AGENDA FOR THE LIGO INTEGRATION MEETING

9 February 1995

Session	Time	Presenter
Facilities	**0900 - 0930**	
• Ground motion measurements at the sites		Lisa Sievers
• Foundation motions and alignment		Mike Gamble
Beam Tube	**0930 - 1030**	
• QT Status & Baffle design issues		Larry Jones
• Tube motion analysis		Mike Gamble
• Beam tube scattering measurements & baffle material options		Rai Weiss
• Synopsis of the Baffle Review Meeting		Albert Lazzarini
BREAK	**1030 -1045**	
Vacuum Equipment	**1045 - 1115**	
• Deferral of getter pump procurement for initial interferometer		Mike Zucker
• Procurement status & update		John Worden
Detector	**1130 - 1245**	
• Length control modeling		Lisa Sievers
		Jordan Camp
		Dave Redding (JPL)
• IFO configuration definition		Yaron Hefetz

Optical Modeling for Length Control

February 9, 1995

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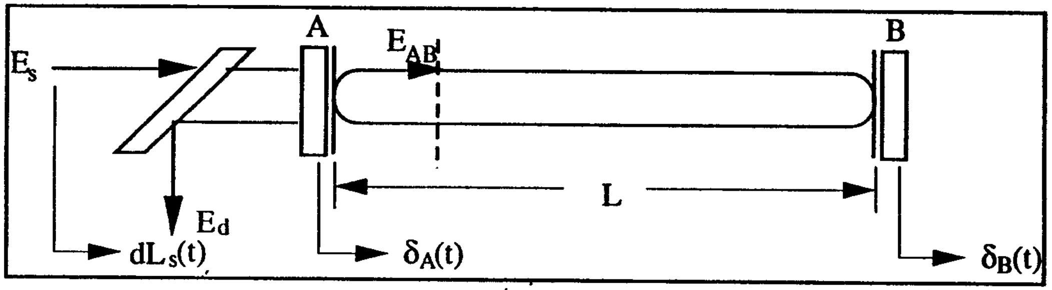
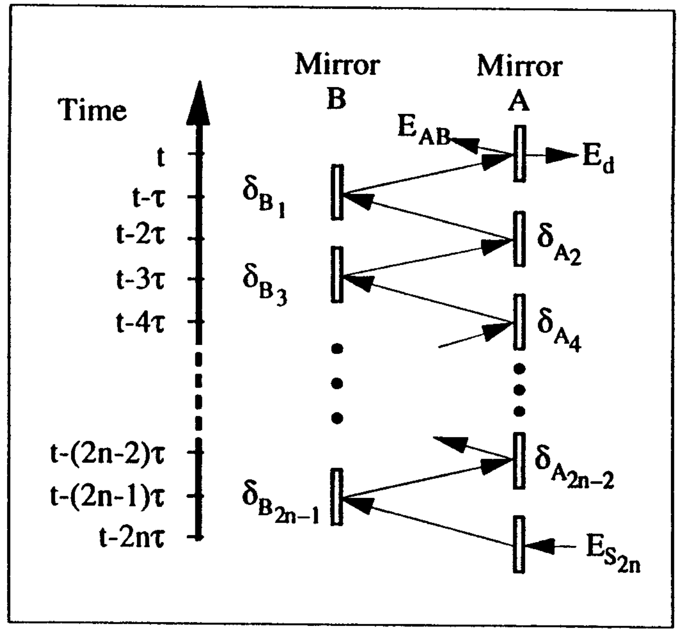
Single-Cavity Simulation



- **"Green's function" model sums impulse response of cavity times source for all past times**
  - Impulse response depends on position of mirrors at all intervening times
  - Three frequencies (carrier and 2 sidebands)



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Single-Cavity Simulation (cont.)

