odot}$  MBHs.
- MBH-MBH binaries resulting from formation of multiple MBHs in many galaxies plus their sinking to the center by dynamical friction.
- MBH-MBH binaries formed by mergers of galaxies or pre-galactic structures that already contain MBHs.
- MBH-MBH binaries formed by growth of several seed BHs in the same galactic nucleus.
- Sudden formation of MBHs.

It appears plausible, or even likely, that at least one of these types of binaries can be detected and studied by LISA.

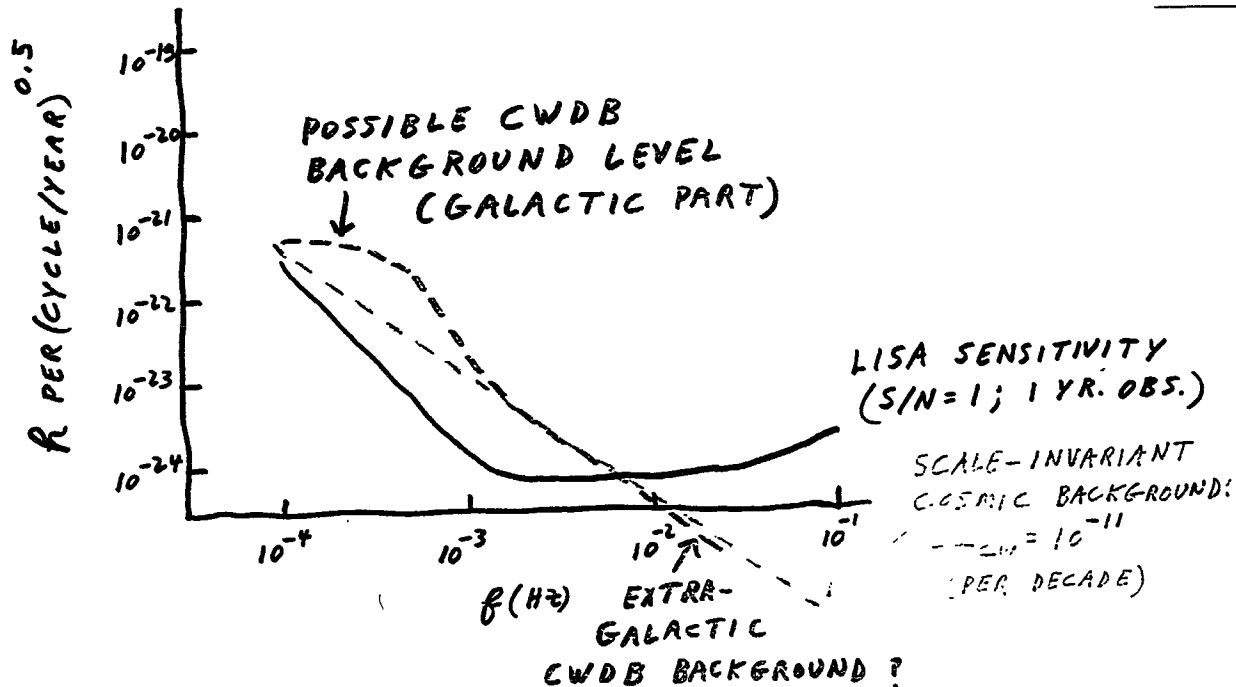




**Objective:** To test general relativity in the high field limit, and to search for cosmological information on gravitational background radiation and possibly on the expansion parameter  $q_0$ .

- If BH-MBH or MBH-MBH coalescence signals are seen, they will provide a unique test of general relativity at extremely high fields. Even slight deviations from the dynamical predictions of the theory would be detectable.
- Cosmological background radiation could be detected near 10 mHz with a sensitivity of  $10^{-10}$  to  $10^{-11}$  of the closure density.
- If several MBH-MBH coalescences with both masses  $\geq 10^5 M_\odot$  occur, the S/N will be high even at cosmological distances, and very accurate luminosity distances can be obtained. If the event directions are measured and can be associated with clusters of galaxies with known redshifts, it appears possible to determine the cosmological expansion parameter  $q_0$ .

## BACKGROUND SIGNALS + LISA SENSITIVITY

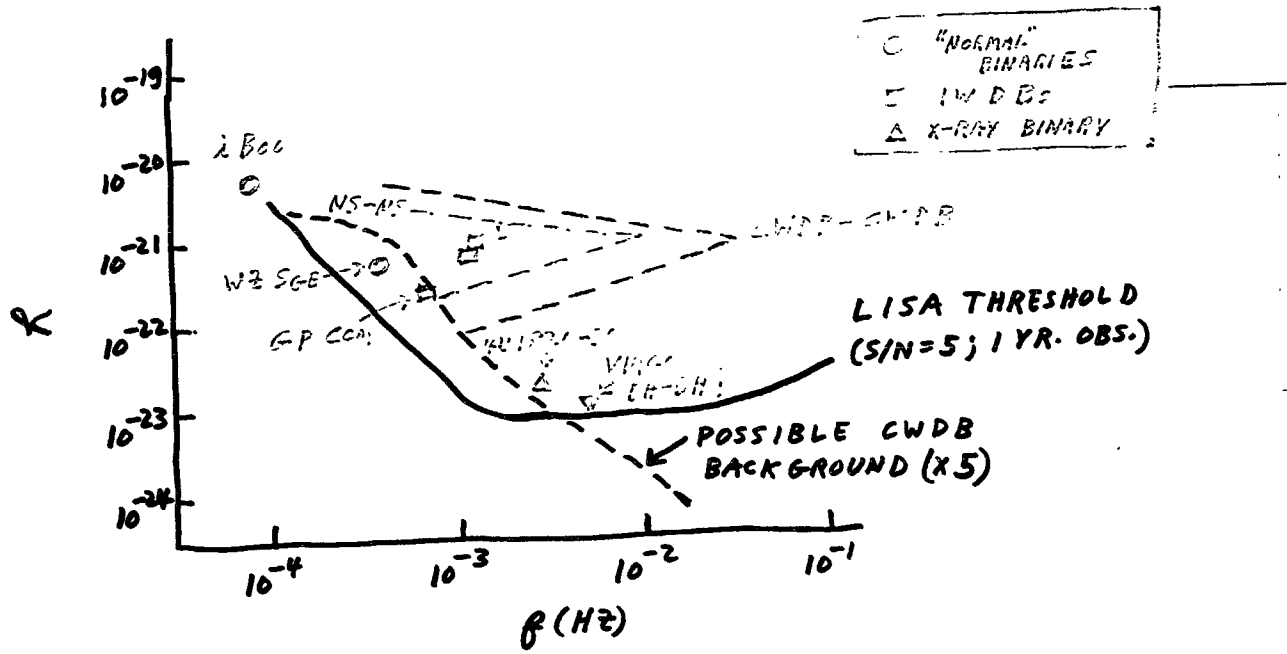


### Galactic Astronomy

**Objective:** To investigate the number and distribution of different types of short period binaries in our galaxy.

- Neutron star binaries: hundreds will be detectable and their locations can be determined throughout the galaxy.
- Close white dwarf binaries (CWDBs): thousands above about 1 mHz are expected to be resolvable.
- Known sources: a few will be detectable, including Boo (normal binary), WZ Sge (cataclysmic variable) and 4U1820-30 (x-ray binary).
- Interacting white dwarf binaries (IWDBs): many will be observable, GP Com is known, and several others are known except for possible frequency ambiguities.
- Other compact binaries: BH-neutron star and BH-BH binaries are expected to be detectable.
- CWDB background: below about 1 mHz, the very large number of CWDBs will give a confusion-limited background.

# LISA SENSITIVITY



---

# LISA

## Laser Interferometer Space Antenna

– a gravitational wave detector  
for low frequency signals

H. Ward, Aspen, January 1996

HW 2/96

1

---

## LISA Science Team

---

- ◆ P. Bender, JILA
- ◆ K. Danzmann, Uni Hannover
- ◆ J. Hough, Uni Glasgow
- ◆ A. Rudiger, MPQ
- ◆ R. Schilling, MPQ
- ◆ R. Stebbins, JILA
- ◆ P. Touboul, ONERA
- ◆ W. Winkler, MPQ
- ◆ I. Ciufolini, CNR
- ◆ W.M. Folkner, JPL
- ◆ D. Robertson, Uni Glasgow
- ◆ M. Sandford, RAL
- ◆ B. Schutz, AEI
- ◆ T.J. Sumner, Imperial Coll.
- ◆ H. Ward, Uni Glasgow
- ◆ J. Cornelisse, Study Manager, ESTEC
- ◆ Y. Jafry, Study Scientist, ESTEC

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2

# Spaceborne Detectors

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- ◆ Space-based detectors are sensitive in the low-frequency range
  - ◆ long baseline
  - ◆ no gravity-gradient wall
- ◆ Two-arm interferometers are much more sensitive than spacecraft tracking

---

## Complementary to other Detectors

---

- ◆ Ground-based (VIRGO / LIGO / GEO) “blind” below 1 Hz
- ◆ Space-based frequency range extends down to  $10^{-4}$  Hz
- ◆ LISA detects local precursors of stellar coalescing binaries
- ◆ LISA + ground-based detectors could provide spectrum of cosmological background

# Measurement Technique

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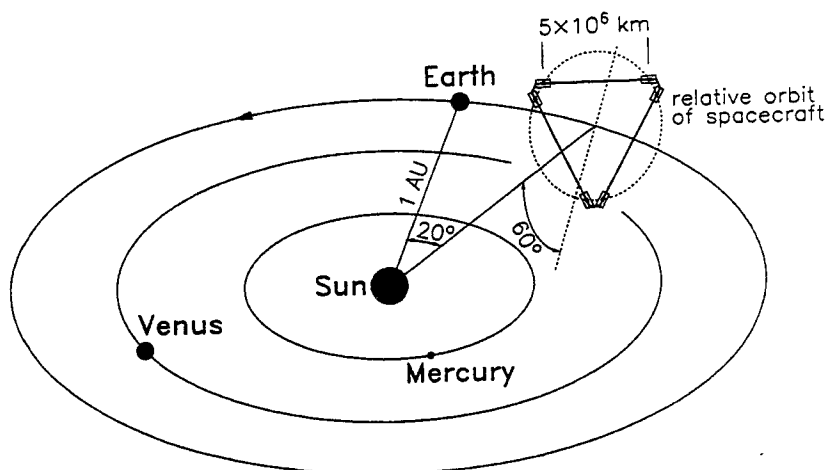
- ◆ Laser interferometry between inertial masses
- ◆ Long baseline :  $5 \times 10^9$  m
- ◆ High measurement sensitivity :
  - ◆  $\delta x < 25 \text{ pm} / \sqrt{\text{Hz}}$  from  $10^{-3}$  to  $10^{-1}$  Hz
  - ◆  $\delta a < 10^{-15} \text{ ms}^{-2} / \sqrt{\text{Hz}}$  from  $10^{-4}$  to  $10^{-3}$  Hz

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# Spacecraft Orbits

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HW 2/96

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# LISA – Interferometry

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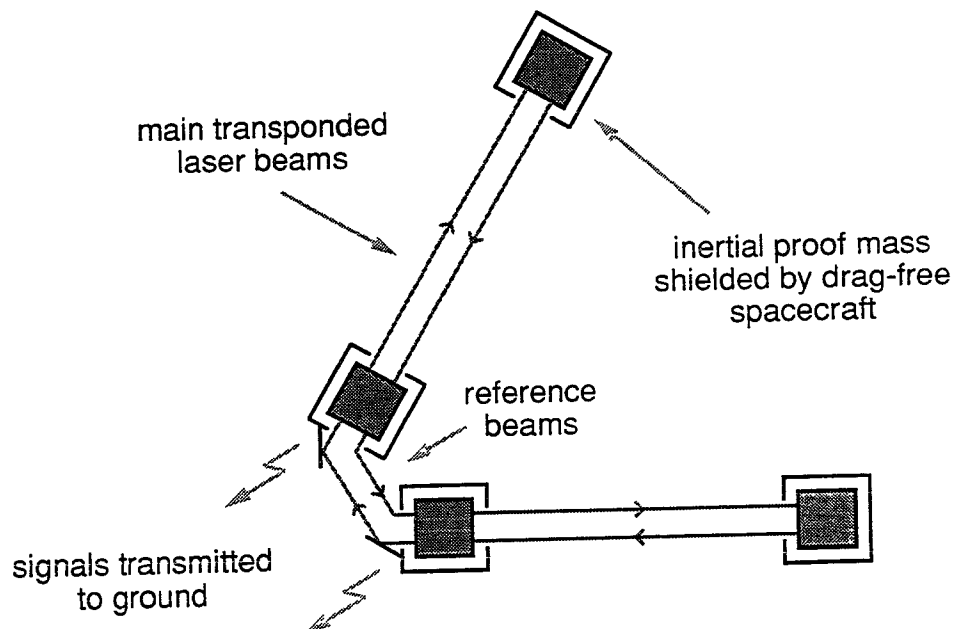
- ◆ Effectively a Michelson interferometer
- ◆ Phase-locked laser transponders at arm ends
- ◆ Lasers in central spacecraft phase-locked

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## Outline Layout of LISA ( 2 arms )

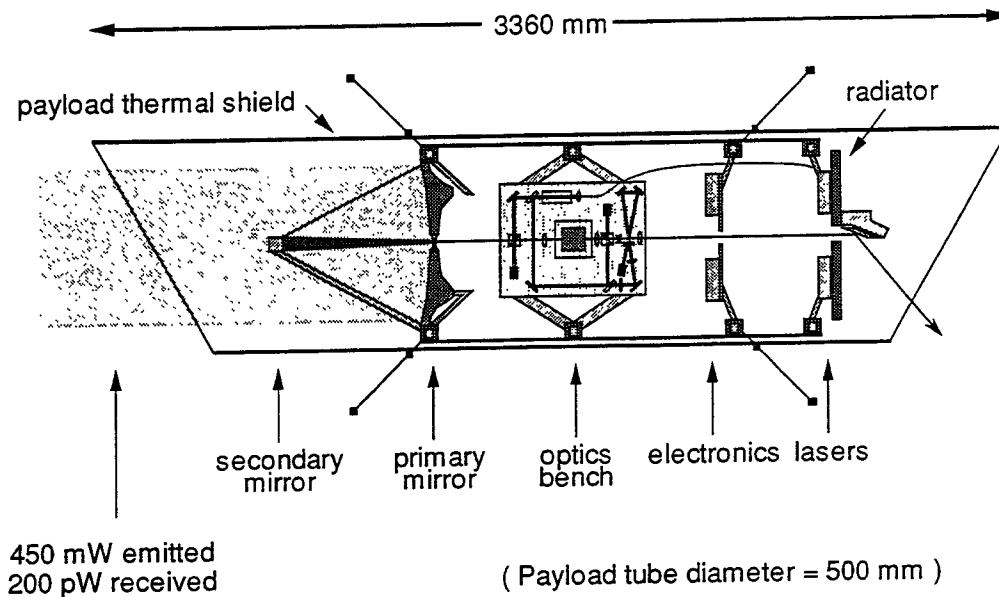
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# LISA Science Payload



HW 2/96

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## Drag - free Control – 1

### ◆ Objective

- ◆ To obtain at the centre of each spacecraft a proof-mass free of spurious accelerations in order to define the interferometer arms

### ◆ Concepts

- ◆ Accelerometer based on a cubic proof-mass (highly accurate geometry, 1.3 kg of Au-Pt alloy)
- ◆ Capacitive position sensing of the 6 degrees of freedom
- ◆ Electrostatic servo control of the proof-mass position and attitude with respect to the ULE accelerometer cage
- ◆ Spacecraft driven by FEEP thrusters to nullify the electrostatic feedback forces on the proof-mass

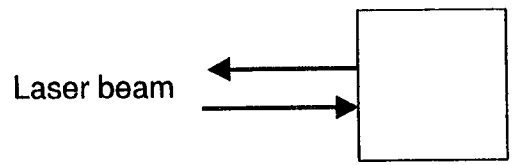
HW 2/96

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# SENSOR PERFORMANCE OBJECTIVES

## ● PROOF-MASS POSITION SENSING :

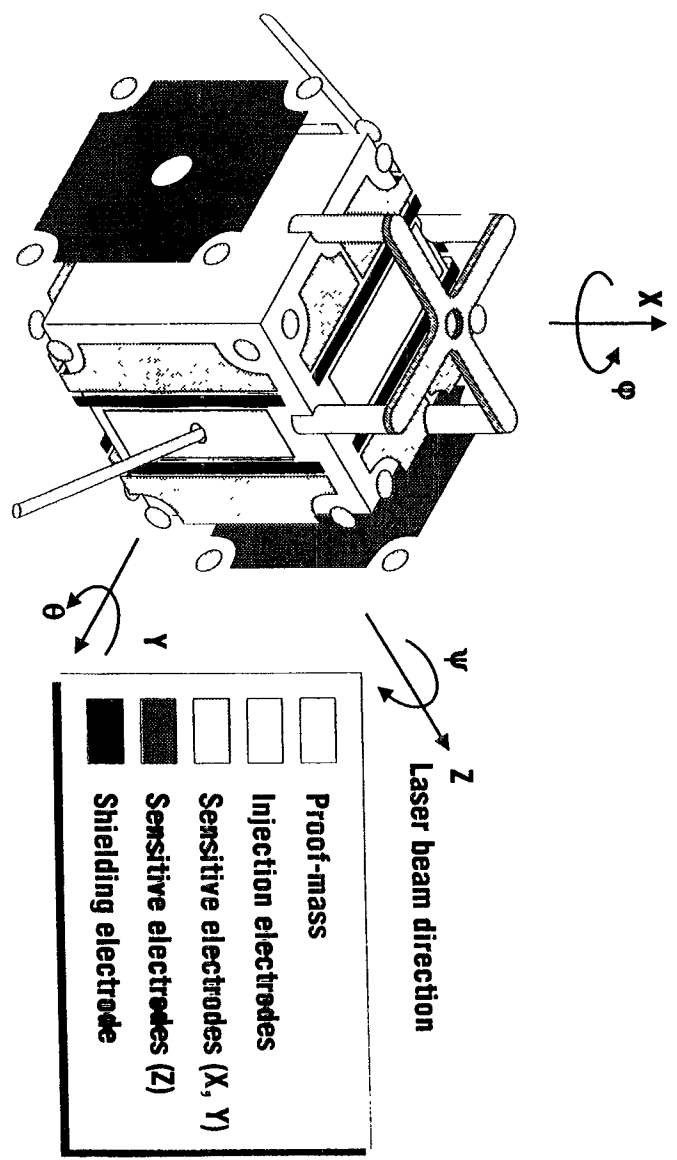


Since the laser beam is reflected on the proof-mass :

- a resolution of  $10^{-9} \text{ m}/\sqrt{\text{Hz}}$  is sufficient to limit the variations of proof-mass disturbances (S/C gravity gradient in particular).
- but the electrostatic stiffness induced by the sensing must be very weak ( $<10^{-6} \text{ N/m}$ ).

## ● PROOF-MASS FREE MOTION :

- All accelerometric disturbances must be less than  $10^{-15} \text{ ms}^{-2}/\sqrt{\text{Hz}}$  in  $[10^{-4} \text{ Hz}; 5 \times 10^3 \text{ Hz}]$ 
  - ➔ optimization of geometrical and electrical configurations
  - ➔ high thermal stability.
- Sensor output kept at null
  - ➔ no stringent requirement on scale factor accuracy, stability and linearity.



# LISA PROOF-MASS ASSEMBLY

## PROOF-MASS DISTURBANCES DUE TO ENVIRONMENT (cn'd)

### ■ PROOF-MASS CHARGE Q :

variations due to flux of particles from cosmic rays and solar flares

evaluated to  $\sim 10^{-17}$  C/s -  $10^{-18}$  C/s

### ■ DISTURBANCES :

due to ● Lorentz force  
● electrical forces when asymmetry of the geometry

→  $Q < 3 \times 10^{-14}$  C      corresponding to 1 mV

$$\delta Q < 10^{-12} \text{ C} / \sqrt{\text{Hz}}$$

Q to be deduced from the measurement of the effect of a sine wave exciting voltage on the electrostatic suspension.

Q to be controlled by use of U.V. lights.

## PROOF-MASS DISTURBANCES DUE TO ENVIRONMENT

### ■ MAGNETIC :

- Lorentz force :  
→ control of the proof-mass charge
- Magnetic moment of the proof-mass  
→ Au Pt alloy :  $\chi = 10^{-6}$

### ■ RESIDUAL GAS :

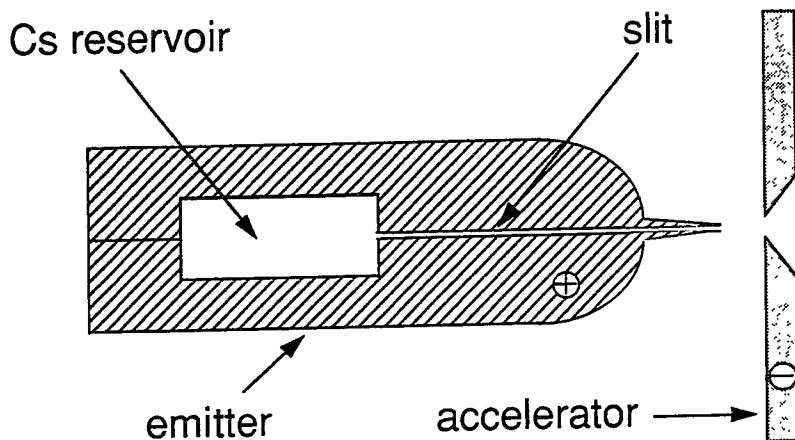
- radiometer effect
- damping
- acoustic transfer
- statistical fluctuations  
→  $P_0 < 10^{-6}$  Pa  
→ internal core opened to space vacuum  
use of low outgassing parts and baked sensor core

### ■ RADIATION PRESSURE

### ■ GEOMETRICAL STABILITY :

- U.L.E. accelerometer cage
- suspended proof-mass  
→ expected thermal stability better than  
 $10^{-6}$  K /  $\sqrt{\text{Hz}}$

# FEEP Thrusters



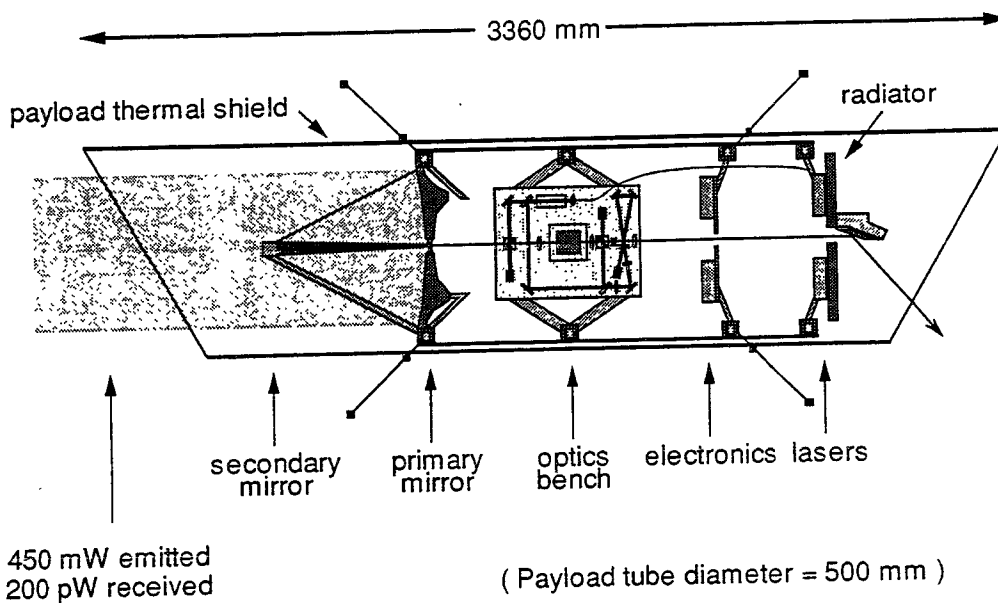
Force range : 1 – 25  $\mu\text{N}$

Response time : 10 – 30 ms

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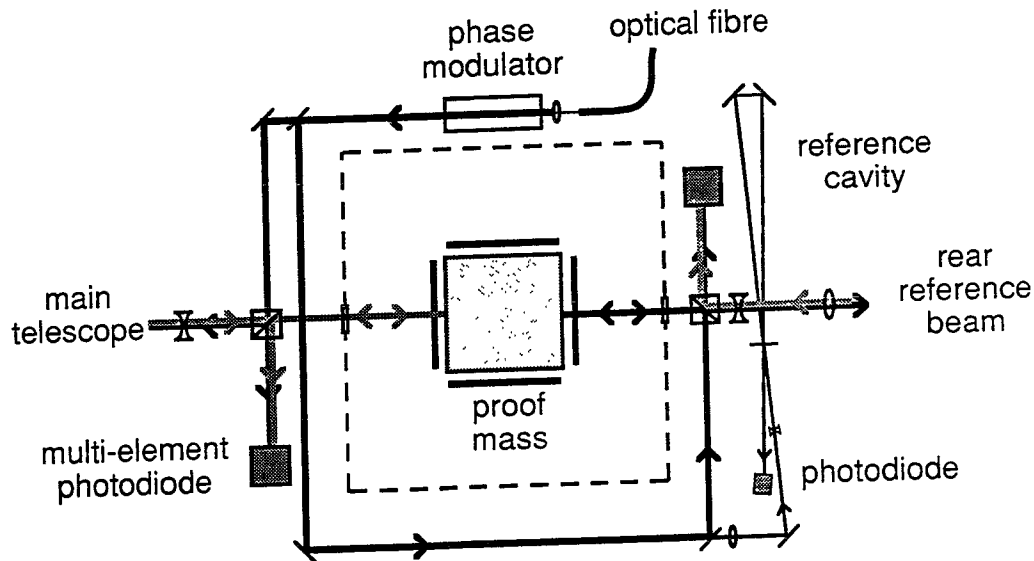
# LISA Science Payload



HW 2/96

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# Optical Layout



HW 2/96

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# Photon Noise

- ◆ Want photo-electron shot noise to contribute a measurement noise less than  $5 \text{ pm} / \sqrt{\text{Hz}}$  for a single traverse of one arm
- ◆ For a given laser power and arm length this requirement sets the mirror diameter needed
- ◆ For  $1.06 \mu\text{m}$  light, effective power  $300 \text{ mW}$ , and an arm length of  $5 \times 10^9 \text{ m}$ , a telescope output mirror diameter of  $38 \text{ cm}$  is required

HW 2/96

18

# Lasers for LISA

---

- ◆ High output power ( $\sim 1$  W)
  - ◆ Low noise
  - ◆ Good reliability
  - ◆ High efficiency
  - ◆ Compact size
- ➔ Diode pumped Nd:YAG lasers

HW 2/96

19

# Laser Power Fluctuations

---

- ◆ Power fluctuations cause acceleration noise since light is reflected from the proof mass
- ◆ Require any component of spurious acceleration noise to be  $< 10^{-16} \text{ m s}^{-2}$
- ◆ For proof mass of 1.3kg and 10mW reflected power this sets a tolerable limit to laser power fluctuations of

$$\delta P/P < 2 \times 10^{-6} / \sqrt{\text{Hz}}$$

HW 2/96

20

## Laser Frequency Noise

---

- ◆ Couples in through difference ( $\Delta x$ ) in arm length
- ◆ Fractional laser frequency noise ( $\delta v/v$ ) translates to apparent displacement noise  $\delta x = \Delta x \cdot (\delta v/v)$ 
  - ◆ For arm length  $5 \times 10^9$  m,  $\Delta x$  could be  $10^9$  m, so for  $\delta x < 2$  pm  $\sqrt{\text{Hz}}$  we need  $\delta v < 6$   $\mu\text{Hz} \sqrt{\text{Hz}}$
- ◆ Stabilisation to a ULE cavity with thermal stability  $\Delta T < 6$   $\mu\text{Hz} \sqrt{\text{Hz}}$  could only give  $\delta v < 10$   $\text{Hz} \sqrt{\text{Hz}}$ 
  - ◆ Not good enough, so further correction algorithm needed

HW 2/96

21

## Frequency Correction Algorithm

---

- ◆ Provided by comparing the mean phase of the light returning in two adjacent arms with the phase of the transmitted light
  - ◆ Since  $\phi = (4\pi/c) \cdot v \cdot L$ ,  $\delta\phi = (4\pi/c)(v \cdot \delta L + L \cdot \delta v)$  and since  $\delta L$  is small,  $\delta\phi \sim (4\pi/c) \cdot L \cdot \delta v$ . So if  $\delta\phi$  is measured,  $\delta v$  can be calculated
  - ◆ Now the gravity wave signal is found from the phase difference between adjacent arms; the measured phase difference can now be corrected for the effect of  $\delta v$

HW 2/96

22

## Doppler Effect – 1

---

- ◆ Relative velocity between craft leads to Doppler beat signals in the phase comparisons
- ◆ Possible velocity of 8 m/s gives a beat in one return arm of 16 MHz
- ◆ Need to reduce this to manageable rate (few Hz) for later signal processing and transmission
  - ◆ Done by beating with a stable on-board reference clock
  - ◆ Clock noise must allow measurement precision of  $\delta x < 2 \text{ pm } / \sqrt{\text{Hz}}$  or  $\delta \phi < 2.5 \times 10^{-5} \text{ rad } / \sqrt{\text{Hz}}$

HW 2/96

23

## Doppler Effect – 2

---

- ◆ Required frequency stability of clock = measurement frequency / clock frequency \* measurement phase noise limit
  - ◆ Thus for observations at  $10^{-3} \text{ Hz}$  and a beating frequency of 16 MHz, a clock frequency stability of  $1.6 \times 10^{-15} / \sqrt{\text{Hz}}$  is required
  - ◆ This is equivalent to an Allan standard deviation of  $\sigma_{\text{Allan}} \sim 4 \times 10^{-17}$  over 1000 second timescales
  - ◆ Typical USO has  $\sigma_{\text{Allan}} \sim 2 \times 10^{-13}$  so clock stabilisation technique is required

HW 2/96

24

## Doppler Effect – 3

---

- ◆ Stabilisation of clock frequency done analogously to the scheme for laser frequency stabilisation – essentially by using the stable arm length as a reference against which to measure the frequency
  - ◆ Clock signal is phase modulated on to the laser light at a high modulation frequency ( $\sim 1$  GHz)
  - ◆ Carrier and sideband signals monitored and their frequency fluctuations deduced from knowledge of approximate common mode and differential arm lengths

HW 2/96

25

## Spacecraft – 1

---

- ◆ Drag-free
  - ◆  $10^{-15} \text{ m s}^{-2}$  (rms) in the band  $10^{-4}$  to  $10^{-1}$  Hz using Caesium FEEP thrusters for control
- ◆ Pointing
  - ◆ few nrad/ $\sqrt{\text{Hz}}$  in above band
- ◆ Payload (each spacecraft)
  - ◆ mass : 67 kg
  - ◆ power : 48 W
  - ◆ size : cylinder 0.5 m diameter by 3.3 m long

HW 2/96

26



## Spacecraft – 2

---

- ◆ Spacecraft - 3 axis stabilised (6 S/C in total)
  - ◆ Mass : 290 kg each spacecraft in orbit
  - ◆ Power : 183 W each spacecraft in orbit
- ◆ Propulsion module
  - ◆ Mass : 216 kg, 2 spacecraft per module
  - ◆ Propellant : 240 – 920 kg, depending on launch date, for 2 spacecraft
- ◆ Total launch mass : 6200 kg

HW 2/96

27

## Spacecraft – 3

---

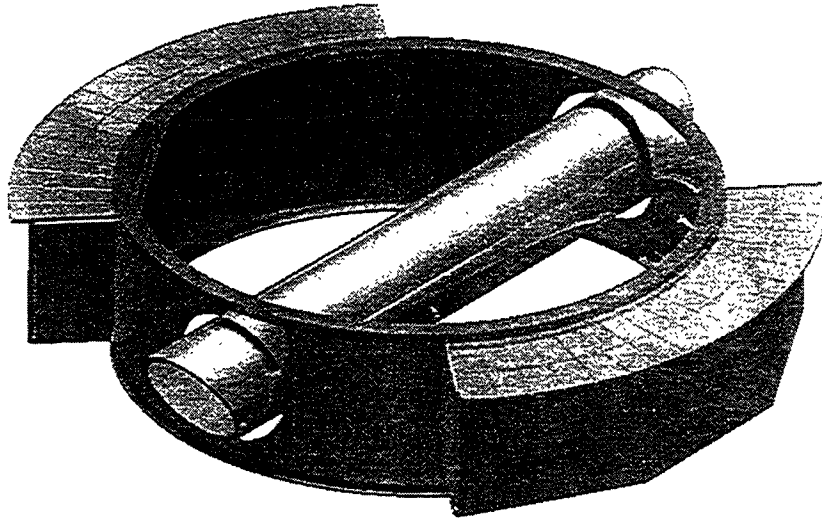
- ◆ Telemetry
  - ◆ 560 bps continuous total
  - ◆ Ground stations : Villafranca and Perth
- ◆ Mission life
  - ◆ Specification : 2 years
  - ◆ Feasible life : 3 to 10 years

HW 2/96

28

# Spacecraft with Payload

---

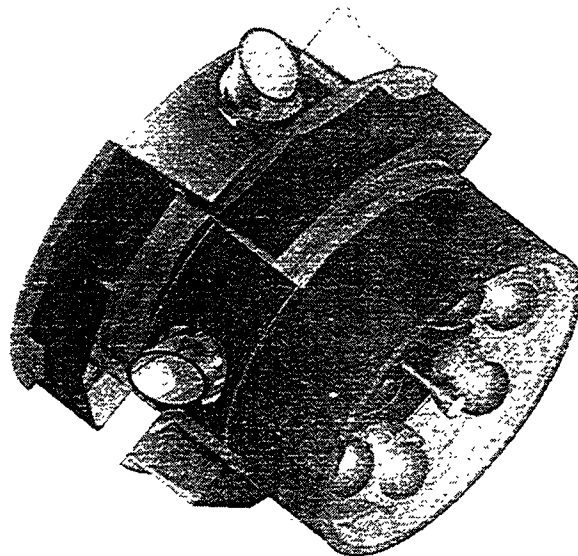


HW 2/96

29

# Propulsion Module + 2 Spacecraft

---



HW 2/96

30

# Launcher

---

- ◆ ARIANE 5, dual launch configuration
  - ◆ two pairs of spacecraft in the lower compartment
  - ◆ one pair in the upper under the short fairing
- ◆ Each pair of spacecraft has its own jettisonable propulsion module
  - ◆ provides a  $\Delta V$  of 1000 m/s for final orbit injection



HW 2/96

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# Conclusion

---

- ◆ LISA will probe the gravitational wave spectrum in a region not accessible to ground-based detectors
- ◆ LISA is complementary to the ground-based experiments and should detect a range of exciting sources
- ◆ LISA's technology is demanding but no breakthroughs are required

HW 2/96

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## BH-MBH BINARIES

P.L. BENDER + D. HILS

LISA IS EXPECTED TO SEE MANY

'GUARANTIED SOURCES'

- NOT ALL ARE AP EXCITING
- COMPACT STAR - MBH BINARIES ARE NOT CERTAIN TO EXIST
- BUT OF GREAT INTEREST TO AP+P

COMPACT  $\star$ : WD OR NS

$$m_c = 1.4 M_{\odot}$$

RECENTLY

COMPACT STAR: BH

$$\underline{m_c = 7 M_{\odot}}$$

ADVANTAGE: LARGER SIGNAL

WORRY: ARE THERE ENOUGH BH'S

DIFFERENCE:  $m_c \gg m_f = M_{\odot}$

MASS SEGREGATION

①

NEED NEAR RADIAL ORBITS

$$\Delta E_{GR} \quad \Delta L_{GR}$$

②

EFFECT OF CLUSTER FIELD  $\star$ 'S:

- DISPERSION  $\Delta L_{RMS}$
- FATAL IF  $L < L_{crit}$  AT PERICENTER
- SHORT DURATION ORBITS

MORE TIGHTLY BOUND  $\star$

HAS SMALLER  $\Delta L_{RMS}$

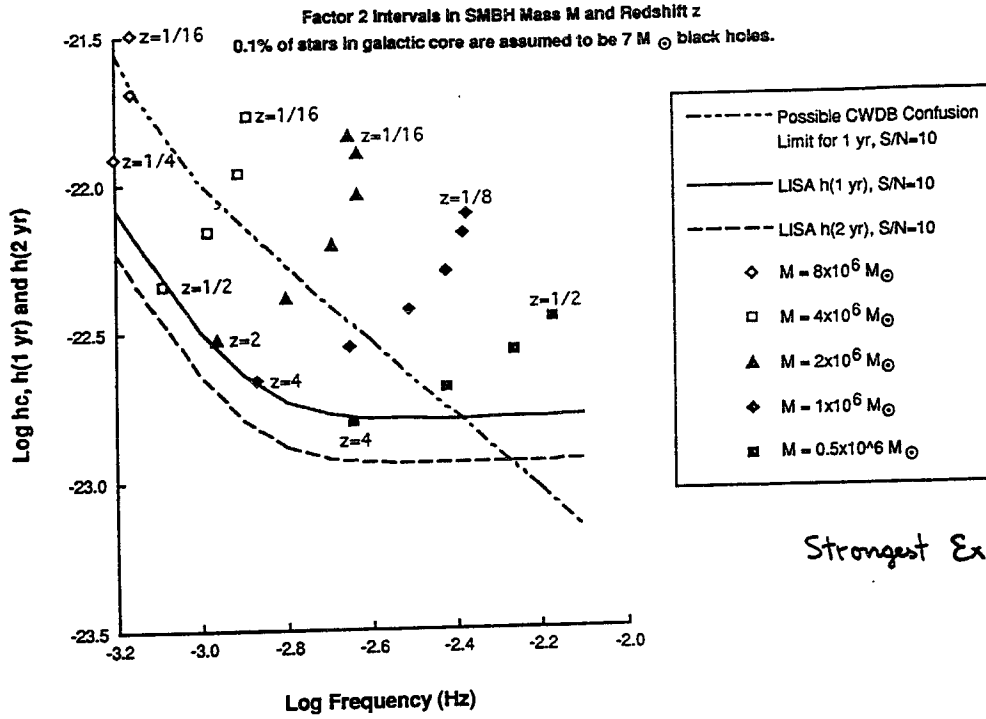
BETTER CHANCE TO RADIATE

ENERGY GRADUALLY BY GR

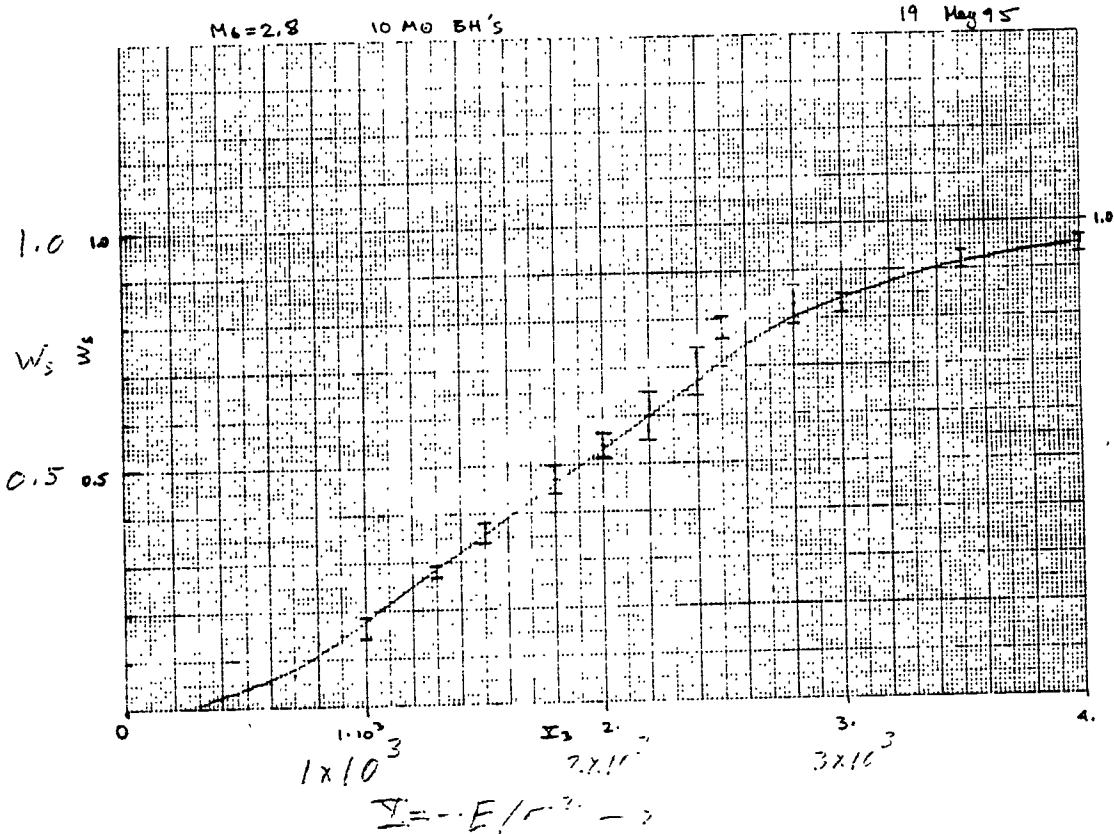
$\Delta E$ : FEW %

LISA CANDIDATE

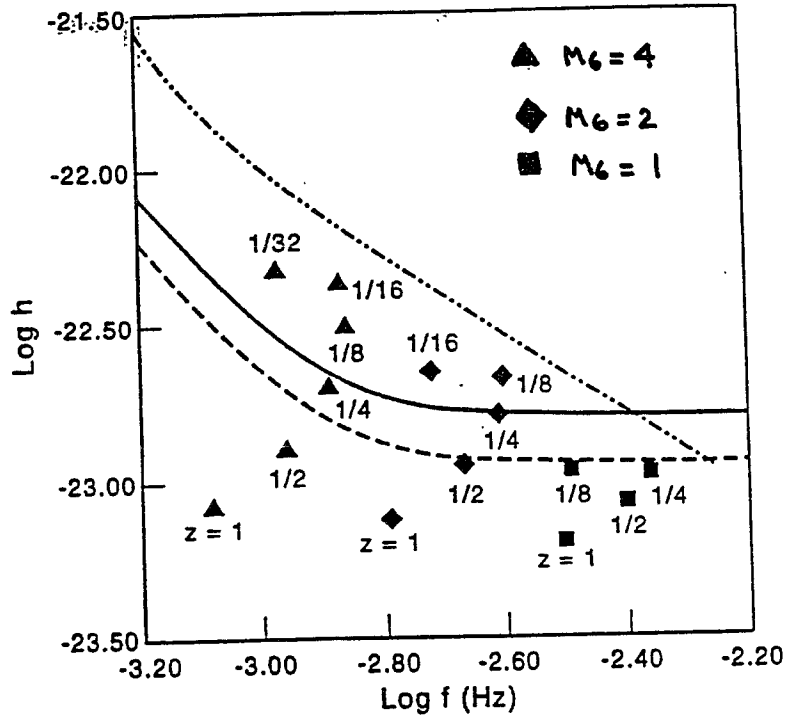
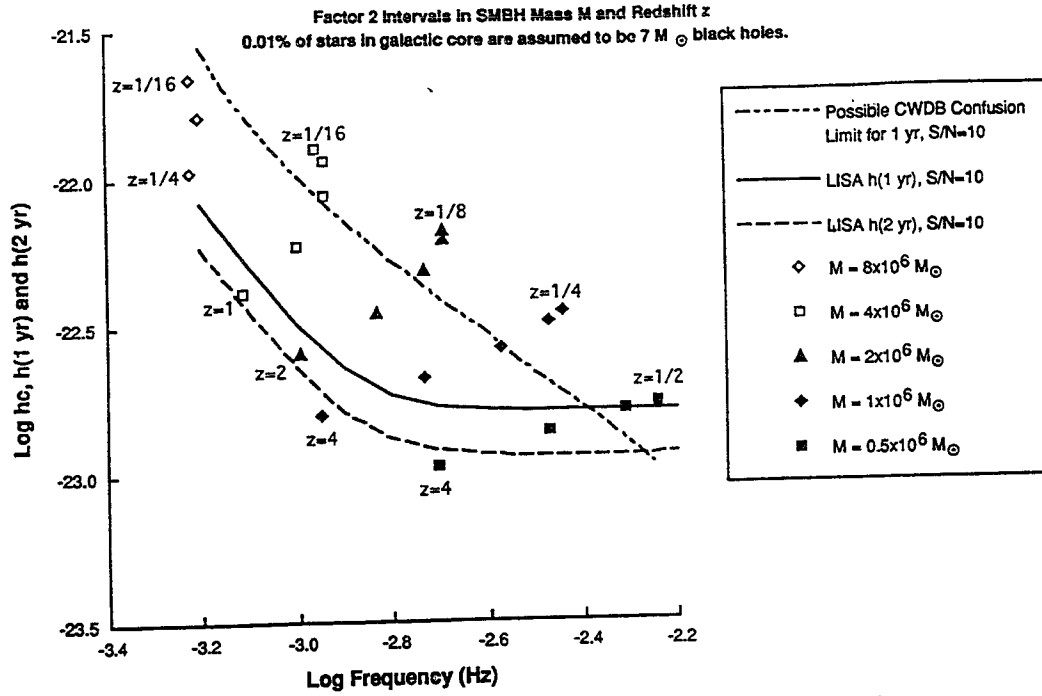
### Expected Signals from BH-SMBH Binaries



Strongest Expected Sources



Expected Signals from BH-SMBH Binaries



Hills + Bender: ApJ 445, L7 (1995)

Figure 1

10% compact stars  
 $m_c = 1.4 M_{\odot}$

Strongest Expected Sources

0.1 % of stars in core are in  $7 M_{\odot}$  BH's

Estimated BH-MBH Coalescence  
Event Rate  
(per year)

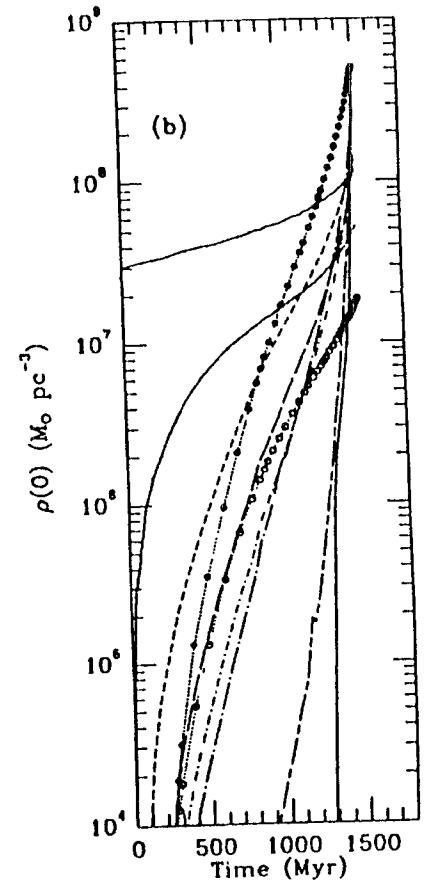
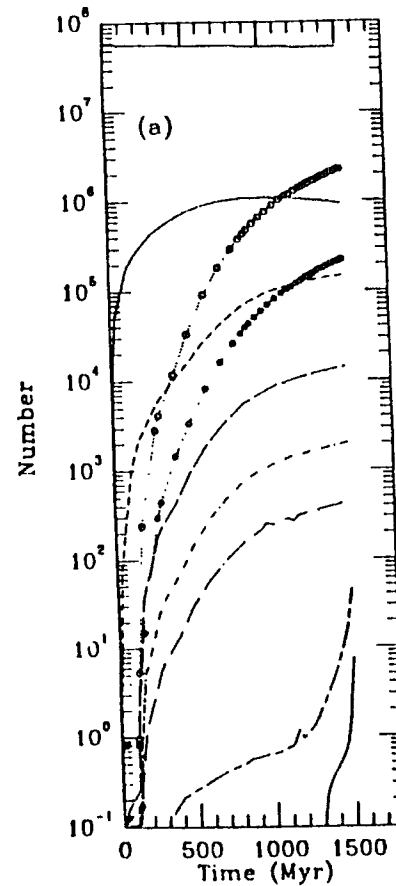
Z	$M_6$				
	0.5	1	2	4	8
1/16	.010	.014	.024	.040	.065
1/8	.056	.093	.16	.26	.42
1/4	.30	.50	.83	1.4	2.2
1/2	1.2	2.0	3.3	5.4	8.8
1	3.0	5.0	8.3	14	22
2	4.1	6.8	11	19	30

Observable Duration of BH-MBH  
Coalescence Events for LISA  
(years)

Z	$M_6$				
	0.5	1	2	4	8
1/16	18	71	280	590	390
1/8	10	39	150	260	65
1/4	5.2	21	73	82	3.0
1/2	2.5	10	30	11	
1	1.1	4.5	6.0	0	
2	0	1.7	0		

Estimated Number of Events Above LISA  
Instrumental Threshold  
(S/N = 10; 1 year)

Z	M <sub>6</sub>				
	0.5	1	2	4	8
1/16	0.2	1.0	7	24	26
1/ 8	0.6	3.6	23	67	27
1/4	1.5	10	60	110	7
1/2	2.9	22	97	61	
1	3.2	20	49	0	
2	0	7	0		



Quinlan + Shafer 1990, <sup>A.P.T.</sup> 356, 483

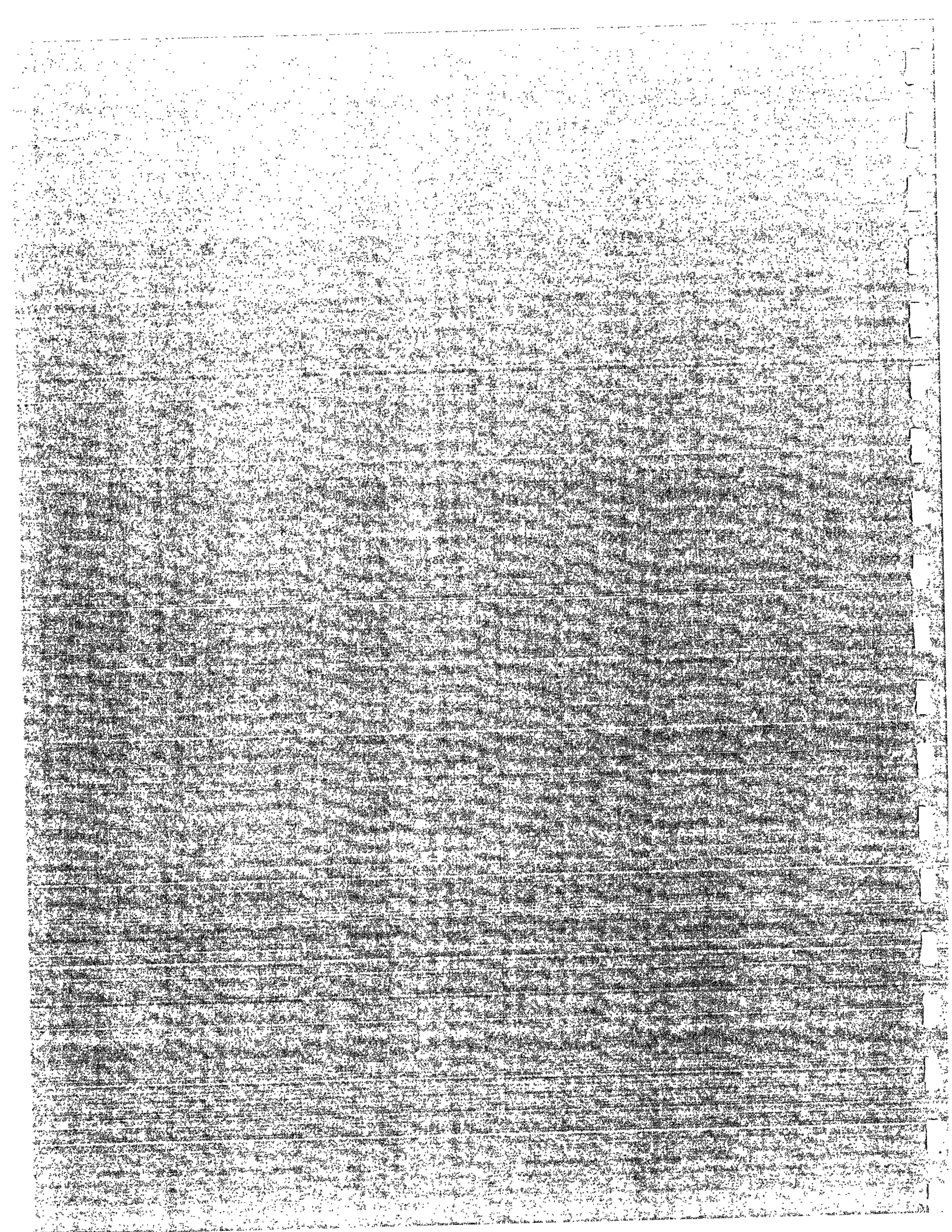




Tuesday PM      Space Based Detectors-LISA II

January 16      Chair: R. Schilling

4:30	B. Willke (Hannover)	Lasers for LISA
5:00	Discussion	
5:10	D. Robertson (Glasgow)	Spacecraft Pointing
5:40	Discussion	
5:50	R. Stebbins (JILA)	LISA Error Budgets
6:20	Discussion	
6:30	Coffee Break	
6:45	D. Robertson (Glasgow)	GEO 600 Developments
7:15	Discussion	
7:25	D. Schnier (Hannover)	Power Recycling
7:40	Discussion	

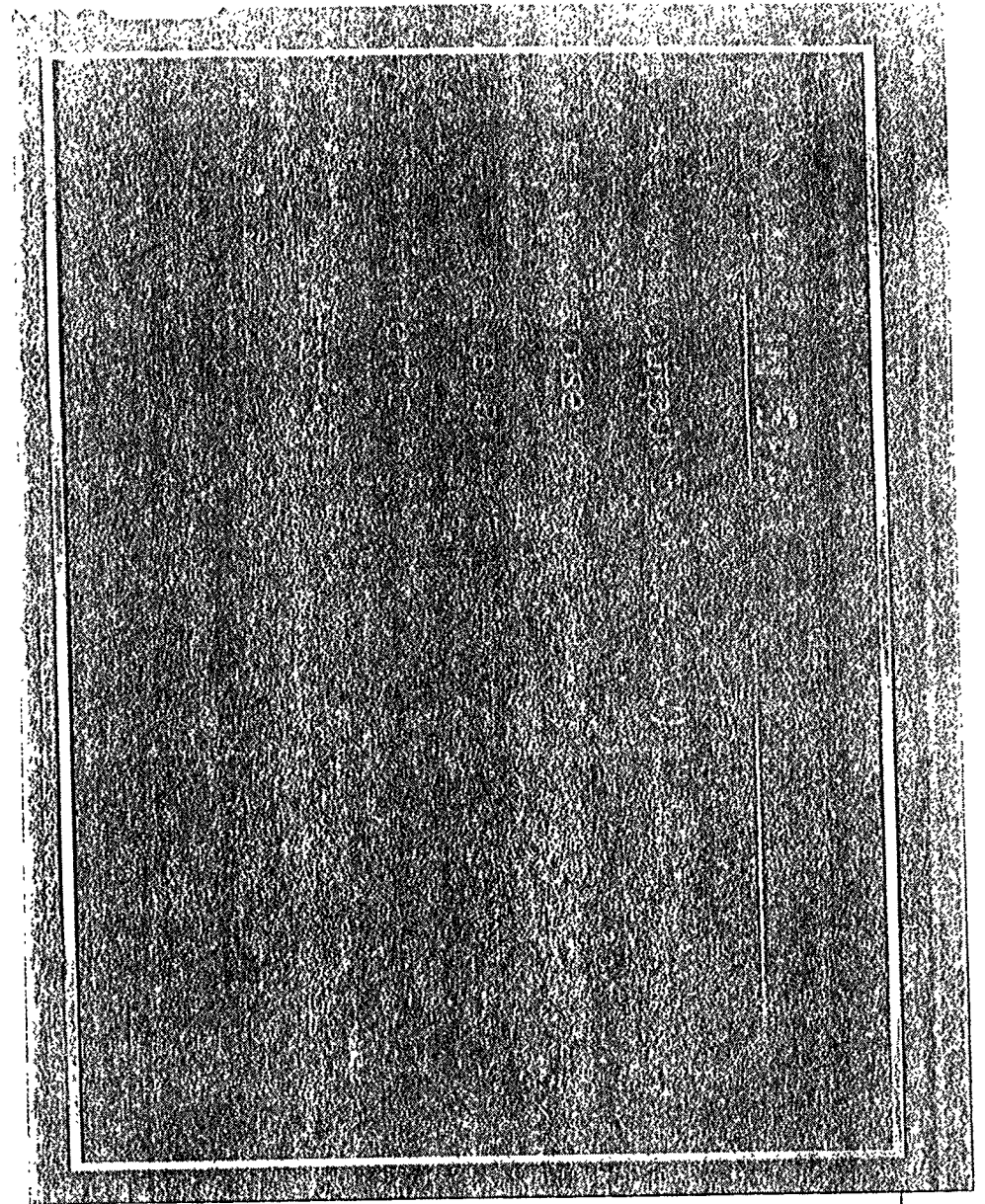


# Lasers for LISA

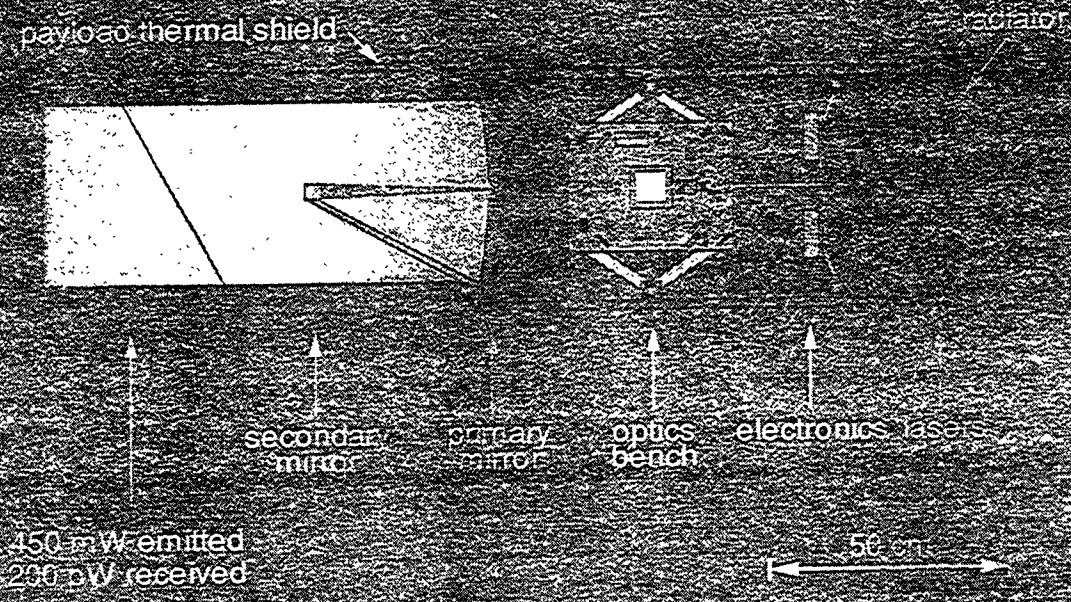
Benno Willke  
University of Hanover

results of the LISA study team  
reported in the Pre-Phase A Report

Aspen Winter Conference 96

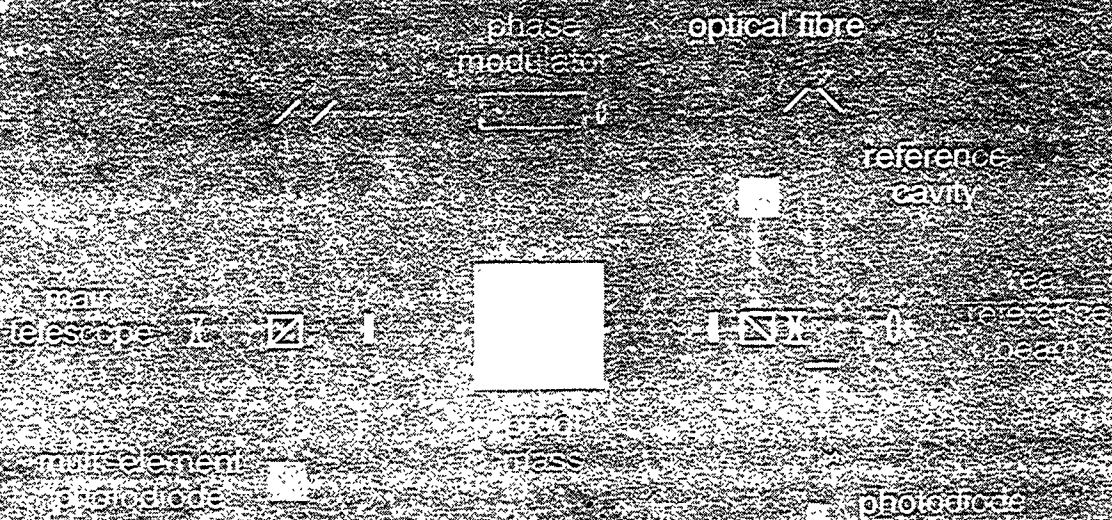


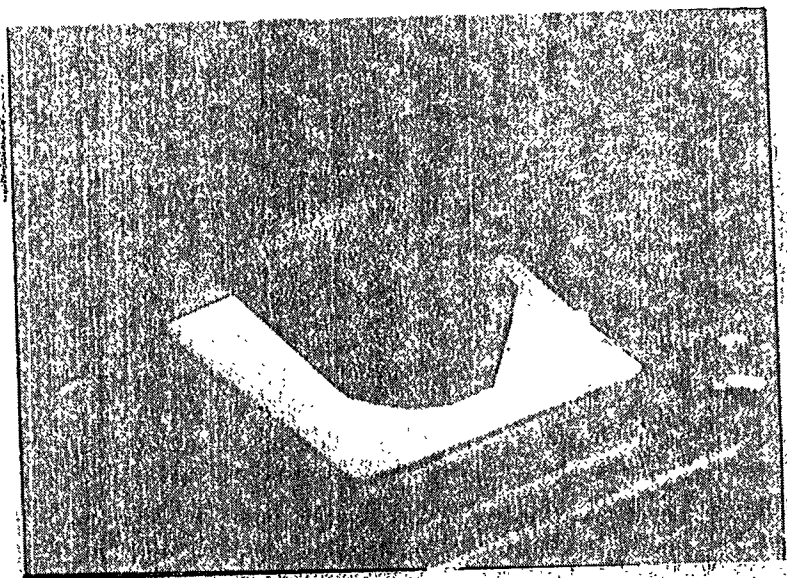
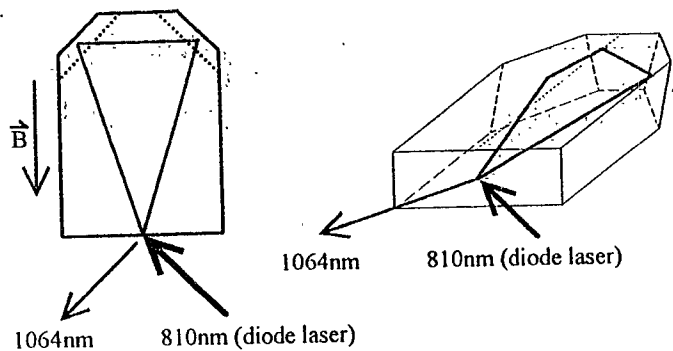
# LISA Science Payload



LEITZ 4704  
RESEARCH INSTRUMENTS

# Optical Layout



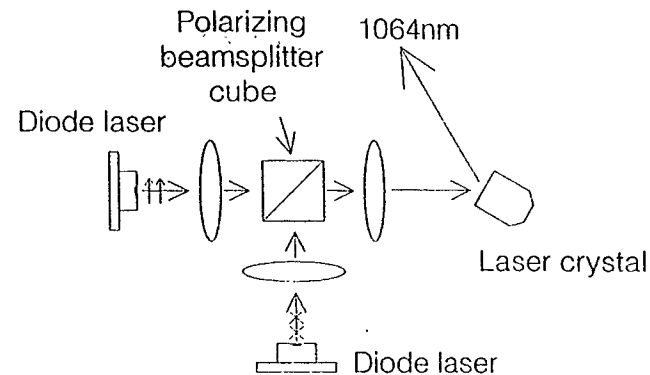


Monolithic Nd:YAG ring laser



EMET 27214

## Pump Scheme



Diode lasers:  
Siemens SFH 487401  
emitting aperture 1x200um  
output power 1W cw

Maximum pump power:	2.0W
Maximum output power:	1.1W
Slope Efficiency:	60%
Electrical/optical Conversion efficiency:	> 15%

2 04046-72 if

©

Pump scheme



required frequency stability

phase of light  $\phi = \frac{2\pi\nu}{c} \ell$

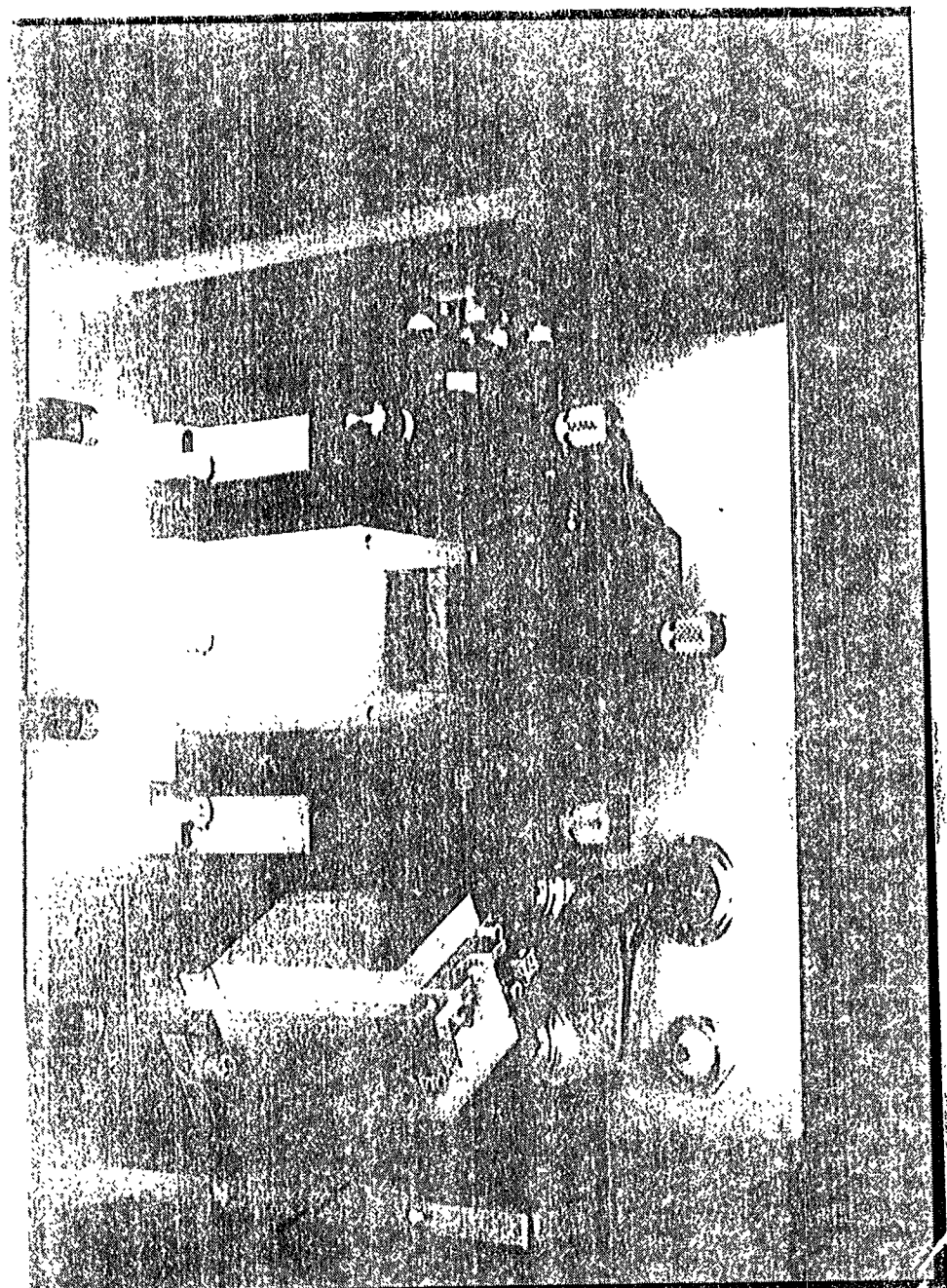
interference  $\Delta\phi = \frac{2\pi\nu}{c} \Delta\ell$

frequency noise  $\Rightarrow d(\Delta\phi) = \frac{2\pi}{c} \Delta\ell \cdot d\nu$

displacement  $\Rightarrow d(\Delta\phi) = \frac{2\pi}{c} \nu \cdot d(\Delta\ell)$

GOAL :  $d(\Delta\ell)_{\text{noise}} < 2 \cdot 10^{-12} \frac{\text{m}}{\sqrt{\text{Hz}}}$

$\sim d\nu = 1 \cdot 10^{-6} \left[ \frac{5 \cdot 10^8 \text{ m}}{\Delta\ell} \right] \frac{\text{Hz}}{\sqrt{\text{Hz}}}$



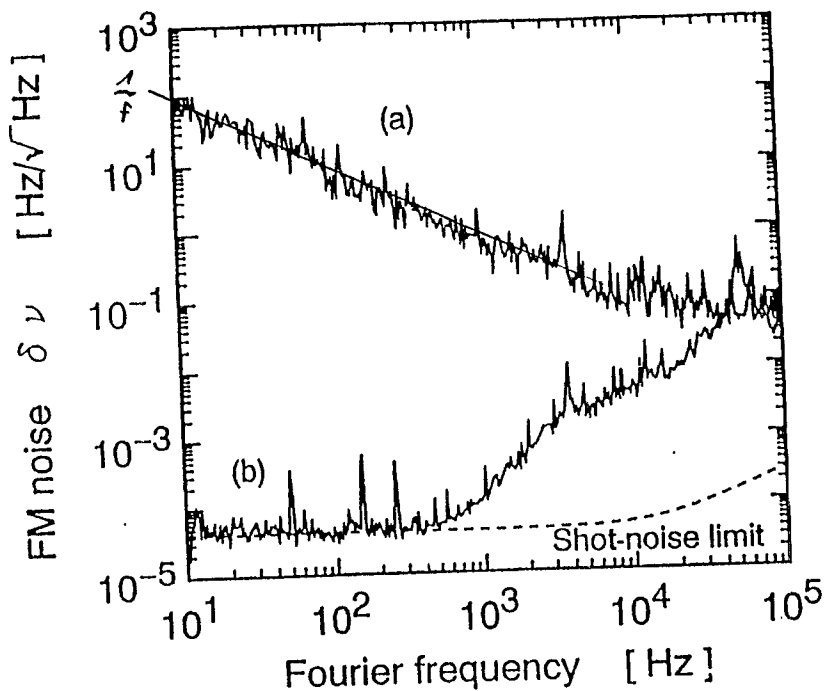


Fig. 3. Relative FM noise spectra at the error signal at a free-running state [curve (a)] at a stabilized state [curve (b)]. The calculated shot-noise limit is  $4.2 \times 10^{-5}$  assuming a  $\frac{1}{f}$  dependency

$$\sim S_{\nu} = 1 \text{ MHz} \cdot \sqrt{1/\text{Hz}} \quad @ 1 \text{ mHz}$$

### frequency stabilisation

stabilization on a cavity mounted to an ULE platform

$$\frac{d\nu}{\nu} = \frac{dL}{L}$$

thermal expansion of the platform

$$\frac{dL}{L} = \alpha \cdot dT$$

$$\sim d\nu = \nu \cdot \alpha \cdot dT$$

linear spectral density of frequency noise

$$d\nu = 8 \cdot \left[ \frac{\alpha}{3 \cdot 10^8 \frac{1}{K}} \right] \cdot \left[ \frac{dT}{10^{-6} K/\sqrt{\text{Hz}}} \right] \frac{\text{Hz}}{\sqrt{\text{Hz}}}$$



required amplitude stability

radiation pressure of 10 mW causes

i) steady acceleration  $a = 6 \cdot 10^{-11} \frac{m}{s^2}$

ii) fluctuating acceleration

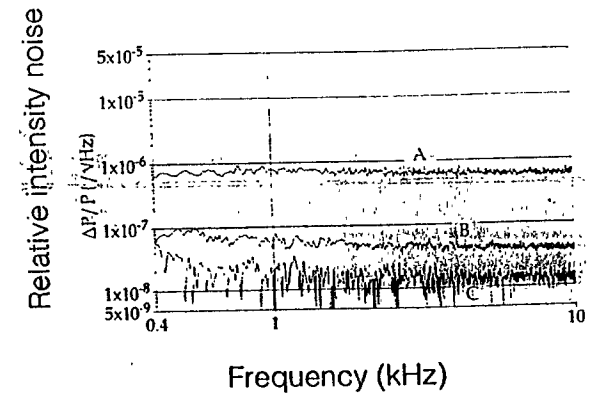
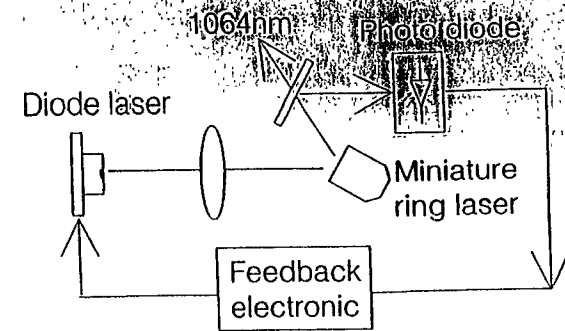
$$\tilde{a} = 6 \cdot 10^{-11} \frac{m}{s^2} \cdot \frac{\tilde{S}_P}{P}$$

to keep  $\tilde{a} \leq 10^{-16} \frac{m}{s^2} \cdot \frac{1}{\sqrt{Hz}}$  we need

$$\frac{\tilde{S}_P}{P} \leq 2 \cdot 10^{-6} \frac{1}{\sqrt{Hz}}$$

HEITZEL 2724

## Suppression of Intensity Noise by Electronic Feedback to the Diode Laser \*



\* Collaboration with the Australian National University  
C.C. Harb, T.C. Ralph, M. B. Gray, H.-A. Bachor  
published in IEEE J. of Quantum Electron. (1994).

2-040311-72-11

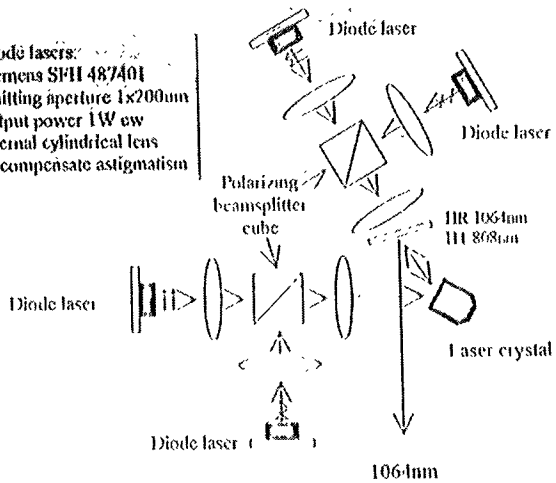
© 1994

Intensity noise suppression



## High power miniature ring laser

Diode lasers:  
Siemens SPH 487401  
emitting aperture 1x200µm  
output power 1W cw  
internal cylindrical lens  
to compensate astigmatism

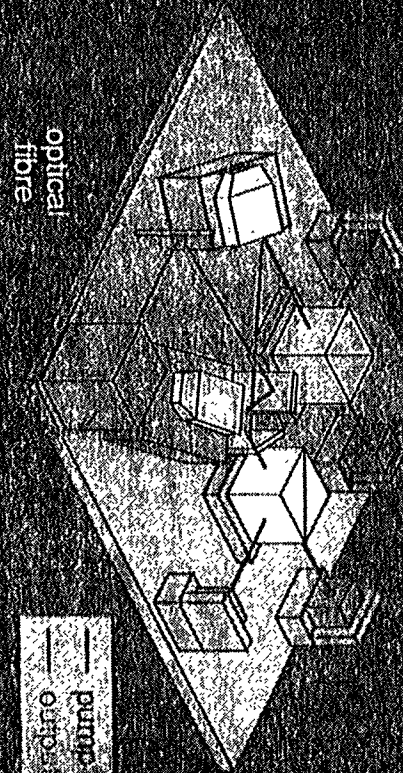


Maximum pump power: 4.0W  
Maximum output power: 2.1W multimode  
1.8W single-frequency  
Conversion Efficiency: 15%

Emitted radiation has low amplitude noise and a small spectral linewidth less than 1kHz.

High power miniature ring laser

**LISA**



Laser Design for LISA

## work to be done :

- ◆ one directional operation without or with low magnetic field
- ◆ injection-locked concept or single Nd-YAG laser
- ◆ reduction of intensity noise due to pointing of the laser at the fiber coupler
- ◆ investigation and stabilisation at low frequency
- ◆ test of new pump concepts (MOPA)
- ◆ longtime test of laser diodes :
  - lifetime
  - wavelength
- ◆ space qualification

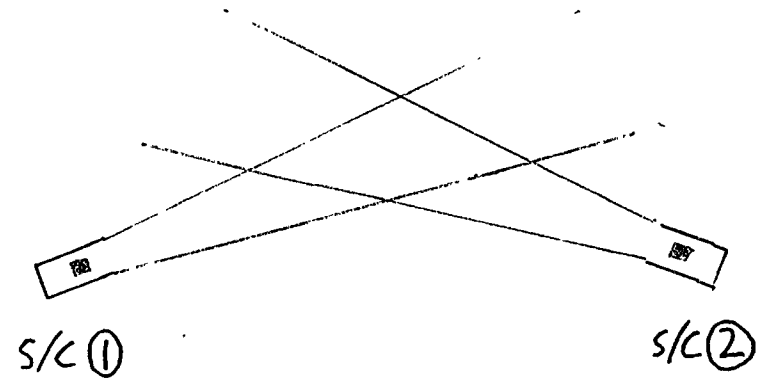
# OPTICAL ALIGNMENT FOR LISA

- ACQUISITION
- REQUIRED ACCURACY
- SENSING
- MODELLING

DAVID ROBERTSON  
U. OF GLASGOW

# POINTING ACQUISITION

LASER BEAM WIDTH  $4 \times 10^{-6}$  rad  
STAR TRACKER ACCURACY  $10^{-4}$  rad



# POINTING ACQUISITION

STAR TRACKER  $10^{-4}$  RAE

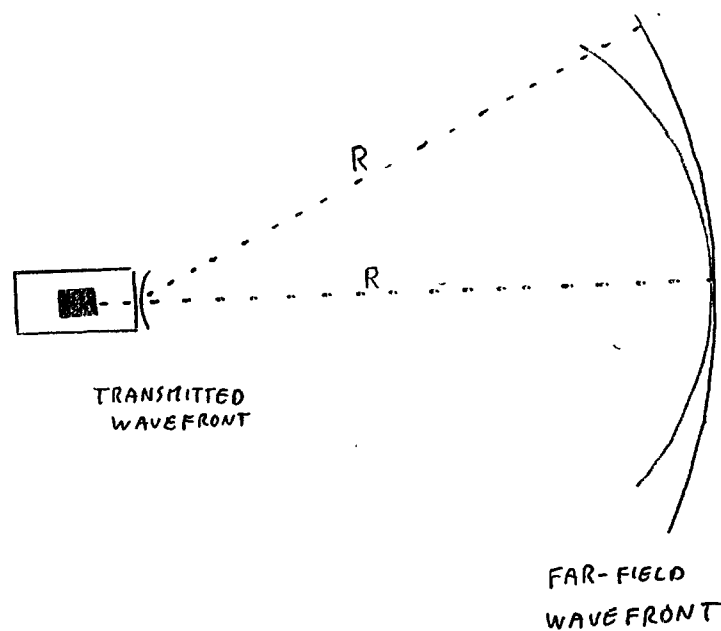
EXPAND BEAMS \*3  $1.2 \times 10^{-5}$  R

SCAN S/C ②  $10 \times 10$  GRID  $10^{-5}$  RAE  
- PROOF PASS AS REFERENCE

ALIGN S/C ① TO RECEIVED WAVEFRONT

ALIGN S/C ② TO RECEIVED WAVEFRONT

# WAVEFRONT DISTORTION



# DEFOCUS ERROR

- STRONGEST COUPLING

$$\delta x = \frac{1}{32} \left( \frac{2\pi}{\lambda} \right)^2 D^2 d \theta_{oc} \delta \theta$$

D - MIRROR DIAMETER [38 cm]  
d - CURVATURE ERROR [TRANSMITTED WAVE FRONT]

# REQUIREMENTS

$$\delta x \leq 10^{-12} \text{ m} / \sqrt{\text{Hz}}$$

$$d \approx \frac{\lambda}{15}$$

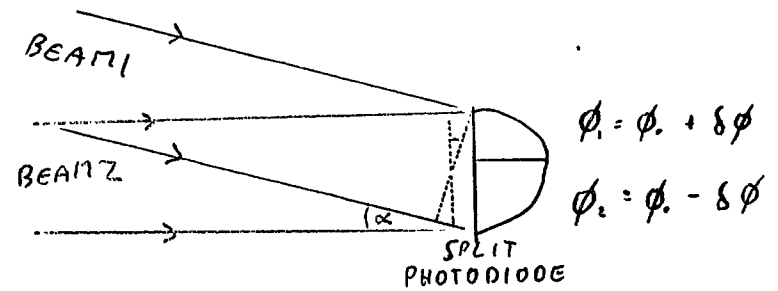
$$\rightarrow \theta_{oc} \delta \theta \leq 90 \times 10^{-18} \text{ rad}^2 / \sqrt{\text{Hz}}$$

$$\text{e.g. } \theta_{oc} \approx 15 \text{ nrad}$$

$$\delta \theta \sim 6 \text{ nrad} / \sqrt{\text{Hz}}$$

# WAVEFRONT SENSING

- DIFFERENTIAL PHASE SENSING  
WITH SPLIT PHOTODIODE



$$\phi_1 + \phi_2 \rightarrow \phi_0$$

$$\phi_1 - \phi_2 \rightarrow \delta\phi \rightarrow \alpha$$

## FINE FOCUS

LOCK INTERFEROMETER

MODULATE POINTING  $\theta$ ,  $\omega_0$

"GW" SIGNAL AT  $\omega_0$

- ADJUST FOCUS TO MINIMISE

## OPTICAL MODELLING

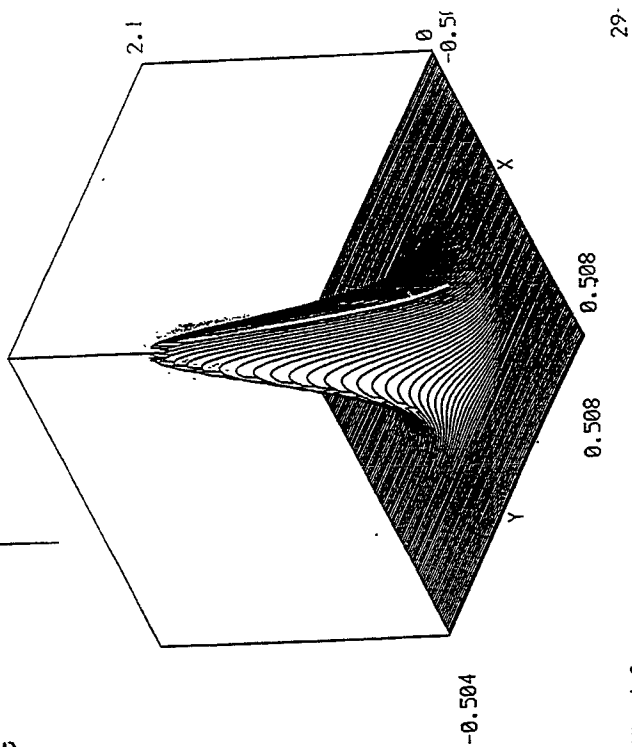
PAUL McNAMARA - GLASGOW

MARTIN CALDWELL - R.A.L.

