Searching for Periodic GW Signals in Space- and Ground-Based Detector Data

John T. Whelan
`john.whelan@aei.mpg.de`

Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Potsdam

Max-Planck-Institut
für Gravitationsphysik
(Albert-Einstein-Institut)

Rochester Institute of Technology CCRG Seminar
2008 March 12
G080072-00-Z

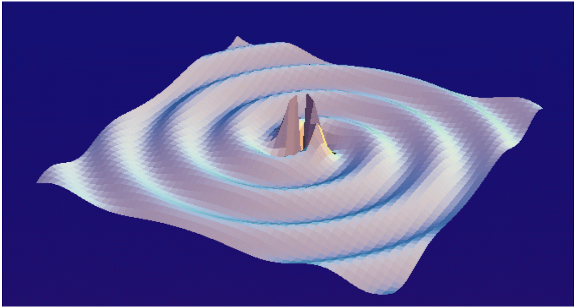
Outline

1. Searches for Gravitational Waves
    - Crash Course in Gravitational Wave Physics
    - Gravitational-Wave Observations & Detectors
    - The Mock LISA Data Challenges
2. Searches for Periodic Gravitational Waves
    - $\mathcal{F}$-Statistic Search Technique
    - AEI $\mathcal{F}$-Statistic Search for White Dwarf Binaries
    - New Cross-Correlation Method for Periodic GW Searches

Outline

1. Searches for Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Observations & Detectors
   - The Mock LISA Data Challenges
2. Searches for Periodic Gravitational Waves
   - $\mathcal{F}$-Statistic Search Technique
   - AEI $\mathcal{F}$-Statistic Search for White Dwarf Binaries
   - New Cross-Correlation Method for Periodic GW Searches

Motivation



- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light
  ⟶ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime:

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

$$= \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\mathrm{tr}} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\mathrm{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

Gravitational Wave as Metric Perturbation

- For GW detection, spin-2 “graviton tensor” $h_{\mu\nu}$ is difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$

($h_{\mu\nu}$ “small” in weak-field regime, e.g. for GW detection)

- E.g. Plane wave propagating in z direction

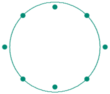
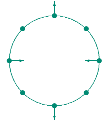
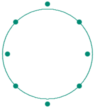
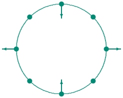
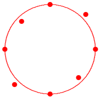
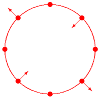
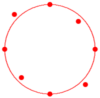
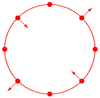
$$\{h_{\mu\nu}\}=\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & h_+ & h_\times & 0\\ 0 & h_\times & -h_+ & 0\\ 0 & 0 & 0 & 0\end{pmatrix} e^{i2\pi f(z-t)}$$

h_+ and $h_\times$ are amplitudes of “plus” and “cross” pol states.

$$\overleftrightarrow{h} = \left[h_+\overleftrightarrow{e}_+ + h_\times\overleftrightarrow{e}_\times\right] e^{i2\pi f(\hat{k}\cdot\vec{r}-t)}$$

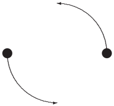
Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization				
Cross (×) Polarization				

Gravitational Wave Generation

- Generated by moving/oscillating mass distribution
- Lowest multipole is quadrupole
- Classic example: orbiting binary system



(e.g., Binary Pulsar 1913+16
– Observed energy loss agrees w/GW prediction)

- Periodic signals with slow freq evolution arise from
  - Early stages of binary evolution
  - Rapidly rotating non-axisymmetric neutron stars

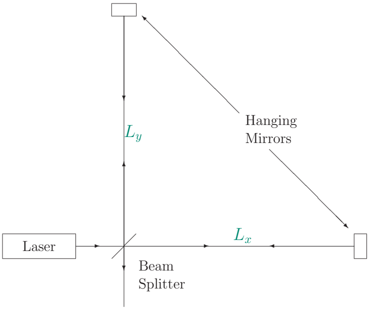
Outline

1. Searches for Gravitational Waves
    - Crash Course in Gravitational Wave Physics
    - Gravitational-Wave Observations & Detectors
    - The Mock LISA Data Challenges

2. Searches for Periodic Gravitational Waves
    - $\mathcal{F}$-Statistic Search Technique
    - AEI $\mathcal{F}$-Statistic Search for White Dwarf Binaries
    - New Cross-Correlation Method for Periodic GW Searches

Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in

$$L_x - L_y = \sqrt{g_{11} L_0^2} - \sqrt{g_{22} L_0^2}$$

$$= \sqrt{(1+h_{11})L_0^2} - \sqrt{(1+h_{22})L_0^2}$$

$$\approx L_0 \frac{h_{11}-h_{22}}{2} \sim L_0 h_+$$

- More gen, $(L_1 - L_2)/L_0 = \overset{\leftrightarrow}{h} : \overset{\leftrightarrow}{d}$ with "response tensor"

$$\overset{\leftrightarrow}{d} = \frac{\hat{n}_1 \otimes \hat{n}_1 - \hat{n}_2 \otimes \hat{n}_2}{2}$$

(also when $\hat{n}_1$ & $\hat{n}_2$ not $\perp$)

Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

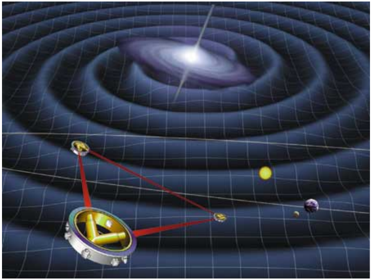
LIGO Livingston (La.)

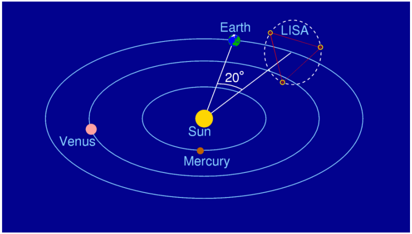
GEO-600 (Germany)

Virgo (Italy)

LISA: Interferometry in Space

- Planned Joint NASA-ESA Mission: to launch 2018 or later
- 3 spacecraft will orbit sun in 5 mio km ▽
  & track each other w/lasers
- Laser phase data combined to simulate IFO:
  "Time-Delay Interferometry" (TDI)

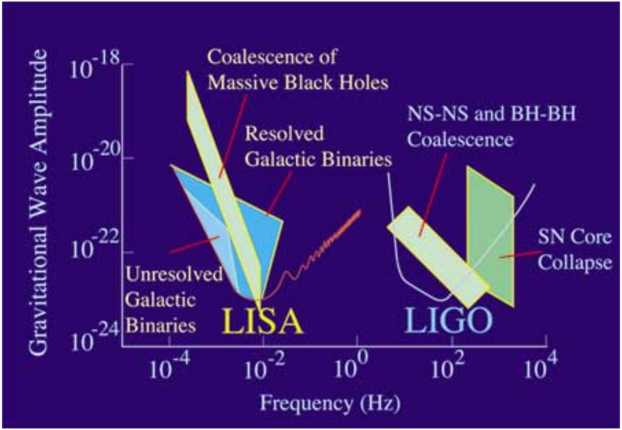




Credits: NASA/JPL; MPI for Gravitational Physics (AEI)/Einstein Online

Differences Between LISA and LIGO

- Diff noise sources & sizes mean diff frequency ranges



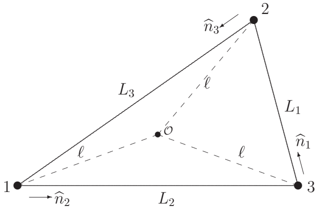
Credit: NASA/JPL

Differences Between LISA and LIGO

- Diff noise sources & sizes mean diff frequency ranges
- LIGO data noise-dominated; can seek one source at a time
  LISA data will contain many strong sources;
  ⟶ must worry about signal extraction
- LISA to observe GWs w/λ comparable to arm length
  ⟶ At higher frequencies, simple IFO picture breaks down
  & response depends on propagation direction

$$\widetilde{X}(f) = \frac{\widetilde{h}(f)}{R(f)} = \frac{\overleftrightarrow{h}(f) : \overleftrightarrow{d}(f, \widehat{k})}{R(f)}$$

LISA Response



- LISA spacecraft 1, 2, 3
- Arm lens L_1, L_2, L_3
  all $\approx L = 5\,\text{million km}$
  vary due to GW & orbit
- TDI vars X, Y, Z comb links
  btwn sc to cancel laser noise
- Convert into “strains”
  $\widetilde{h}^X(f) = R(f)\widetilde{X}(f) = \overleftrightarrow{h}(f) : \overleftrightarrow{d}^X(f, \widehat{k})$
  where in long-λ limit
  $\overleftrightarrow{d} \approx \frac{1}{2}(\hat{n}_2 \otimes \hat{n}_2 - \hat{n}_3 \otimes \hat{n}_3)$
  (etc. for Y & Z)

