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Displacement-noise-free gravitational-wave detection with a single Fabry-Perot cavity

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Outline

Idea of displacement-noise-free measurement

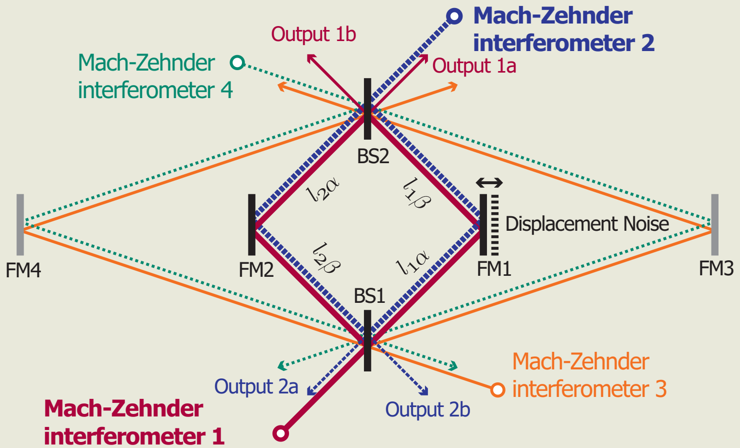
Idea

Kawamura and Chen with colleagues proclaimed the idea to exclude displacement noise in GW detectors using distributed nature of GW[a]. They consider several variants.

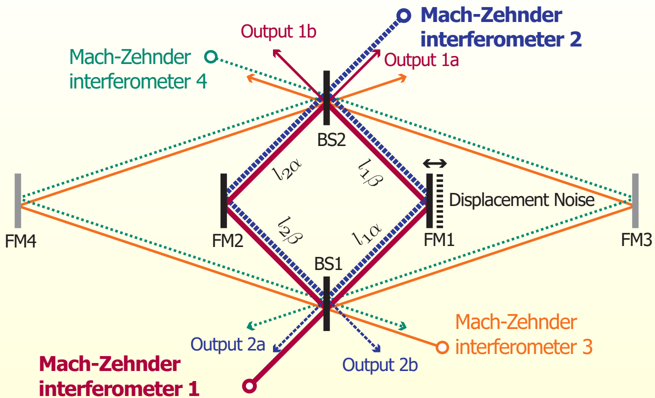
[a] S. Kawamura, Y. Chen, PRL, **93**, 211103 (2004),
Y. Chen, S. Kawamura, PRL, **96**, 231102 (2006),
Y. Chen, A. Pai, K. Somiya, S. Kawamura, S. Sato, K. Kokeyama, R. Ward, K. Goda and E. Mikhailov, PRL, **97**, 151103 (2007),
S. Sato, K. Kokeyama, R. Ward, S. Kawamura, Y. Chen, A. Pai, K. Somiya, PRL, **98**, 141101 (2007)

2D Mach-Zehnder scheme

Kawamura, Chen *et al* analysed several variants, for example:



Manipulating by outputs one **can exclude information on displacement of each 6 mirrors** and keep information on GW signal.



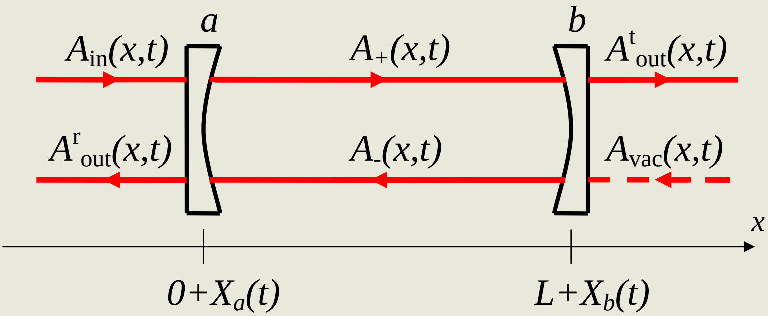
Shortage

In low-frequency region ($L \ll \lambda_{GW}$) the displacement-noise-free response signal decreases as $(f_{\rm GW} L/c)^3$.