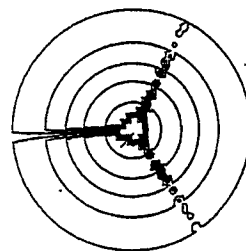
A.S.A.P. - RAY ANALYSIS

CODE V - TOLERANCING

Field of fiber spider and secondary mirror have been cut out

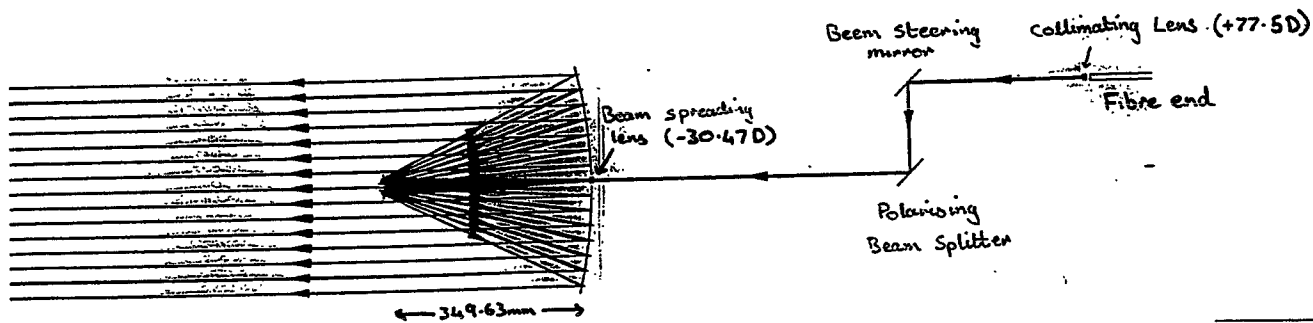


ASAP v4.0



Contour plot of above.

ASAP v4.0



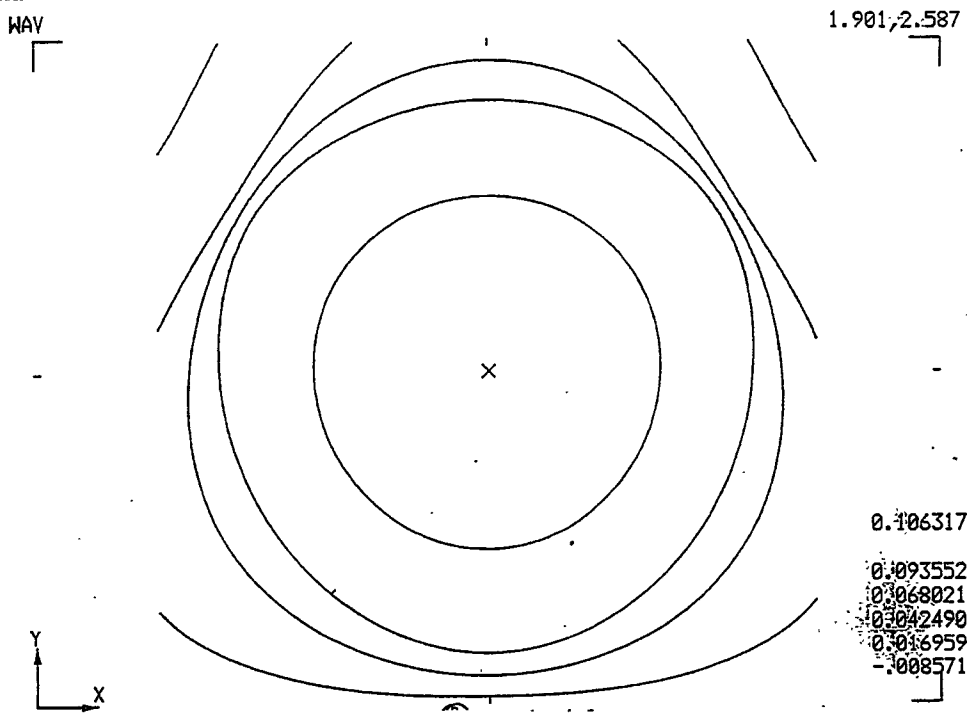
Primary : Diameter = 380mm  
Focal Length = 380mm

Secondary : Diameter = 40mm  
Focal length = 33mm

Z  
7203, - .1000E-2

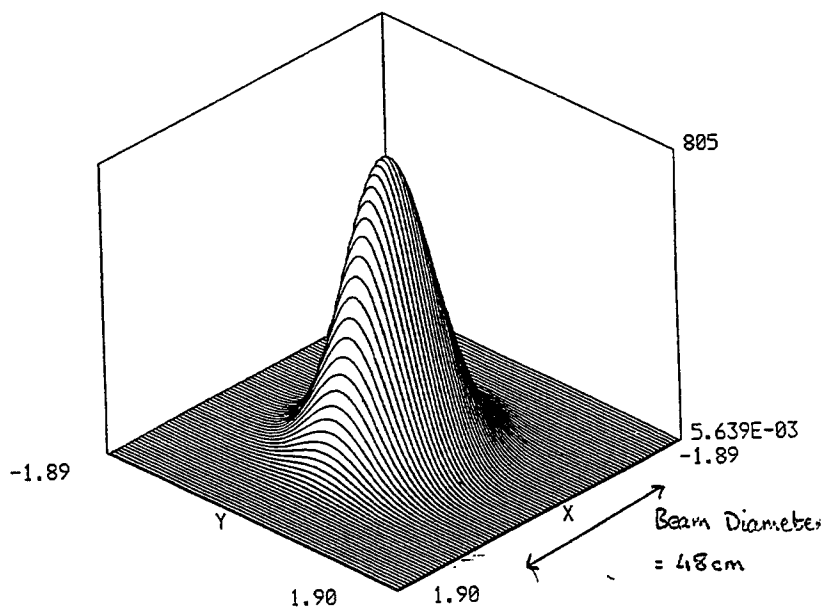


Contour plot of wavefront error, showing signs of spider obscuration



Field after propagation (at 5x Confocal distance)

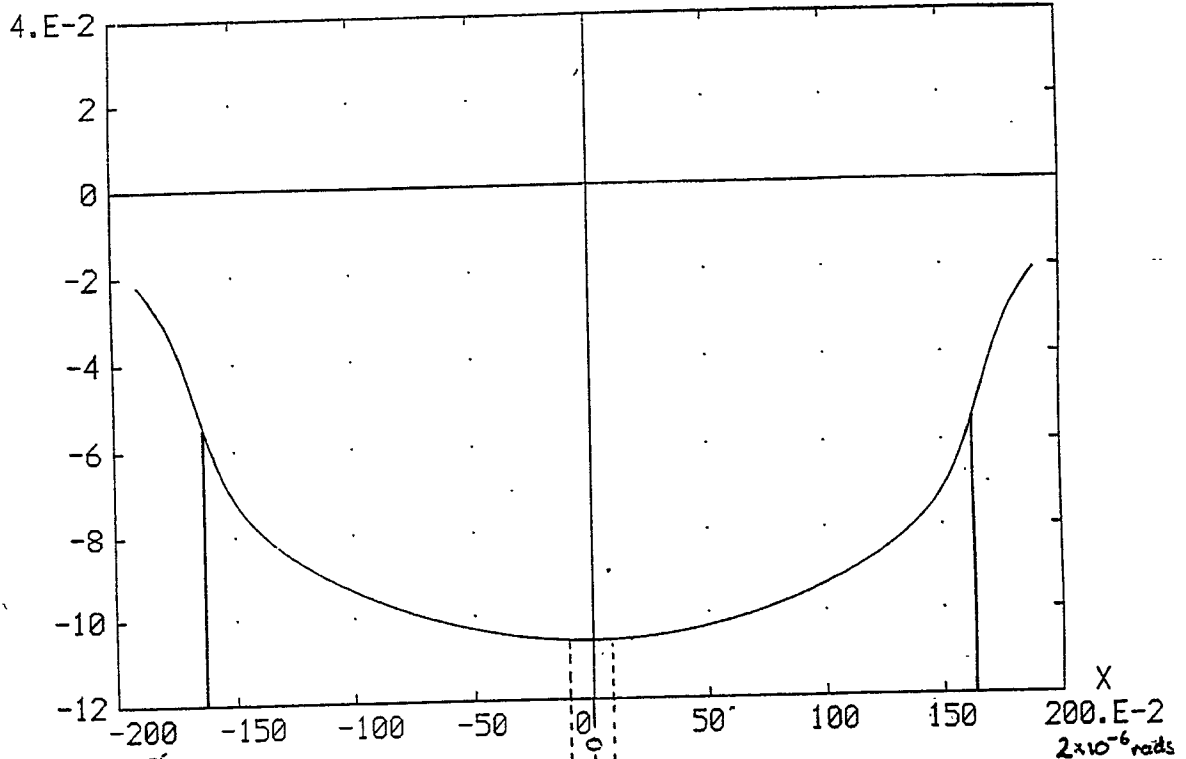
ISOMETRI



GRAPH

WAV

Wavefront Error



Gradient  $\frac{dz}{d\theta}$  :  
(m/rad)

-0.097 m/rad

$3 \times 10^{-4}$  m/rad

0.097 m/rad

$\frac{dz}{d\theta} < 0.001$  m/rad ( $0.1 \mu\text{rad} < \theta < 0.1 \mu\text{rad}$ )

## **LISA Error Budgets**

Robin T. Stebbins  
JILA/University of Colorado

Aspen Winter Conference on Gravitational  
Waves and Their Detection  
Aspen Center for Physics

Aspen, Colorado  
16 January 1996

## **Outline**

- LISA - A different kind of interferometer
- Noise types, error allocation and combination
  - Optical-path noise
  - Acceleration-like noise
  - Allocation, budget and all that
  - Combining error allocations
- Optical-path noise
  - Detector shot noise
  - Master clock noise
  - Residual laser phase noise
  - Laser beam-pointing noise
  - Scattered-light effects
- Acceleration noise
  - Thermal distortion of spacecraft
  - Thermal distortion of payload
  - Payload gravity noise
  - Thermal gradients in test mass cavity
  - Electrical force from charging and patch fields
  - Lorentz force from charging and interplanetary fields
  - Residual gas impacts
  - Telescope thermal expansion
  - Magnetic force on test mass from fluctuating interplanetary field
  - Other substantial effects
  - Other insubstantial effects

## A Different Interferometer

- Test masses are in free fall.
- The solar orbit is a very benign environment.
- The drag-free system further reduces external disturbances.
- LISA has a comparatively simple optical system.

## Noise Types, Error Allocation and Combination

- Optical-path noise: unwanted effects that make the apparent optical path change. These generally have a white spectrum.
- Acceleration noise: unwanted effects that are, or look like, forces on the test masses, i.e., that have  $1/f^2$  frequency dependence when displayed as a displacement. Warning: Those which are driven by solar insolation are different by a small fractional power of  $f$ .
- We have constructed a budget by identifying the major sources of errors, picking an error allocation that appears sufficient - or even generous - and making an allocation for other smaller effects, known and unknown.
- We have adopted a noise combination procedure that was recommended to us by Dan Debra as one that worked well in other precision measurement satellites at Stanford.
  - Simply add the five largest errors, which are not known to be uncorrelated
  - Add the remainder quadratically.

This is an attempt to be conservative and reflect our ignorance.

### Major Sources of Optical-Path Noise

<u>Source</u>	<u>Error</u> $10^{-12} \text{ m}/\sqrt{\text{Hz}}$	<u>Number</u>
Detector shot noise (random)	7	× 4
Master clock noise (random)	10	× 1
Residual laser phase noise after correction	4	× 1
Laser beam pointing instability	4	× 4
Laser phase measurements and offset lock	2	× 4
Scattered light effects	2	× 4
Other substantial effects	2	× 60
<b>Total Error Allocation for L<sub>2</sub>-L<sub>1</sub></b>	<b><math>31 \times 10^{-12} \text{ m}/\sqrt{\text{Hz}}</math></b>	

### Major Sources of Acceleration Noise

<u>Source</u>	<u>Error</u> $10^{-16} \cdot (10^{-4} \text{ Hz}/f)^{1/3}$	<u>Number</u>
Thermal distortion of outer layer	5	× 4
Thermal distortion of payload (internal)	2.5	× 4
Payload gravity noise, plus spacecraft displacement	2.5	× 4
Temperature difference variations across cavity	2.5	× 4
Electrical force on charged proof mass	2.5	× 4
Lorentz force on charged proof mass from fluctuating interplanetary field	2.5	× 4
Residual gas impacts on proof mass	2.5	× 4
Telescope thermal expansion	2.5	× 4
Magnetic force on test mass	1	× 4
Other substantial effects	2.5	× 20
Other smaller effects	1	× 100
<b>Total Error Allocation for Second Time Derivative of L<sub>2</sub>-L<sub>1</sub></b>	<b><math>3 \times 10^{-15} \cdot (10^{-4} \text{ Hz}/f)^{1/3}</math> <math>\text{m}/\text{s}^2/\sqrt{\text{Hz}}</math></b>	

## **Conclusions**

- We have made a fairly extensive, initial survey of possible error sources.
- The major contributors have been estimated.
- There are no insurmountable obstacles.
- These conservative budgets give a very rewarding sensitivity.

## Glasgow Group

Jim Hough

Gavin Newton

Norna Robertson

Harry Ward

David Robertson

Sheila Rowan

Ken Skeldon

Ken Strain

Alison McLaren - M. PLISSI

Morag Casey

Paul McNamara

Stuart Killbourn

Sharon Twyford

Colin Craig

Allan Latta

Angus McKellar

## PROTOTYPE DETECTOR \*

- THERMAL NOISE
- TEST GEO600 ITEMS + STRATEGIE

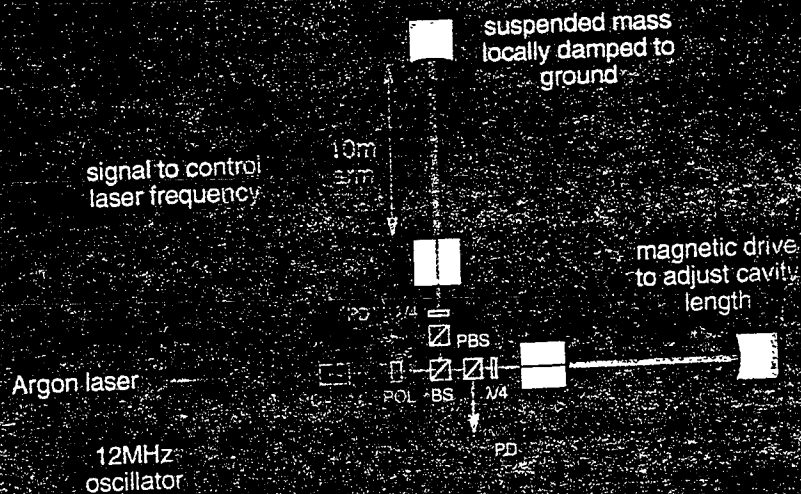
## SUSPENSION DEVELOPMENT

- MODELLING
- BUILDING
- MONOLITHIC, HIGH Q \*

## OPTICAL MODECLEANER \*

## LASER AMPLIFIER

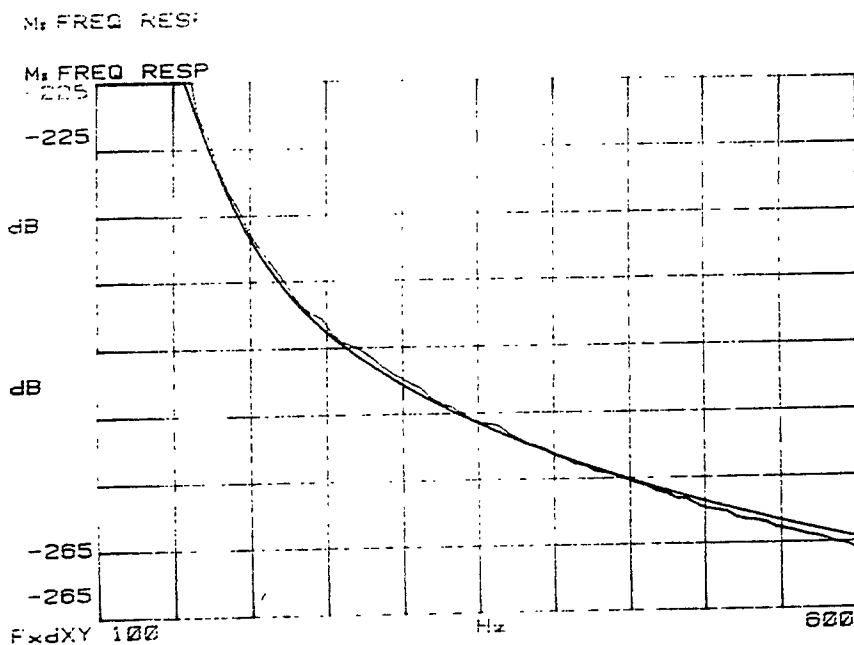
# Glasgow Prototype Detector Layout



- PBS = polarising beam splitters
- POL = polariser
- PD = photodiode and preamp
- PC = Pockels cell phase modulator

differential displacement output signals

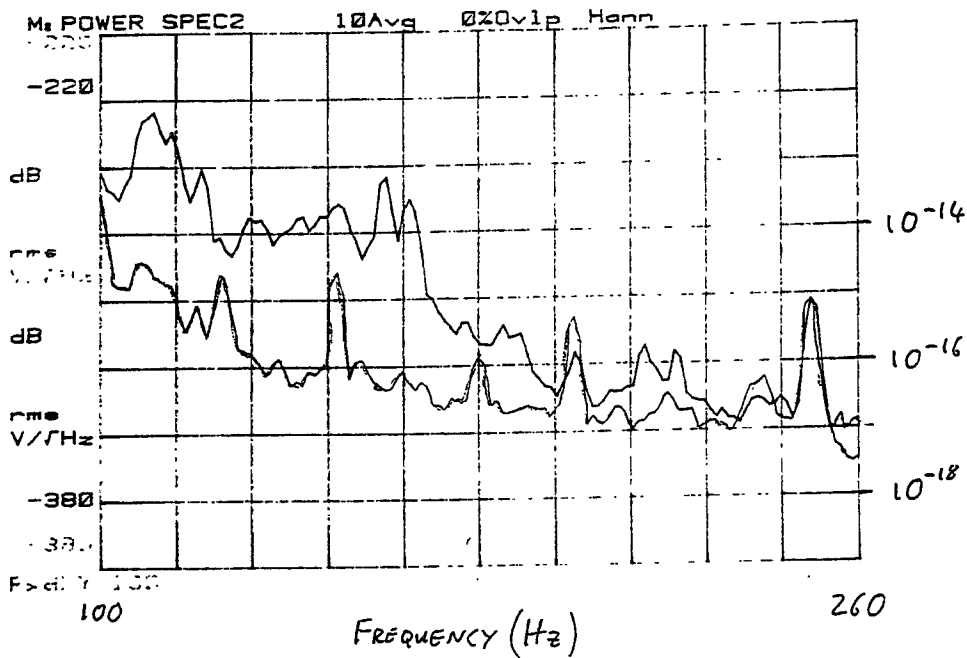
## CALIBRATION CROSS-CHECK





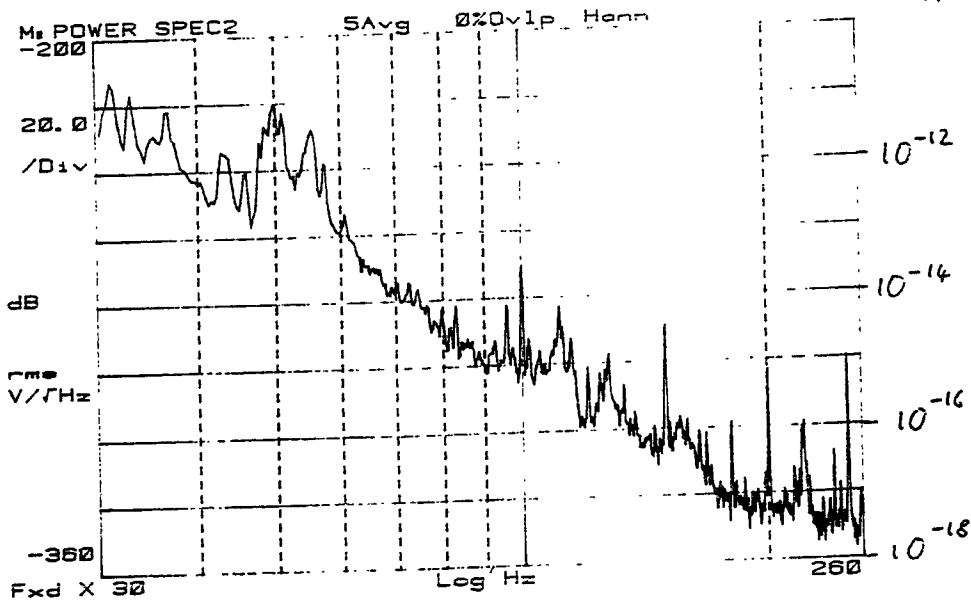
# DISPLACEMENT NOISE

$m/\sqrt{Hz}$

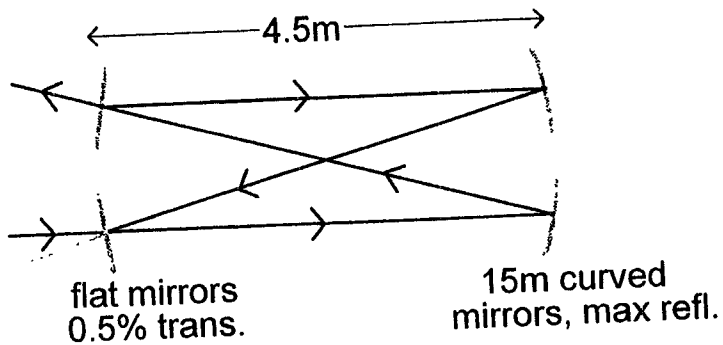


# DISPLACEMENT NOISE

$m/\sqrt{Hz}$



# Glasgow Prototype Modecleaner



*finesse = 600*  
 *$FSR_{\frac{1}{2}} = 8.3 \text{ MHz}$*   
*effective cavity length = 9 m*  
*optical path length = 18 m*  
*TEM10 suppression = 350*  
*TEM20 suppression = 300*

# Glasgow Modecleaner

## Progress Report December 1995

K. Skeldon

### Acknowledgments

K. Strain  
A. Grant  
H. Ward  
J. Hough  
A. Latta  
D. Edwards  
et al...

## Coupling of Beam Jitter through Beam Splitter Misalignment

Require!

$$\frac{\Delta x}{w} < 10^{-10} / \sqrt{\text{Hz}}$$

*Implies 4 orders of magnitude suppression*

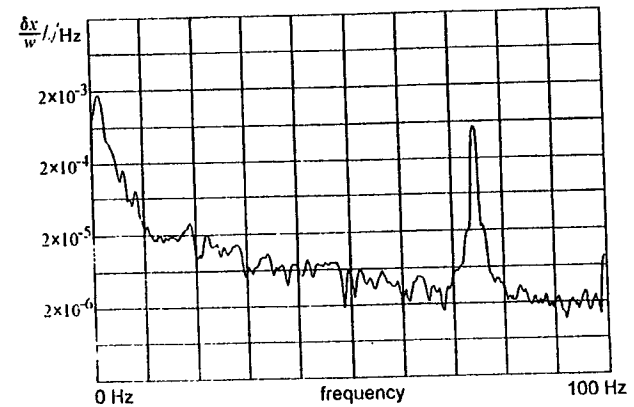
## Coupling of Beam Width Pulsation through Mirror Curvature Mismatch or Effects of Scattered Light

Require!

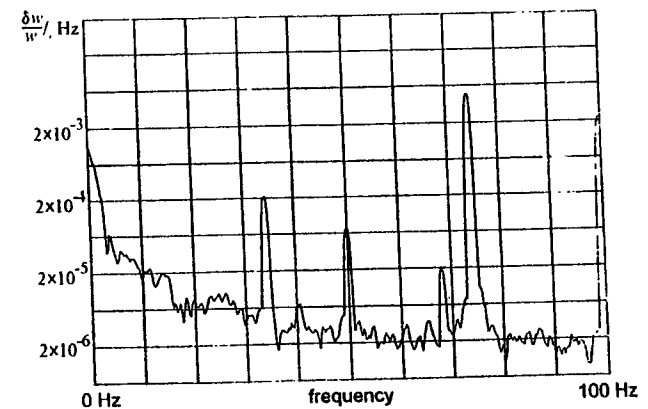
$$\frac{\Delta w}{w} < 10^{-11} / \sqrt{\text{Hz}}$$

*Implies about five orders of magnitude suppression*

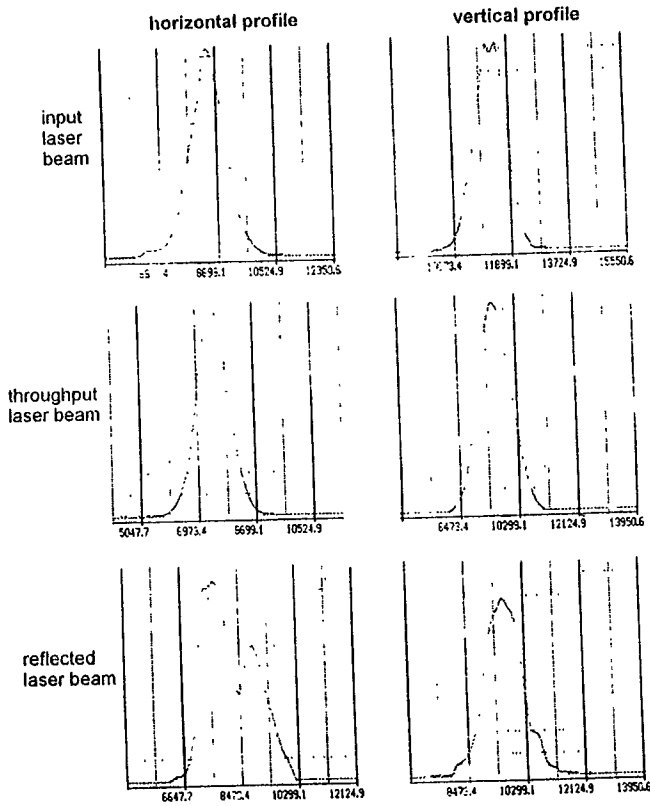
## Typical Nd:YAG Laser Beam Jitter Noise



## Typical Nd:YAG Laser Beam Width Pulsation Noise

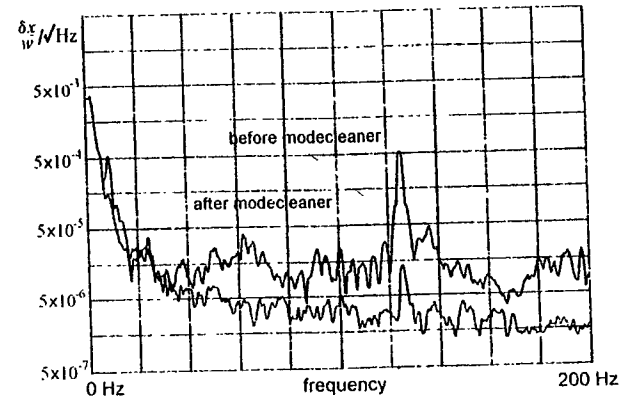


## Spatial Filtering Action

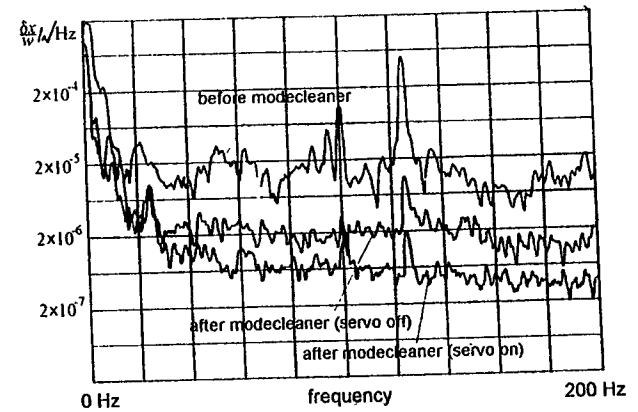


CHALLENGE NO. 6 / 088 0

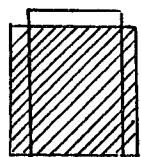
## Reduction of Applied Jitter Peak



## Reduction of Applied Jitter Peak (Intensity Servo after Modecleaner)



FUSED SILICA  
MICROSCOPE  
SLIDE  
PREFORM



TORSEAL +  
STAINLESS STEEL  
SANDWICH

NECK

FIBRE

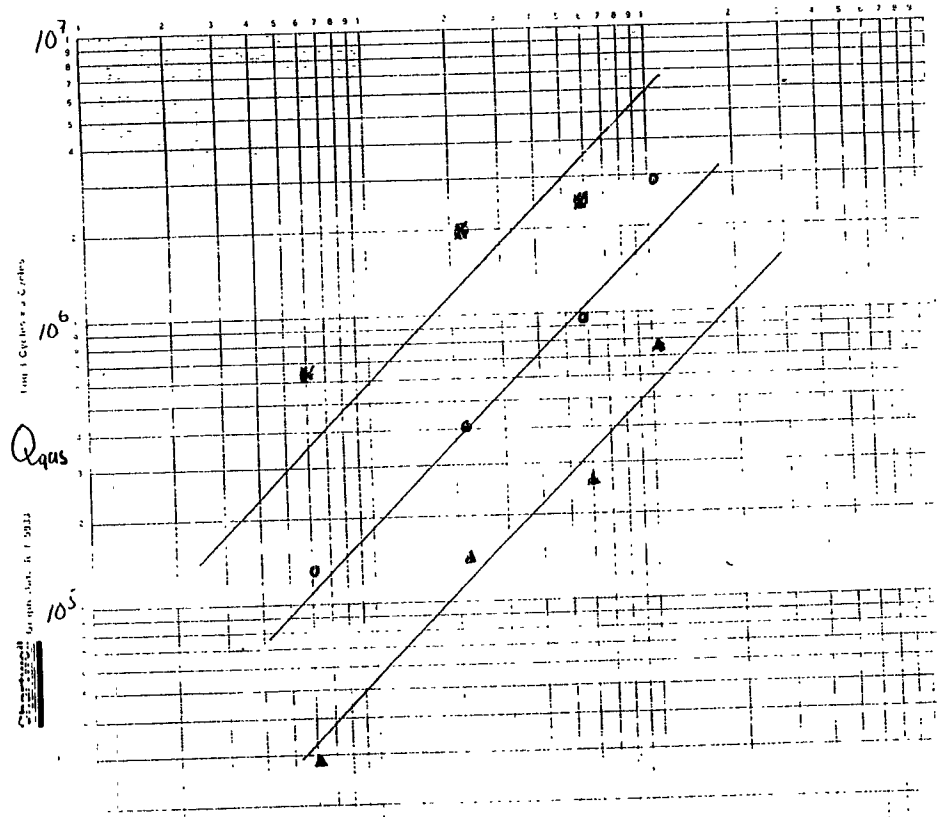
DIRECT MEASUREMENT OF  
MATERIAL Q FOR FUSED  
SILICA RIBBON FIBRES

S.M. TWYFORD  
A.C. MCLAREN

R. HUTCHINS  
J. HOUGH  
N.A. ROBERTSON

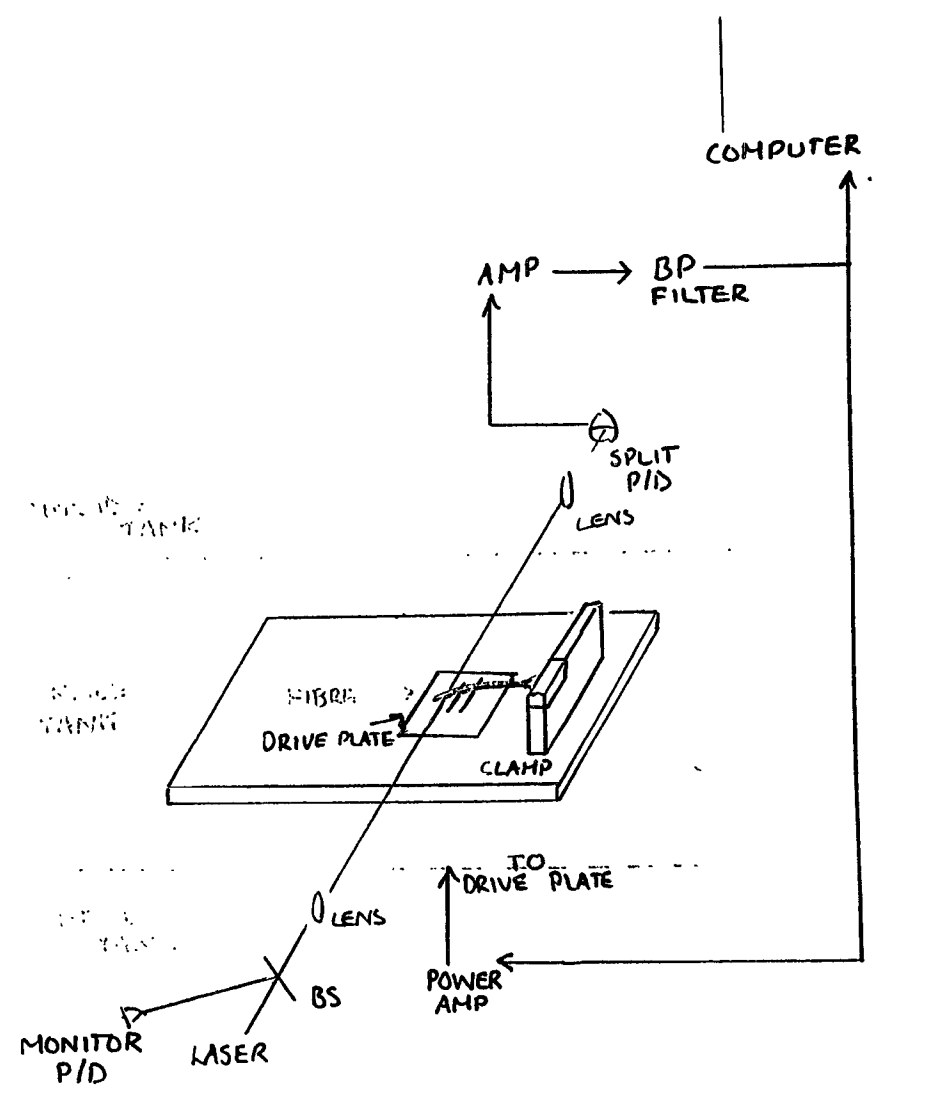
J. FALLER.

$Q_{GAS}$  vs  $f$



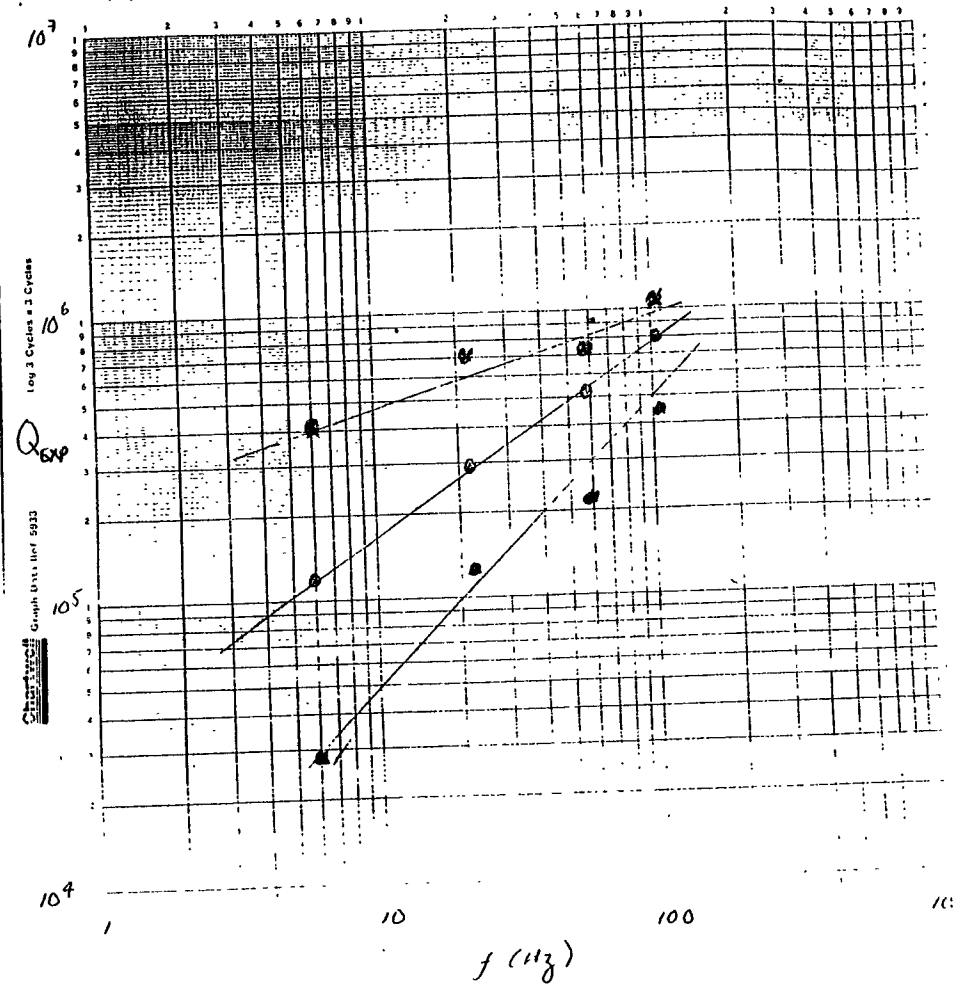
$10^4$  1 10 100  
 $f$  (Hz)

- $Q_{GAS}$  FOR  $\sim 1 \times 10^{-6}$  mbar.
- $Q_{GAS}$  FOR  $\sim 1 \times 10^{-5}$  mbar.
- ▲  $Q_{GAS}$  FOR  $\sim 1 \times 10^{-4}$  mbar.



+ SEISMOMETER TO MONITOR GROUND VIBRATIONS

Q EXPERIMENTAL vs f



- : Q EXPERIMENTAL FOR  $\sim 1 \times 10^{-6}$  mbar
- : Q EXPERIMENTAL FOR  $\sim 1 \times 10^{-5}$  mbar
- ▲ : Q EXPERIMENTAL FOR  $\sim 1 \times 10^{-4}$  mbar

STWYFORD

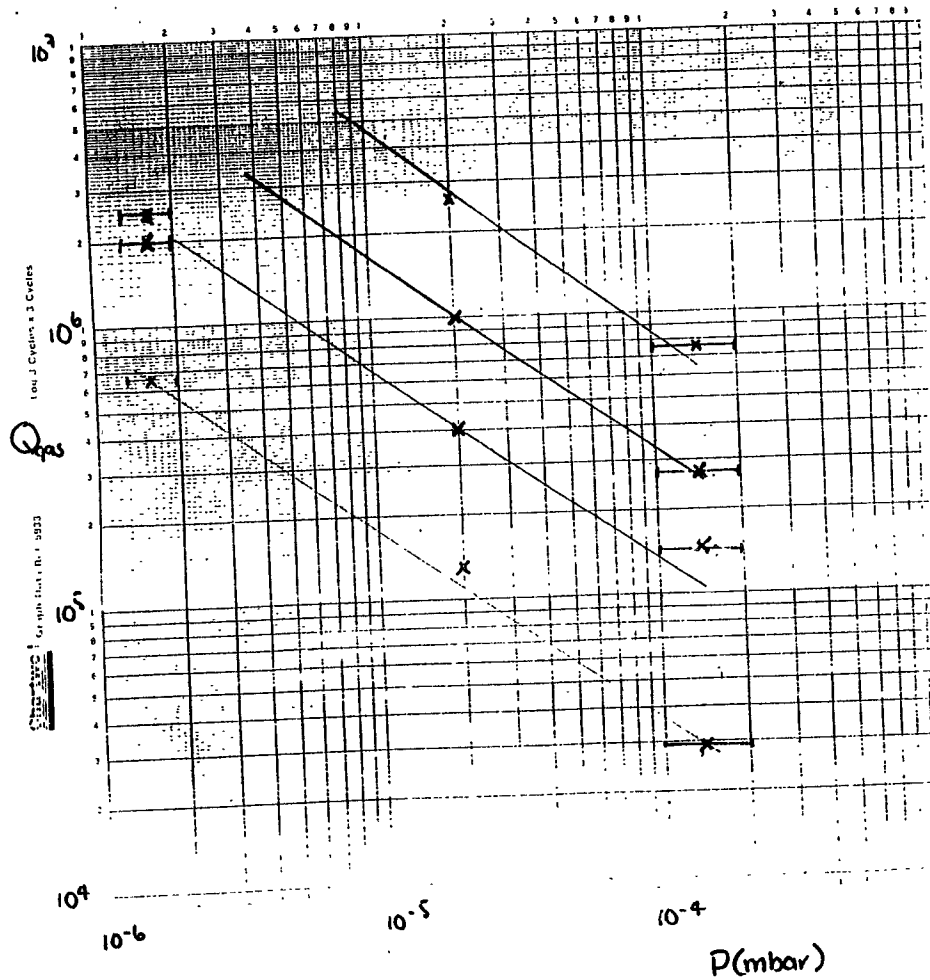
$$\frac{1}{Q_{\text{gas}}} = \frac{1}{Q_{\text{experiment}}} - \frac{1}{Q_{\text{material}}}$$

$Q_{\text{gas}}$  = Q UPPER LIMIT ASSUMING GAS DAMPING (FOR A GIVEN FREQ. AT A GIVEN PRESSURE).

$Q_{\text{experiment}}$  = Q OBTAINED FROM EXPERIMENT

$Q_{\text{material}}$  = ASSUMED MATERIAL Q =  $1.1 \times 10^6$  OBTAINED FROM BEST EXPERIMENTAL RESULT.

$Q_{gas}$  vs P



x : 6.06 Hz  
 x : 22.8 Hz  
 x : 59.6 Hz  
 x : 106 Hz

RESULTS SHOW

$$Q_{gas} \propto f$$

AGREEMENT WITH SIMPLIFIED STATIST  
 GAS DAMPING MODEL

$$Q_{gas} = \frac{\omega_n m}{2PA} \sqrt{\frac{RT}{M}}$$

$\omega_n$  = MODE FREQUENCY

$m$  = FIBRE MASS

$P$  = TANK PRESSURE

$A$  = SURFACE AREA.

$T$  = TEMP.

$M$  = MOLECULAR MASS OF GAS  
 PRESENT IN TANK



## THERMOELASTIC DAMPING

FREQ. OF MAX. DAMPING GIVEN

BY :-

$$f_{\text{char}} = \frac{\sqrt{K}}{2} \sqrt{\frac{1}{\rho c l^2}} \quad (\text{RIBBON})$$

$f_{\text{char}}$  = CHARACTERISTIC FREQ

$$K = 1.38 \text{ W/mK} \quad (\text{AT } 20^\circ\text{C})$$

$$\rho = 2.2 \times 10^3 \text{ kg/m}^3$$

$$c = 772 \text{ J/kg.K}$$

$$l = 100 \mu\text{m} \quad \text{: - ESTIMATE}$$

Q AT  $f_{\text{char}}$  GIVEN BY :-

$$\phi(\omega) = \frac{\Delta}{2}$$

$$Q = \frac{1}{\phi(\omega)}$$

$\phi(\omega)$  : LOSS ANGLE

$\Delta$  = RELAXATION STRENGTH

$$= \frac{E \alpha^2 T}{\rho c}$$

$$\sim \begin{aligned} E &= 7 \times 10^{10} \text{ N/m}^2 \\ \alpha &= 5.1 \times 10^{-7} / \text{K} \end{aligned}$$

FROM SIMPLE GAS DAMPING,  
EXPECT :

$$Q_{\text{gas}} \propto \frac{1}{P}$$

EXPERIMENTAL RESULTS SHOW :

$$Q_{\text{gas}} \propto P^{-2/3}$$

THE RELAXATION TIME,  $\tau$ , FOLLOWS  
FROM  $f_{char}$

$$\tau = \frac{1}{2\pi f_{char}}$$

FOR REMAINING PTS. ON THERMOELASTIC  
DAMPING CURVE :-

$$\phi = \frac{\Delta W \tau}{1 + \omega^2 \tau^2}$$

$$Q = \frac{1}{\phi}$$

# Power Recycling at the 30m Prototype Detector in Garching

Dietmar Schnier

Mathias Schrempel

Harald Lück

Karsten Danzmann

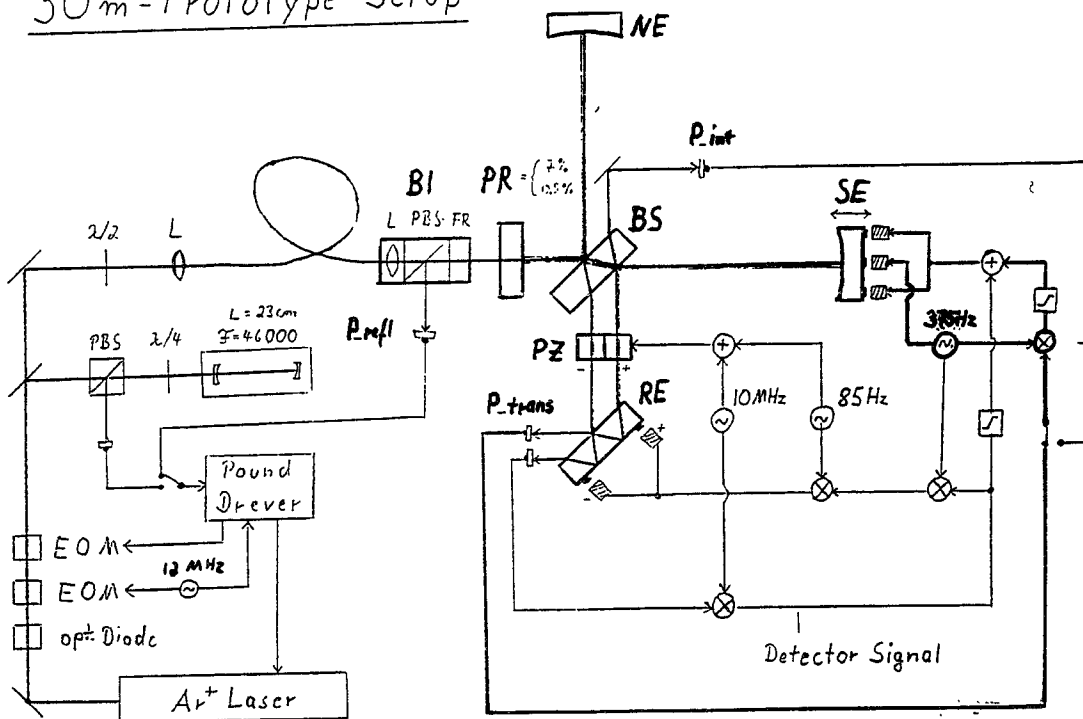
Walter Vinkler

Roland Schilling

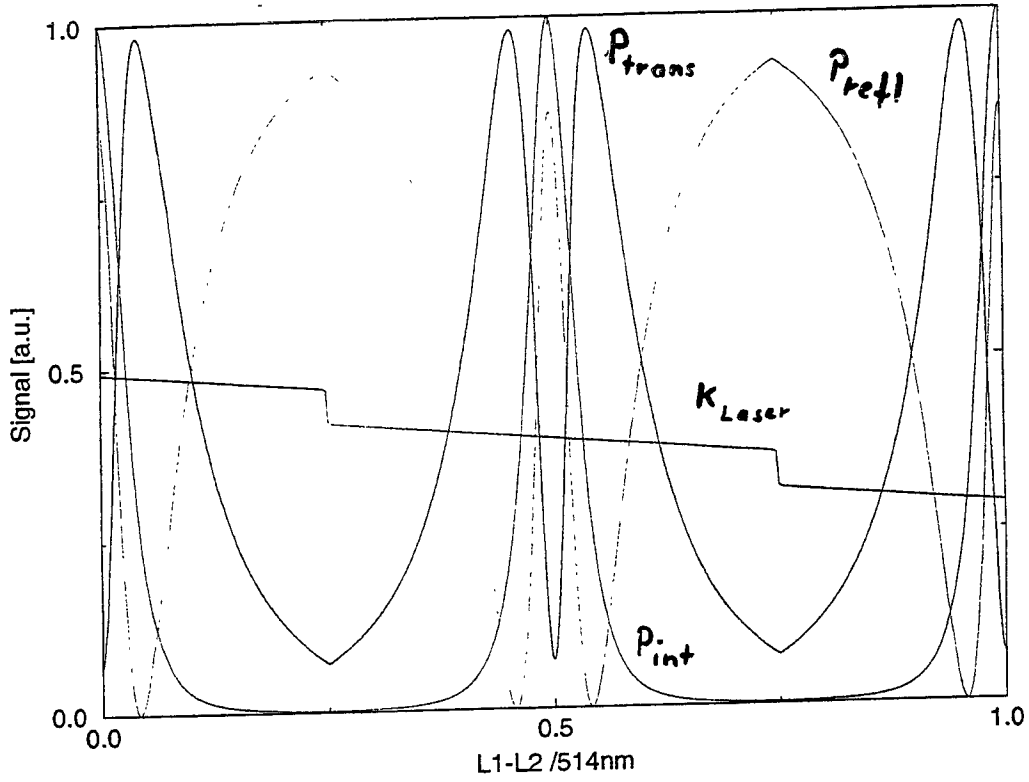
Jun Mizuno

Gerhard Heinzel

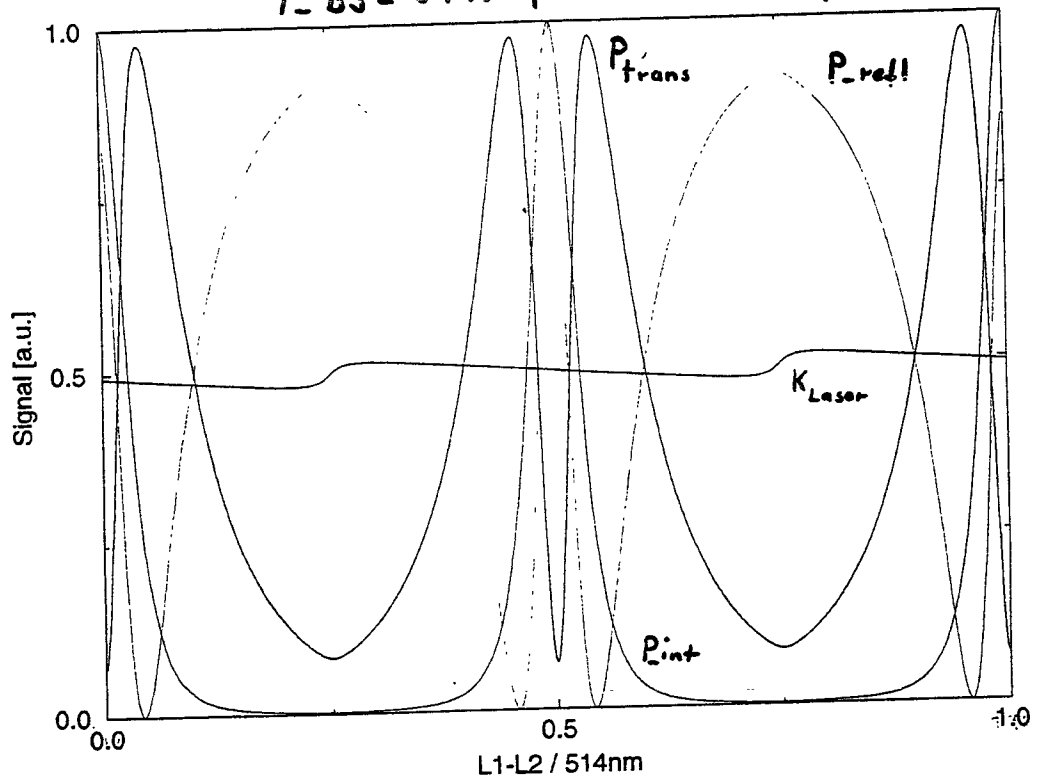
## 30m-Prototype Setup



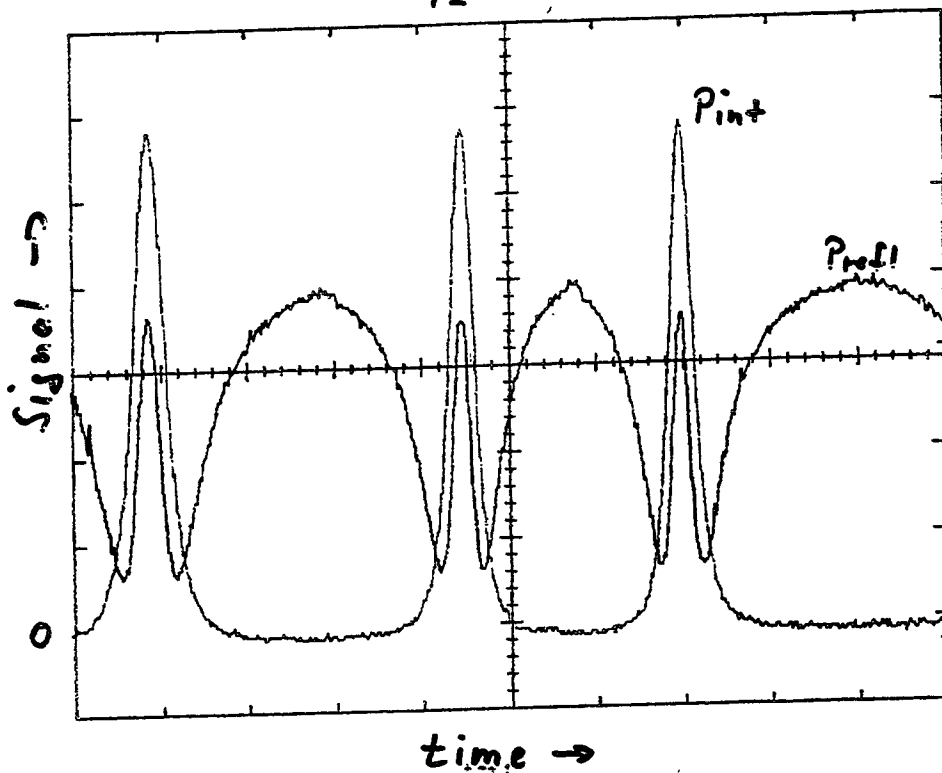
50% Beamsplitter,  $T_{PR}=7\%$ , Losses= $.3\%$



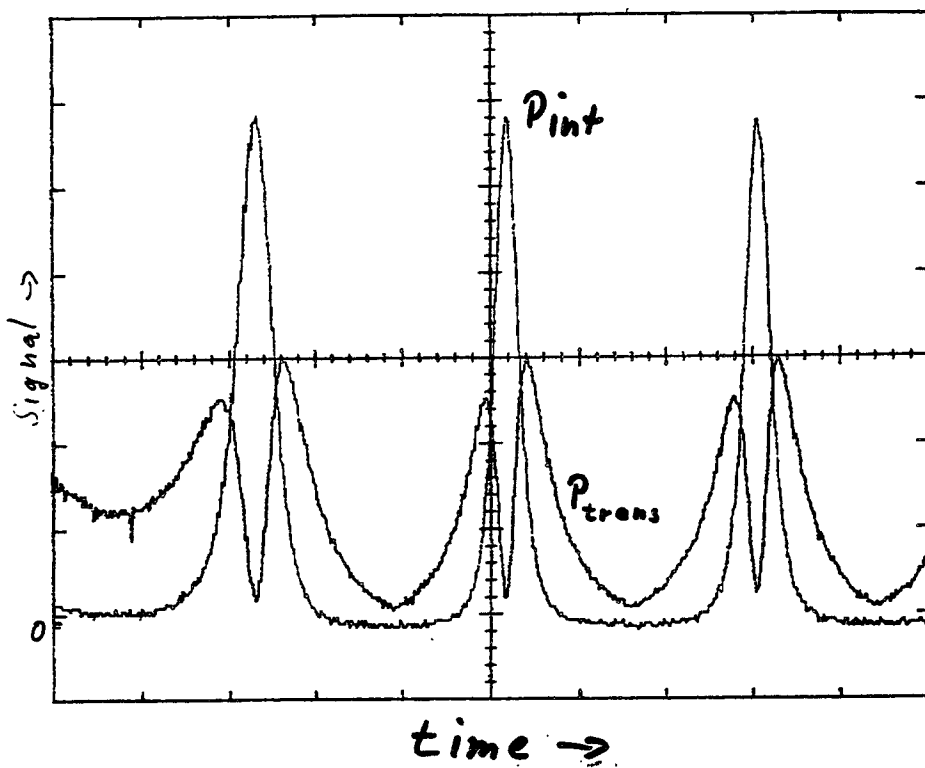
$T_{BS} = 54\%$  ,  $T_{PR} = 7\%$  ,  $L = 0.3\%$



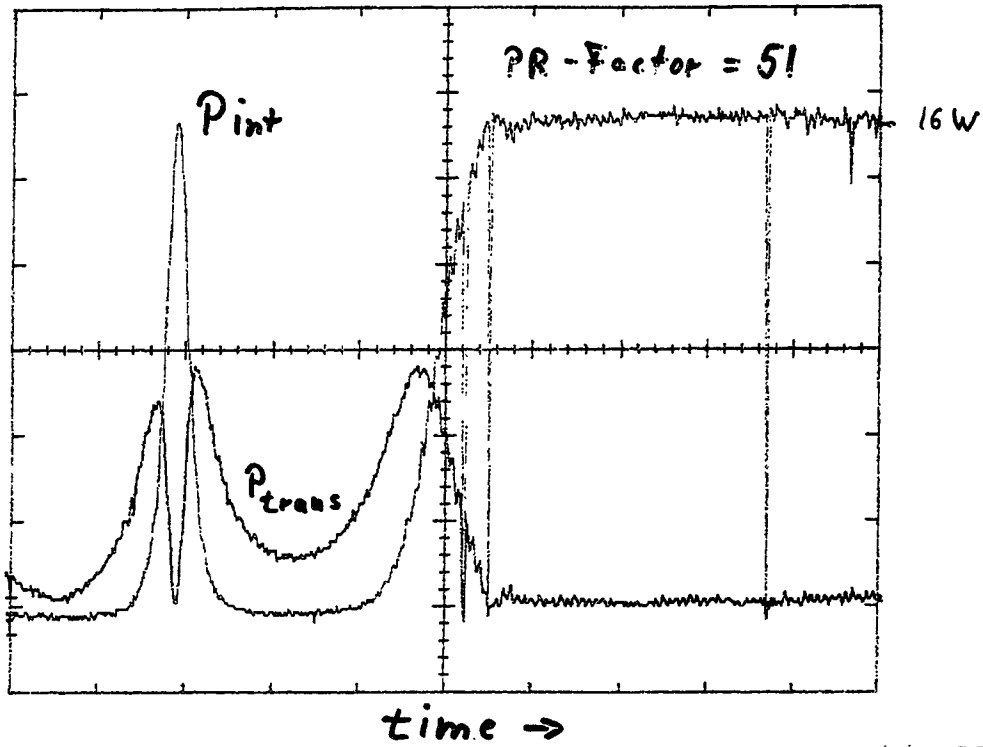
$T_{PR} = 7\%$



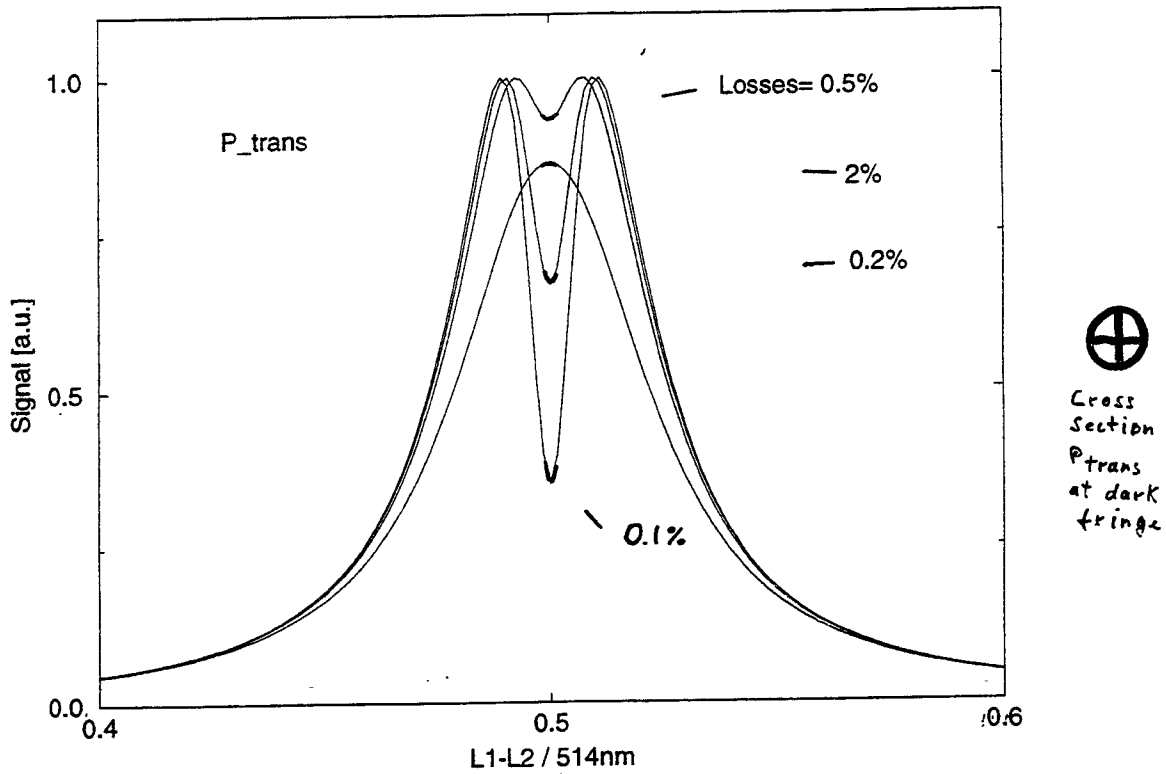
$T_{PR} = 7\%$



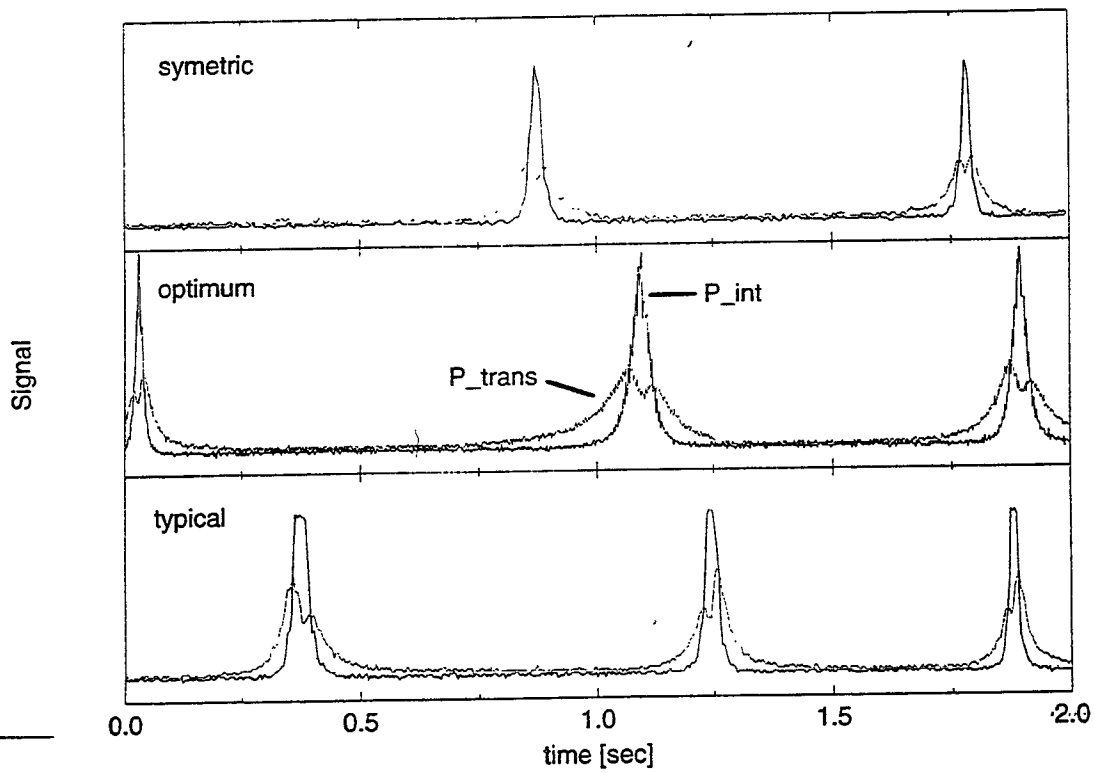
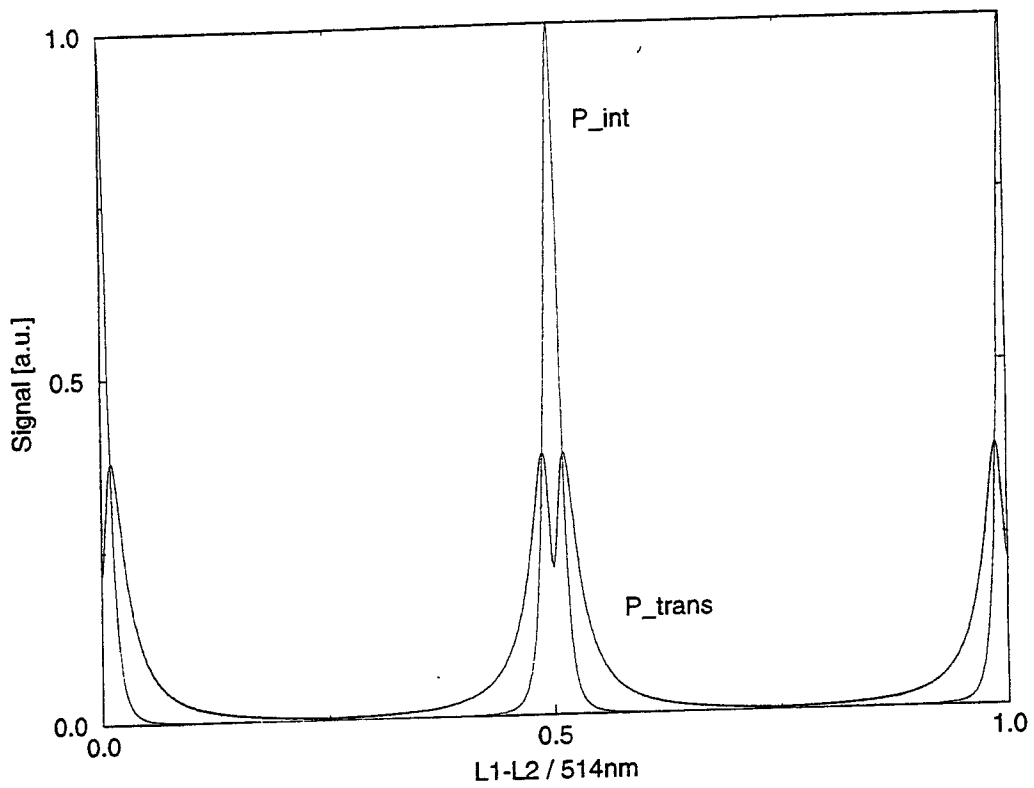
$T_{PR} = 7\%$

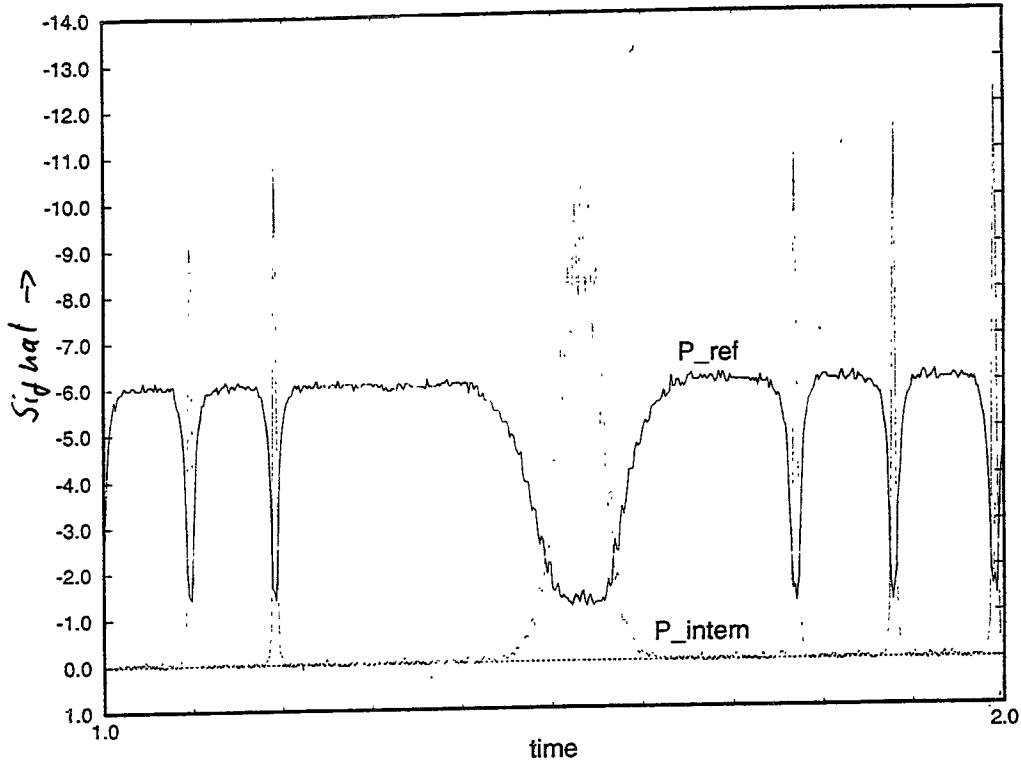


Transmission PR-Mirror = 0.5%

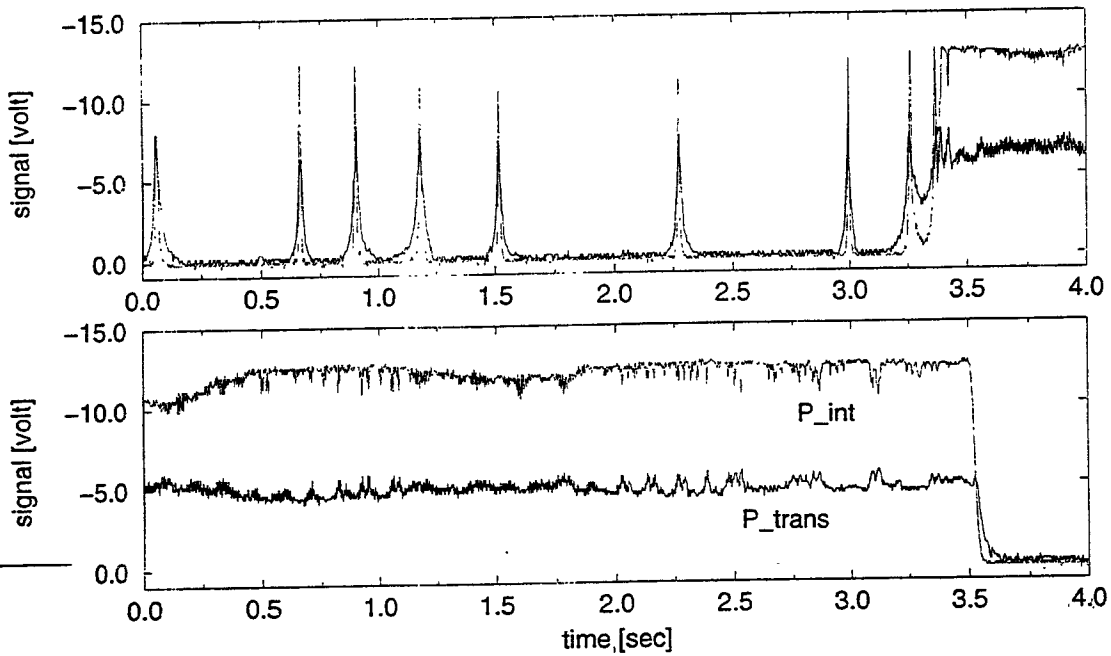


$T_{PR}=0.5\%$ ,  $T_{BS}=54\%$ ,  $Losses=0.3\%$



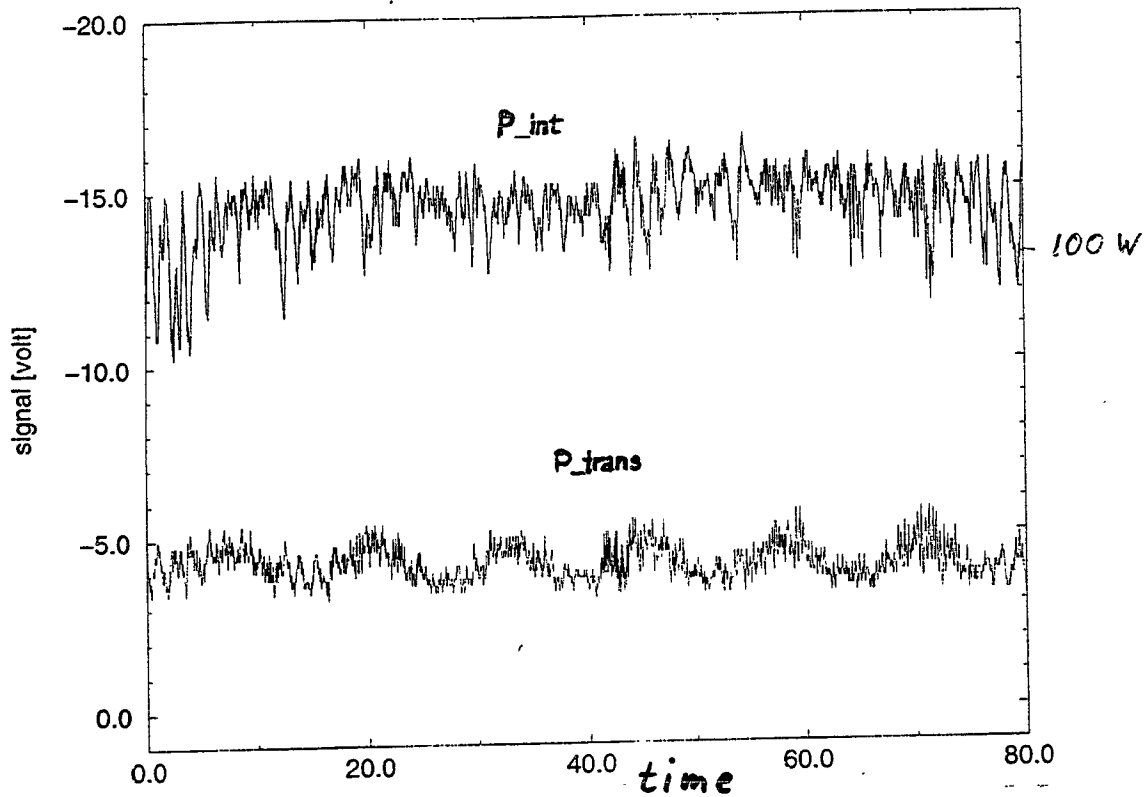


PR-Mirror T=0.5%, Prelock with P\_int





$\tau_{PR} = 0.5\%$  PR-Factor  $\sim 320$



### Open Questions

- Cross Section of  $P_{trans}$  at dark fringe :  $\oplus = ?$
- $\oplus \leftrightarrow$  Mirror Deformation
- Asymmetry of  $P_{trans}$

:

### Future Work

- Laser Power Stabilization
- External Modulation with P. R.
- Auto Beam Alignment

:

o

Wednesday AM LIGO Research Community (LRC)  
January 17 Chair: H. Ward

8:00 G. Sanders (Caltech) The View From LIGO  
8:20 Discussion  
8:30 Meeting of LIGO Research Community and Discussion  
1:30 Meeting of LIGO Research Community (continued)

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# LIGO Research Community: The View From LIGO

Gary Sanders  
Aspen Workshop  
January, 1996

---

# LIGO Users Community

Gary H. Sanders  
California Institute of Technology  
Aspen Winter Conference on  
Gravitational Waves and Their  
Detection  
January 24, 1995

# Traditional “Users” Models

---

- High Energy and Nuclear Physics
  - » Accelerators
    - CERN, FNAL, TRIUMF, SIN, Frascati, Saclay
  - » Reactors
    - Grenoble
- Astronomy
  - » Telescopes
    - Palomar, Arecibo, Submillimeter, Owens Valley, Hubble
- Magnetic Fusion Machines
  - » TFTR, MFTF, ITER
- Materials Science
  - » Light Sources
  - » Spallation Sources

## “User” Models (recited by a high energy physicist)

---

- Facility capabilities supported by in-house team
- Access to facility generally open to international community
- Research can be collaboration between in-house and community scientists or can be completely independent
- Research proposals are collaborative and submitted to peer review
- Review includes scientific/technical review and agency funding review
  - » Scientific review generally held by facility Director/Principal Investigator
  - » Agency holds funding review
- Facility creates a Program Advisory Committee
- Users organize a Users Group with elected leadership and charter
  - » Users Group becomes the “customers” voice

# Immediate User Issues

---

- User requirements for facilities
  - » Visitor accommodations - offices, labs, food, sleeping quarters
  - » Clean room storage and work areas for test and assembly - “staging” areas and labs
  - » Computing
  - » Intellectual climate
- Computing infrastructure
  - » hardware environment
  - » software tools
    - AVS vs. Khoros
- Review of proposals
  - » Early LIGO program
  - » Advanced detectors

# National/International

---

- US groups
  - » NSF funding proposals
  - » LIGO project deliverables funded by LIGO
- International Gravity Wave Network
  - » Interlaboratory agreements to share data for combined results
  - » Share in technology development
- International joint projects
  - » Government to government

# AVS 5 vs. Khoros 2

---

- Price
  - » AVS is \$7K - \$25K/user
  - » Khoros is "free"
- Support
  - » AVS offers commercial type support
  - » Khoros is informal and unpredictable
    - LIGO may have to hire support staff
- Distribution licenses
  - » AVS run-time modules cost \$
  - » Khoros executable modules free

LIGO considering selection within 1995

(LIGO has negotiated "community" price 40% lower)

# Aspen Discussions

---

- "Users Group Charter" strawman (Berley/BNL model) discussed
  - » "Users" changed to "Research Community"
  - » Changes made to Executive Committee and nominating process
  - » Revised draft charter agreed to at end of day
- Communique sent internationally by Syd Meshkov
  - » Conference proceedings and communique (L950365) available
  - » comments on role of LIGO Research Community
  - » comments on nominating process
  - » comments on composition, membership, and organization
  - » names submitted for Nominating Committee and LIGO Pre Program Advisory Committee
- Response was very supportive and constructive



## LIGO Research Community

---

The LIGO Research Community(LRC) is currently in an early stage of existence. The first steps towards its organization were made at the Aspen Winter Conference on Gravitational Waves and Their Detection, Jan. 22-28, 1995. A [communiqué](#) (postscript) describing what happened at the meeting was circulated shortly thereafter. As a result of the views expressed at the meeting, a [LIGO Pre-Program Advisory Committee\(LPPAC\)](#) was formed recently. News items that affect the LIGO Research Community will be posted in this part of the LIGO Home Page as they occur. The next major meeting of the LRC will be at the 1996 Aspen Winter Conference on Gravitational Waves, Jan. 15-21, 1996. A preliminary program for the meeting will also be posted here when it is formulated.