Outline

1. Searches for Gravitational Waves
    - Crash Course in Gravitational Wave Physics
    - Gravitational-Wave Observations & Detectors
    - The Mock LISA Data Challenges
2. Searches for Periodic Gravitational Waves
    - $\mathcal{F}$-Statistic Search Technique
    - AEI $\mathcal{F}$-Statistic Search for White Dwarf Binaries
    - New Cross-Correlation Method for Periodic GW Searches

Mock LISA Data Challenges

- LISA data analysis presents unusual challenges;
  Need to coördinate searches for different types of signals
  Need plan worked out before LISA flies
- LISA International Science Team (LIST) has organized MLDCs to build community expertise
  Extract simulated signals from simulated LISA noise

Challenge	Dates	Results Presented
MLDC1	2006 Jun-Dec	GWDAW 11, Potsdam
MLDC2	2007 Jan-Jun	GR 18 / Amaldi 7, Sydney
MLDC1B	2007 Jul-Dec	GWDAW 12, Boston
MLDC3	2008 Jan-Dec	GWDAW 13, Arecibo

First Mock LISA Data Challenge

- MLDC1 Results submitted December 2006
- MLDC1B Results submitted December 2007
- Data sets:
  - Challenge 1.1: White Dwarf Binaries: Periodic Sources
  - Challenge 1.2: Super-Massive Black Hole Inspirals
  - Challenge 1.3: Extreme Mass Ratio Inspirals (deadline postponed until MLDC2)
- Entries submitted by ten groups each time
- AEI group of Reinhard Prix & JTW searched for WD binaries w/$\mathcal{F}$-statistic method w/Deepak Khurana for MLDC1B

Second Mock LISA Data Challenge

- Results submitted June 2007
- Data sets:
  - Challenge 1.3: Extreme Mass Ratio Inspirals
  - Challenge 2.1: Galactic Binaries (30 Million)
  - Challenge 2.2: “Whole Enchilada”: Galaxy + EMRIs + BHB
- Entries submitted by thirteen groups
- AEI group of Reinhard Prix & JTW
  searched for WD binaries w/$\mathcal{F}$-statistic method
  (improved pipeline to distinguish sources)

Third Mock LISA Data Challenge

- Results due December 2008
- Data sets:
    - Challenge 3.1: Galactic WDB w/frequency evolution
    - Challenge 3.2: SMBH binary + galaxy
    - Challenge 3.3: EMRIs
    - Challenge 3.4: Bursts
    - Challenge 3.5: Stochastic Background

Outline

1. Searches for Gravitational Waves
    - Crash Course in Gravitational Wave Physics
    - Gravitational-Wave Observations & Detectors
    - The Mock LISA Data Challenges
2. Searches for Periodic Gravitational Waves
    - $\mathcal{F}$-Statistic Search Technique
    - AEI $\mathcal{F}$-Statistic Search for White Dwarf Binaries
    - New Cross-Correlation Method for Periodic GW Searches

Periodic Gravitational Waves

(Quasi-)periodic GW @ solar-system barycenter (SSB):

$$\overleftrightarrow{h} = A_+ \cos[\phi_0 + \phi(\tau)]\, \overleftrightarrow{e}_+(\beta,\lambda,\psi) + A_\times \sin[\phi_0 + \phi(\tau)]\, \overleftrightarrow{e}_\times(\beta,\lambda,\psi)$$

Periodic Gravitational Waves

(Quasi-)periodic GW @ solar-system barycenter (SSB):

$$\overleftrightarrow{h} = A_+ \cos[\phi_0 + \phi(\tau)]\, \overleftrightarrow{e}_+(\beta,\lambda,\psi) + A_\times \sin[\phi_0 + \phi(\tau)]\, \overleftrightarrow{e}_\times(\beta,\lambda,\psi)$$

- SSB time τ related to detector time t by doppler shift, which depends on sky position $\{\beta,\lambda\}$
- $\phi(\tau)$ from frequency & derivs $\{f,\dot{f},\dots\}$
- Doppler params $\theta = \{\beta,\lambda,f,\dot{f},\dots\}$ will determine signal templates