Fabry-Perot cavity formed by two moved mirrors

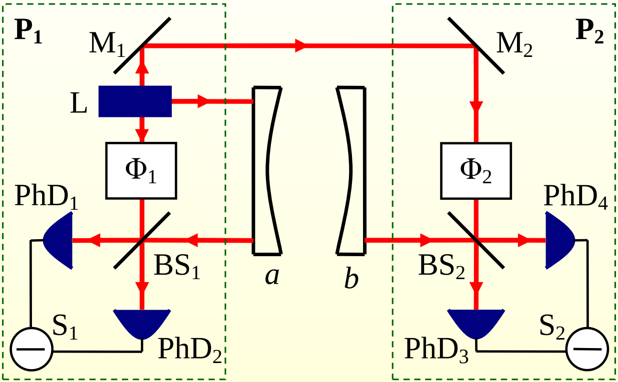
Analyzed scheme

We analyse a detuned Fabry-Perot cavity, two mirrors may move as free masses, its transmittances T and reflectivities R are the same.



Experimenter can manipulate by linear combination of the reflection-output $A^{\rm r}_{\rm out}(x,t)$ and transmission-output $A^{\rm t}_{\rm out}(x,t)$ signals.

Detailed scheme of Fabry-Perot cavity



Assumptions: rigid installation on platform

Laser L, mirrors M_1, M_2, beamsplitters and homodyne detectors are assumed to be **rigidly installed on not moving** platforms P_1 and P_2. We use local Lorentz (LL) gauge.

Double-pumped Fabry-Perot cavity: manipulation with 4 outputs

The pump wave through mirror *a* has detuning δ_1, polarization in the plane of incidence and denote it with $A_{\rm in}$; the pump wave through mirror *b* has different detuning δ_2, polarization orthogonal to the plane of incidence and is denoted with $B_{\rm in}$. Corresponding vacuum pumps through mirrors *b* and *a* are denoted with $A_{\rm vac}$ and $B_{\rm vac}$.

Small output amplitudes (double-pumped FP cavity)

$$a^{\rm r}_{\rm out} = \mathcal{R}_1 a_{\rm in} + \mathcal{T}_1 a_{\rm vac} - \frac{RT^2\mathcal{A}e^{2i\delta_1\tau}2ik_0\Big[(X_b + X_{\rm gw})e^{i\Omega\tau} - \sigma_1 X_a\Big]}{\big(1-R^2e^{2i\delta_1\tau}\big)\big(1-R^2e^{2i(\delta_1+\Omega)\tau}\big)},$$

$$a^{\rm t}_{\rm out} = \mathcal{T}_1 a_{\rm in} + \mathcal{R}_1 a_{\rm vac} + \frac{R^2T^2\mathcal{A}e^{3i\delta_1\tau}\,2ik_0\big[(X_b + X_{\rm gw})e^{2i\Omega\tau} - X_a e^{i\Omega\tau}\big]}{\big(1-R^2e^{2i\delta_1\tau}\big)\big(1-R^2e^{2i(\delta_1+\Omega)\tau}\big)}.$$

...and the similar formulas for amplitudes $b^{\rm r}_{\rm out},\ b^{\rm t}_{\rm out}$

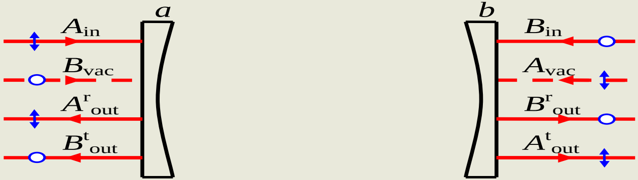
Important that coefficients $\sigma_1,\ \sigma_2 \neq 1$:

$$\sigma_1 \simeq 1 + 2i\delta\tau\,\frac{\gamma - i(\delta_1+\Omega)}{\gamma}$$

In opposite case the displacement-noise cancellation is impossible.

Cancellation of displacement noise

Proper linear combination of the reflection-output and transmission-output signals.