---

### News:

- We are currently gathering information for the [LIGO Research Community database](#). [New!](#)
  - Executive Committee: [Election results](#)
  - The Aspen Center for Physics announces the [Aspen Winter Conference on Gravitational Waves and Their Detection](#)
  - [Invitation to Join the LIGO Research Community](#)
  - [Charter of the LIGO Research Community](#)
  - [LPPAC Formation Announcement](#)
- 

return to the [LIGO Home page](#)

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last modified 12 Dec 95

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For information about LIGO, contact [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)

## External Users and Advisors

---

- LIGO involvement with the scientific community
  - » All who are interested in exploiting the scientific opportunities offered by LIGO
  - » Nominating Committee
  - » Executive Committee

- Pre Program Advisory Committee
- Program Advisory Committee
- External Advisory Committee

LIGO Decided to ask Pre-Program Advisory Committee to facilitate formation of EAC, PAC, LRC Ex. Comm.



## LPPAC Formation Announcement

---

At the Aspen Winter Conference on Gravitational Waves and their Detection in January, it was the sense of the meeting that a LIGO Pre-Program Advisory Committee (LPPAC) should be formed. Taking into account the many suggestions that have been received, this committee has been constituted with the following membership:

- P. Saulson (Syracuse) - Chair
- S. Finn (Northwestern)
- A. Giazotto (Pisa)
- J. Hall (JILA)
- W. Hamilton (LSU)
- C. Prescott (SLAC)
- A. Ruediger (MPI-Garching)

The LPPAC will exist only for a year or two. During its brief life it will act as both a LIGO Program Advisory Committee (LPAC) and as an External Advisory Committee (EAC). Before it goes out of existence it will help design a final LPAC and EAC. We hope to convene the first meeting some time in the fall.

At Aspen, it was also agreed that a Nominating Committee be formed to finalize a charter and organize the initial leadership for the LIGO Research Community, consisting of all scientists interested in LIGO's scientific opportunities. We propose that the LPPAC should play this role as well. This makes efficient use of the time of the members, and reflects the strong overlap in suggestions of members made to us by the research community contacted after the Aspen meeting. In the interim it will concern itself with:

1. Initial interferometer design technical advice
2. Commissioning Plan
3. Community Research Proposals
4. Establishing a LIGO Visitors Program, as well as a Users Program

As soon as the LPPAC meets, you will be informed about its deliberations, as well as about other activities.

We plan to hold another Aspen Winter Conference on Gravitational Waves from Jan. 15-21, 1996. The subject has not been finalized and we welcome your suggestions.

---

Return to the [LIGO Research Community page](#)  
Return to the [LIGO Home Page](#)

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*webmaster@ligo.caltech.edu / 15 Jul 95*



## Executive Committee: election results

---

The ballots for election to the Executive Committee of the LIGO Research Community have been tabulated. The Executive Committee has the following membership, listed in a ranked order by the number of votes received:

1. David Shoemaker
2. Sam Finn
3. Bill Hamilton
4. Harry Ward
5. Joan Centrella
6. Eric Gustafson
7. Bruce Allen

Thus Shoemaker, Finn, and Hamilton have won terms on the Executive Committee of three sessions; Ward and Centrella terms of two sessions; and Gustafson and Allen single session terms.

At a teleconference meeting of these seven members, they elected Sam Finn to be the Chair of the Executive Committee.

The next general meeting the LIGO Research Community and of the Executive Committee will be held at the Aspen Center for Physics Winter Conference on Gravitational Waves and Their Detection, Jan. 14-20, 1996, Aspen Colorado.

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Return to the [LIGO Home Page](#)  
Return to the [Research Community Home Page](#)

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*last modified 28 Nov 95*

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*For information about LIGO, contact [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)*





## Invitation to Join the LIGO Research Community

---

I am writing to invite you to join a new organization, called the LIGO Research Community, or LRC. It has two goals:

1. to facilitate communication between the Laser Interferometer Gravitational-Wave Observatory Project (LIGO) and the community of people interested in the field of gravitational wave detection, and
2. to serve as an advocacy group for the study of gravitational waves.

Membership in the LIGO Research Community is open to all persons interested in these goals. We expect this to include

1. experimental physicists and engineers working in any branch of gravitational wave detection, or who have an interest in the technology,
2. theoretical physicists, numerical relativists, and astronomers interested in astrophysical sources of gravitational waves or in what they may reveal about the universe,
3. students interested in these areas, and
4. anyone else.

Membership is open to people from all countries, without regard to affiliation (or lack thereof) with any gravitational wave project.

The organization has the acronym "LIGO" in its name because we expect it to serve as a kind of users' group for the LIGO Project. But LIGO needs advice from the widest possible community of actual and potential "users" of all kinds. In addition, we hope that the LIGO Research Community will play an important role in promoting and fostering the study of gravitational waves.

The first work toward creating the LRC was carried out by the LIGO Project, culminating in a series of discussions at the January 1995 Aspen Winter Conference on Gravitational Waves and Their Detection. A communic and draft charter were circulated after that meeting for comments. Final work on organizing the LIGO Research Community was performed by the LIGO Pre-Program Advisory Committee, which met on September 8 and 9, 1995.

Joining the LIGO Research Community is simple -- just send a message to [lrc@ligo.caltech.edu](mailto:lrc@ligo.caltech.edu), indicating your interest. Or, if you prefer, send regular paper mail to LIGO Research Community, c/o LIGO Project, Mail Code 51-33, California Institute of Technology, Pasadena, CA 91125, USA.

When you join, you should also vote for members of the Executive Committee of the LIGO Research Community. This is the governing body of the LRC. For detail on its role, and on other aspects of the functioning of the LRC, please see the [LIGO Research Community Charter](#), in the LaTeX file attached to the end of this message.

The Executive Committee will have seven members. A slate of nine nominees, broadly representative of the gravitational wave community, has been prepared by LIGO's Pre-Program Advisory Committee. The nominees for seats on the LRC Executive Committee are:

- Bruce Allen, Wisconsin-Milwaukee
- Joan Centrella, Drexel
- L. Sam Finn, Northwestern
- Eric Gustafson, Stanford
- William Hamilton, LSU
- C. Norman LIGO

- Robin T. Stebbins, JILA
- Henry Ward, Glasgow.

Please cast your votes by giving the seven names from this list whom you would most like to have serve on the Executive Committee. Include your votes in your message to [lrc@ligo.caltech.edu](mailto:lrc@ligo.caltech.edu).

In order for your vote to count, please send it before Friday, November 3. The seven who receive the most votes will be named to the Executive Committee. For further details, please refer to the LRC Charter, especially to Section 6.

Each year there will be at least one general membership meeting of the LIGO Research Community. The first such meeting will be held in Aspen, Colorado, as part of the Aspen Winter Conference on Gravitational Wave Sources and their Detection, January 15-20, 1996. For more information on this meeting, please contact Syd Meshkov at [syd@theory.caltech.edu](mailto:syd@theory.caltech.edu).

I hope you will want to join the LIGO Research Community, and that you will cast your ballot soon. If you have any questions, please contact me at [saulson@suhep.phy.syr.edu](mailto:saulson@suhep.phy.syr.edu).

Please share this message with anyone else who may be interested in joining the LIGO Research Community. This message will also be posted on the World Wide Web, via a link from the LIGO Home Page, at the location <http://ligo.caltech.edu>.

*Sincerely yours,*

*Peter Saulson*

*Chair, LIGO Pre-Program Advisory Committee  
for the Committee:*

*Sam Finn, Adalberto Giazotto, John Hall, William Hamilton, Charles Prescott, and Albrecht Ruediger*

---

Return to the Research Community Home Page  
Return to the LIGO Home Page

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17 Oct 95

For problems or suggestions about Web material, contact [webmaster@ligo.caltech.edu](mailto:webmaster@ligo.caltech.edu)  
For information about LIGO, contact [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)



# Charter of the LIGO Research Community

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September 26, 1995

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## Contents:

- 1> Name and Goals
  - 2> Membership
  - 3> Offices and Committees
  - 4> Procedures
  - 5> Amendments
  - 6> Exceptions
- 
- 

## 1> Name and Goals

### 1. Name

The name of this organization is the **LIGO Research Community**. The LIGO Research Community is also referred to as the LRC.

### 2. Goals

The goals of the LIGO Research Community are to

1. provide an organized channel for the interchange of information between the LIGO management and those who utilize the scientific opportunities afforded by LIGO;
  2. to serve as an advocacy body for the study of gravitational waves and related physics and astronomy.
- 
- 

## 2> Membership

# Discussions with VIRGO

---

- Barish/Sanders meeting with Giazotto/Brillet in March
- Broad agreement on need for intimate collaboration to optimize physics output of LIGO/VIRGO
- Draft of Memorandum of Understanding underway
- Agreement to exchange personnel, information, technical advice, technology
- Plan to form working groups on data collection, data analysis protocols, observing/maintenance cycles
- Possibility to jointly form LIGO/VIRGO Research Community with rotating meetings

# Memoranda of Understanding

---

- Standardized format
  - » MOU - broad areas of agreement, legal principles, description of general programs, no fixed term
  - » Attachments - written as needed with fixed terms and describing specific tasks, deliverables, dates and payments, if relevant. These may be rewritten and replaced without modifying the MOU
- NSF visibility/concurrence
  - » domestic, <\$100K - NSF provided copies
  - » domestic, >\$100K - NSF concurrence
  - » international, NSF concurrence

---

## Plans

---

- LIGO Research Community
- MOU's
- LIGO/VIRGO working groups
- LIGO Visitor's Program
- Program Advisory Committee
- External Advisory Committee

## Agenda: LIGO Research Community

Sam Finn

- What is the LRC?

Aims/Goals

Relationship to LIGO

Members and membership

- Questions/Discussion

What is LRC?

How does it operate?

- Coffee Break

- Defining a Research Environment:  
LIGO and the Grav. Physics Research Community

- Questions/Discussion

- Adjourn

## What is the LRC?

From Charter:

Provide an organized channel for the interchange of information between LIGO management and those individuals who utilize the scientific opportunities afforded by LIGO;

To serve as an advocacy body for the study of gravitational waves and related physics and astronomy

Examples:

Facilitate exchange within user community (hardware, data analysis, theory)

Two-way communication about design decisions today & science program decisions in future

To insure LIGO functions as part of an international community

The Voice of those who will "use" LIGO or are interested in its functioning!

## Relationship with LIGO

- Independent & Separate

## Members & Membership

- Who should want to be a member?

Anyone who...

Agrees with aims

Is interested in keeping abreast of LIGO plans

Wants to insure LIGO remains a healthy part of the international gravitational research community

- Who can be a member?

Anyone. The LRC is an organization of individuals

- How do I join?

E-Mail to: [LRC@LIGO.CALTECH.EDU](mailto:LRC@LIGO.CALTECH.EDU)



## Issue Identification Groups

- Sources, Data and Analysis

Chair: Joan Centrella

E-Mail: LRC...SDA@HOLMES.ASTRO.NWU.EDU

- Hardware Development/Installation/Operations

Chair: Eric Gustafson

E-Mail: LRC.HARDWARE@HOLMES.ASTRO.NWU.EDU

- People

Chair: Harry Ward

E-Mail: LRC...PEOPLE@HOLMES.ASTRO.NWU.EDU

- Chairs select group members from volunteers

Charge: Canvas Members  
Assemble Contributions  
Report to Executive Committee



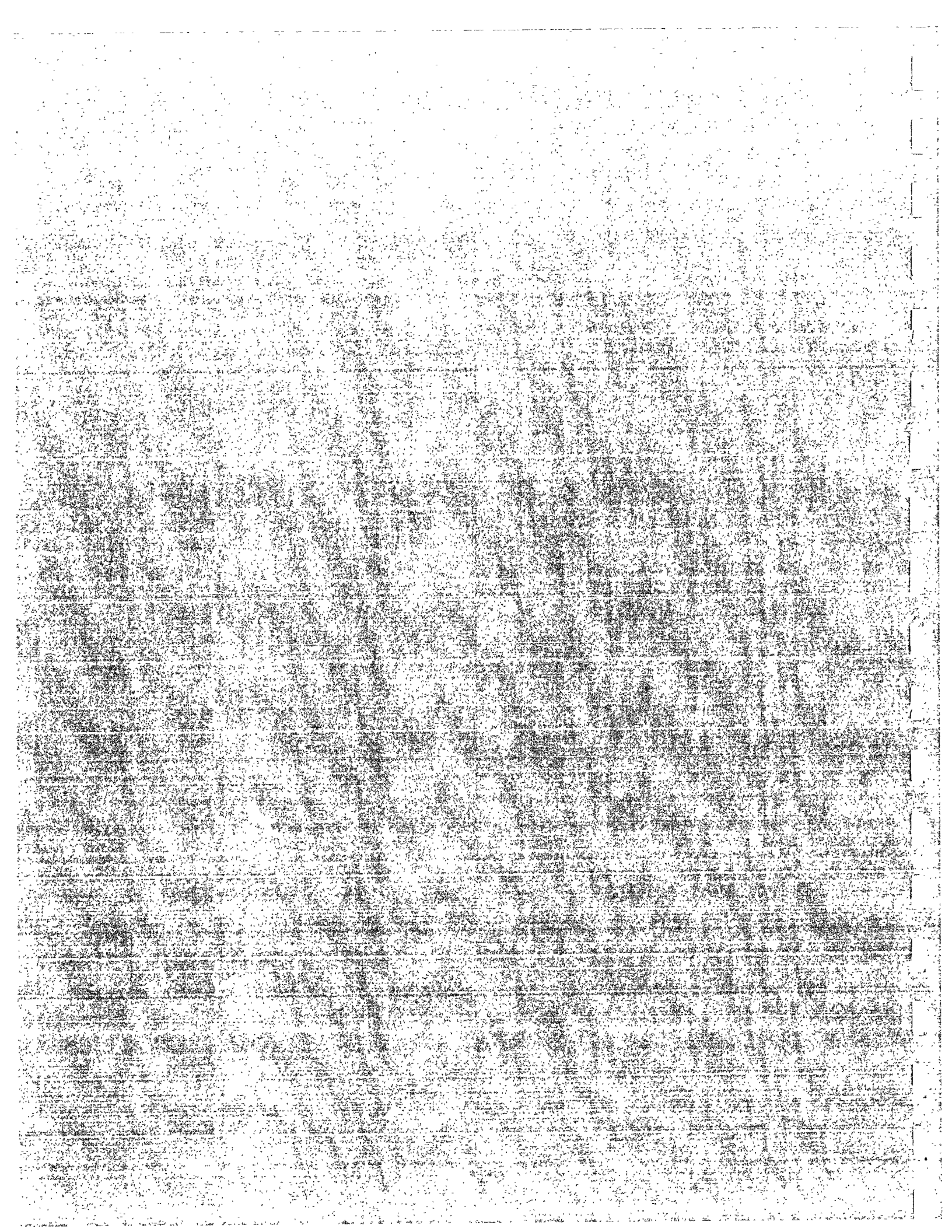


Wednesday PM Shot Noise; Detector Improvement  
January 17 Chair: D. Shoemaker

4:30 A. Ruediger (Garching) History and Derivation of Shot Noise Equations  
5:00 Discussion  
5:10 J. Hall (JILA) Dynamic Light-induced Mirror Birefringence  
5:40 Discussion

Wednesday Evening: Public Lecture, Wheeler Opera House  
Chair: S. Meshkov

8:00 L. Krauss The Physics of Star Trek  
(Case-Western Reserve)



# Shot Noise Equation <sup>Rüdiger</sup> ①

## History and Derivation

In laser-interferometric GW detectors  
fundamental limit set by shot noise

Recognized by pioneers Forward, Weiss (71/72)  
did first calculations

Since then, other authors  
copied  
interpreted  
estimated  
calculated anew.

Led to host of — sadly to say —  
spectrum of differing equations

wide spread in numerical factors ②  
started idea to derive, write up, publish  
"correct equation"

### Shot Noise in Interferometers used for gravitational wave measurements

A. Rüdiger, J. Mizuno, R. Schilling, W. Winkler, and K. Danzmann\*

Max-Planck-Institut für Quantenoptik, D-85748 Garching  
(\* also Universität Hannover, D-30167 Hannover)

#### Abstract

The phenomenon of shot noise sets a severe limitation to the sensitivity of (laser illuminated) Michelson-type interferometers, as used or proposed for the detection of gravitational waves. Despite the high significance of this noise source, and though the general scaling laws with light power  $P$ , wave length  $\lambda$ , detector efficiency  $\eta$ , etc., are generally agreed upon, there is a wide variety of different numerical factors used in describing the precise magnitude of the effect. This paper is intended to clarify the correct magnitude, once and for all. The result will be an apparent fluctuation  $\delta L$  in the path length difference  $\Delta L = L_2 - L_1$  that is described by a single-sided (power) spectral density of

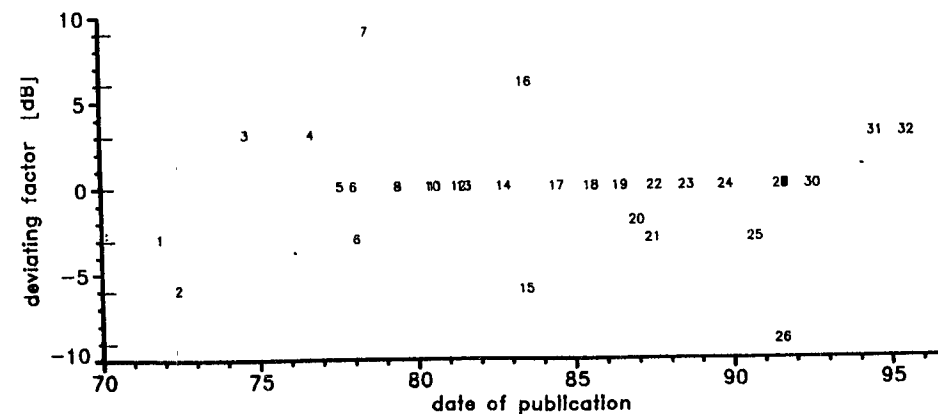
$$S_{\delta L}(f) = (\overline{\delta L(f)})^2 = \frac{\hbar c}{\pi} \frac{\lambda}{\eta P}$$

## 1 Introduction

### 1.1 A historical survey

In the many (almost 25) years since first publications on laser interferometers for the detection of gravitational waves, it has been recognized that *shot noise* would be one of the most fundamental and most stringent limitations to achieving the high sensitivities required to detect the astrophysical sources of interest.

It is quite natural that this shot noise limit has found appropriate attention in almost all of the early papers and proposals, and also later in textbooks, review articles, and proposals for large scale antennas.



plans for write-up on shot noise

Mentioned to Syd Meshkov

--- promptly recruited me for  
talk at Aspen Winter Conference  
--- and I agreed.

But then

Because:

more to it than just "write it up":  
different derivations  
different formulations  
wide variety of different cases  
much literature search

Yet:

usefulness of such a collection,  
if only to save others time and pain in  
deriving equations anew  
working through conflicting sources  
perhaps stumbling into pitfalls  
before finding the true answer,  
or answer to particular case.

Write-up

Found no time [LISA!]

to tackle this ambitious program

Put before you only skeleton,  
flesh to be added in next months:

careful derivations  
discussion of different approaches  
clear statements of validity  
consistent nomenclature, notation

Request to all interested for assistance.

Nomenclature, notation important  
to prevent misinterpretations

make lavish use of

= geometric arm length

= total optical path =

$N$  = number of passes in DL

(elsewhere:  $N = \cdot \cdot$ ,  $2N = \cdot \cdot$ ,  $b = N$ ;  $b = N - 1$ )

$\mathcal{P}$  = single-sided power spectral density of  $\mathcal{P}$

$\mathcal{P}_L = \sqrt{\quad}$  = "linear" spectral density of  $\mathcal{P}$

$\Lambda = \frac{c}{f_{GW}}$  = wave length of GW

## One simple derivation (5)

Michelson interferometer (simple, DL)  
Response to gravitational wave  $h$

$$\delta L = h \cdot L$$

Effect of path difference fluctuation  $\delta L$   
on photo current:

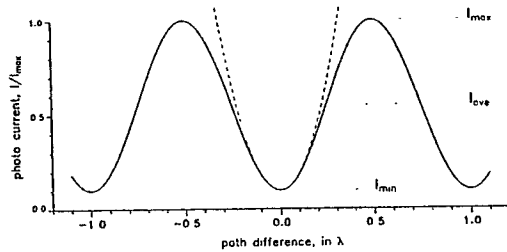


Fig. 3. Output current of single photodiode, vs. phase difference  $\Delta\phi$ .

mid-slope detection scheme:

$$\begin{aligned} \delta I &= \frac{I_{\max} - I_{\min}}{2} \cdot \sin \delta\phi \\ &\approx \frac{I_{\max} - I_{\min}}{2} \cdot \delta L \cdot \frac{2\pi}{\lambda} \end{aligned}$$

Schottky equation for (Poisson distributed) current  $I_{\text{ave}}$

$$\delta I(f) = \sqrt{2e I_{\text{ave}}} = \sqrt{2e \frac{I_{\max} + I_{\min}}{2}}$$

by equating the  $\delta I$ , and with  $I = \eta \cdot P \cdot \frac{e}{h\nu}$ :

$$\delta L_1(f) = \sqrt{2 \frac{hc}{\pi} \cdot \frac{\lambda}{\eta P}} \cdot \sqrt{\frac{1+\mu}{(1-\mu)^2}}$$

$I_{\min} \sim 1$

Result was: (6)

$$\delta L_1(f) \approx \sqrt{2 \frac{hc}{\pi} \cdot \frac{\lambda}{\eta P}} \cdot \sqrt{\frac{1+\mu}{(1-\mu)^2}}$$

Perfectly legal derivation,  
but result not yet complete:

Can do measurements at  
both output ports:  
take the difference of the two:  
signal response doubled

Shot noise contributions  
add only in quadrature.

slightly counter-intuitive  
might expect anti-correlation

but Poissonian distribution is "robust"

By monitoring both output ports

$$\delta L(f) = \sqrt{\frac{hc}{\pi} \cdot \frac{\lambda}{\eta P}} \times \left( \text{QI-dependent correction, } \geq 1 \right)$$

e.g. =  $\sqrt{\frac{1+\mu}{(1-\mu)^2}}$

for mid-slope scheme

QI = quality  
of interference

have to consider

main concern

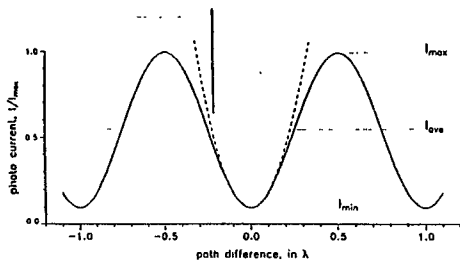


Fig. 3. Output current of single photodiode, vs. phase difference  $\Delta\phi$ .

dark-fringe scheme

prerequisite for power recycling

$$I(\Delta\phi) = I_{min} + (I_{max} - I_{min}) \cdot \sin^2\left(\frac{\Delta\phi}{2}\right)$$

needs modulation to read out GW signal

simplest case:  $I_{min} = 0$

then for purely quadratic function (parabola)  
same as for mid-slope with  $\mu=0$ .

for realistic sinusoidal response  
same only for small modulation

With finite dark-fringe current  $I_{min}$ :

optimum modulation swing  
is function of  $I_{min}$

furthermore have to specify  
method and wave shape of modulation

$$G_{min} = \sqrt{\frac{2e}{(1-L)SIL}} \cdot E \cdot F$$

General		Without Recycling	With Recycling
		$F = 1$ for $M=0$	$F = \sqrt{L}$ for $M=0$
INTERNAL	Sine wave	$E = 1.0$	---
	Square wave	$E = \frac{I}{I_{\text{avg}}} = 1.111$	---
EXTERNAL	Sine wave	$E = \frac{1}{\sqrt{2} \cdot I_{\text{avg}}} = 1.215$	$F = \frac{1 + \sqrt{M(1-L)S}}{\sqrt{1-2M}}$
	Square wave	$E = \frac{I}{I_{\text{avg}}} = 1.111$	$F_{\text{ext}} = \frac{\sqrt{M(1-L)S}}{\sqrt{L+M(1-L)S} + \sqrt{M(1-L)S}}$
	SSB (A=0)	$E = \sqrt{2} = 1.414$	$F_{\text{int}} = \frac{\sqrt{M(1-L)S} [L+M(1-L)S]}{1 - [L+M(1-L)S]}$

Gain in light power:  $G_p = 10 \cdot \log \frac{P_r}{P_i}$

Gain in sensitivity:  $G_s = 20 \cdot \log \frac{\sqrt{(1-L)S}}{E \cdot F}$

Get back to front factor

(9)

$$S = \frac{hc}{\pi} \cdot \frac{\lambda}{\eta P}$$

In literature observe some diversity in numerical factor to it.

Why is that?

Many pitfalls, particularly factors of 2 (in power representation) lurking

e.g.

- consider only one mid-slope photo detector
- $h = 2\delta l/l$  as opposed to  $h = \delta L/L$
- in derivation via number of photons,  $n = \frac{P \cdot \tau}{h\nu}$ :  
from measurement time  $\tau$  to bandwidth:  $\Delta f = 1/2\tau$
- number of bounces  $b = N$ , but also  $b = N/2$
- simple Michelson:  $N = 2$
- ... and many more

(10)

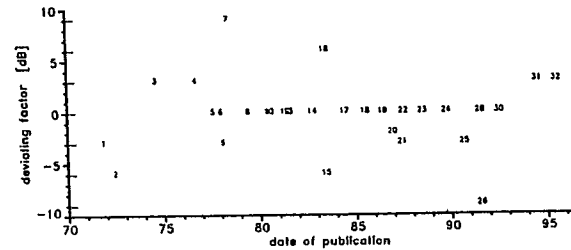


Fig. 1. Semigraphical display of the deviating numerical factors in some selected publications, presented in logarithmic scale (dB). Owing to its definition, the presentation in these units of dB covers the comparison in "amplitude" ( $20 \cdot \log \frac{a}{a_0}$ ) as well as in "power" ( $10 \cdot \log \frac{P}{P_0}$ ) spectral densities. The ominous factors of 2 in power (or  $\sqrt{2}$  in amplitude) correspond to a distance of 3 dB. With the exception of one reference, [20], this one having a deviation factor  $\pi^2/16$  in power (2.1 dB), all factors are off by integer multiples of 3 dB.

and truly enough:

nearly all deviations from true value  
are integer powers of 2 (in power)  
or integer powers of  $\sqrt{2}$  (in amplitude)

but important

observe carefully what author meant

Important: quantum-mechanical approach  
(C. Caves)

11

Other topics to be discussed

⋮

### 3.3 Experimental evidence, for MP interferometers

- ... calibration with DC incandescent light
- ... Shoemaker 38, calibration, 250 mW (?), measured 86

- ... very close, only a few dB off
- ... Niebauer: even closer
- ... would be in severe need of explanation

← Schnupp: Non-stationary shot noise

### 4 Fabry-Perot cavities in the arms

- ... not terribly different from DL,
- ... requires phase response with  $\mathcal{F}$

$$N_{\text{eff}} \approx \frac{4}{\pi} \mathcal{F}^2$$

$$\mathcal{F} = \text{finesse}$$

particularly for FP:  
assistance from other groups required

12

## 5 Other Michelson configurations

### 5.1 Advanced interferometers (ground-based)

#### 5.1.1 Power recycling

- ... assumed in several of the papers
- ...  $(1 - R)$  perhaps oversimplified

#### 5.1.2 Signal recycling

#### 5.1.3 Resonant sideband extraction

### 5.2 Shot noise theorem

← Jun Mizuno

- ... not make this central part of this paper
- ... will be topic of further paper

### 5.3 Squeezed light

- ... discussed by Caves
- ... investigated ...
- ... no significant improvement expected
- ... Kimble (Aspen)

### 5.4 Interferometers in space

- ... also goes back to early proposals, [...]
- ... only recently coming within reach [...]

#### 5.4.1 Space projects

- ... configuration, long distances,
- ... requires active transponders instead of mirrors
- ... extremely low light power

#### 5.4.2 Shot noise in LISA

- ... David Robertson ?
- ... 60° ?



The "Shot Noise Theorem"  
(Jun Mizuno)

(13)

$$\tilde{h} = \frac{\delta \mathcal{L}}{\mathcal{L}} = \frac{1}{N \cdot e} \cdot \sqrt{\frac{hc}{\pi} \cdot \frac{\lambda}{\eta P_{BS}}}$$
$$= \sqrt{\frac{hc}{\pi} \cdot \frac{\lambda}{\eta P_{BS} \cdot (N \cdot e)^2}}$$

$$\tilde{h} = \sqrt{2 \frac{hc}{\pi c} \cdot \frac{\Delta f}{\eta \mathcal{E}}}$$

$\mathcal{E}$  = stored energy

$$\Delta f = \text{"bandwidth"} = \frac{c}{2N \cdot e}$$

covers very wide variety of  
interferometer configurations

DL, FP

recycled (power, signal, RSE)

Question:

(14)

would such a write-up be useful?  
does something already exist?

how far into details, cases  
should it go?

would take not only my time,  
but also that of others.

Is it worth that?

# "Dynamic Photo-refractive Birefringence in Gyro/LIGO quality Mirrors"

Jun Ye  
John Hall  
JILA · NIST & University of Colorado

a Glitch on the long road to "Measurement  
of the Magnetically-Induced Birefringence  
of the Vacuum"

\* S.A. Lee  
W. M. Fairbank, Jr  
W. H. Toki } Colorado State Univ

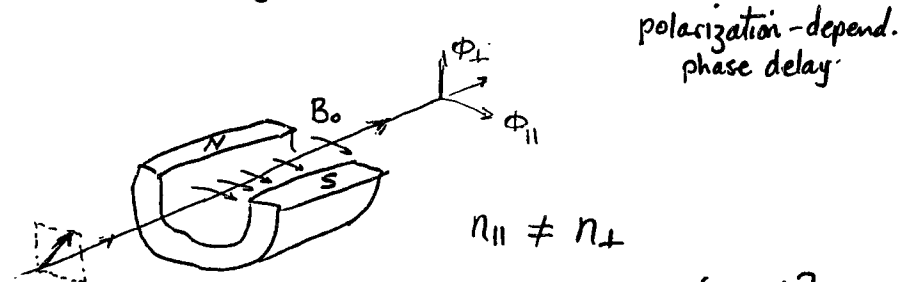
Tariq S. Jaffery - SSC

Frank Nezrick  
P. Colestock  
H. Kautzky  
V. Cupps  
M. Kuchnir } Fermi Labs

HAR

JILA

## Magnetically-Induced Birefringence



$$\Delta n \equiv n_{\parallel} - n_{\perp} = \left(\frac{\alpha}{30\pi}\right) \left(\frac{B}{B_{\text{cr}}}\right)^2$$

where  $B_{\text{cr}} = 4.4 \times 10^9 \text{ T}$

$$\psi_{\text{QED}} = \Phi_{\parallel} - \Phi_{\perp} = \left(\frac{\alpha}{30}\right) \left(\frac{B}{B_{\text{cr}}}\right)^2 \frac{L N}{\lambda}$$

for  $B = 6 \text{ Tesla}$

$$\rightarrow \Delta n = 1.4 \times 10^{-22} \leftarrow$$

---

E. Zavattini, A. Melissinos, F. Nezrick .....

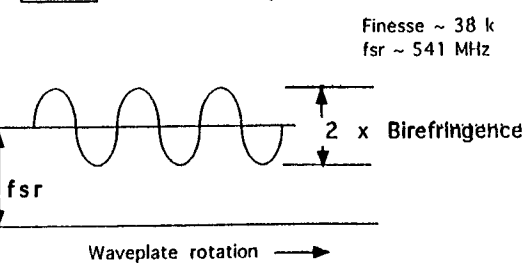
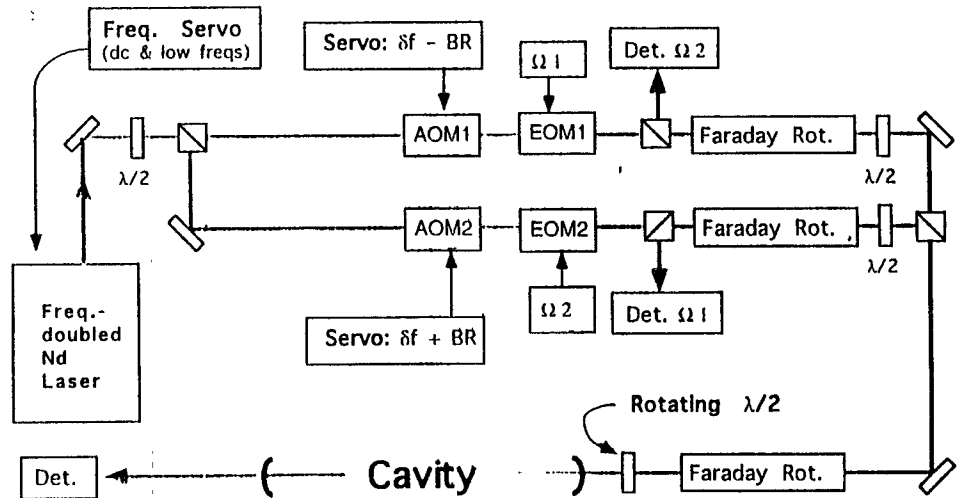
Phys Rev D 1994

"The Brookhaven Experiment"

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E. Zavattini, Carlo Rizzo .....

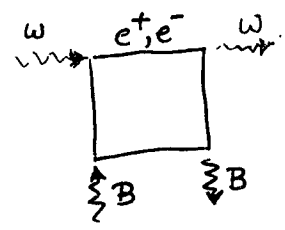
a new expt. in Legnaro - starts ~1997  
(using rotating SC magnet)



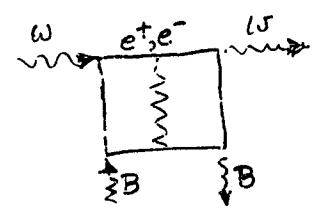
### Testing Electro-optic measurement techniques for QED / Axion expt

Jun Ye & John Hall - JILA Oct 94

### QED Origin of Magnetic Effect



Dominant Term  
"Light-by-light" scattering



1.45% Radiative Correction

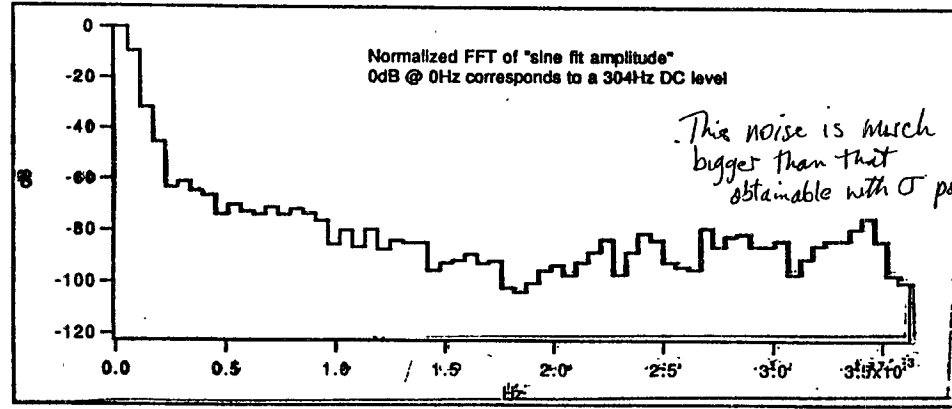
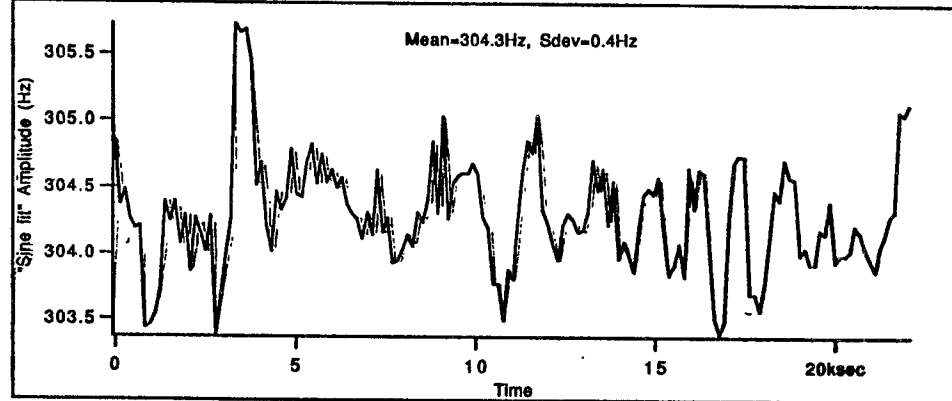
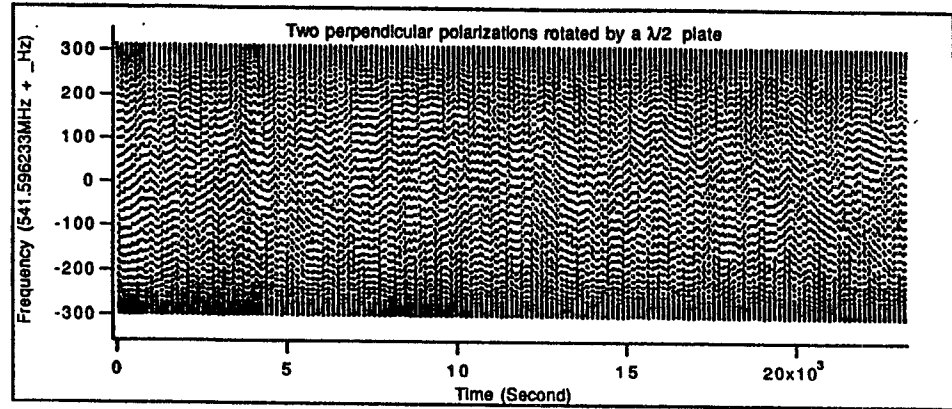
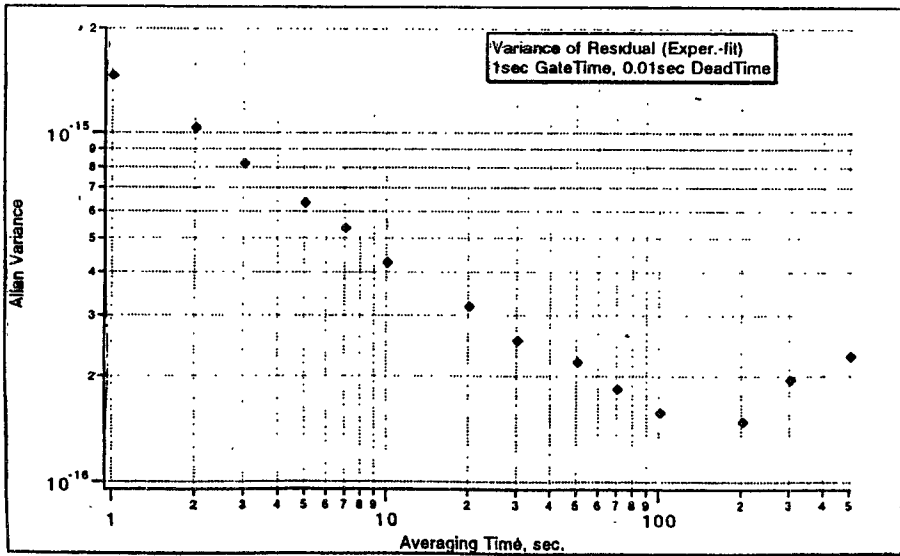
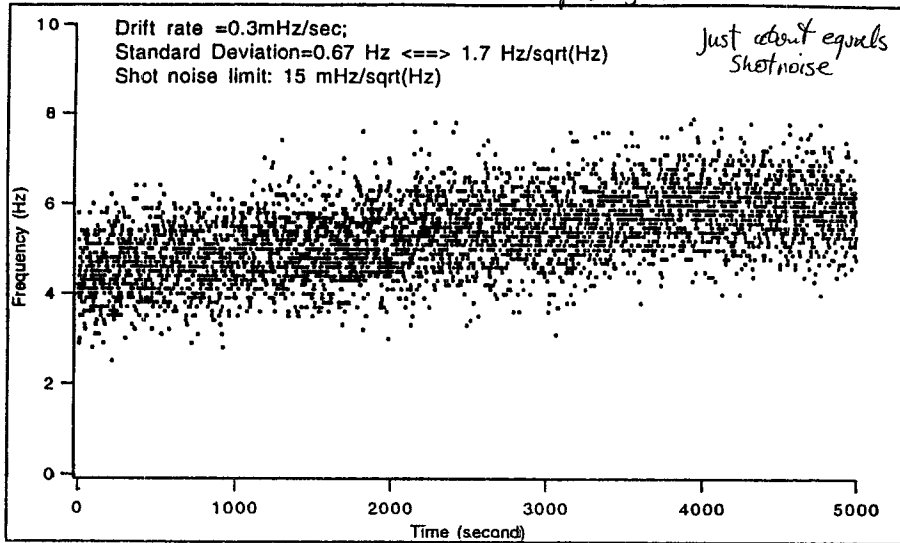
(1936) Euler-Heisenberg Lagrangian for EM field in vac.

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \left(\frac{2\alpha^2}{45m_e^4}\right) [(\mathbf{E}^2 - \mathbf{B}^2)^2 + 7(\mathbf{E} \cdot \mathbf{B})^2]$$

lowest order QED term

Weisskopf 1936  
Schwinger 1951

*Circular polarization*



What does this mean?

Noise is bigger with linear polarization by ~5 fold. Why?

Static birefringence 300 Hz

1 fsc → 540 MHz

5.5 ppm of 1 order

~ 0.9 microradians phase diff/bounce

Noise of "static" birefringence: -100dB

→ 9 picoradians phase noise

effective measurement time  $2 \times 10^4$  sec

equiv 3 nanorads /  $\sqrt{\text{Hz}}$

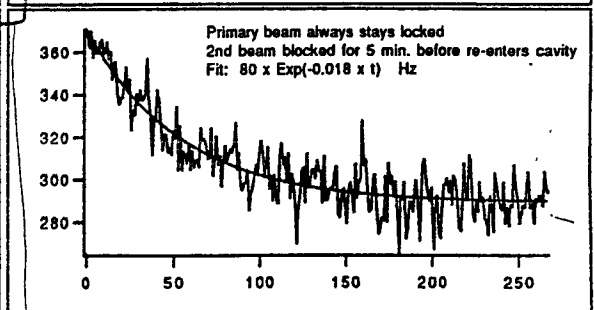
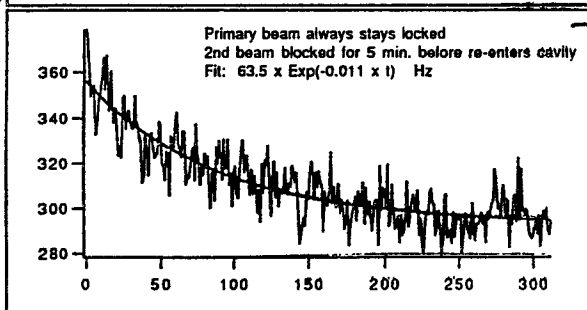
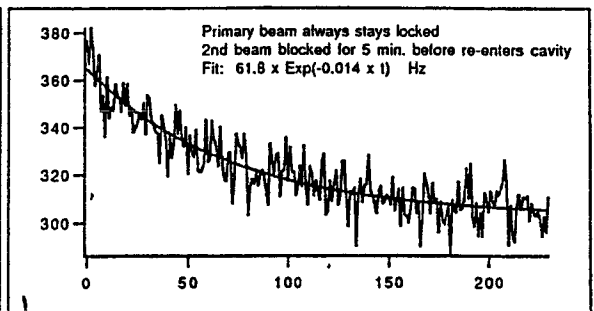
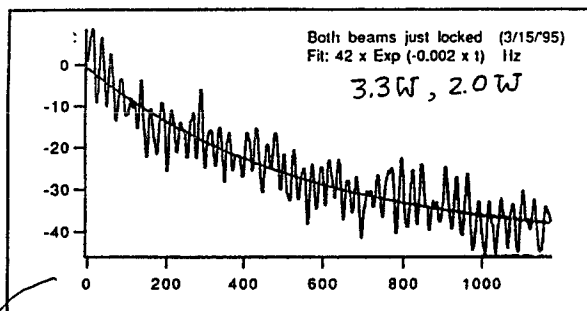
Frey Equivalents

Noise is ~ 3 milliHertz - This is shot noise at 1 sec NOT at 20,000 s

QED signal  $\frac{\Delta f}{f} = \frac{\Delta N}{N} = 10^{-22} \times 1.4$

→ 80 NANO Hertz

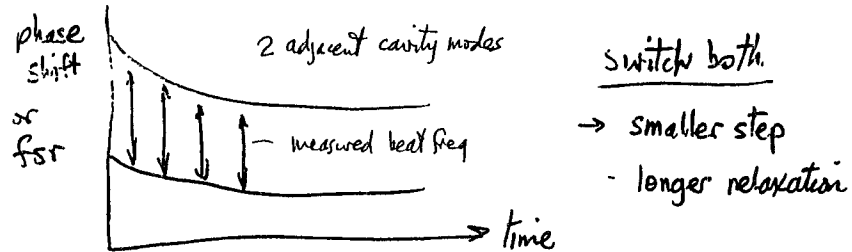
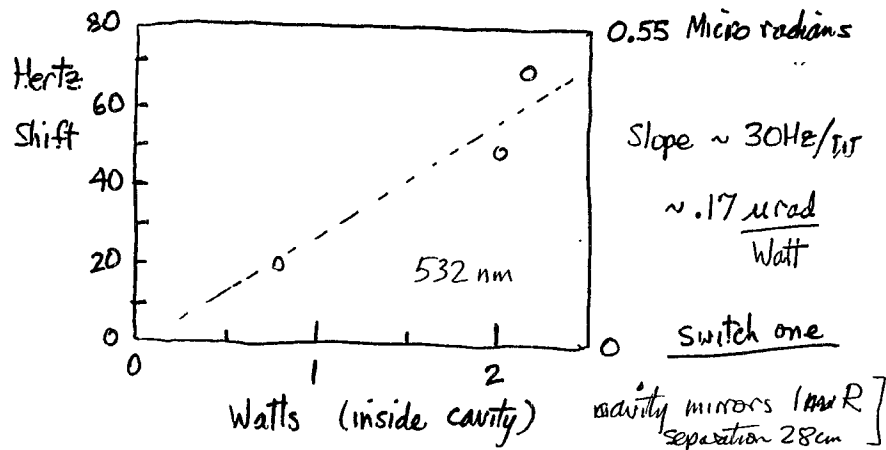
Time Dependence?  
Power Dependence?  
PROBLEMS HERE



Both Beams Switched  
jump ~ 40 Hz  
relax time ~ 500 sec

jump ~ 70 Hz 0.4  $\mu$ rods  
relaxation rate ~ 0.014  $s^{-1}$  (70 sec)

One Beam Switched (~ 2W)



Mirrors coated by Ojai Research  $\sim 1986$   
 $T \approx 25 \text{ ppm}$   $A+S \approx 50 \text{ ppm}$   $F \approx 42,000$

Use  $\sigma$ -pol  $\rightarrow$  UNDETECTABLE SMALL EFFECT  
 crossed linear polarization  $\rightarrow$  dynamic BR  
 photorefractivity; orientationally-aware traps  
 $\rightarrow$  Need 2-sets of 2-level systems?

### Bottom Line

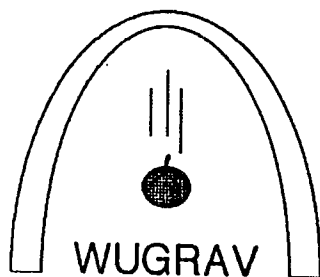
1. Mirrors know if they have been fluxed with linear polarized light.
2. Circular polarization can "erase".
3. some evidence that relaxation time is power dependent
4. Weak depolarized component in LIGO may give non-thermal time-dependent contrast
5. May be possibility of mode coupling in high Q systems

Thursday AM      Data Analysis I. Search for Compact Object Binaries  
January 18      Chair: S. Finn (Northwestern)

- 8:00      C. Will (Washington U)      Compact Binary Inspiral: A Post-Newtonian Playground
- 8:25      Discussion
- 8:35      J. Wilson (LLNL)      Neutron Star Binaries Hydrodynamics
- 9:00      Discussion
- 9:10      A. Wiseman (Caltech)      Final Wave Forms for Binary Inspiral
- 9:35      Discussion
- 9:45      Coffee Break
- 10:00      K. Thorne (Caltech)      The Final Merger of Compact Binaries: Information Content, Waveform Computations, and Data Analysis Changes
- 10:25      Discussion
- 10:35      R. Matzner (UT-Austin)      Grand Challenge Update
- 11:00      Discussion

# COMPACT BINARY INSPIRAL: A POST-NEWTONIAN PLAYGROUND

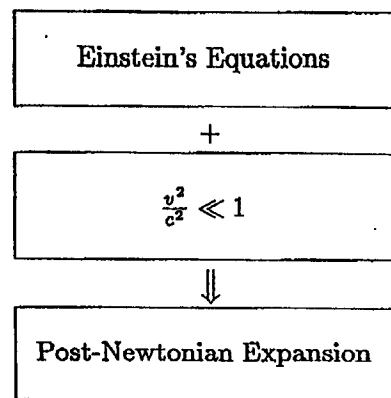
Clifford M. Will



WASHINGTON UNIVERSITY GRAVITATION GROUP

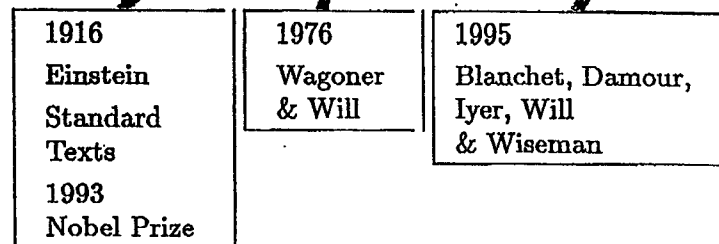
- The Challenge to Post-Newtonian Theorists
- An Extended Epstein-Wagoner Framework
  - *C. M. Will & A. G. Wiseman (Caltech)*
- The BDI post-Minkowskian Framework
  - *L. Blanchet, T. Damour & B. R. Iyer*
- The Struggle for Higher Orders
  - How convergent is the PN Series?

## A Post-Newtonian Playground



Evolution of orbital frequency caused by  
gravitational radiation energy-loss

$$\frac{df}{dt} = \left(\frac{df}{dt}\right)_q \left\{ 1 + O\left(\frac{v^2}{c^2}\right) + O\left(\frac{v^3}{c^3}\right) + O\left(\frac{v^4}{c^4}\right) \dots \right\}$$





## THE RELAXED EINSTEIN EQUATIONS

Field Definition:

$$h^{\alpha\beta} = \eta^{\alpha\beta} - \sqrt{-g}g^{\alpha\beta}$$

Harmonic Coordinate Condition:

$$\partial_\beta h^{\alpha\beta} = 0$$

"Reduced" Field Equations:

$$\square h^{\alpha\beta} = -16\pi(-g)T^{\alpha\beta} - \Lambda^{\alpha\beta}(h) \equiv -16\pi\tau^{\alpha\beta}$$

$$\partial_\beta \tau^{\alpha\beta} = 0$$

Formal Solution:

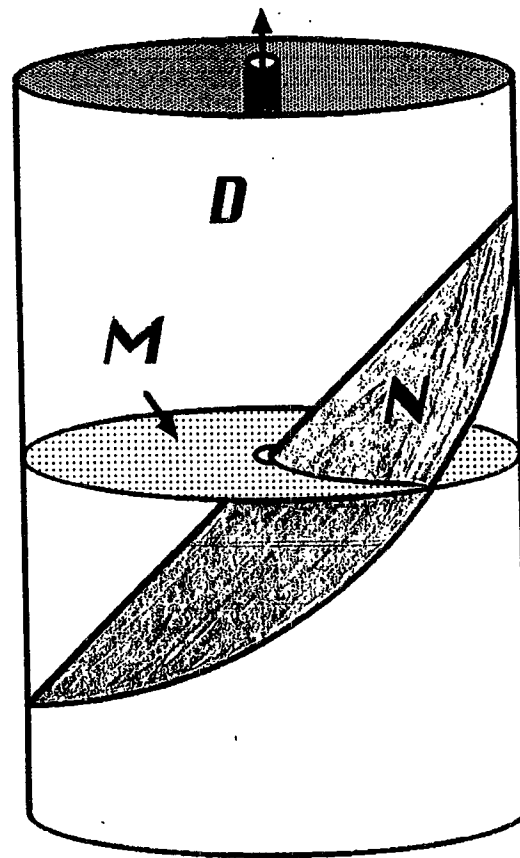
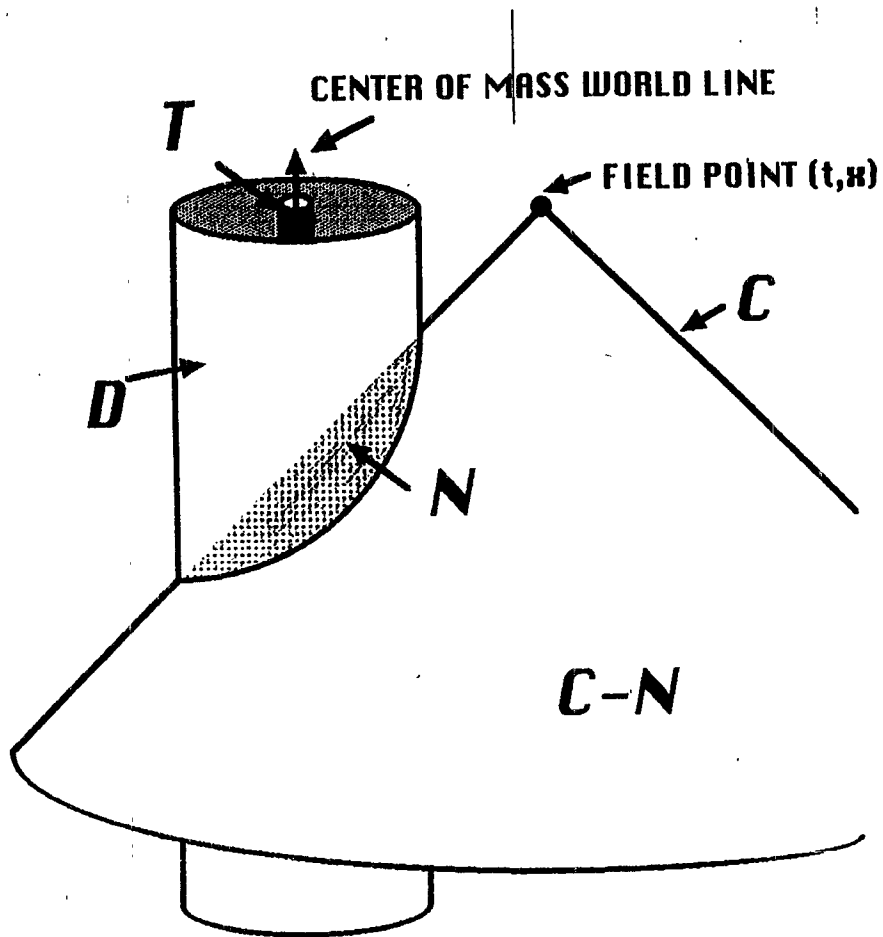
$$h^{\alpha\beta} = 4 \int_{\mathcal{C}} \frac{\tau^{\alpha\beta}(t', \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \delta(t' - t + |\mathbf{x} - \mathbf{x}'|) d^4 x'$$

Divide into Inner ( $\mathcal{N} = \{|\mathbf{x}'| < \mathcal{R}\}$ ) and Outer ( $\mathcal{C} - \mathcal{N} = \{|\mathbf{x}'| > \mathcal{R}\}$ )

integrals, where  $\mathcal{R} \sim \lambda$ .

Characteristics of Inspiralling Binaries

Type of Binary	$2 \times 1.4M_\odot$	$10M_\odot + 1.4M_\odot$	$2 \times 10M_\odot$
$r/m$ when $f=10\text{Hz}$	180	70	47
$v/c$ when $f=10\text{Hz}$	0.07	0.12	0.15
$r/m$ when $f=1000\text{Hz}$	8	-	-
$v/c$ when $f=1000\text{Hz}$	0.3	-	-
$f$ at coalescence (Hz)	1,400	360	190
Time to coalescence (s)	1,030	270	40
Number of GW cycles to coalescence	16,500	3,520	600



## RADIATION ZONE SOLUTION: NEAR ZONE INTEGRAL

$$R \gg \mathcal{R}$$

Radiative Field:

$$h_{\mathcal{N}}^{ij}(t, \mathbf{x}) = \frac{4}{R} \sum_{m=0}^{\infty} \frac{1}{m!} \frac{\partial^m}{\partial t^m} \int_{\mathcal{M}} \tau^{ij}(u, \mathbf{x}') (\hat{\mathbf{N}} \cdot \mathbf{x}')^m d^3 x' + O(R^{-2})$$

$$u \equiv t - R$$

Harmonic Identities ( $\tau^{\alpha\beta}_{, \beta} = 0$ ):

$$\tau^{ij} = \frac{1}{2} (\tau^{00} x^i x^j)_{,00} + 2(\tau^{k(i} x^{j)})_{,k} - \frac{1}{2} (\tau^{kl} x^i x^j)_{,kl}$$

$$\tau^{ij} x^k = \frac{1}{2} (2\tau^{0(i} x^{j)}) x^k - \tau^{0k} x^i x^j)_{,0} + (\tau^{l(i} x^{j)}) x^k - \frac{1}{2} \tau^{lk} x^i x^j)_{,l}$$

Multipole Expansion of Radiative Field:

$$h_{\mathcal{N}}^{ij} = \frac{2}{R} \frac{d^2}{dt^2} \sum_{m=0}^{\infty} \hat{N}_{k_1} \dots \hat{N}_{k_m} I_{EW}^{ijk_1 \dots k_m}(u)$$

Epstein-Wagoner Moments:

$$I_{EW}^{ij} = \int_{\mathcal{M}} \tau^{00} x^i x^j d^3 x + I_{SURF}^{ij}$$

$$I_{EW}^{ijk} = \int_{\mathcal{M}} (2\tau^{0(i} x^{j)}) x^k - \tau^{0k} x^i x^j) d^3 x + I_{SURF}^{ijk}$$

$$I_{EW}^{ijkl} = \int_{\mathcal{M}} \tau^{ij} x^k x^l d^3 x$$

$$I_{EW}^{ijk_1 \dots k_m} = \frac{2}{m!} \frac{d^{m-2}}{dt^{m-2}} \int_{\mathcal{M}} \tau^{ij} x^{k_1} \dots x^{k_m} d^3 x$$

## RADIATION ZONE SOLUTION: RADIATION ZONE INTEGRAL

Change Variables:

$$u' \equiv t' - r'$$

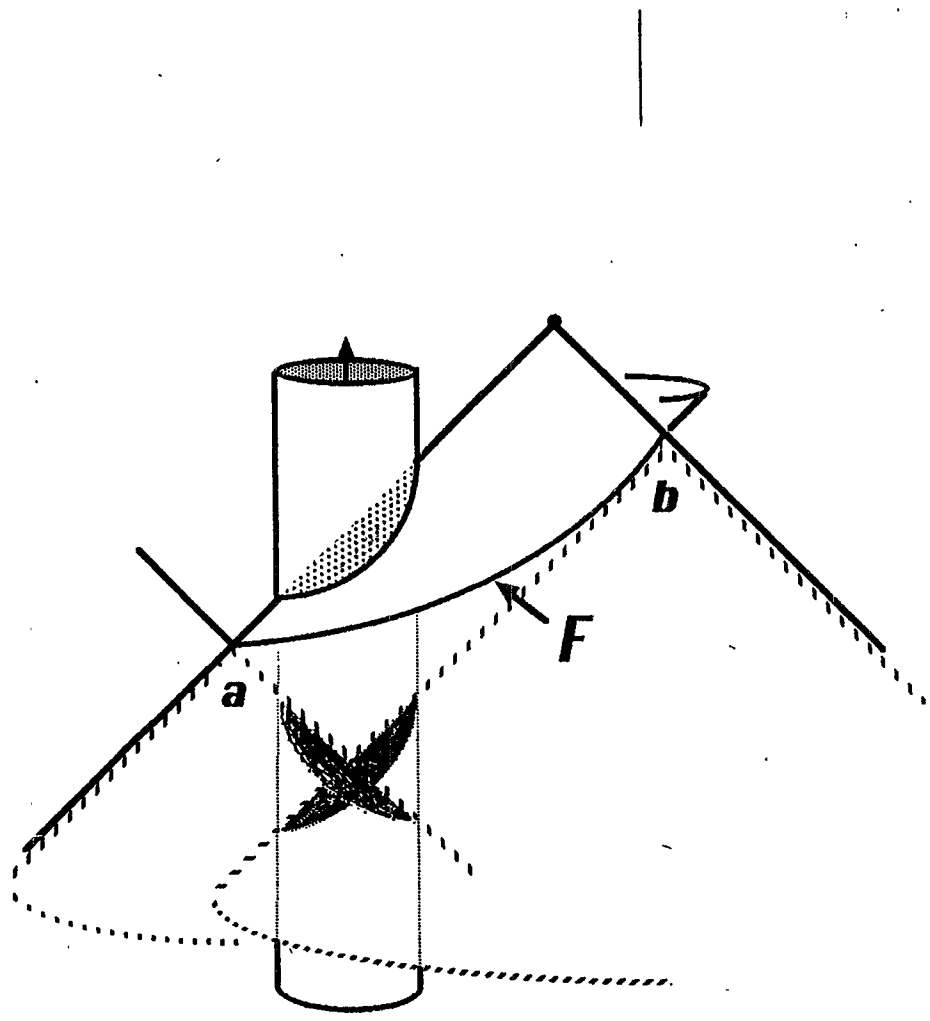
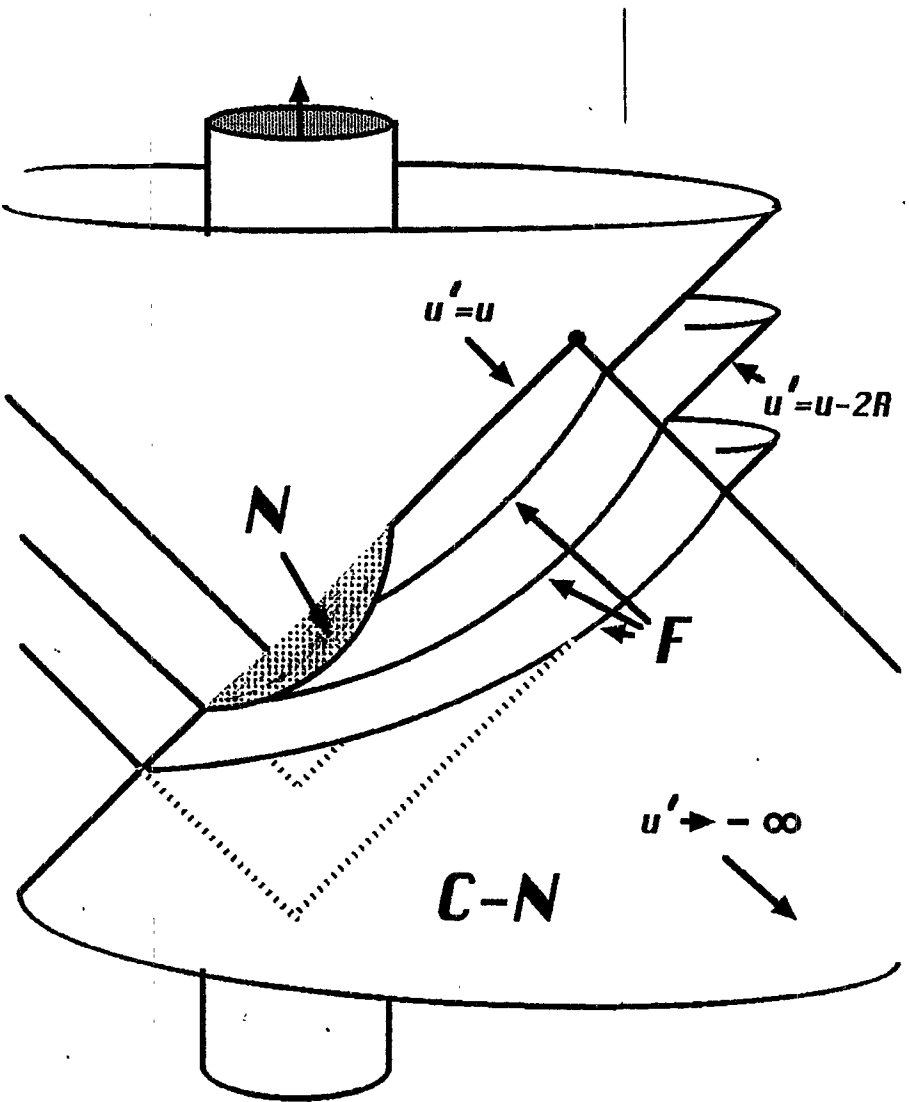
Radiative Field:

$$h_{TT}^{ij} = 4 \int_{-\infty}^u du' \int \frac{\tau^{ij}(u' + r', \mathbf{x}')}{t - u' - \mathbf{n}' \cdot \mathbf{x}} [r'(u', \Omega')]^2 d^2 \Omega'$$

where

$$t - u' = r' + |\mathbf{x} - \mathbf{x}'|$$

$$r'(u', \Omega') = \frac{r^2 - (t - u')^2}{2(\mathbf{n}' \cdot \mathbf{x} - t + u')}$$



## NEAR ZONE SOLUTION

$$|\mathbf{x}| < \mathcal{R}$$

Source Densities:

$$\sigma = T^{00} + \sum_i T^{ii}$$

$$\sigma_i = T^{0i}$$

$$\sigma_{ij} = T^{ij}$$

Expansion Parameter:

$$\epsilon \sim v^2 \sim (m/r)$$

Retarded Potentials:

$$V(t, \mathbf{x}) \equiv \int_c \frac{d^3 x'}{|\mathbf{x} - \mathbf{x}'|} \sigma(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}')$$

$$\dot{V}_i(t, \mathbf{x}) \equiv \int_c \frac{d^3 x'}{|\mathbf{x} - \mathbf{x}'|} \sigma_i(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}')$$

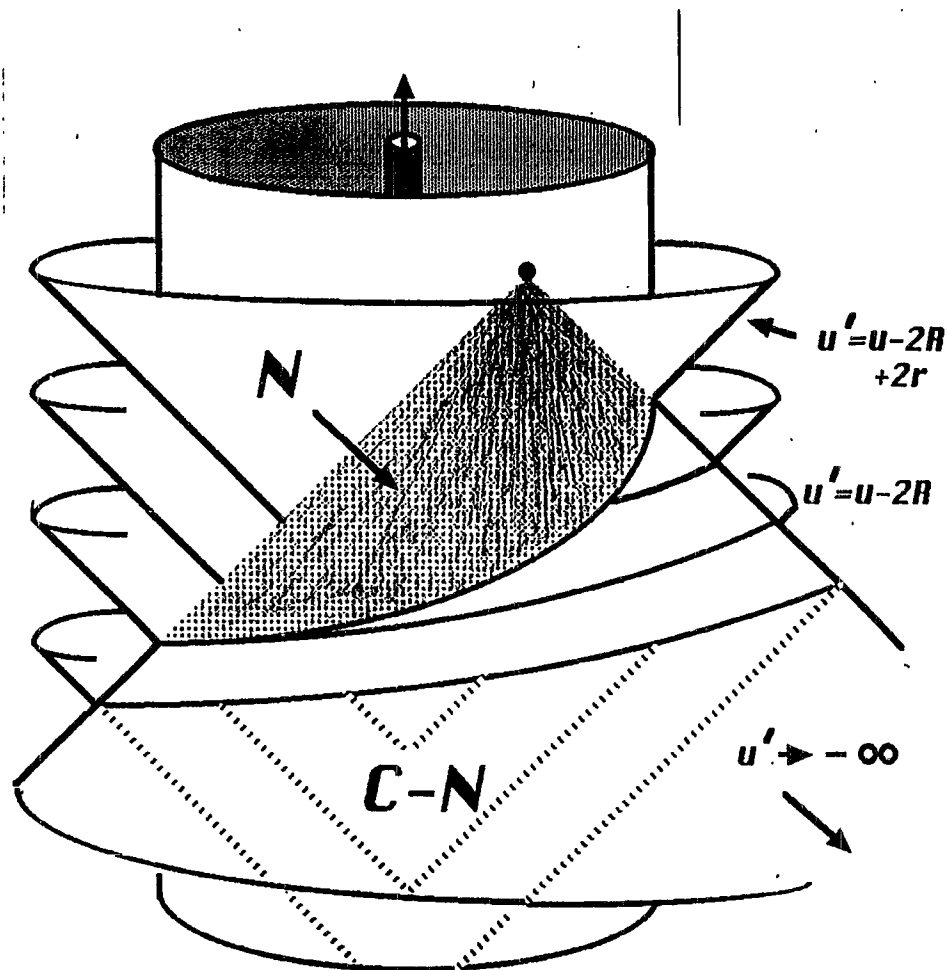
$$W_{ij}(t, \mathbf{x}) \equiv \int_c \frac{d^3 x'}{|\mathbf{x} - \mathbf{x}'|} \left[ \sigma_{ij} + \frac{1}{4\pi} (V_{,i} V_{,j} - \frac{1}{2} \delta_{ij} V_{,k} V_{,k}) \right] (t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}')$$

Near Zone Field

$$h^{00} = -4V + 4(W_{ii} - 2V^2) + O(3)$$

$$h^{0i} = -4V_i + O(5/2)$$

$$h^{ij} = -4W_{ij} + O(3)$$



## RESULTS OF EXTENDED EW FRAMEWORK

- **Finite and Well-defined**

- \* Outer integrals convergent
- \* No infinite or ill-defined terms

$h_{\mathcal{N}}^{ij}(t, \mathbf{x}) = \text{finite EW terms} + 1/\mathcal{R}$  terms

$$h_{\mathcal{C}-\mathcal{N}}^{ij}(t, \mathbf{x}) = \frac{4m}{R} \int_0^\infty ds {}^{(4)}Q^{ij}(u-s) \left[ \ln\left(\frac{s}{2R+s}\right) + \frac{11}{12} \right] \\ + \frac{4m}{3R} \hat{N}^k \int_0^\infty ds {}^{(5)}Q^{ijk}(u-s) \left[ \ln\left(\frac{s}{2R+s}\right) + \frac{97}{60} \right] \\ - \frac{16m}{3R} \epsilon^{(i)ka} \hat{N}^k \int_0^\infty ds {}^{(4)}J^{aj}(u-s) \left[ \ln\left(\frac{s}{2R+s}\right) + \frac{7}{6} \right] \\ + \frac{1912}{315} \frac{m}{R} {}^{(4)}Q^{ij}(u)\mathcal{R}$$

- **3/2PN and 2PN tail terms**

- \* Correct constants in harmonic coordinates
- \* No dependence on "matching" radius

- **Propagation along "true" null cones**

- \*  $R$  dependence in tail integral gives circular orbit waveform phase

$$\psi = \omega\{t - R - 2m \ln R - 2m[\gamma + \ln(4\omega e^{-11/12})]\}$$

- **2PN results for general systems**

Energy Loss Formula:

$$\frac{dE}{dt} = \frac{1}{5} \left\{ {}^{(3)}\mathcal{I}_{STF}^{ij} {}^{(3)}\mathcal{I}_{STF}^{ij} + \frac{5}{189} {}^{(4)}\mathcal{I}_{STF}^{ijk} {}^{(4)}\mathcal{I}_{STF}^{ijk} + \frac{5}{9072} {}^{(5)}\mathcal{I}_{STF}^{ijkl} {}^{(5)}\mathcal{I}_{STF}^{ijkl} \right. \\ \left. + \frac{16}{9} {}^{(3)}\mathcal{J}_{STF}^{ij} {}^{(3)}\mathcal{J}_{STF}^{ij} + \frac{5}{84} {}^{(4)}\mathcal{J}_{STF}^{ijk} {}^{(4)}\mathcal{J}_{STF}^{ijk} \right\}$$

Relation between STF Moments and EW moments:

$$\mathcal{I}_{STF}^{ij} = [I_{EW}^{ij} + \frac{1}{21}(11I_{EW}^{ijkk} - 12I_{EW}^{k(ij)k} + 4I_{EW}^{kkij}) \\ + \frac{1}{63}(23I_{EW}^{ijkkl} - 32I_{EW}^{k(ij)kll} + 10I_{EW}^{kkijll} + 2I_{EW}^{kkljij})]_{STF} \\ + \mathcal{I}_{TAIL}^{ij}$$

$$\dot{\mathcal{I}}_{STF}^{ijk} = [3I_{EW}^{ijk} + 3I_{EW}^{ijkll} - 3I_{EW}^{lljk} + I_{EW}^{llijk}]_{STF} + \mathcal{I}_{TAIL}^{ijk}$$

$$\ddot{\mathcal{I}}_{STF}^{ijkl} = [12I_{EW}^{ijkl} + \frac{72}{55}(13I_{EW}^{ijklmm} - 12I_{EW}^{immjkl} + 4I_{EW}^{mmijkl})]_{STF}$$

$${}^{(3)}\mathcal{I}_{STF}^{ijklm} = [60I_{EW}^{ijklm}]_{STF}$$

$${}^{(4)}\mathcal{I}_{STF}^{ijklmn} = [360I_{EW}^{ijklmn}]_{STF}$$

$$\mathcal{J}_{STF}^{ij} = [\epsilon_{ipq}(\frac{1}{2}I_{EW}^{jqp} + \frac{9}{28}I_{EW}^{jqpmm} - \frac{3}{28}I_{EW}^{qmmjp})]_{STF} + \mathcal{J}_{TAIL}^{ij}$$

$$\dot{\mathcal{J}}_{STF}^{ijk} = [\epsilon_{ipq}(2I_{EW}^{jqpk} + \frac{28}{15}I_{EW}^{jqpkmm} - \frac{8}{15}I_{EW}^{qmmjpjk})]_{STF}$$

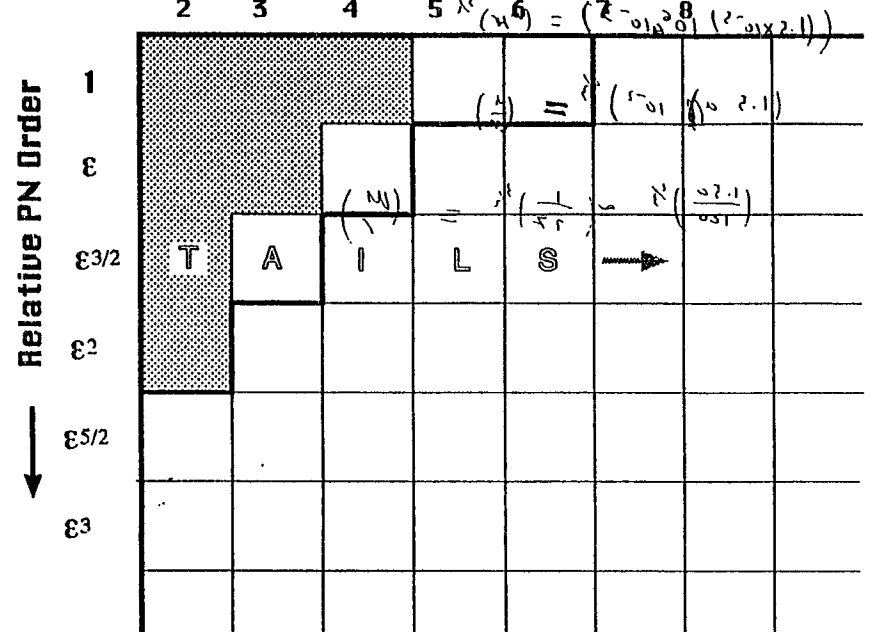
$$\ddot{\mathcal{J}}_{STF}^{ijkl} = [9\epsilon_{ipq}I_{EW}^{jqpkl}]_{STF}$$

$${}^{(3)}\mathcal{J}_{STF}^{ijklm} = [48\epsilon_{ipq}I_{EW}^{jqpklm}]_{STF}$$



# The post-Newtonian Playground

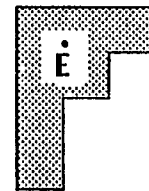
Multipole moments  $(I^L, J^L) = (\dots)$



Simon & Poisson (1996)

Convergence (test body) if

$$\frac{v}{c} < \sqrt{\frac{2}{e}}$$





# NEUTRON STAR BINARIES HYDRODYNAMICS I

WILSON  
MATHEUS  
MIRONETTI } N.D.

## FIELD EQUATIONS

$$d\tau^2 = -(d^2 - \beta_i \beta^i) dt^2 + 2\beta_i dx^i dt + \varphi^4 S_{ij} dx^i dx^j$$

$$K_{ij} = (D_i \beta_j + D_j \beta_i - \frac{2}{3} \varphi^4 S_{ij} D_k \beta^k) / 2\alpha$$

$$t_{ik} = 0 \quad \frac{\dot{\varphi}}{\varphi} = \frac{1}{6} D_k \beta^k$$

ASSUMPTIONS { CONFORMAL FLATNESS  
STATIONARITY

HAMILTONIAN EQ.

$$\nabla^2 \varphi = -2\pi \varphi^5 \left( \rho W + \rho G W (\Gamma W - \frac{(\rho E)}{W}) + \frac{K_{ij} \beta^i \beta^j}{16\pi} \right)$$

$$K \Rightarrow \nabla^2 (\alpha \varphi) = 2\pi \alpha \varphi^5 \left( \rho (\beta W^2 - 2) + \rho E (3\Gamma W - 5) + \frac{7}{16\pi} K_{ij} \beta^i \beta^j \right)$$

MOMENTUM CONSTRAINT

$$\nabla^2 \beta^i = \frac{\partial}{\partial x^i} \left( \frac{D_k \beta^k}{3} \right) + 16\pi \alpha \varphi^5 S^i_j + \frac{\partial}{\partial x^i} \left[ \frac{1}{2} \rho A \left( \beta_i \beta^i - \frac{2}{3} S_{ij} \beta^j \right) \right]$$

$$A = \alpha / \varphi^6$$

$$W = \alpha U t$$

GIVEN  $\rho, e, S^i_j$  solve these eq. for  $\alpha, \varphi, \beta^i$

PERFECT FLUID

EQUATIONS

$$T_{\mu\nu} = (\rho + \epsilon \rho + P) U_\mu U_\nu + P g_{\mu\nu}$$

$$D = W \rho$$

$$E = W \rho E$$

$$S_i = (D + \Gamma E) U_i$$

$$\Gamma = 1 + \frac{P}{\rho E}$$

$$\text{THEN: } \frac{\partial D}{\partial t} = -6 D \frac{\partial \ln \varphi}{\partial t} - \frac{1}{\varphi^6} \frac{\partial}{\partial x^i} (\varphi^6 D v^i)$$

$$\frac{\partial E}{\partial t} = -6 \Gamma E \frac{\partial \ln \varphi}{\partial t} - \frac{1}{\varphi^6} \frac{\partial}{\partial x^i} (\varphi^6 \Gamma E v^i) + P \left( \frac{\partial v^i}{\partial t} + \frac{1}{\varphi^6} \frac{\partial}{\partial x^j} (\varphi^6 v^j v^i) \right)$$

$$\frac{\partial S_i}{\partial t} = -6 S_i \frac{\partial \ln \varphi}{\partial t} + \frac{1}{\varphi^6} \frac{\partial}{\partial x^j} (\varphi^6 S_j v^i) - \alpha \frac{\partial P}{\partial x^i}$$

$$+ 2\alpha (D + \Gamma E) \left( \frac{W^2 - 1}{W} \right) \frac{\partial \ln \varphi}{\partial x^i} - W (D + \Gamma E) \frac{\partial v^i}{\partial x^i} + S_j \frac{\partial v^j}{\partial x^i}$$

CHOOSE  $\beta^i \rightarrow \infty$  such that  $v^i$  is minimized.  
on shell  $\vec{\beta} = R \times \vec{\omega} + \vec{\beta}_d$

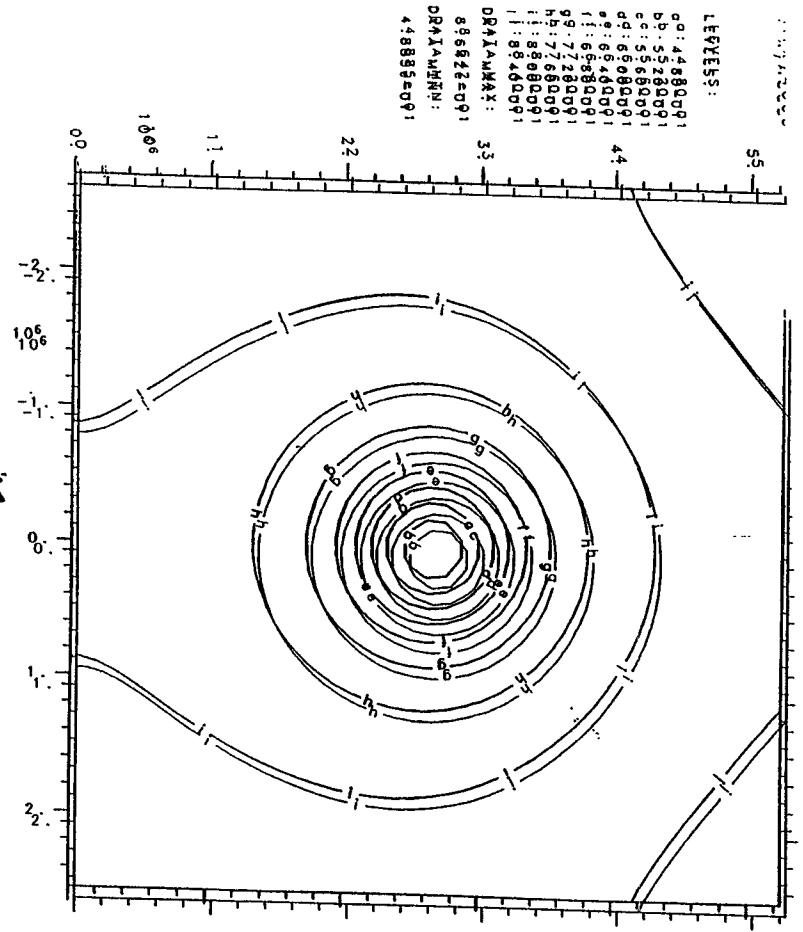
$\omega$  is coordinate rotation speed

DISCRETIZE THESE EQUATIONS AND INSERT INTO COMPUTER.