Periodic Gravitational Waves

(Quasi-)periodic GW @ solar-system barycenter (SSB):

$$\overleftrightarrow{h} = A_+ \cos[\phi_0 + \phi(\tau)]\, \overleftrightarrow{e}_+(\beta,\lambda,\psi) + A_\times \sin[\phi_0 + \phi(\tau)]\, \overleftrightarrow{e}_\times(\beta,\lambda,\psi)$$

$\{\overleftrightarrow{e}_{+,\times}\}$ is pol basis $\perp$ prop direction

Constructed rel to source (e.g., orbital plane or NS ang mom)

Related to fiducial basis $\{\overleftrightarrow{\varepsilon}_{+,\times}\}$ (from ecliptic or equator) by rotation

$$\overleftrightarrow{e}_+(\beta,\lambda,\psi) = \overleftrightarrow{\varepsilon}_+(\beta,\lambda)\cos 2\psi + \overleftrightarrow{\varepsilon}_\times(\beta,\lambda)\sin 2\psi$$
$$\overleftrightarrow{e}_\times(\beta,\lambda,\psi) = -\overleftrightarrow{\varepsilon}_+(\beta,\lambda)\sin 2\psi + \overleftrightarrow{\varepsilon}_\times(\beta,\lambda)\cos 2\psi$$

Periodic Gravitational Waves

(Quasi-)periodic GW @ solar-system barycenter (SSB):

$$\overleftrightarrow{h} = A_+ \cos[\phi_0 + \phi(\tau)]\, \overleftrightarrow{e}_+(\beta,\lambda,\psi) + A_\times \sin[\phi_0 + \phi(\tau)]\, \overleftrightarrow{e}_\times(\beta,\lambda,\psi)$$

Allows factorization ($\sum_{\mu=1}^{4}$ implicit) $\overleftrightarrow{h} = \mathcal{A}^\mu \overleftrightarrow{h}_\mu(\tau;\theta)$ where

$$\begin{aligned}
\mathcal{A}^1 &= A_+ \cos\phi_0 \cos 2\psi - A_\times \sin\phi_0 \sin 2\psi \\
\mathcal{A}^2 &= A_+ \cos\phi_0 \sin 2\psi + A_\times \sin\phi_0 \cos 2\psi \\
\mathcal{A}^3 &= -A_+ \sin\phi_0 \cos 2\psi - A_\times \cos\phi_0 \sin 2\psi \\
\mathcal{A}^4 &= -A_+ \sin\phi_0 \sin 2\psi + A_\times \cos\phi_0 \cos 2\psi
\end{aligned}$$

and

$$\begin{aligned}
\overleftrightarrow{h}_1(\tau) &= \overleftrightarrow{\varepsilon}_+ \cos\phi(\tau)\,, &\qquad \overleftrightarrow{h}_2(\tau) &= \overleftrightarrow{\varepsilon}_\times \cos\phi(\tau)\,, \\
\overleftrightarrow{h}_3(\tau) &= \overleftrightarrow{\varepsilon}_+ \sin\phi(\tau)\,, &\qquad \overleftrightarrow{h}_4(\tau) &= \overleftrightarrow{\varepsilon}_\times \sin\phi(\tau)\,.
\end{aligned}$$

$\mathcal{F}$-Stat Search for Periodic GWs (JKS 1998)

- Measured strain (= noise + signal) is $x(t;\mathcal{A},\theta) = n(t) + \mathcal{A}^\mu h_\mu(t;\theta)$
  $n(t)$ & $h_\mu(t;\theta) = \overset{\leftrightarrow}{h}_\mu : \overset{\leftrightarrow}{d}$ depend on detector, $\mathcal{A}$ does not
- Jaranowski, Królak, Schutz 1998: Log-likelihood
  $$-\int \frac{|\widetilde{x}(f) - \mathcal{A}^\mu \widetilde{h}_\mu(f)|^2}{S_n(f)} df + \int \frac{|\widetilde{x}(f)|^2}{S_n(f)} df = -\mathcal{A}^\mu \mathcal{M}_{\mu\nu} \mathcal{A}^\nu + 2\mathcal{A}^\mu x_\mu$$
  quadratic in $\mathcal{A}$; maximize analytically
- log-likelihood maximized by amplitude parameters $\mathcal{A}^\mu_{\text{MLE}} = \mathcal{M}^{\mu\nu} x_\nu$; max value is $2\mathcal{F} = x_\mu \mathcal{M}^{\mu\nu} x_\nu$
- $\mathcal{F}$-stat search technique:
  - Make a grid of doppler params θ (freq & sky pos)
  - For each choice of θ, calculate $2\mathcal{F}$ from data
  - High values are candidate sources w/amp params $\mathcal{A}_{\text{MLE}}$