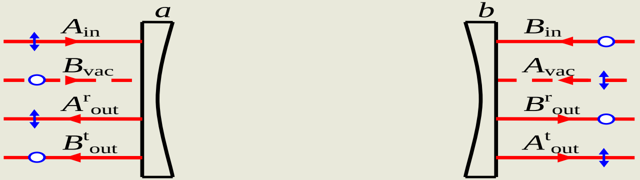
Step 1: cancelllation of information about X_a

From the first pair of signals $a_{\rm out}^{\rm r,t}$:

$$\begin{aligned} s_1 &= Re^{i(\delta_1+\Omega)\tau} a_{\rm out}^{\rm r} + \sigma_1 a_{\rm out}^{\rm t} \\ &= s_1^{\rm fl} + \frac{R^2 e^{i\delta_1\tau}\left(1-e^{2i\delta_1\tau}\right)}{\left(1-R^2 e^{2i\delta_1\tau}\right)} \mathcal{A}\, 2ik_0 (X_b + X_{\rm gw}) e^{2i\Omega\tau}\,. \end{aligned} \tag{1}$$

Cancellation of displacement noise (cont.)

Step 2: cancellation of information about $(-X_a + X_{\rm gw})$



From the second pair of signals $b^{\rm r,t}_{\rm out}$:

$$
\begin{aligned}
s_2 &= Re^{i(\delta_2+\Omega)\tau} b^{\rm r}_{\rm out} + b^{\rm t}_{\rm out} \\
&= s_2^{\rm fl} - \frac{R^2 e^{i\delta_2\tau}\left(1 - e^{2i\delta_2\tau}\right)}{\left(1 - R^2 e^{2i\delta_1\tau}\right)} \mathcal{B}\, 2ik_0 X_b e^{i\Omega\tau}. \qquad (2)
\end{aligned}
$$

Cancellation of displacement noise (cont.)

Step 3: cancellation of information about X_b from the pair of $s_{1,2}$

We assume that mean amplitudes of waves in cavity are equal to each other:

$$\frac{\mathcal{A}}{\left(1 - R^2 e^{2i\delta_1\tau}\right)} = \frac{\mathcal{B}}{\left(1 - R^2 e^{2i\delta_2\tau}\right)}$$

Then we can cancel the information about X_b from combinations $s_{1,2}$:

$$\begin{aligned} s &= s_1 + \frac{e^{i\delta_1\tau}\left(1 - e^{2i\delta_1\tau}\right)}{e^{i\delta_2\tau}\left(1 - e^{2i\delta_2\tau}\right)}\, s_2 e^{i\Omega\tau} \\ &= s^{\mathrm{fl}} + \frac{R^2 e^{i\delta_1\tau}\left(1 - e^{2i\delta_1\tau}\right)}{\left(1 - R^2 e^{2i\delta_1\tau}\right)} \mathcal{A}\, 2ik_0 X_{\mathrm{gw}} e^{2i\Omega\tau}. \end{aligned} \tag{3}$$

DFI response signal s **does not contain any information about displacement noise of the test masses.**

Particular cases

Equal detunings $\delta_1 = \delta_2$ and equal pumps $\mathcal{A} = \mathcal{B}$

In the narrow-band approximation

$$s\big|_{\delta_2=\delta_1} \approx a_{\rm in} + b_{\rm in} + a_{\rm vac} + b_{\rm vac} - \frac{i\delta_1}{\gamma - i\delta_1}\,\mathcal{A}\,2ik_0 Lh. \tag{4}$$

γ is the cavity half-bandwidth.

Opposite detunings $\delta_1 = -\delta_2$ and equal pumps amplitudes $\mathcal{A} = \mathcal{B}$

In the narrow-band approximation

$$s\big|_{\delta_2=-\delta_1} \approx a_{\rm in} - b_{\rm in} + a_{\rm vac} - b_{\rm vac} - \frac{i\delta_1}{\gamma - i\delta_1}\,\mathcal{A}\,2ik_0 Lh \tag{5}$$

Discussion

Key role

Key roles in isolation of the GW signal from displacement noise is played by the equivalence principle in terms of the LL gauge or by the distributed nature of GWs in terms of the TT gauge.

