Parametric Instability

Comparison between
Mesa beams and Gaussian beams
And
Modal misalignment suppression for
Advanced LIGO

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Comparison between Mesa beams and Gaussian beams

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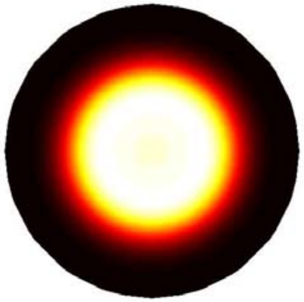
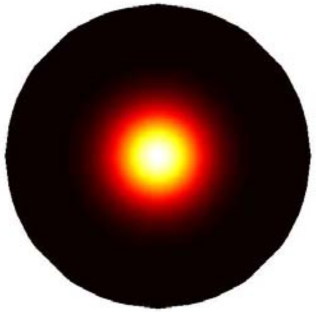
Cavity types

Mesa beam



Gaussian Beam



$$u_{00}^{MF}(D,\vec{r}) = \int_S \exp\left[\frac{-k(\vec{r}-\vec{r}\,')^2(1+i)}{2L}\right] d^2\vec{r}\,'$$

$$u_{00}^{GF}(R,\vec{r}) = \exp\left(-i\left[\frac{kL}{2}+\frac{k|\vec{r}|^2}{2R}-\arctan\left(\frac{2R-L}{L}\right)^{-1/2}\right]\right)$$

$$D = 4\sqrt{\frac{L}{k}}$$

Diffraction losses
σ = 19.5 ppm

R = 2050.6 m

Cavity length L = 4000 m T_{ITM}=2.3% T_{ETM} =5.0 ppm

Test mass: r = 0.16 m, t = 0.13 m, Sapphire M-axis, wedge 0.5deg

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Parametric Instability- Simple cavity

$$R_j = \frac{4PQ_{mj}}{McL\omega_{mj}^2}\left(\frac{Q_{1i}\Lambda_{1i}}{1+\Delta\omega_{1i}^2/\delta_{1i}^2} - \frac{Q_{1ai}\Lambda_{1ai}}{1+\Delta\omega_{1ai}^2/\delta_{1ai}^2}\right)$$

Fixed parameters:

- circulating power P = 830 kW
- acoustic mode Q-factor $Q_m = 10^7$
- mass of the test mass M = 40 kg

Λ - overlapping parameter
(optical and acoustic mode shapes must be known)
Q_1 ,δ_1 – optical Q-factor and half line-width of the TEMs
(diffraction losses should be taken into account)

2 possible processes:

$\omega_1-\omega_0 = \sim \omega_m$: damping $\rightarrow \Delta\omega_a \equiv \omega_1-\omega_0-\omega_m$

$\omega_0-\omega_1 = \sim \omega_m$: excitation $\rightarrow \Delta\omega \equiv \omega_0-\omega_1-\omega_m$

optical $\omega_{0,1}$ and acoustic ω_m frequencies must be determined
$\rightarrow$ cavity structure and acoustic mode calculation required

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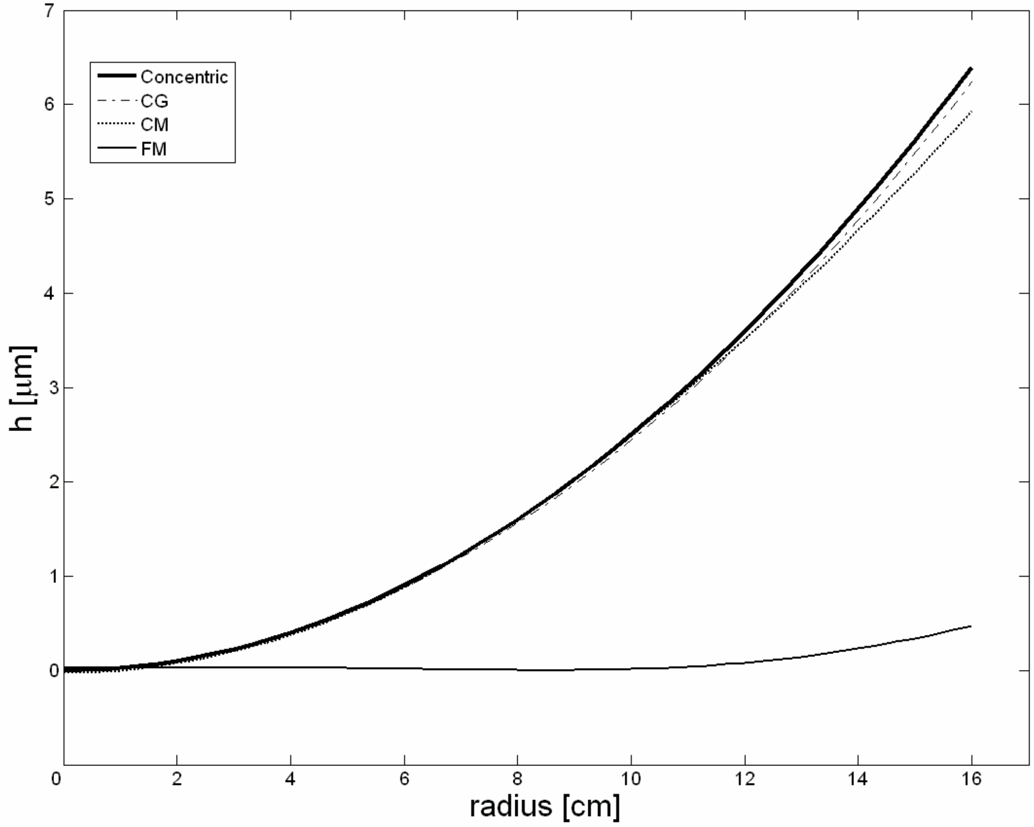
Cavity structure - Mirror shape



$$kh^F(\vec{r}) = Arg(u(D,\vec{r}))$$

$$h^F(\vec{r}) + h^C(\vec{r}) = \frac{r^2}{L}$$

Only near concentric cavities have small tilt instability thus only this cavity type is considered in PI analysis.

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Cavity structure - Eigenvectors

The Fresnel-Kirchhoff propagation equation:

$$E_2(\vec{r}_2, L) = -\frac{ik}{2\pi}\iint_S G(\vec{r}_2, \vec{r}_1) E_1(\vec{r}_1, 0) F(\theta) d^2\vec{r}_1$$

For a fine meshgrid of the mirror surface the electric field E across element area d^2r becomes quasi-steady. The above equation can be written as an eigenequation.

$$\gamma E_1 = M_{21} M_{12} E_1$$

$$M_{12} = \frac{ik}{2\pi}\iint_S G(\vec{r}_2, \vec{r}_1) d^2\vec{r}_1$$

G – propagation kernel

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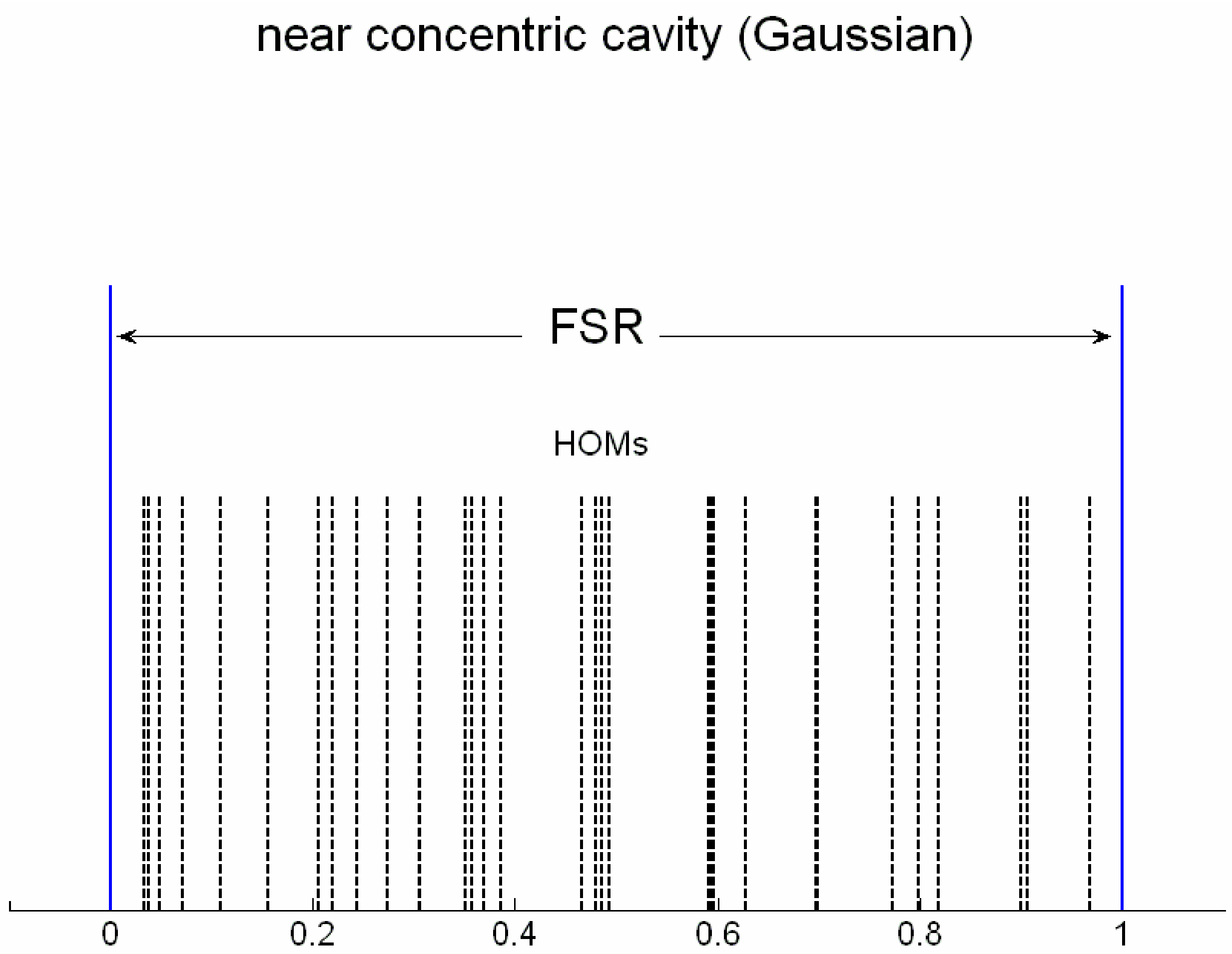
Cavity structure – Gaussian beam



