

Robbie,
this is the final
draft of my
transparencies.
David has
approved it.

-Mandi- 930603

INTERFEROMETER TOPOLOGY AND CONTROL SYSTEMS

**CANDIDATE LIGO INTERFEROMETER
OPTICAL AND SERVO TOPOLOGY:
ASYMMETRY MODULATION**

**M. Regehr
June 8, 1993**

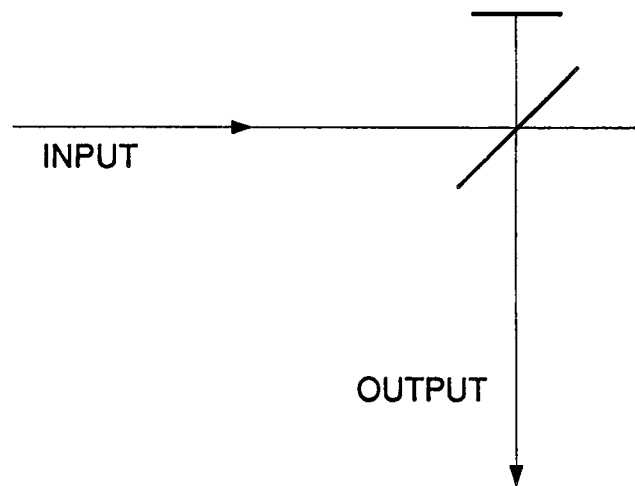
“Asymmetry” Scheme for Gravitational Wave Signal Extraction and Auxiliary Sensing

- **Method of extracting gravitational wave signal and auxiliary signals for control of mirror positions**
- **Status; Work remaining to qualify scheme for LIGO**
- **Caltech tabletop prototype**

“Asymmetry” Scheme for Gravitational Wave Signal Extraction

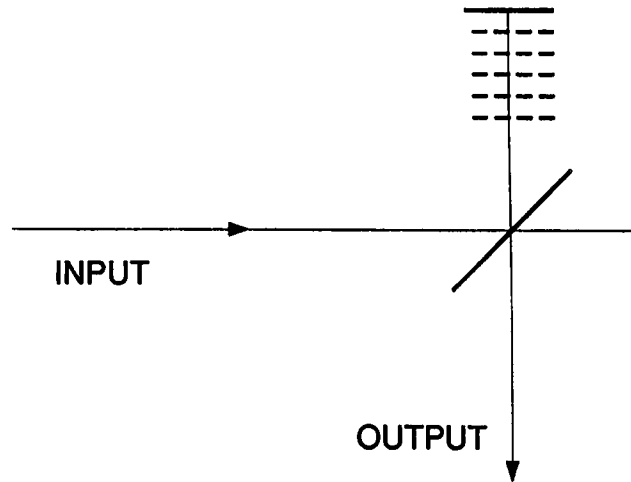
(Schnupp, '86 or '87)

Symmetric Michelson interferometer:



- Output dark for all wavelengths if dark for any.

Introduce asymmetry by shifting one mirror away from beamsplitter by a (large) integral number of half-wavelengths of green light:



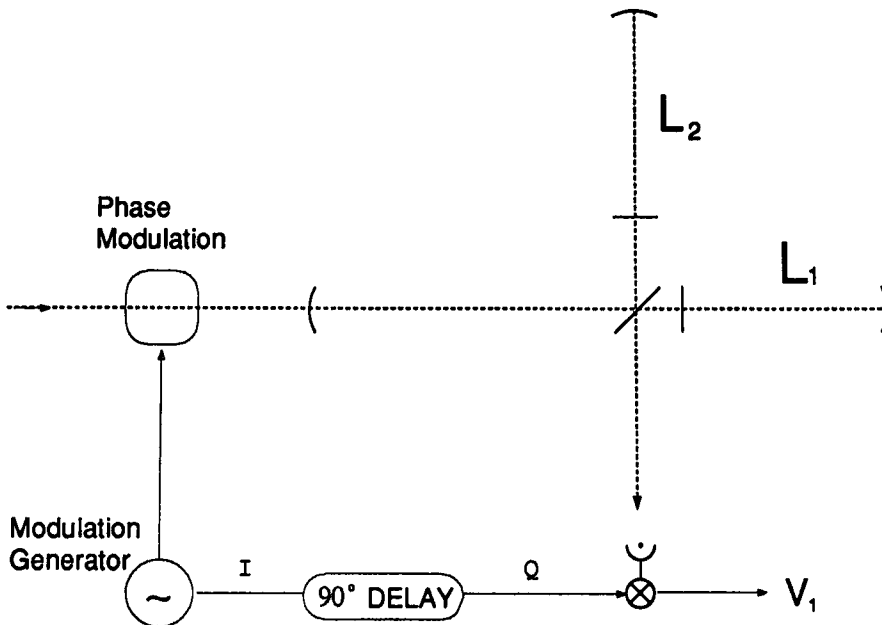
Now output is dark for green, not dark for other wavelengths.