FOR PRESENT FIND STATIONARY SOLUTIONS ONLY.

000000  
150091

$J = 2.5 \times 10^{14} \text{ cm}^2$



## RESULTS

UL

WE HAVE SELECTED 3 VALUES OF ANGULAR MOMENTUM AND SEEN WHAT HAPPENS.

$$M_G^0 = 1.45 M_\odot$$

EOS SUCH THAT  $M_G^{\text{CRIT}} = 1.59 M_\odot$   
PUT STARS ON GRID. EVOLVE UNTIL STARS SETTLE DOWN.

$$J = 2.2 \times 10^{14} \text{ cm}^2 \quad (G = c = 1)$$

ORBIT UNSTABLE.  
STARS START TO SPIRAL IN.

$$\frac{d\rho}{\rho} = 9.2 \quad m = 2 M_G^0 \quad \text{INSIDE LAST STABLE ORBIT.}$$

$$J = 2.3 \times 10^{14} \text{ cm}^2 \quad \frac{J}{M^2} = 1.2 G$$

ORBIT STABLE.  
BUT CENTRAL DENSITY HAS RISEN TO  $2.7 \times 10^{15} \text{ gm/cc}$  (CRITICAL DENSITY FOR THIS EOS)  
FREQUENCY  $\approx 310 \text{ Hz}$

$$J = 2.7 \times 10^{14} \text{ cm}^2$$

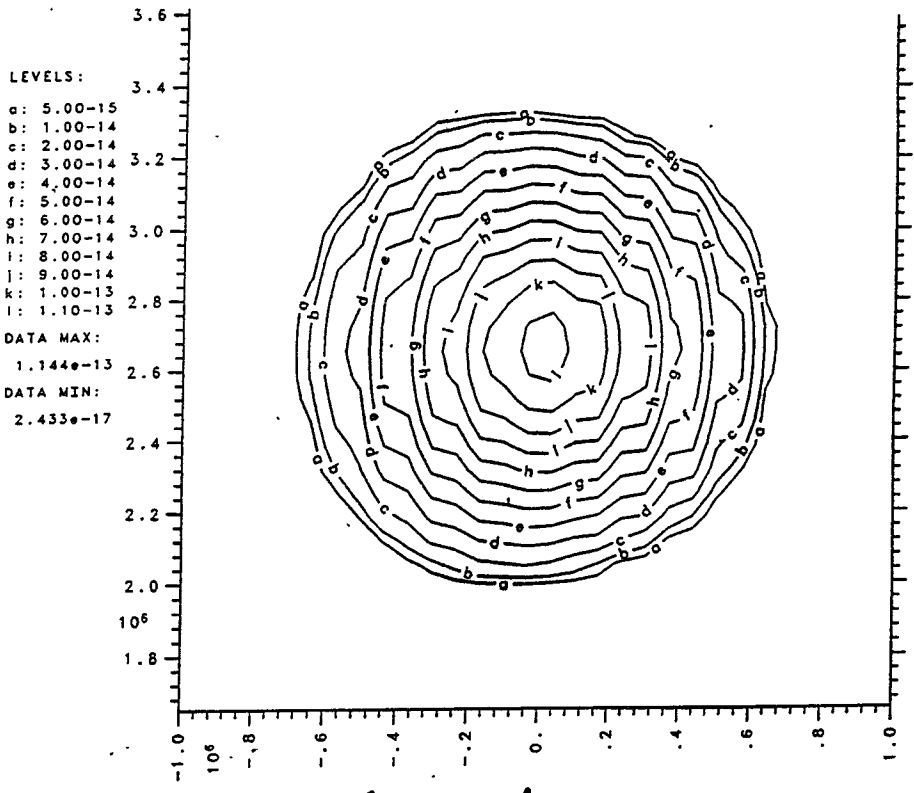
ORBIT STABLE  
CENTRAL DENSITY  $\rho = 2 \times 10^{15}$

$$\text{INITIAL CENTRAL DENSITY} = 1.4 \times 10^{15} \text{ gm/cc}$$

$$J/\omega J \approx 10^{-4}$$



5



00000  
1 2 3 4 5 6 7 8 9 10

iso d

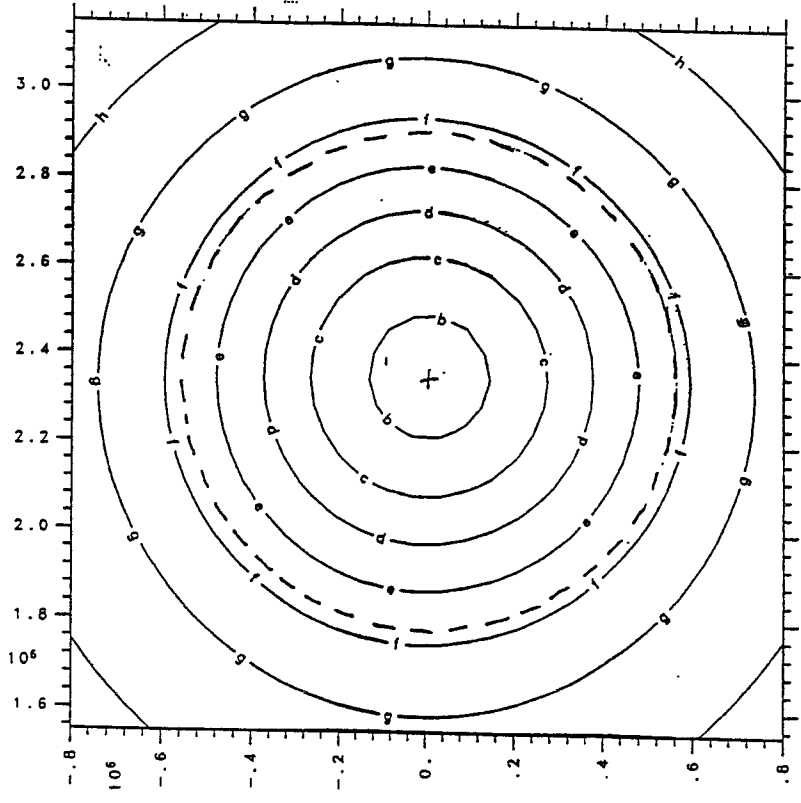
density

mhxy6400

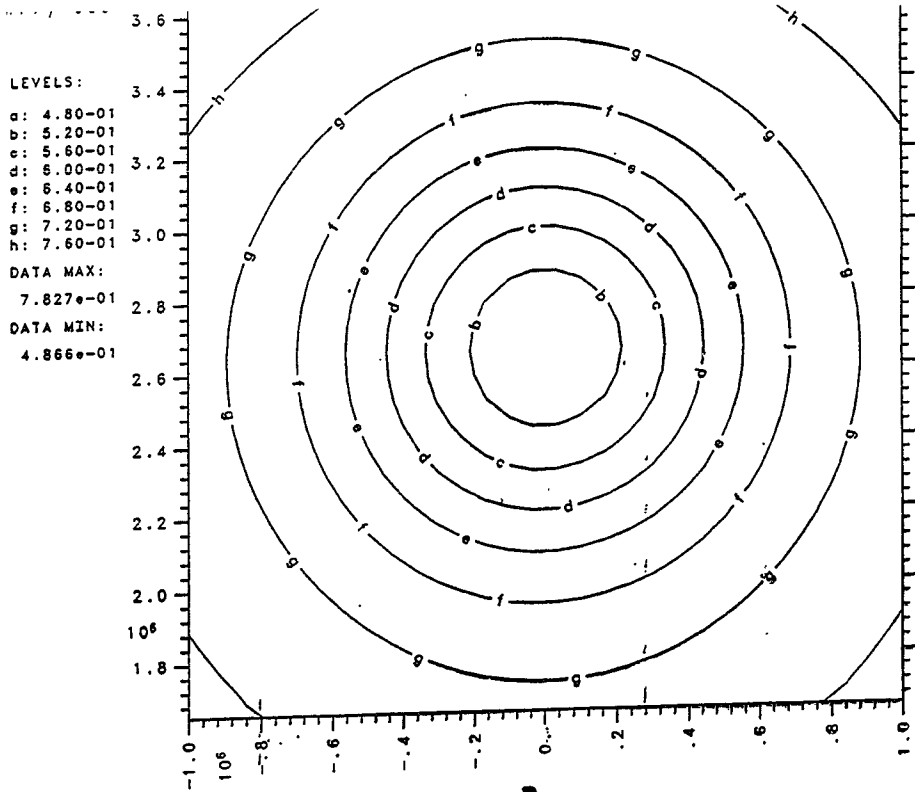
LEVELS:  
 a: 4.40-01  
 b: 4.80-01  
 c: 5.20-01  
 d: 5.60-01  
 e: 6.00-01  
 f: 6.40-01  
 g: 6.80-01  
 h: 7.20-01

DATA MAX:  
 7.494e-01

DATA MIN:  
 4.635e-01



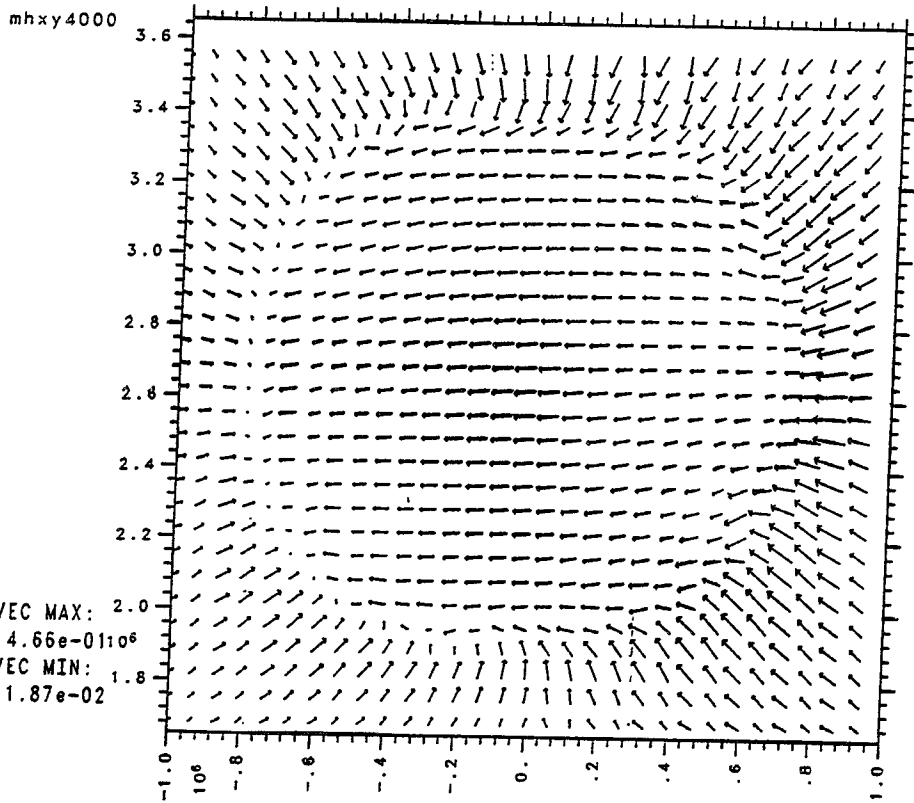
2



7

00000 iso al  
 4.2470e+08 BOX/ID: box 003 MTV run25 11:00:07 01-12-96 v REGION: o11

*SP2*



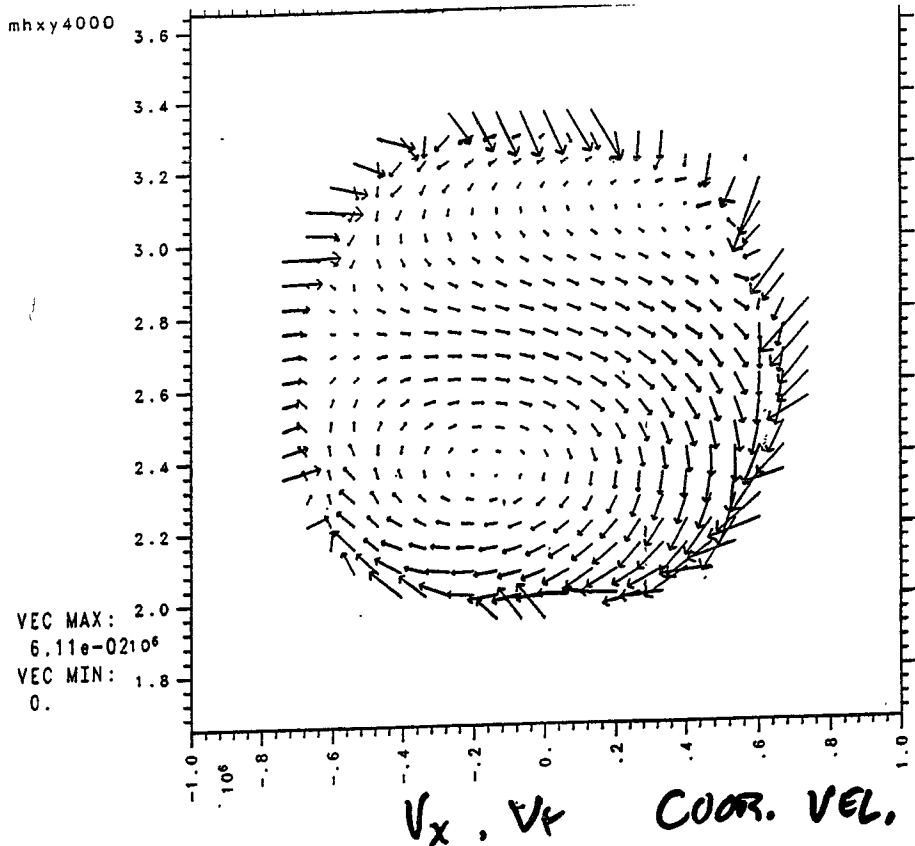
6

00000 vector ux uy  
 4.2470e+08 BOX/ID: box 003 MTV run25 11:03:53 01-12-96 v REGION: o11

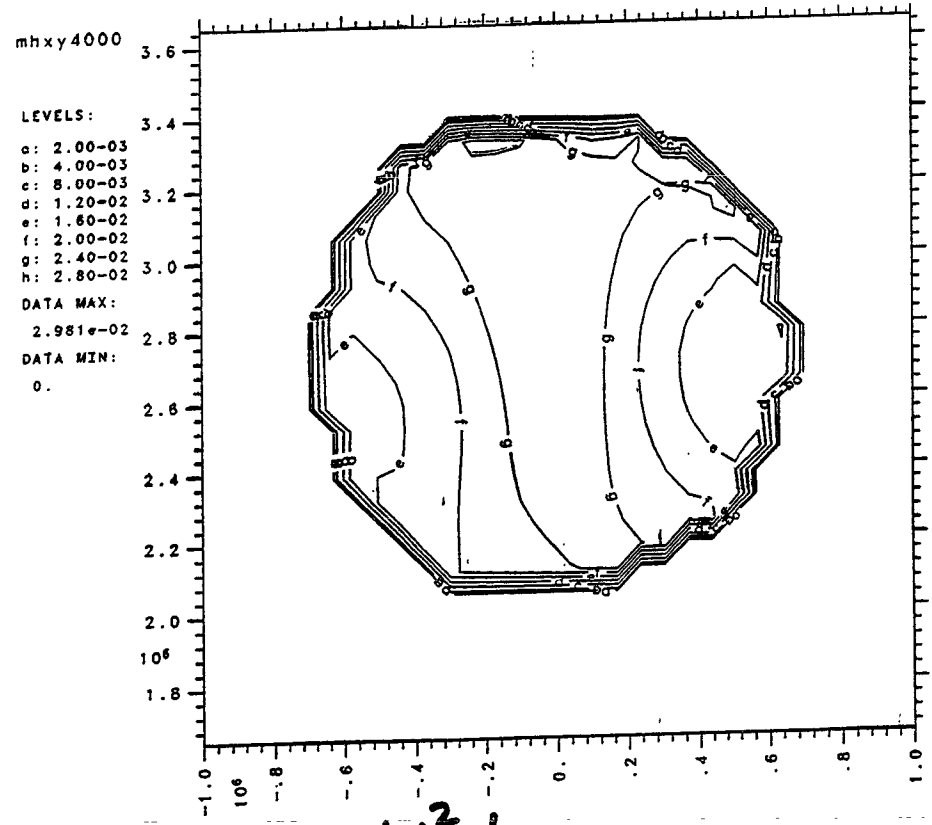
$U_x, U_y$  FOUR VEL.

IN STAR  $|U| \approx .3$

7



00000 vector vx vy  
4.2470e+08 BOX/ID: box c03 MTV run25 11:02:73 01-12-95 v REGION: all



00000 iso wsq  
4.2470e+08 BOX/ID: box c03 MTV run25 11:01:31 01-12-95 v REGION: all

$w^2 - 1$

## CONCLUSIONS.

IV

ORBITS UNSTABLE AT  
PERHAPS A LARGE RADIUS.

STARS ARE UNSTABLE IF  
EQUATION OF STATE NOT STIFF  
ENOUGH.

ORBITAL FREQUENCY PERHAPS  
LOW.

OUR EQUATION OF STATE IS  
FROM WILSON MAYBE SUPERNOVA MODELING  
 $M_{\text{CRIT.}} = 1.59 M_{\odot}$

BUT

RECENT WORK FAVORS A LOWER MASS

BETHE BROWN

$M_{\text{CRIT.}} = 1.50 - 1.55 M_{\odot}$

OBSERVED N.S. MASSES CONSISTANT  
WITH  $M_{\text{CRIT.}} = 1.5$  J. FINN

## WHY DENSITY INCREASES.

V

REWRITE FIELD SOURCES:

FOR  $\varphi$

$$\rho = \rho + \rho \epsilon + (W^2 - 1)(\rho + \Gamma \rho \epsilon)$$

FOR  $\alpha \varphi$

~~$$\rho = \rho + \rho \epsilon + (W^2 - 1)(\rho + \Gamma \rho \epsilon)$$~~

$$\rho = \rho + \rho \epsilon (6\Gamma - 5) + (W^2 - 1)(3\rho + 3\Gamma \rho \epsilon)$$

MOMENTUM EQUATION HYDROSTATICS

$$\frac{\partial \rho}{\partial x^i} = -(\rho + \rho \epsilon \Gamma) \frac{\partial \log \alpha}{\partial x^i}$$

$$+ (W^2 - 1)(\rho + \rho \epsilon \Gamma) \left[ 2 \frac{\partial \log \varphi}{\partial x^i} - \frac{\partial \log \alpha}{\partial x^i} \right]$$

AS WE SAW IN FIGURES

$W^2 - 1$  IS SOMEWHAT CONSTANT  
OVER THE STARS.

TO STUDY FURTHER THE VI  
DENSITY ENHANCEMENT EFFECT.  
WE WROTE 1D SPHERICAL CODE  
WITH  $W^2-1$  PUT INTO SOURCES  
AND FORCES.

RESULTS:

EOS  $M_{\text{CRIT}} = 1.57 M_{\odot}$

IF  $M_g^0 = 1.40 M_{\odot}$

$W^2-1 \rightarrow .035$  UNSTABLE

$M_g^0 = 1.45 M_{\odot}$

$W^2-1 > .020$  UNSTABLE

at  $\rho = 1.8 \times 10^{15} \frac{\text{g}}{\text{cm}^3}$

EXAMPLE 3D

$J = 2.5 \times 10^{51} \text{cm}^2$

$W^2-1 \approx .024 - .030$

IF EOS SOFT GET BLACK HOLES

CAVEAT. IN 3D CALCULATIONS WE  
HAVE TAKEN ZERO TEMPERATURE BY  
EDICT.

VII  
LOTS OF ENERGY MUST BE  
DISAPPEARED.

ESTIMATE  $\approx 6 \times 10^{52}$  ergs.

ENOUGH TO POWER  $\gamma$ -ray source.  
or at least produce high  
velocity wind.

$$\frac{1}{\sqrt{1-\beta^2}} \approx 10$$

QUESTION: HOW IS OUR  
ENERGY DISAPPEARED?

VISCOSITY, SHOCKS  $\Rightarrow$  NEUTRINO  
EMISSION

IS CONNECTION OF  
NEUTRON STAR BINARIES

AND  $\gamma$ -ray BURSTS IMPORTANT FOR  
G.W.O.?



FUTURE  
WHAT NEXT?

VIII

I IMPROVE NUMERICS SO MORE  
QUANTITATIVE RESULTS.

II STUDY COLLAPSE IN 1D and 3D

III PERSUE  $\gamma$ -RAY MODELING

IV CONNECT TO P.N. CALCULATIONS

V DEVELOP 3D G-R SUPERNOVA  
COMPUTER PROGRAM

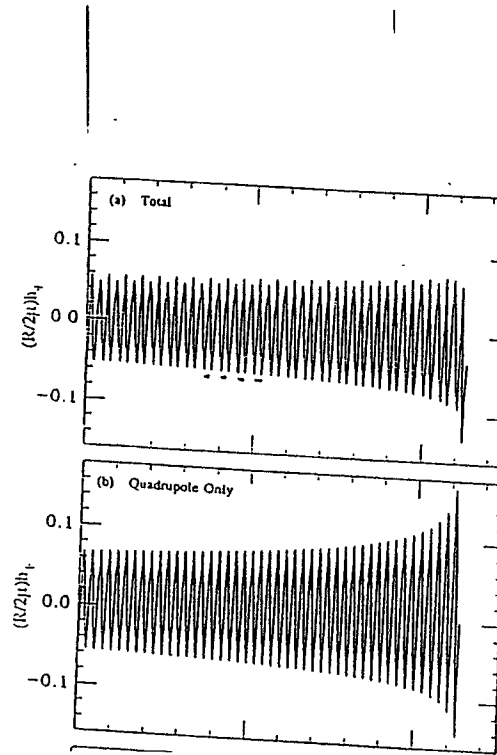
VI STUDY EOS PARAM

## 2<sup>nd</sup> Post-Newtonian Waveforms

What are they?

What are they good for?

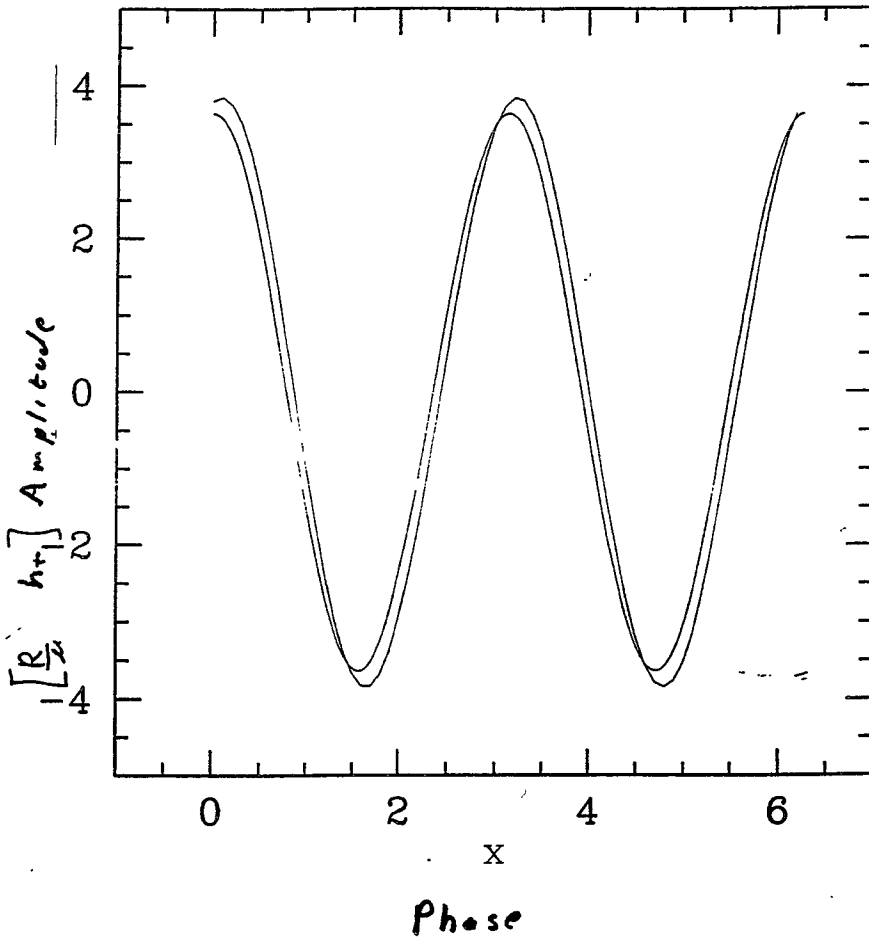
(Alan Wiseman) (Caltech)  
Chris Will (Wash U. St. L.)  
Luc Blanchet (Munich)  
Thibault Damour (Munich)  
Bela Iyer (RRI, IIT)



- $m_1/m_2 = 10$
- circular orbit
- Note true waveform does <sup>not</sup> grow as fast as Qudd.

Wiseman

Wiseman



.8 M<sub>⊙</sub>      f<sub>GW</sub> ≈ 200 Hz  
 2. M<sub>⊙</sub>

Wiseman

From  $\theta \Sigma \omega \omega$

$$h_{+, \times} = \frac{2Gm\eta}{c^2 R} \left( \frac{Gm\omega}{c^2} \right)^{3/2} \left\{ H_{+, \times}^{(0)} + z^{3/2} H_{+, \times}^{(1/2)} + z H_{+, \times}^{(1)} + z^{3/2} H_{+, \times}^{(3/2)} + z^2 H_{+, \times}^{(2)} + z^{5/2} H_{+, \times}^{(5/2)} \right\} \quad (2)$$

$$H_{+}^{(0)} = (1 + c^2) \cos 2\psi$$

$$H_{+}^{(1/2)} = \frac{2}{3} \frac{\delta m}{m} \left[ (5 + c^2) \sin \psi + 9(1 + c^2) \sin 3\psi \right]$$

$$H_{+}^{(1)} = \frac{1}{6} \left[ -19 - 9c^2 + 2c^4 + \eta(19 - 11c^2 - 6c^4) \right] \cos 2\psi - \frac{4}{3} c^2 (1 + c^2) (1 - 3\eta) \cos 4\psi$$

$$H_{+}^{(3/2)} = \frac{2}{192} \frac{\delta m}{m} \left\{ [-57 - 60c^2 + c^4 + 2\eta(49 - 12c^2 - c^4)] \sin \psi + \frac{27}{4} [-73 - 40c^2 + 9c^4 + 2\eta(25 - 8c^2 - 9c^4)] \sin 3\psi - \frac{625}{2} (1 - 2\eta) c^2 (1 + c^2) \sin 5\psi \right\} + 2\pi(1 + c^2) \cos 2\psi$$

$$H_{+}^{(2)} = \frac{1}{120} \left[ -22 - 396c^2 - 145c^4 + 5c^6 + \frac{5}{3} \eta(-706 - 216c^2 + 251c^4 - 15c^6) + 5\eta^2(98 - 108c^2 + 7c^4 + 5c^6) \right] \cos 2\psi + \frac{2}{15} c^2 \left[ 59 + 35c^2 - 8c^4 + \frac{5}{3} \eta(-131 - 59c^2 + 24c^4) + 5\eta^2(21 - 3c^2 - 8c^4) \right] \cos 4\psi + \frac{81}{40} (1 - 5\eta + 5\eta^2) c^4 (1 + c^2) \cos 6\psi + \frac{2}{40} \frac{\delta m}{m} \left\{ [11 + 7c^2 + 10(5 + c^2) \ln 2] \cos \psi + 5\pi(5 + c^2) \sin \psi + 27 \left[ 7 - 10 \ln(3/2) \right] (1 + c^2) \cos 3\psi + 135(1 + c^2) \sin 5\psi \right\} \quad (3a)$$

$$H_{\times}^{(0)} = 2c \sin 2\psi$$

$$H_{\times}^{(1/2)} = -\frac{3}{4} \frac{\delta m}{m} \left[ \cos \psi + 3 \cos 3\psi \right]$$

$$H_{\times}^{(1)} = \frac{c}{3} \left[ -17 + 4c^2 + \eta(13 - 12c^2) \right] \sin 2\psi - \frac{8}{3} (1 - 3\eta) c^2 \sin 4\psi$$

$$H_{\times}^{(3/2)} = \frac{2c}{96} \frac{\delta m}{m} \left\{ [63 - 5c^2 + 2\eta(-23 + 5c^2)] \cos \psi + \frac{27}{2} [47 - 15c^2 + 2\eta(-19 + 15c^2)] \cos 3\psi + \frac{625}{2} (1 - 2\eta) c^2 \cos 5\psi \right\} + 4\pi c \sin 2\psi$$

$$H_{\times}^{(2)} = \frac{c}{60} \left[ -68 - 226c^2 + 15c^4 + \frac{5}{3} \eta(-572 + 490c^2 - 45c^4) + 5\eta^2(56 - 70c^2 + 15c^4) \right] \sin 2\psi + \frac{4}{15} c^2 \left[ 55 - 12c^2 + \frac{5}{3} \eta(-119 + 36c^2) + 5\eta^2(17 - 12c^2) \right] \sin 4\psi + \frac{81}{20} (1 - 5\eta + 5\eta^2) c^4 \sin 6\psi + \frac{3}{20} \frac{\delta m}{m} \left\{ [3 + 10 \ln 2] \sin \psi - 5\pi \cos \psi + 9 \left[ 7 - 10 \ln(3/2) \right] \sin 3\psi - 45\pi \cos 3\psi \right\} \quad (3b)$$

$$\psi = \phi - \frac{1}{\omega} \left\{ \frac{3715}{8064} + \frac{55}{96\eta} \right\} \Theta^{3/2} - \frac{3\pi}{4} \Theta^{-1/2} + \left( \frac{9275405}{14450688} + \frac{284875}{258048\eta} + \frac{1855}{2048\eta^2} \right) \Theta^{5/2} + O(\ln \Theta) \quad (6)$$

and the frequency (satisfying  $\omega = d\psi/dt$ )

$$\omega = \frac{c^3}{8Gm} \left\{ \Theta^{-3/2} + \left( \frac{743}{2688} + \frac{11}{32\eta} \right) \Theta^{-1/2} - \frac{3\pi}{10} \Theta^{-3/2} + \left( \frac{1855009}{14450688} + \frac{56975}{258048\eta} + \frac{371}{2048\eta^2} \right) \Theta^{1/2} + O(\ln \Theta) \right\} \quad (7)$$

We use in these expressions the dimensionless time variable defined by

Wiseman

### Contributions to G.W. Phase

Reductive Ord.	# of GW Cycles	Harmonic that carry Power	"Physical" Origin
Leading Ord.	16050	2	Simple Quad.
$(v/c)^2$	439	1, 2, 3	Current, Quadrup.
$(v/c)^3$	-208	2	Tail!
$(v/c)^4$	9	1, 2, 3, 4	....
	$\approx 17$		Spin Orbit
	$\approx 2$		Spin-Spin

- Two N.S. 1.4  $M_{\odot}$
- Spiraling from  $f_{orb} = 5 \text{ Hz}$  to  $500 \text{ Hz}$   
 $f_{GW} = 10 \text{ Hz}$  to  $1000 \text{ Hz}$

? What about convergence properties of the post-Newtonian expansion?

Higher Harmonics Carry off (Orbital) Energy

$\Rightarrow$  Orbital Radius Shrinks

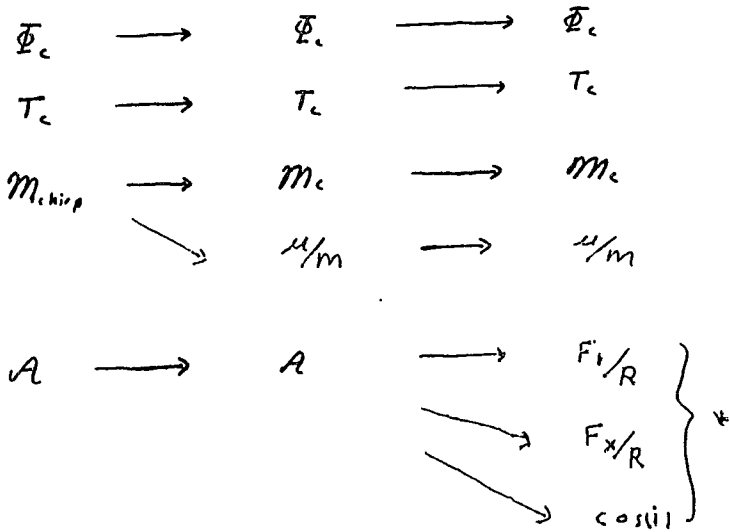
$\Rightarrow$  Orbital Frequency Rises (Hence G.W. Freq. Rises)

$\infty$  Tracking G.W. phase (or orbital phase) over many cycles (a long time) requires accurate knowledge of the energy-loss rate

$\infty$  Need to know the harmonic structure of the emitted wave!

Higher Harmonics don't just carry energy - They carry Information.

Quad. Sig. Analysis. $h = A_c \cos 2\Phi_{Quadr}$	Restricted P.N. Sig. Analysis $h = A_c \cos 2\Phi_{PN}$	Signal Analysis $h = \sum_{n_1, n_2, p} A_{nm} \cos M\Phi$
--	--	---



Finn + Chernof

Cutler + Flanagan  
Krolak

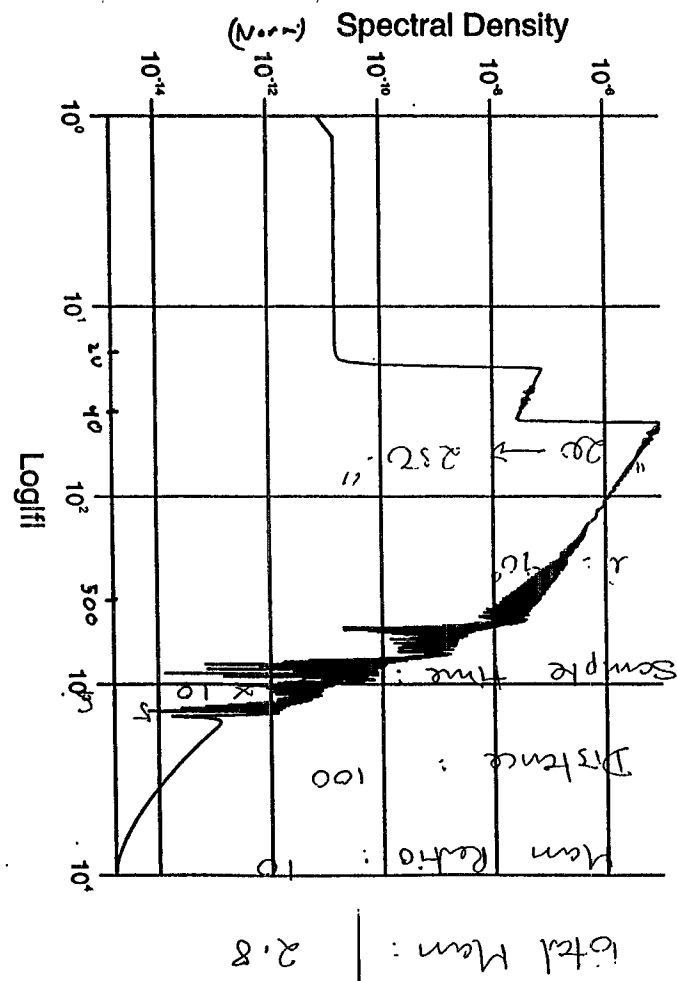
In Progress  
{ Allen, Besty, Finn, AGW }

\* Weakly, but still determined harmonics 1, 3, 4

Wiseman

Wiseman

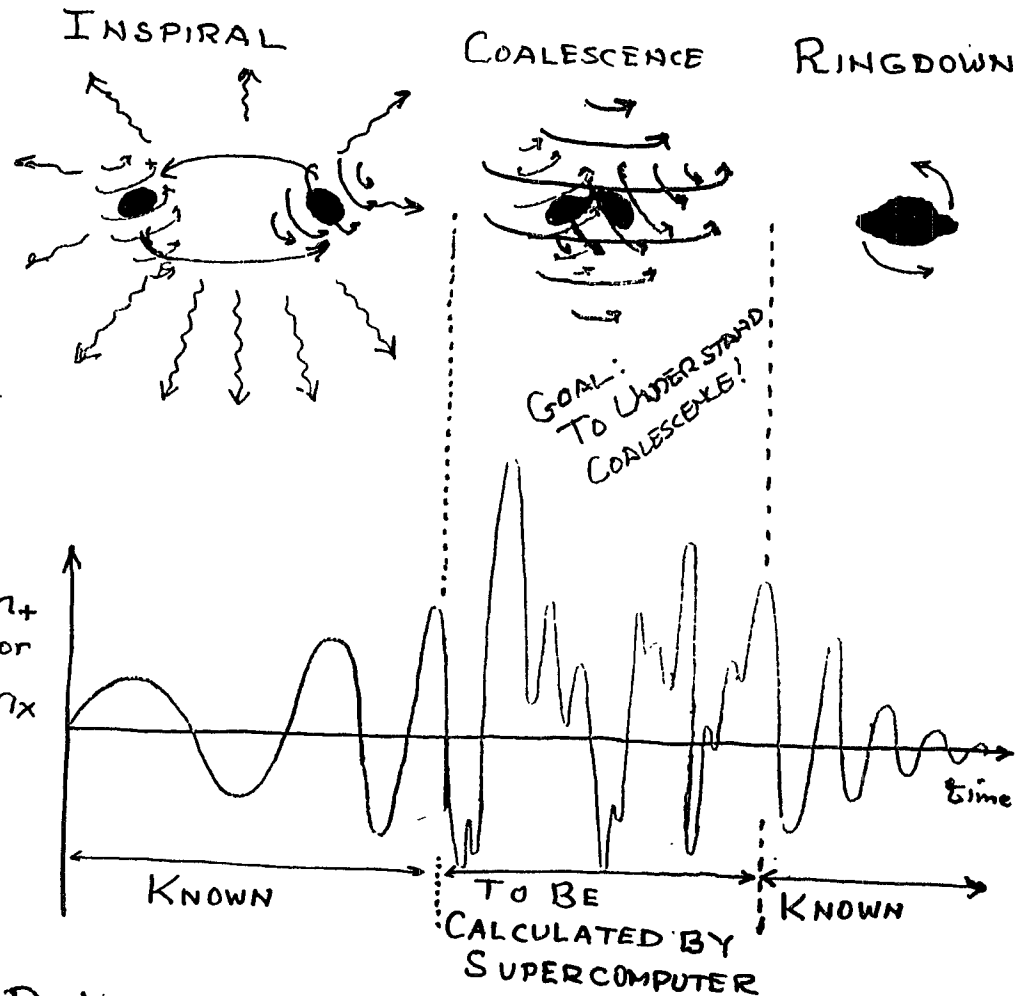
$M_{total} = 2.3 \text{ M}\Theta$   
 $m_1/m_2 = 10$   
*i* inclination =  $90^\circ$   
 Span 20Hz - 250Hz



# BLACK-HOLE COALESCENCE

Kip Thorne

"The Final Merger of Compact Binaries:  
Information Content, Waveform Computations,  
and Data Analysis Issues

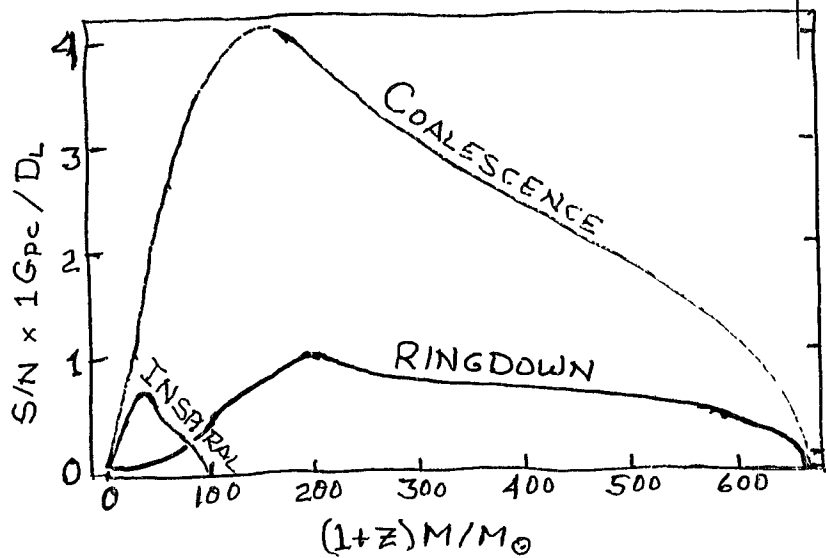


! Detectable throughout the Universe

**LIGO/VIRGO**: for  $M_1, M_2 \sim 15$  to  $1000 M_\odot$

**LISA**: for  $M_1, M_2 \sim 3 \times 10^4$  to  $10^8 M_\odot$

# FOR FIRST LIGO INTERFEROMETERS & HUGHES



Number of bits & information in noise-contaminated coalescence waveform

$$= \tau \int \log_2 \left( 1 + \frac{1}{\tau} \frac{d(S/N)^2}{df} \right) df$$

duration of coalescence

$$\tau \Delta f = \text{Number of cycles in waveform (since } \Delta f \sim \bar{f} \text{)}$$

$$\approx N_{\text{cyc}} \log_2 \left[ 1 + \frac{(S/N)^2}{N_{\text{cyc}}} \right]$$

$$\approx N_{\text{cyc}} \log_2 \left[ \frac{(S/N)^2}{N_{\text{cyc}}} \right] \quad \text{for } N_{\text{cyc}} \ll (S/N)^2$$

$$\approx 1.4 (S/N)^2 \quad \text{for } N_{\text{cyc}} \gg (S/N)^2$$

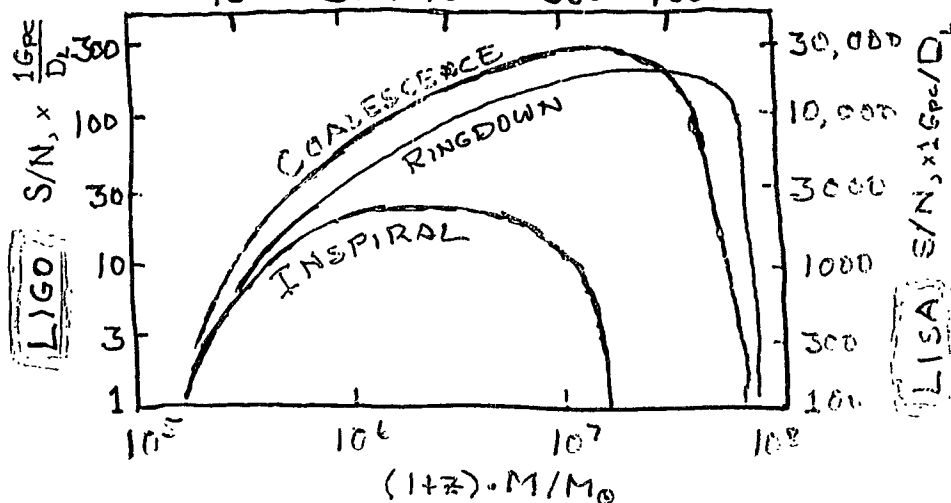
# S/N For INSPIRAL, COALESCENCE, RINGDOWN

[Eanna Flanagan & Scott Hughes]

[ $M_1 = M_2 = M/2$ ;  $(a/M)_{\text{Final}} = 0.98$ ; 10% efficiency]

"ADVANCED" INTERFEROMETERS (1+z)(M/M\_sun)

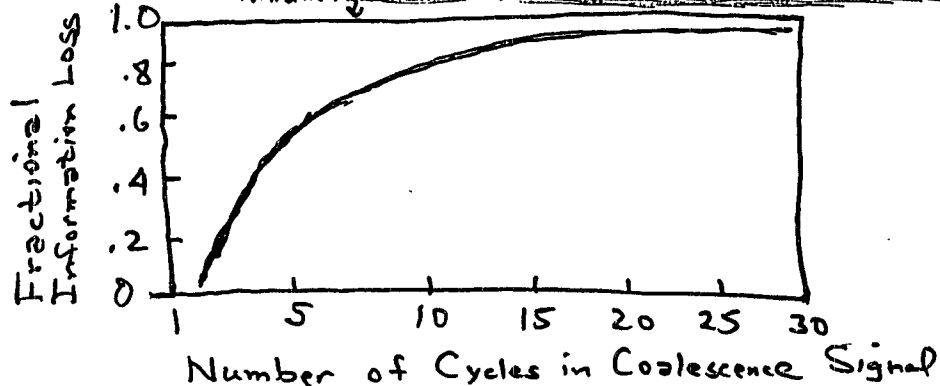
10	30	100	300	1000
----	----	-----	-----	------



For LIGO/VIRGO:

If BBH Project fails to give us waveform information

We need theoretical waveforms as aid in extracting coalescence information!! (BBH Grand Challenge)



## OTHER KINDS OF COMPACT OBJECTS

$$R \lesssim 10 \text{ GM}/c^2$$

## 1. BOSON & SOLITON STARS

- Condensates of (hypothetical) bosons, held together by gravity (& possibly also particle interactions) [cf. Little & Madsen, Int. J. Mod. Phys., 1, 101 (1992)]

• Examples: ← Golpi, Shapiro, Wasserman, PRL, 57, 2485 (1986)

★ Complex scalar field  $\phi$ , of mass  $m$ ,  $|\partial\phi|^4$  coupled

$$M_{\text{max}} \approx 0.06 \lambda^{1/2} \frac{m_{\text{pl}}^3}{m^2} \approx 0.1 M_{\odot} \frac{\lambda^{1/2}}{(m/1 \text{ GeV})^2}$$

- "Boson ★"

★ Higgs type scalar field of mass  $m$  coupled to a complex scalar field or Fermion field

$$M_{\text{max}} \sim \frac{m_{\text{pl}}^4}{m^3} \sim \frac{10^{12} M_{\odot}}{(m/300 \text{ GeV})^3}$$

- "Soliton ★"

## 1. Naked Singularities

• Conventional Wisdom: Cosmic Censorship [Penrose] - they don't exist

• But: "Choptuons" [Hawking vs Preskill & Thorne]

Horowitz's Inocuous Singularities

## How SEARCH FOR EXOTIC COMPACT OBJECTS?

0.01 to 0.2  $M_{\odot}$

LIGO/VIRGO: Binary Inspiral

Note: for  $M_c \approx 0.01 M_{\odot}$

- Several months to sweep thru LIGO/VIRGO band
- verify radii  $R \lesssim 10 \text{ km}$
- "advanced" interferometers: search out to Virgo
- enormous computational task

30-300  $M_{\odot}$  - LIGO/VIRGO

$3 \times 10^5 - 3 \times 10^7 M_{\odot}$  - LISA

Watch a  $\sim 1 M_{\odot}$  WR, NS, BH spiral into the massive compact object

... Map the massive object's spacetime geometry (measure its multipole moments)

... If it is a BH: "No Hair"

Mass & Spin  $\Rightarrow$  All Other Moments

$\uparrow$   $M_0$        $\uparrow$   $S_2$        $\uparrow$   $M_2, M_3, M_4, \dots$   
 $S_2, S_3, S_4, \dots$



IDEALIZED EXAMPLE [Fintan Ryan: Phys Rev D in press]

- Axisymmetric Central Object 

Moments  $M_0, M_2, M_4, M_6, \dots$   
 $S_1, S_3, S_5, S_7, \dots$

- Circular Orbit in Equatorial Plane

- Monitor Evolution of Orbital Phase  
 [to accuracy  $\sim 1/10^3$  for LIGO,  $\sim 1/10^6$  for LISA]  
 as WD, NS, or BH spirals from  $r \sim 10M$  to  $r \sim 1M$

$$v = (\pi M_c f)^{1/3}$$

$$\frac{N_{\text{cycles}}}{\ln f} = \frac{5}{96\pi} \frac{1}{(\pi M_c f)^{5/3}} \left[ 1 + \frac{747}{336} v \right]$$

$$\left( -4\pi + \frac{113}{12} \frac{S_1}{M_c^2} \right) v^3$$

GW Tail - Poisson

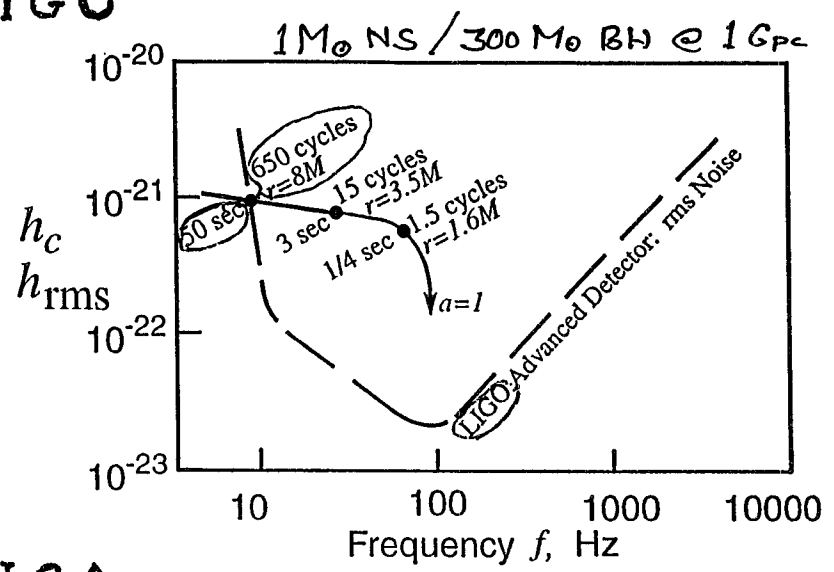
$$\left( \frac{3058673}{1016064} - \frac{1}{16} \frac{S_1^2}{M_c^4} + 5 \frac{M_2}{M_c^3} \right) v^4$$

$$(\dots) v^5 + (\dots) v^6$$

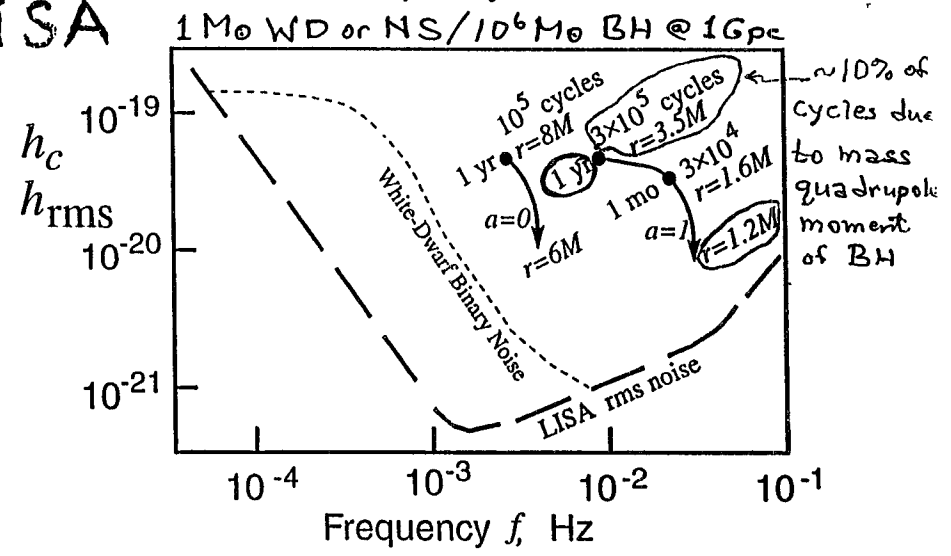
$$\left( \dots - 22 \frac{S_3}{M_c^4} \right) v^7 + \left( \dots - \frac{45}{4} \frac{M_4}{M_c^5} \right) v^8$$

TWO EXAMPLES

LIGO



LISA



Can probably measure lowest few moments to very high accuracy

## COMPLICATIONS:

### Theoretical:

- To compute nonlinear terms in series extremely difficult:

Components of curvature/metric don't decouple

No separation of variables

- Series may not be accurate enough.

### Astrophysical [especially for LISA]:

- Orbit should be nonequatorial & nonspherical
- Accretion disk will perturb orbit

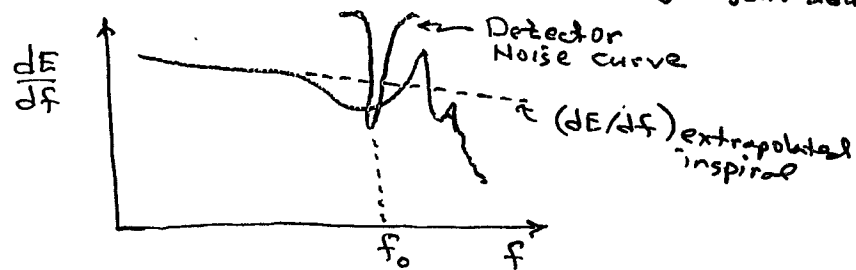
$$\frac{\dot{L}_{Acc}}{\dot{L}_{GW}} \sim \frac{(\text{Mass in disk inside orbit})}{M_0} \left(\frac{r}{M_0}\right)^{5/2}$$

↑ very roughly

Will drive orbit into equatorial plane...

## COMPARISON OF DUAL RECYCLED INTERFEROMETERS &

### SPHERICAL BARS OR TIGAS [Daniel Kennefick & Dustin Laurence]



### ■ Interferometers:

$$\frac{S}{N} = \frac{\sqrt{(dE/df)_0}}{\sqrt{(dE/df)_{\text{extrapolated inspiral}}}} \left(\frac{200 \text{ Mpc}}{r}\right) \left(\frac{1 \text{ kHz}}{f_0}\right) 0.7 \sqrt{\frac{\text{Circulating power}}{100 \text{ kWatts}}}$$

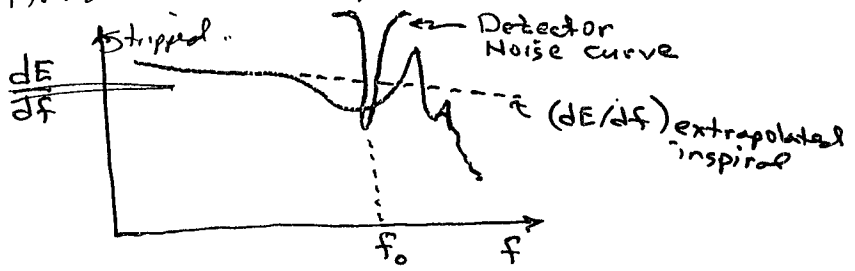
### ■ Bars:

$$\frac{S}{N} = \frac{\sqrt{(dE/df)_0}}{\sqrt{(dE/df)_{\text{extrapolated inspiral}}}} \left(\frac{200 \text{ Mpc}}{r}\right) \left(\frac{1 \text{ kHz}}{f_0}\right) 0.3 \sqrt{\frac{hf_0}{kT_{\text{noise}}}}$$

# COMPARISON OF DUAL RECYCLED INTERFEROMETERS &

## SPHERICAL BARS OR TIGAS [Daniel Kennefick & Dustin Laurence]

Has done a nice paper on this



### Interferometers:

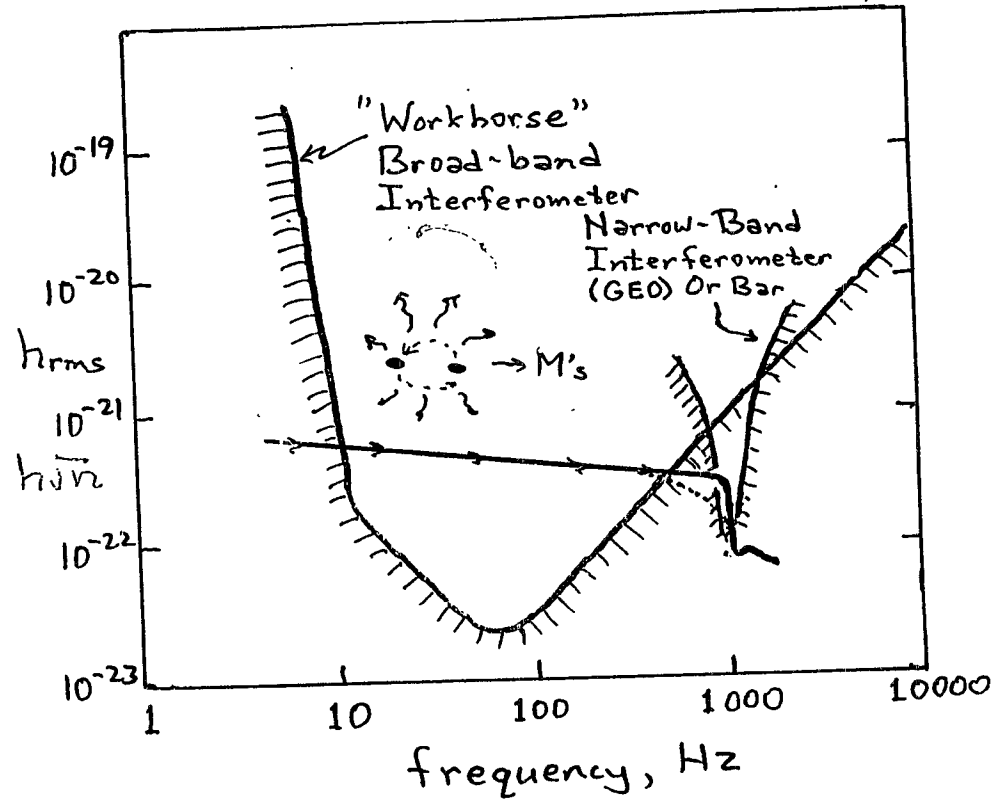
$$\frac{S}{N} = \frac{\sqrt{(dE/df)_0}}{\sqrt{(dE/df)_{\text{extrapolated inspired}}}} \left(\frac{200 \text{ Mpc}}{r}\right) \left(\frac{1 \text{ kHz}}{f_0}\right) 0.7 \sqrt{\frac{\text{Circulating power}}{100 \text{ kWatts}}}$$

### Bars:

$$\frac{S}{N} = \frac{\sqrt{(dE/df)_0}}{\sqrt{(dE/df)_{\text{extrapolated inspired}}}} \left(\frac{200 \text{ Mpc}}{r}\right) \left(\frac{1 \text{ kHz}}{f_0}\right) 0.3 \sqrt{\frac{h f_0}{k T_{\text{noise}}}}$$

Comments: NS explosion  
- Nadezha  
To detect: specially detectors  
- optimize  
Wang:  $\Gamma = 1.1$

# NS/NS COALESCENCE

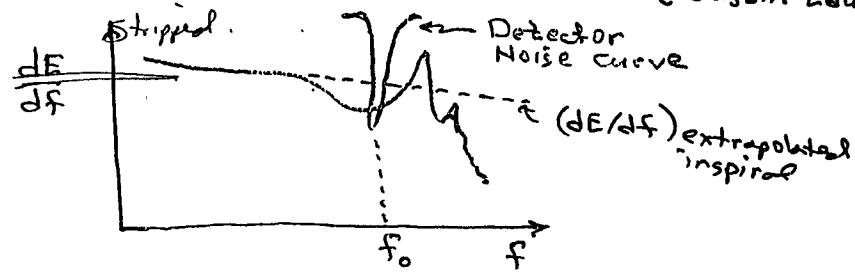


■ OBJECTIVE: GET INFORMATION ABOUT NS EQN OF STATE FROM COALESCENCE WAVES

- OPPOSITE EXTREME FROM INSPIRAL
  - can only get a few bits of information
  - must jointly design interferometer & data analysis to get stiffness of Eqn of State
  - For this need: Numerical Simulations - Dependence of Waveforms on Eqn of State

# COMPARISON OF DUAL RECYCLED INTERFEROMETERS &

SPHERICAL BARS OR TIGAS [Daniel Kennefick & Dustin Laurence]



## Interferometers:

$$\frac{S}{N} = \frac{\sqrt{(dE/df)_0}}{\sqrt{(dE/df)_{\text{extrapolated inspiral}}}} \left( \frac{200 \text{ Mpc}}{r} \right) \left( \frac{1 \text{ kHz}}{f_0} \right) 0.7 \sqrt{\frac{\text{Circulating Power}}{100 \text{ kW/Hz}}}$$

## Bars:

$$\frac{S}{N} = \frac{\sqrt{(dE/df)_0}}{\sqrt{(dE/df)_{\text{extrapolated inspiral}}}} \left( \frac{200 \text{ Mpc}}{r} \right) \left( \frac{1 \text{ kHz}}{f_0} \right) 0.3 \sqrt{\frac{h f_0}{k T_{\text{noise}}}}$$

Comments: NS application  
 - Nadykhi  
 To detect: specially selected  
 - optimize  
 Noise:  $\Gamma = 1.1$

“Black Hole Binaries: Coalescence and Gravitational Radiation”

UPDATE - R MATZNER

Computational Grand Challenge Program NSF ASC/PHY 9318152  
(ARPA Supplemented)

PI: Richard Matzner           The University of Texas at Austin, Austin, TX 78712

Co PIs:

J. Browne                    The University of Texas at Austin, Austin, TX 78712

L.L. Smarr, H.E. Seidel      National Center for SuperComputing Applications, University of Illinois, Urbana, IL

P.E. Saylor, F. Saied        University of Illinois, Urbana, IL

G. Fox                        Northeast Parallel Architecture Center, Syracuse University, Syracuse, NY

S. Teukolsky                 Center for Radio Physics and Space Research, Cornell University, Cornell, NY

S.L. Shapiro                 Center for Astrophysics and Relativity, Cornell, NY

J.W. York, C.R. Evans       University of North Carolina, Chapel Hill, NC

L.S. Finn                     Northwestern University, Evanston, IL

P. Laguna                    Pennsylvania State University, University Park, PA

J. Wmicour                  University of Pittsburgh, Pittsburgh, PA

## Binary Black Hole Grand Challenge

Purposes:

To develop a Problem Solving Environment [RNPL + Dynamic Adaptive Parallel infrastructure/PAMRSL]

To provide controllable convergent algorithms to compute gravitational waveforms from Black Hole encounters (relevant to detectors)

To provide representative examples of computational waveforms.

## BLACK HOLE COALESCENCE

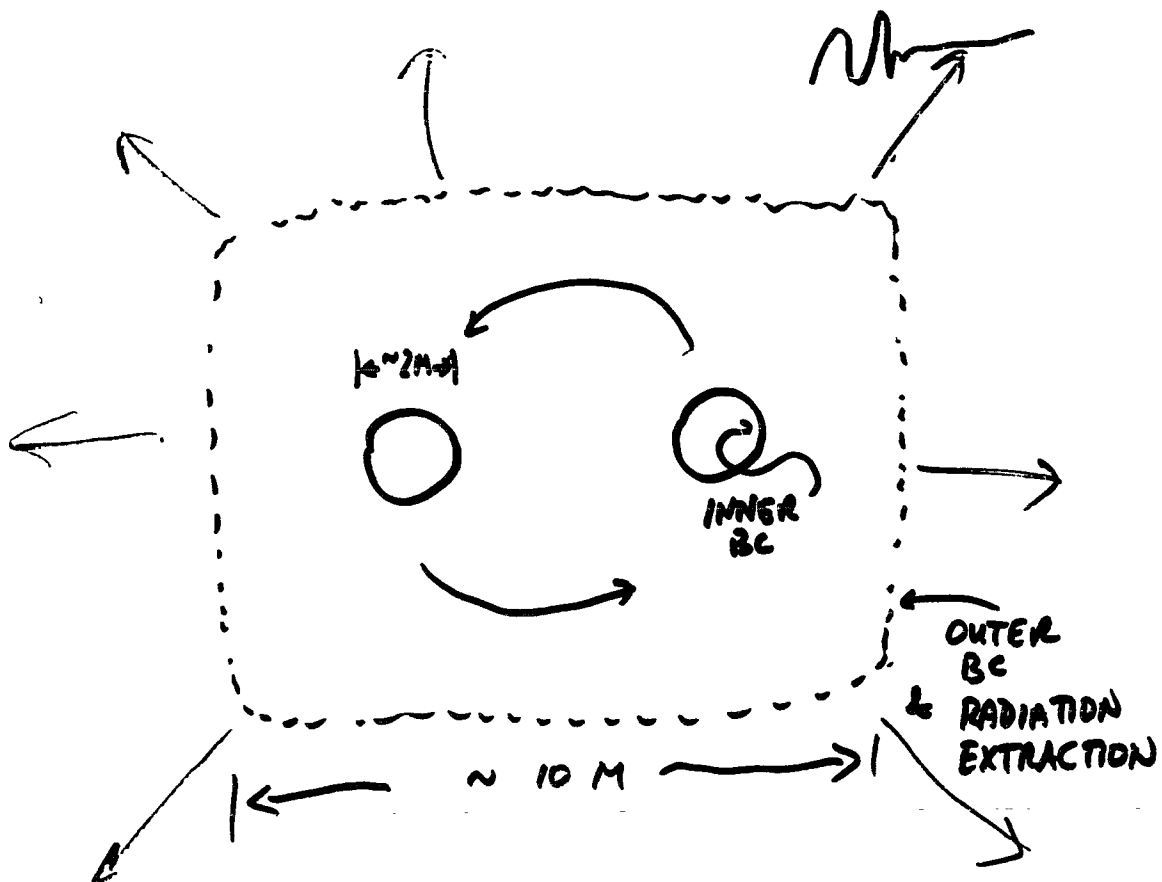
~ 1 TO 10 SOLAR MASS BLACK HOLES.

BINARIES FORMED IN SN?

VERY STRONG SOURCES OF GRAVITATIONAL RADIATION:

$$\phi \sim 1$$

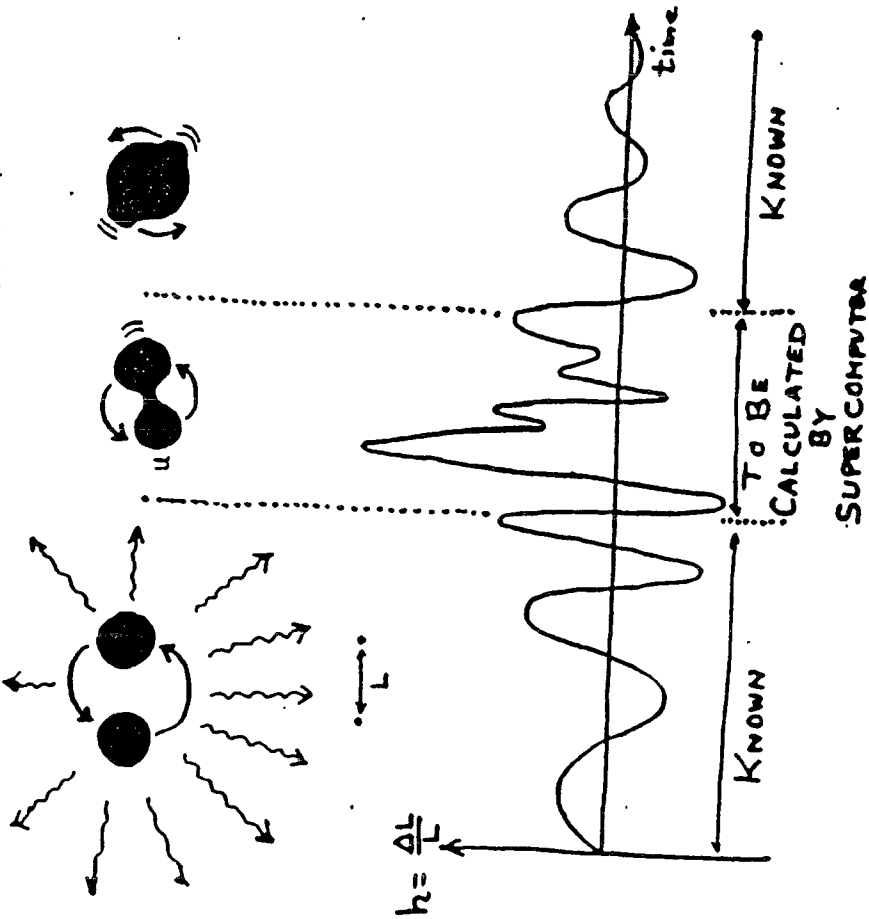
[NEUTRON STARS  $\phi \sim 0.1$ ]



BLACK-HOLE EXISTENCE & DYNAMICS

!! [The "Ultimate test" of general relativity]

COALESCENCE OF A BLACK-HOLE BINARY

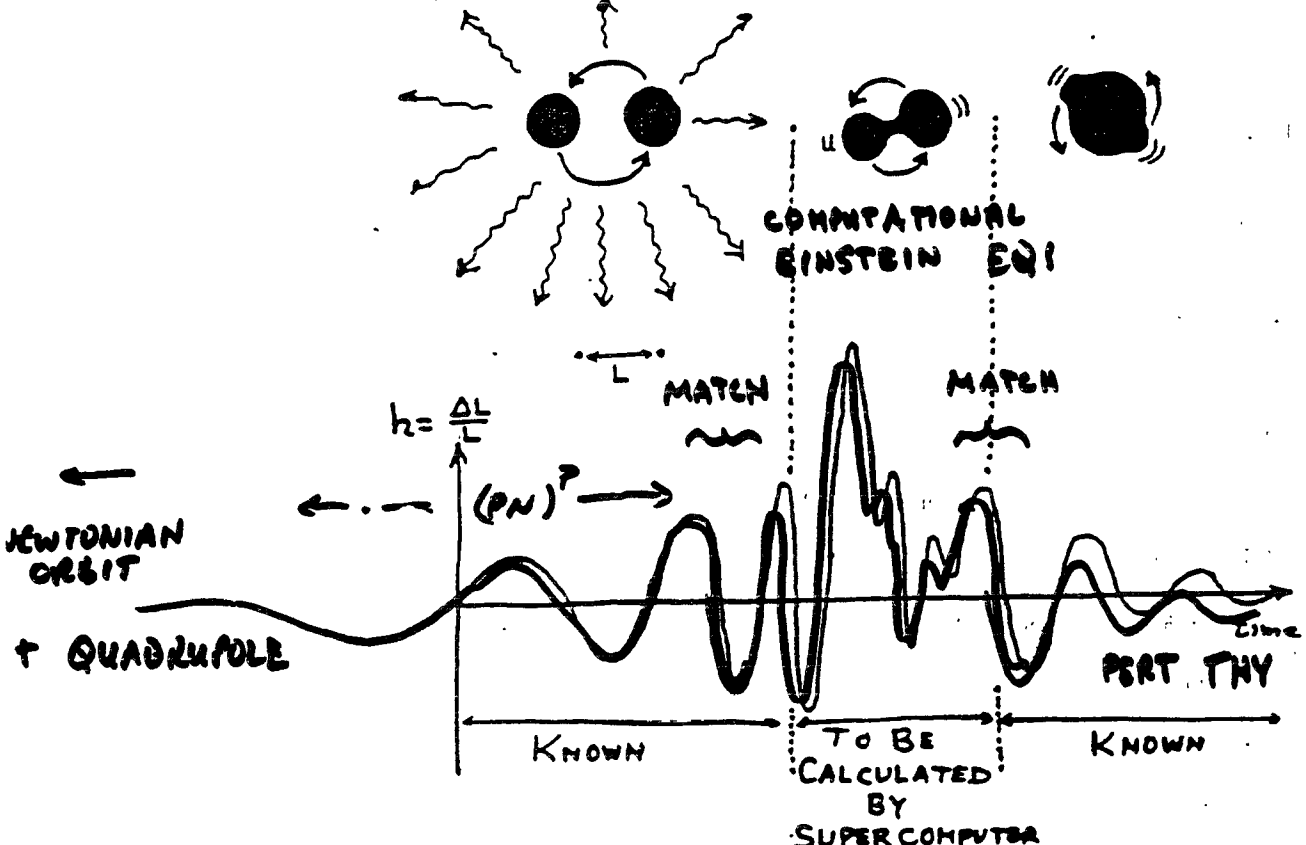


LIGO could detect such events throughout the Universe for hole masses 10-1000  $M_{\odot}$

K. THORNTON

!! [The "Ultimate test" of general relativity]

COALESCENCE OF A BLACK-HOLE BINARY



- CONTROLLABLE CONVERGENT ALGORITHMS.