Circulating field:

$$E_{AB} = t_A \left[1 + \sum_{n=1}^{n_{beams}} \left[\prod_{m=1}^{n} g_m \right] \right] E_s$$

Round-trip gain:

$$g_m = r_A \, r_B \, e^{-jk\left(dL_{s2m} + 2\delta_{B(2m-1)} - 2\delta_{A2m}\right)}$$

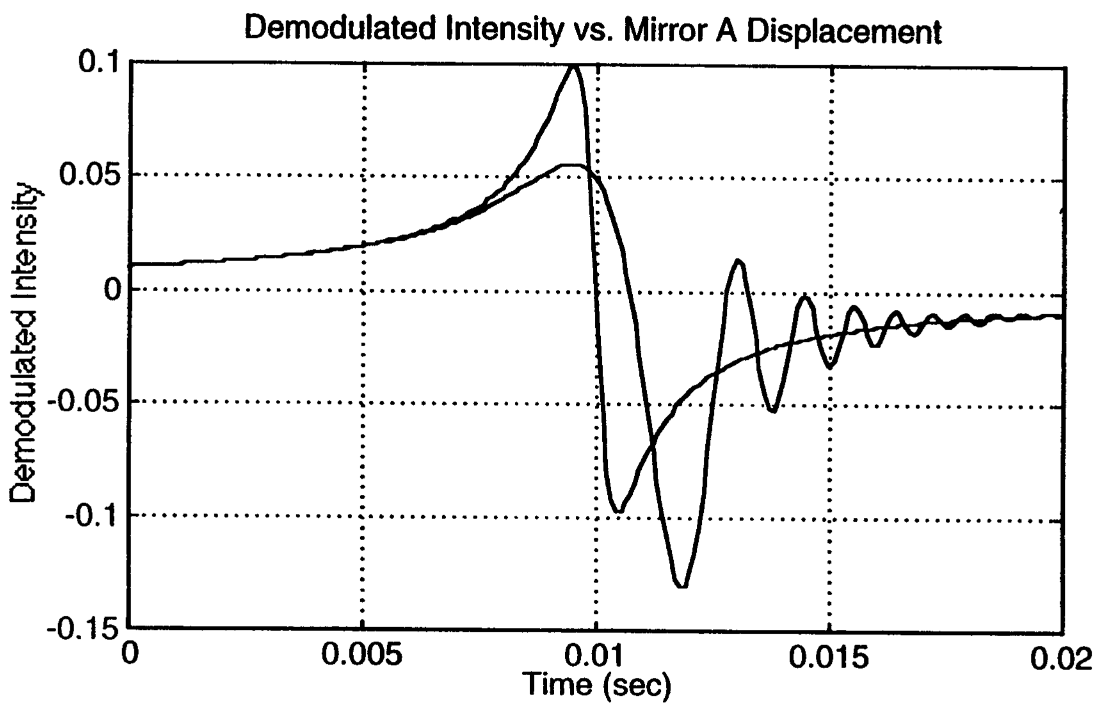
Arrange into recursive form:

$$\begin{aligned} E_{AB} &= t_A \left[1 + g_1 + g_1 g_2 + \cdots + g_1 g_2 \cdots g_{n_{beams}} \right] E_s \\ &= t_A \left[1 + g_1 \left[1 + g_2 \left[\cdots \left[1 + g_{n_{beams}} \right] \cdots \right] \right] \right] E_s \end{aligned}$$

Update equation is carried forward in time together with dynamics and control equations:

$$(E_{AB})_i = t_A (E_s)_i + g_i (E_{AB})_{i-2}$$

Typical Response



Time-Domain Simulation

- Recursive form allows faster evaluation for LIGO cavities (high finesse, long length, high BW control)
  - Time step is a sub-multiple of the one-way light time
- Formulation assumes that light time is constant
  - Error if mirrors are far from nominal position and moving rapidly:

$$\frac{\delta\Delta L_{eff}}{fringe\ width} < \frac{8\ finesse^2\ v\ dL}{\pi\lambda c}$$