In particular: Modulate phase of input light at radio frequency

- **Modulation imposes sidebands on incident light.**
- **Effect of gravitational wave is slow motion of mirrors. Carrier (“green”) light at output beats against sidebands, producing amplitude modulation.**
- **Gravitational wave is detected by demodulation.**

Full interferometer exactly analogous.

- **High sideband transmission (from laser to output) possible because sidebands made to resonate in recycling cavity.**



Signal at output:

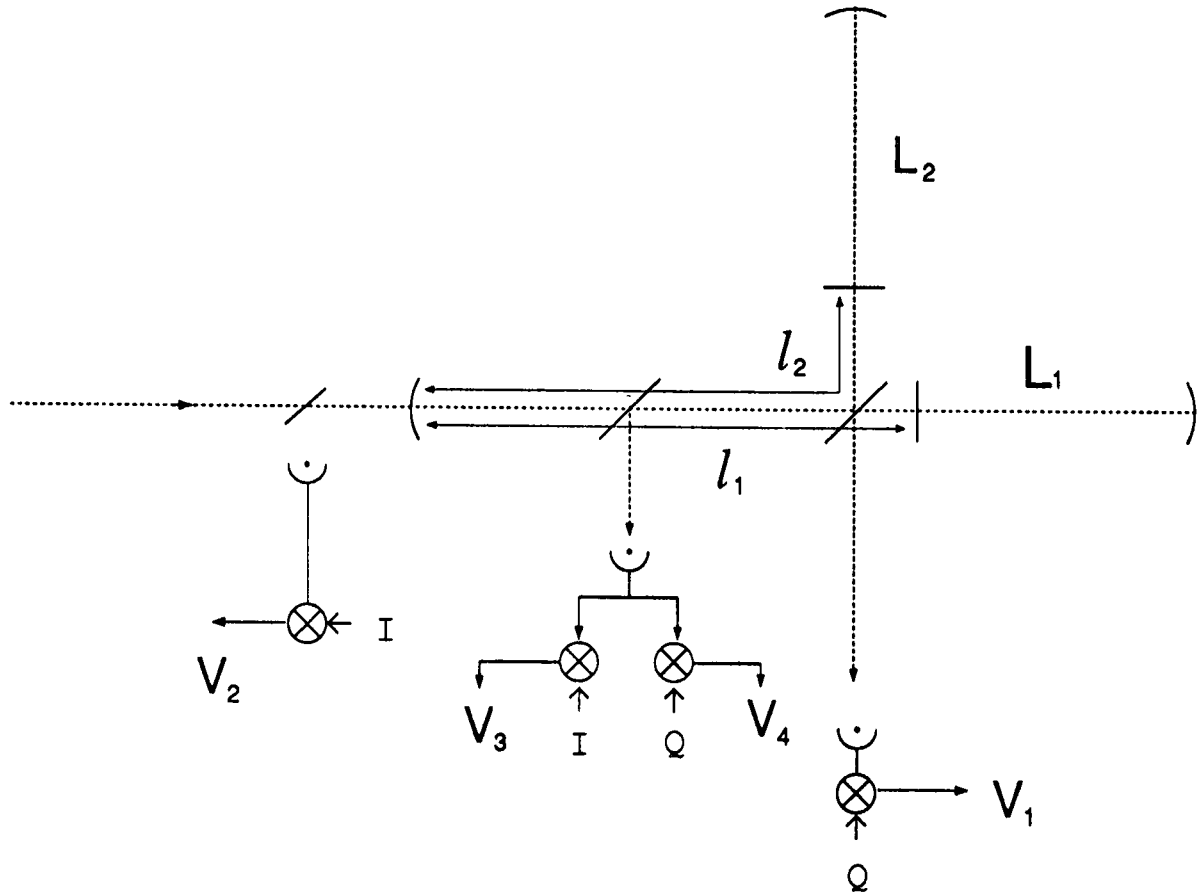
$$V_1 \propto L_1 - L_2$$

\propto Gravitational Wave Signal

- **Planned asymmetry: 60 cm**
- **Shot noise level insensitive to details of modulation scheme**
- **No known mechanisms by which asymmetry would degrade noise performance measurably**