DISCRETIZATION  $\Delta h$

$$u_N = u_a + k(x, t)(\Delta h)^p + \dots$$

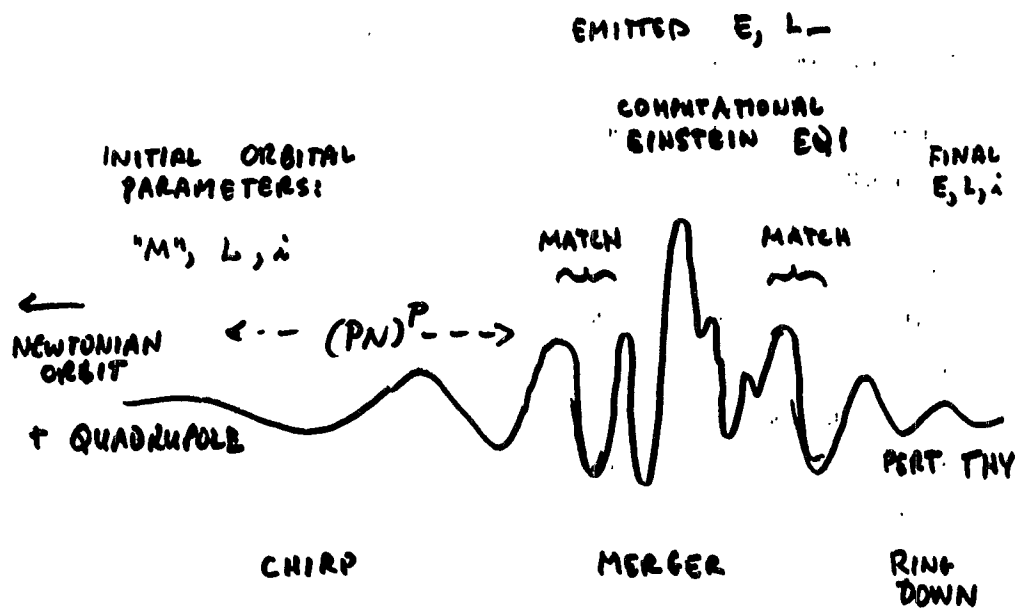
$p = 2$  2nd ORDER

$$u_h = u_a + k(\Delta h)^2 + \dots$$

$$u_{2h} = u_a + 4k(\Delta h)^2 + \dots$$

$$3u_a = 4u_h - u_{2h} + \dots$$

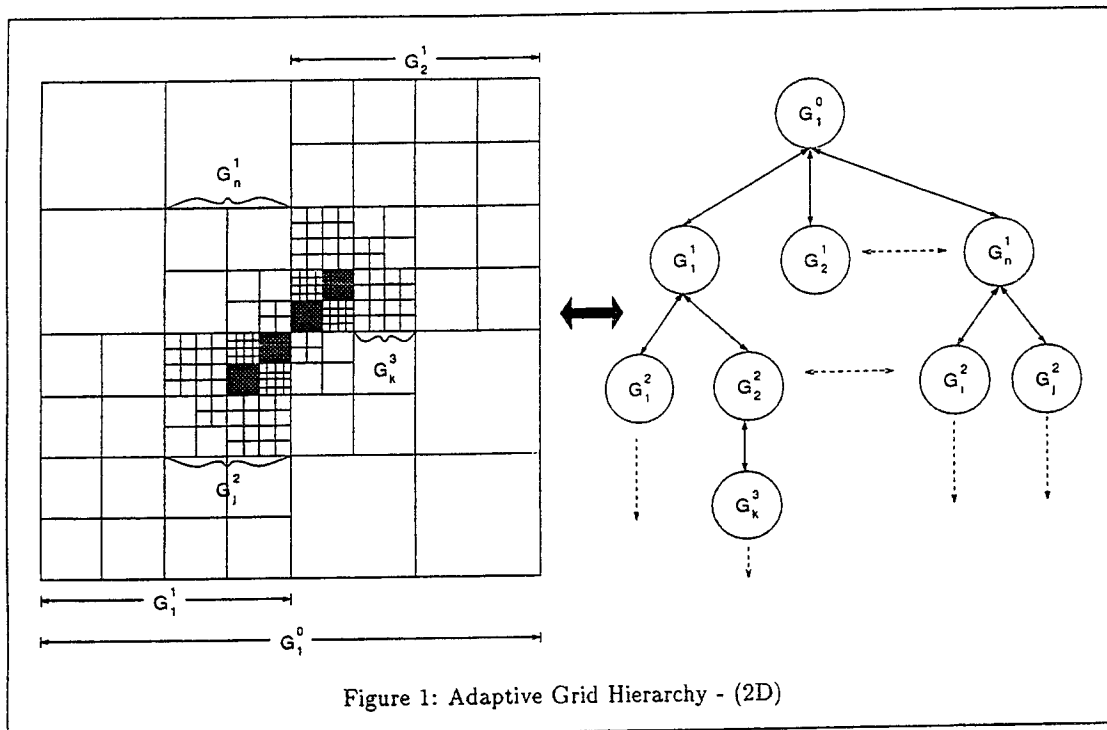
RICHARDSON EXTRAPOLATION  
(1911)





## Distributed Adaptive Grid Hierarchy

- C, C++, MPI wrapper
- Recycles "dusty deck" FORTRAN, C
- Automatically (spacetime) refines/derefines. [Shadow Hierarchy].
- Efficient  $\sim 90\%$ ; better than hand coded in all cases checked.



```

#include "DAGH.h"
#include "sorfortran.h"

#define DIM 3
#define MaxLev 1

void main(INTEGER argc, char *argv[])
{
    MPI_Init( &argc, &argv );

    INTEGER nx, ny, nz;
    DOUBLE h, p, omega, epsilon;
    f_readinput(&nx, &ny, &nz, &h, &p, &omega, &epsilon);

    // Set up the Grid Structure
    INTEGER shape[DIM];
    DOUBLE bb[2*DIM];
    shape[0] = nx;
    bb[0] = 1.0;
    bb[1] = 1.0*(nx+1);
    shape[1] = ny;
    bb[2] = 1.0;
    bb[3] = 1.0*(ny+1);
    shape[2] = nz;
    bb[4] = 1.0;
    bb[5] = 1.0*(nz+1);

    INTEGER t = 0, l = 0;

    GridHierarchy GH(DIM, NON_CELL_CENTERED, MaxLev);
    SetBaseGrid (GH, bb, shape);
    ComposeHierarchy(GH);

    INTEGER t_sten = 0; INTEGER s_sten = 1;
    GridFunction(DIM) <DOUBLE> u("u", t_sten, s_sten, GH);
    GridFunction(DIM) <DOUBLE> rhs("rhs", t_sten, s_sten, GH);
    GridFunction(DIM) <DOUBLE> res("res", t_sten, s_sten, GH);

    while (norm2 > epsilon) {

        cnt++;

        Sync(u, t, l);

        pass = 1;
        forall(u, t, l, c)

            f_relax (
                FBA(u(t, l, c)), shape,
                FDA(u(t, l, c)), FDA(rhs(t, l, c)), FDA(res(t, l, c)),
                &h, &p, &omega, &epsilon, &norm, &pass);

        end_forall

        Sync(u, t, l);

        pass = 2;
        forall(u, t, l, c)

            f_relax (
                FBA(u(t, l, c)), shape,
                FDA(u(t, l, c)), FDA(rhs(t, l, c)), FDA(res(t, l, c)),
                &h, &p, &omega, &epsilon, &norm, &pass);

        end_forall

        norm2 = Norm2(res, t, l);
        maxval = MaxVal(u, t, l);
        minval = MinVal(u, t, l);

        if (me == 0) {
            cout << cnt << " " << norm2 << " " << norm << " ";
            cout << maxval << " " << minval << " ";
            cout << "\n" << flush;
        }
    }
}

```

```
/* Fortran Interfaces */
```

```
#define f_readinput FORTRAN_NAME(sor_readinput_, SOR_READINPUT, sor_readinput)  
extern "C" {  
void f_readinput(  
    INTEGER *, INTEGER *, INTEGER *,  
    DOUBLE *, DOUBLE *, DOUBLE *, DOUBLE *);  
}
```

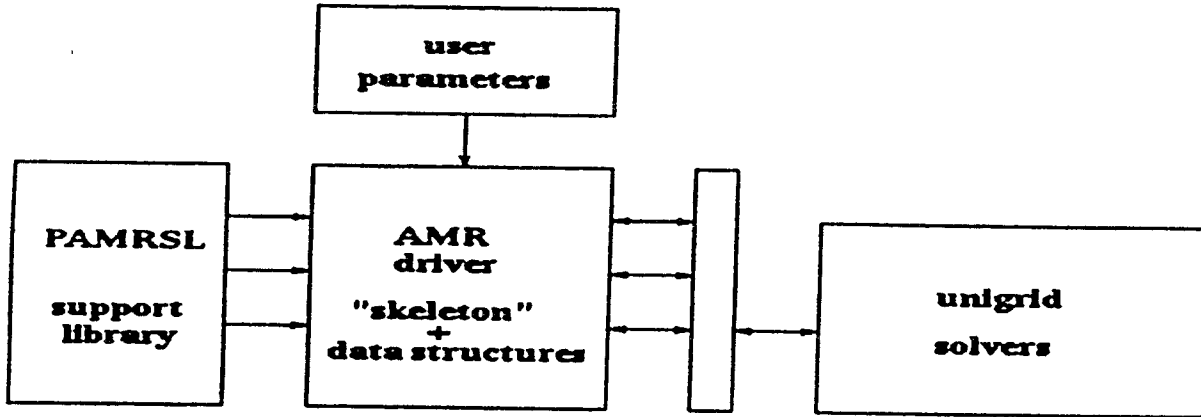
```
#define f_initial FORTRAN_NAME(sor_init_, SOR_INIT, sor_init)  
extern "C" {  
void f_initial(  
    BI,  
    DOUBLE *, DOUBLE *, DOUBLE *,  
    FDI(DOUBLE), FDI(DOUBLE), FDI(DOUBLE),  
    DOUBLE *, DOUBLE *, DOUBLE *, DOUBLE *);  
}
```

```
#define f_relax FORTRAN_NAME(sor_relax_, SOR_RELAX, sor_relax)  
extern "C" {  
void f_relax(  
    BI, INTEGER *,  
    FDI(DOUBLE), FDI(DOUBLE), FDI(DOUBLE),  
    DOUBLE *, DOUBLE *, DOUBLE *, DOUBLE *, DOUBLE *,  
    INTEGER *);  
}
```

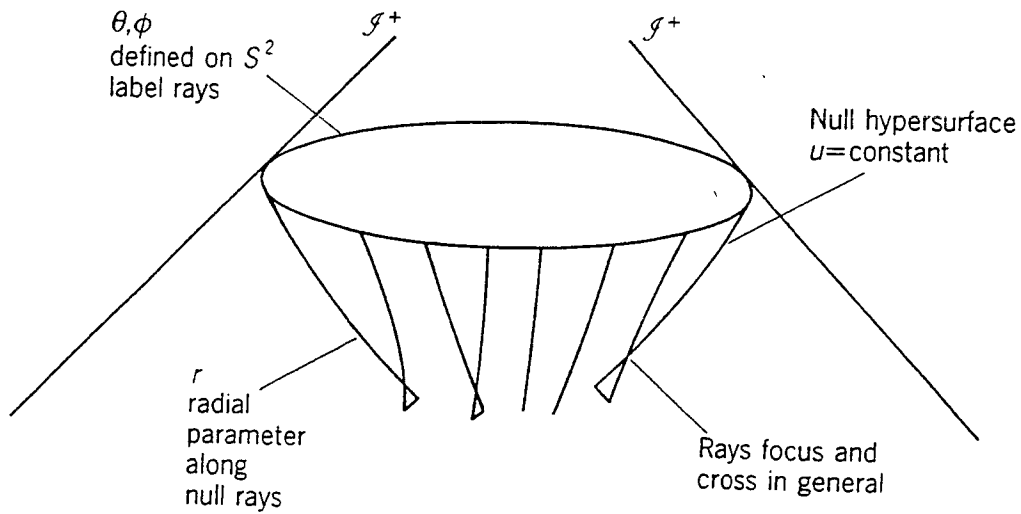
```
C-----  
      subroutine sor_relax(lb,ub,shape,gshape,  
    &          u,rhs,res,  
    &          h, p, omega, epsilon, norm, pass)  
      implicit none  
  
      integer lb(3),ub(3),shape(3),gshape(3)  
      real*8 u(shape(1),shape(2),shape(3))  
      real*8 rhs(shape(1),shape(2),shape(3))  
      real*8 res(shape(1),shape(2),shape(3))  
  
      real*8 h, p, omega, epsilon, norm  
      real*8 norm2  
      integer pass  
  
      call relaxc(lb,ub,shape,gshape,  
    *          u,rhs,res,  
    *          h, p, omega, pass)  
  
      if (pass .eq. 2) then  
        norm = norm2( shape(1)*shape(2)*shape(3) , res )  
      endif  
  
      return  
      end  
C-----
```

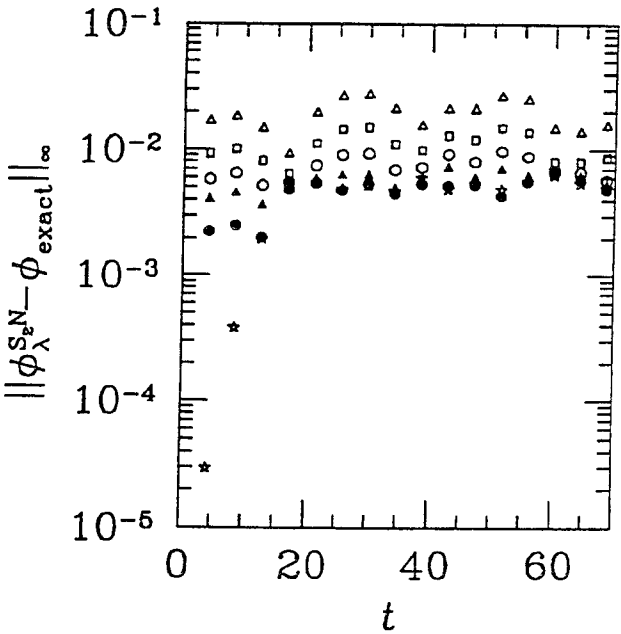
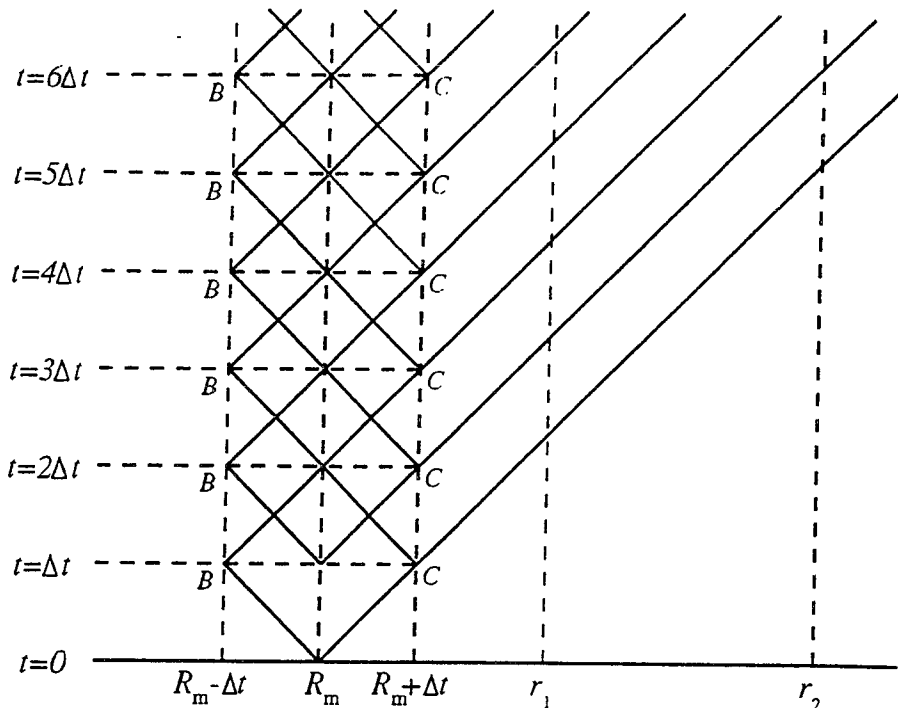
PARALLEL ADAPTIVE MESH REFINEMENT SUPPORT LIBRARY

SYSTEM ARCHITECTURE

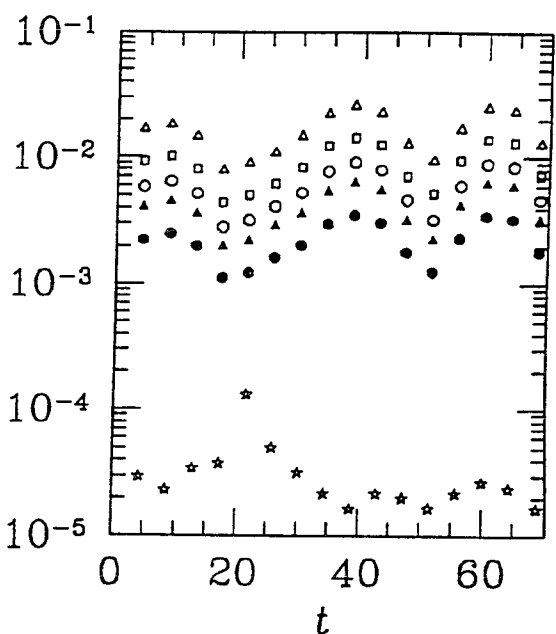


HPF/FORTRAN 90.

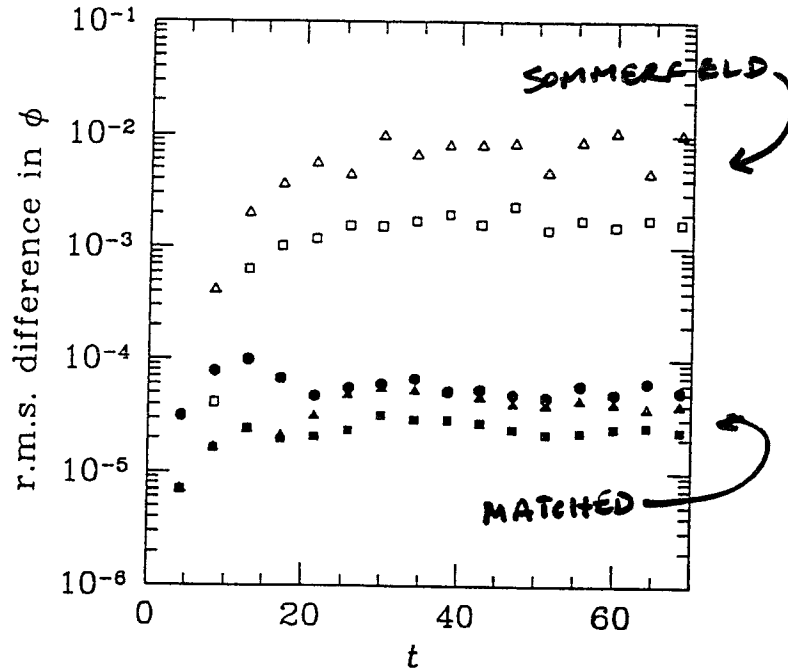




SOMMERFELD BOUNDARY



MATCHED BOUNDARIES



SENSITIVITY TO BOUNDARY  
POSITION: COMPARE  
AT 2 OUTER  
RADI

### YORK/CHOQUET-BRUHAT HYPERBOLIC FORMULATION

#### MAXWELL EXAMPLE

$$\begin{aligned}
 \partial_0 E_i - (\nabla \times B)_i &= 0 \\
 \partial_0 B_i + (\nabla \times E)_i &= 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} \partial_0 E_i - (\nabla \times B)_i &= 0 \\ \partial_0 B_i + (\nabla \times E)_i &= 0 \end{aligned}} \right\} \text{EVOLUTION EQS}$$

$$\begin{aligned}
 \nabla \cdot E &= 0 \\
 \nabla \cdot B &= 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} \nabla \cdot E &= 0 \\ \nabla \cdot B &= 0 \end{aligned}} \right\} \text{CONSTRAINTS}$$

"OLD WAY":

$$E_i = -\partial_0 A_i - \nabla_i \phi$$

$$B_l = (\nabla \times \mathbf{A})_l$$

$$\begin{aligned} -\partial_0^2 A_i - \nabla_i \partial_0 \phi &= (\nabla \times \nabla \times \mathbf{A})_i \\ &= -\nabla^2 A_i + \nabla_i (\nabla \cdot \mathbf{A}) \end{aligned}$$

$$-\partial_0^2 A_i + \nabla^2 A_i = \nabla_i (\underbrace{\partial_0 \phi + \nabla \cdot \mathbf{A}})$$

SPECIFY (eg. to zero) by gauge

**CONSERVATION OF GAUGE MUST  
BE EXPLICITLY CHECKED**

"NEW WAY":

$$\left. \begin{aligned} \mathcal{G}_{0i} &= \partial_0 E_i \\ \mathcal{G}_{ij} &= \nabla_i E_j \end{aligned} \right\} \begin{array}{l} \text{DEFINE} \\ \mathcal{G}_{0i}, \mathcal{G}_{ij} \end{array}$$

$$\begin{aligned} \partial \mathcal{G}_{0i} &= \partial_0 \partial_0 E_i = (\nabla \times \partial_0 \mathbf{B})_i \\ &= -(\nabla \times (\nabla \times \mathbf{E}))_i \\ &= \nabla^2 E_i - \nabla_i (\nabla \cdot \mathbf{E}) \quad \text{zero by constraint} \\ &= \nabla^j \nabla_j E_i \\ &= \nabla^j \mathcal{G}_{ij} \end{aligned}$$

$$\begin{aligned}\partial_o \mathcal{G}_{oi} &= \nabla^j \mathcal{G}_{ij} \\ \partial_o \mathcal{G}_{ji} &= \partial_o \nabla_j E_i = \nabla_i \partial_o E_i \\ &= \nabla_j \mathcal{G}_{oi}\end{aligned}$$



$$\partial_o E_i = \mathcal{G}_{oi} \leftarrow \text{time integration}$$

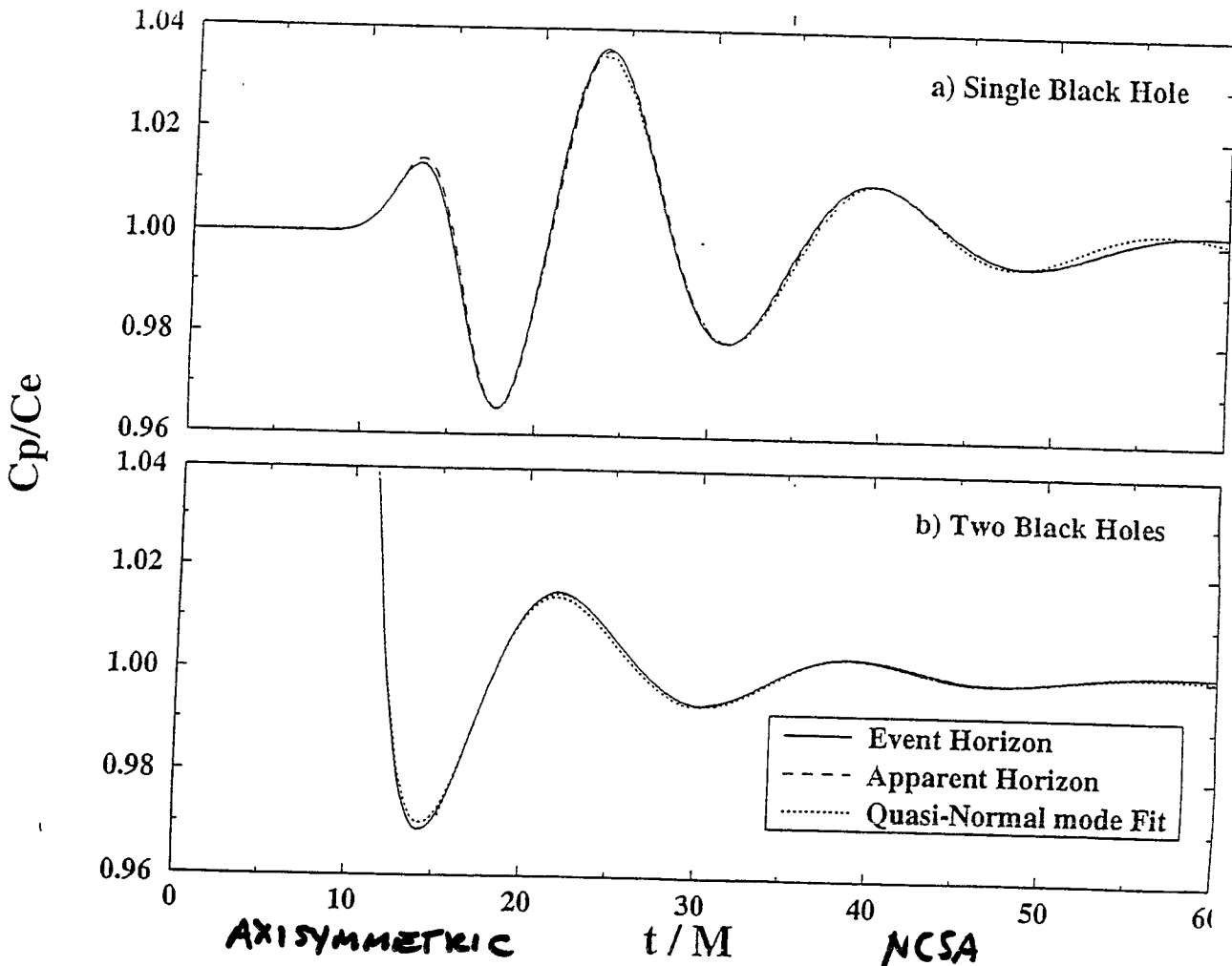
$$\begin{aligned}\partial_o \mathcal{G}_{oi} &= \nabla^j \mathcal{G}_{ji} \\ \partial_o \mathcal{G}_{ji} &= \nabla_j \mathcal{G}_{oi}\end{aligned} \left. \vphantom{\begin{aligned}\partial_o \mathcal{G}_{oi} \\ \partial_o \mathcal{G}_{ji}\end{aligned}} \right\}$$

$$\hookrightarrow \partial_o \partial_o \mathcal{G}_{oi} = \nabla^j \nabla_j \mathcal{G}_{oi}$$

CONSTRAINT IS ESSENTIAL.

GAUGE IS IRRELEVANT

- EVOLVE TIME DERIVATIVE.





Thursday PM Data Analysis II. Time-Frequency Techniques and Other Issues  
January 18 Chair: C. Will

- 4:30 B. Owen (Caltech) Searching for Coalescing Binaries:  
Templates, Strategies, Computing Requirements
- 4:55 Discussion
- 5:05 J. Centrella (Drexel) Coalescence Wave Forms and Rotational  
Instabilities
- 5:30 Discussion
- 5:40 A. Krolak (Warsaw) Estimating Parameters of  
GW Signals Using Wavelet Analysis
- 6:05 Discussion
- 6:15 Coffee Break
- 6:30 A. Vecchio (Cardiff) Why Realistic Errors in Parameter  
Extraction Don't Follow Cramer-Rao Bounds
- 6:55 Discussion
- 7:05 H. Yamamoto (Caltech) LIGO Simulation Environment
- 7:30 Discussion

Introduction

A. The gist.

This is a summary of current conventional wisdom.

I'll describe implementations of matched filtering, for 50 years known as the optimal linear technique - for well-known signals.

Recently unconventional wisdom has grown: neural nets, adaptive filtering algorithms, other nonlinear methods.

These are interesting but little quantitative analysis exists - yet.

3. Basics of matched filtering.

SNR is defined as the cross-correlation of whitened ~~informed~~ data stream with the <sup>template</sup> expected signal.

This statistic is compared to a threshold to decide if a signal is there. Threshold is set to control false alarms.

Basics of mismatched filtering.

We don't know the two-body solution in GR but approximate it.

An imperfect template reduces the mean SNR as its cross-correlation with the actual signal.

→ So it's good to know how accurate templates need to be to keep a certain

Parameterized filtering.

Also we expect many different <sup>signals</sup> ~~templates~~; same <sup>functional form</sup> ~~waveforms~~ but different <sup>fraction of event rate</sup> ~~parameters~~.

We need to construct many filters spanning parameter space, <sup>and use them all.</sup>

To detect most signals we space filters <sup>parameters</sup> very finely, but this means many filters and lots of computing power.

So it's worth some effort to find the optimum trade-off between event rate and computing power, (i.e. fine ~~grid~~ and coarse grid).

Hierarchical search strategy.

We can combine the low CPU power of a coarse <sup>grid</sup> of templates with the high event rate/low false alarm rate of a fine grid by making two passes.

The first pass, with a coarse grid selects ~~likely~~ likely candidates for an inspiral signal, ~~most~~

Candidates are then sent through a fine grid in the second pass, and more rigorously tested for false alarms.

II. Imperfection of Waveform Templates

Transp. A. The post-Newtonian expansion, and others.

Cliff Will/Alan Wiseman have much more to say about this.

~3 lines

PN is the workhorse for constructing templates - an approximation currently good to  $O(v/c)^4$  beyond leading RR.

Test-mass perturbation of exact GR ~~is up to~~ <sup>(extreme mass ratio)</sup> is up to  $O(v/c)^8$  in some cases and rapidly getting higher - as Tajiri said Tuesday.

This is a lot of PhD-hours for each order - how much is needed?

Cutler & Flanagan estimate  $O(v/c)^4$  for parameter extraction - but search templates should be less finicky.

B. The fitting factor.

For search templates we don't want to use v/c as a criterion.

FF = max...

Instead, Apostolatos introduced the fitting factor, which gives the fraction of optimal SNR obtained by an ~~approximate~~ <sup>approximate</sup> family of templates.

$\Delta\Phi$  graph

The max appears because an ~~infinite~~ <sup>approximate</sup> template with one set of parameters will in general have its greatest cross-correlation with an exact waveform which ~~it~~ had a different set of parameters.

geometric graph

That's bad for extraction - leads to systematic errors in parameters - but good for detection because ensures FF is (much) higher than the simple cross-correlation.

Harris' table

C. Estimates of fitting factors

We don't really know FF because we don't know the signal - estimate FF by taking best ~~approximate~~ <sup>approximate</sup> waveform for signal and seeing how lesser templates do.

Best estimate now is Apostolatos' manuscript using  $O(v/c)^4$  PN templates as the signal - his results indicate that  $O(v/c)^2$  is good enough to keep well over 90% of optimal ER ( $\sim$  cube of FF).

As expected, search templates don't require as much accuracy as extraction templates.

## Parameterized Filtering

### A. The goal.

We want to find an expression for  $N$  and  $P$  in terms of event rate - i.e. if 10% ER loss is acceptable, how much will that cost us?

### B. The method.

For each template shape  $U_2$  we compute the following function... this SNR ~~is~~ <sup>notes: ~~is~~ ~~for~~ ~~all~~ ~~the~~ ~~possibility~~ ~~of~~ ~~different~~ ~~various~~ ~~possible~~ ~~coalescence~~ ~~times~~ ~~and~~ ~~phases~~ ~~the~~ ~~signal~~ ~~might~~ ~~have~~.</sup> tells us if a stretch of data exceeds threshold for a certain template shape, which is numerically efficient  $\propto N \log N$

In a brute-force search we do this for  $N$  different values of  $\lambda$ ; so it's useful to define the match as...

The match is the fraction of optimal SNR we get by searching for  $U_2$  with  $U_2$  <sup>chosen</sup> neighbors.

We ~~choose~~ our filter spacing so that the match between neighbors is some fixed amount.

### C. The minimal match.

Analogous to the FF, we define the MM <sup>to be</sup> ~~the~~ <sup>the</sup> worst case match between signal and template (assuming our templates are good enough that  $FF \approx 1$ ).  $\rightarrow ER \sim MM^3$

For a brute-force search, or 2nd pass of a hierarchical search, ~~we~~  $MM \approx 1$  in order to keep most of the event rate - so expand the match to get a metric defined on the intrinsic parameter space.

~~Thus~~  $ds^2$  between nearby template parameters is <sup>fraction of</sup> optimal SNR lost due to ~~parameter~~ difference.

### D. Achieving the goals.

Now getting  $N$  is a trivial ratio of volumes - lets us decide what <sup>part of</sup> parameter space, too.

For a ~~two~~ two-parameter family of templates (PN, two masses) searching for signals from objects of mass  $M_{min}$  or greater, we get...

This corresponds to a computing power of...

We see that even a brute force search is possible for nonspinning binaries of minimum NS mass or greater.

Believe  $N$  and  $P$  to  $\sim 3$ : I used restricted PN to  $(\theta(L/c))^2$ , no ecc. no spins.

### E. Precessing binaries.

Apostolatos' new manuscript <sup>suggests</sup> that significant spin-orbit precession necessitates 3 more parameters - multiplies  $N, P$  by  $10^6$ !

Don't know yet what range of spins is a problem - too new & complicated.

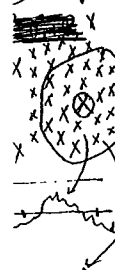
Also don't know feasibility of considering spins only for the most common masses, e.g. (1.4  $M_{\odot}$ ).

or approximating simpler functions.

Too new to say - got the manuscript Monday - but warns of a possible major problem.

## IV. Hierarchical Search Strategy (Work in progress) <sup>Dharmadhar Mohanty, Thorne, & van</sup>

### A. ~~The~~ <sup>transp.</sup> The method.



First the data are passed through a coarse grid of filters and compared to a relatively low threshold.

For every template shape that triggers in the first pass, a second pass is made in which all shapes on a finer grid around it are ~~not~~ tested for some range of coalescence times including the range that triggered the first-pass template.

A detection is registered only if one of these second-pass templates ~~reaches~~ achieves a SNR above a higher threshold - roughly that which would have been used for a brute force search, to prevent false alarms.

### B. Calculating $P$ .

$P$  depends on ~~MM, MM2, MM1~~ (false alarm rate) and  $p_2$  (false alarm rate on 1st pass) ~~and MM1, MM2~~ (false alarm rate on 2nd pass).

Fixing ~~overall~~ <sup>overall</sup> false alarm rate

We have four variables  $p_1, p_2, MM_1, MM_2$  characterizing the two passes. <sup>false alarm rate</sup>

Because the second pass is conditional on the results of the first, we lose one independent variable.

Setting overall false alarm and false dismissal rates reduces us to one independent variable, which is convenient to parameterize as roughly  $\xi = \frac{1-MM_1}{1-MM_2}$ .

Find minimum  $P$  by setting  $\frac{dP}{d\xi} = 0$ .

Crappy details make it hard to do exactly, but a rough approximation (current best guess) indicates a savings of tenfold (1st LIGO) or less (advanced).

## V. Conclusions

It is possible to achieve <sup>over 90</sup> ~~with~~ <sup>few</sup> percent of ideal event rate ~~well~~ within the limits of end-of-the-decade computing technology.

Except... ~~high-spin systems are a problem~~ <sup>high-spin systems are a problem</sup>, and Kip now wants to look for objects down to  $10^{-2} M_{\odot}$  for reasons he will tell you about.

Not only does my scaling law predict horrible computing requirements, but filters will need to last for months.

Thus we have Doppler shifts, sky patches, all the headaches of pulsar searches in addition to binary searches.

# Searching for Inspiring Binaries: Strategies, Templates, and Computing Requirements

Ben Owen  
Caltech

matched filtering:

$$\rho = 4 \operatorname{Re} \left[ \int_0^{\infty} dt \frac{S^*(t) [A \tilde{s}(t) + i \tilde{r}(t)]}{S_0(t)} \right] = \operatorname{Re} [(s, A s + n)]$$

$(s, s) = 1$ ;  $A =$  constant amplitude of signal

$$\bar{\rho} = A \text{ (optimal)}$$

mismatched filtering:

template  $u$  such that  $(u, u) = 1$

$$\bar{\rho} = \operatorname{Re} [(A s, u)]$$

$$\bar{\rho} / \bar{\rho}_{\text{opt}} = \operatorname{Re} [(s, u)]$$

parameterized filtering:

$u_k$ : template with shape parameters  $\lambda$

$$\rho = \max_k \operatorname{Re} [(A s + n, u_{\lambda(k)})]$$

$$k = 1, 2, \dots, N$$

Approximations to two-body GR:

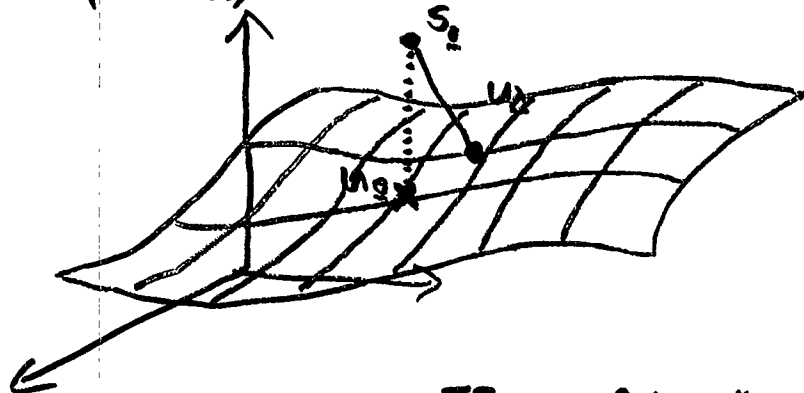
Post-Newtonian known to  $\mathcal{O}(v/c)^4$

BH Perturbation subject to weekly changes!

How well do we need to know these approximations?

Cutler & Flanagan: less than  $\mathcal{O}(v/c)^6$

Fitting Factor:  $FF(\theta) = \max_{\lambda} \max_{A t_c} |(s_{\theta}, u_{\lambda} e^{i 2 \pi f A t_c})|$   
(Apostolatos)



Template Family	FF with $\mathcal{O}(v/c)^4$	
	NS/NS	BH/NS
"Newtonian"	.531	.669
$\mathcal{O}(v/c)^2$	.620	.729
$\mathcal{O}(v/c)^3$	.993	.990

# Template Counting Sathyaprakash & Dhurandhar, Owen

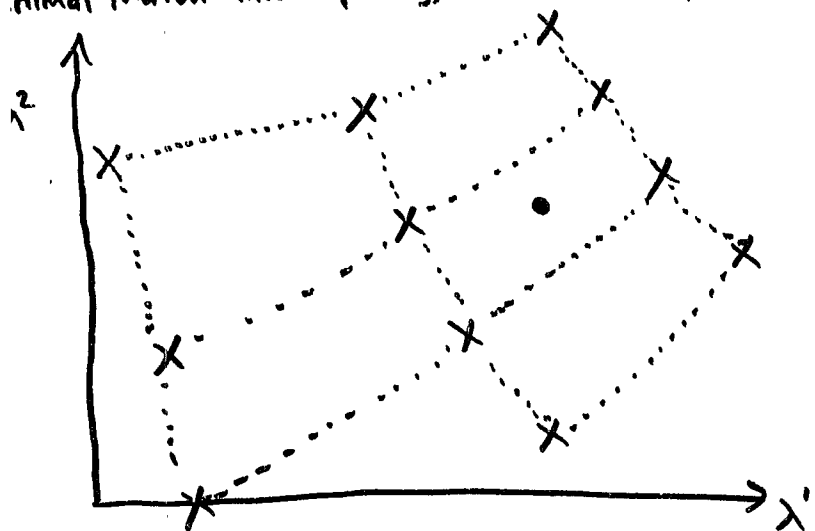
$$P_{\Delta}(t) = \left| 4 \int_0^{\infty} \frac{[A^2(f) + \gamma^2(f)]^2}{S_n(f)} u_{\Delta}(f) e^{i2\pi f t} \right|$$

$$= |(A \sin, u_{\Delta} e^{i2\pi f t})|$$



Match:  $M(\lambda, \Delta) = \max_t |(u_{\lambda}, u_{\Delta} e^{i2\pi f t})|$

- accounts for covariance between  $\lambda$  and time, phase  
 Minimal Match fixes spacing, number of templates



Fine grid approximation:

$$M(\lambda, \lambda + \Delta\lambda) \approx 1 - O(\Delta\lambda)^2$$

$$g_{ij} = -\frac{1}{2} \frac{\partial^2 M}{\partial \lambda^i \partial \lambda^j} \Big|_{\Delta\lambda=0}$$

Differential geometry makes it easy:

$$N = \frac{\text{total volume of parameter space}}{\text{volume per template}}$$

$$= \frac{\int d^n \lambda \sqrt{\det \|g_{ij}\|}}{(2 \sqrt{(1-MM)/N})^n}$$

Loss of event rate  $\mathcal{L} \approx 3(1-MM)$  for  $MM \approx 1$

- Example templates:
- (two) mass parameters only
  - restricted PN to  $O(v/c)^2$
  - objects with mass  $\geq M_{min}$

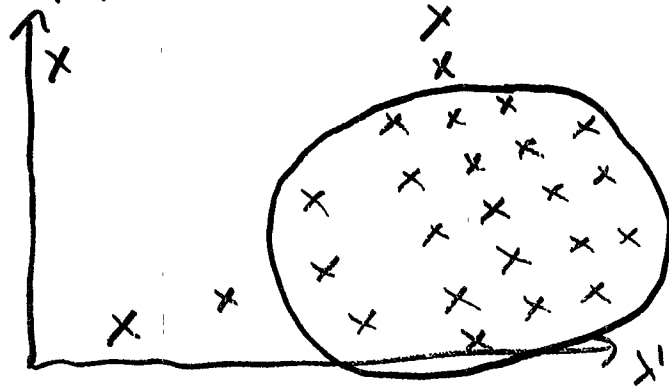
$$N \approx \left(\frac{0.1}{\mathcal{L}}\right) \left(\frac{0.2 M_{\odot}}{M_{min}}\right)^{2.7} \times \begin{cases} 2 \times 10^5 & \text{first LIGO} \\ 8 \times 10^6 & \text{advanced LIGO} \end{cases}$$

$$P \approx \left(\frac{0.1}{\mathcal{L}}\right) \left(\frac{0.2 M_{\odot}}{M_{min}}\right)^{2.7} \times \begin{cases} 30 \text{ Gflops} & \text{first LIGO} \\ 400 \text{ Gflops} & \text{advanced LIGO} \end{cases}$$

→ Heavy (but bearable) demands

## Hierarchical Search Strategy

In preparation: Dhurandhar; Mohanty, Thorne; Owen



total computing power:  $P = P_1 + P_2$

Four parameters:  $p_1, p_2, MM_1, MM_2$

- $P_2 \propto$  false alarm rate in first pass
- Set fixed overall false alarm rate  
... and false dismissal rate
- One remaining independent parameter:

$$\xi = \frac{1 - MM_1}{1 - MM_2}$$

• Minimize  $P(\xi)$ :  $dP/d\xi = 0$

$\sim \frac{1}{10}$  of brute force  $P$  (first LIGO)

$\sim \frac{1}{3}$  for advanced LIGO

## Conclusions:

- We can achieve over 90% of optimal event rate with end-of-decade computing technology for some common binary systems.
- Detecting rapidly spinning binaries may be more difficult.
- Kip's new suggestion - to search for  $10^{-2} M_\odot$  objects - will be even more difficult.

# Binary Coalescence

and

# Rotational Instabilities

Joan Centrella

Drexel University

3-D numerical models of astrophysical  
GW sources

Use 2 different hydrodynamical codes:  
Eulerian - grid

Lagrangian - particles - SPH

Current simulations use Newtonian  
gravity, with GW in quadrupole  
approximation

## Coalescing Binary Neutron Stars

Zhuge, Centrella, McMillan 1994 PRD 50, 6247;  
1996, in preparation

- Model NS as spherical polytropes of mass  $M$ ,  
radius  $R$ , EOS  $P = K\rho^\Gamma = K\rho^{\Gamma+1/n}$
- Start out in point-mass limit at wide  
separation  $\geq 4R$  on circular orbit
- To cause inspiral, mimic grav. radiation reaction
  - add "frictional" terms to eqns of motion
  - removes energy from orbit at rate given  
by point-mass inspiral
  - turn frictional terms off when Newtonian  
tidal forces dominate  $\rightarrow$  coalescence
- Simulations carried out using smoothed  
particle hydrodynamics - SPH
- Standard Run :  $M_1 = M_2 = 1.4 M_\odot$   
 $R_1 = R_2 = 10 \text{ km}$   
 $\Gamma = 2$  ; no spin
- Vary:  $R$ ,  $\Gamma$ ,  $\frac{M_2}{M_1}$ , spin

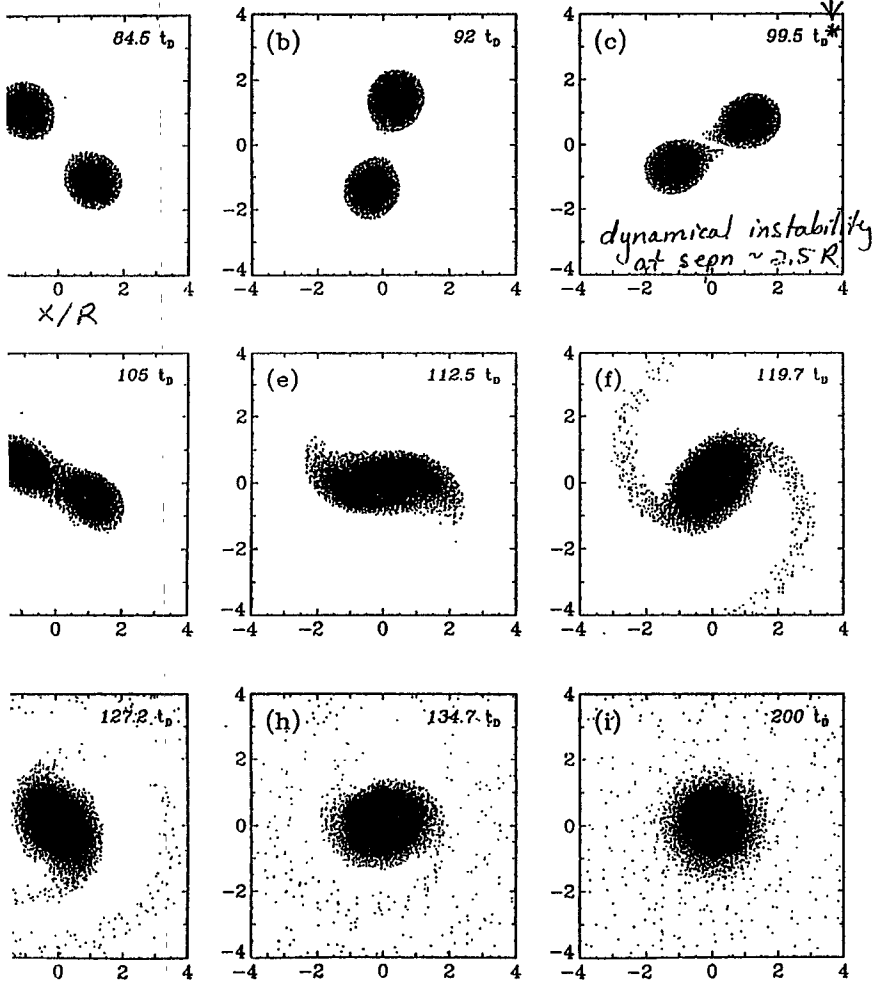
$M_1 = M_2 = 1.4 M_\odot$   
 $p = 2$   
 $R = 10 \text{ km}$

$$t_D = \left( \frac{R^3}{GM} \right)^{1/2} = .073 \text{ ms}$$

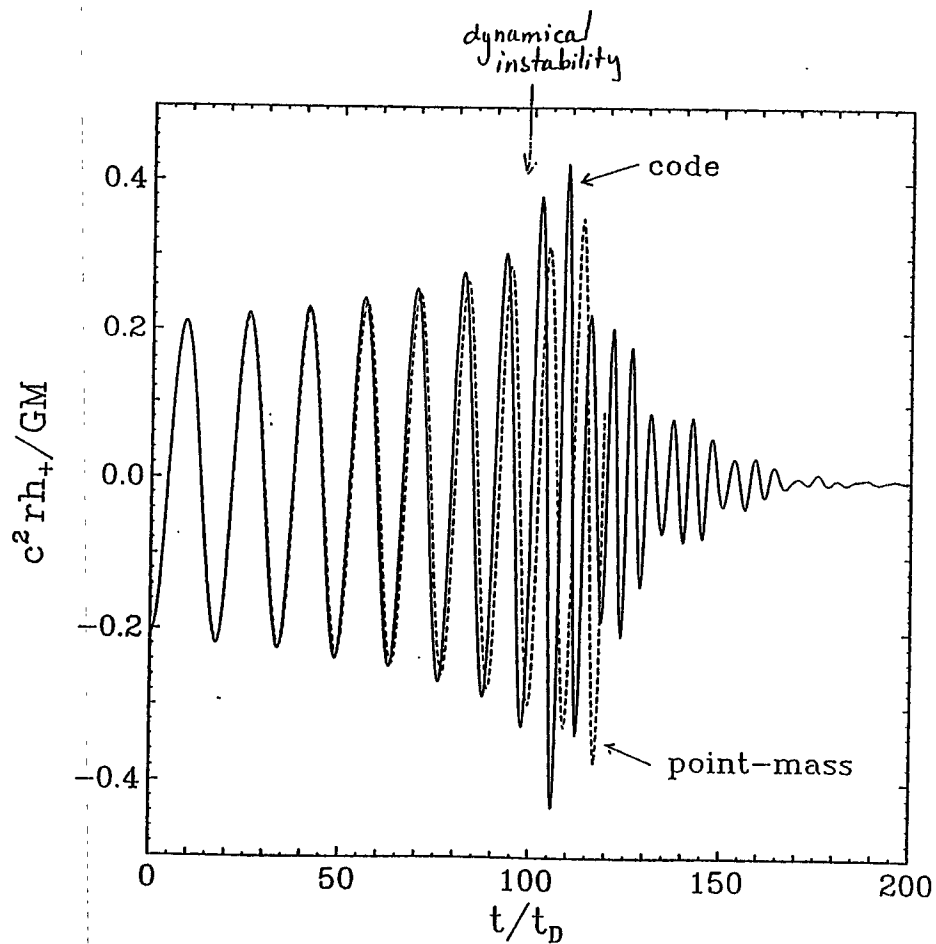
$N = 7046 \text{ particles/star}$

turn off  
 "friction" term

stars orbit counterclockwise



$M_1 = M_2 = 1.4 M_\odot$   
 $p = 2$   
 $R = 10 \text{ km}$

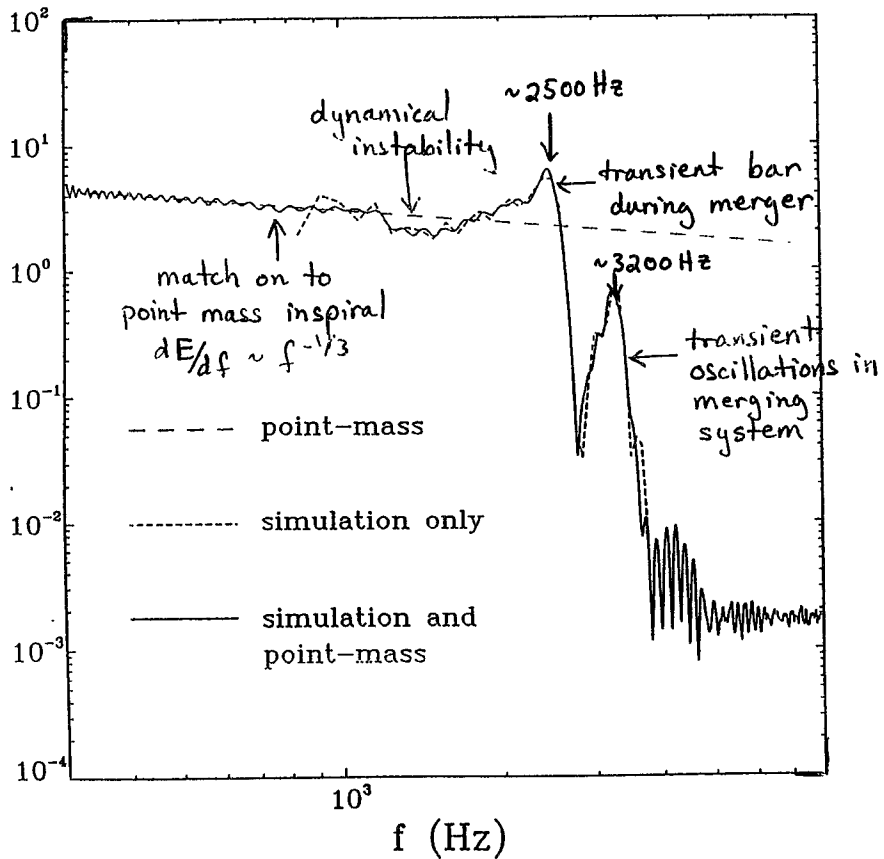




$$M_1 = M_2 = 1.4 M_\odot$$

$$\Gamma = 2$$

$$R = 10 \text{ km}$$



•• point mass inspiral  $\propto a^{-3}$   
 $a = \text{separation}$

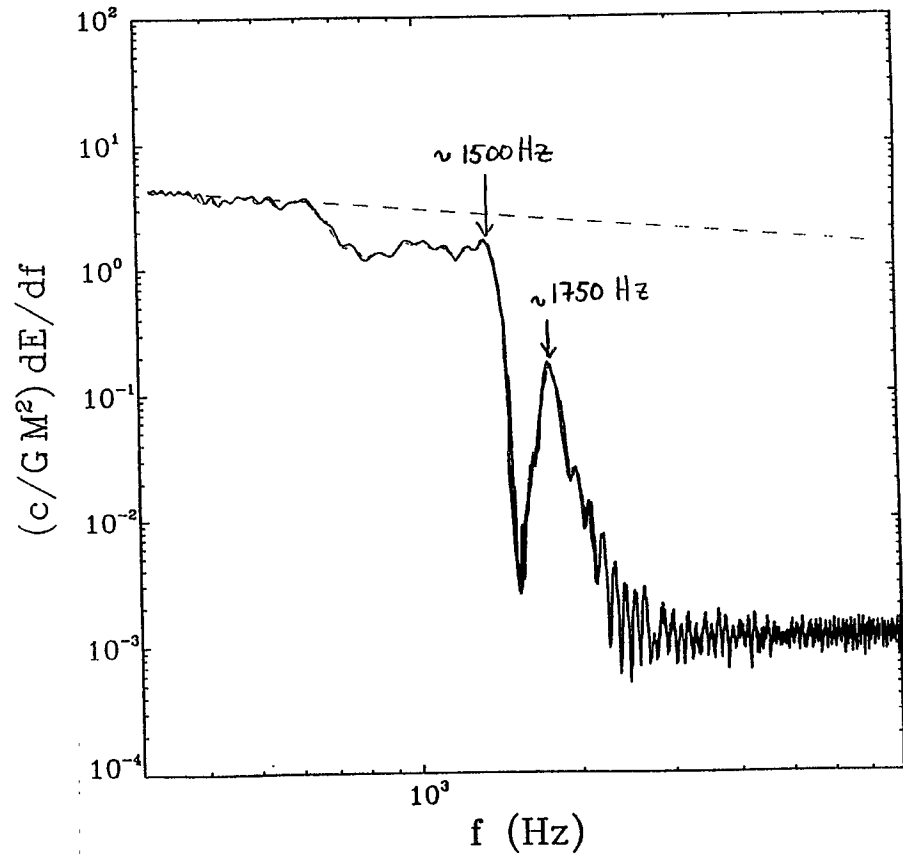
•• features in spectrum show this scaling:

$$M_1 = M_2 = 1.4 M_\odot$$

$$\Gamma = 2$$

$$R = 15 \text{ km}$$

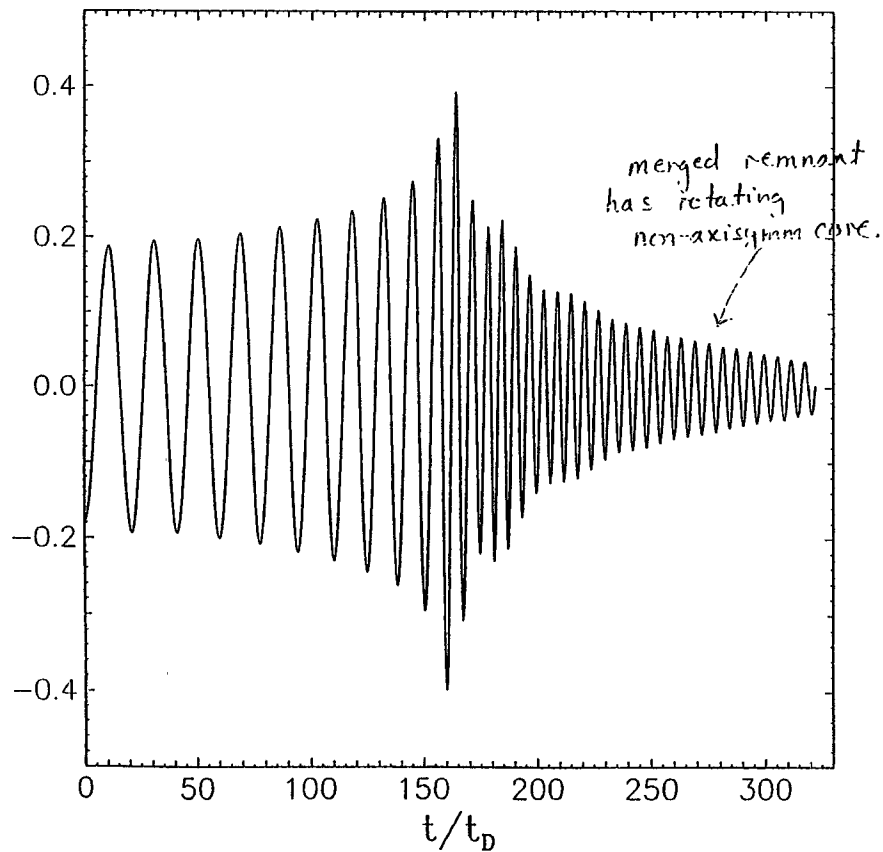
$$\frac{f_{R=10 \text{ km}}}{f_{R=15 \text{ km}}} \sim \left( \frac{R=10 \text{ km}}{R=15 \text{ km}} \right)^{-3/2} \sim 1.8$$



$$M_1 = M_2 = 1.4 M_\odot$$

$$R = 10 \text{ km}$$

$$\Gamma = 3$$



$$M_1 = M_2 = 1.4 M_\odot$$

$$R = 10 \text{ km}$$

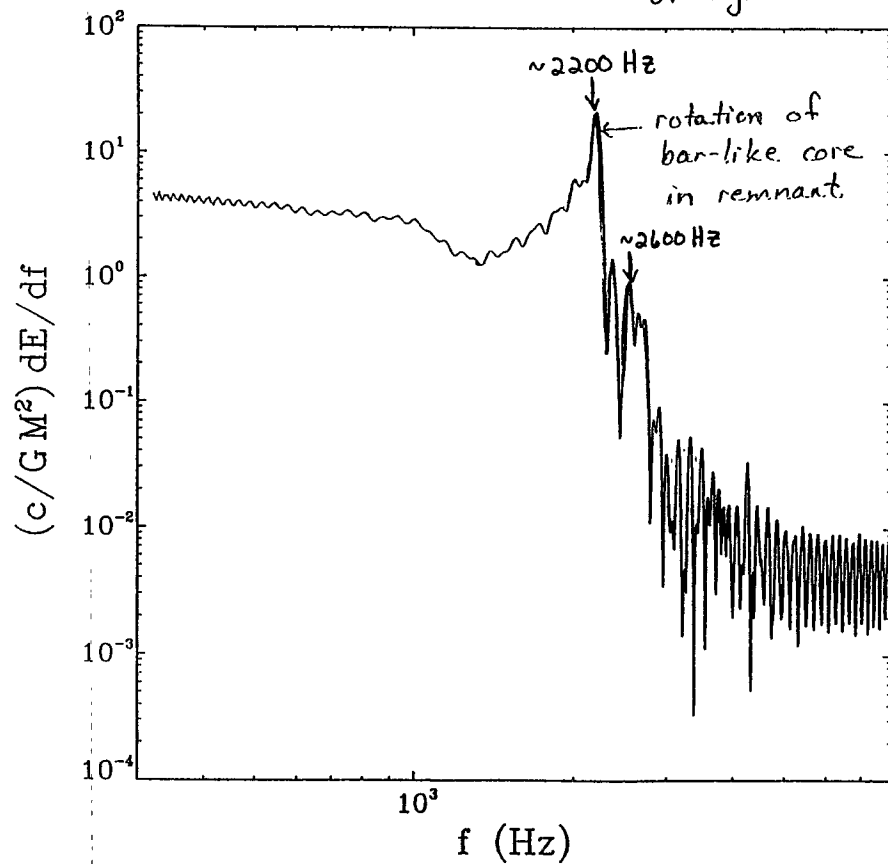
$$\Gamma = 3$$

frequency scaling:

$$\frac{f_{\Gamma=2}}{f_{\Gamma=3}} \sim 1.2$$

$$f_{\Gamma=3}$$

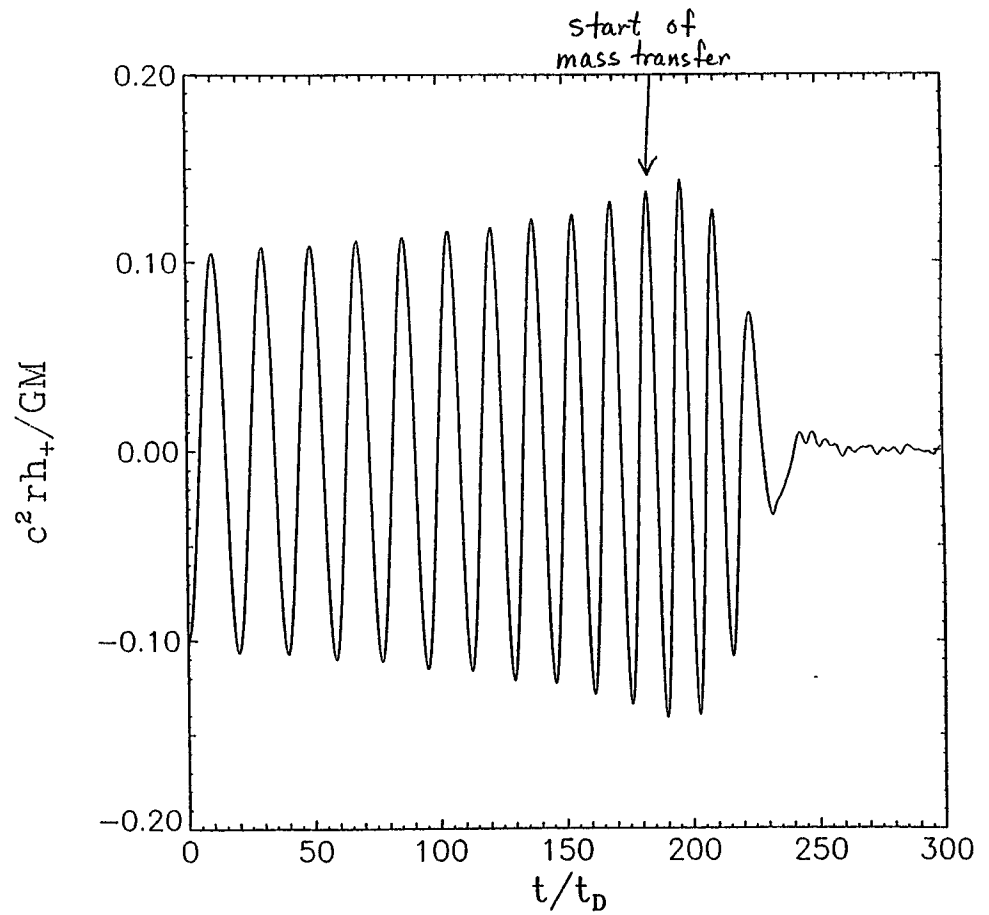
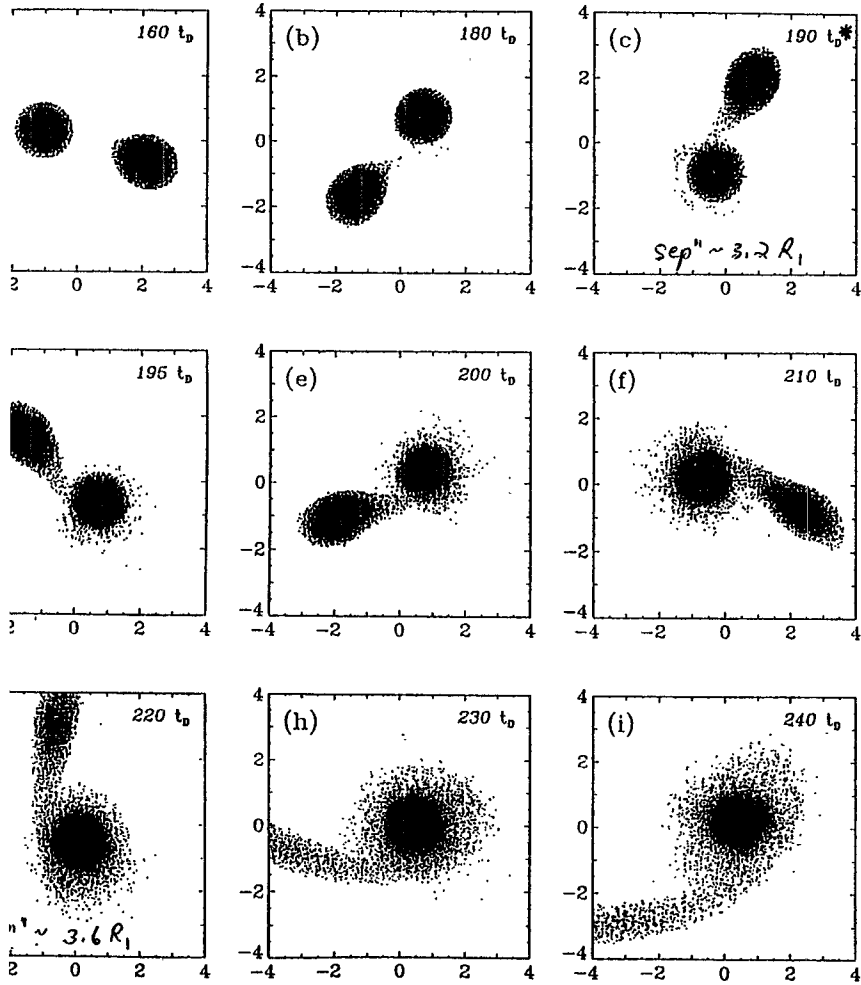
cf. Lai, Rasio + Shapiro -  
freq scaling for onset  
of dynamical instab



$N = 4696$  particles/star

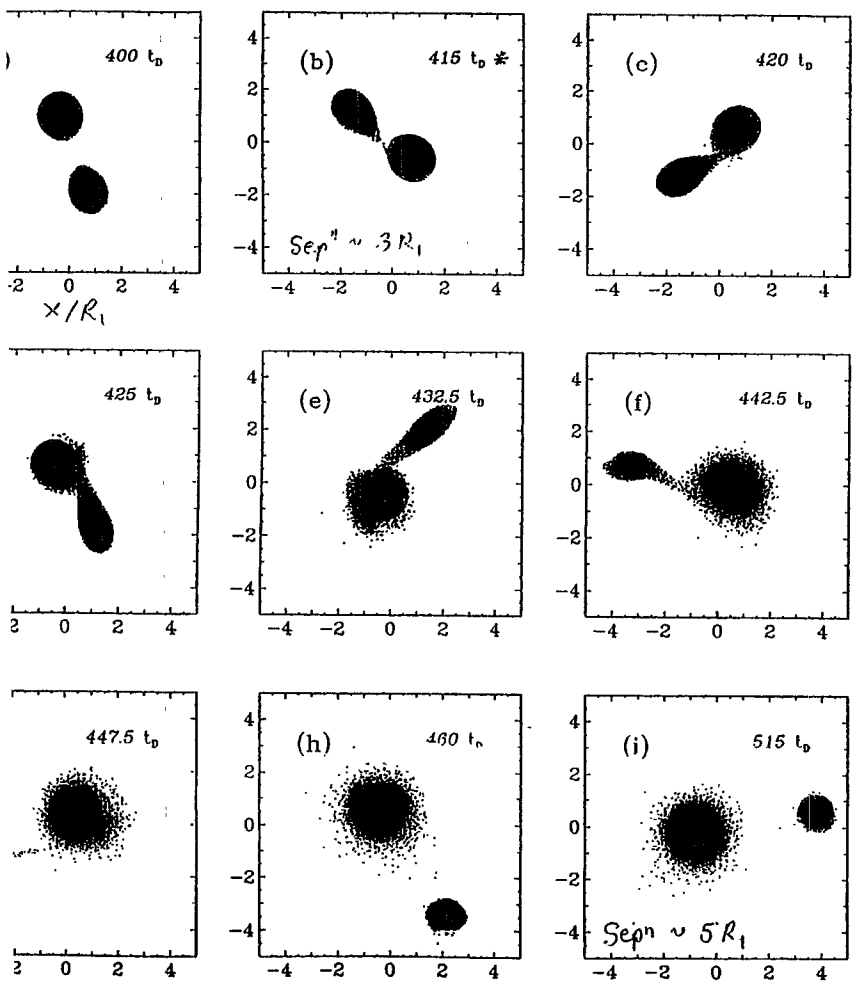
$$\left. \begin{array}{l} M_1 = 1.4 M_\odot \\ M_2 = \frac{1}{2} M_1 = 0.7 M_\odot \\ \Gamma = 2 \end{array} \right\} \begin{array}{l} R_1 = 10 \text{ km} \\ R_2 = 10 \text{ km} \\ K_1 = K_2 \end{array}$$

$$\left. \begin{array}{l} M_1 = 1.4 M_\odot \\ M_2 = \frac{1}{2} M_1 = 0.7 M_\odot \\ \Gamma = 2 \end{array} \right\} \begin{array}{l} R_1 = 10 \text{ km} \\ R_2 = 10 \text{ km} \end{array}$$

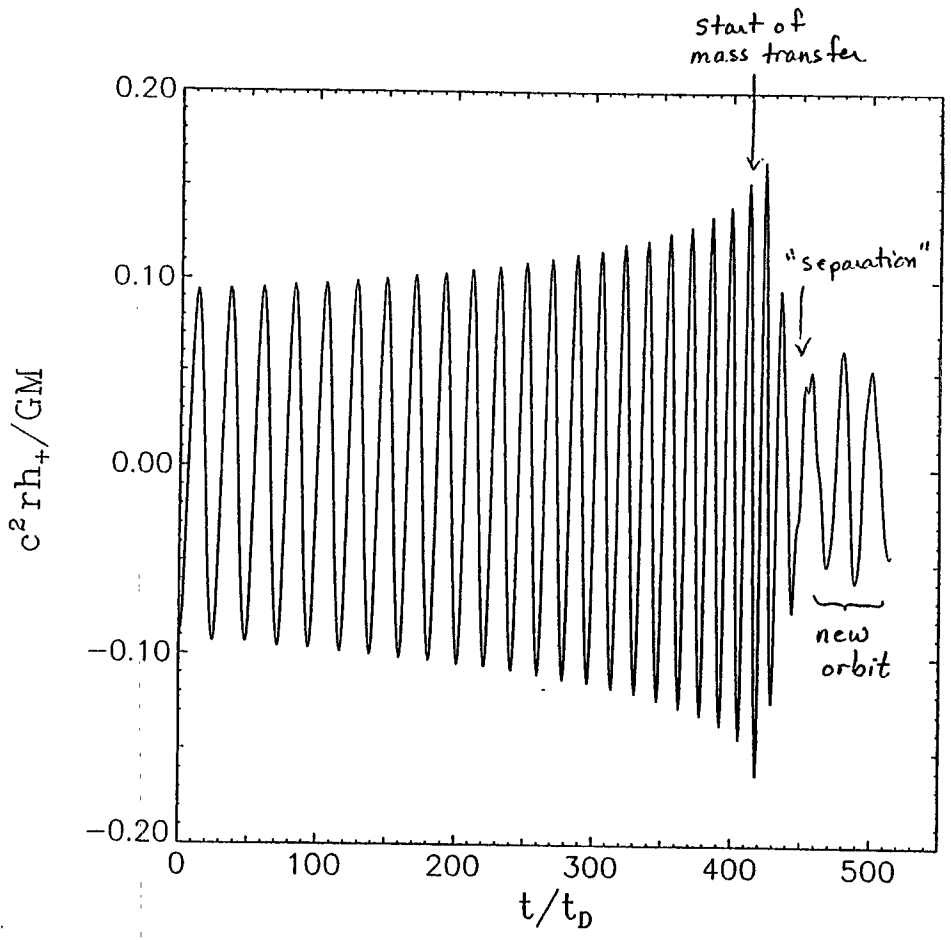


$N = 4076$  particles/star

$M_1 = 1.4 M_\odot$        $R_1 = 10 \text{ km}$   
 $M_2 = \frac{1}{2} M_1 = 0.7 M_\odot$        $R_2 = 8.7 \text{ km}$   
 $\Gamma = 3$



$M_1 = 1.4 M_\odot$        $R_1 = 10 \text{ km}$   
 $M_2 = \frac{1}{2} M_1 = 0.7 M_\odot$        $R_2 = 8.7 \text{ km}$   
 $\Gamma = 3$



## axisymmetric Rotational Instabilities

Houser, Centrella, + Smith 1994 PRL 72, 1314  
Smith, Houser, Centrella, 1996 ApJ (in press)

axisymmetric rotational instabilities @  $\Omega \sim \omega$   
• arise in rapidly rotating axisymmetric fluids

•  $\beta = \frac{\text{rotational KE}}{\text{grav PE}}$  sufficiently large

•  $m=2$  "bar mode"

scenarios:

- Rotating, collapsing stellar core - prevented from collapsing to NS by centrifugal forces - "centrifugal hang-up"
- Rotating stellar spun up by accretion

axisymmetric instability occurs  
→ burst of GW

relations: start with axisymmetric equilibrium models, polytropes  $P = K\rho^n$ , mass  $M$ , equatorial radius  $R$

models run using Eulerian code + SPH

## Models:

- $\mu = 5/3$ ,  $\beta = .30$ , run using SPH + Eulerian code

highly flattened:

polar radius  $\sim .2 R$

differential rotation

$N \approx 32,000$  particles  
system rotates counterclockwise

Figure 10

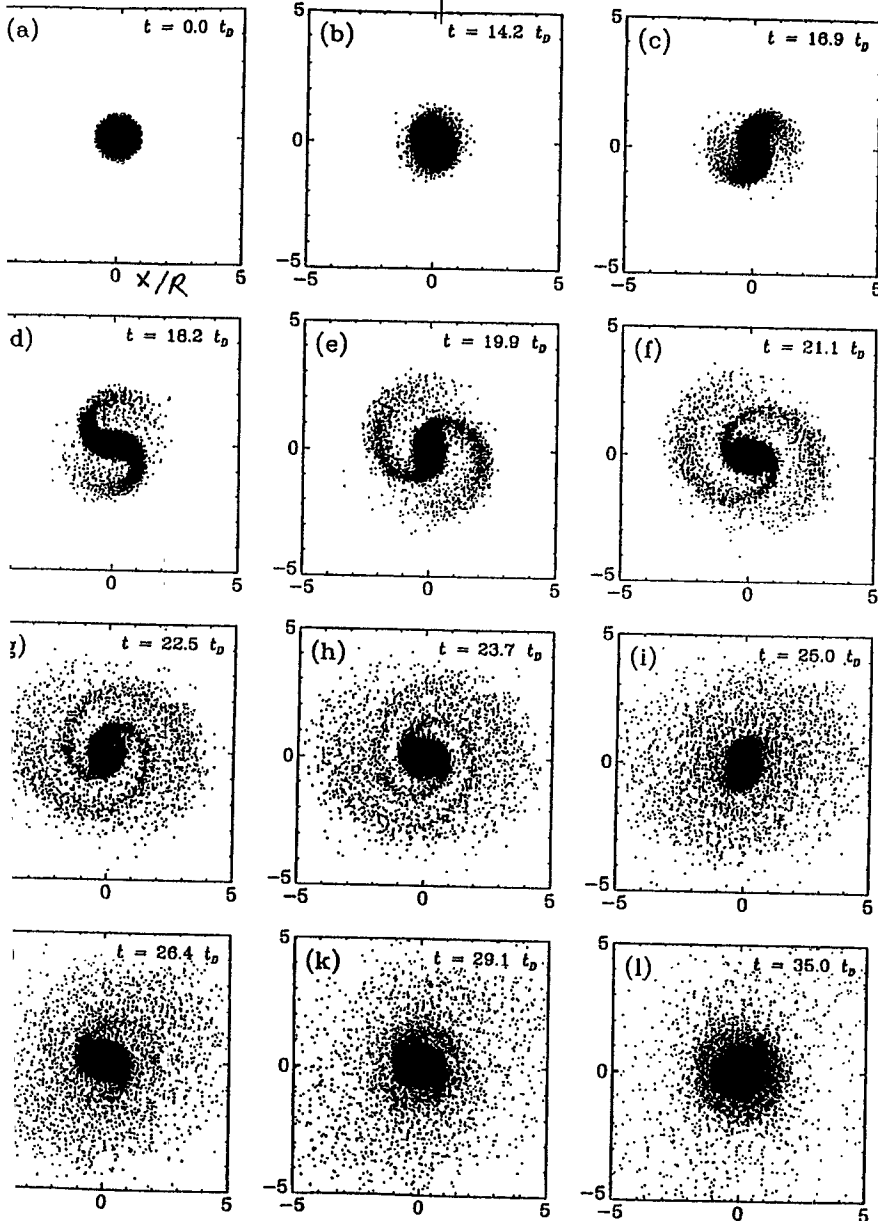
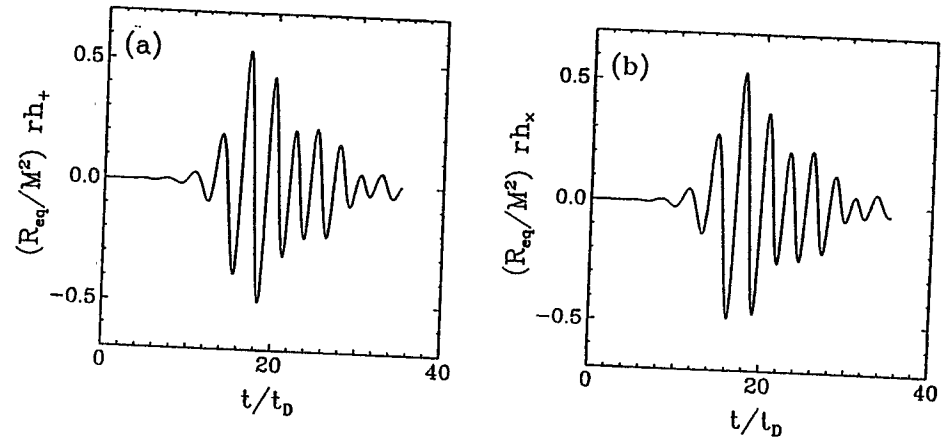


Figure 18

Grav Waves for observer  
on axis at  $\Theta = \psi = 0$   
( $c = G = 1$ )



$$T_{GW} \sim 3.5 t_D$$

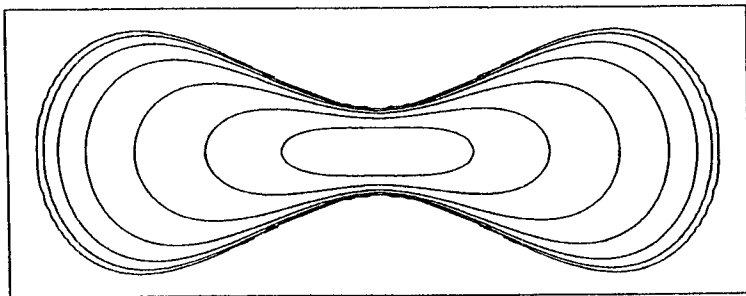
$$t_D = \left[ \frac{R^3}{GM} \right]^{1/2}$$

$N=3$

$$\Gamma = 4/3, \quad \beta = .30$$

run using Eulerian code

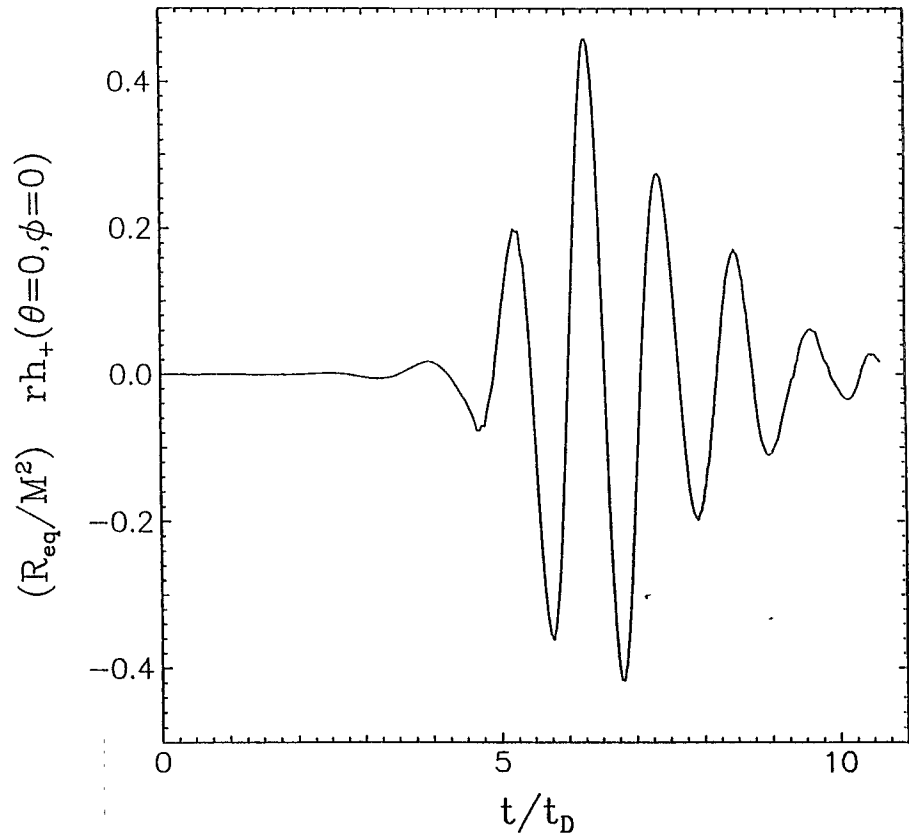
initial axisymm. equilibrium model



$N=3$

Gravitational Waveform  
( $c = \zeta = 1$ )

$$T_{GW} \sim t_D$$



scaling - Amplitude + frequency of GW depend  
on physical parameters when instability occurs

$$1.4 M_{\odot}, R = 10^3 \text{ km}$$

$$r = 20 \text{ Mpc}, h \sim 3 \times 10^{-24}$$

$$r = 10 \text{ kpc}, h \sim 6 \times 10^{-21}$$

$$f \sim 10 \text{ Hz}$$

$$2 M_{\odot}, R = 4 \times 10^3 \text{ km}$$

$$r = 20 \text{ Mpc}, h \sim 1.5 \times 10^{-24}$$

$$r = 10 \text{ kpc}, h \sim 3 \times 10^{-21}$$

$$f \sim 1 \text{ Hz}$$

$$1.4 M_{\odot}, R = 10^2 \text{ km}$$

$$r = 20 \text{ Mpc}, h \sim 3 \times 10^{-23}$$

$$r = 10 \text{ kpc}, h \sim 6 \times 10^{-20}$$

$$f \sim 300 \text{ Hz}$$

$$1.4 M_{\odot}, R = 10 \text{ km}$$

$$r = 20 \text{ Mpc}, h \sim 3 \times 10^{-22}$$

$$r = 10 \text{ kpc}, h \sim 6 \times 10^{-19}$$

$$f \sim 4 \text{ kHz}$$

→ Robust numerical codes able to  
carry out detailed astrophysical  
modeling in the "Golden Age"  
of Gravitational Wave Astronomy



Directions ...

rotational instabilities: effects of  
different rotation laws + collapse

more realistic modeling of EOS

General Relativistic effects:

- for binaries, more rapid plunge?
- collapse to Black Hole
  - final remnant?
  - earlier - cf. J. Wilson - ?
- rotational instabilities + collapse

⋮

# Estimating parameters of ~~quasi-periodic~~ wave signals using wavelet analysis

A. KRÓLAK

Aspen Winter Physics Conference - January 1996

A common way to analyse a signal  $s(t)$  to decompose it with respect to a certain set of functions  $\Psi(t, p)$

$$Tf(p) = \int_{-\infty}^{+\infty} s(t) \Psi^*(t, p) dt$$

$$= \int_{-\infty}^{+\infty} \hat{s}(\omega) \hat{\Psi}^*(\omega, p) d\omega$$

uncertainty principle:  $\Delta t \Delta \omega \geq \frac{1}{2}$

von sampling:  $\psi(t, t_0) = \delta(t - t_0) \quad \hat{\psi}(\omega, t_0) = \frac{1}{\sqrt{2\pi}} e^{i\omega t_0}$

transform:  $\psi(t, \omega_0) = \frac{1}{\sqrt{2\pi}} e^{-i\omega_0 t} \quad \hat{\psi}(\omega, \omega_0) = \delta(\omega - \omega_0)$

transform:  $\psi(t, t_0, \omega_0) = \frac{1}{\sqrt{2\pi}} e^{-i\omega_0 t} e^{-\frac{(t-t_0)^2}{2\sigma^2}}$

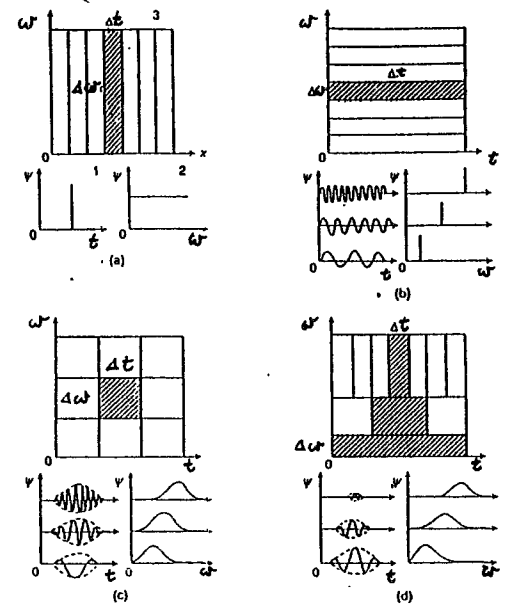
$$\hat{\psi}(\omega, t_0, \omega_0) = \sigma e^{-\frac{(\omega-\omega_0)^2 \sigma^2}{2}}$$

transform:  $\psi(t, t_0, a) = \frac{1}{\sqrt{2\pi}} \frac{1}{a} e^{-i\frac{2\pi(t-t_0)}{a}} e^{-\frac{(t-t_0)^2}{2a^2}}$

+ wavelet)

$$\hat{\psi}(\omega, t_0, a) = e^{i\frac{2\pi t_0}{a}} e^{-\frac{(\omega-\omega_0)^2 a^2}{2}}$$

(2)



A comparison of the way different transforms balance physical and wavenumber-space resolution. (a) Shannon transform. Perfect physical resolution, no wavenumber-space resolution. (b) Fourier transform. Perfect wavenumber-space resolution, no physical-space resolution. (c) Gabor transform. Balance between wavenumber- and physical-space resolution: the same at all length-scales. (d) Wavelet transform. Balance between wavenumber- and physical-space resolution varies with length-scale. Smaller length-scales are more finely resolved: mathematical microscope

retroscattered wave signal from a source in  
 multipole approximation

$$s(t) = \frac{A}{(t_c - t)^{1/4}} \cos \phi(t), \quad A = \text{const}$$

$$\phi(t) = -2 \left( \frac{t_c - t}{5M} \right)^{5/8}$$

$$t_c = \frac{5}{256} \frac{1}{(\pi f_i)^{3/2}} \frac{1}{M^{5/2}} \quad f_i = f(0)$$

stationary frequency:

$$f(t) = \frac{1}{2\pi} \phi'(t) = \frac{1}{8\pi M} \left( \frac{t_c - t}{5M} \right)^{-3/8}$$

important characteristic of the signal is

- bandwidth product BT

$$BT = T_s B_s = \frac{\int_{-\infty}^{+\infty} t^2 |h(t)|^2 dt}{\int_{-\infty}^{+\infty} |h(t)|^2 dt}$$

$$B_s = \frac{\int_{-\infty}^{+\infty} f^2 |\hat{h}(f)|^2 df}{\int_{-\infty}^{+\infty} |\hat{h}(f)|^2 df}$$

$$BT = 581 \frac{1}{(\text{NO } f_i \text{ } 100\text{Hz})^{5/2}} \left( \frac{f_u}{f_i} \right)^{1/2}$$

typical GW signal from a coalescing binary  
 used by laser interferometers will be  
 characterized by a LARGE BT product.