Currently the basis of LIGO searches for spinning neutron stars

Outline

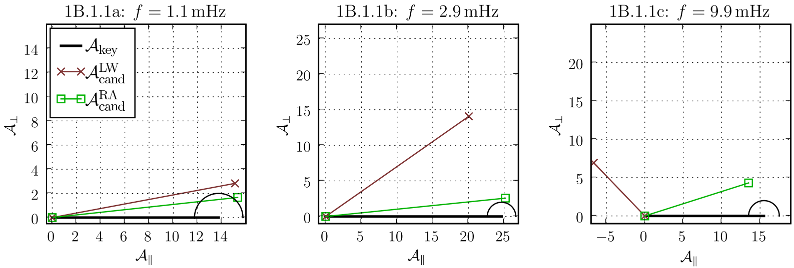
1. Searches for Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Observations & Detectors
   - The Mock LISA Data Challenges
2. Searches for Periodic Gravitational Waves
   - $\mathcal{F}$-Statistic Search Technique
   - AEI $\mathcal{F}$-Statistic Search for White Dwarf Binaries
   - New Cross-Correlation Method for Periodic GW Searches

Prix/Whelan/Khurana MLDC Searches

We used open source
LAL & LALAPPS (LIGO) code,
with slight LISA mods

- MLDC1, Prix & JTW, *CQG* **24**, S639 (2007) (arXiv:0704.2983)
- MLDC2, Amaldi Poster LIGO-G070462-00-Z
- MLDC1B, GWDAW Poster LIGO-G070818-01-Z

Challenge 1(B).1.1: Isolated Binaries

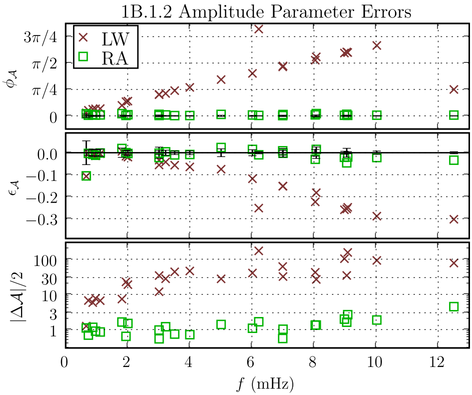


	f (mHz)	Δf(nHz)		ϕ_{sky}(mrad)		ϵ_θ	
		LW	RA	LW	RA	LW	RA
a	1.1	−0.7	−0.7	61.9	46.1	0.5	0.3
b	2.9	0.9	0.9	12.3	7.7	1.1	0.9
c	9.9	1.8	1.8	5.1	7.5	0.4	0.5

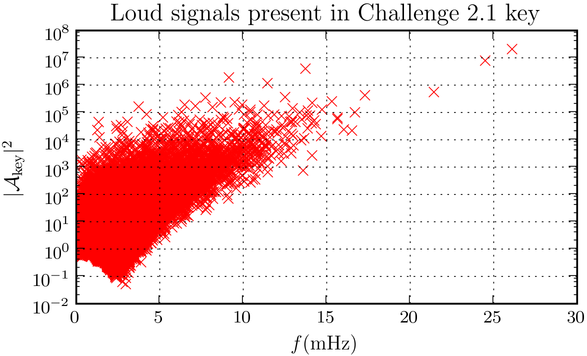
Good sky position even w/long-wavelength response
Rigid adiabatic response needed to get amp params

Challenge 1(B).1.2: Verification Binaries

25 verification binaries
w/known dop params;
amp params well fit
if RA response used.

