# Bayesian burst detection better

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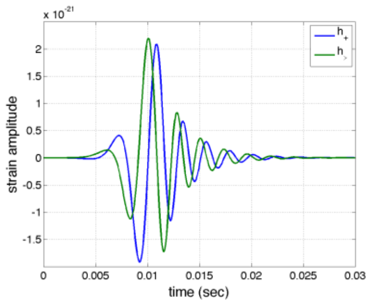
*A Bayesian approach to the detection of un-modelled bursts of gravitational radiation has been predicted to outperform existing methods. Monte-Carlo simulations confirm this and demonstrate that the improvement is significant.*

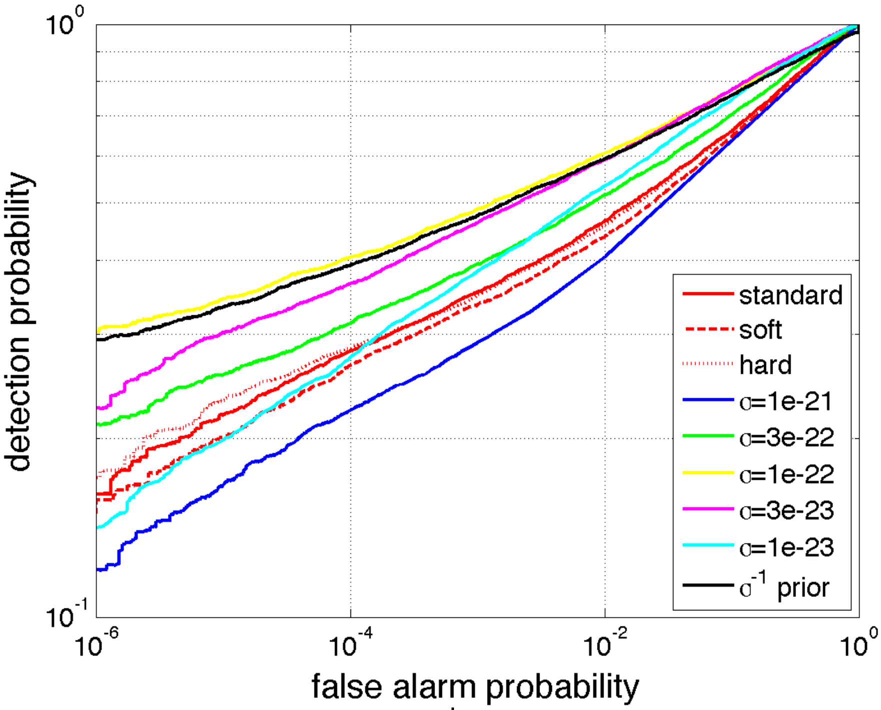
In [arXiv:0712.0196] we (with G Woan) presented a Bayesian analysis of the problem of detecting a gravitational wave burst of unknown waveform with a network of ground-based interferometric gravitational wave detectors, and demonstrated that several previously proposed coherent burst methods were equivalent to unphysical choices of Bayesian signal prior. This implied that previously proposed methods were optimal for unrealistic simulations, and that they should therefore be less efficient than a Bayesian method whose signal priors better reflected our state of knowledge.

If the problem of network burst detection is formulated as a choice between the hypotheses $\{H_0\colon \mathbf{x} = \mathbf{e},\ H_1\colon \mathbf{x} = \mathbf{Fh} + \mathbf{e}\}$, the Bayesian analysis follows immediately from specification of prior plausibility distributions for noise $p(\mathbf{e})$ and signal $p(\mathbf{h})$. In the absence of an explicit signal distribution, various arguments lead to different statistics: the excess power statistic checks if $H_0$ is "falsified", the Gursel-Tinto statistics looks for falsifying power only in span $\mathbf{F}$ where gravitational waves could be responsible for it and the soft and hard constraint differently weight the power arising from the two polarizations. Unfortunately these arguments are implicitly equivalent to proposing that signals are infinite or infinitesimal; the Bayesian method should perform better because it (quite literally) doesn't waste (signal) energy overcoming physically inaccurate assumptions about the universe.

To validate and quantify the expected improvement in detection efficiency, we conducted a Monte-Carlo simulation to produce ROC curves for the standard Gursel-Tinto, soft constraint, hard constraint and Bayesian statistics. The Bayesian statistic uses Gaussian noise and signal population models that it is the optimal statistic for (in the sense that it maximises the detected fraction for a given false alarm fraction).

*Figure 1: (below) Injected waveform*




However, we instead used a particular 20-20 solar mass inspiral Lazarus waveform source from random directions and fixed 100 MPc distance, injected into simulated identical detectors with H1, L, V, G's orientations. The average SNR in the network was 5. The Bayesian method has no special relationship with this more structured source population; the outcome is not predetermined.

For approximately 5,000 injections, we computed the Gursel-Tinto, soft constraint, hard constraint and five Bayesian odds ratios for five logarithmically distributed characteristic signal amplitudes σ. Combining the five Bayesian statistics marginalizes away the dependence on the nuisance signal amplitude with a scale invariant prior, producing a new Bayesian statistic that doesn't propose a particular signal size.

We can see in Figure 2 that the non-Bayesian statistics all perform quite similarly. The Bayesian statistics that propose that the signal has a particular amplitude perform better or worse than the non-Bayesian methods, depending on how close their proposed amplitude is to the true value. However, the marginalized Bayesian statistic performs almost as well as the best particular amplitude Bayesian statistic, and approximately twice as well as any of the non-Bayesian statistics (in the low false alarm regime). This means that we don't have to know the exact signal size to produce an efficient Bayesian test.

Limited trials suggest the Bayesian method also produces superior parameter estimation; in Figure 3 the Bayesian analysis correctly localises an injection to 1/10000th of the sky with 95% confidence, while the non-Bayesian statistics identify the wrong direction.

In conclusion, we have demonstrated that a Bayesian analysis significantly outperforms several previously proposed statistics, using a traditional Frequentist assessment of efficiency.

*Figure 2: (above) ROC for various statistics*

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*Figure 3: (below) Signal plausibility as a function of direction for Bayesian, Tikhonov, GT and soft statistics. White is plausible, black implausible; circle is the injection, square is the estimated direction*