  - Worst case (40 m cavity) error ratio < 1e-6 for acquisition control, much less near resonance
  - No qualitative change in system response
- Code written in Fortranfor execution speed, linked to Matlab control design environment for ease of use
- Approach generalizes straight-forwardly to coupled cavities, recombined cavities, recycled interferometer

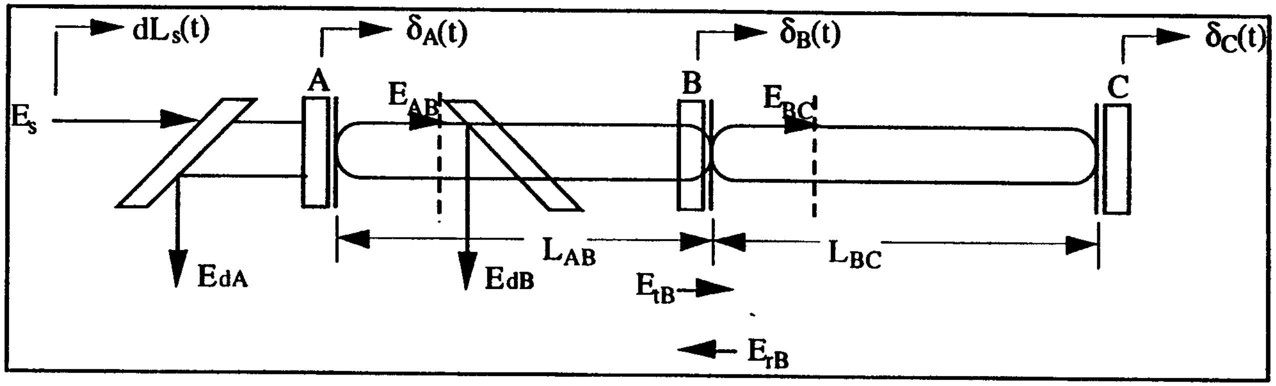
Frequency Response

- **Frequency response obtained by Laplace transforming time-domain equations**

$$\frac{I_d}{\delta_A}(s) = 8kE_cE_{sb}\left[-t_A^2\frac{2r_Ar_B}{1-r_A^2r_B^2} - t_A^2\sum_{i=1}^{\infty}\left(\frac{(r_Ar_B)^{i+1}}{1-r_Ar_B} - \frac{(-r_Ar_B)^i}{1+r_Ar_B}\right)e^{-2i\tau s}\right.$$
$$\left. - \frac{t_A^4}{r_A^2}\frac{r_Ar_B}{1-r_A^2r_B^2}\left[\sum_{i=1}^{\infty}\left((r_Ar_B)^{i+1} + (-r_Ar_B)^{i+1}\right)e^{-2i\tau s}\right]\right]$$

- **Agree with M. Regher's results for single cavity**
- **Rational form developed by L. Sievers**