$$\Delta\nu_{p0} = \frac{FSR}{\pi}(\arg(\gamma_p) - \arg(\gamma_0))$$

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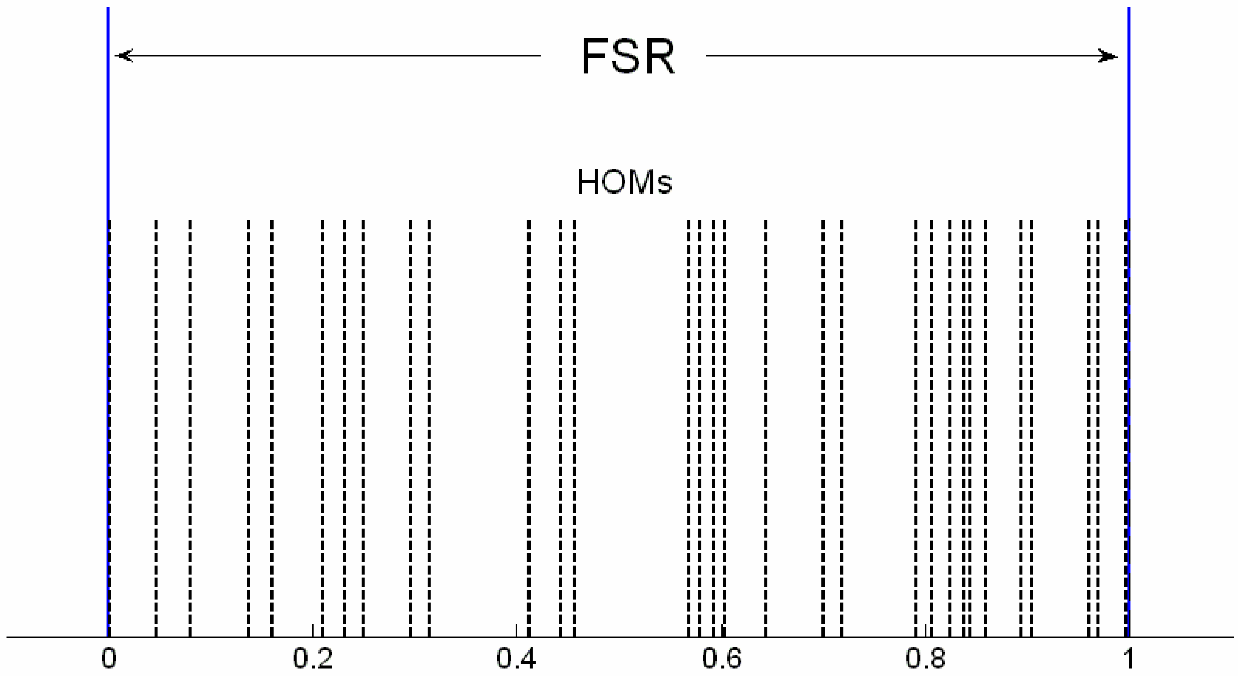
Cavity structure – Mesa beam

near concentric cavity (Mesa)

Near concentric cavity was obtained from near planar cavity using duality relation



Duality relation:

$$\gamma^{C} = e^{-4ikL}\left(\gamma^{F}\right)^{*} \rightarrow \quad \Delta\nu^{C} = M \cdot FSR - \Delta\nu^{F} \quad with \quad M = 1$$

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Acoustic modes analysis

$$M_{ab}\frac{\partial^2 u_i^b}{\partial t^2}+K_{aibk}u_k^b=0$$

Global stiffness matrix:

$$K_{aibk}=\int_S C_{ijkl}\frac{\partial\varphi^a(\vec{\xi})}{\partial\xi_j}\frac{\partial\varphi^b(\vec{\xi})}{\partial\xi_l}dV$$

Global mass matrix:

$$M_{ab}=\rho\int_S \varphi^a\varphi^b dV$$

C – compliance tensor (due to the crystal symmetries can be reduced to the 6x6 matrix

For sapphire M-axis:

$$C=\begin{pmatrix} 498.1 & & & & & \\ 110.9 & 496.8 & & \text{Symmetric} & & \\ 110.9 & 163.6 & 496.8 & & & \\ 0 & 0 & 0 & 153.1 & & \\ 0 & -23.5 & 23.5 & 0 & 147.4 & \\ 0 & 0 & 0 & -23.5 & 0 & 147.4 \end{pmatrix}$$

assuming $u=\mu_i\cos\omega_i t$

we obtain an eigenequation

$$\left|K-\omega_i^2 M\right|=0$$

Solved with ANSYS – 140k nodes

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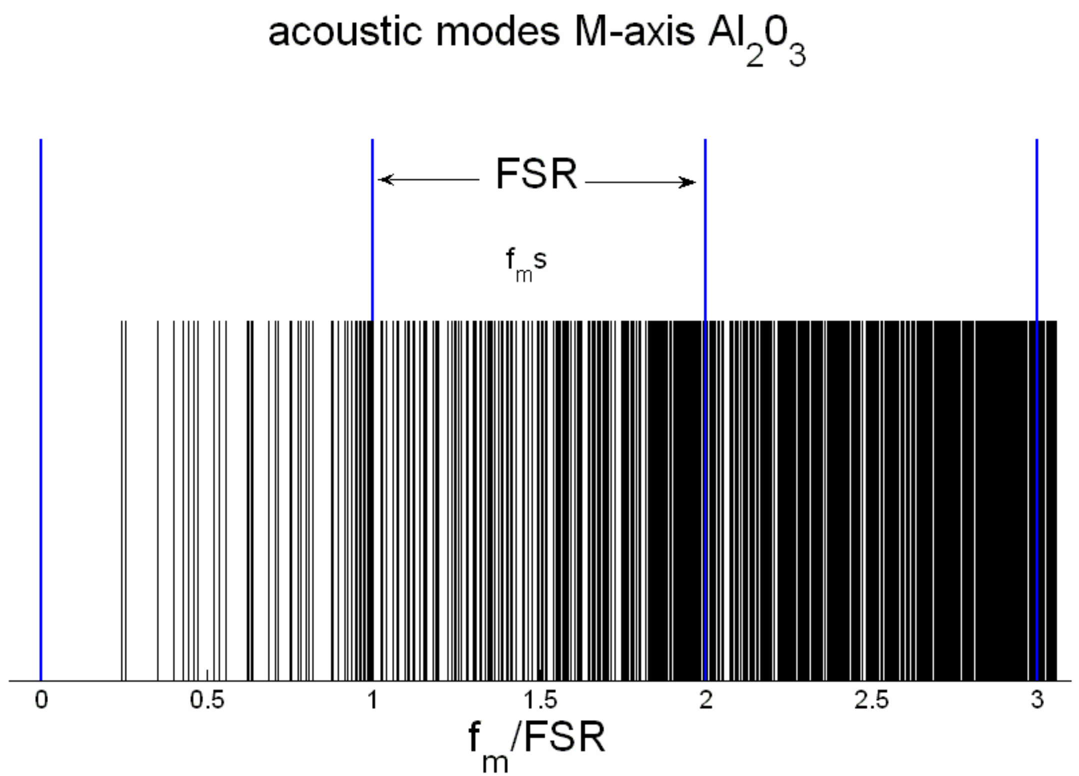
Acoustic modes frequencies



