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Verification of conditional quantum state in GW detectors

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Outline

1. MQM experiment and “philosophy” of verification
2. Time scales and precision of verification experiment
   - “White” noise
   - Non-“white” noise
3. Conclusion

What is MQM experiment? Why do we need verification?

GOAL of MQM experiment

Prepare, manipulate and observe quantum state of macroscopic test masses, thereby testing quantum mechanics in macroscopic world, using interferometric gravitational wave detectors

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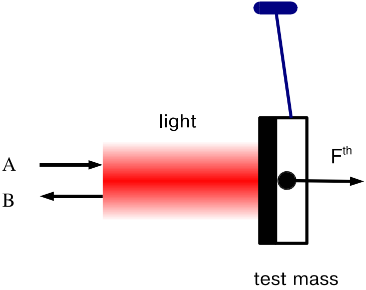
What the success of verification depends on, and why do we believe in it?

- On classical noise budget being below the SQL in the frequency band of interest;
- Use of QND measurement techniques will allow to probe quantum state with sub-SQL precision!

Simple conceptual scheme of the MQM experiment

Linear Gaussian system coupled with light $\Longrightarrow$ Heisenberg equations:



Heisenberg equations for our system

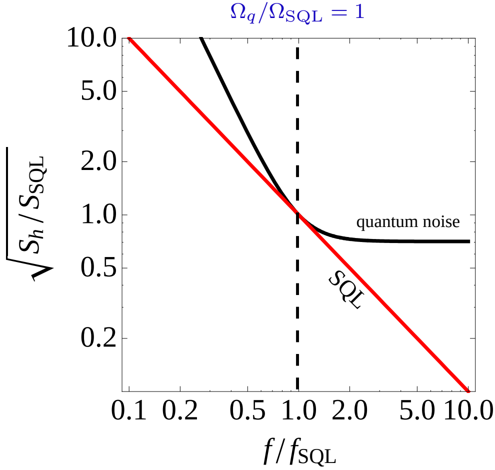
$$
\begin{aligned}
m\ddot{\hat{X}} + 2\gamma_p \dot{\hat{X}} + \omega_p^2 \hat{X} &= \alpha \hat{A}_1 + F^{\rm th}\,, \\
\hat{B}_1 &= \hat{A}_1\,, \\
\hat{B}_2 &= \hat{A}_2 + \frac{\alpha}{\hbar}(\hat{X} + X^{\rm th})\,.
\end{aligned}
$$

$\alpha = \sqrt{\hbar m \Omega_q}$: measurement strength,
$\Omega_q \sim 1/\tau_q$: measurement timescale.

How big is the probability to observe the quantum behavior of the test mass?

The answer depends on how strong is it coupled to the environment, *i.e.* how big are thermal and quantum noises.

"White" noise



Quantum noise

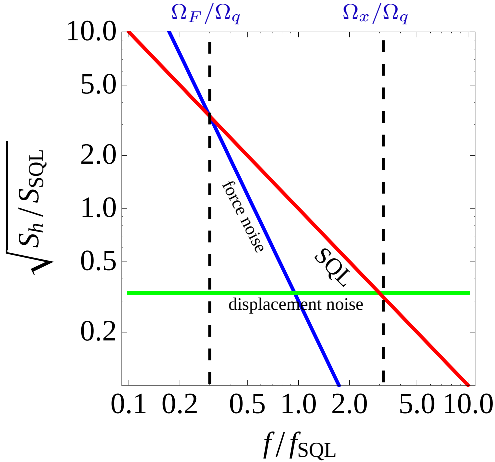
Shot and radiation pressure noises $\Longrightarrow$ white:

$$S_x^q = \frac{\hbar}{m\Omega_q^2} = const$$

$$S_F^q = \hbar m\Omega_q^2 = const$$

$$S_{\rm GW}^q(\Omega) = S_x^q + \frac{S_F^q}{m^2\Omega^4} = \frac{\hbar}{m\Omega^2}\left[\frac{\Omega^2}{\Omega_q^2} + \frac{\Omega_q^2}{\Omega^2}\right]$$