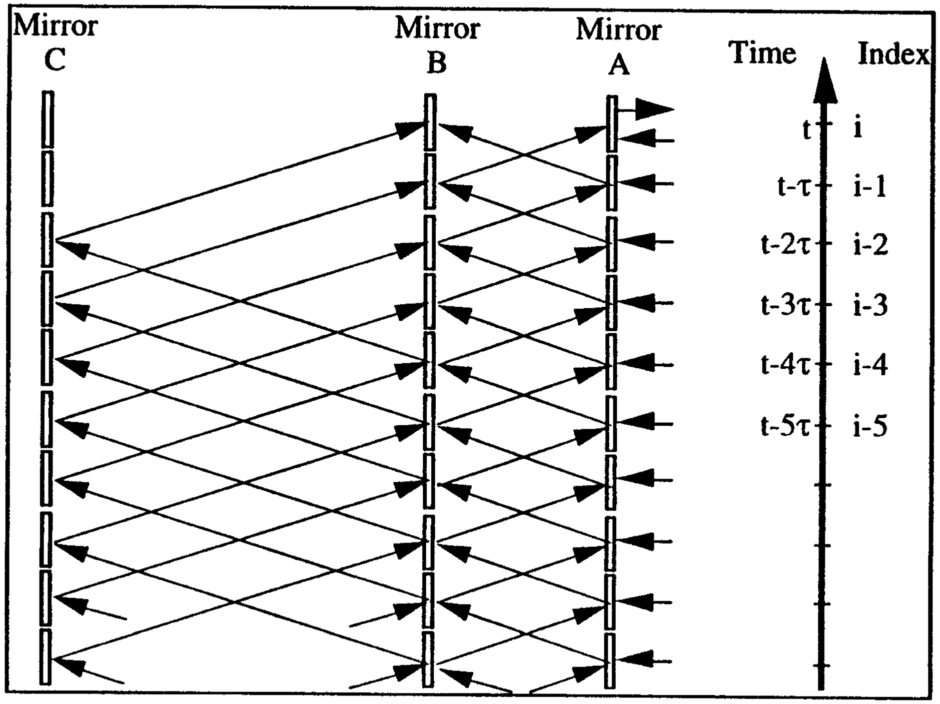
Coupled Cavity Simulation



l (mm)	LAB (m)	LBC (m)	RA	RB	RC	fmod (mHz)
0.5145	6.000000216	3990.0001436	0.9499	0.9699	0.9998	12.4875

- **Coupled cavity interferometer presents programming challenges similar to those expected for recycled interferometer**

Coupled Cavity Simulation (cont.)



- Green's function approach would require summing large numbers of beams
- Recursive form avoids this problem
  - Accumulate fields in both cavities as a running product
  - Time step a submultiple of one-way light time in short cavity

Coupled Cavity Simulation (cont.)

Update equations:

Circulating field in front cavity:

$$(E_{AB})_i = t_A (E_s)_i + g_{ABi} (E_{AB})_{i-2} + h_i (E_{BC})_{i-1}$$

$$g_{ABi} = -t_{BS}^2 r_A r_B e^{-jk2(\delta_{B(i-1)} - \delta_{Ai})}$$

Circulating field in back cavity:

$$(E_{BC})_i = t_{BS} t_B (E_{AB})_{i-1} + g_{BCi} (E_{BC})_{i-2n}$$

$$g_{BCi} = r_B r_C e^{-jk2(\delta_{C(i-2n)} - \delta_{Bi})}$$

Field at detectors:

$$E_{dAi} = r_A e^{-jk2\delta_{Ai}} E_{si} + t_{BS}^2 t_A r_B e^{-jk2\delta_{B(i-1)}} E_{AB(i-2)} + t_{BS}^2 t_A t_B r_C \frac{t_B}{r_B} e^{-jk2\delta_{C(i-1-n)}} E_{BC(i-1-2n)}$$