800 modes: 8.9 kHz – 114.6 kHz

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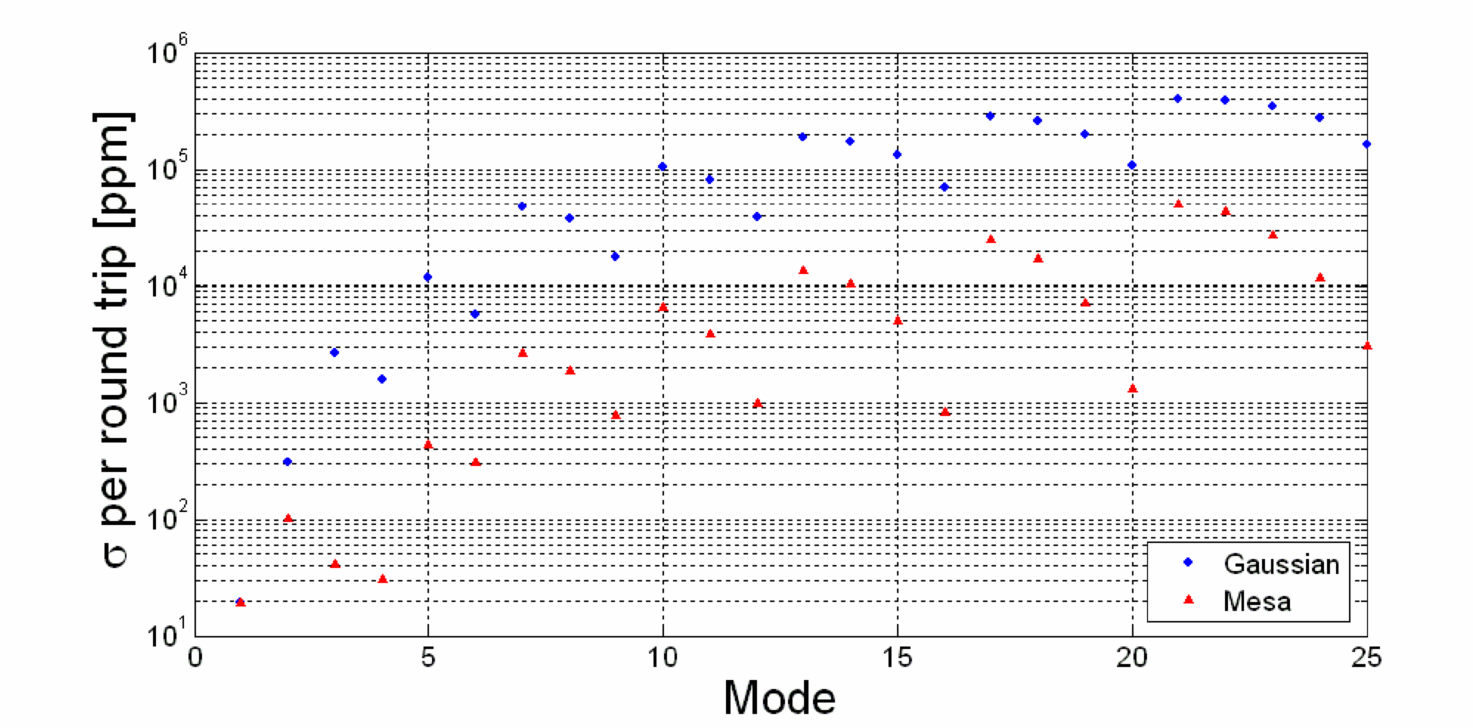
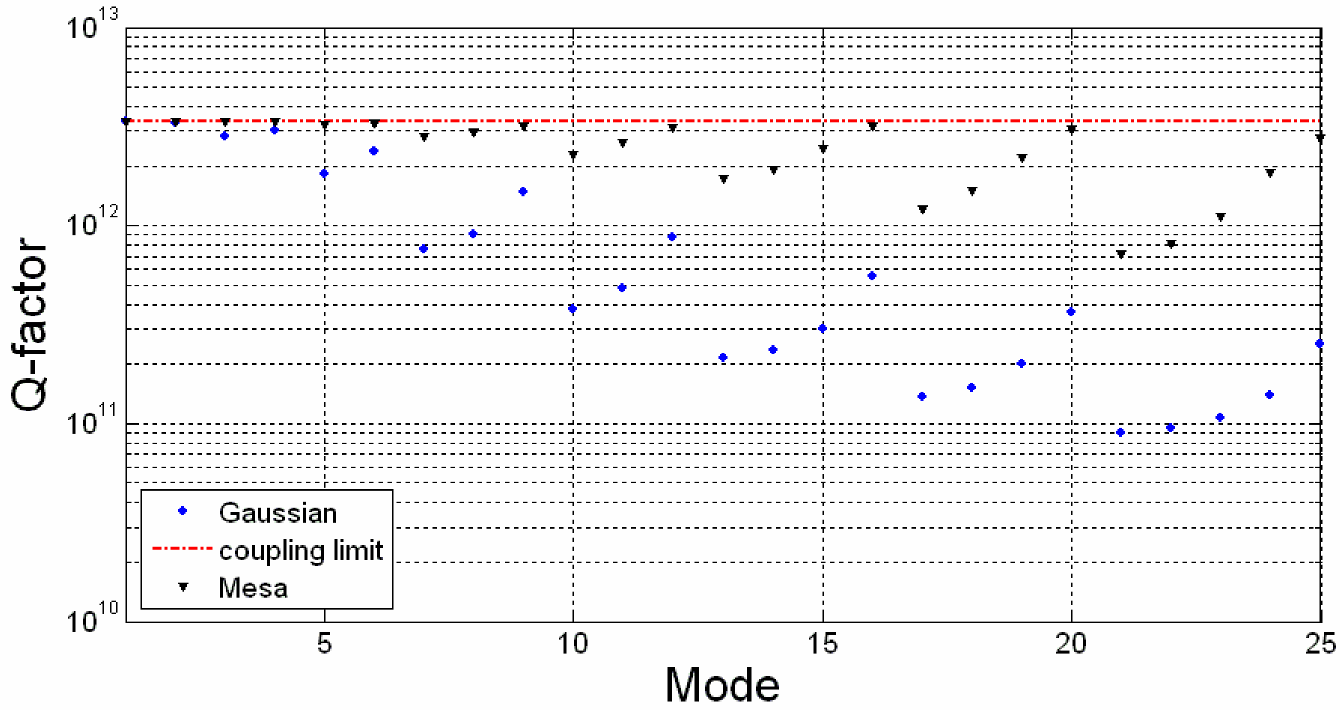
Diffraction losses

$$\sigma_p = 1 - |\gamma_p|^2$$



Optical modes Q-factors

$$Q_p = \frac{-2\pi\nu_p}{FSR \ln\left[\left(1-\sigma_p\right)R_1 R_2\right]}$$



Q-factors of the mesa TEMs are close to the coupling limit. Diffraction losses have substantially smaller effect on Q in comparison to the Gaussian cavity.

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Overlapping parameter

Parametric gain R is also set by the overlapping parameter Λ. The spatial match between HOM and fundamental mode must be conserved.

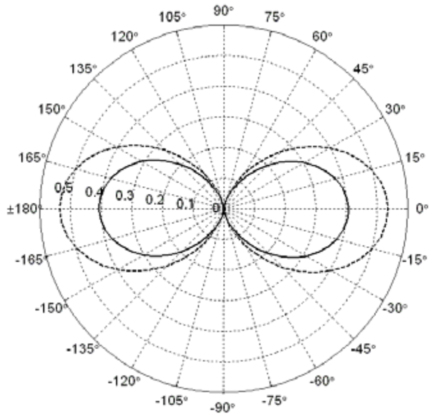
$$\Lambda_{ij} = \frac{V\left(\int (\vec{E}^{0} \circ \vec{E}^{i}) \mu^{j}{}_{\perp} d\vec{r}_{\perp}\right)^{2}}{\int \left|\vec{E}^{0}\right|^{2} d\vec{r}_{\perp} \int \left|\vec{E}^{i}\right|^{2} d\vec{r}_{\perp} \int \left|\vec{\mu}^{j}\right|^{2} d\vec{r}}$$

- spatial match is defined as an integral $\int (\vec{E}^{0} \circ \vec{E}^{HOM}) \mu_{\perp} d\vec{r}_{\perp}$ The higher the integral value the better is a match between modes.

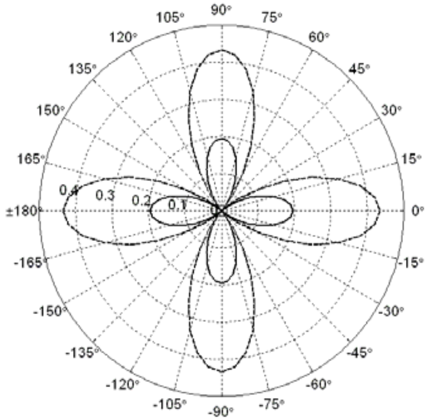
- $V / \int \left|\vec{\mu}\right|^{2} d\vec{r}$ is a mass ratio of the mass of the test mass and the effective mass of the acoustic mode. The effective mass refers to the excitation susceptibility of an acoustic mode.

Optimal overlapping parameter

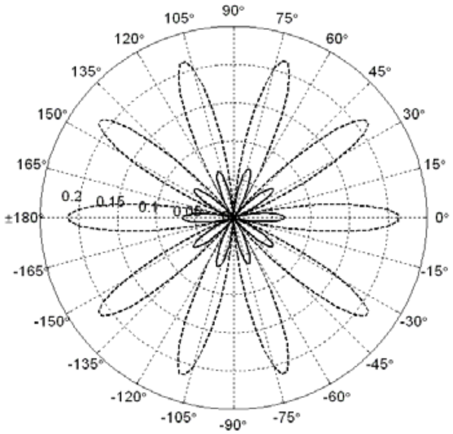
Λ_{MAX} → rotation of the acoustic mode on the optical mode