“Asymmetry” Scheme for Auxiliary Sensing

Signals corresponding to remaining three degrees of freedom needing to be controlled also available:



$$V_2 \propto \delta L_1 + \delta L_2 + \varepsilon_2(\delta l_1 + \delta l_2)$$

$$\varepsilon_2 \simeq 0.006$$

$$V_3 \propto \delta L_1 + \delta L_2 + \varepsilon_3(\delta l_1 + \delta l_2)$$

$$\varepsilon_3 \simeq 0.002$$

$$V_4 \propto \varepsilon_4(\delta l_1 - \delta l_2)$$

$$\varepsilon_4 \simeq 0.002$$

Challenges:

- “Ill conditioned” system: V_2, V_3 (and, if demodulator phase not quite right, V_4) all more sensitive to $L_1 + L_2$ than to other two degrees of freedom.

Solution: separate signals by taking linear combinations, or use high gain in one loop

- When not near resonant state, gain in each loop depends on other degrees of freedom: difficult to understand acquisition (and to diagnose problems if not working)

May require use of alternate auxiliary sensing scheme for lock acquisition

Benefits:

- Extreme optical simplicity

Progress:

- **Low frequency analysis and numerical model completed**
- **Frequency response analysis nearly complete; Numerical model being debugged**
- **Prototype assembled and functioning**

The Table-Top Prototype

Differences from LIGO:

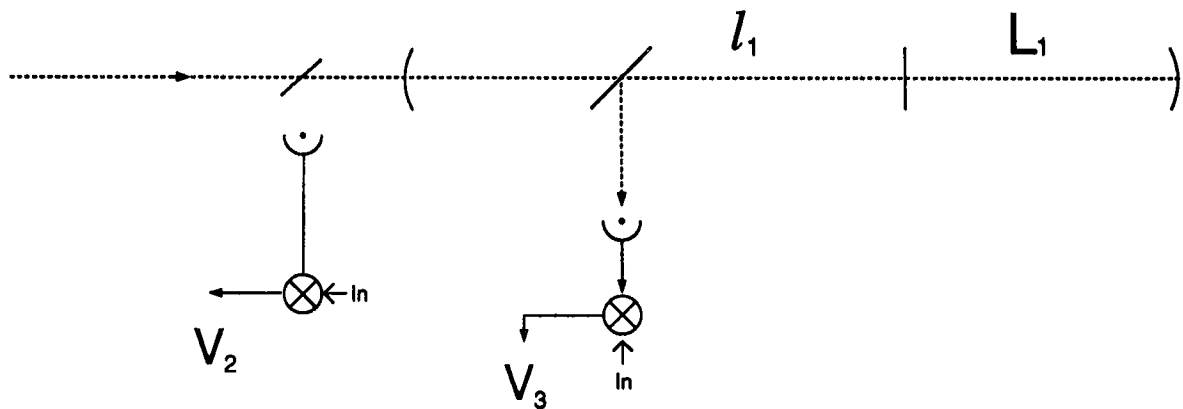
- **Lengths: arm cavities 6 m instead of 4000 m**
- **Characteristic frequencies 20 kHz, 200 kHz instead of 5 Hz, 90 Hz**
- **Spectra of disturbances ~ 500 Hz (acoustic) instead of ~ 5 Hz (seismic/stack)**

Similarities:

- **Mirror transmissions: 20% recycling mirror, 10% cavity input mirrors (3%, 3% respectively in LIGO)**
- **Modulation frequency: 12 MHz**

Results

- **Model verification for the 'coupled cavity' subsystem**



- **Lock acquisition for full interferometer**
- **Low frequency model verification in progress**

Work remaining:

- **Complete experimental verification of low frequency model**
- **Verify frequency response analysis and numerical model against each other, and possibly against prototype**
- **Design a controller for LIGO and verify performance by modelling**

[0?] Introduction: statement (in about two sentences) summarizing what is about to be presented

- method of extracting gravitational wave signal and auxiliary signals
- work done; work remaining to qualify scheme for ligo
- Caltech tabletop prototype