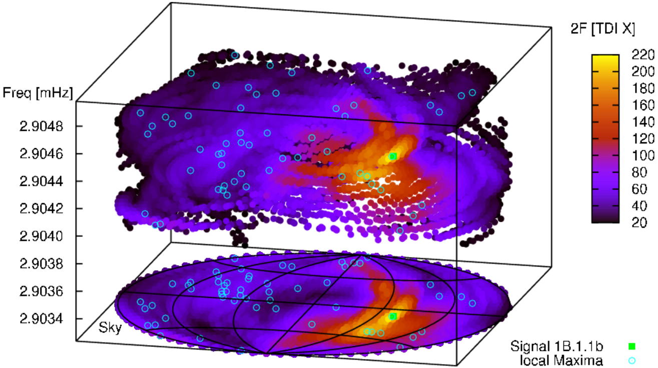


Galactic Binaries Injected in MLDC2



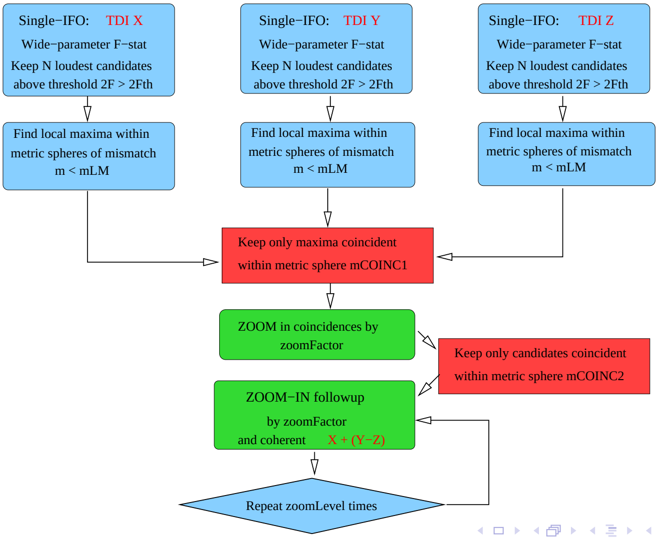
Challenge 2.1 has 26 million galactic WD binaries, of which 59401 designated as “bright” sources

Secondary Maxima in Doppler Parameter Space

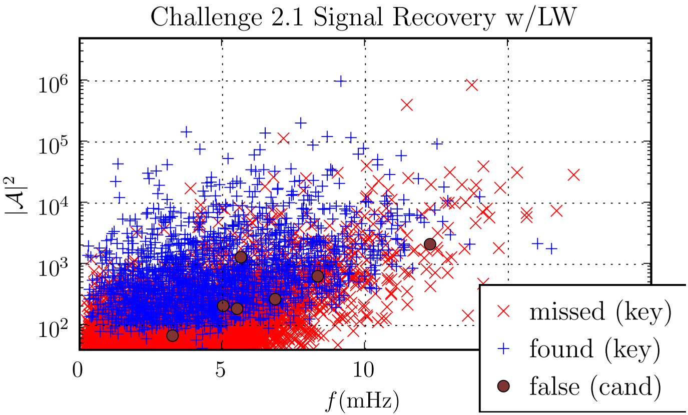


True signals identified by coïncidence btwn TDI vars

Pipeline for Prix/Whelan/Khurana MLDC Searches

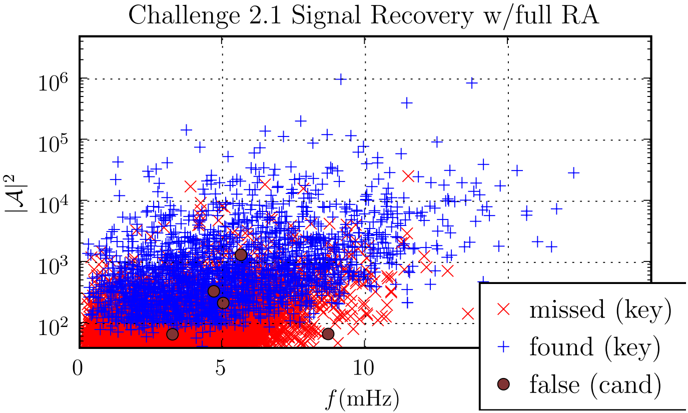


Overview of Galactic Signals Recovered (LW)



Found many signals, but still missed some bright ones (especially at higher f), using long-wavelength response

Overview of Galactic Signals Recovered (RA)