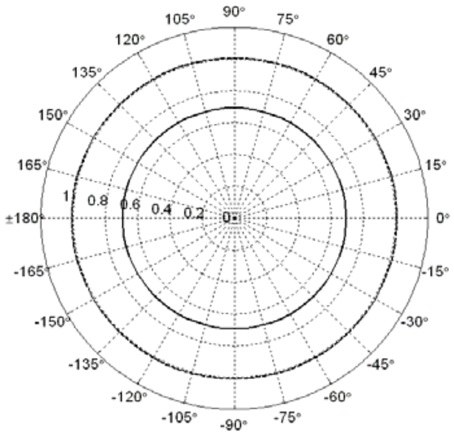
TEM 01



TEM 02



TEM 18



TEM 10

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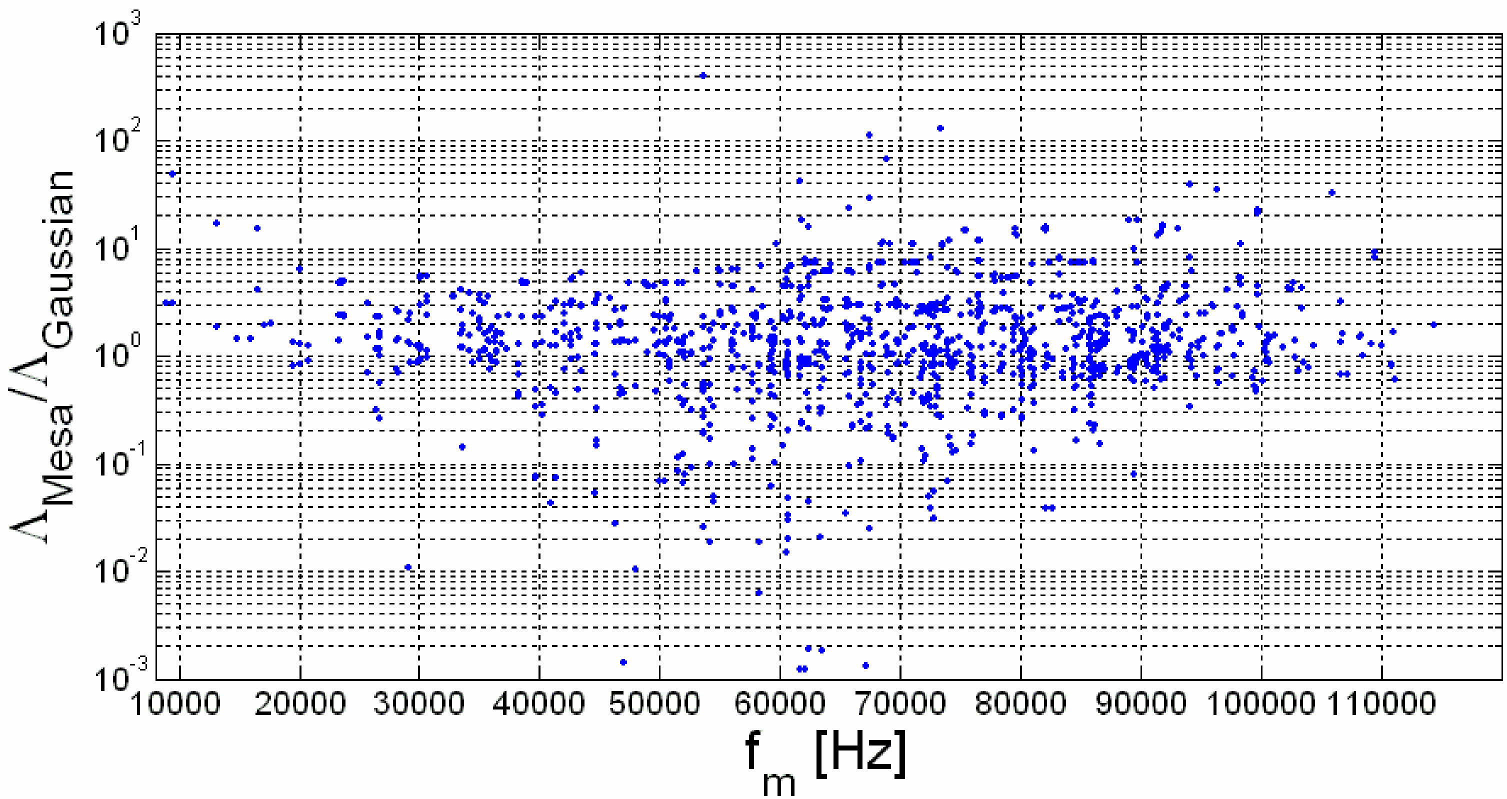
Overlapping parameter

Λ selection criteria: $\Lambda \geq \omega_m^2 / \vartheta Q_m$, $\vartheta = \dfrac{4PQ_o}{mLc}$ (only opto-acoustic interactions with such Λ can be dangerous)

- $\Lambda_{Mesa} > \Lambda_{Gaussian}$: **772**
- $\Lambda_{Gaussian} > \Lambda_{Mesa}$: **404**

⟶ ~2 times more overlaps for mesa beam



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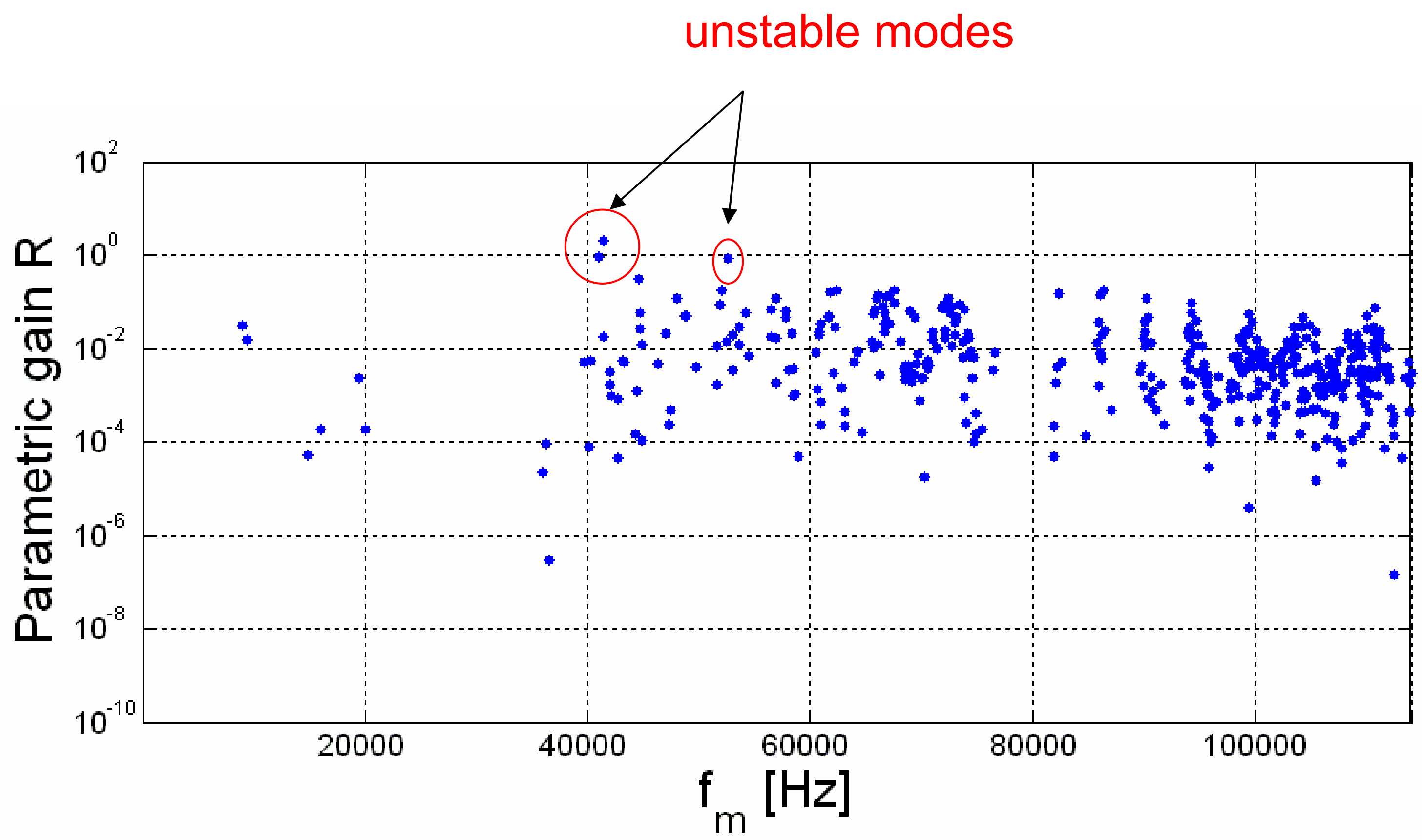
R-value for Gaussian cavity



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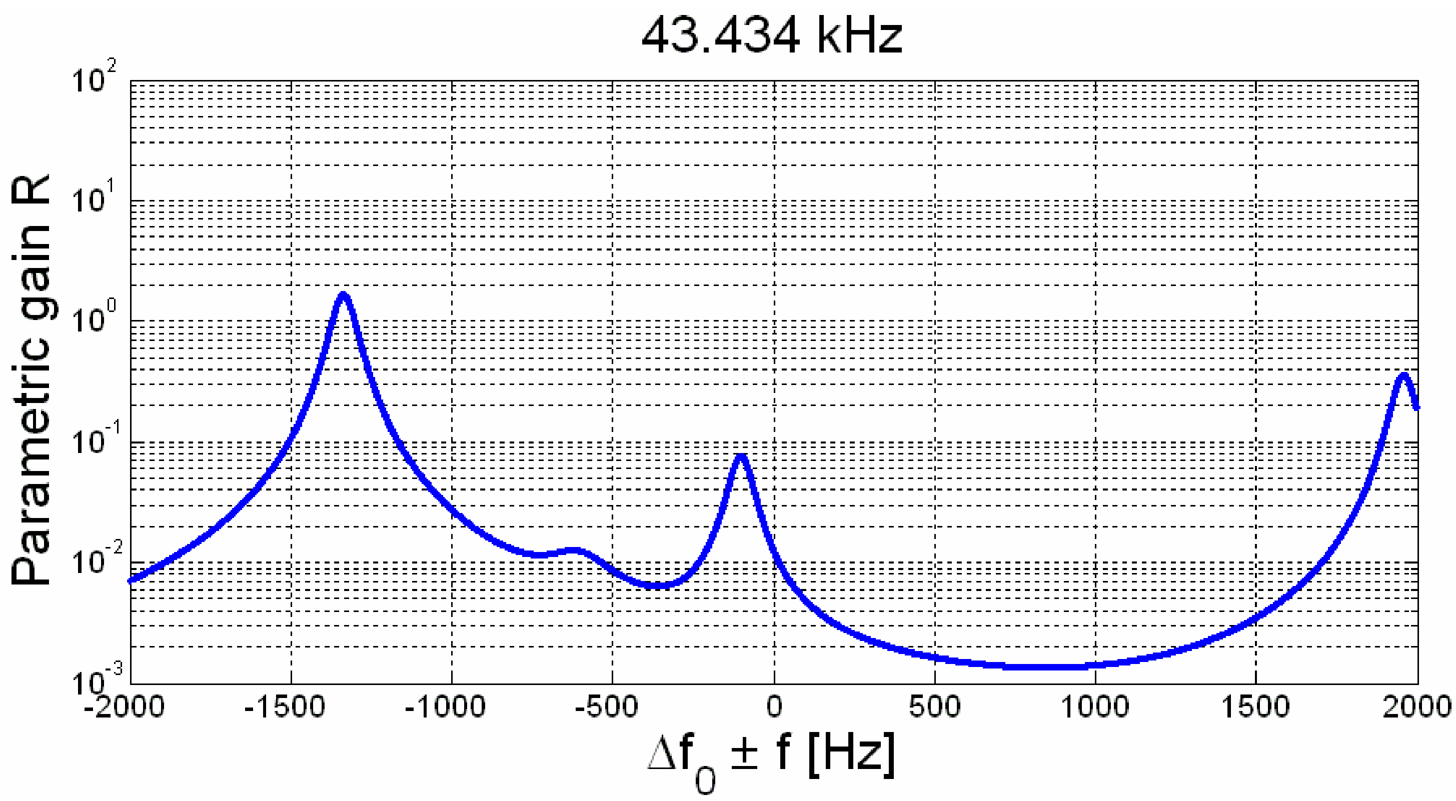
R-value vs. Δω uncertainty