Gravitational Wave Signal Extraction

[2] mechanism - one transparency (??); use sideband (preferred) or common mode/differential phase modulation explanation

- mention fact that sidebands resonate in recycling cavity (hard to explain in common mode/differential phase modulation framework)

[1]- size of asymmetry planned - shot noise identical to within a few percent for all modulation techniques considered - no known mechanisms by which asymmetry could degrade noise performance measurably - advantage: extreme optical simplicity

Auxiliary Signal Extraction

[1] - schematic showing where light extracted and demodulated - statement of dependence of signals on lengths in interferometer

[1]- challenges:

- "ill conditioned" system: V2, V3 (and, if demod phase not quite right, V4) all more sensitive to L1+L2 than to other two degrees of freedom.

Solve by decoding or high gain in one loop

- when not near resonant state, gain in each loop depends on other degrees of freedom: difficult to understand acquisition (and to diagnose problems if not working)

May require use of alternate auxiliary sensing scheme for lock acquisition

[1] Strategy of verifying adequacy for ligo

- low frequency analysis and numerical model
- frequency response analyzed; numerical model being debugged
- prototype

[1] The Experiment

- differences from ligo

lengths, characteristic frequencies

spectra of disturbances, bandwidth limitations in actuators

- similarities:

mirror reflectivities (list values)

modulation frequency

[1] - results:

model verification (including servo model and proposed method of dealing with ill conditioning) for coupled cavity experiment (show diagram)

[1] lock acquisition for full interferometer (show photograph)

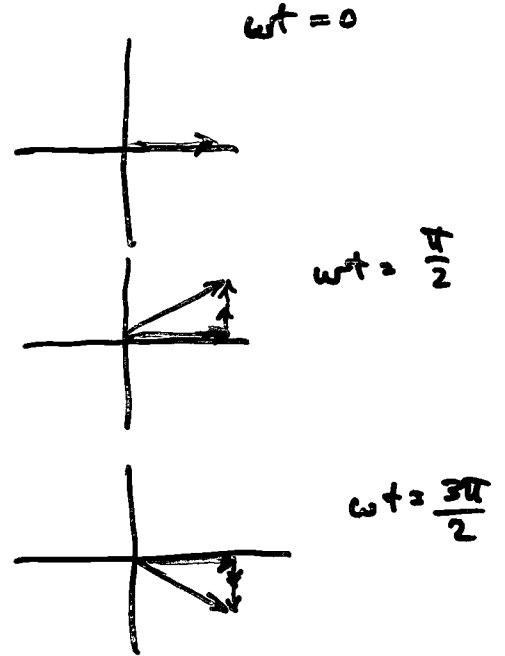
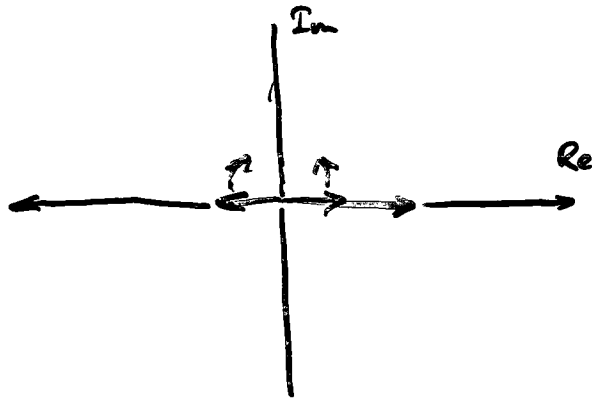
low frequency model verification in progress; would like to test frequency response as well