Rigid adiabatic response improves signal recovery

Loudest “misses” now found

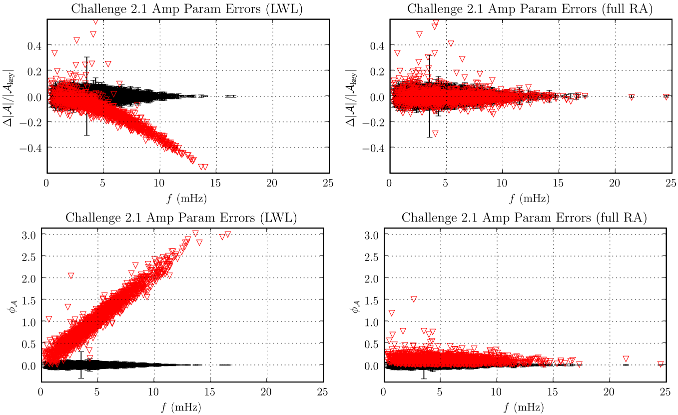
Statistics of Galactic Signals Recovered

Focus on sources w/expected $2\mathcal{F} > 40$

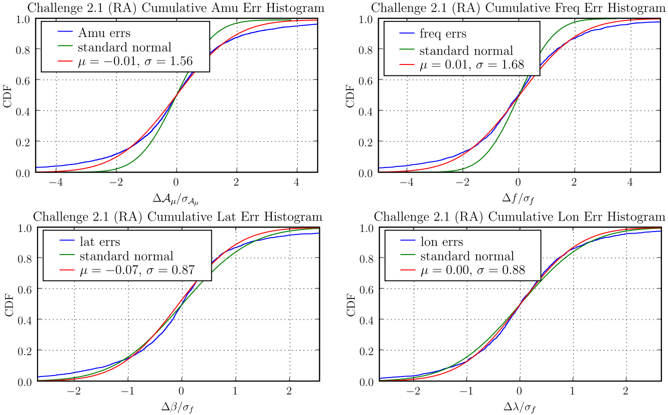
Freqs	Signals	Found		False	
	$(\lvert\mathcal{A}\rvert^2 > 40)$	LW	RA	LW	RA
0–5 mHz	4443	982	1025	1	2
5–10 mHz	1966	652	822	5	3
10–15 mHz	163	68	133	1	0
15–20 mHz	7	2	7	0	0
20–27 mHz	3	0	2	2	0

Improved response improves efficiency
Future searches will subtract found bright signals & iterate

Amplitude Accuracy w/LWL and RA



Cumulative Histograms: Err/Sigma



Plans for the Future

- Crowder et al find $\sim 10\times$ as many galactic binaries
  We may need source subtraction
  to iteratively find "buried" signals
- Alternatives to coïncence condition
  to eliminate secondary maxima
- MLDC3 includes $\dot{f}$;
  fully coherent template bank may become prohibitive;
  may need to use semi-coherent method

Outline

1. Searches for Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Observations & Detectors
   - The Mock LISA Data Challenges
2. Searches for Periodic Gravitational Waves
   - $\mathcal{F}$-Statistic Search Technique
   - AEI $\mathcal{F}$-Statistic Search for White Dwarf Binaries
   - New Cross-Correlation Method for Periodic GW Searches

Cross-Correlation Search

- Full coherent search impossible on high-dim param space
- Semi-coherent methods incoherently combine short-time coherent searches
- Cross-correlation search designed for localized stochastic sources used to search for periodic signals from Low-Mass Xray Binaries (Ballmer, *CQG* **23**, S179 (2006); LSC, *PRD* **76**, 082003 (2007))
- Not ideal–neglects doppler-shift & long-term coherence
- Dhurandhar, Krishnan, Mukhopadhyay & JTW, arXiv:0712.1578 generalizes to include cross-correlation of different times from same detector; if all pairs included, approximates $\mathcal{F}$-stat; can be customized to adjust coherence time.

Conclusions

- Similar tech to search for rotating NS in LIGO/Virgo/etc & WD binary in LISA
- Mock LISA Data Challenge Searches (Prix, JTW, Khurana)
  - $\mathcal{F}$-statistic method to find doppler-shifted periodic signals applied to mock LISA data
  - Had to model LISA response beyond long-λ limit to get accurate amplitude param recovery
  - Weaker signals can be mistaken for secondary maxima partially overcome by coïncidence condition Probably need signal subtraction to go further
  - MLDC3 adds $\dot{f}$ dim to param space may need to use semi-coherent methods
- Cross-correlation technique proposed for LMXB searches (Dhurandhar, Krishnan, Mukhopadhyay, JTW)