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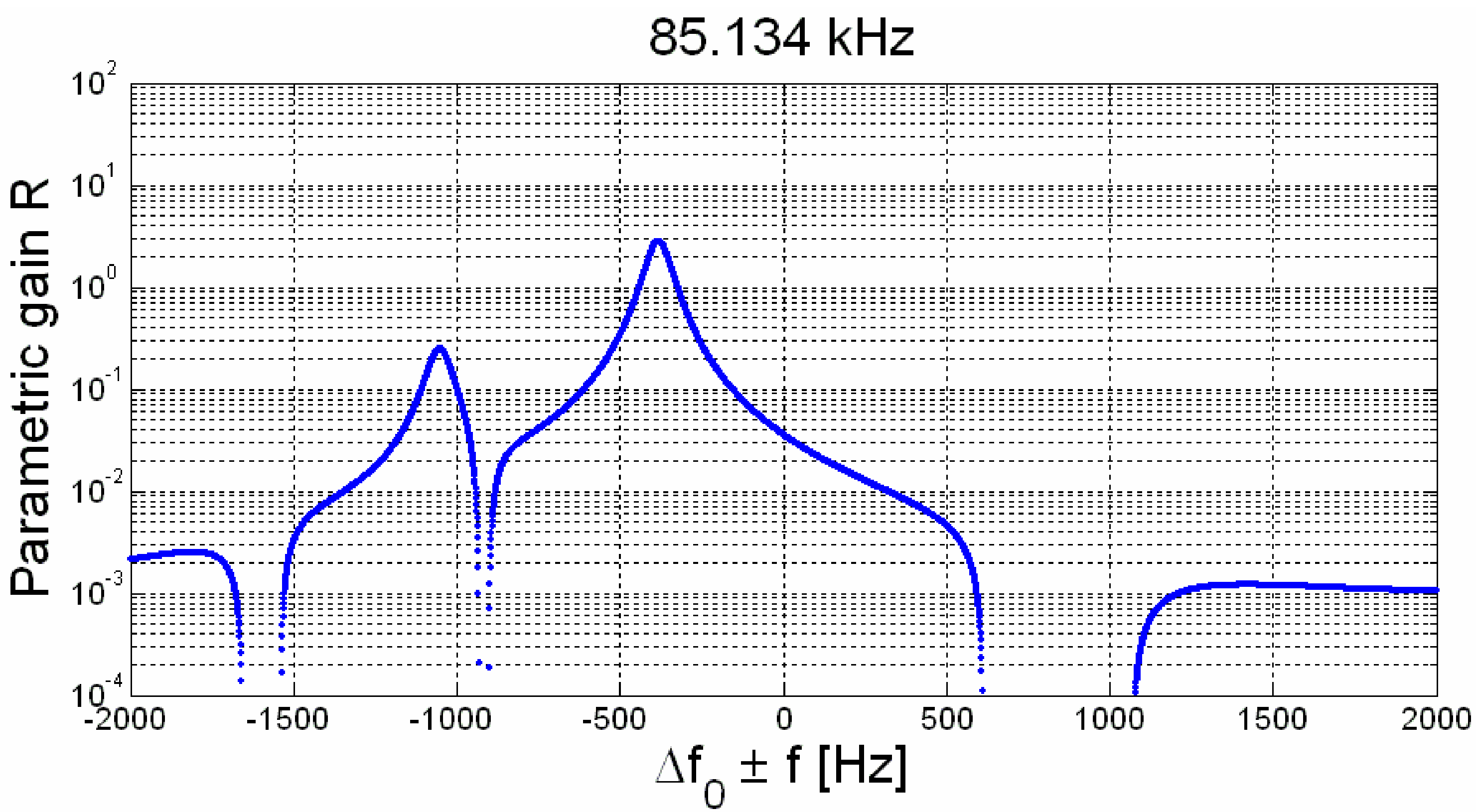
R-value vs. Δω uncertainty



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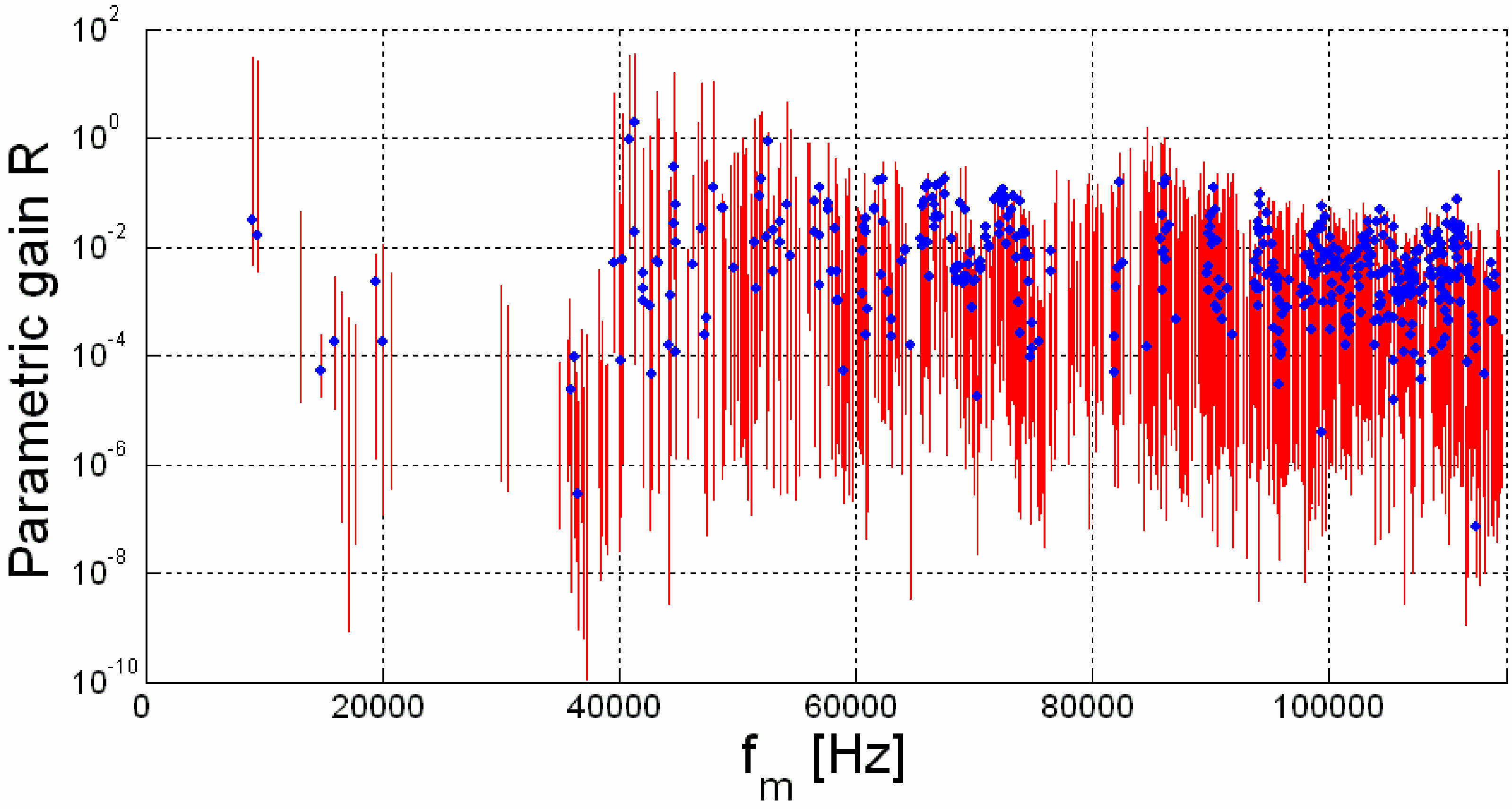
R-value for Gaussian cavity