[1]Work Remaining

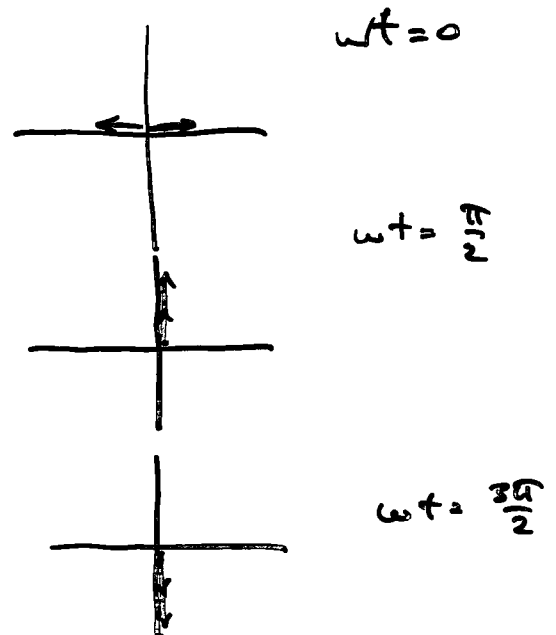
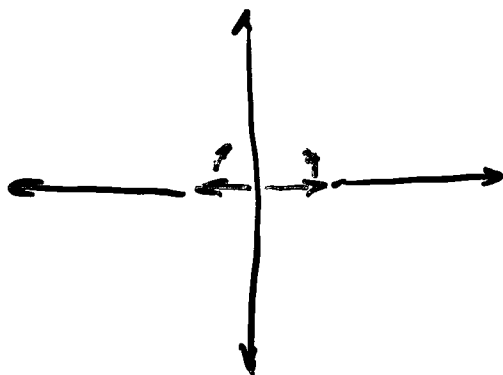
verify frequency response analysis and computer model against each other

design a controller for ligo and verify performance by modelling

Represent with a phasor diagram in which carrier is stationary and sidebands rotate:

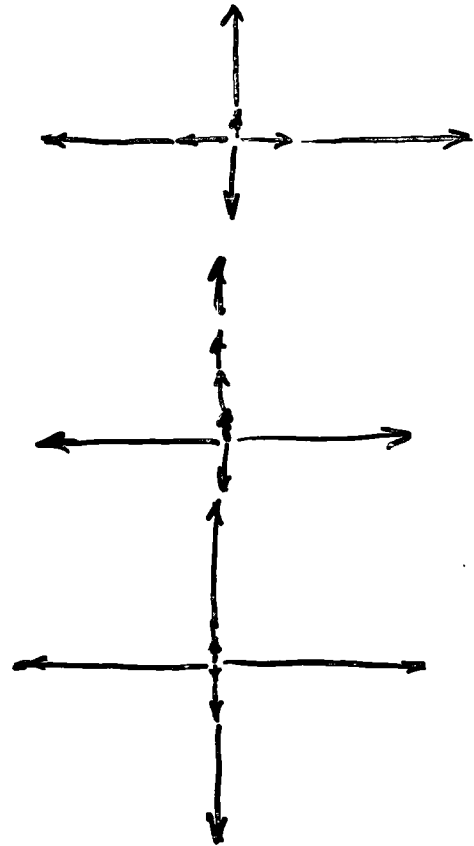
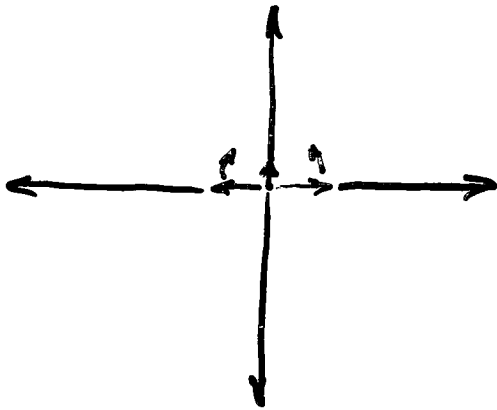


In absence of gravitational wave, only sidebands at output (sideband transmission maximum for quite large $(\lambda_m/4)$ asymmetry):



- Intensity of output light has only DC and 2ω frequency components.