Impossible to apply DNFI to register non-gravitational force

Let the external non-gravitational force $\mathbf{F}(t)$ acts on mirror b along the x-axis. We denote the corresponding displacement of the mirror as X_Fs:

$$a^{\rm r}_{\rm out} \sim (X_b + X_F)e^{i\Omega\tau} - \sigma X_a, \quad a^{\rm t}_{\rm out} \sim (X_b + X_F)e^{2i\Omega\tau} - X_a e^{i\Omega\tau},$$
$$b^{\rm r}_{\rm out} \sim - X_a^{i\Omega\tau} + \sigma(X_b + X_F), \quad b^{\rm t}_{\rm out} \sim -X_a e^{2i\Omega\tau} + (X_b + X_F)e^{i\Omega\tau}.$$

Force-induced displacement X_F cannot be separated from X_b in all the output signals.

Comparison

Sensitivity

Sensitivity of scheme with double pumped FP cavity is no better or worse than a simple one-round-trip detector.

Comparision with conventional FP cavity

In LIGO the signal is greater due to resonance gain.
So to reach SQL sensitivity in our double pumped FP cavity we need light amplitude approximately finesse times larger than in scheme with conventional FP cavity (not noise-free).

Comparison with displacement-noise-free Mach-Zehnder topology

Sensitivity of displacement-noise free topology with the Mach-Zander interferometer is worse by factor $(\Omega L/c)^3$ than with our double pumped FP cavity.

Laser noise cancelation

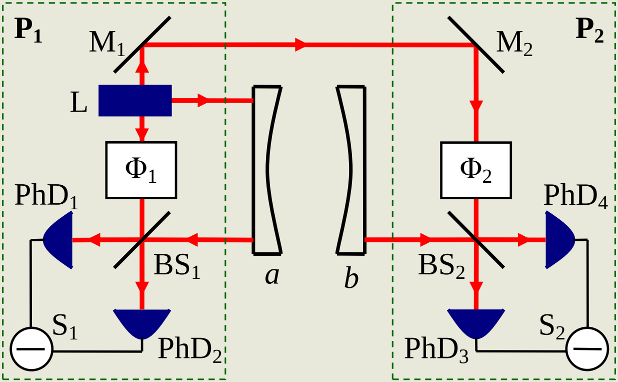
Dispacement-noise-free with Mach-Zahnder topology

Recal that Kawamura, Chen and colleagues proposed to subtract laser noise in displacement-noise free topology with the Mach-Zander interrferometer. In particlar, it allows to cancel noise produced by possible laser displacements.

Dispacement-noise-free with double-pumped FP cavity

The Laser noise cancelation is also possible in our scheme:
It is subject of separate analysis.
The major problem — there are the additional beamsplitters and mirrors producing displacement noise.

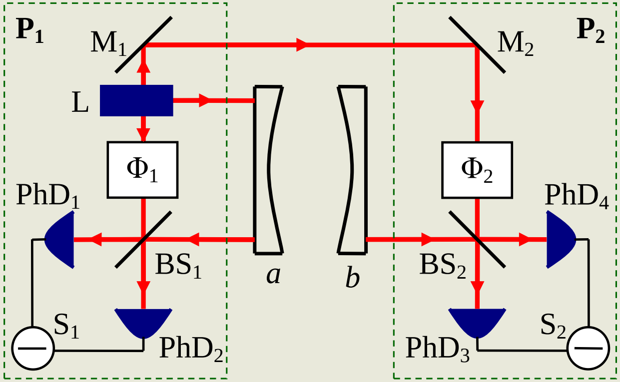
No displacement noise from beamsplitters and additional mirrors



In LIGO there is resonance gain of signal,
in displacement-noise-free configuration — no resonance gain.

The vulnerable assumptions (cont.)

Platforms can not move



Under consideration:

mirrors are rigidly attached to movable platforms.

Acknowledgments

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