A large time-bandwidth product  
 allows application of the ridge extraction  
 method from wavelet transform and  
 consequently determination of the frequency  
 modulation law of the signal.\*

Phase of the wavelet transform  
 (calculated by means of stationary phase  
 approximation) and using the Morlet wavelet  
 is given by

$$T(b, a) = \frac{1}{a} \int_{-\infty}^{+\infty} s(t) \overline{g\left(\frac{t-b}{a}\right)} dt$$

stationary point:  $f(t_s) = \frac{1}{a}$

$$\varphi_s(b, a) = \arg T(b, a) = \phi(t_s) - \omega_0 \frac{t_s - b}{a} + \frac{1}{2} (t_s - b)^2 \frac{\Phi''(t_s)}{1 + a^4 \Phi''(t_s)} + \frac{1}{2} \arctan [a^2 \Phi''(t_s)]$$

$$\Phi''(t_s) = 2\pi f'(t_s) = \frac{192\pi^2}{5} a^{-11/2} (\pi M)^{5/2}$$

$$t_s = \frac{5}{256\pi} \frac{1}{(\pi M)^{5/2}} \left[ \frac{1}{f_i^{3/2}} - a^{3/2} \right]$$

\* Delprat et al. IEEE Trans. Inform. Theory 38, 644 (1992).  
 General theory  
 Innoent & Vinet Cardiff Workshop, April 1992  
 Application to GW signal from a binary

Ridge of the transform is defined by (5)

$$t_r(b, a) = b$$

$$\Rightarrow \frac{1}{d_r(b)} = \frac{1}{8\pi a^2} \left( \frac{t_c - b}{5M} \right)^{-3/2} = f(b)$$

2. ridge determines the frequency modulation

$$\frac{1}{a} \left. \frac{d\tilde{\Psi}}{db} \right|_{a=\text{const}} = \frac{1}{a} + (t_s - b) \frac{\frac{1}{2\pi} \Phi''(t_r)}{(1 + a^4 \Phi''(t_r))^2}$$

On the ridge

$$\frac{1}{2\pi} \left. \frac{d\tilde{\Psi}}{db} \right|_{a=\text{const, ridge}} = \frac{1}{d_r}$$

This provides a way to extract numerically ridge of the wavelet transform and

the frequency modulation law.

Reconstruction of the signal.

$$s(b) = \sqrt{\frac{2}{\pi}} (1 + a^4 \Phi''(b)^2)^{1/4} \exp[-\frac{i}{2} \arctan(a^2 \Phi''(b))] T(b, ar)$$

↑  
skeleton of  
the wavelet  
transform

$$t) = \text{Re } Z_s(t)$$

can be used as a matched filter

Estimation of the chirp mass (6)

A numerical algorithm consisting of

1. Linearization:  $\ell(t) = 1 - \left( \frac{f(t)}{100} \right)^{-8/3}$
2. Smoothing:  $t(b) = \frac{(800a)^{3/2}}{5} \sqrt{t}$
3. Linear regression

Estimation of the chirp mass from the frequency modulation extracted from the wavelet transform.

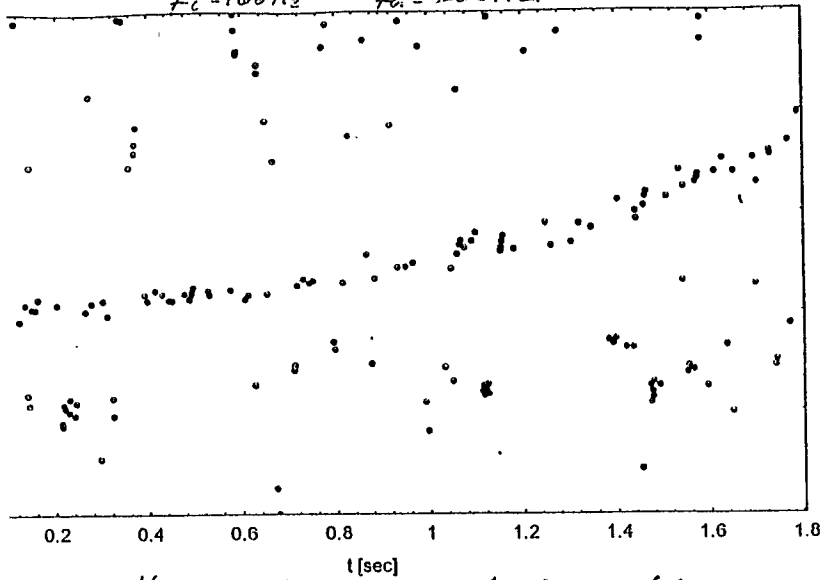
$$m_1 = m_2 = 1.4 M_\odot \quad M_\odot \approx 1.22$$

50 ms for each signal-to-noise ratio

$d_0$	$M_\odot$	bias	$\sigma_{M_\odot}$
84	1.2157	0.0031	0.0023
25	1.2155	0.0033	0.0087
8	1.2156	0.0022	0.021

$$\frac{\sigma_{M_\odot}}{M_\odot} \approx \frac{2.0}{d_0} \%$$

Ridge of the wavelet transform  
 $f_0 = 100 \text{ Hz}$      $f_0 = 200 \text{ Hz}$



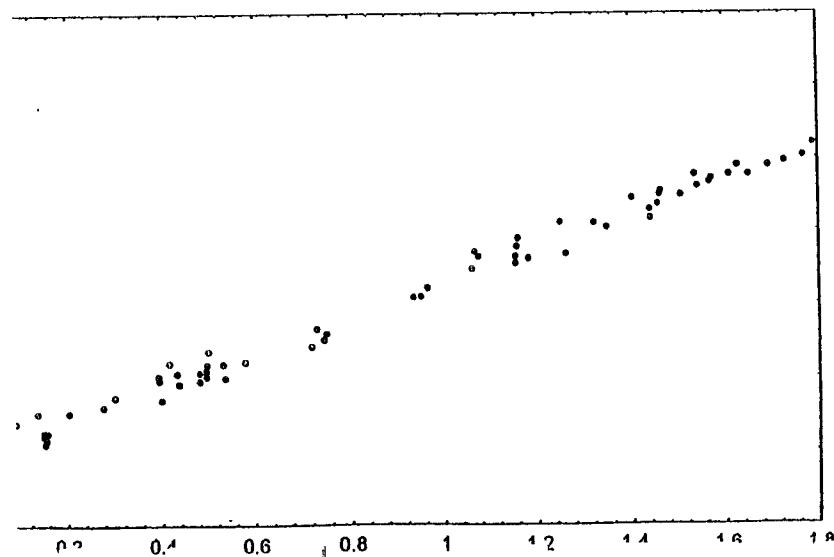
responding optimal signal-to-noise  
 No  $d_0 = 25$

We have analyzed the signal with the 1<sup>st</sup> PN correction to see whether by wavelet analysis one can discriminate small corrections to the signal

Estimation of generalized chirp mass

$d_0$	$\overline{M_{90}}$	bias	$\sigma_{M_{90}}$
84	1.1584	0.06042	0.002733
25	1.1584	0.06096	0.01298
8	1.1648	0.05397	0.02952

Linearization and smoothing of the ridge



Comparison with the non-periodic filter analysis  
 Analytic formula for the generalized chirp mass:

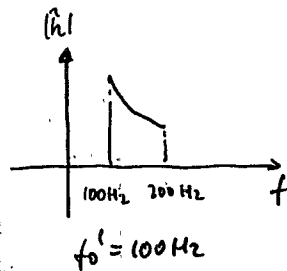
$$M_{90} = \frac{1}{k}^{3/5}$$

$$k = k_1 + k_2 (\bar{\alpha} f_0)^{2/3}$$

$$k_1 = \frac{1}{M^{2/5}}$$

$$k_2 = \frac{1}{M} \left( \frac{743}{336} + \frac{11}{4} \frac{M}{m} \right)$$

$$d^2 = 4 \int_{f_0}^{\infty} \frac{|\dot{h}|^2}{f^4} dt = \text{const} \int_{f_0}^{\infty} \frac{dt}{f^5}$$



$$M_{90} = 1.1655$$

There now exist better numerical  
 methods to extract the ridge of  
 wavelet transform and reconstruct  
 signal when the noise is high;

p. simulated annealing algorithm.

R.A. Carmona, W.L. Hwang, S. Torresani  
 "Ridge detection with continuous  
 wavelet transform" *IEEE Trans. on*

Application of wavelet analysis to  
 detection of GW signals from coalescing  
 binaries

Flaminio, L. Martonetti, B. Moura, S. Fissot,  
 Verkindt, M. Yvert, *Astroparticle Physics* 2,  
 35 (1994)

③

### Non-linear wavelet analysis ?

The idea is to generalize the wavelet  
 analysis by introducing a non-linear  
 dilation in the wavelets

$$g(b, a) = \frac{1}{a^n} g\left(\frac{t-b}{a^k}\right)$$

where  $n$  and  $k$  can be any real number.  
 (In the standard wavelet analysis  $k = \pm 1$ .)

The motivation to consider such  
 a generalization is that by suitably  
 choosing the number  $k$  one can after  
 application of the ridge extraction algorithm  
 get the ridge as a LINEAR function of  $b$

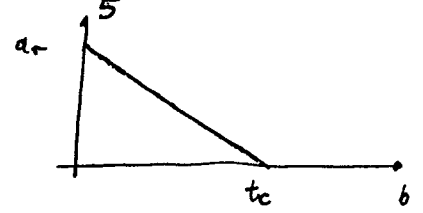
In our case we have

$$f(t) = \frac{1}{8\pi M} \left(\frac{t_c - t}{5M}\right)^{-3/8}$$

$$f(b) = \frac{1}{a_r^k}$$

choosing  $k = 3/8$

$$a_r = \left(\frac{8\pi}{5}\right)^{8/3} M^{5/3} (t_c - b)$$



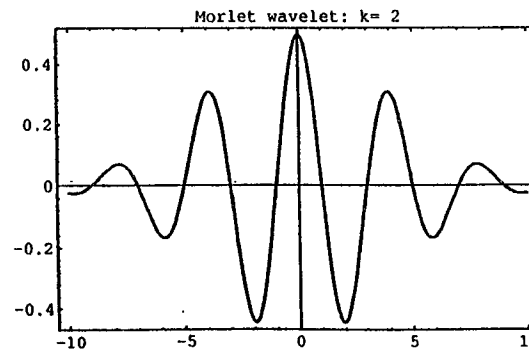
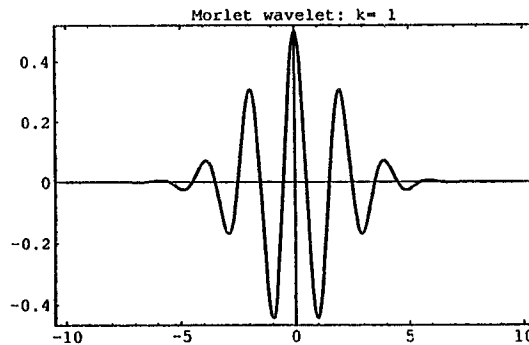
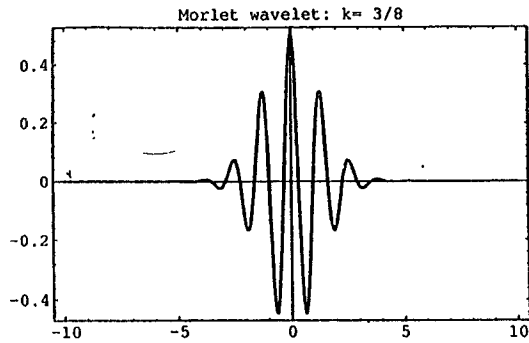
Generalized Morlet wavelets

$$\psi_k = e^{-\frac{t^2}{2a^k}} \cos \frac{bnt}{a^k}$$

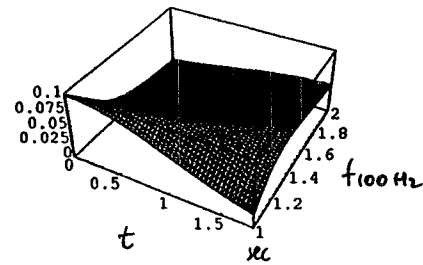
(11)

Modulus of the transform

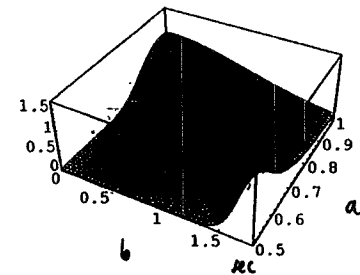
(12)



Gabor transform

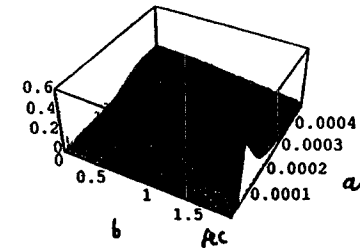


Wavelet transform



Non-linear wavelet transform

$$k = 3/8$$



# LOWER BOUNDS ON THE MINIMUM MEAN SQUARE ERROR

David Nicholson & Alberto Vecchio

*Department of Physics and Astronomy  
University of Wales - College of Cardiff*

Gravitational wave observations:

$$s(t) = h(t; \theta) + n(t)$$

where  $\theta$  is a M-dimensional vector parameter.

## Error on the estimation of $\theta$ ?

For any estimator  $\hat{\theta}$  the estimator error is  $\epsilon = \hat{\theta} - \theta$  and the error correlation matrix is given by:

$$R_{\epsilon} = \langle \epsilon \epsilon^T \rangle$$

THE GOAL:

We are interested in lower bounding  $a R_{\epsilon} a^T$  for any M-dimensional vector  $a$ . If  $a$  unit vector  $\rightarrow$  bound on the MSE of  $\theta$

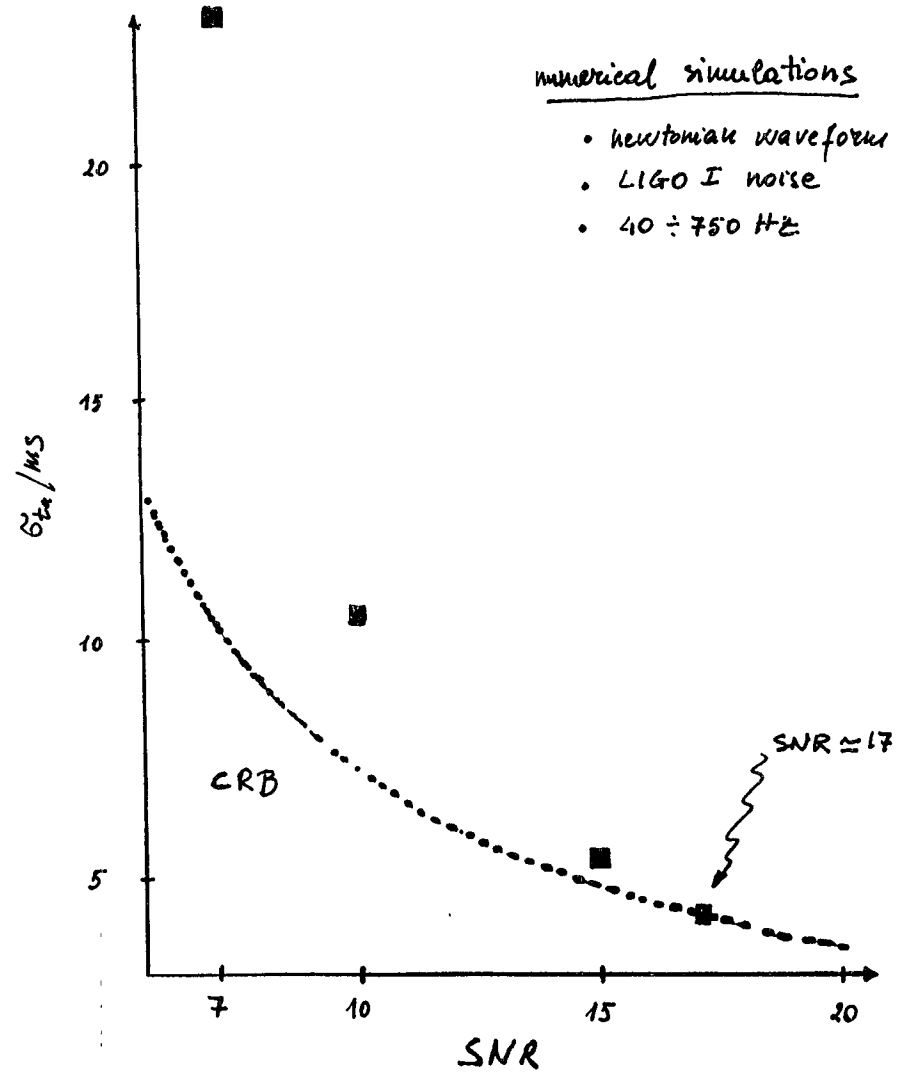
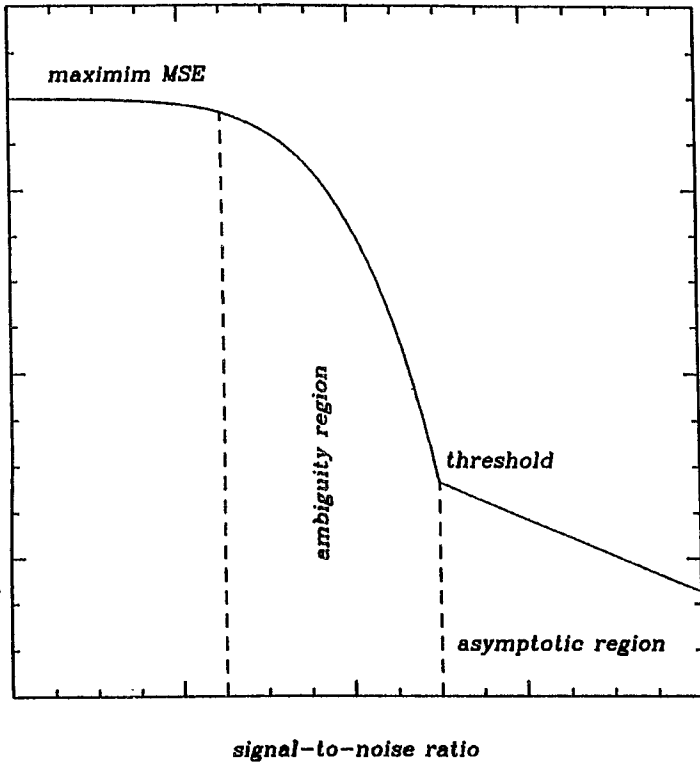
The **minimum MSE estimator** is the conditional mean estimator:

$$\hat{\theta} = \langle \theta | x \rangle = \int \theta p(\theta | x) d\theta$$

Its MSE is the greatest lower bound



MSE behaviour in non linear estimation problems



evaluation of the exact minimum MSE is often difficult or even possible  $\leadsto$  good computable bounds:

### local

- Cramer-Rao (Cramer 1945; Rao 1946)
- Barankin (Barankin 1949)

### global

- Weiss-Weinstein (Weiss & Weinstein 1985a, 1985b, 1986)
- Ziv-Zakai (Ziv & Zakai 1969; Chazan, Zakai & Ziv 1975; Bellini & Tartara 1974; Bell 1995)

### Weiss-Weinstein and Ziv-Zakai bounds (GB):

Bayesian

global

independent on the estimation process

free from regularity conditions

they can incorporate any *a priori* information

in the limit  $\text{SNR} \gg 1$ :

$$\left. \begin{array}{l} \text{WWB} \\ \text{ZZB} \end{array} \right\} \rightarrow \text{CRB}$$

in the limit  $\text{SNR} \ll 1$ :

$$\left. \begin{array}{l} \text{WWB} \\ \text{ZZB} \end{array} \right\} \rightarrow \text{a priori information}$$

Used so far  $\implies$  Cramer-Rao's bounds:

- Theoretical problems:
  1. local estimate
  2. they can not incorporate general *a priori* information
- Results of numerical simulations (Balasubramanian, Sathyaprakash & Dhurandhar 1995)

$\Downarrow$

New bounds:

- Good performance in the whole SNR region
- Accurate prediction of the location of the threshold
- Useful to compare numerical simulations and to test *information extraction algorithms*

$$\int \psi(x, \theta) p(x, \theta) d\theta = 0$$

$$\epsilon^2 \geq \frac{\langle \theta \psi(x, \theta) \rangle^2}{\langle \psi^2(x, \theta) \rangle}$$

$$L(x; \theta + \delta, \theta) - L(x; \theta, \theta)$$

WWB

$\forall \delta, 0 < \epsilon < 1$

$$L(x; \theta, \theta') = \frac{p(x, \theta)}{p(x, \theta')}$$

$$ww \geq \frac{\delta^2 e^{2\mu(\epsilon, \delta)}}{e^{\mu(2\epsilon, \delta)} + e^{\mu(2-2\epsilon, -\delta)} - e^{\mu(\epsilon, -2\delta)}}$$

$$L(\epsilon, \delta) = \ln \langle L^\epsilon(x; \theta + \delta, \theta) \rangle$$

TEST POINT

(displacement from the true parameter value)

$$\underline{\theta} = (\theta_1, \theta_2, \dots, \theta_n)$$

M parameters

$$\langle \underline{\epsilon} \underline{\epsilon}^T \rangle \approx \Delta Q^{-1} \Delta^T$$

$$\Delta = [\underline{\delta}_1, \underline{\delta}_2, \dots, \underline{\delta}_n]$$

(M x P)  
test points

$$Q_{ij} = 2 \frac{e^{\mu(\frac{1}{2}, \underline{\delta}_i - \underline{\delta}_j)} - e^{\mu(\frac{1}{2}, \underline{\delta}_i + \underline{\delta}_j)}}{e^{\mu(\frac{1}{2}, \underline{\delta}_i)} e^{\mu(\frac{1}{2}, \underline{\delta}_j)}} \quad (P \times P)$$

$$\mu(\frac{1}{2}, \underline{\delta}) = -\frac{1}{4} \rho^2 [1 - \gamma(\underline{\delta})] + \ln \int_{-\infty}^{+\infty} d\underline{\theta} [p(\underline{\theta} + \underline{\delta}) p(\underline{\theta})]^{1/2}$$

$$\gamma(\underline{\delta}) = \frac{(h(\underline{\theta} + \underline{\delta}) | h(\underline{\theta}))}{(h(\underline{\theta}) | h(\underline{\theta}))}$$

(.|.): inner product

simple example

preliminary results:

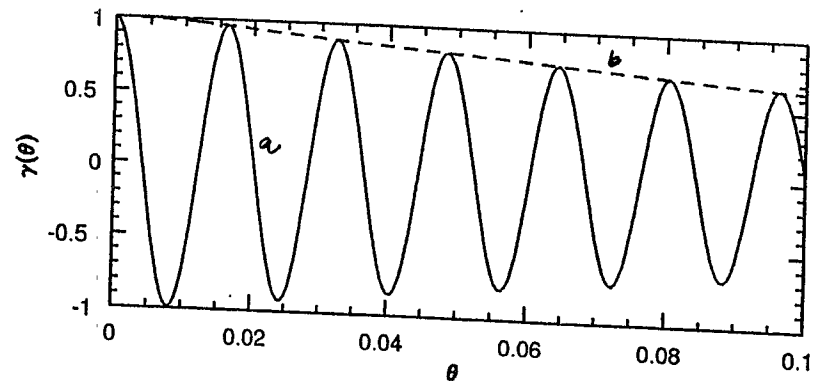
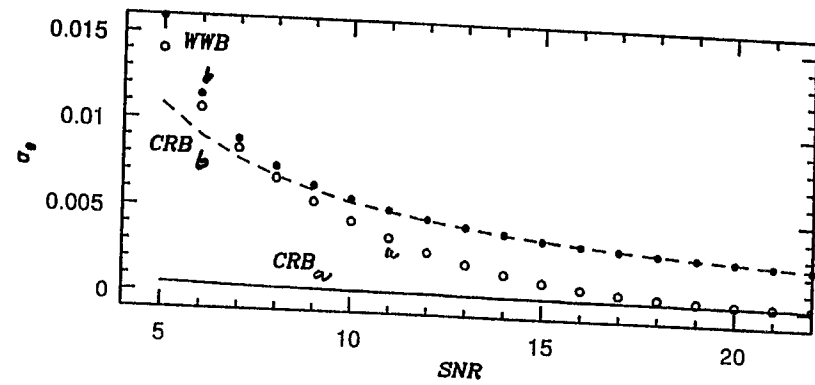
Newtonian waveforms:

2D problem  $(t_a, \tau)$

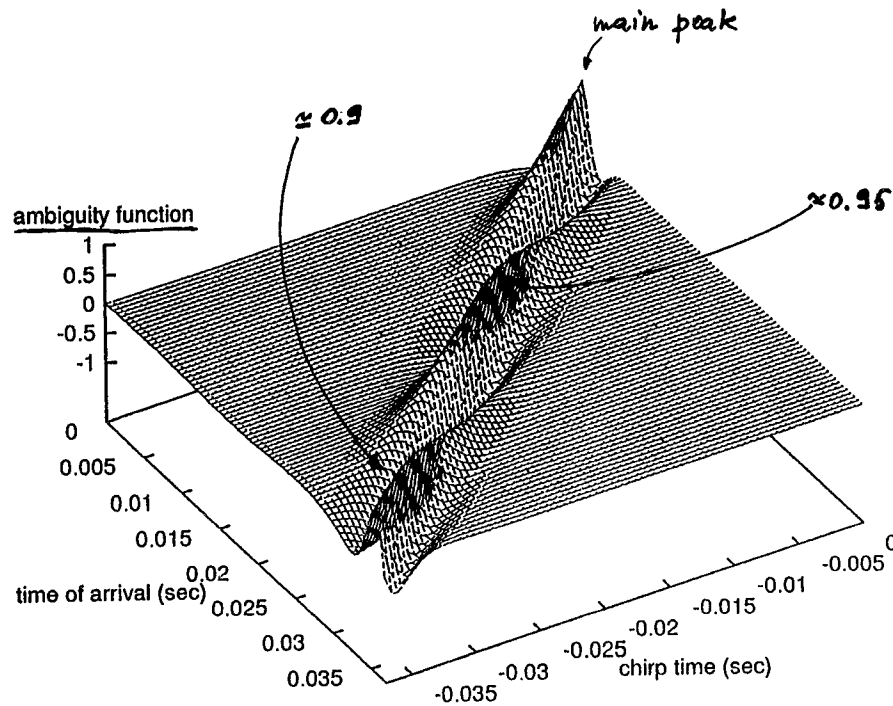
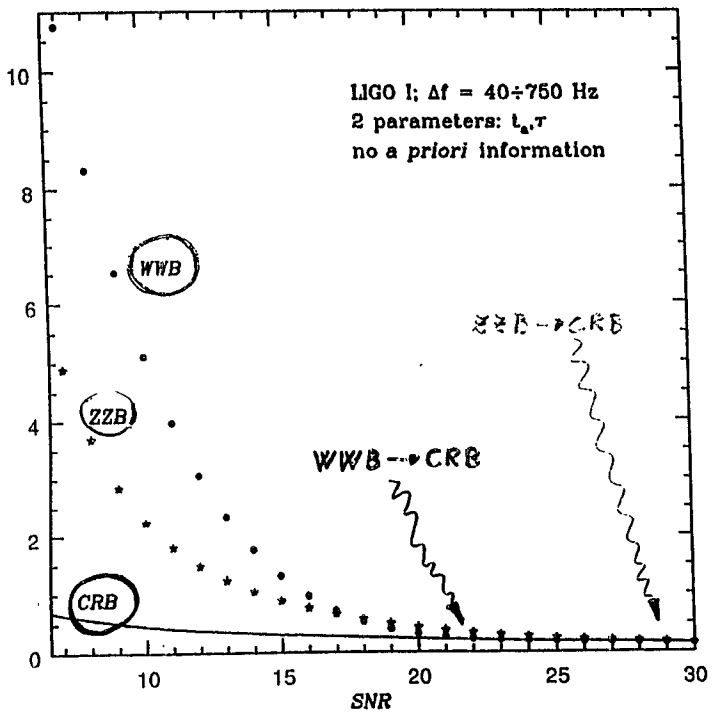
3D problem  $(t_a, \tau, \phi_a)$

LIGO I noise

range of parameters



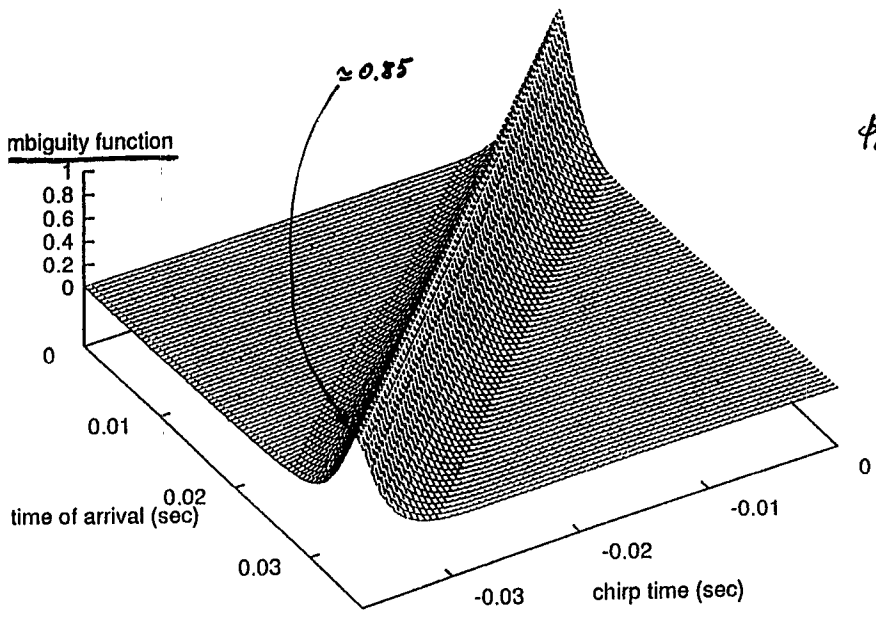
LIGO I noise  
40-750 Hz



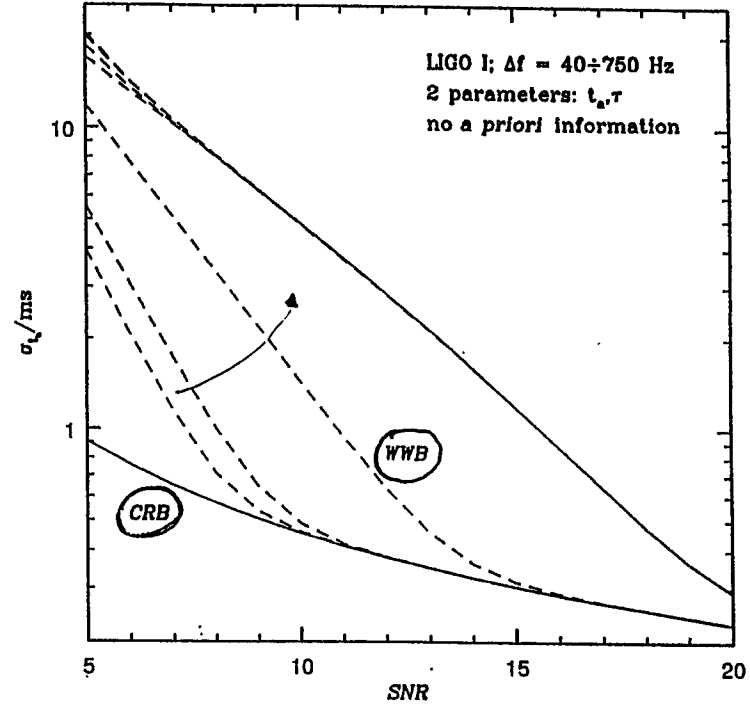
12.

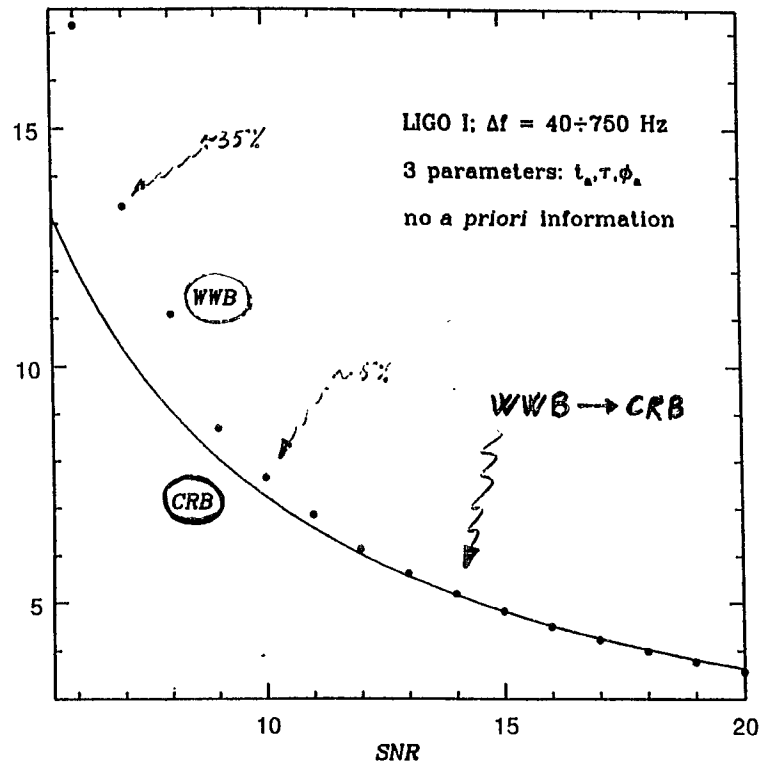
Newtonian signal  
 $(t_a, \nu, \phi_a)$

LIGO I noise  
 $40 \div 750 \text{ Hz}$

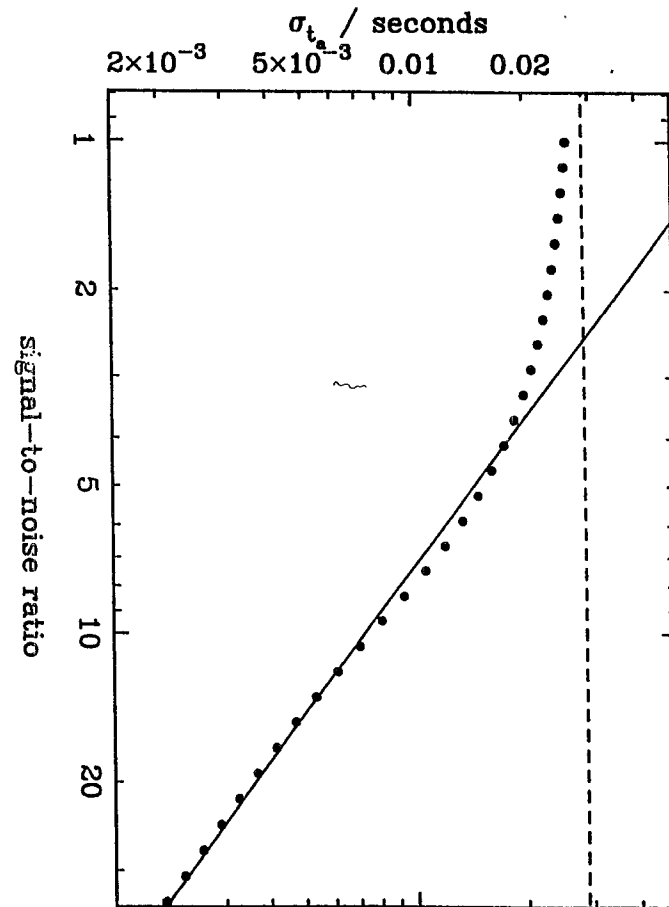


$\phi_a = \phi_a^{(max)}$





15.



16.

## CONCLUSIONS

The general behaviour of WWB and ZZB applied to gravitational wave observations of coalescing binaries is in agreement with the results obtained in other fields

The discrepancy between CRB and GB depends on the shape of the ambiguity function

The knowledge of the parameter range before the estimation problem is important to reduce the errors

Numerical simulations predict greater errors and thresholds

## FUTURE WORK

- Thorough study of the ambiguity function
- Extension to all the parameters
- Ligo II noise
- A priori probability
- Optimal bandwidth
- Coincidence experiments
- Comparison with numerical simulations



# LIGO Simulation Environment

Jan. 18, 1996 Hiroaki Yamamoto / Caltech

---

- LIGO End-to-End model
  - ›› We are developing integrated modular environment
    - SIESTA of VIRGO or GRID of GEO
- Physics models
  - ›› Building blocks - Interferometer(IFO) and Noise sources
- AVS
  - ›› Software framework of LIGO simulation environment
- Environment
  - ›› Modeling and Simulation Environment using AVS



1 of 9

LIGO-G950000-00-M

---

## LIGO End-to-End model - Why do we need it ? -

---

- Understand apparatus behavior
- Design trade study
  - ›› Predict performance
  - ›› Try different models
- Easy integration of work in LIGO
  - ›› Work going on using different flavors - c/fortran/  
MatLab/Mathematica
  - ›› Use same environment to reuse work
- Data analysis
  - ›› Compare models' predictions and data
  - ›› On-line preselection algorithm
  - ›› off-line search algorithm
- Make life easier
  - ›› Friendlier interface
  - ›› Data visualization

LIGO-G950000-00-M

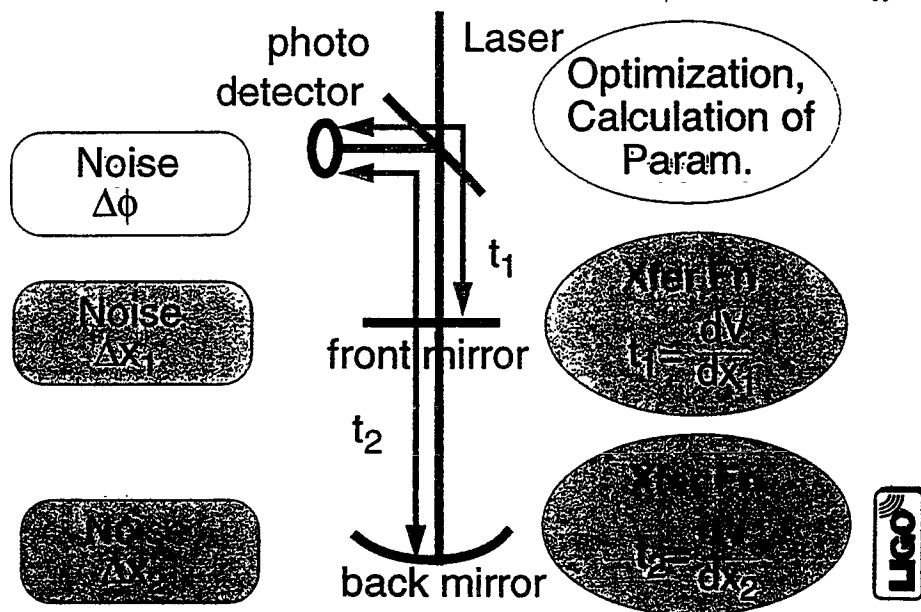
2 of 9

# LIGO End-to-End model = IFO + Noise

LIGO-G950000-00-M

$$\Delta\phi(P_{in}, 1-c, G\dots) \oplus t_1 \otimes \Delta x_1 \oplus t_2 \otimes \Delta x_2$$

3 of 9



## Physics models - Interferometer -

Static model		InLock model	Lock acquisition model	
FFT model	Modal model	Twiddle	Single mode	Spatial multi mode
		frequency dom.	time domain	
J.Y. Vinet etc (VIRGO) and Y. Hefetz etc (MIT/LIGO)	N. Mavalvala and Y. Hefetz (MIT)	Martin M. Regehr (CIT grad)	D. Reding (JPL)	D. Reding (JPL) and R. Beausoleil (CYGNUS Laser Corp)
FFT (128,256), Paraxial apprx.	Hermit Gauss (00,01,10), small misalignm	Single mode steady state linear apprx.	Convergence in time space (*3)	Small/Large amplitude misalignment
all static quantities, realistically deformed/tilted mirrors	low frequency alignment control design	Transfer function	Lock acquisition control design	Alignment control design
CPU/Memory demanding, 1 day w/ SS20 (*1)		base of End-to-End model (*2)	FP, CC, Recombined IFO done, Recycled IFO Feb~Mar	just starting

4 of 9

LIGO-G950000-00-M



## Physics models - Super Computers -

- CSCC (Concurrent Supercomputing Consortium) hardware at Center for Advanced Computing Research of Caltech

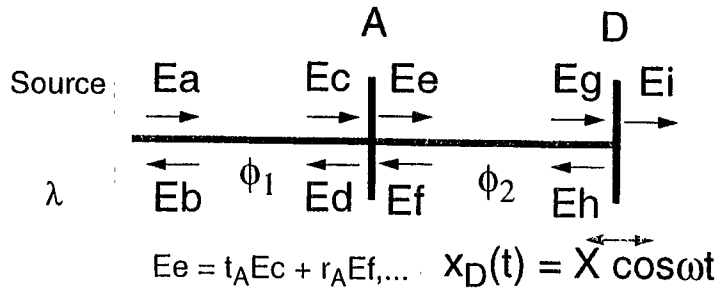
>> Massively Parallel Processors (MPP)  
>> effectively x 10 ~ 100 times faster than SPARC20

FEATURES	DELTA	TREX	RAPTOR	JPL CRAY
MODEL	PROTOTYPE	XPS L38	XPS A4	T3D
GFLOPS	30.7	38.4	4.3	38.4
NODES, PE'S PER NODE	513, 1PE	512, 1 PE & 1 Comms Procsr	57, 1 PE & 1 Comms Procsr	128, 2 PE's
CPU	i860 XR	i860 XP	i860 XP	DEC 21064
SPEED	40 MHZ	50 MHZ	50 MHZ	150 MHZ
MFLOPS/CPU	60	75	75	150
MB/NODE	16	32	32	64
TOTAL GB	8.2	16.4	1.8	16.4
DISKS IN GB'S	93 (RAID0)	67.2 (RAID3)	14.4 (RAID3)	103
TOPOLOGY	2D (16X36)	2D (16X36)	2D (16X4)	3D TORUS

# Twiddle

single mode InLock state model in freq. domain

Mach-Zehnder, example

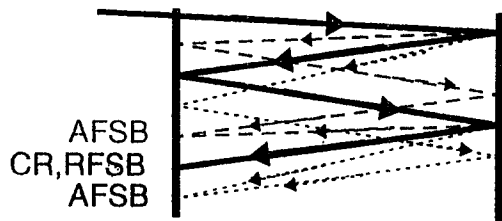


equations

$$M_{DC}(r, t, \phi(\lambda)) \begin{bmatrix} E_a \\ E_b \\ \dots \\ E_i \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix} \Rightarrow M_{AF}(r, t, \phi(\lambda, \omega)) \begin{bmatrix} E_a \\ \dots \\ E_h \\ E_i \end{bmatrix} = \begin{bmatrix} 0 \\ \dots \\ S \\ 0 \end{bmatrix} \quad S = \frac{X}{\lambda} E_0$$

approximation

$$\gg \frac{X}{\lambda} \ll 1$$



# Implementation of Twiddle in Mathematica and AVS

- Mathematica code for FP

```
s1 = source[gamma]
m1 = mirror[r1,t1]
m2 = endmirror[r2,t2]
connect[s1,1,m1,1,length1,phase1]
connect[m1,2,m2,1,length2,phase2]
shake[m2]
findfields[omega]
```

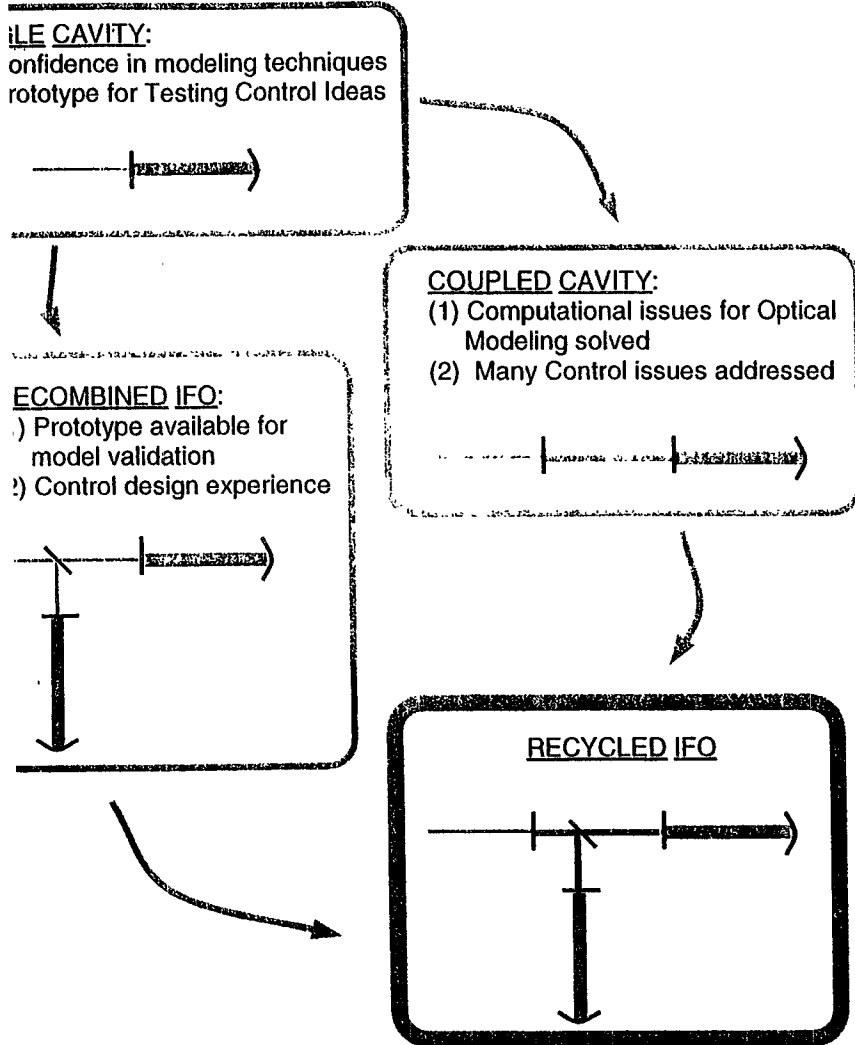
- Nice feature of Twiddle

- ›› Automatic construction of the matrix M
- ›› Easy to build various IFO configurations

- AVS version of Twiddle

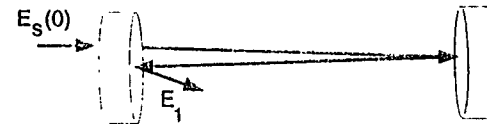
- ›› Twiddle Mathematica code is ported to Fortran
- ›› Grag optics elements and link to construct IFO
- ›› GUI parameter specification

## Overview of Lock Acquisition Modeling Program: Building Block Approach

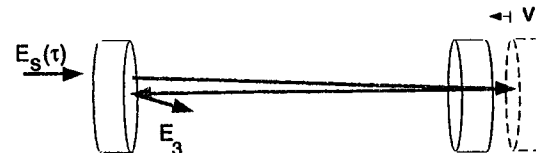
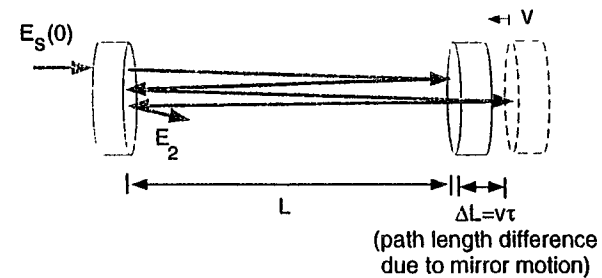


## Method for Calculating Optical Response (Single Cavity)

E field in cavity at  $\tau$ :  $E(\tau) = E_1 + tE_s$



E field in cavity at  $2\tau$ :  $E(2\tau) = E_2 + E_3 + tE_s$



- Want to calculate E field in cavity as mirrors move.
- Sum contribution of E field due to light entering cavity at discrete intervals  $t, t-\tau, t-2\tau, \dots$

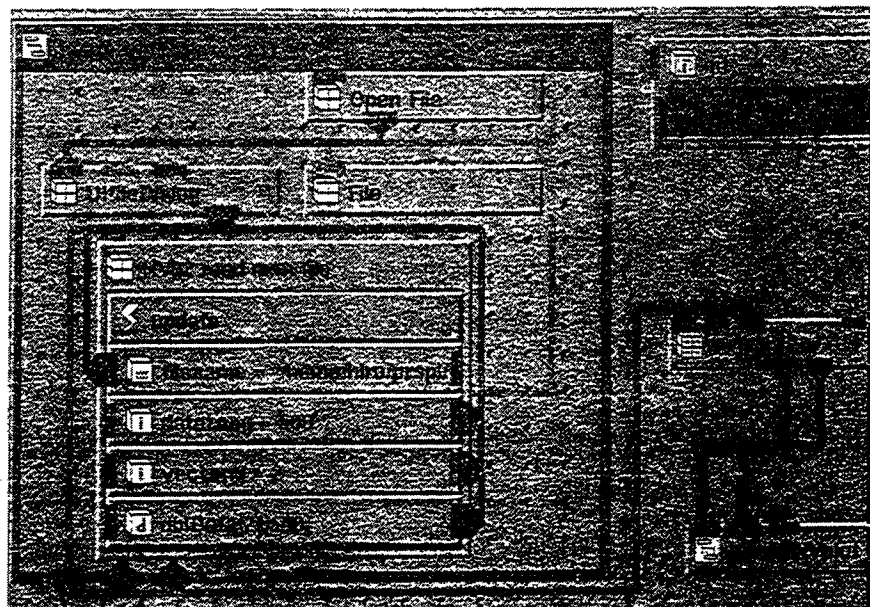
# Physics models - Noise -

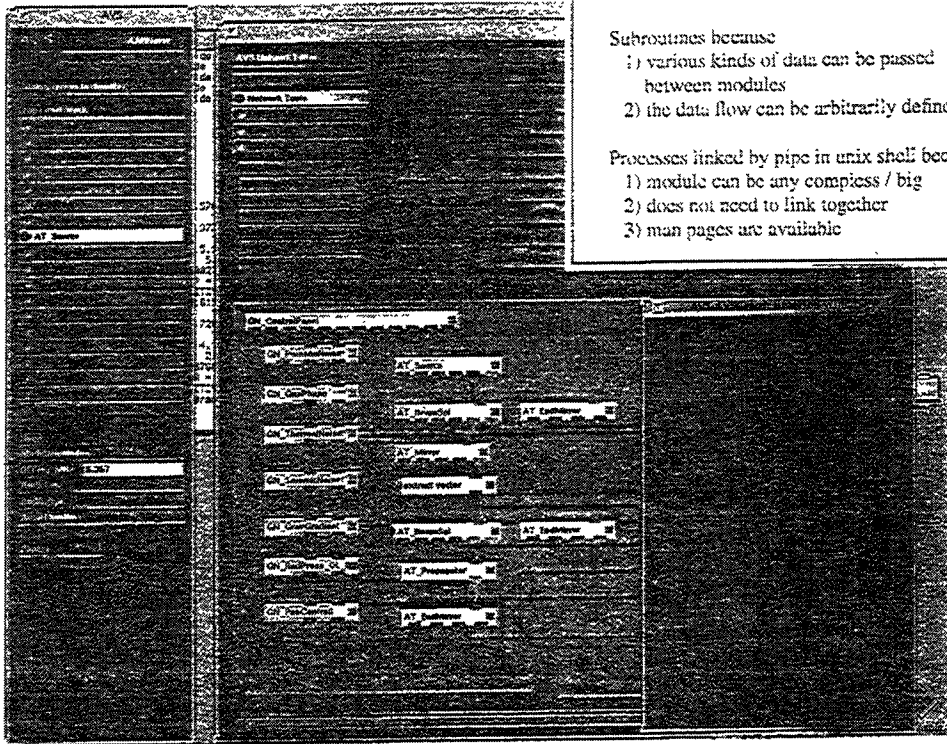
Seismic Noise	Model to use either the LIGO standard spectrum or optional data from sites along with the transfer function of the seismic isolation stacks. Issues such as possible correlations between different testmasses to be studied for inclusion in model.	
Thermal Noise	Model developed out of code written J.R. Hutchinson to calculate frequencies of modes of a solid cylinder and code by A. Gillespie to calculate effective mass coefficients for these modes. Together the frequencies and coefficients determine the thermal noise of the internal modes of the test mass. Model for thermal noise for the violin and pendulum resonances based on calculations and code by A. Gillespie and F. Raab.	
Shot Noise	Model based on calculation for Recycled Unbalanced Fabry Perot LIGO IFO by Torrey Lyons and Martin Regehr.	
	<ul style="list-style-type: none"> <li>* Radiation Pressure and Quantum Noise,</li> <li>* Residual Gas Noise, * Laser Noise,</li> <li>* Phase Noise Due to Scattering and Stray Beams,</li> <li>* Magnetic and Electric Field Noise,</li> <li>* Technical Noise</li> </ul>	Rainer Weiss gravnoise program



# AVS - software framework -

- (\*4)-
- 1) Object Oriented
  - 2) Visual Program
  - 3) Open/Extensible
  - 4) Application Framework
  - 5) GUI builder
  - 6) Library  
2D/3D





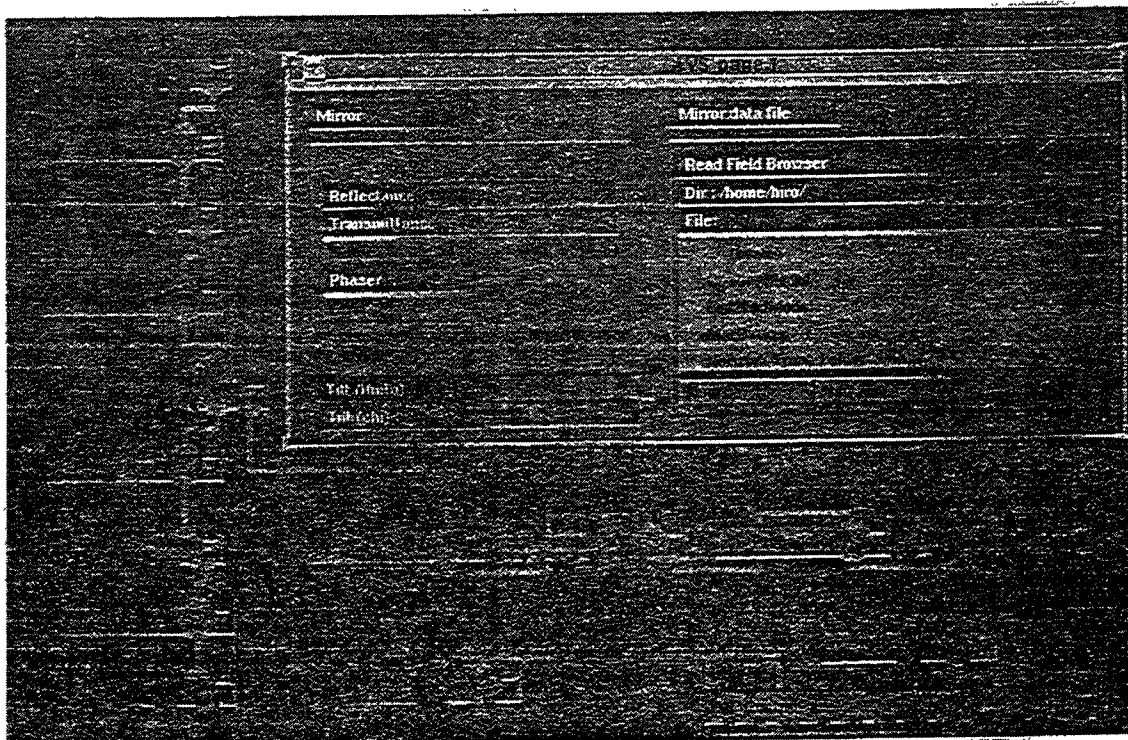
Modules are like

Subroutines because

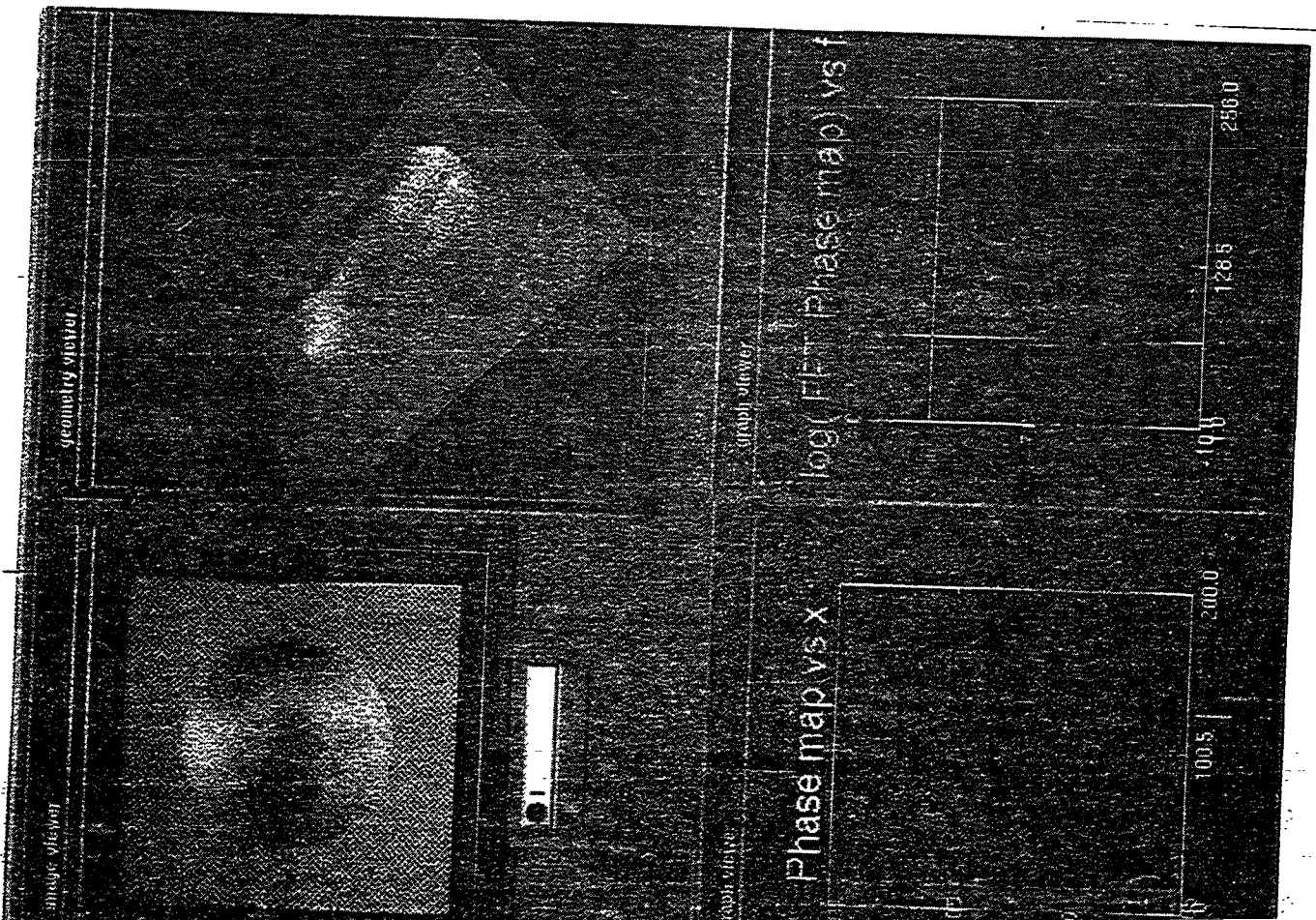
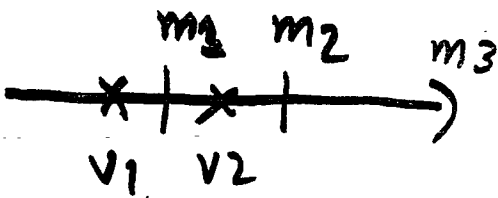
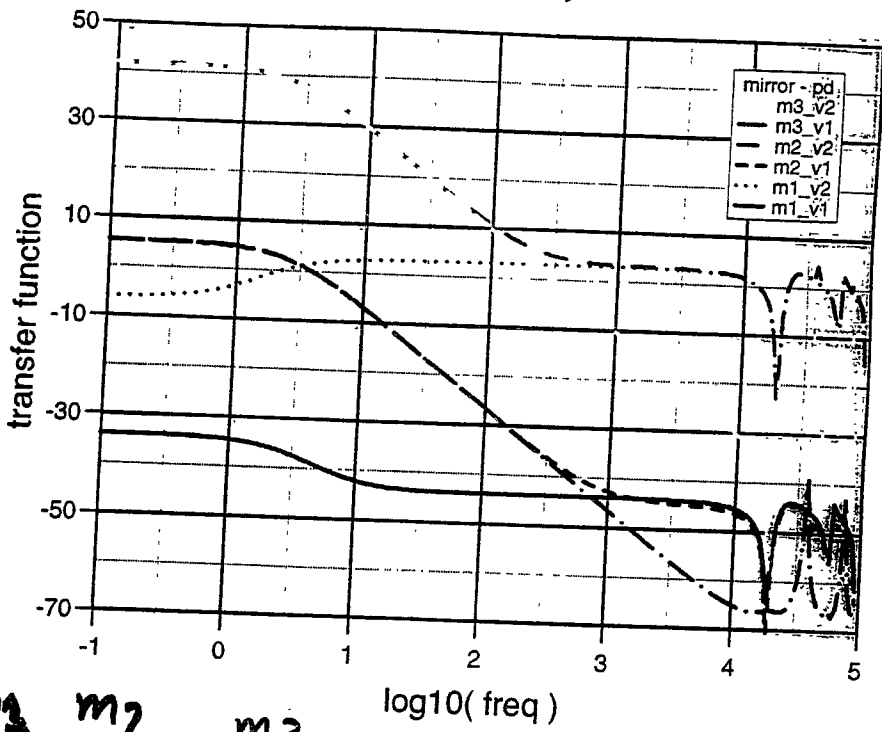
- 1) various kinds of data can be passed between modules
- 2) the data flow can be arbitrarily defined

Processes linked by pipe in unix shell because

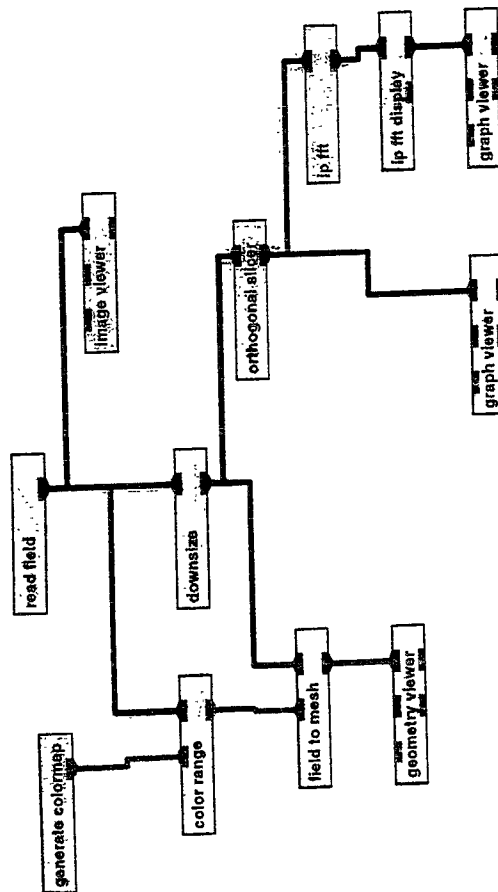
- 1) module can be any complex / big
- 2) does not need to link together
- 3) man pages are available



# Coupled Cavity



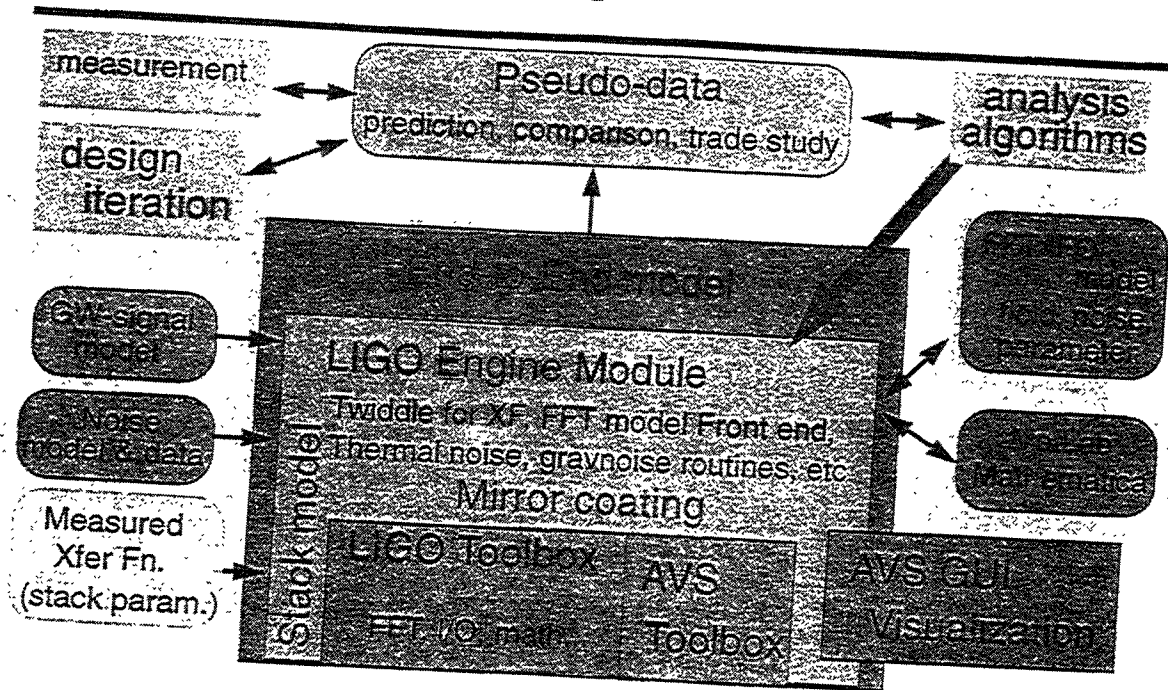




## AVS - how to use -

- End user
  - >>Use AVS module without modification
  - >>Custom-made interface - easy to use
- Network programmer
  - >>Use network editor to build network and use
  - >>Object Oriented feature allows very flexible programming
  - >>Build IFO using Twiddle, define what to measure
- Low level programmer
  - >>Extend/create modules using C/C++/Fortran
  - >>Object Oriented Application Framework makes it easy to combine work done independently
  - >>Build IFO model, implement noise, then build End-to-End model

# Modeling and simulation environment using AVS



Friday AM      Data Analysis III. Pulsar and Other Periodic Source Searches  
Stochastic Background Searches

January 19      Chair: F. Raab

8:00      D. Nicholson (Cardiff)      Realistic Search of One Year's Data  
8:30      Discussion  
8:40      G. Jones (Cardiff)      Results of a Practical Search of a Large Patch  
of Sky for a Pulsar-type Signal  
9:10      Discussion  
9:20      Coffee Break  
9:30      B. Allen (Milwaukee)      Stochastic Background Searches  
10:00      Discussion  
10:10      R. Fakir (UBC)      The GAP Concept  
10:15      Discussion

D. Nicholson, Aspen Jan 14-21 1996

## ANALYSING INTERFEROMETER DATA (~1yr)

### FOR CONTINUOUS WAVE SOURCES

B. Schutz, G. Jones, DN

University of Wales, Cardiff, UK

- targetted searches
- wideband search across the sky
- open issues

## TARGETTED SEARCHES

### ① Selected radio pulsars (Crab, J0437-4715)

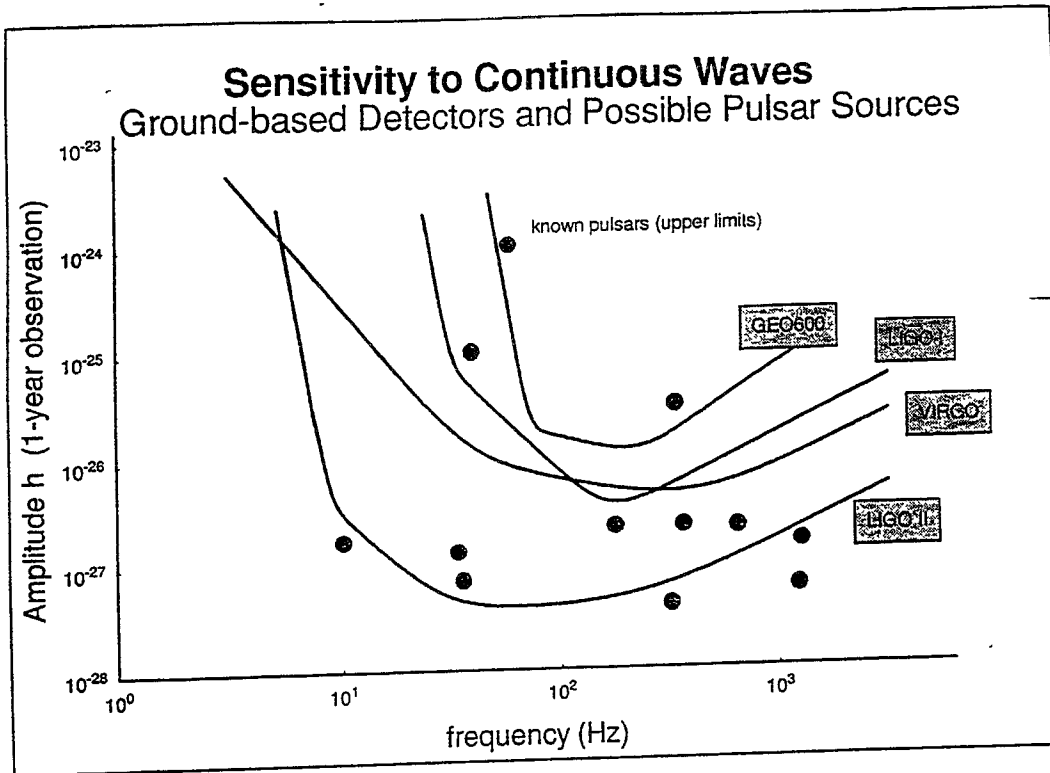
- $f_{gw}$ ,  $(\theta, \phi)$  known
- analysis is 'straightforward'
  - data compression
  - Doppler demodulation
  - Fourier transform
  - statistical peak search
- no severe computational demands

### ② Massive X-ray binary systems

- $f_{gw}$  unknown,  $(\theta, \phi)$  known
- analysis significantly more difficult!
  - Doppler demodulation
  - wideband FFT of very long time-series
  - statistical peak search
- enormous computational demand on memory

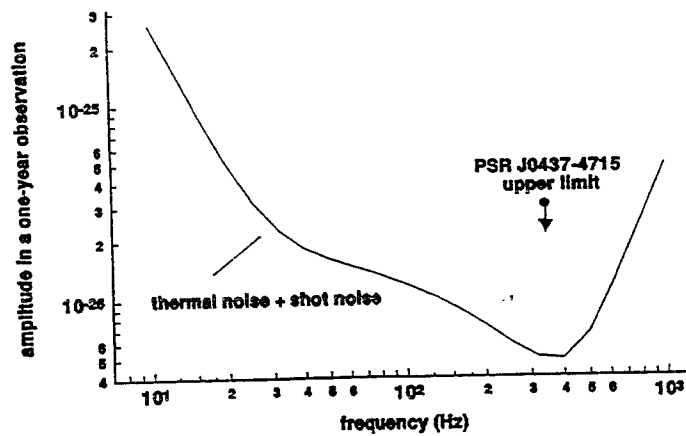
Options:

- slice time-series into  $M \times L (= N)$  chunks  
operation count:  $M \cdot N \log_2 N$  (slow!)
- incoherent FT (loss in sensitivity)
- process data on a parallel machine  
operation count:  $N \log_2 N + N$  ] v. efficient!  
+ 1 communications burst



4

#### GEO600 SEARCH FOR PSR J0437-4715 Narrow-band operation



## ALL SKY/FREQUENCY SEARCH

- $f_{gw}, (\theta, \varphi)$  unknown
- analysis looks outrageously difficult
- no symmetry in Doppler motions

Angular resolution: ( $T_{obs} \gtrsim 2$  days)

$$\Delta\theta = 1 \times 10^{-6} \left( \frac{f}{1\text{kHz}} \right)^{-1} \left( \frac{T_{obs}}{10^7\text{s}} \right)^{-2}$$

$$N_{patch} = \frac{4\pi}{(\Delta\theta)^2} \sim 10^{13} \left( \frac{f}{1\text{kHz}} \right)^2 \left( \frac{T_{obs}}{10^7\text{s}} \right)^4$$

Computational costs:

- search patches by 'stepping around the sky' (Schutz)

$$\tilde{S}_b(f'_b, \theta', \varphi') = \int_{-\infty}^{\infty} \tilde{S}_b(f_b, \theta, \varphi) q(\theta', \varphi', f'_b; \theta, \varphi, f_b) df_b$$

→  $q$  is a peaked function → sets angular resolution

- assume 'stepping' operation count = 10/data point

$$\rightarrow \text{analysis time } T' = 2.5 \times 10^{16} \left( \frac{f}{1\text{kHz}} \right)^3 \left( \frac{T_{obs}}{10^7\text{s}} \right)^5 \left( \frac{S}{100\text{MFlop}} \right)^{-1}$$

$$\sim \frac{1}{10} H_0^{-1} !!$$

$$\rightarrow \text{observation time allowed} = 4.4 \times 10^4 \left( \frac{f}{1\text{kHz}} \right)^{-3/4} \left( \frac{S}{100\text{MFlop}} \right)^{1/4} \text{ s.}$$

## HIERARCHIAL SEARCH

or chase the signal

- $N$  Gaussian samples of data @ 2kHz,  $N_{patch}$   
Allow 1 false-alarm in the full, wideband search

$$\rightarrow \tilde{h} \text{ threshold} \sim 11$$

- Break long time-series into  $M$  chunks

- Doppler demodulations on chunk data

$$N_{patch} \rightarrow N_{patch}/M^4$$

- compute power spectra

- form incoherent average

- set  $\tilde{h}$  threshold at  $\tilde{h}_T/M^{1/4}$

- statistical peak search

$$\rightarrow \text{candidate } f_{gw}, (\theta, \varphi)$$

- Search full data around  $f_{gw}, (\theta, \varphi)$  candidates

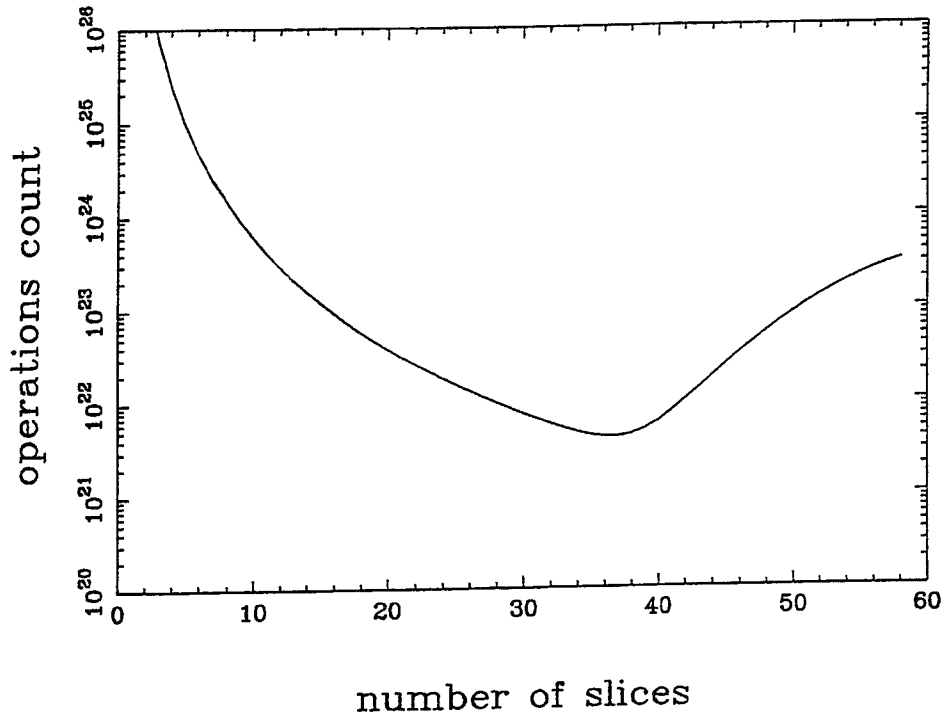
- Select  $M$  to trade off sky patch size against # false-alarms

$$M \sim 35 \text{ appears optimal}$$

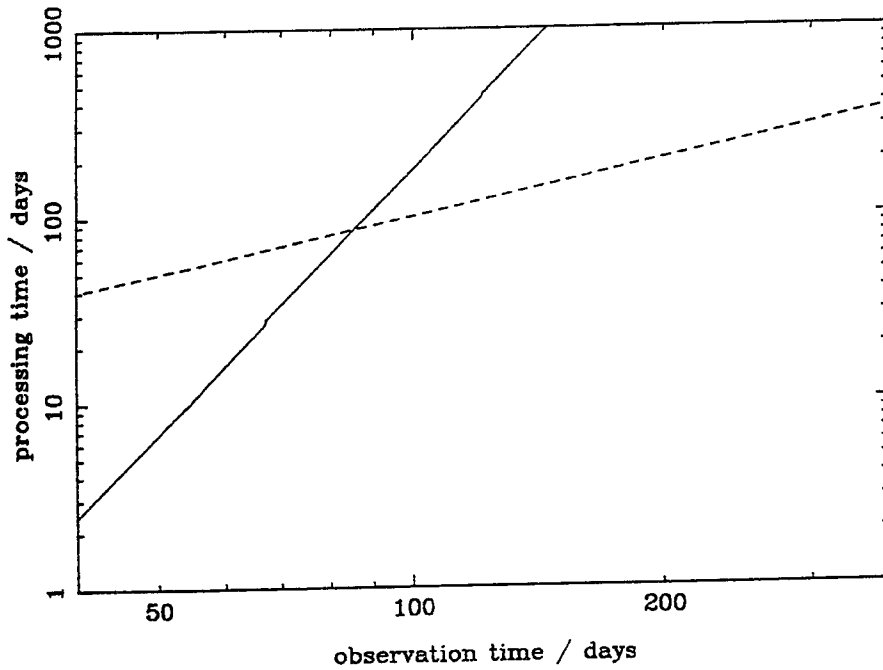
$$\text{allowed observation time} \sim 3 \text{ months}$$

- possibly we can develop more hierarchial stages  
statistical analysis will not be easy

search of 1 yr of data,  $f < 2$  kHz



1 yr. search of optimally sliced data on Teraflop supercomputer



## OPTIMIZING LONG FT EFFICIENCY

- FT's of very long time-series underpin our analysis
- LIGO, VIRGO may archive short FT's from on-line analysis
- would these improve efficiency of pulsar searches?