possible unstable modes: 18

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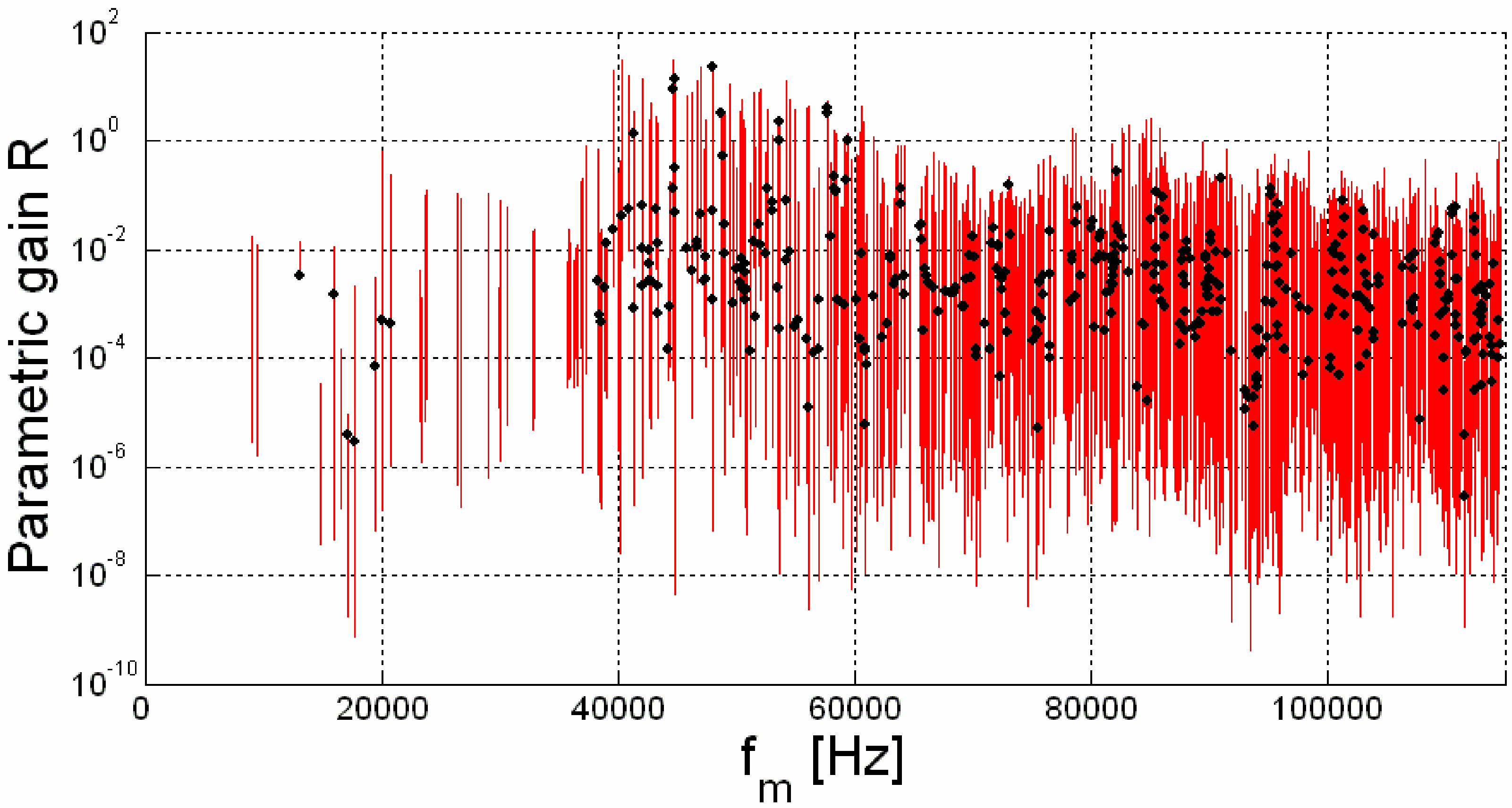
R-value for mesa cavity



possible unstable modes: 46

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Conclusions

•Diffraction losses of the mesa HOMs lower than for the Gaussian HOMs by a factor of ~ 15.

•HOMs Q-factor in the mesa cavity set by coupling losses.

• The mesa cavity has ~2 times more overlaps with value bigger than for Gaussian cavity.

•The mesa cavity is ~2.6 times more susceptible to parametric instability than the Gaussian cavity.

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Modal misalignment suppression for Adv. LIGO

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Parametric Instability

$$R = \frac{2PQ_m \Lambda \omega_s}{McL\omega_m^2} \mathrm{Re}\left[\frac{1}{(1+\chi^2)}\left(\frac{-1}{i\Delta\omega + \lambda_1} + \frac{-\chi^2}{i\Delta\omega + \lambda_2}\right)\right]$$

Optical mode frequency:

$$\omega_s = \frac{1}{2}(\omega_{s1} + \omega_{s2})$$

Arm tuning parameter:

$$d = \frac{1}{2}(\omega_{s1} - \omega_{s2})$$

IFO configurations:

1) IFO with PRM and symmetrical arms
2) IFO with PRM and asymmetrical arms
3) IFO with PRM, SRM and symmetrical arms
4) IFO with PRM, SRM and asymmetrical arms

Model limitations: PRC unstable, no diffraction losses allowed

PARAMETERS

T_ITM = 1.4%
T_ETM = 5ppm
T_PRM = 2.5%

SRM = 20%
δ = 11 deg
(see T070247-01)

Wavelength = 1.064e-06 m
Arm length = 3994.75 m
Mirror radius = 0.17 m (coated 0.16.8 m)
Optical modes: up to order 12
Total number of optical modes = *60*
Estimated numerically
Axial mode order = 5
Total number of acoustic modes = *5507*
(range: 5.8 kHz – 140 kHz)
RoC step: ***0.1m***

RoC_ITM = 1971 m
RoC_ETM = 2191 m

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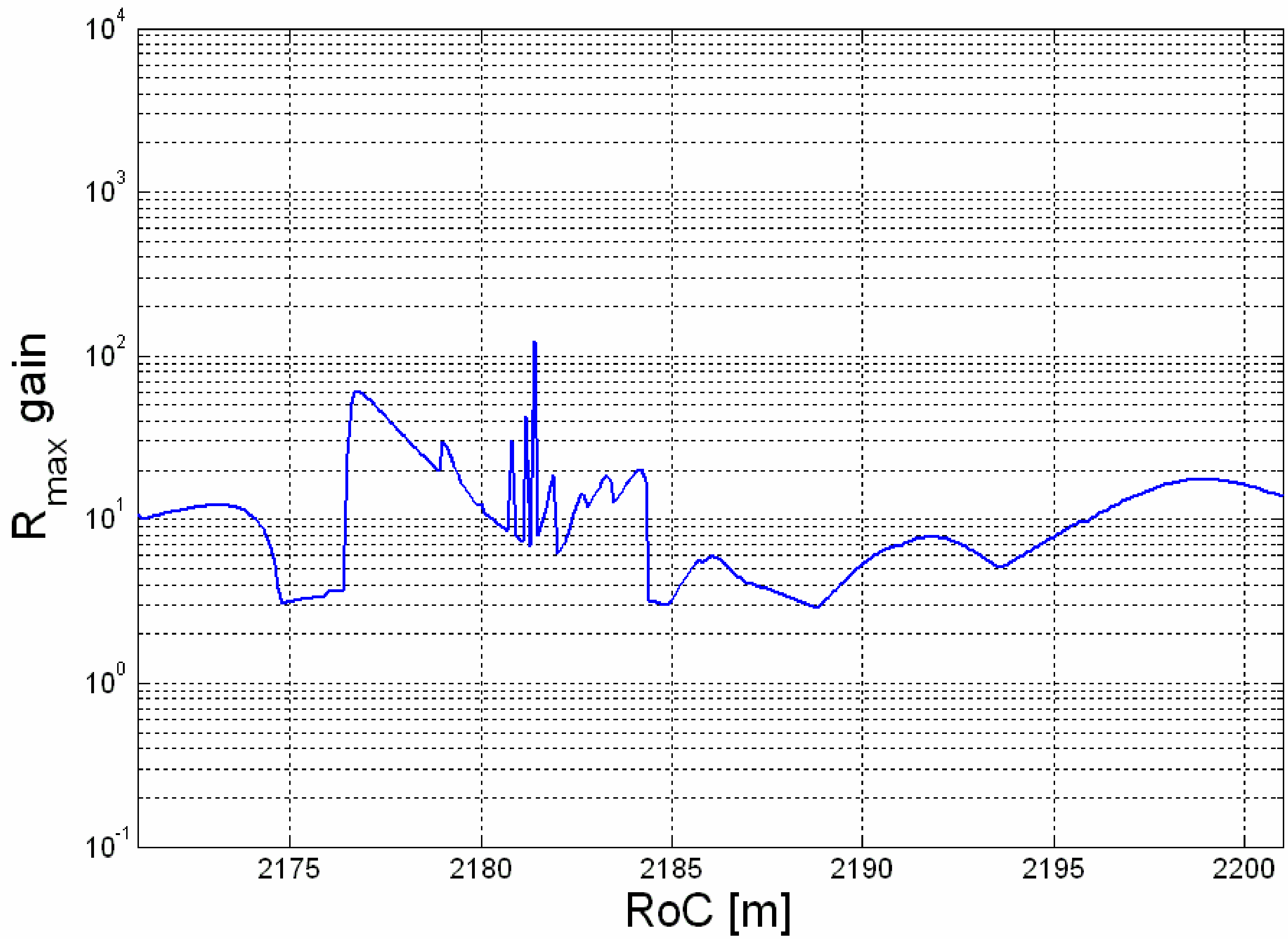
IFO with PRM