$$E_{dBi} = r_{BS} E_{ABi}$$

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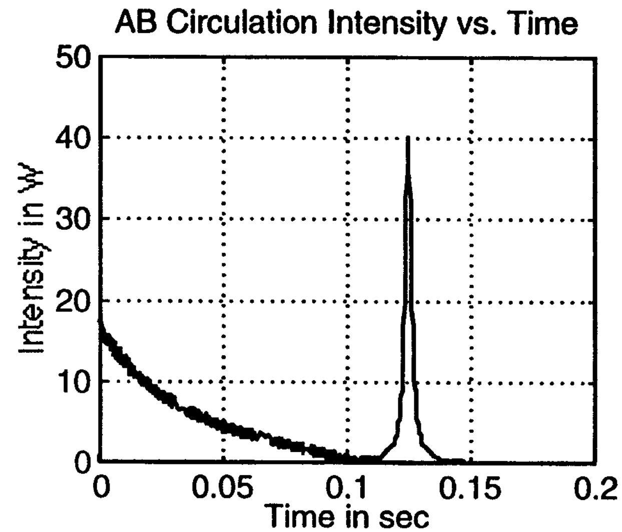
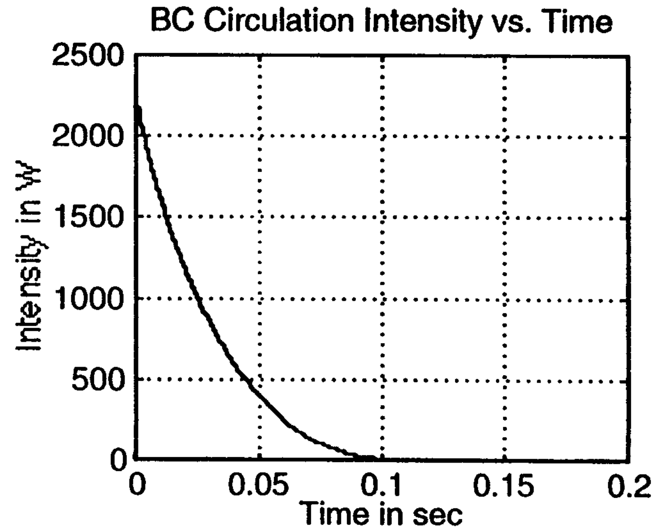
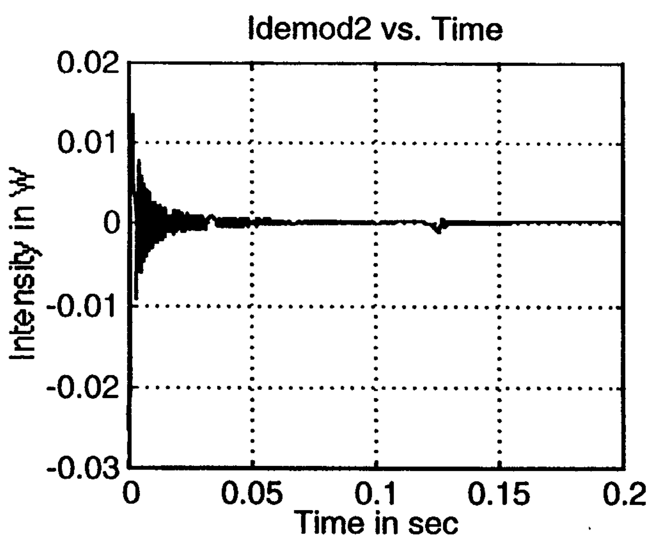
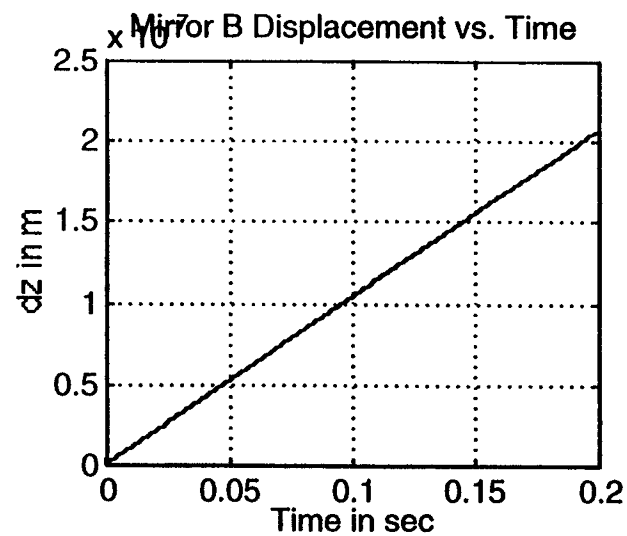
Typical Response









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Continuing Work

- Coupled cavity
  - Frequency response validation
- Recombined cavity
  - Programming issues were resolved in single-cavity work
  - Provides a validation point
- Recycled interferometer cavity
  - Programming issues were resolved in coupled-cavity work
  - Predicts length control performance of full-up LIGO