Arrange  $N$  samples of data in a matrix:

$$\begin{matrix} & \rightarrow n \\ \downarrow m & \begin{pmatrix} x_0 & x_1 & \dots & x_{n-1} \\ x_n & x_{n+1} & \dots & x_{2n-1} \\ \vdots & \vdots & & \vdots \\ x_{n(m-1)} & & & \end{pmatrix} \end{matrix} \quad nm = N$$

- 1. FT on rows  $\rightarrow$  short FT's forming our start point
- 2. FT on columns  $\rightarrow C_{b\beta} \quad \begin{matrix} b \in (0, \dots, n-1) \\ \beta \in (0, \dots, m-1) \end{matrix}$

We can show,

$$C_{b\beta} = \sum_c x_{\beta+cm} S_{c-b,\beta} \quad c \in (0, \dots, n-1)$$

$$\text{where, } S_{c-b,\beta} = \frac{1}{n} \left[ \frac{1 - e^{2\pi i(c-b-\beta/m)}}{1 - e^{2\pi i(c-b-\beta/m)/n}} \right]$$

+ suspect 90% accuracy for  $\begin{matrix} 0 < \beta < m/4 \\ 3m/4 < \beta < m-1 \end{matrix}$

- $\rightarrow$  half the long FT approximately right
- $\rightarrow$  bca savings:  $\log_2 m / \log_2 (mn)$

## REALISTIC SEARCH

### OF ONE YEAR OF DATA (?)

- start with short (archived) FT's aligned along rows of a data matrix
- assign columns to parallel processors
- optimally slice column data
- Doppler demodulate for some  $(\theta, \varphi)$
- FT's on processors
- implement stepping algorithm
- weed out candidate f<sub>gw</sub>,  $(\theta, \varphi)$ 's
- refine search

For  $T_{\text{obs}} \sim 3 \times 10^7 \text{ s}$

$T_{\text{analyze}} \sim T_{\text{obs}} \quad ?$

Looks possible with a Teraflop parallel supercomputer



## QUESTION

- for LRC (SDA)

• A state-of-the-art parallel machine seems essential for pulsar data analysis

• Should LIGO, VIRGO, GEO600 have proprietary machines? Purchase when?

• Long FT's underpin the data analysis

- special chips can be configured for efficient computation of FT's
- dedicated pulsar search computers
- are these preferable

Issues re: LRC (SDA)  
special interest group?

Results of a practical search of a large patch of sky for a pulsar-type signal

G.S. Jones, D. Nicholson, B.F. Schutz  
Department of Physics and Astronomy, University of Wales College Cardiff  
PO BOX 913  
CARDIFF

January 13, 1996

#### Abstract

A method for a search for continuous gravitational waves in data produced by laser interferometers is presented in this talk. An algorithm is shown that can be used to correct for the Doppler shifts introduced by the Earth's accelerations, and that can 'search' many separate positions in the sky without the need of more costly techniques. How this algorithm is implemented is presented.

A new method of compressing data to a heterodyned narrow frequency bandwidth time series is described. This method, called Fourier domain sampling, reconstructs an effectively re-sampled and filtered time series from short period Fourier transforms. As these short period Fourier transforms are produced from the data taken by detectors for other research, then this method is much more efficient than other ways of producing heterodyned narrow frequency bandwidth time series.

These procedures were applied to 100 hours of data taken with the Glasgow gravitational wave detector to search the Large Magellanic Cloud (an area 52 degrees in R.A. and 10 degrees in declination with 171 individual 'patches') and the SN1987A remnant, over a 5Hz bandwidth centred around 934Hz for sources of continuous gravitational waves. No significant evidence was found for sources of gravitational waves of strain amplitudes greater than of order  $2 \times 10^{-21}$  for all the patches searched and no significant evidence was found for sources of strain amplitudes greater than of order  $2 \times 10^{-21}$  for the region about SN1987A taking into account the spin down rate of a pulsar claimed to have been observed.

These results have been presented in Phd thesis 'Search for continuous gravitational wave sources' by Gareth S. Jones

## Search for Continuous Gravitational Wave Sources

### Talk Plan

- 'pulsar' search
- $q$  kernel
- Data compression
- Application to real data
- Problems to be solved in future

G. Jones @ ASTRO.CF.AC.UK

URL: WWW.ASTRO.CF.AC.UK/~Gareth.Jones/

## 'STEPPING' AROUND THE SKY

1. Correct data for Doppler broadening for a particular region in the sky
2. Fourier transform data
3. Calculate  $q$  kernel for step
4. Convolve  $q$  with Fourier domain
5. repeat steps 3 and 4

## CONVENTIONAL SEARCH TECHNIQUE

1. Correct data for Doppler broadening for a particular region in the sky
2. Fourier transform data
3. Repeat steps 1 and 2

## Stepping in Fourier Space

$$\hat{S}_b[f'_b, \theta', \phi'] = \int_{-\infty}^{\infty} \hat{S}_b[f_b, \theta, \phi] q(\theta', \phi', f'_b; \theta, \phi, f_b) df_b$$

$$q(\theta', \phi', f'; \theta, \phi, f) = \exp\left(2\pi i f \frac{R_{\oplus}}{c} \cos \psi (\cos \theta - \cos \theta') + 2\pi i (f - f')(t_o + T_{obs}/2)\right) S$$

$$S = 2 \sum_{k=-\infty}^{\infty} J_k(B) \exp(-ik\alpha - ik\Omega t_o - ik\Omega T_{obs}/2) \frac{\sin(T_{obs}(2\pi(f - f') - k\Omega)/2)}{(2\pi(f - f') - k\Omega)}$$

$$A^2 = \sin^2 \theta' + \sin^2 \theta - 2 \sin \theta \sin \theta' \cos[\phi' - \phi]$$

$$B = 2\pi f \frac{R_{\oplus}}{c} A \sin \psi$$

$$\alpha' = \tan^{-1} \left[ \frac{\sin \theta'' \cos \phi'' - \sin \theta \cos \phi}{\sin \theta'' \sin \phi'' - \sin \theta \sin \phi} \right]$$

## FOURIER DOMAIN SAMPLING

1. Fourier transform short periods of data  
(For coalescing binaries search)
2. Take region of Fourier transform
3. Inverse Fourier transform region
4. Re-combine sections of data, applying phase corrections

## COMPLEX HETERODYNE TECHNIQUE

1. Heterodyne data
2. Filter and resample

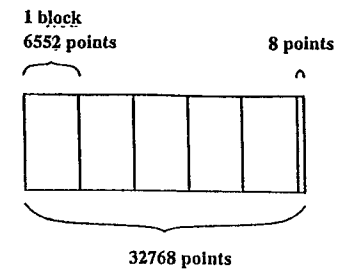


Figure 4.3: Diagram of block structure in a group of data with padded zeros

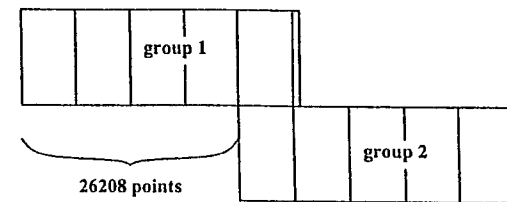


Figure 4.4: Diagram of how consecutive groups contain data from previous group

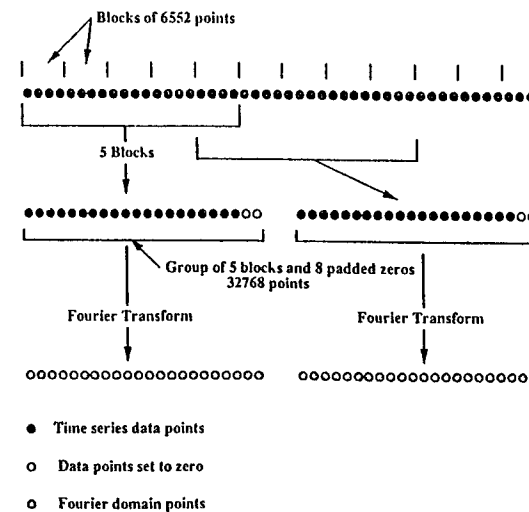


Figure 4.5: Diagram of how Fourier transforms are obtained from the data stream

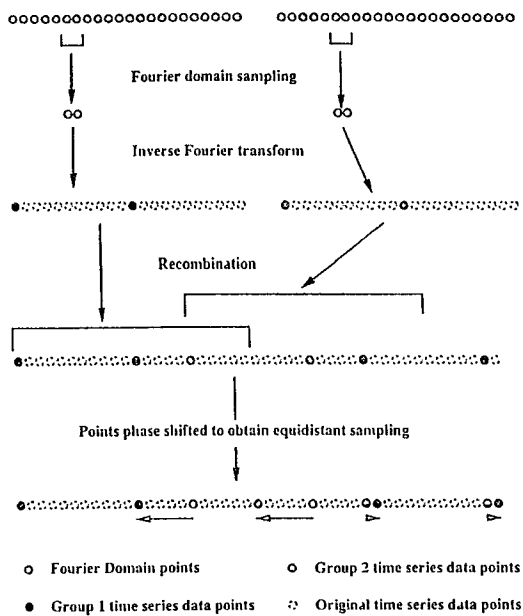


Figure 4.10: Fourier domain sampling and re combination: a somewhat idealised diagram of how any number of Fourier domain sampling can be used with phase shifts

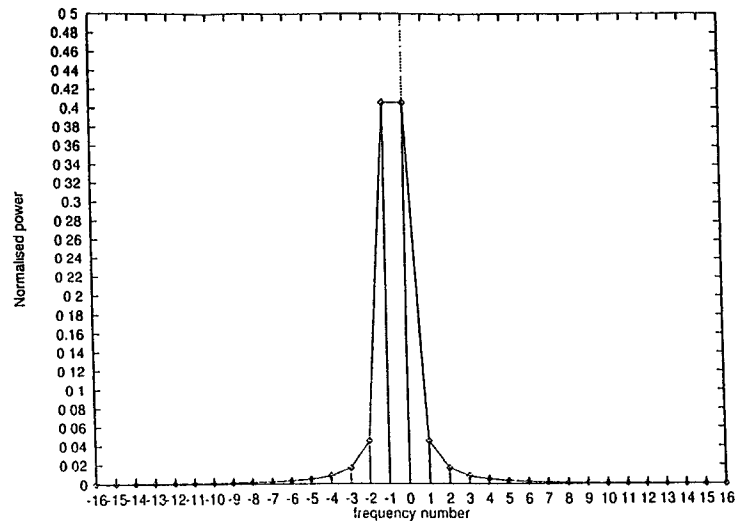


Figure 4.11: Discrete Fourier transform of example signal: Plot of 32 sample points taken from a Fourier transform sample with a signal present at frequency index of  $k_s = -0.5$ .

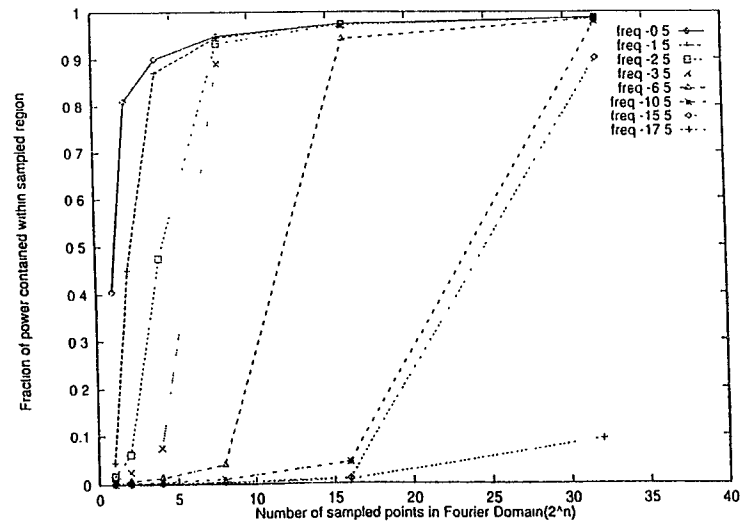


Figure 4.12: Comparison of different compression schemes: Plot of the amount of power within the sampled region caused by different frequency signals. A signal with frequency outside the region would ideally have no power within it and vice versa.

# GLASGOW 100 Hour DATA RUN

- DATA TAKEN BETWEEN

to 2nd MARCH 1989 22<sup>h</sup> 51<sup>m</sup> 41.971<sup>s</sup>  
6<sup>th</sup> MARCH 1989 19<sup>h</sup> 04<sup>m</sup> 03.34<sup>s</sup>

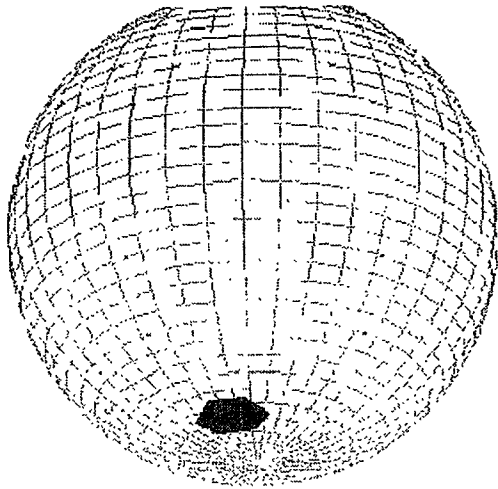
SAMPLED AT 20KHz  
+ HOUSEKEEPING DATA

## SEARCH OF LARGE MAGELLANIC CLOUD

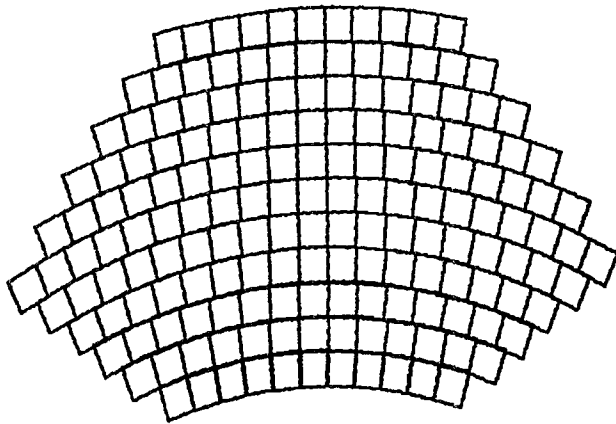
- Data corrected for signals coming from region of SN1987A
- 'Stepping' with  $q$  kernel 10 steps away from central patch
- Total of 171 patches examined

## SEARCH FOR SN1987A 'PULSAR' SIGNAL

- Data corrected for signals coming from region of SN1987A
- Data corrected for assumed spin-down rate of pulsar frequency
- PREDICTED STRAIN  $h \sim 10^{-26}$  (UPPER LIMIT)



Celestial sphere with area being searched superimposed over it.



Graphical representation of patches being searched

## RESULTS

### SN1987A remnant search

- A frequency range of  $\pm 2.5\text{Hz}$  around  $934\text{Hz}$  was searched
- Data Doppler corrected for SN1987A
- Data corrected for 'measured' spin-down of 'pulsar'
- The characteristics of the noise about  $934\text{Hz}$  was noted
- No significant evidence was found for sources of strain amplitudes greater than a strain amplitude of  $2 \times 10^{-21}$

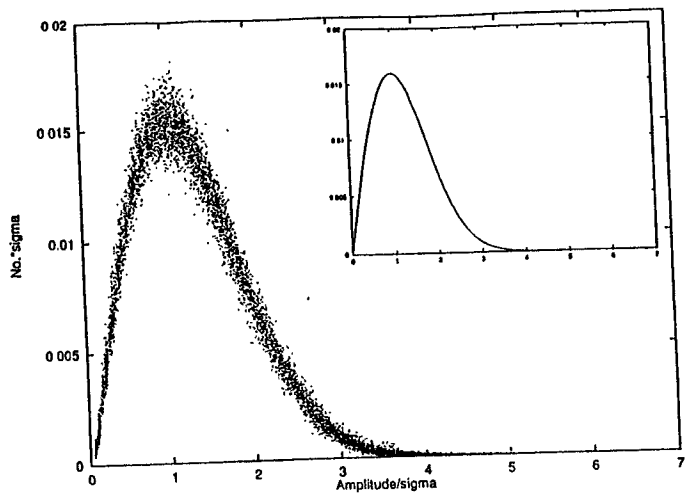


Figure 6.12: Histogram of data from central patch with all corrections for pulsar. The predicted Rayleigh distribution is shown as the insert.

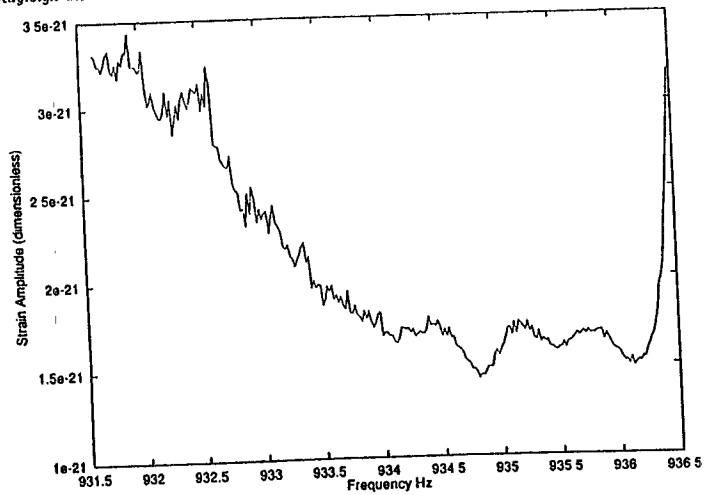
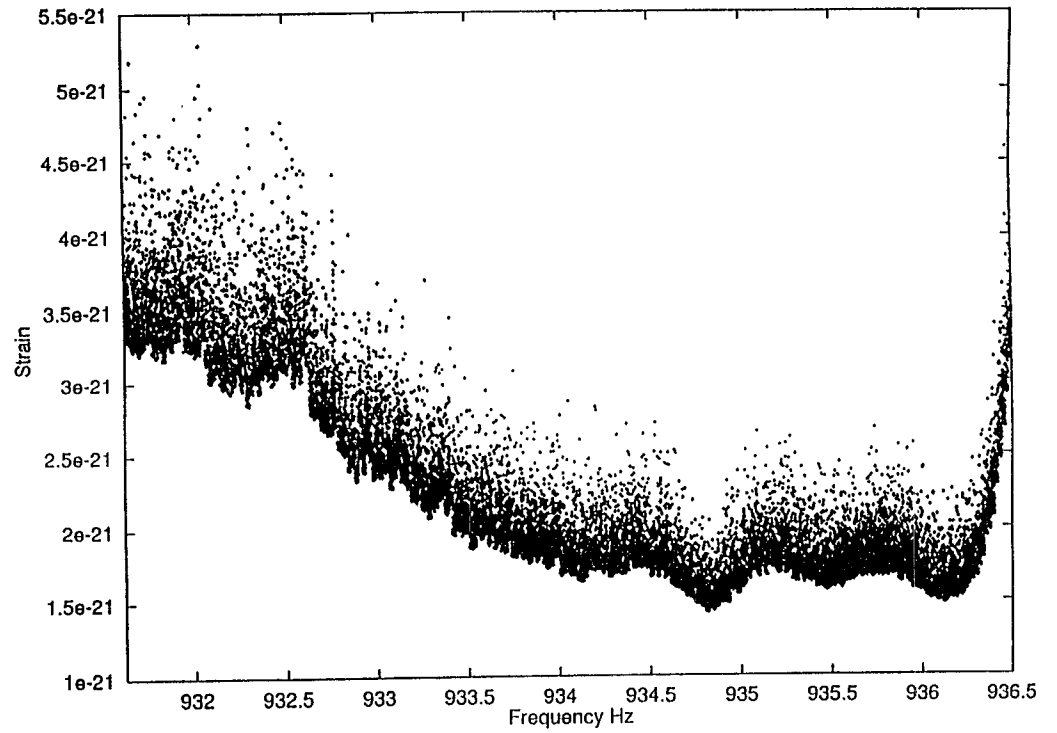


Figure 6.13: Upper limit for non-observed Gravitational waves. Upper limit for a source of gravitational wave for it not to be detected at a 99% confidence level, corrected for the SN1987A pulsar.

Figure 6.10: Fourier spectrum of data from central patch with all corrections for pulsar.





## RESULTS

### Large Magellanic Cloud Search

- A frequency range of  $\pm 2.5\text{Hz}$  around  $934\text{Hz}$  was searched
- 171 patches were examined covering the entire LMC
- The characteristics of the noise about  $934\text{Hz}$  was noted
- No significant evidence was found for sources of strain amplitudes greater than a strain amplitude of  $2 \times 10^{-21}$

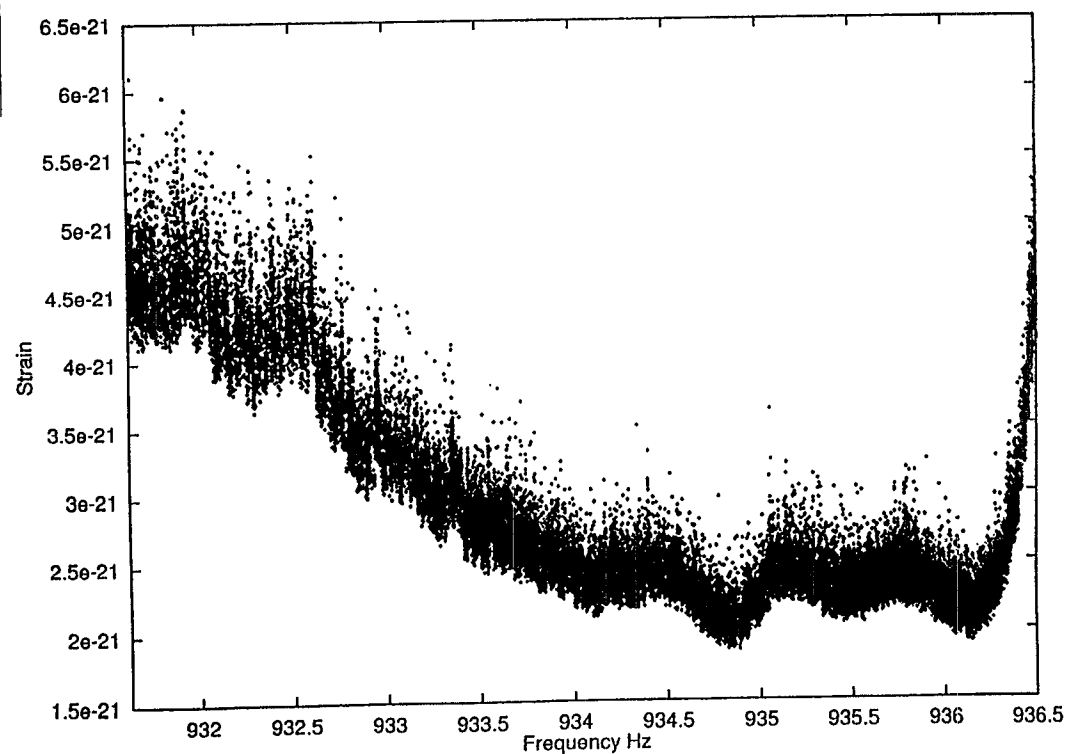


Figure 6.16: Fourier spectrum of data from ALL patches.

## Future work

- Current  $q$  kernel only valid for times  $< 100$  hours

## Searching for unknown sources

- Pulsar spin-down
- Binary system
- Proper motion

## Data handling

- Fourier transform averaging
- Use of parallel machines

Aspen  
1/19/96

## STOCHASTIC BACKGROUND SEARCHES

Bruce Allen  
U. Wisconsin - Milwaukee

1. Define  $\Omega_{gw}$
  2. Some Sources (brief)
- 
3. How to detect
  4. Optimal Filtering
  5. A Data Analysis Pipeline  
What next?

Aspen  
1/19/96

## STOCHASTIC BACKGROUND SEARCHES

Bruce Allen  
U. Wisconsin - Milwaukee

1. Define  $\Omega_{gw}$
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- 
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  5. A Data Analysis Pipeline  
What next?

## Stochastic Background

Assumed: isotropic, stationary, Gaussian

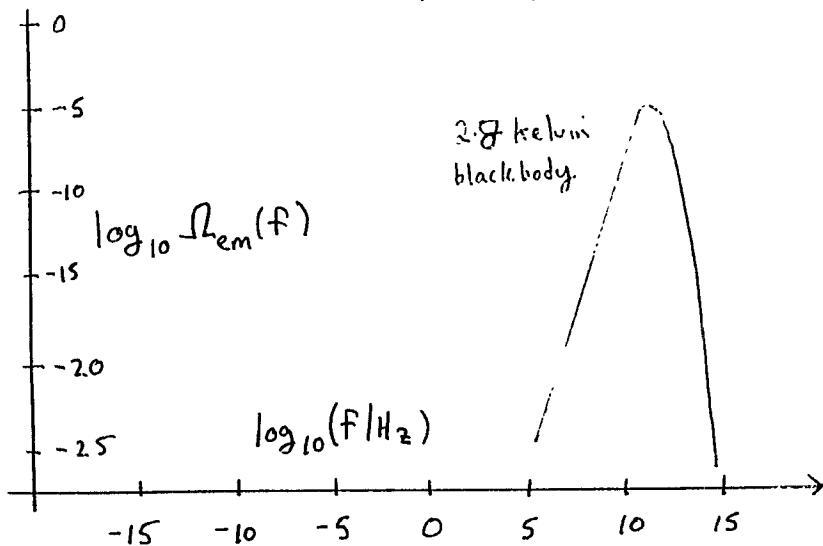
↑  
not essential

$$\Omega_{gw}(f) = \frac{1}{\rho_{\text{close-universe}}} \frac{d\rho_{gw}}{d \ln f} \quad (\text{dimensionless})$$

$$h_c(f) = 1.3 \times 10^{-20} h \sqrt{\Omega_{gw}(f)} \left( \frac{100 \text{ Hz}}{f} \right)$$

↑  
Hubble: 100 km sec<sup>-1</sup> Mpc<sup>-1</sup>

For comparison, consider electromagnetic background (CMBR)



## Observational Limits on $\Omega(f)$

nucleo-synthesis:  $\frac{\rho_{gw}}{\rho_c} < 10^{-5} \Rightarrow \int_{-\infty}^{\infty} \Omega(f) d(\ln f) < 10^{-5}$

pulsar timing:  $\Omega(f = \frac{1}{10 \text{ yr}}) < 6 \times 10^{-8}$

CMBR + COBE:  $\Omega < 7 \times 10^{-11} \left( \frac{10^{-18} \text{ Hz}}{f} \right)^2$   $10^{-18} \text{ Hz} < f < 3 \times 10^{-17} \text{ Hz}$

## Possible Early Universe Sources

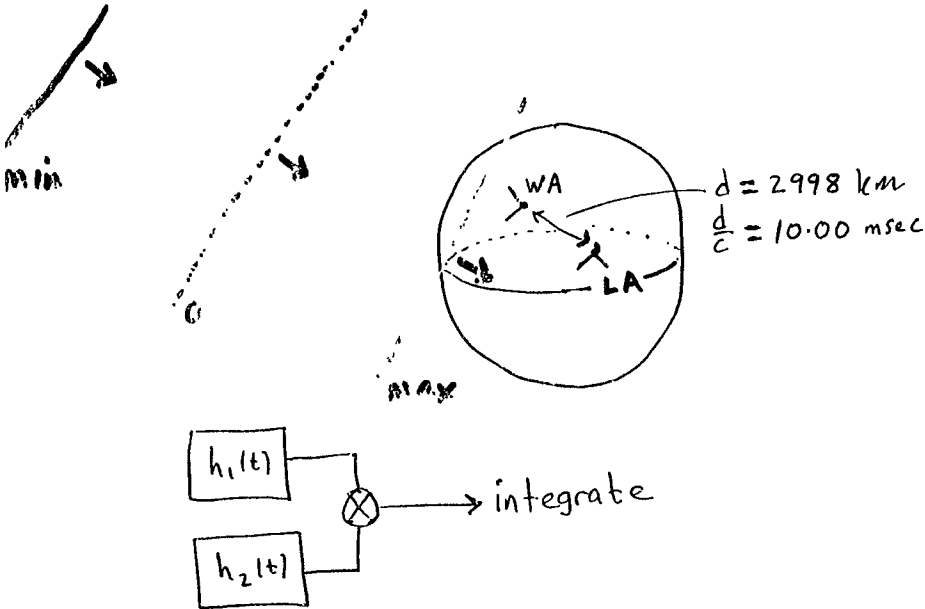
"standard" inflation  $\Omega \lesssim 10^{-14}$  wide freq band

cosmic strings:  $\Omega < 10^{-9}$  in LIGO band.  $\Omega \propto f^{\gamma}$   $\gamma < 0$

First order phase transition:  $\Omega < 10^{-6}$ , bump in spectrum

"stringy" inflation:  $\Omega < 10^{-5}$ , slope  $\Omega \propto f^3$  in LIGO band

# HOW TO DETECT STOCHASTIC BACKGROUND



For waves with  $\frac{\lambda}{2} > 3000 \text{ km}$  ( $f \lesssim 50 \text{ Hz}$ )  
 detector arms move in phase (together) so  
 average product  $\langle h_1(t)h_2(t) \rangle > 0$

In absence of background (and other signals)  
 average product  $\langle h_1(t)h_2(t) \rangle \rightarrow 0$ .

Michelson, Mon. Not. Roy. Astron. Soc. 227 (1987) 933.  
 Christensen, Phys. Rev. D46 (1992) 5250.  
 Flanagan, Phys. Rev. D48 (1993) 2389.

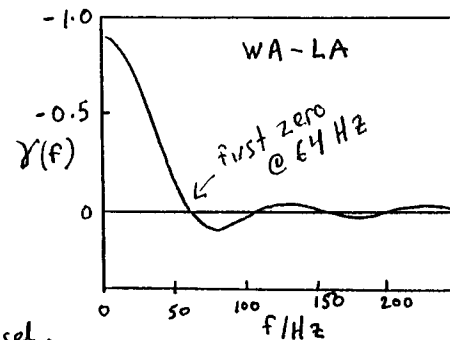
# Overlap Reduction Function $\gamma(f)$

Sensitivity reduced by factor  $\gamma$

- (1) non-parallel alignment
- (2) separation time delay  $\Rightarrow$  signals out of phase

$$\gamma(f) = \frac{5}{8\pi} \int d^2\Omega \underbrace{e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x}}}_{\text{unit vector}} \underbrace{(F_1^+ F_2^+ + F_1^x F_2^x)}_{\text{detector separation}}$$

$$F_i^{+,x} = \underbrace{(X_i^a X_i^b - Y_i^a Y_i^b)}_{\text{defines } \perp \text{ arms of detector}} e_{ab}^{+,x}(\Omega) \quad \underbrace{\uparrow}_{\text{spin 2 polarization tensor}}$$

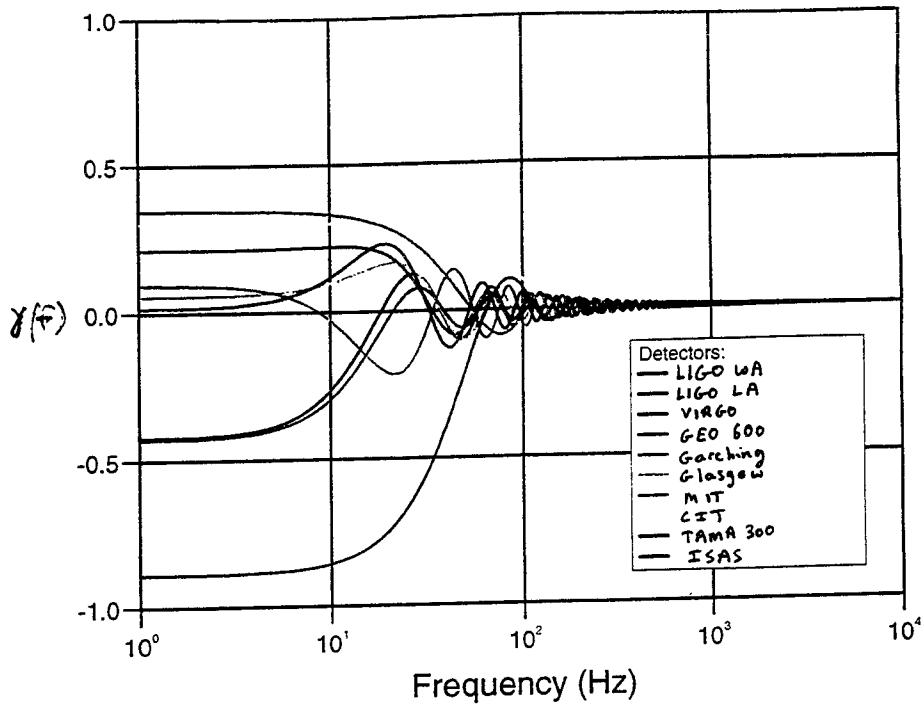


$$\gamma(f) = A \underbrace{J_0(\alpha)}_{\text{Bessel}} + B \underbrace{J_1(\alpha)}_{\frac{\alpha}{d}} + C \underbrace{J_2(\alpha)}_{\frac{\alpha^2}{d^2}} \quad \alpha = 2\pi f |\Delta \vec{x}|$$

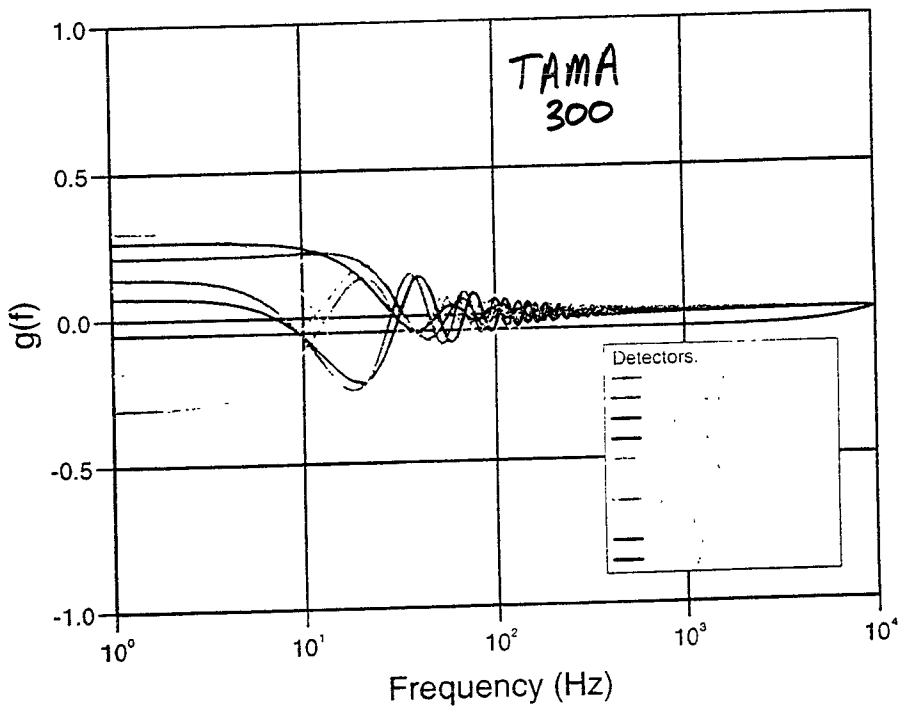
$A, B, C$  depend upon arm orientations, constant.

LIGO WA

Overlap Reduction Function  $g(f)$



Overlap Reduction Function  $g(f)$



# OPTIMAL FILTERING

(Sorry, Sam  
Frequencies!)  $\rightarrow$

$$\text{Signal: } S = \int_0^T dt \int_0^T dt' h_1(t) h_2(t') Q(t-t')$$

$\uparrow$        $\uparrow$        $\uparrow$   
 W/A    L/A    Optimal Filter

Typically  $Q(\Delta t) \rightarrow 0$  for  $\Delta t \geq d/c = 10^{-2}$  sec

Freq Domain  $S = \int_{-\infty}^{\infty} df \tilde{h}_1(f) \tilde{h}_2^*(f) \tilde{Q}(f)$

Fourier Transform is  $\tilde{g}(f) = \int_{-\infty}^{\infty} e^{2\pi i f t} g(t) dt$

To find optimal  $\tilde{Q}(f)$ : assume isotropic + unpolarized source.

Spectral properties of stochastic background, detector noise:

$$\langle \tilde{h}_i(f) \tilde{h}_j^*(f') \rangle = \delta(f-f') \frac{3H_0^2}{10\pi^2} F^{-3} \Omega_{gw}(f) \gamma(f)$$

$\uparrow$   
overlap reduction

$$\langle \tilde{n}_i(f) \tilde{n}_j^*(f') \rangle = \delta(f-f') \delta_{ij} S_i(f)$$

$\uparrow$   
 $f_n$

label sites: 1 = W/A      detector noise power spectrum  
                  2 = L/A

Expected signal:  $\langle S \rangle = \frac{3H_0^2}{10\pi^2} T \int_{-\infty}^{\infty} df \gamma(f) F^{-3} \Omega_{gw}(f) \tilde{Q}(f)$

Expected Noise:  $\langle N \rangle = 0$

$$\text{Noise}^2: \langle N^2 \rangle = T \int_{-\infty}^{\infty} df S_1(f) S_2(f) |\tilde{Q}(f)|^2$$

HOW DO WE CHOOSE  $\tilde{Q}$  FOR LARGEST S/N?

# OPTIMAL FILTERING (CONT)

Define  $(A, B) \equiv \int_{-\infty}^{\infty} df A(f) B^*(f) S_1(f) S_2(f)$

Signal =  $\langle S \rangle = \frac{3H_0^2}{10\pi^2} \left( \tilde{Q}, \frac{\gamma \Omega_{gw}}{F^3 S_1 S_2} \right) T$

$\langle N^2 \rangle = (\tilde{Q}, \tilde{Q}) T$

Extremize  $\frac{\langle S \rangle^2}{\langle N^2 \rangle} = \left( \frac{3H_0^2}{10\pi^2} \right)^2 T \frac{\left( \tilde{Q}, \frac{\gamma \Omega_{gw}}{F^3 S_1 S_2} \right)^2}{(\tilde{Q}, \tilde{Q})}$

Solution (Optimal Filter)

$$\tilde{Q} = \frac{\gamma \Omega_{gw}}{F^3 S_1 S_2}$$

$$\left( \frac{S}{N} \right)^2 = \frac{9H_0^4}{100\pi^4} T \int_{-\infty}^{\infty} df \frac{\gamma^2(f) \Omega_{gw}^2(f)}{F^6 S_1(f) S_2(f)}$$

$H_0$  = Hubble expansion rate,  $\text{sec}^{-1}$

$T$  = integration time, sec

$\gamma(f)$  = overlap reduction function

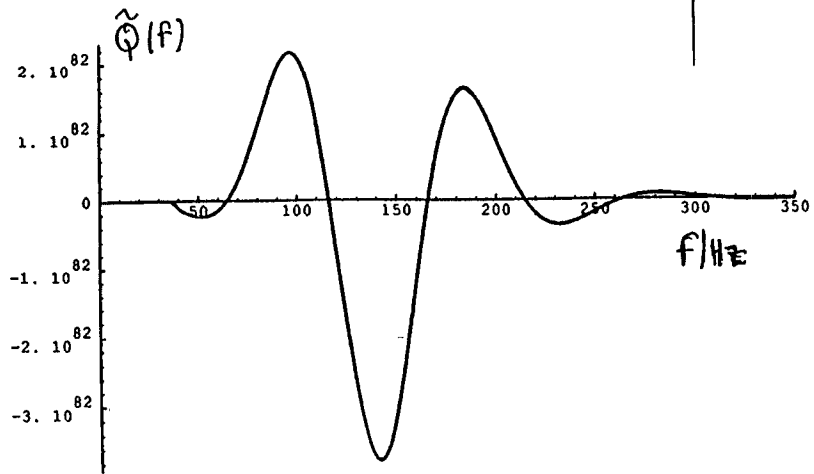
$\Omega_{gw}(f)$  = fractional energy density in GW

$S_i(f)$  = noise power,  $\text{Hz}^{-2} = \text{sec}$

$T = 4$  month,  $S(f) = \text{Science '92}$

$$\Omega_{gw \text{ min}} = \frac{5.1 \times 10^{-6}}{h^2} \text{ (initial)} \quad \frac{5 \times 10^{-11}}{h^2} \text{ (advanced)}$$

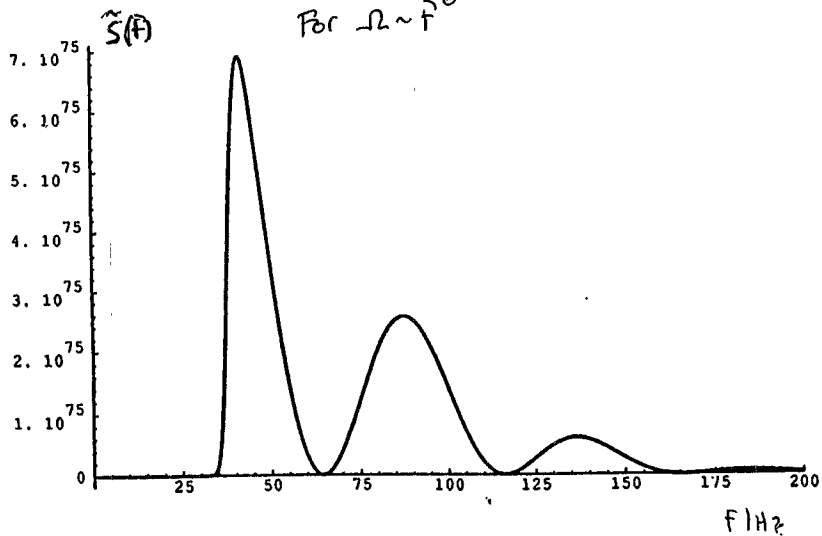
OPTIMAL FILTER (INITIAL LIGC)



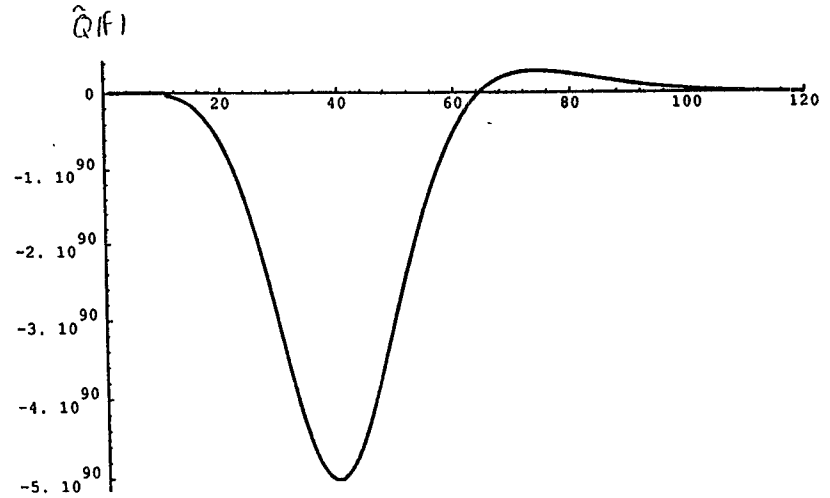
Initial LIGC optimal filter  $\tilde{Q}(f) = \frac{\gamma(f)}{f^3 S_1(f) S_2(f)}$

Expected signal  $\tilde{S}(f) = \frac{\delta^2(f)}{f^6 S_1(f) S_2(f)}$

For  $\Omega \sim f^0$

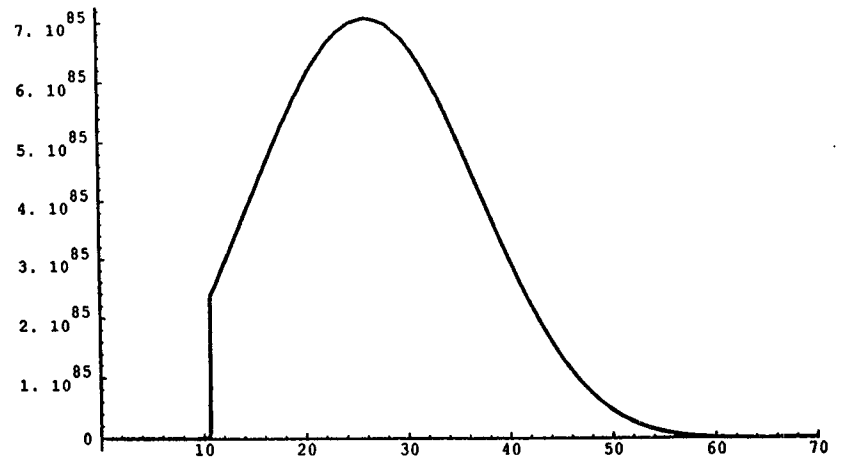


OPTIMAL FILTER (ADVANCED LIGC)



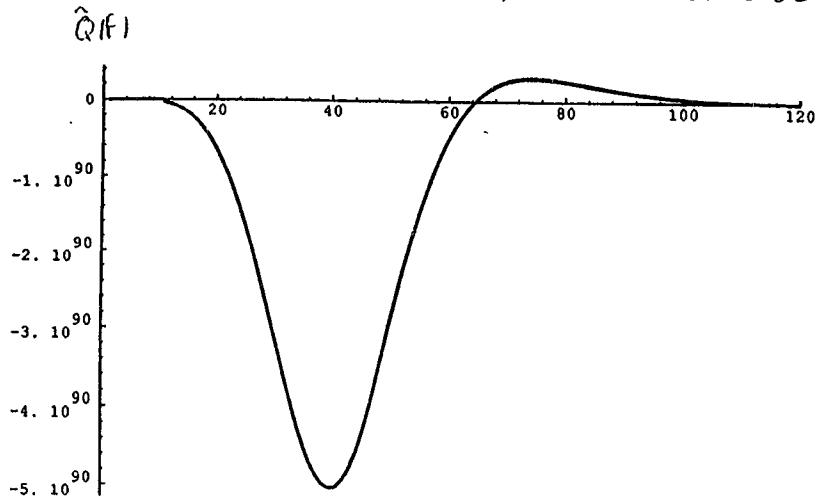
Advanced LIGC optimal filter  $\hat{Q}(f) = \frac{\gamma(f)}{f^3 S_1(f) S_2(f)}$

Expected signal  $\tilde{S}(f) = \frac{\gamma^2(f)}{f^6 S_1(f) S_2(f)}$  for  $\Omega \sim f^0$



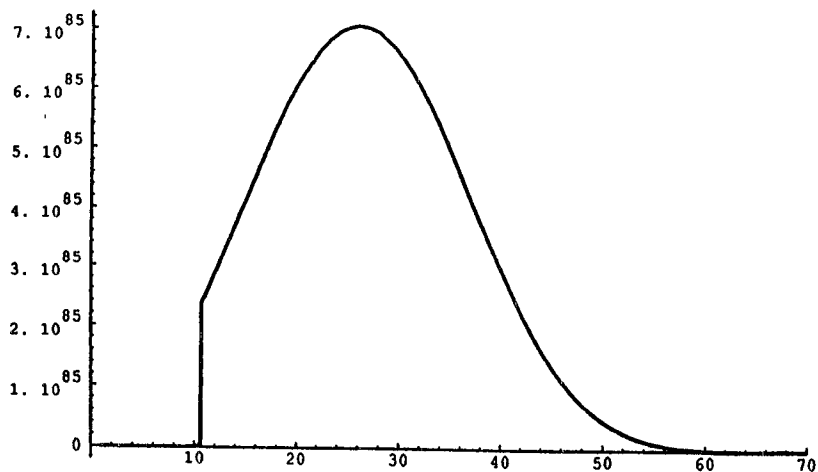


### OPTIMAL FILTER (ADVANCED LIGO)

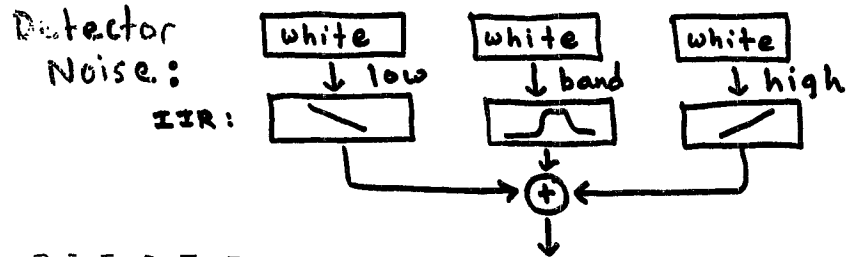


Advanced LIGO optimal filter  $\tilde{Q}(f) = \frac{\gamma(f)}{f^3 S_1(f) S_2(f)}$

Expected signal  $\tilde{S}(f) = \frac{\gamma^2(f)}{f^6 S_1(f) S_2(f)}$  for  $\Omega \sim f^0$



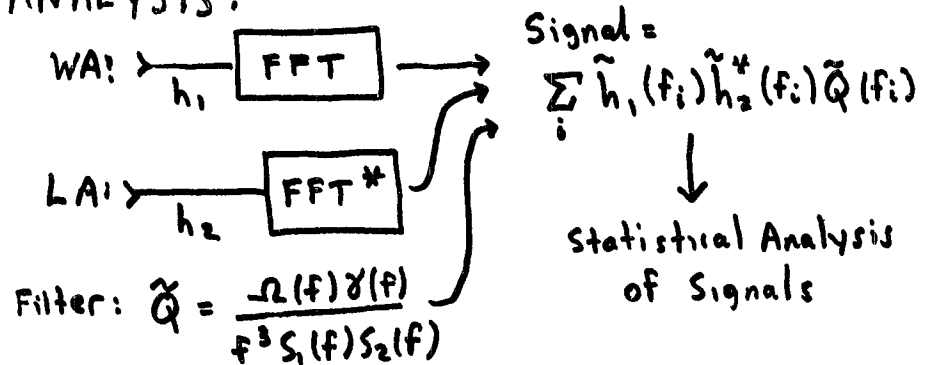
### Data Analysis Pipeline (+ simulated data)



### Stochastic Background:

- (1) choose random sources on celestial sphere.
- (2) generate time domain signals  $h_+$ ,  $h_x$
- (3) find time delay  $\Delta t$  between detectors
- (4) add  $h_+ F_+ + h_x F_x$  into output data streams

### ANALYSIS:



Note: 3.28 sec = 65,536 samples

filters: predicted vs. on-the-fly

Simulate + Analyze at 1/3 real-time rate

## WHAT NEXT:

- ① TESTING
- ② REDUCED BANDWIDTH
- ③ USE AC-IN NOISE
- ④ see Lewis Garching/Glasgow data
- ⑤ "systematic" corruption, correlated  $\bar{B}$
- ⑥ ANISOTROPIC stochastic BG
- ⑦ NETWORKS OF  $N > 2$  DETECTORS
- ⑧ OPTIMAL SLOPE DETERMINATION  
 $\Omega(f) \propto f^p$
- ⑨ WEIGHTING/REJECTION OF "BAD" DATA

min

# The GAP concept

Redouane FAKIR

tails and documents → fakir@physics.ubc.ca

Goal: direct, low cost, timely detection of gravity waves at frequency  $\leq 10^{-2}$  Hz

Limited ambition: individual, galactic, periodic sources only

(e.g. not aiming at LISA targets)

Strategy: 3 excellent, very high precision experiments (already designed or up and running) in astronomy → turn one or more of these into gravity wave detectors

HOW? →

(2)

→ Need to find new effects linking strongly gravity waves to these experiments

## Findings:

3 different g.w. detection schemes with (equivalent) sensitivities of  $10^{-21}$  for binary stars and  $10^{-26}$  for neutron stars

(degree of interest ① < ② < ③ below)

① Based on new version of time delay effect (R.F. Phys. Rev. D)

→ could see neutron star waves

Experiment: of  $h \sim 10^{-26}$  and  $f < 10^3$  Hz

- Using existing pulsar timing experiments and possibly existing data sets
- Sensitivity evaluation (precise): awaiting response of pulsar timing experimental

(2) New astrometric effect of g.w.s :  
(R.F. Ap.J.)  
Periodic shifts in apparent  
stellar positions of  
amplitude  $5 \times 10^{-9}$  to  $5 \times 10^{-8}$  arcsec

• Astrometric experiments :

- Hipparcos (ESA, completed 1994)  
sensitivity  $10^{-4}$  arcsec for 100 sec integr.  
→ potential  $10^{-7}$  arcsec for 3 years  
(fell only 1 order of mag. short of g.w.)
- GAIA (ESA, approved, partially designed)  
Sensitivity  $5 \times 10^{-6}$  arcsec for 100 sec integr.  
→ Potential  $5 \times 10^{-9}$  arcsec for 3 years (Projector 5 years)  
design to be modified for g.w. detection
- POINT (NASA), status?  
Sensitivity  $10^{-6}$  arcsec for 100 sec integratn  
Potential  $10^{-9}$  arcsec for 3 years integratn  
could work as g.w. detector as is

(3) New interstellar scintillation  
effect of gravity waves :  
g.w. modulation of pulsar-  
interstellar-scintillation-pattern  
velocities. (Heavily studied since 1970's)

g.w. displacement of pattern :  
up to 100 km! → macroscopic effect  
of gravity waves!

Amplitude of g.w. modulation of  
pattern speed 0.1 to 1.0 m/sec

Experiment : measurement of pattern speed  
mastered and carried out since 1970  
Hardware → two radio antennae  
on the ground.  
e.g. Penticton-Jodrell Bank Experiment (1973)  
Sensitivity : 100 m/sec for 1000 sec  
→ Potential 0.3 m/sec for 3 years

(4)

→ Possible detection of gravity waves with  $10^{-5} \text{ Hz} < f < 10^{-2} \text{ Hz}$  from the ground!

→ Not affected by seismic noise in same way)

N.B. Galactic background does not limit detectability of individual sources as it does for LISA.

### Epilogue

Several observational astronomy techniques become relevant to gravity wave detection

→ Astronomers to be involved in GAP could become members of L.R.C.

By the way: G.A.P. concept → Gravity wave detection using Astronomical Phenomena

From "A macroscopic gravity wave effect" by Redouane FAKIR... (available upon request) fakhir@physics.ubc.ca

Figure captions

Figure 1.- Blowup of the double trigger mechanism of scheme 1 leading from the phase modulated intensity  $I(t)$  to the amplitude modulated intensity-peak duration  $\tau(t)$  to the pulse train  $p(t)$ , which is to be spectrally analyzed.

\* Figure 2.- As predicted by eq (2), the detection (through scheme 2) of a gravity wave signal of amplitude  $v_{gw}$  and angular frequency  $\omega_{gw}$  buried in random noise of loudness  $\sigma$  is successful all the way up to the realistic range  $\sigma \sim 100v_{gw}$ . No period folding or periodogram variance reduction were used.

$$* \text{ eq. (2): } \text{SNR} = \frac{1}{\sqrt{3}} \frac{\Delta t}{t_c} \frac{s^2}{\sigma^2} \frac{\tau_0^2}{\sigma^2} \exp\left(-\frac{\tau_0^2}{2\sigma^2}\right)$$

$\Delta t$ : duration of experiment

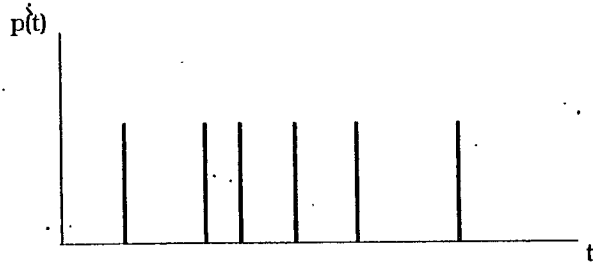
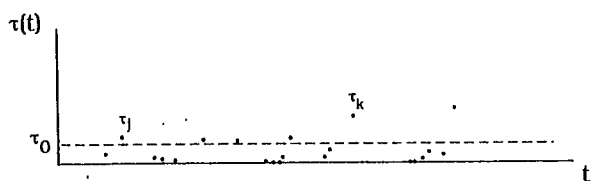
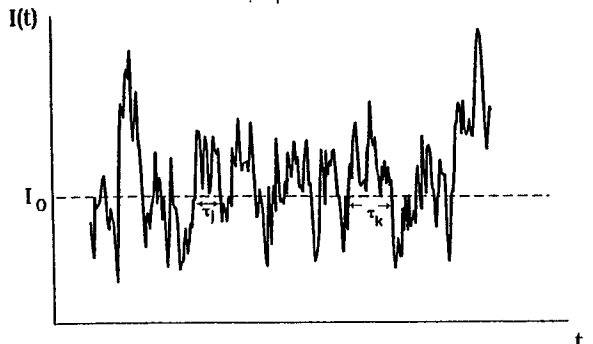
$t_c \sim \frac{1}{\text{Nyquist frequency}}$   
 $s$ : amplitude of signal  
 $\sigma$ : r.m.s. size of noise  
 $\tau_0$ : threshold of amplitude modulated data (velocity  $v_{gw}$  in scheme 2, peak duration  $\tau$  in scheme 1).

New gravity wave data processing technique (periodic waves), here applied to toy model of Gaussian white noise.

seems to perform better than non linear adaptive filtering at least for toy case.

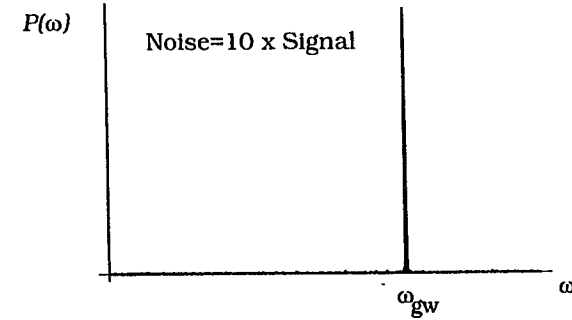
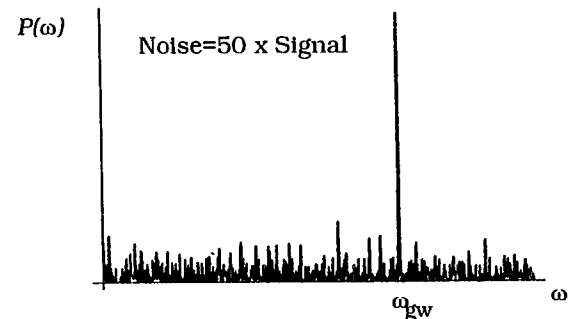
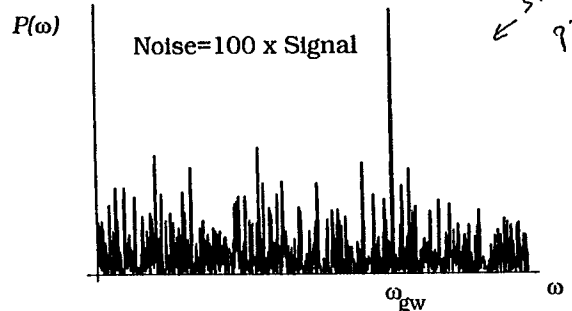
7

Figure 1



8

Figure 2



← SNR predicted  
 $SNR = \sqrt{10}$



(1)

## 100 Hour Test Run

2-6 March 1989

The main objectives for an extended test run have been:

- to prove that continuous operation of interferometric detectors are possible,
- to test the requirements on the stability of the adjustments and other hardware components,
- to provide long term data that can be analysed for various (non-gaussian) noise contributions,
- to rehearse the logistics of data acquisition, data exchange and archiving,
- and only as an extremely faint possibility the idea of finding some gravitational waves, or some other hidden correlations between the data of distant detectors.

(2)

## 100 Hour Test Run

2-6 March 1989

Total run time 100 hours  
Data on tape **91.34 hours  $\cong 91^h 20^m 36^s$**   
Total amount of data **14.5 G bytes**  
in 657 672 records of 22048 bytes each,  
written on 94 reels of tape (6250 bpi),  
each tape storing 7200 records (158.7 MB),  
corresponding to one hour of run time.

Each record represents  $1/2$  second of data, containing

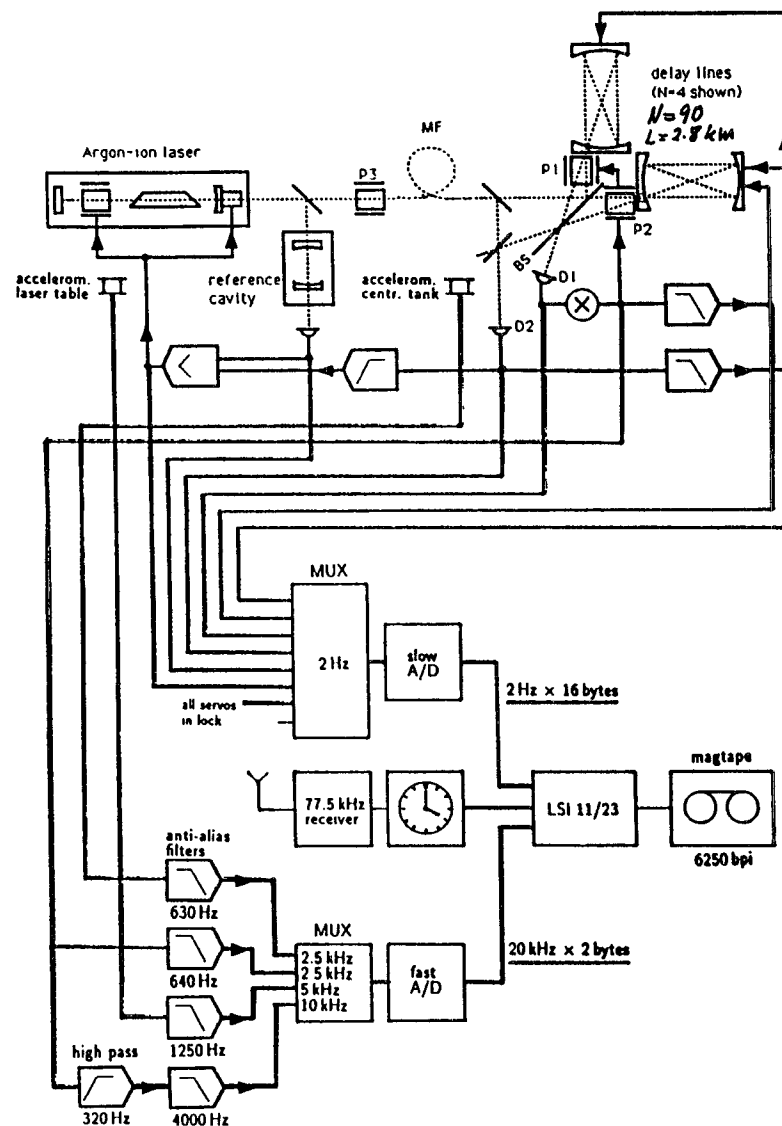
- 5000 samples of interferometer data,
- 2500 samples of accelerometer data from laser table,
- 1250 samples of accelerometer data from central tank,
- 1250 samples of low frequency interferometer data,
- plus 8 different kinds of housekeeping data, sampled once per each  $1/2$  second record.



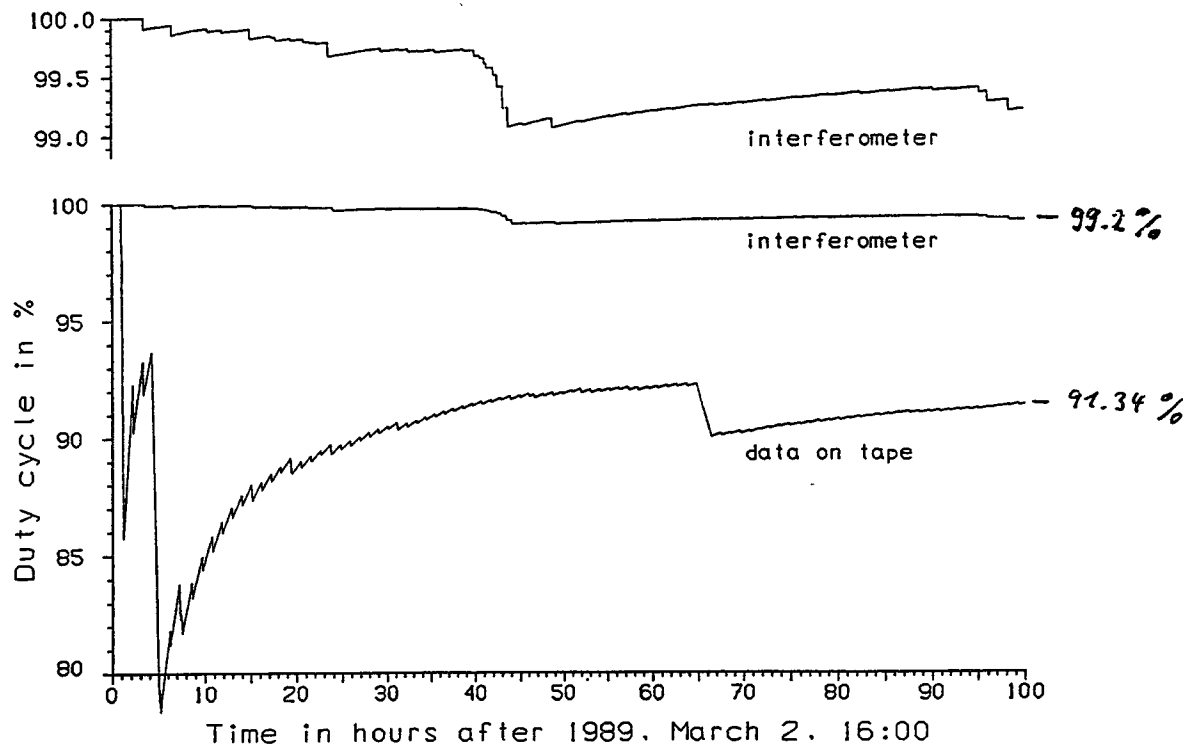
## Housekeeping data

Each 1/2 second record contains the following housekeeping data:

- The actual time from DCF 77,
- A logical signal indicating that the three main servo loops are in lock,
- A signal corresponding to the laser power,
- The low frequency correction signal from the frequency stabilization,
- A signal corresponding to the photocurrent in the diode for the absolute arm length control,
- The low frequency correction signal from the absolute arm length control,
- A signal corresponding to the photocurrent in the diode for the main interferometer output,
- The low frequency correction signal from the armlength difference control.



## Data taking run 1989, March 2 to March 6



Roland Schilling, MPQ Garching, 12. 1. 1996 - 13:56

✂

⑥

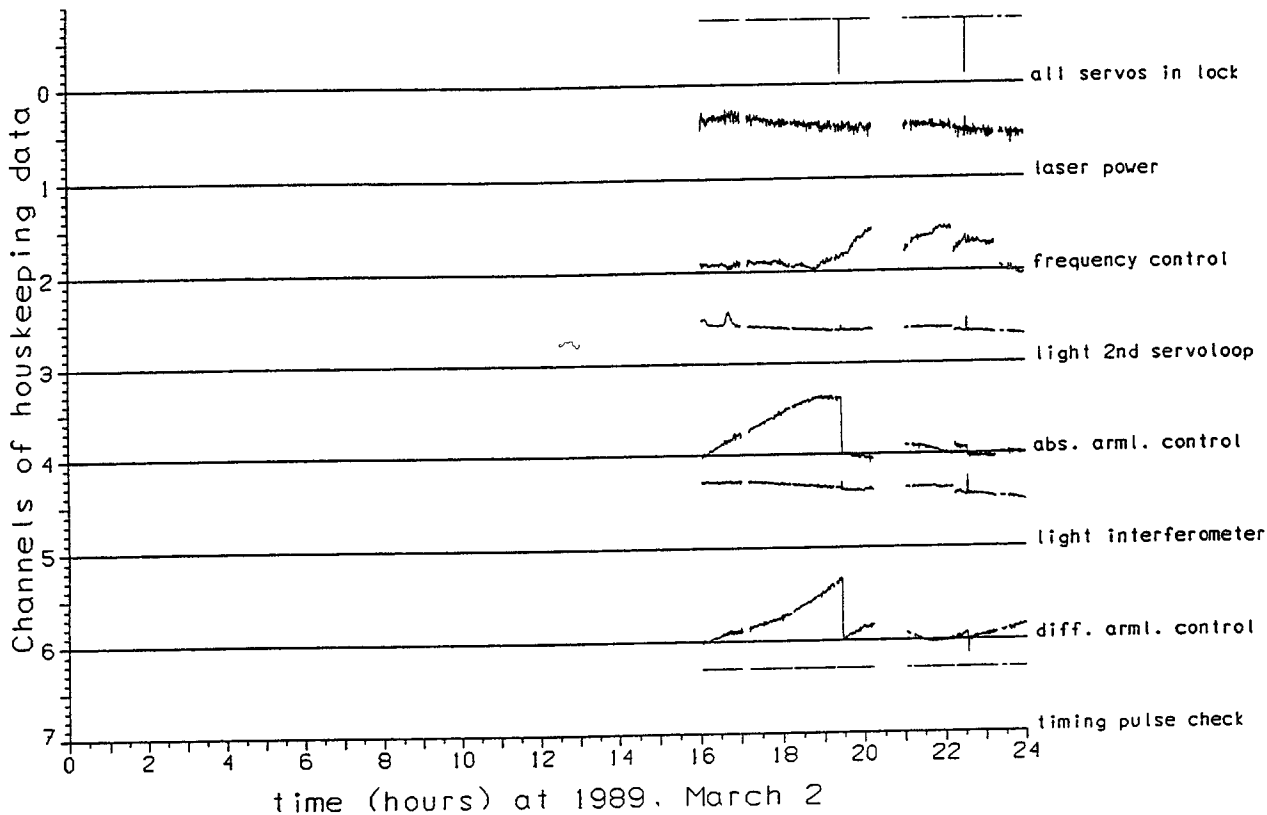
## Investigations of recorded data

We looked at:

- Housekeeping data,
- Noise spectra for selected time slices,
- 1-minute *rms* averages,
- 0.1-second *rms* values of selected time slices,
- Amplitude statistics for selected time slices.

Data taking run 1989, March 2 to March 6

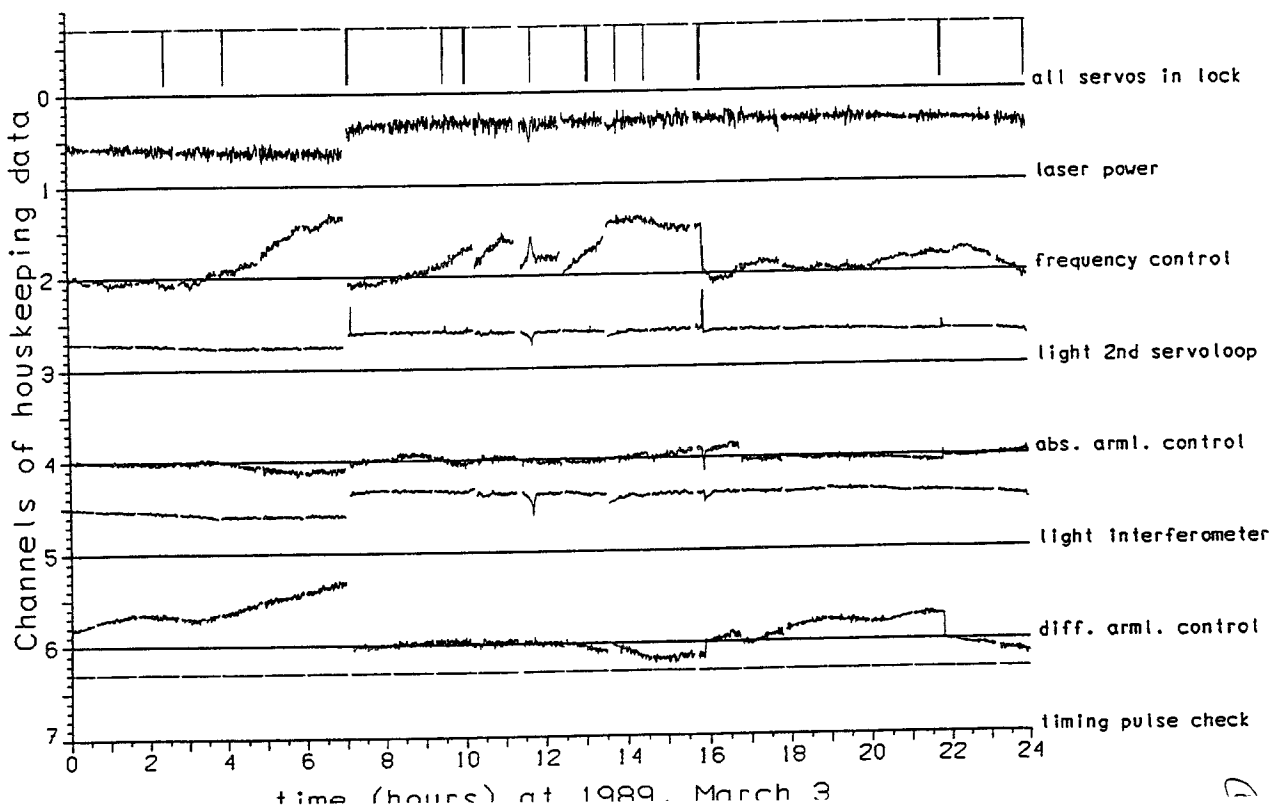
sheet 1



(7)

Data taking run 1989, March 2 to March 6

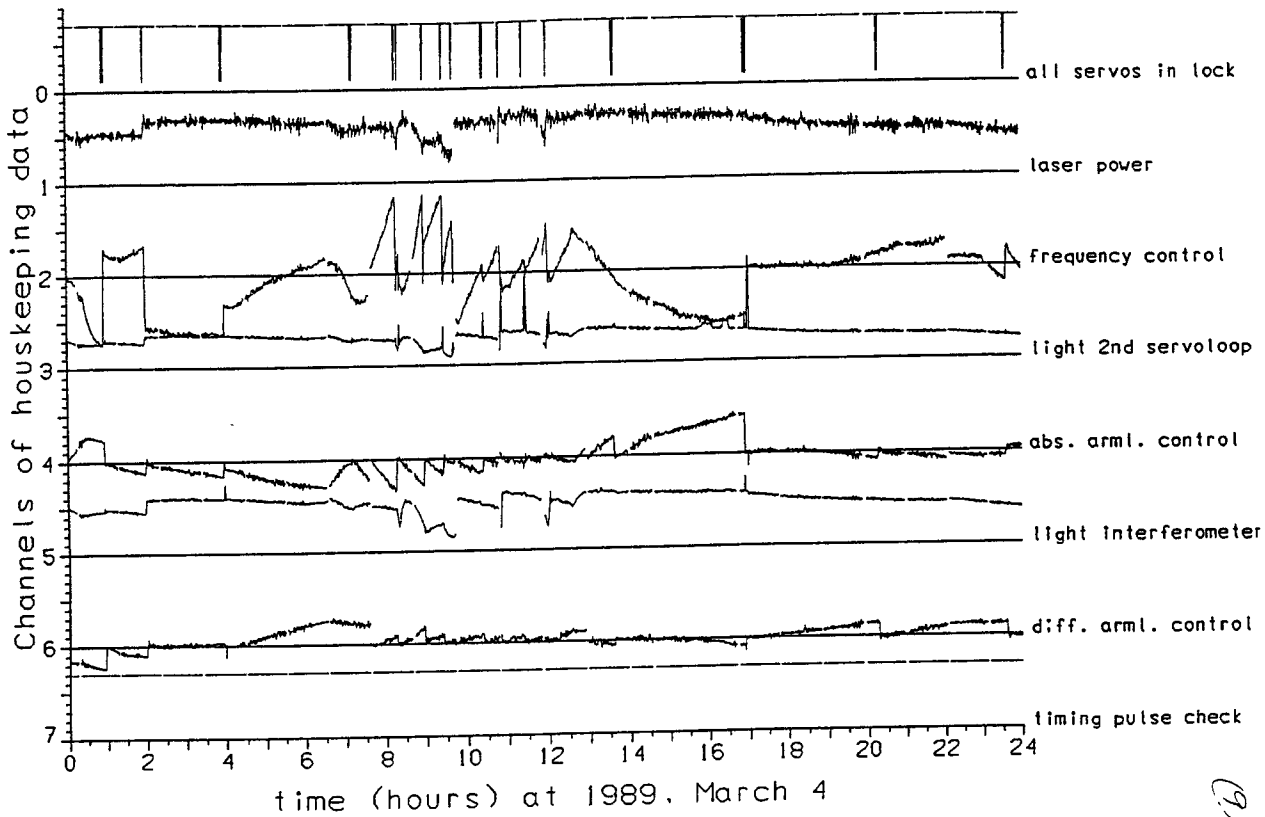
sheet 2



D

### Data taking run 1989, March 2 to March 6

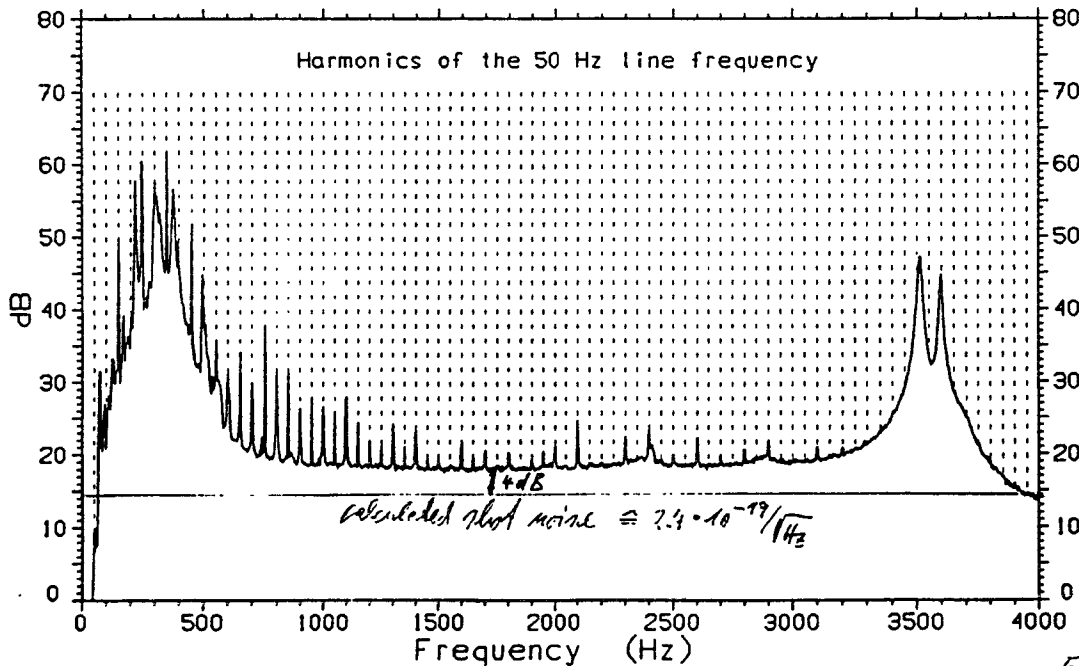
sheet 3



(9)

### Interferometer Spectrum

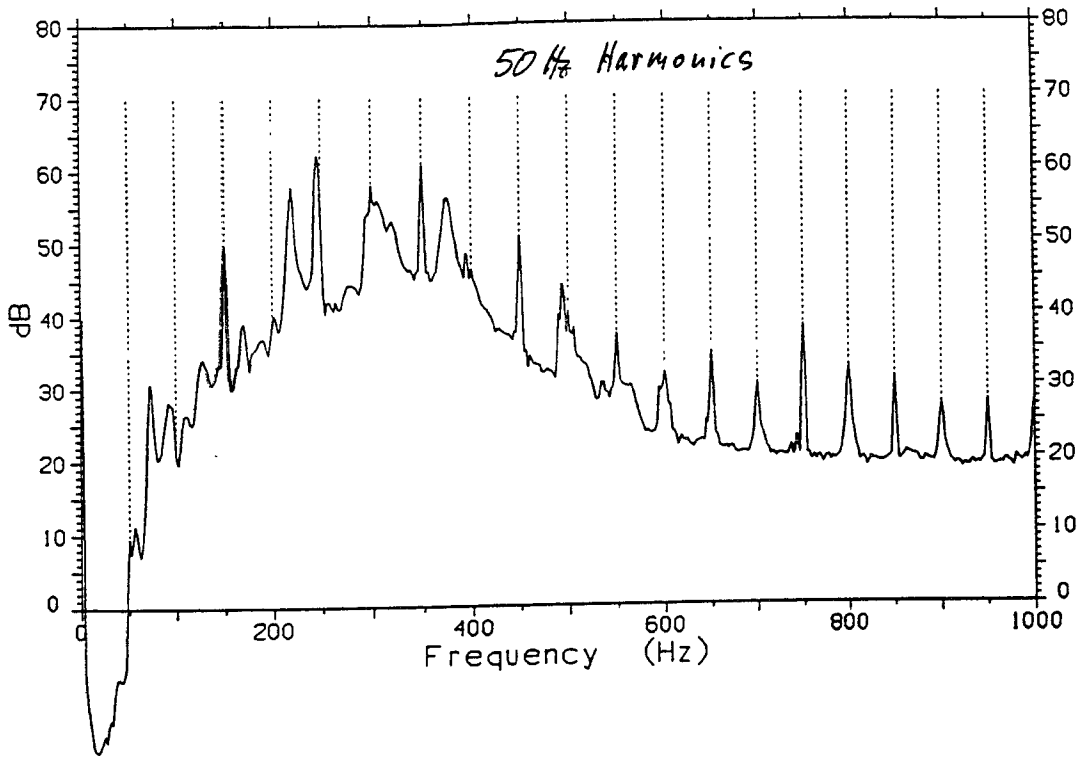
### Data taking run 1989, March 2 to March 6