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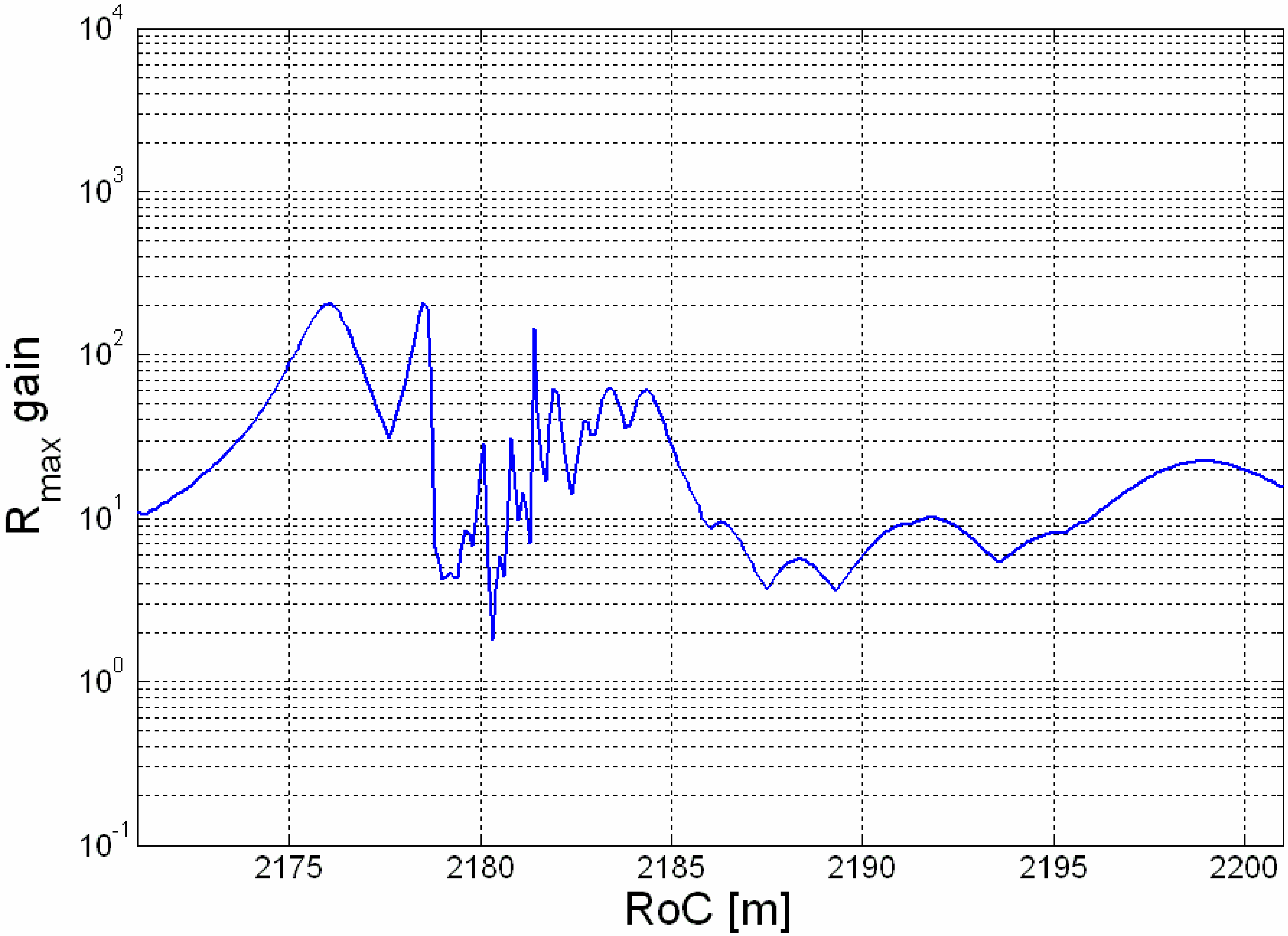
IFO with PRM and SRM



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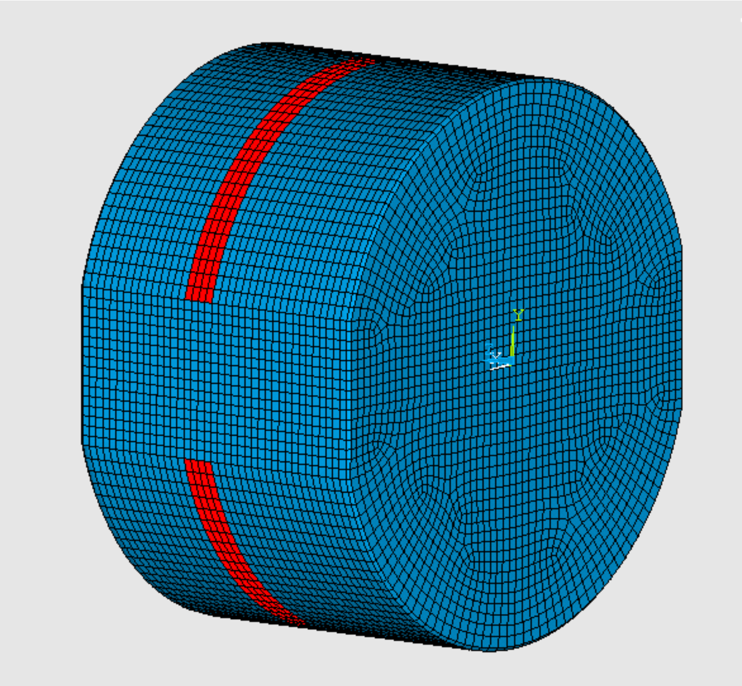
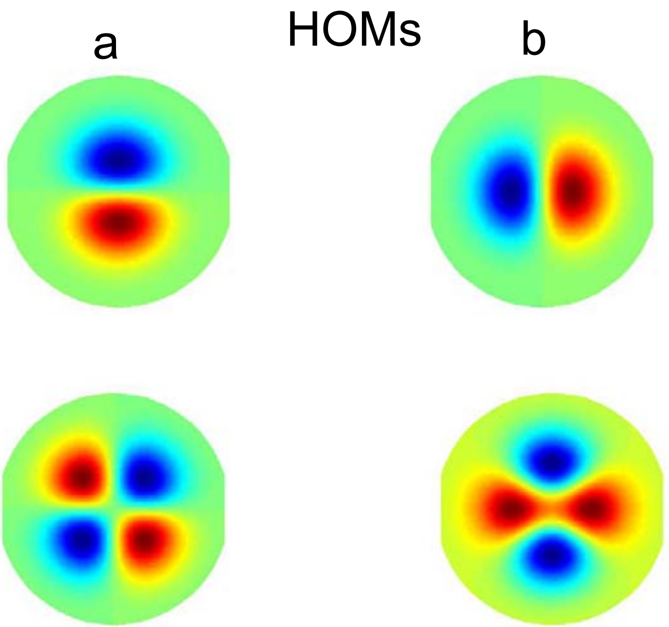
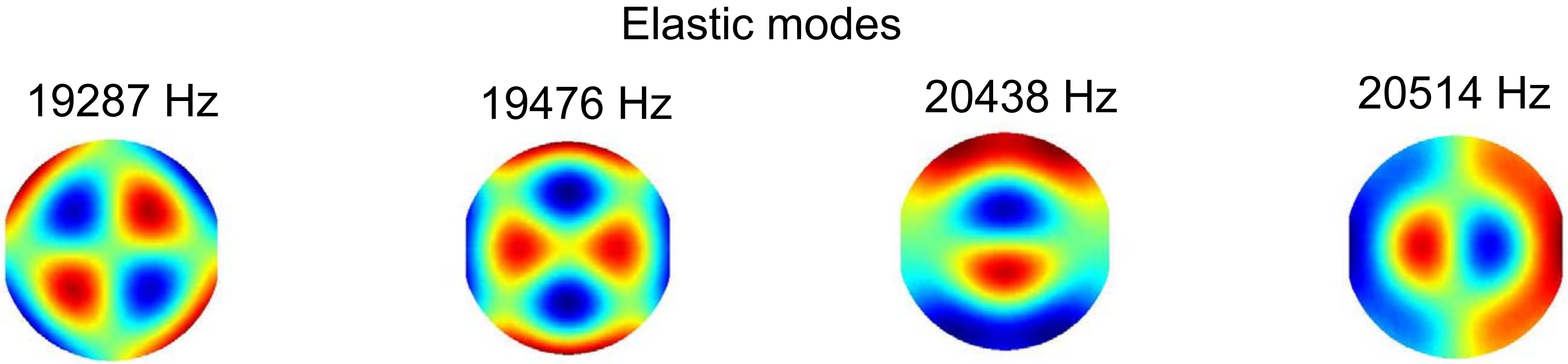
Optical mode orientation







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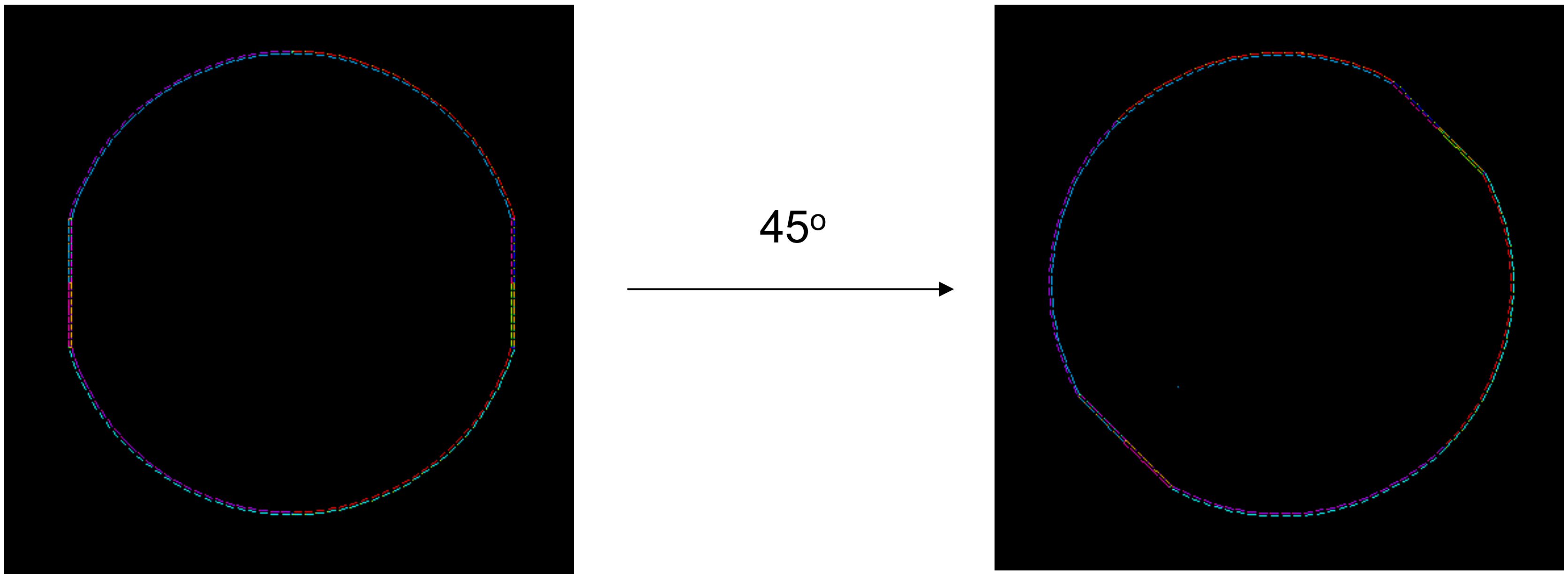
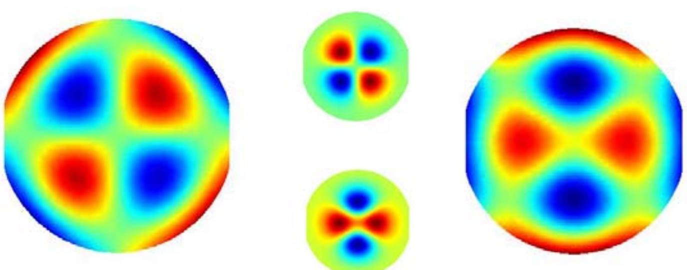
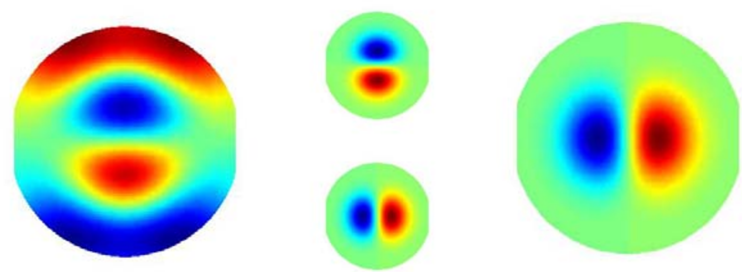
Coating rotation