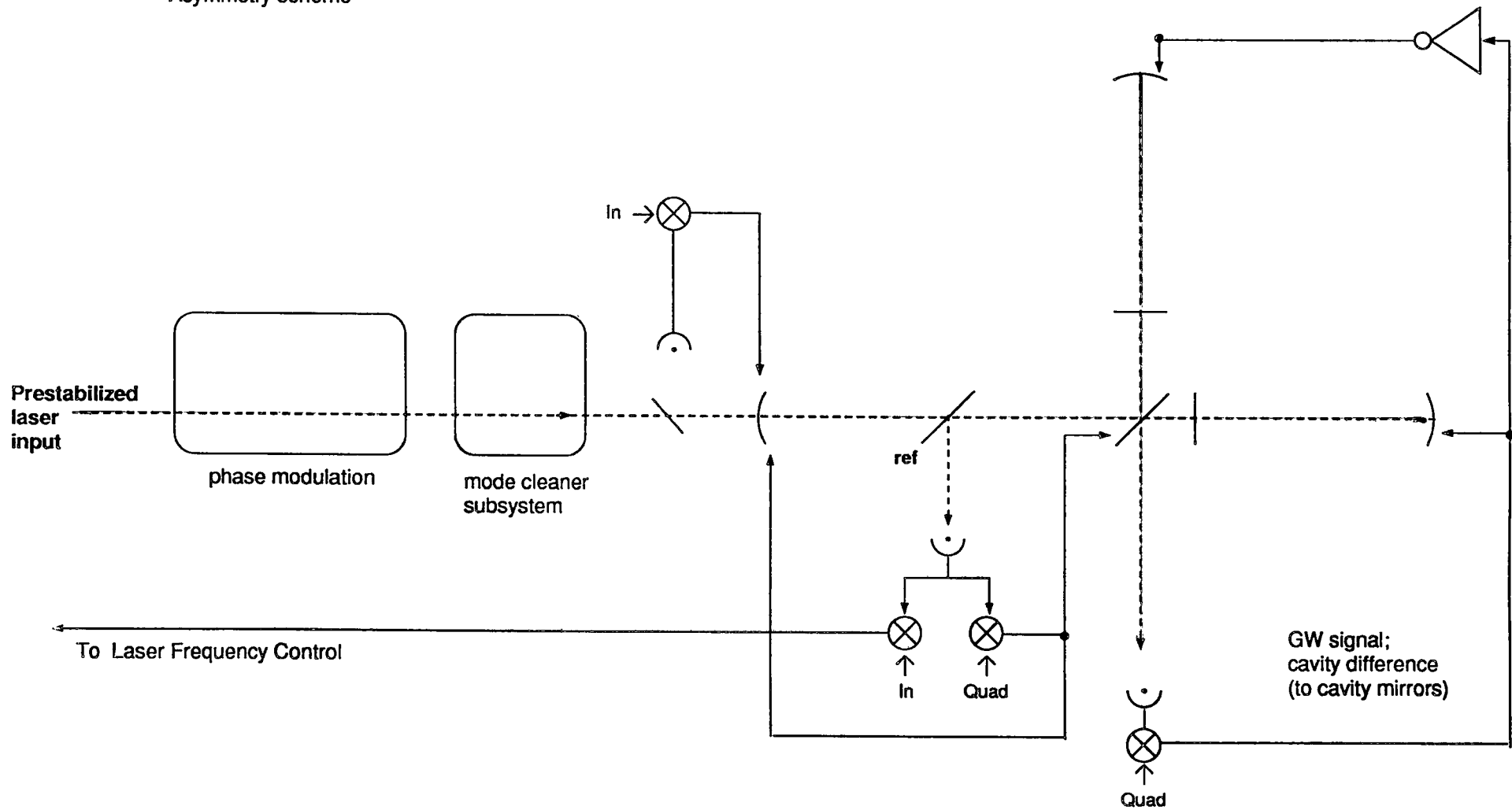
In presence of gravitational wave, carrier plus sidebands:



Output intensity contains component at frequency ω , proportional to gravitational wave amplitude.

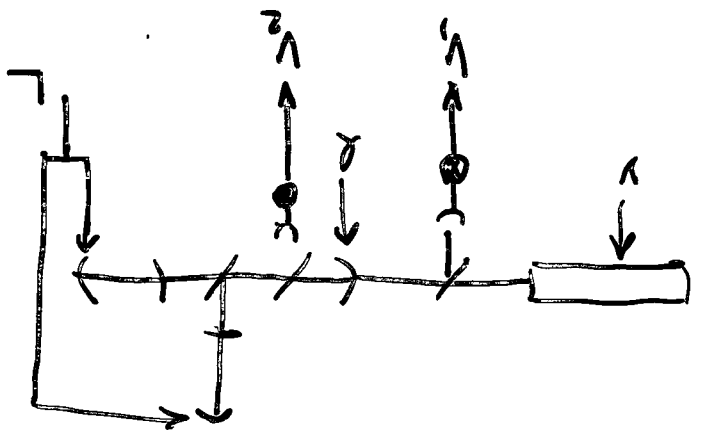
Detectable by demodulating at frequency ω .