5 min after start

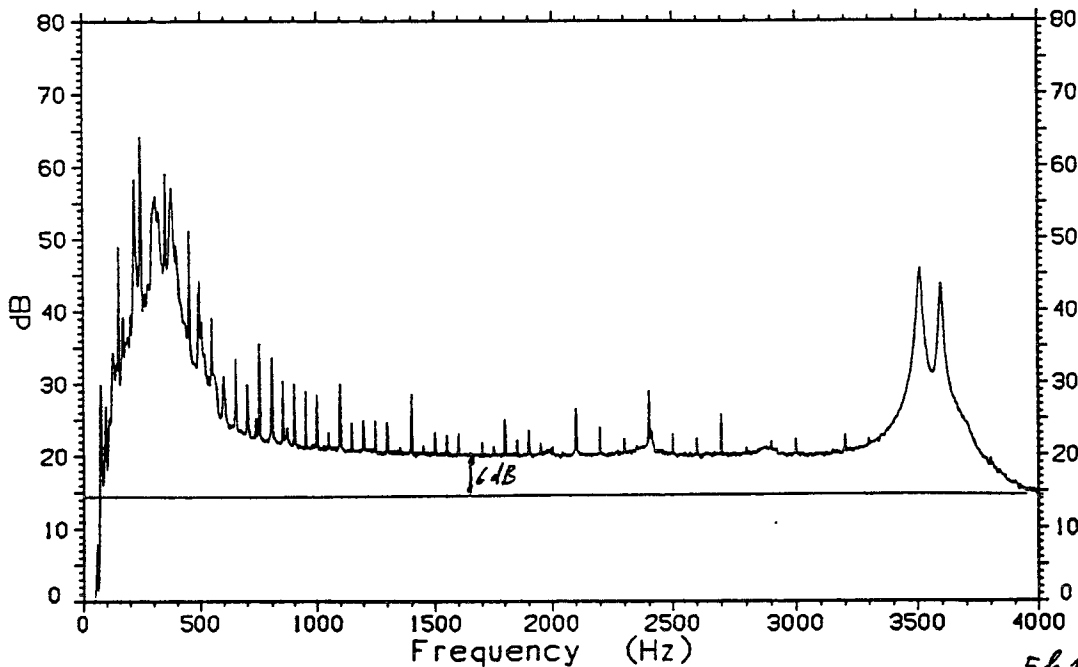
# Interferometer Spectrum

Data taking run 1989, March 2 to March 6



(11)

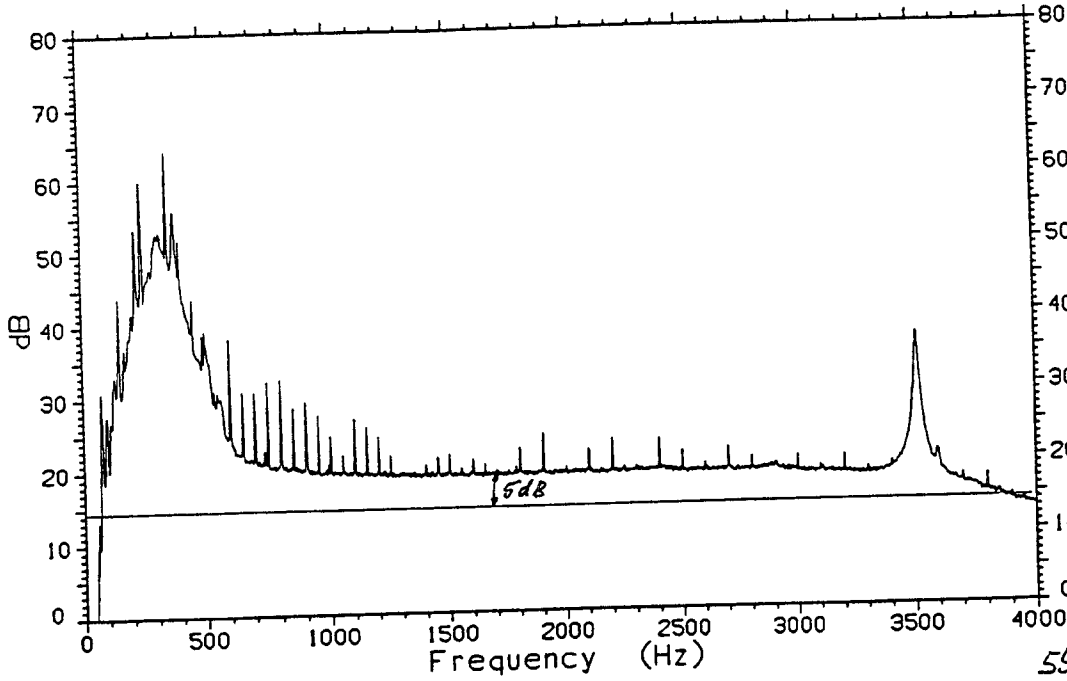
Data taking run 1989, March 2 to March 6



5 h 15 min after start

5000 point FFT, averaged over 600 records, starting at tins = 39040

Data taking run 1989, March 2 to March 6



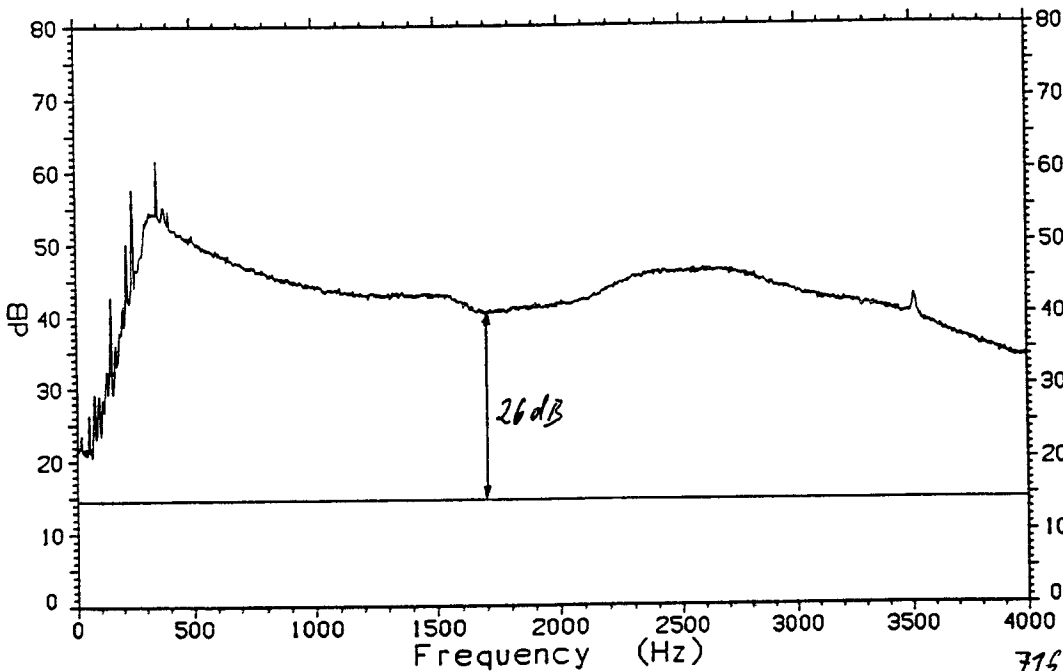
55 h 38 min after start

13

5000 point FFT, averaged over 600 records, starting at tins = 400800

1730's

Data taking run 1989, March 2 to March 6



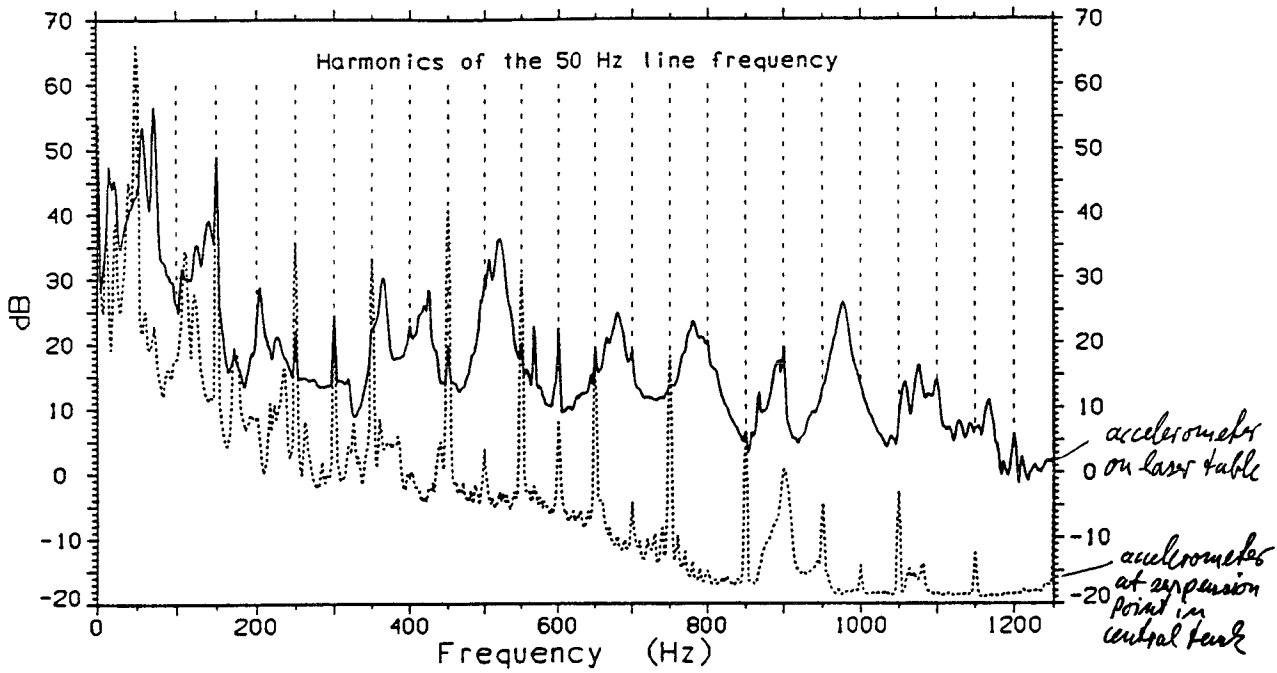
714 49 min after start

14

5000 point FFT, averaged over 600 records, starting at tins = 517320

# Accelerometer Spectra

Data taking run 1989, March 2 to March 6



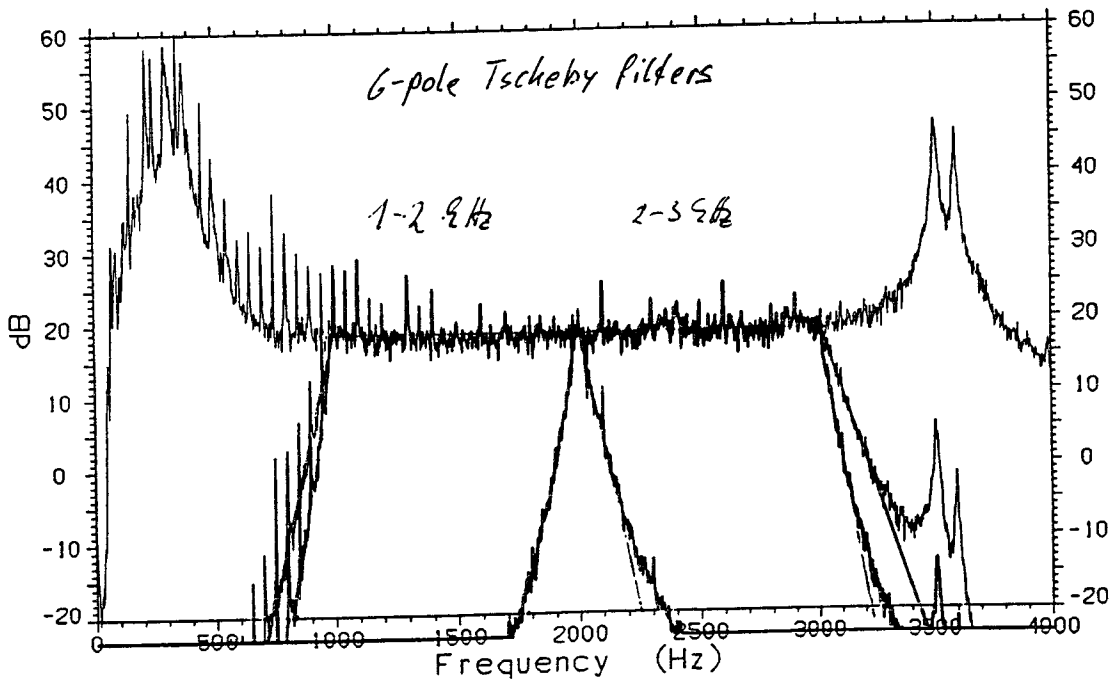
2500 point FFT, averaged over 600 records, starting at tins = 1440

15

88.05.89

14.42:14

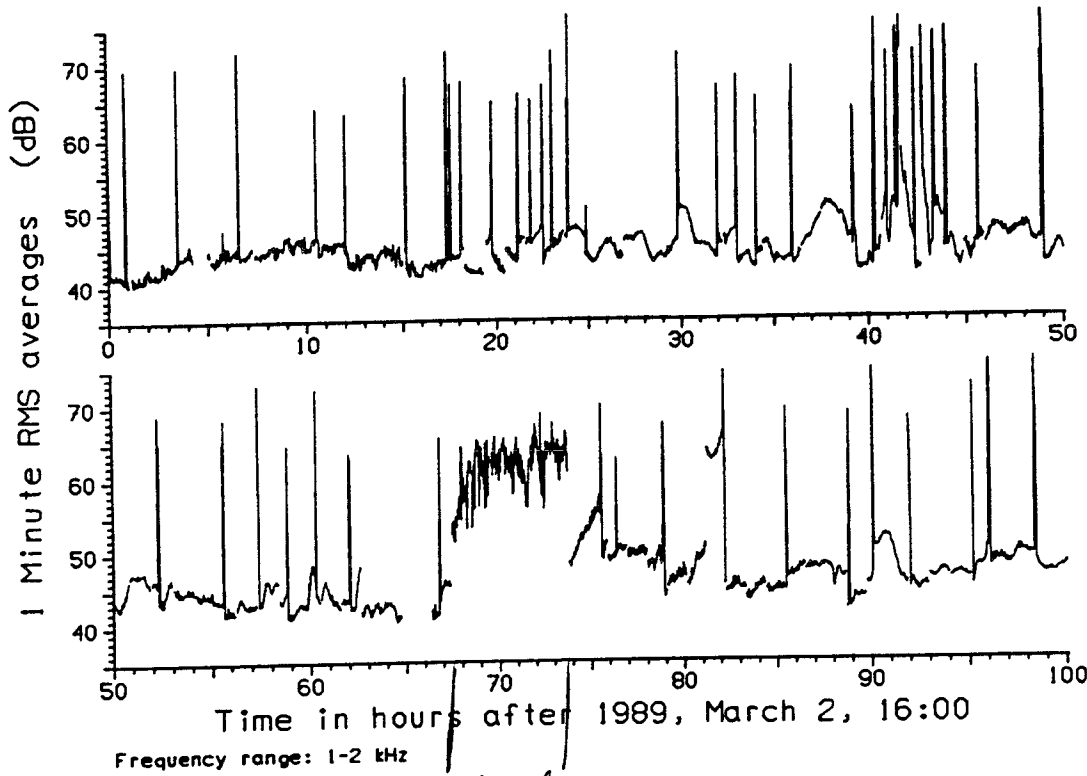
Data taking run 1989, March 2 to March 6



5000 point FFT, averaged from nrec = 1000 to nrec = 1020

16

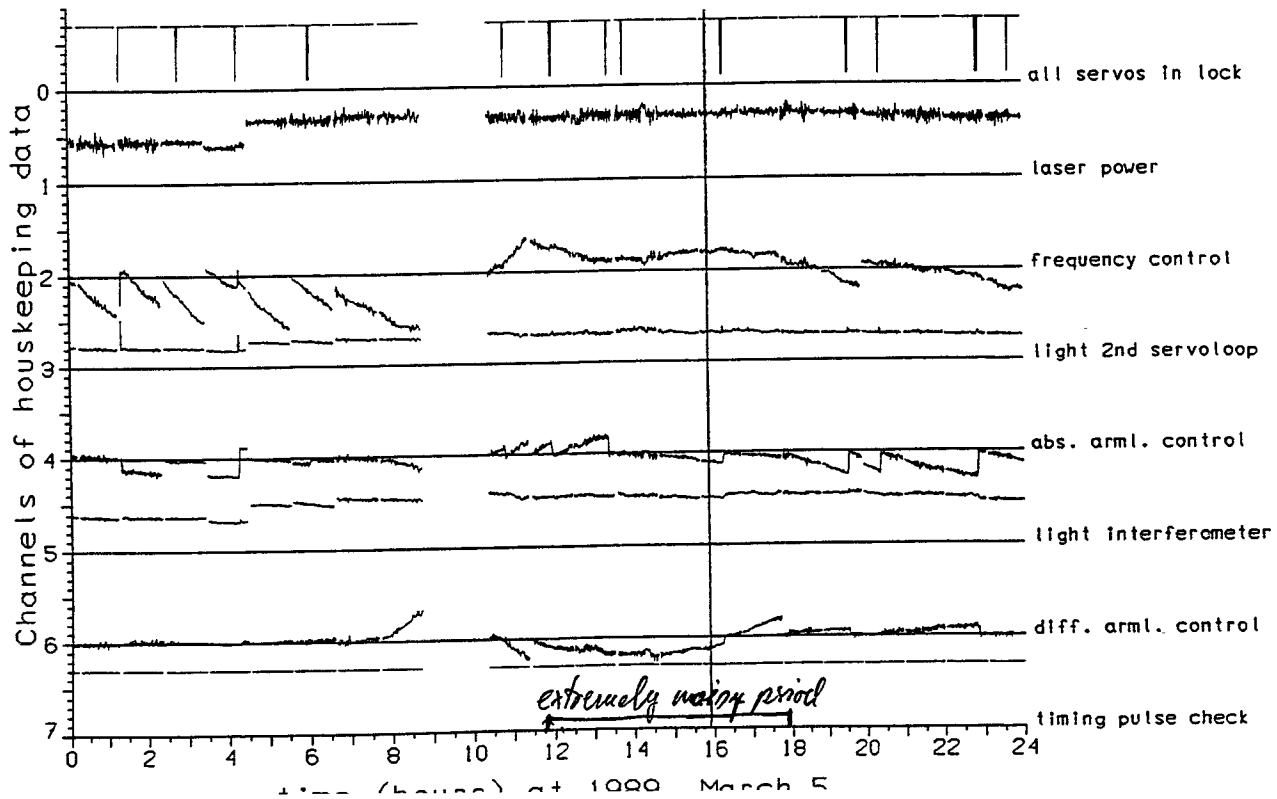
Data taking run 1989, March 2 to March 6



17

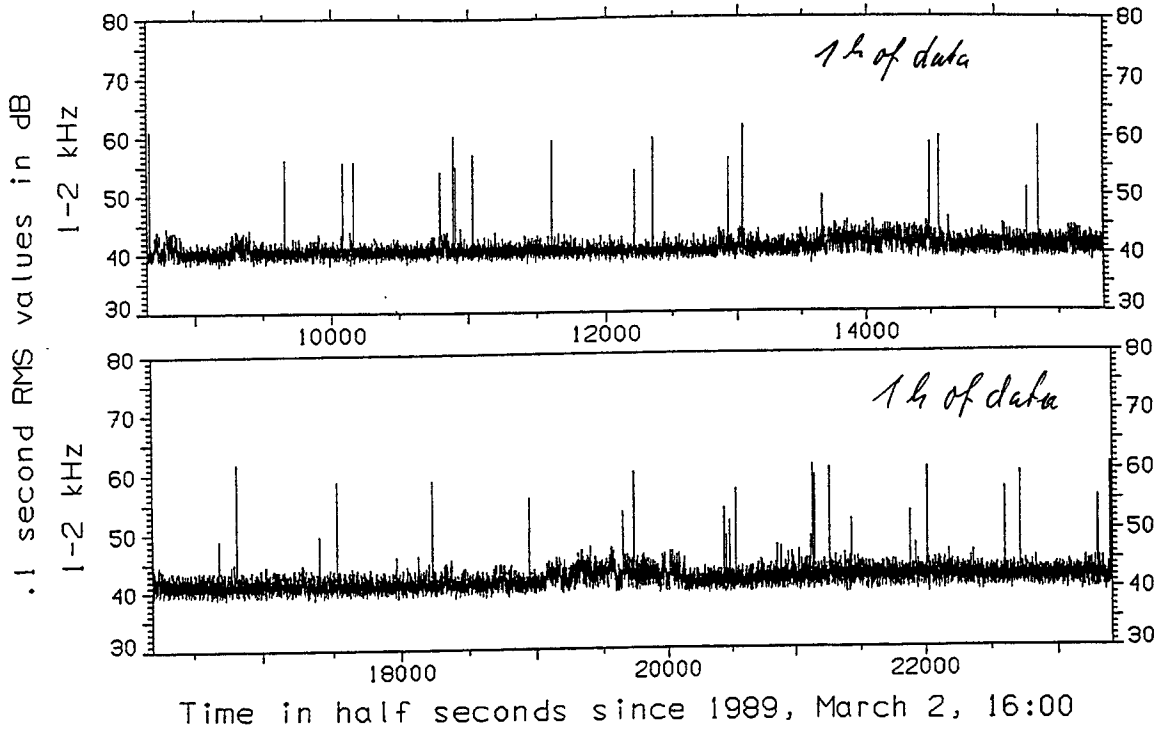
Data taking run 1989, March 2 to March 6

sheet 4



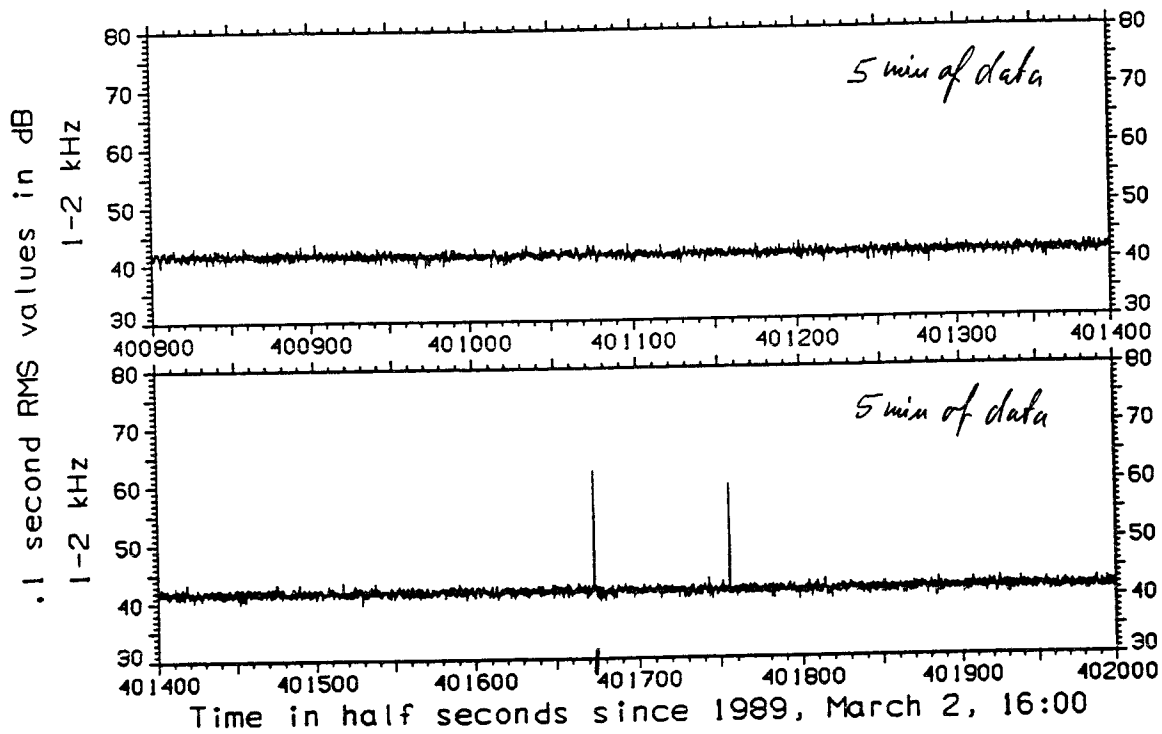


Data taking run 1989, March 2 to March 6



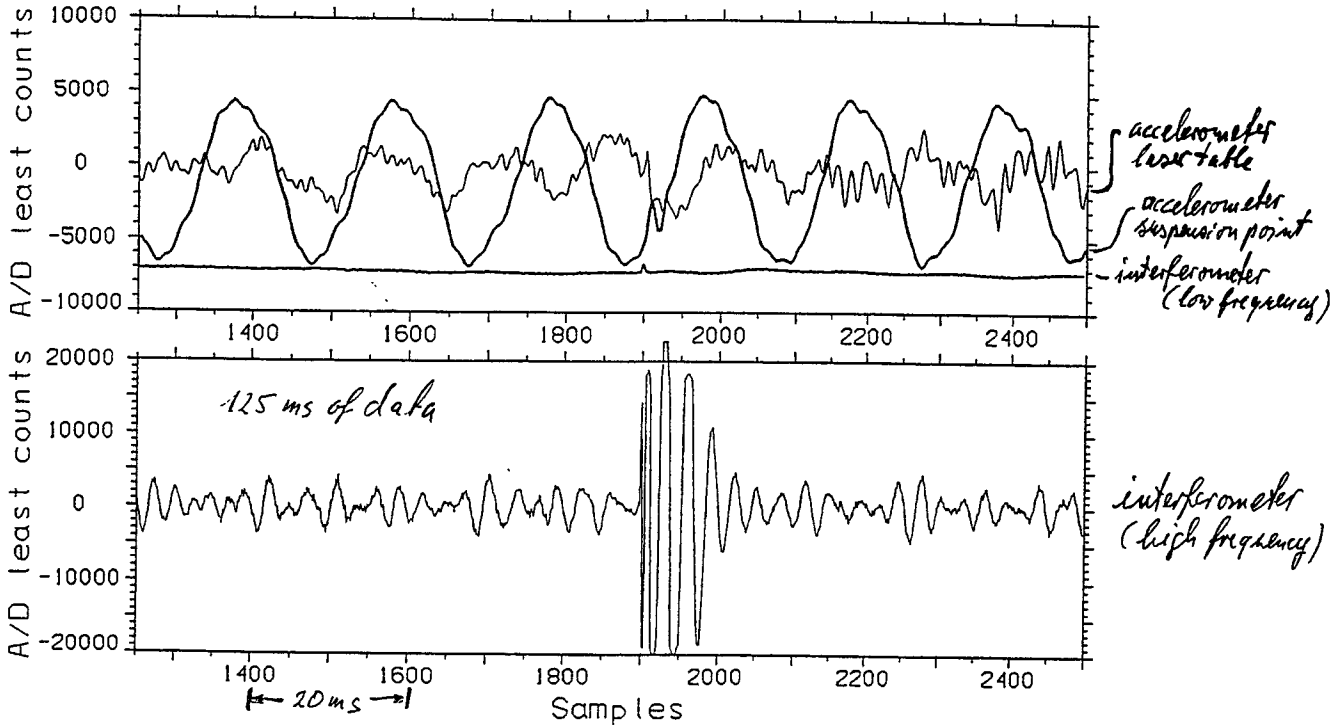
(17)

Data taking run 1989, March 2 to March 6



# Original Data

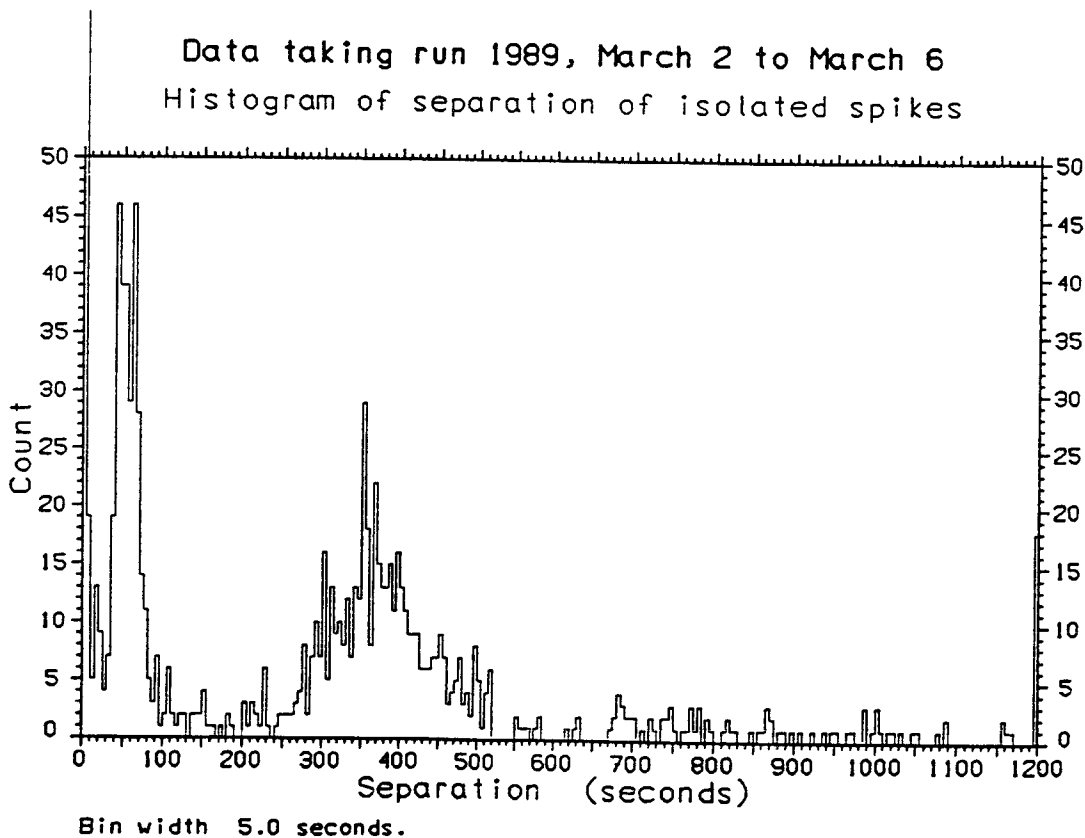
Data taking run 1989, March 2 to March 6



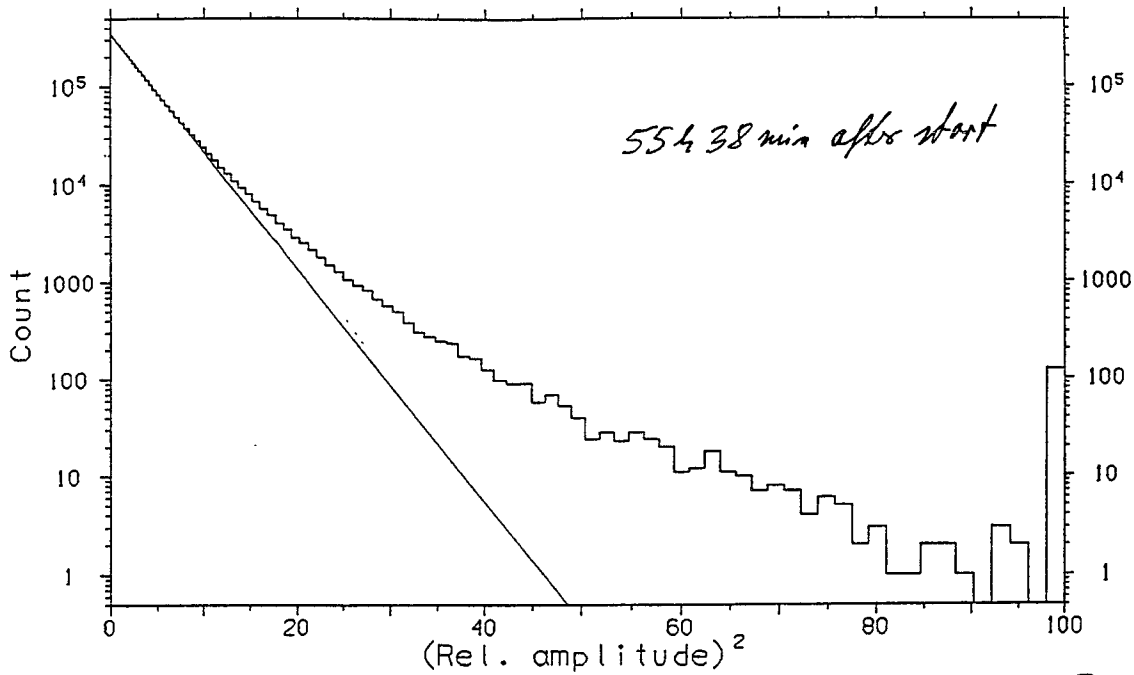
Cassette B00088, GW-file 310, record 874, tins = 401673

(21)

Data taking run 1989, March 2 to March 6  
Histogram of separation of isolated spikes



Data taking run 1989, March 2 to March 6



Amplitude statistic of 3 million data points *~ 5 min of data*

Fitted RMS value = 42.6 dB

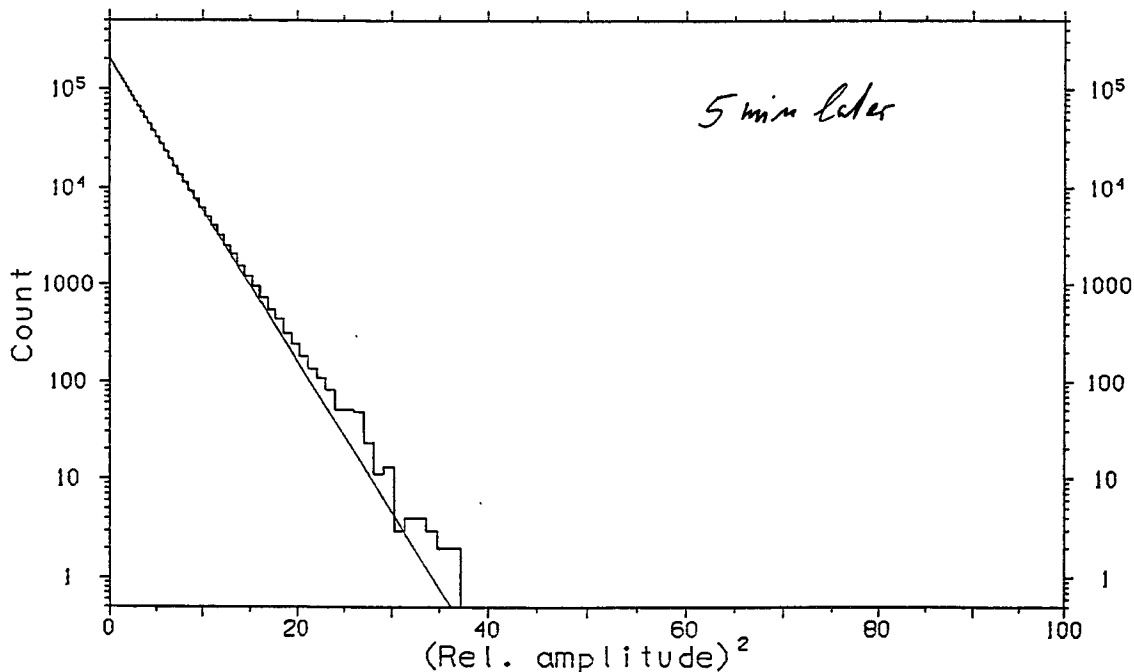
Record no. = 1235

Deviation from Gaussian: 59.

*1-2 kHz*

(23)

Data taking run 1989, March 2 to March 6



Amplitude statistic of 3 million data points

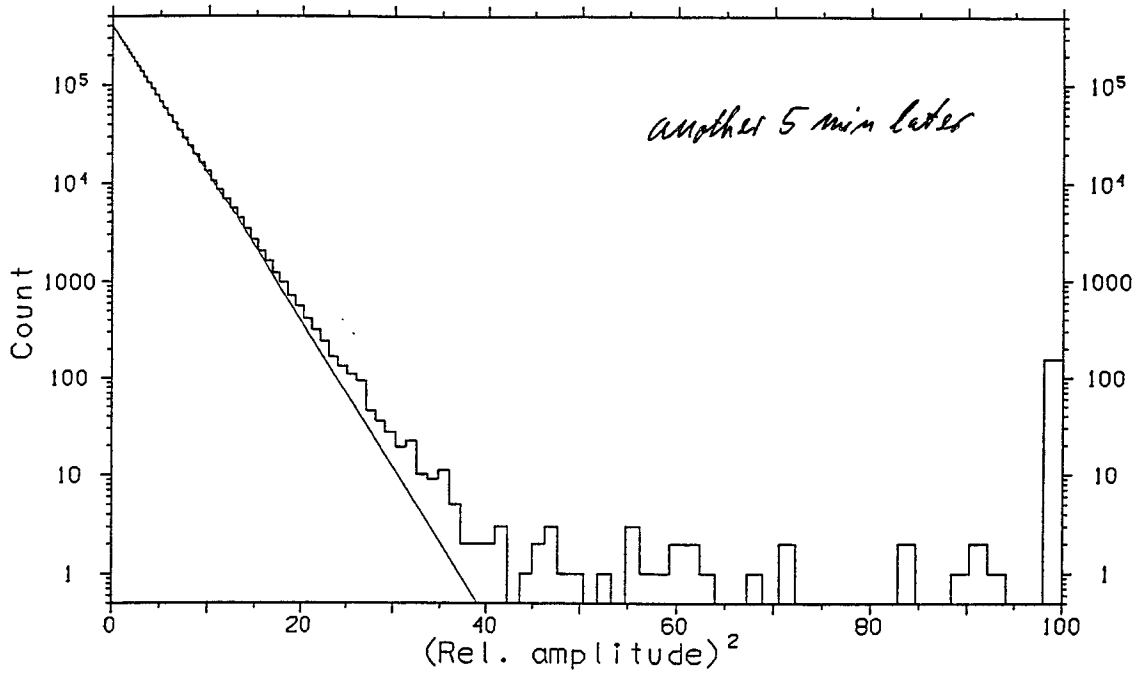
Fitted RMS value = 41.5 dB

Record no. = 1237 *= start of burst = 900800*

Deviation from Gaussian: 27

*1-2 kHz*

Data taking run 1989, March 2 to March 6



Amplitude statistic of 3 million data points

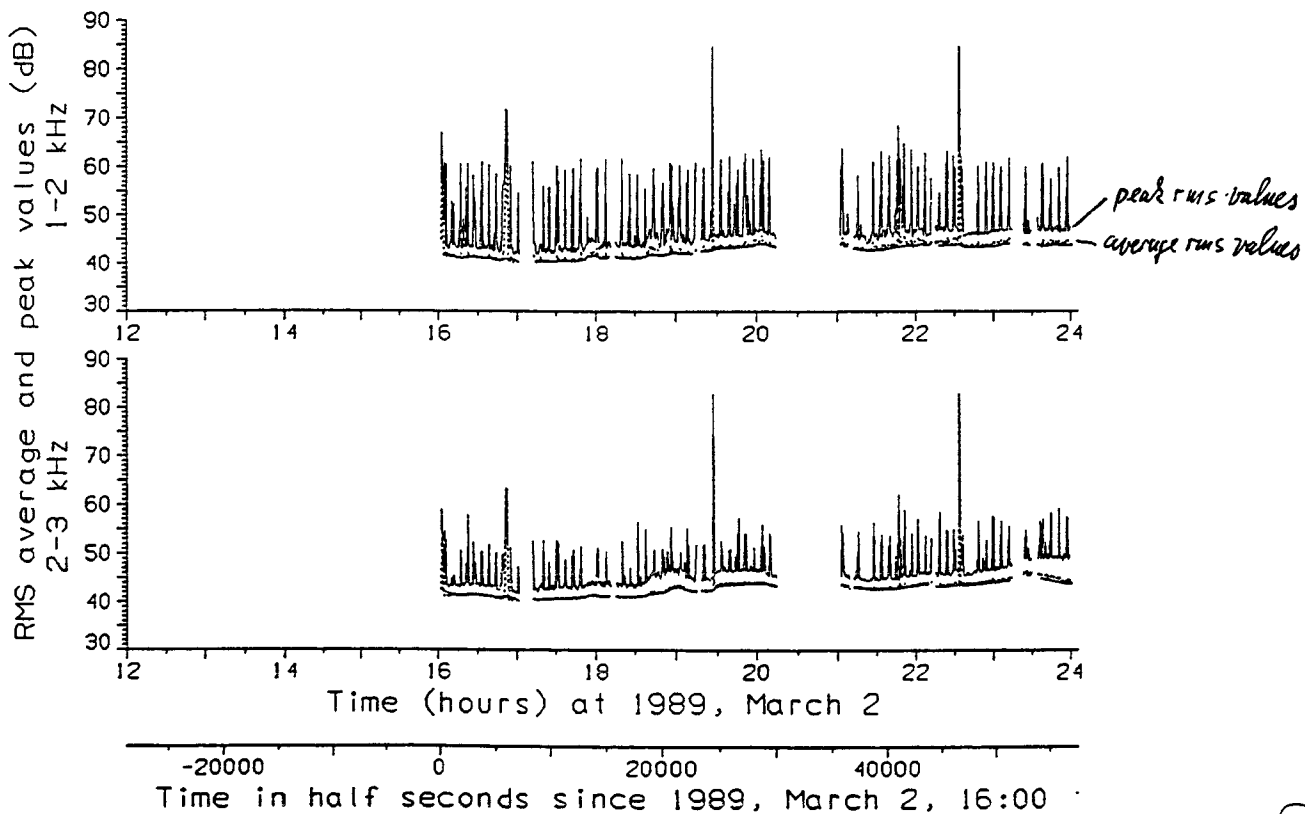
$\sqrt{\text{Fitted RMS value}} = 41.6 \text{ dB}$   
 Record no. = 1239  
 Deviation from Gaussian: 47.

1-2 kHz

(25)

Data taking run 1989, March 2 to March 6

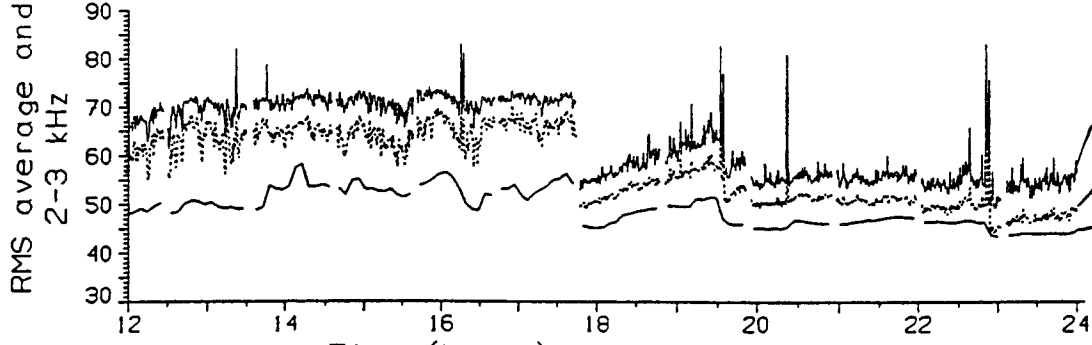
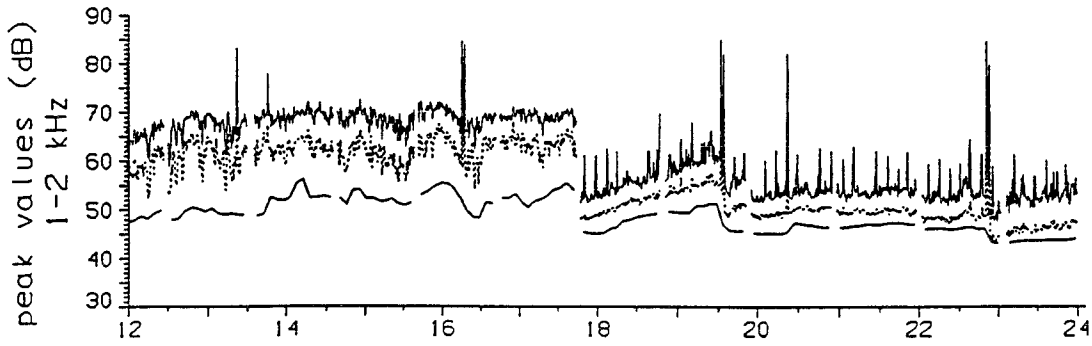
sheet 1



(26)

Data taking run 1989, March 2 to March 6

sheet 7



*peak rms values*  
*average rms values*  
*rms values from amplitude statistics*

Time (hours) at 1989, March 5

50000 52000 54000 56000  
 Time in half seconds since 1989, March 2, 16:00

(27)

# Data Analysis with Multiple Data Streams

Sam Finn  
Northwestern

Multiple Cryogenic Resonant Instruments

Multiple Interferometers

Correlated Noise

Multiple transducers: TIGA

LISA

Hanford 4 x 2 km Interferometers

Heterogeneous Data Streams

Resonant: interferometric instruments

Delay-line vs. Fabry-Perot

Signal recycling vs. broadband

- I Single data stream data analysis
- II Multiple detector data analysis
- III Comments
- IV Multiple data stream, single detector DA
- V Conclusions / Exhortation

# Single Data Stream Data Analysis

$$C_n(T) \equiv \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt n(t) n(t)$$

$$S_n(f) \xleftrightarrow{\text{F.T.}} C_n(\tau)$$

Detector Output  $g(t) = \begin{cases} n(t) \\ n(t) + s(t, \mu) \end{cases}$

Probability  $g(t)$  includes  $s(\mu)$

$$P(\mu, s | g) = \frac{\Lambda(\mu, s)}{\Lambda + P_{(0,0)}/1 - P_{(0,0)}}$$

$$P(s|g) = \int d\mu P(\mu, s|g)$$

CI:  $P(\mu, s|g) > \delta$   
 $\mu$  maximizes  $P(\mu, s|g)$

$$\Lambda(\mu, s) \equiv P(\mu | s) \frac{P(g - s | \mu | 0)}{P(g | 0)}$$

Discrete Sampling

$$P(g(t_0) | 0) = \exp\left[-\frac{1}{2} \frac{g(t_0)^2}{C_n(t_0)}\right] / [2\pi C_n(t_0)]^{1/2}$$

$$P(g(t_1), g(t_2), \dots | 0) = \exp\left[-\frac{1}{2} C_{ij}^{-1} g(t_i) g(t_j)\right] / [(2\pi)^n \det \|C_{ij}\|]^{1/2}$$

$$C_{ij} \equiv C_n(t_j - t_i)$$

Continuum Limit

$$P(g|0) \propto \exp[-\langle g, g \rangle]$$

$$\langle g, h \rangle \equiv \int_{-\infty}^{\infty} \frac{\tilde{g}(f) \tilde{h}^*(f)}{S_n(f)} df$$

one-sided

$$\Lambda(\mu, s) = P(\mu | s) \exp\left[2\langle g, s(\mu) \rangle - \langle s(\mu), s(\mu) \rangle\right]$$

Frequentist:  $\Lambda(\mu, s)$  for thresholding

## Multiple Detector Data Analysis

$$\hat{\mu}_i^\alpha \equiv \left( \begin{array}{l} \text{Maximum Likelihood} \\ \text{estimator of } \mu_i \text{ in} \\ \text{detector } \alpha \end{array} \right) \quad \text{Source } \vec{v}, \vec{\mu} = \vec{\mu}(\vec{v})$$

Different detectors, different noise:  $\hat{\mu}_i^\alpha \neq \hat{\mu}_i^\beta$  ( $\{\alpha\} \neq \{\beta\}$ ) if  $\alpha \neq \beta$

Find "Best fit"  $\hat{\mu}$  to  $\{\hat{\mu}_i^\alpha\}$  (or  $\vec{v}$  to  $\{\hat{\mu}_i^\alpha(\vec{v})\}$ )

Example: Binary Inspiral in several intfs [Jarmanowski; Krolak]

$$\begin{aligned} \hat{M} &\propto \frac{\text{orientation angles}}{d_i} \quad \text{Luminosity distance} \\ \hat{p} &\propto \text{orbital phase at coalescence} \\ \hat{\phi} &= \hat{\phi}(\theta, \varphi, i, \psi, \Phi) \\ \hat{T} &= \hat{T}(\theta, \varphi, T_0) \quad \text{coalescence time at S.S. Bargeester} \end{aligned}$$

If  $\neq$  detectors,  $\geq 3$ ,  $M, d_i, T_0, \theta, \varphi, i, \psi, \Phi$  over-determined

Find "best fit" in "least squares" sense

## Comments

"Best fit" is ad hoc prescription

Least squares gives different  $\mu$  than "Least quartic", or  $L_1$ , or ...

Detection probability? Confidence intervals?

Meaningless since "best fit" is ad hoc

Correlated detector noise?

What if  $\hat{\mu}_i^\alpha - \hat{\mu}_i^\beta$  correlated with  $\hat{\mu}_i^\alpha - \hat{\mu}_i^\beta$ ?



Separate analysis, Separate "sources"



Single analysis, Single source

# Multiple Data Streams, Single Detector Data Analysis

Detector Output  $\vec{g}(t) = \begin{cases} \begin{pmatrix} z_n \\ \vdots \end{pmatrix} (t) \\ \begin{pmatrix} z_n \\ \vdots \end{pmatrix} (t) + \begin{pmatrix} s \\ \vdots \end{pmatrix} (t, \vec{\mu}) \end{cases}$

Probability density

$$P(\vec{\mu}, s | \vec{g}) = \frac{\Lambda(\vec{\mu}, s)}{\Lambda + P_{10}/(1-P_{10})}$$

$$\Lambda(\vec{\mu}, s) = P(\vec{\mu} | s) \exp[2 \langle \vec{g}, \vec{s}(\vec{\mu}) \rangle - \langle \vec{s}(\vec{\mu}), \vec{s}(\vec{\mu}) \rangle]$$

$$\Lambda = \int d^N \mu \Lambda(\vec{\mu})$$

Example: Independent noise  $\alpha\beta C_n(\tau) = 0$  if  $\alpha \neq \beta$

$$\alpha\beta S_n(f) = \begin{pmatrix} S_n(f) & 0 \\ 0 & S_n(f) \\ & \vdots \end{pmatrix}$$

$$\alpha\beta S_n^{-1}(f) = \begin{pmatrix} S_n^{-1}(f) & 0 \\ 0 & S_n^{-1}(f) \\ & \vdots \end{pmatrix}$$

$$\Lambda(\vec{\mu}) = P(\vec{\mu} | s) \prod_{\alpha} \exp[2 \langle \vec{g}, \alpha \vec{s}(\vec{\mu}) \rangle - \langle \alpha \vec{s}(\vec{\mu}), \alpha \vec{s}(\vec{\mu}) \rangle]$$

$\vec{\mu}$  maximizes  $\Lambda(\vec{\mu}) [P(\vec{\mu}, s | \vec{g})]$

$$\rho^2 = \sum_{\alpha} \alpha \rho^2(\vec{\mu}) \leq \sum_{\alpha} \alpha \rho^2(\alpha \vec{\mu})$$

$$P(\vec{g}(t_s) | 0) = \exp[-\frac{1}{2} \alpha\beta C_n^{-1}(0) \vec{g}(t_s) \cdot \vec{g}(t_s)] / [(2\pi)^N \det \| \alpha\beta C_n(0) \|]$$

discrete sampling  $\alpha\beta C_n(\tau) \equiv \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \alpha n(t) \beta n(t+\tau) dt$

Note:  $\alpha\beta C_n(\tau) = \beta\alpha C_n(-\tau)$

$$P(\vec{g}(t_1), \vec{g}(t_2), \dots) | 0) = \exp[-\frac{1}{2} \alpha\beta C_{ij}^{-1} \vec{g}(t_i) \cdot \vec{g}(t_j)] / [(2\pi)^{NM} \det \| \alpha\beta C_{ij} \|]$$

sample correlation matrix

$$\alpha\beta C_{ij} \equiv \alpha\beta C_n(t_j - t_i)$$

$$\alpha\beta C_{ij} = \begin{pmatrix} C_{11} & C_{12} & \dots \\ C_{21} & C_{22} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

- 1 Hanford 2 km
- 2 Hanford 4 km
- 3 Livingston
- 4 Pisa

continuum limit

$$P(\vec{g} | 0) \propto \exp[-\langle \vec{g}, \vec{g} \rangle]$$

$$\langle \vec{g}, \vec{h} \rangle \equiv \int_{-\infty}^{\infty} \alpha\beta S_n^{-1}(f) \alpha \vec{g}(f) \beta \vec{h}^*(f)$$

$$\alpha\beta S_n(f) \equiv 2 \int_{-\infty}^{\infty} d\tau e^{2\pi i f \tau} \alpha\beta C_n(\tau)$$

Two-sided, Complex, Hermitian



## Conclusions

Treat Multiple Instruments as Single Detector!

Probability density  $P(\vec{\mu}, s | \vec{g})$

No ad hoc estimators or prescriptions

Heterogeneous instruments/data streams

Correlated instrument noise

$${}^{\omega}\beta_{C_n}(\tau) = \lim_{T \rightarrow \infty} \int_0^T dt {}^{\omega}n(t) A n(t+\tau)$$

$${}^{\omega}\beta_{S_n}(f) = 2 \int_{-\infty}^{\infty} dt e^{2\pi i f \tau} {}^{\omega}\beta_{C_n}(\tau)$$

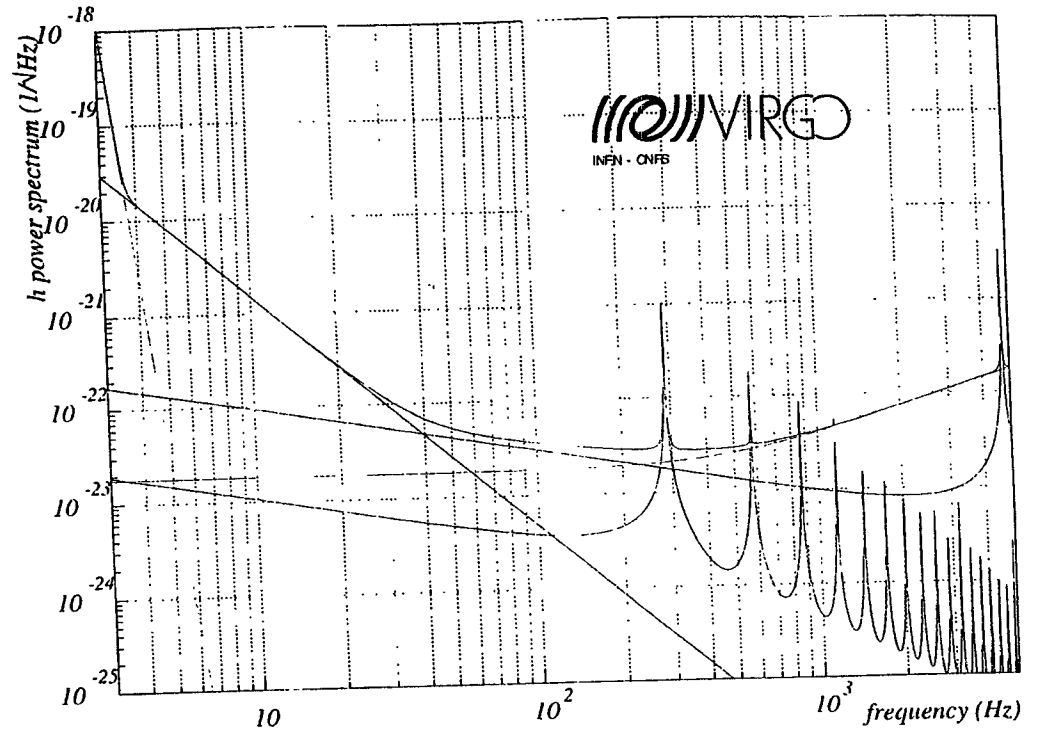
Treat Multiple Instruments as Single Detector!

# Data Analysis Preparation for

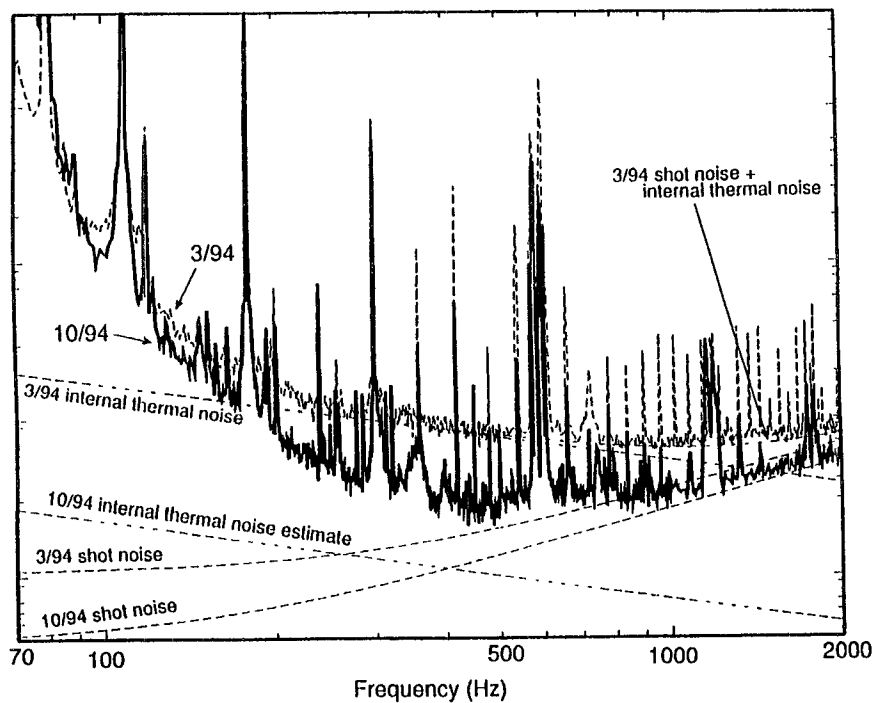


(Annecy, Frascati, Lyon, Napoli, Orsay,  
ESPCI-Paris, Pisa, Perugia, Roma)

- The Experiment and the constraints
- The Online System
- The Calibration
- The Simulation
- Pulsar search



## *Real life*



LIGO Technical Report T94-7

## *Constraints*

Main signal sampling: up to 20 kHz

⇒ lot of data

Need to record additional monitoring signals

⇒ more data

Raw data rate: 1-5 Mbytes/sec

Online system:

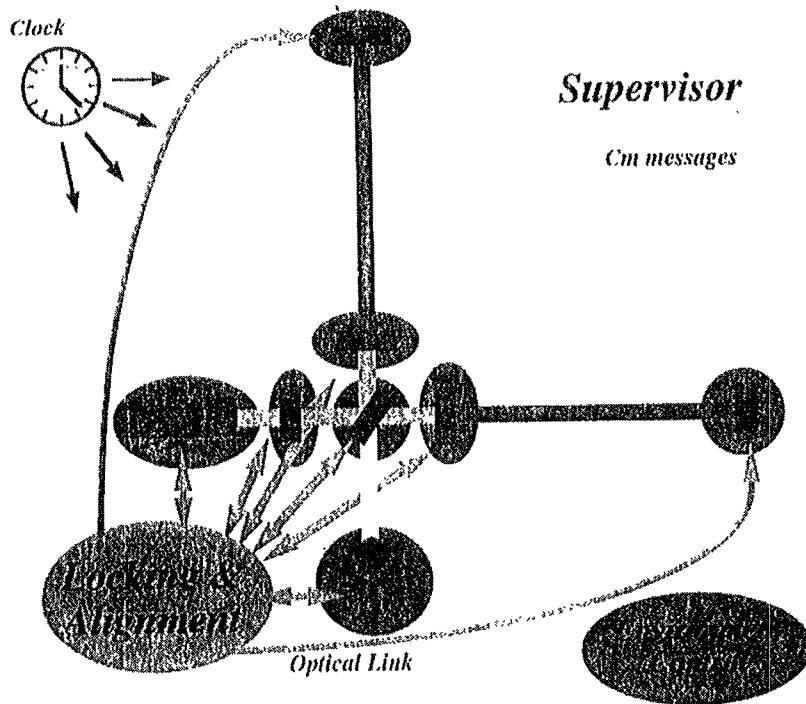
- Robust
- Flexible
- Data reduction : Trigger
- Data organisation : The Frame structure (online/ offline compatibility)

Data analysis will need a very good understanding/modelling of the detector noise

⇒ Calibration

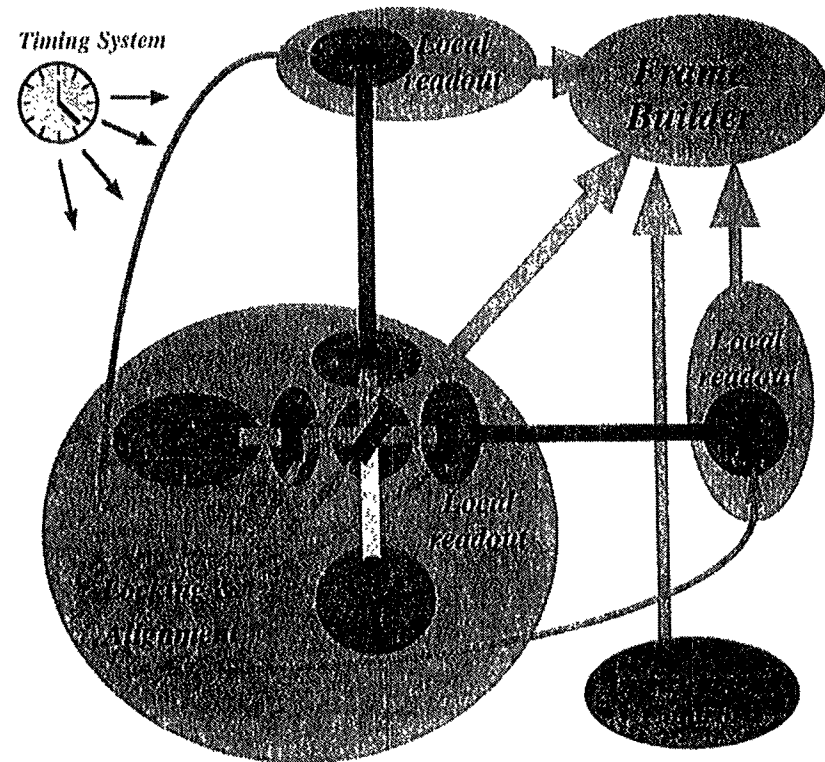
⇒ Simulation

## VIRGO Controls



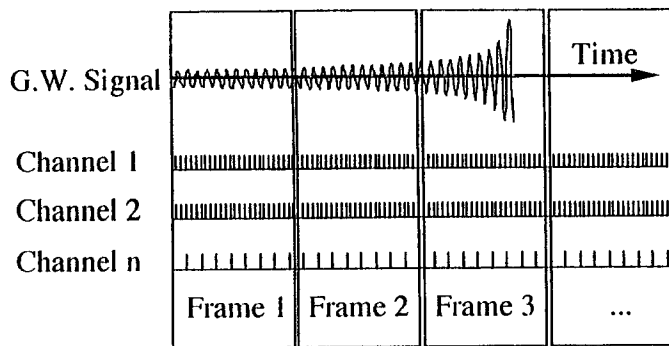
- Digital controls
- Local controls + global controls
- Digital Optical Link
- Clock
- Coordination by the Supervisor
- Slow monitoring by standalone stations

## Virgo DAQ



A Frame = a few seconds of organized data

## Data Format

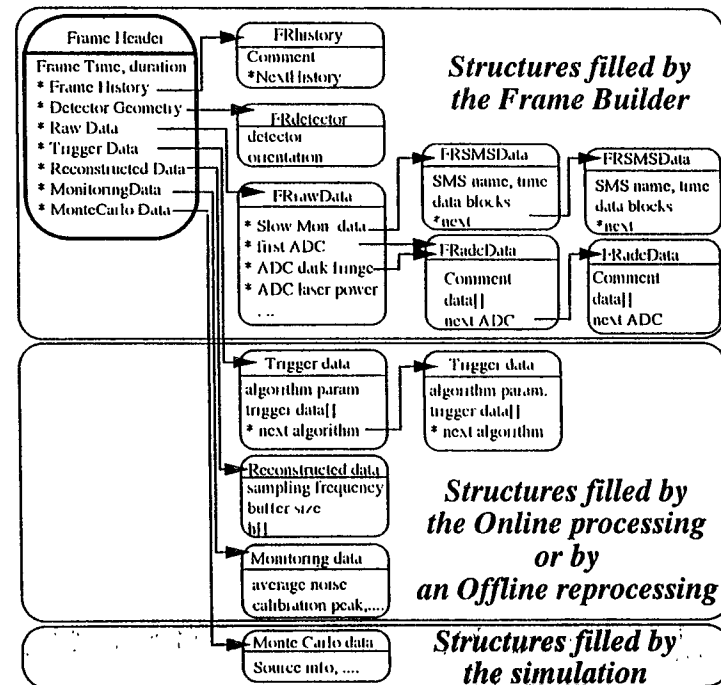


Data taking is a continuous process  
Channels are sampled at different frequencies  
⇒ Use a 'Frame' structure

A Frame is an organised data set  
of few seconds length

One G.W. event correspond to several Frames

## The Frame Structure



A Frame has a tree structure:

- Individual blocks are C structures
- Possible add/drop structure
- Used from online to offline

## *Online Processing*

---

### *Frame Builder*

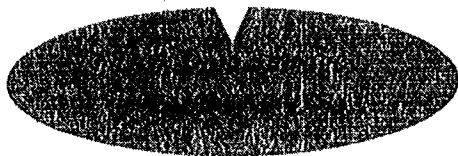
Typical data  
flow = 1-5 Mbyte/s

*Full Raw Data  
Archiving*

*Online  
Data  
Quality*

*Online Trigger*

about 20 kbytes/s  
or  
1 Mbyte/s for burst candidates



## *Online Trigger*

---

Data flow: 1-500 Gbytes/day  
⇒ Overflow any data analysis  
⇒ Need Online data reduction : trigger

Algorithms will work on reconstructed 'h'  
'h' reconstruction: convert raw data (i.e. ADC  
counts) to physical quantities (h)

Trigger algorithms will search for burst events.  
⇒ select 'time windows'

keep:

- all the raw data for selected frames
- 'h' for the others (pulsars searches).

Algorithms

- simple
- robust
- multiples
- upgrades ⇒ data reprocessing

(D. Verkindt et al. Astroparticle physics 2 (1994) 235-248)

## *Calibration*

---

### Goal:

- Convert the ADC values to  $h$
- Study of the Virgo sensitivity
  - ⇒ temporary signals
- Permanent monitoring of the interferometer
  - check the interferometer
  - check the data acquisition system
  - ⇒ few permanent sine waves

### How:

Move a mirror by a well known amplitude

Need to know:

- the force applied on the mirror,
- the mechanical transfer function

### Forces:

- Magnets (locking actuators)
- Radiation pressure from an additional laser

The calibration signal will be provided by an external and independent signal generator.

## *Simulation*

---

### Goals:

- Design of the detector
- Detector commissioning
- Data analysis

Tool: SIESTA

(Simulation of Interferometric Experiment Sensitive To gravitational waves)

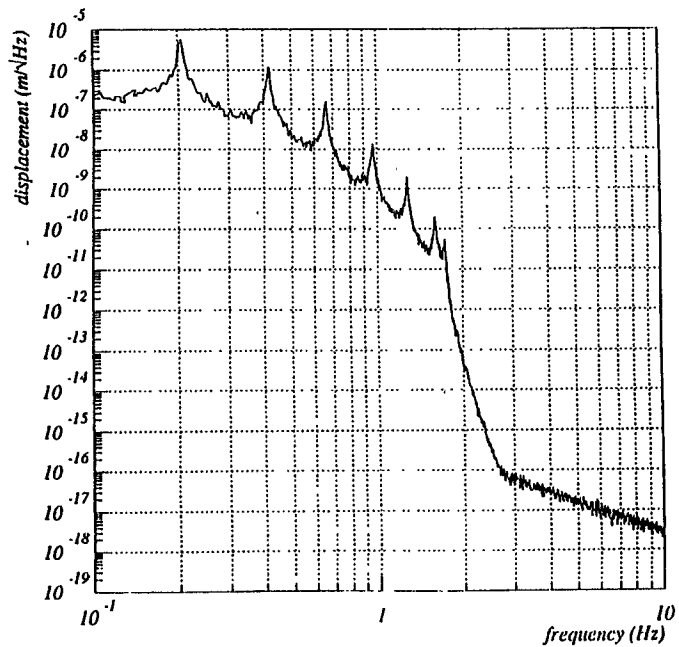
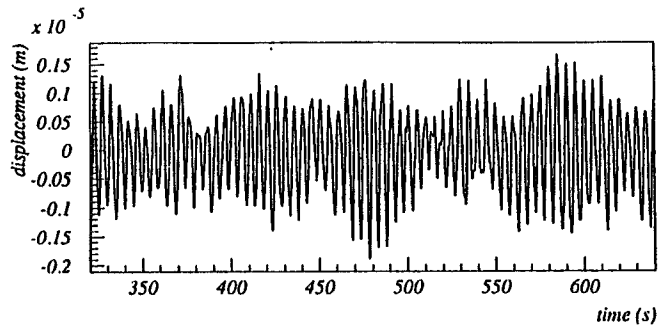
Integrated, general purpose program:

- Time dependent simulation
- Metric, Mechanic, Optic, Electronic
- Various levels of accuracy

Object Oriented Programming: Modularity

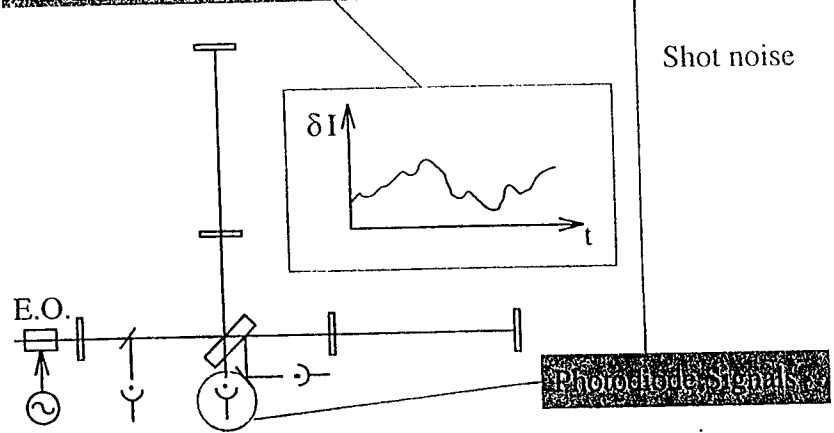
# Mechanical simulation

Ex. simulation of mirror motion due to seismic and thermal noise  
(in time domain)

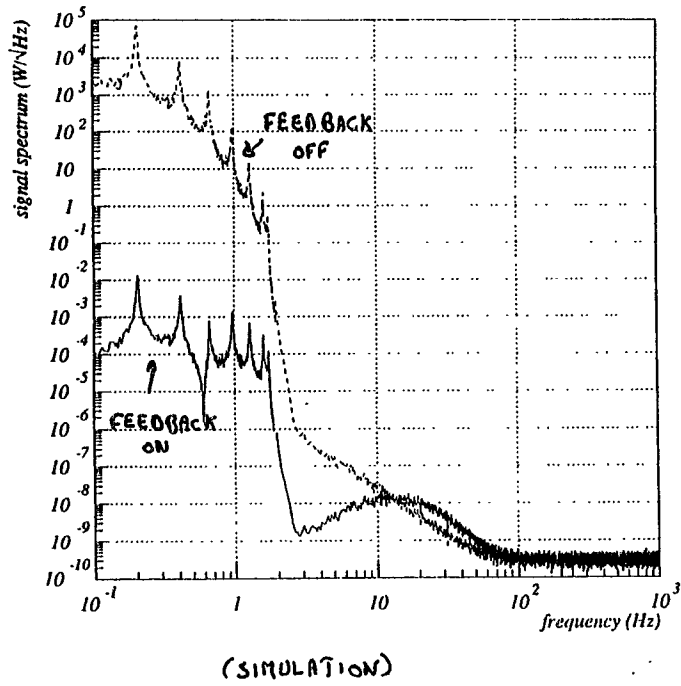


# Optical response

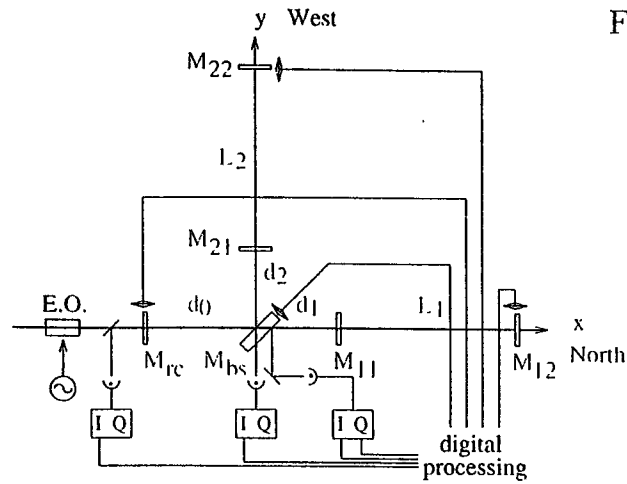
• Analytical simulation: wavefront propagation, diffraction, and waveguide detectors.  
 • Modal expansion of beam fields, waveguide and cavity modes.  
 • Fast simulation: matrix relationships between detector signals and mirror translations, in the linear regime.







## Feedback Simulation



Feedback system:

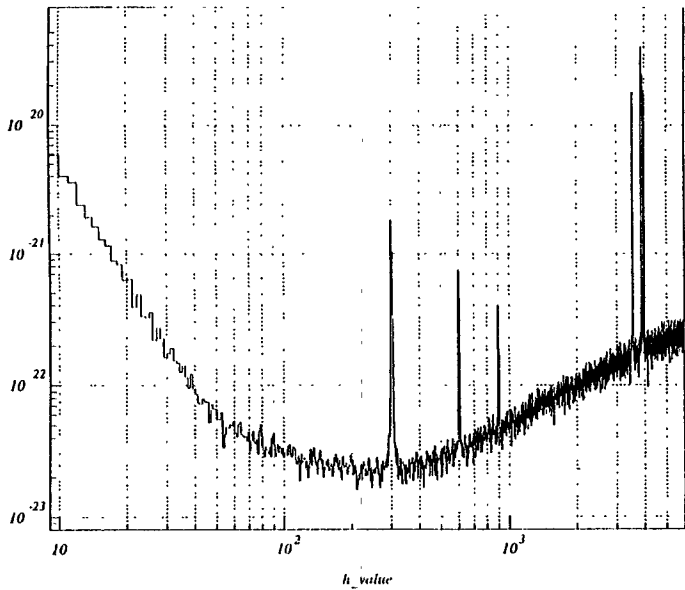
- high gain ( $\sim 10^5, 10^6$ ) at low frequency ( $< \sim 1 Hz$ )
- no noise introduced in the VIRGO frequency range ( $10 Hz - 1 kHz$ )

# Fast Data Simulation

(Example)

This includes:

- Thermal noise (4 mirrors)
- Shot noise



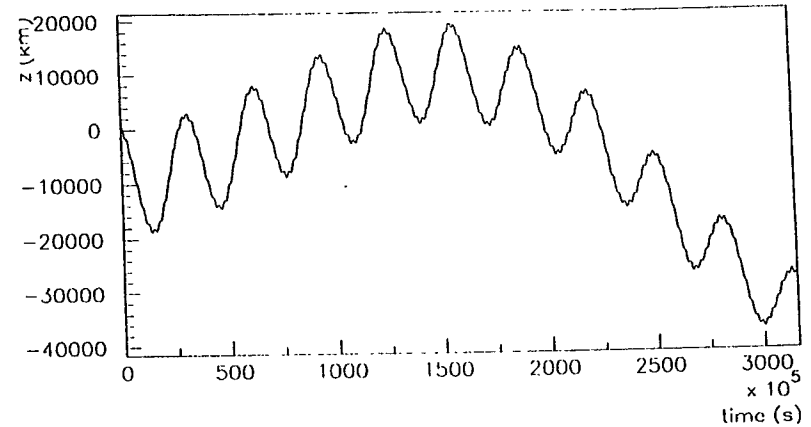
(1 sec of Data @ 20KHz= 6 sec real time)

# Pulsar search

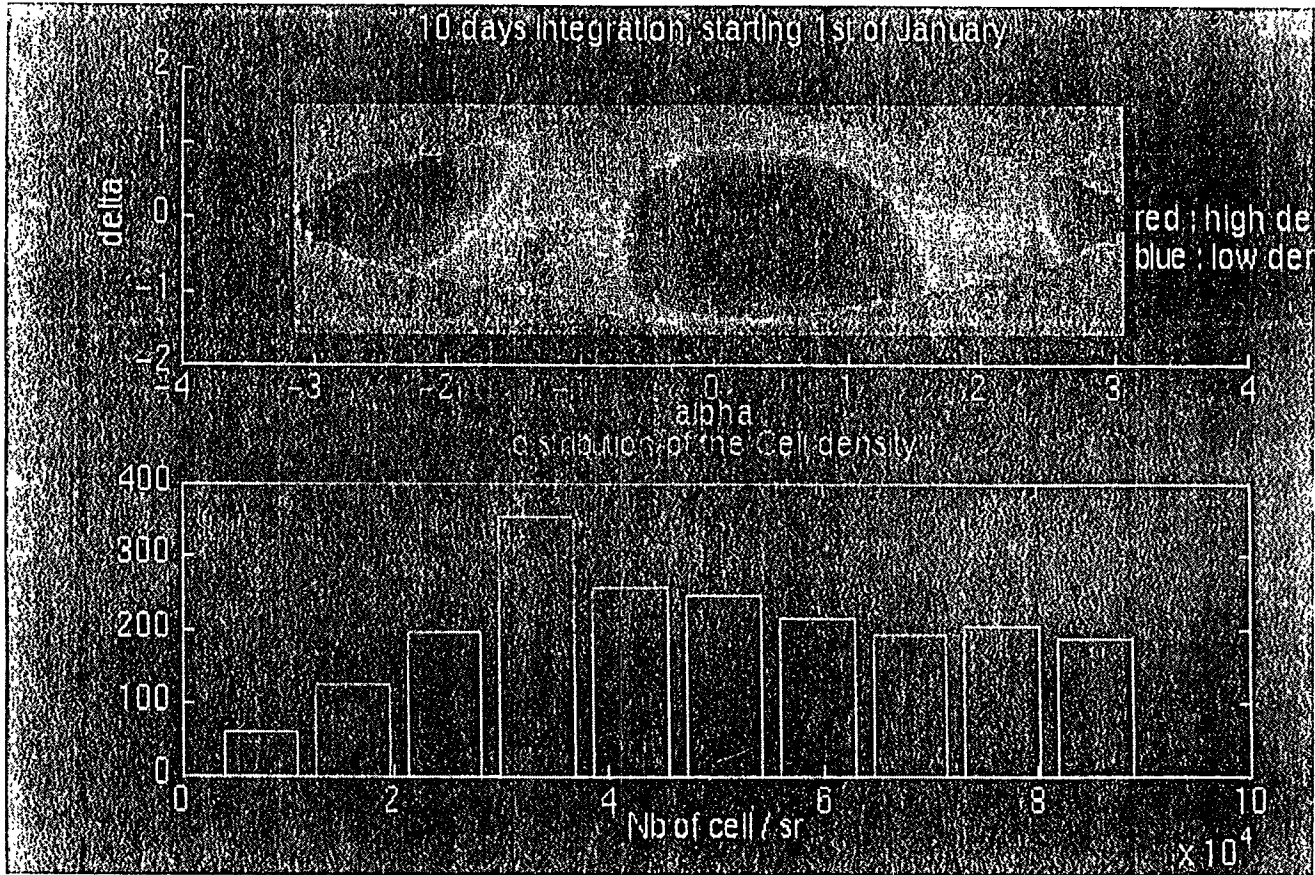
Doppler effect

⇒ Need earth position  
precision: 15km(1kHz/frequency)  
(5% losses)

⇒ Need some care



Earth position out of the ecliptic plan (10 years)

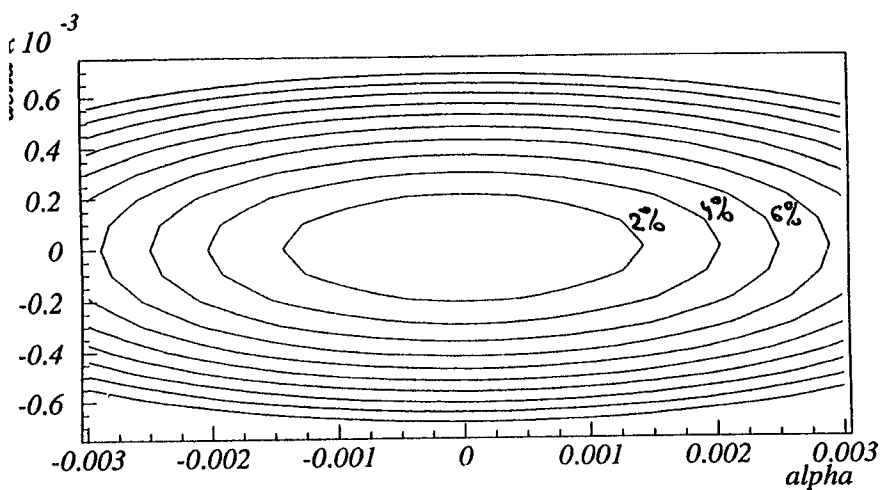


# Pulsar search

Doppler effect

⇒ Need source direction

Example:



- 10 days of integration
- contour lines = 2% losses

# # of Search Directions