does not work



work →Λ reduced

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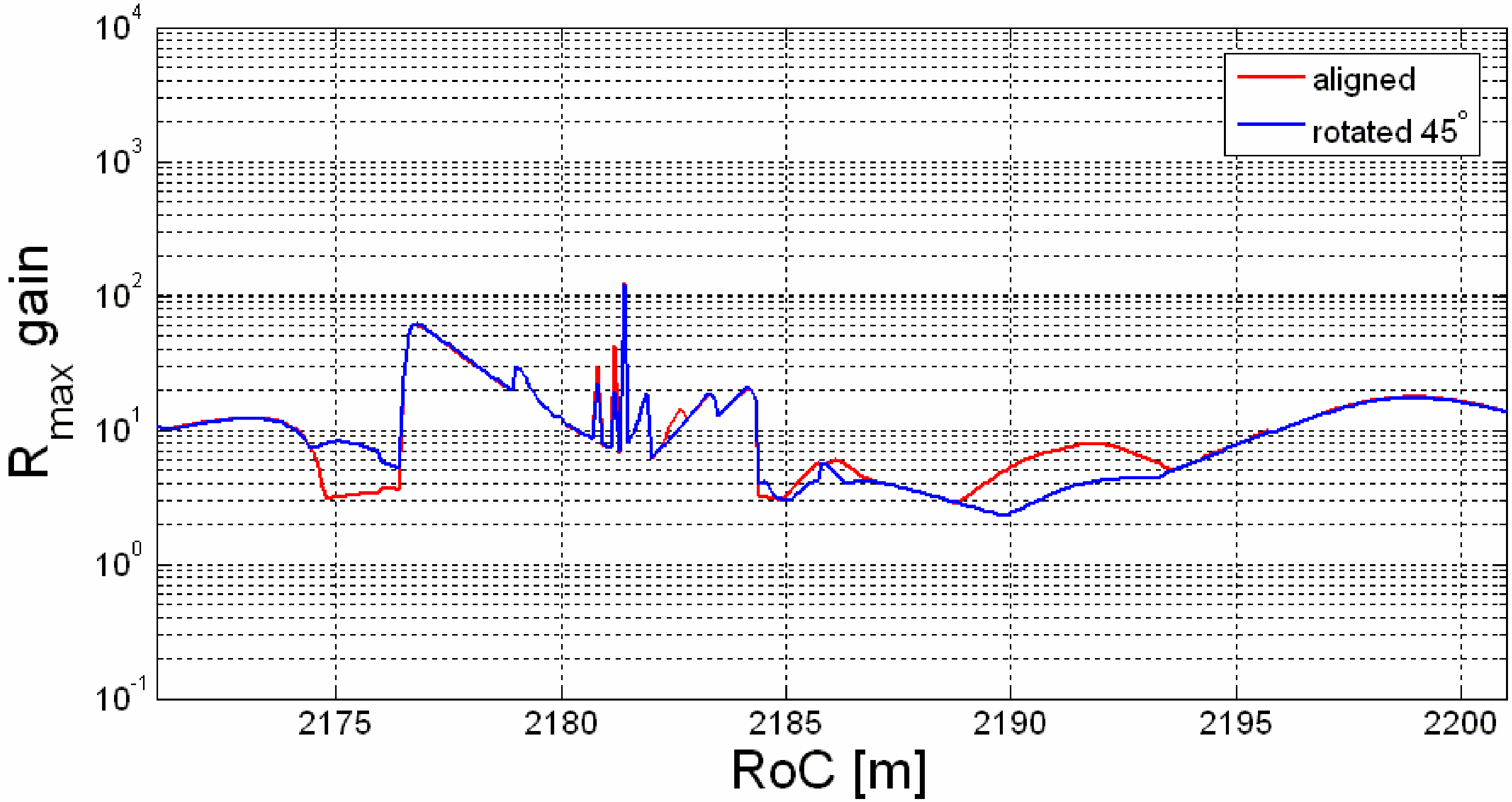
Modal misalignment –IFO with PRM



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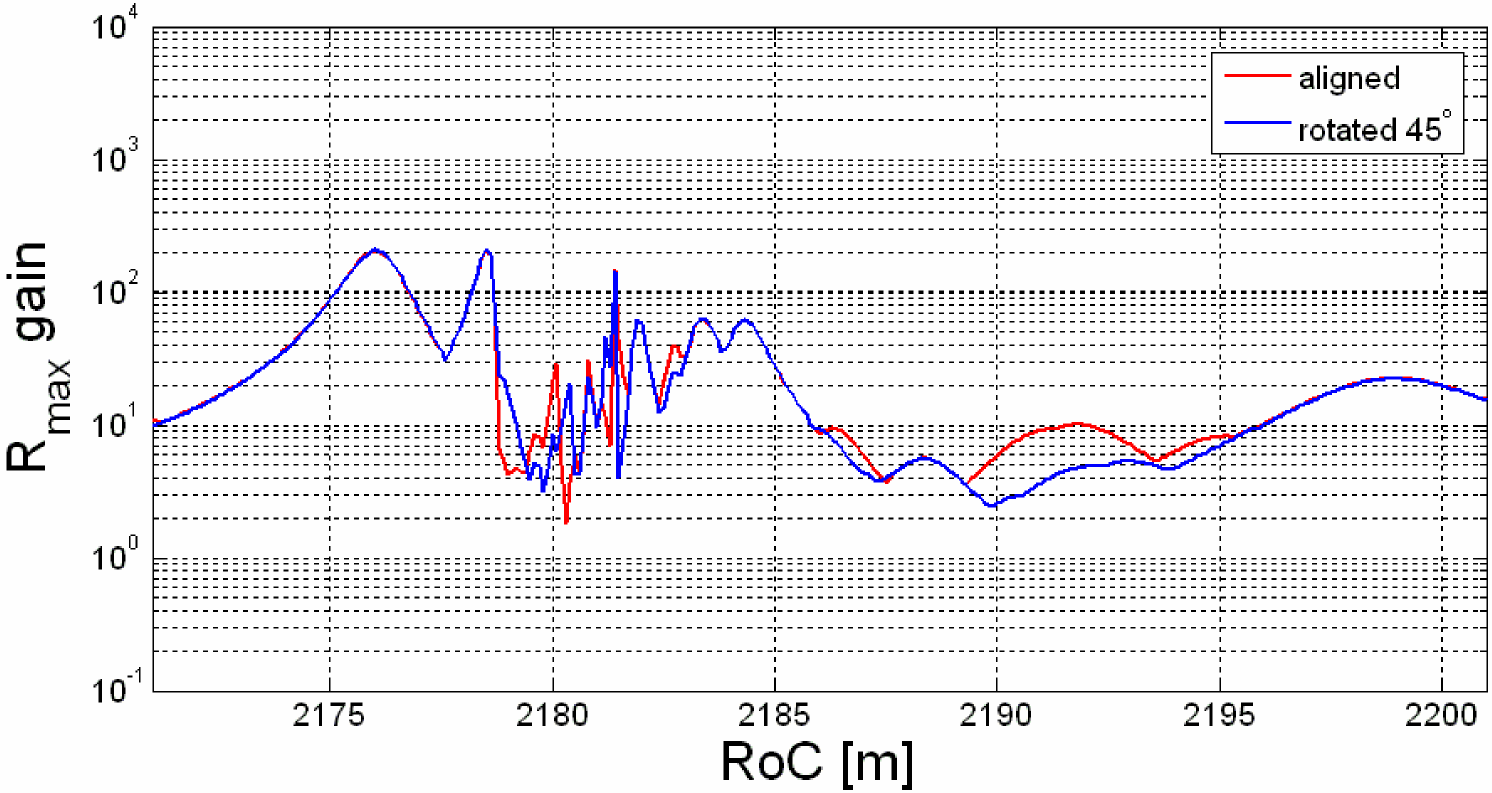
Modal misalignment –IFO with PRM and SRM



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Conclusions

-Arm asymmetry substantially lower parametric gain, R ~ 10-20 in large RoC range.

-Modal misalignment has some effect on reduction of R- value.

- Further R reduction expected from diffraction losses (only for HOMs above 3rd order), unfortunately we do not have a mathematical model for it.

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