Asymmetry scheme



Numerical Example

Interformer

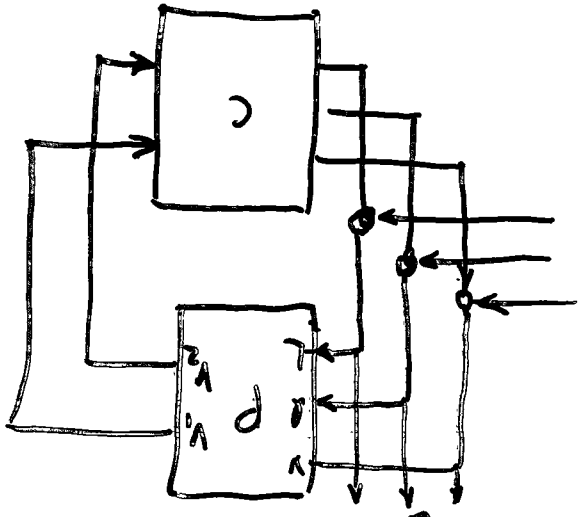


$$P \begin{bmatrix} L \\ V \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

"Plant" P

$$C \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L \\ V \end{bmatrix}$$

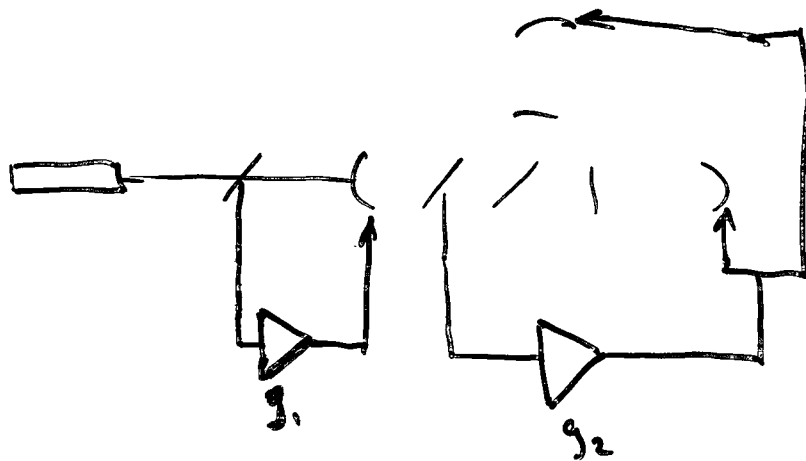
Feed back with controller C



$$e = [I + CP]^{-1} d$$

$$P = \begin{bmatrix} -589 + \frac{400}{s} & 1 \\ -72 + \frac{400}{s} & 1 \end{bmatrix}$$

Case 1: Feedback to masses without "inversion"



$$C = \begin{bmatrix} 0 & 0 \\ g_1 & 0 \\ 0 & g_2 \end{bmatrix}$$

Set g_1, g_2 s.t. gain through driven masses ≈ 100 (ie. $[1+CP]_{22}^{-1} = [1+CP]_{33}^{-1} = \frac{1}{101}$)

measure of performance: weighted disturbance suppression

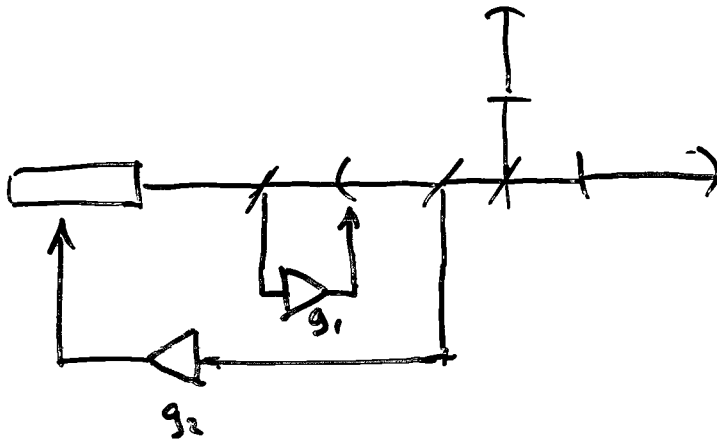
$$W [1+CP]^{-1}$$

$$\text{take } W = \begin{bmatrix} -\frac{1}{400} & 1 & 0 \\ -150 & 0 & 150 \end{bmatrix}$$

in this case, max element of $W [1+CP]^{-1}$

is 1.4

Case 2: High gain feedback to laser:



Same constraint on gain through driven mass,
but allow high gain in feedback to laser

$$[1 + CP]_{11}^{-1} \approx \frac{1}{16000}$$

Now max element of $W [1 + CP]^{-1}$

is 0.01.