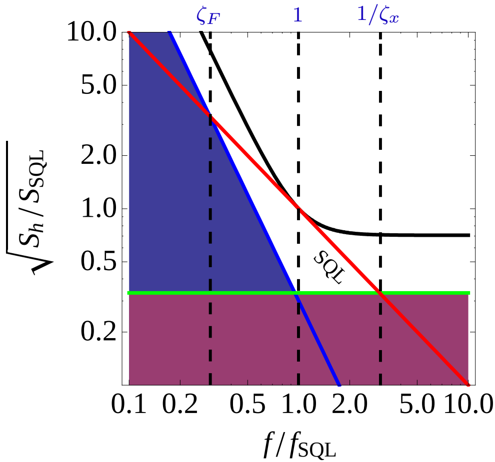
"White" noise



Classical noise

Classical suspension and mirror internal $\Longrightarrow$ white:

$$
\begin{aligned}
S_x^{\rm th} &= \frac{2\hbar}{m\Omega_x^2} = const\,, \\
S_F^{\rm th} &= 2\hbar m \Omega_F^2 = const\,, \\
S_{\rm GW}^{\rm th}(\Omega) &= S_x^{\rm th} + \frac{S_F^{\rm force}}{m^2\Omega^4}\,.
\end{aligned}
$$



Total noise

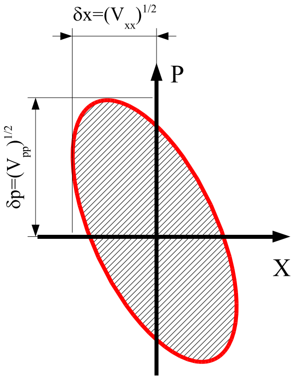
Spectral density of the total noise will be then:

$$S^{\rm tot}_{\rm GW}(\Omega) = S^{q}_{\rm GW}(\Omega) + S^{\rm th}_{\rm GW}(\Omega) =$$
$$= S^q_x(1+2\zeta_x^2) + \frac{S^q_F(1+2\zeta_F^2)}{m^2\Omega^4}\,,$$

where

$$\zeta_x = \frac{\Omega_q}{\Omega_x} \quad \text{and} \quad \zeta_F = \frac{\Omega_F}{\Omega_q}\,.$$

Gaussian states $\Longleftrightarrow$ covariance matrix



$$\mathbb{V}(t) = \begin{bmatrix} \langle \delta\hat{x}^2 \rangle & \langle \delta\hat{x}\delta\hat{p} \rangle_{\rm sym} \\ \langle \delta\hat{p}\delta\hat{x} \rangle_{\rm sym} & \langle \delta\hat{p}^2 \rangle \end{bmatrix}$$

If the test mass is in

- Pure quantum state $\Longrightarrow \det \mathbb{V} = \hbar^2/4$
- Classical mixed state $\Longrightarrow \det \mathbb{V} \gg \hbar^2/4$

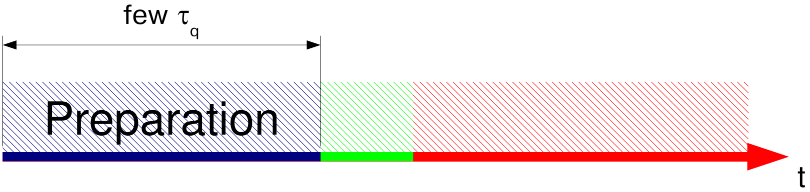
Scheme of state preparation

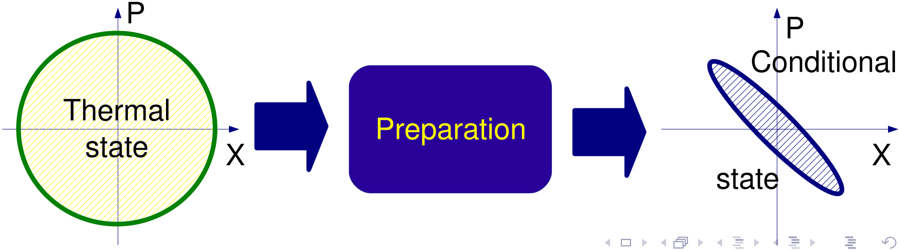
How long does quantum state survive?

State survival timescales $\Longleftrightarrow$ noise budget.
Survival time $\Longrightarrow$ $\tau_F \sim 1/\Omega_F$

Preparation time:

$\tau_\mathrm{P} < \tau_F$





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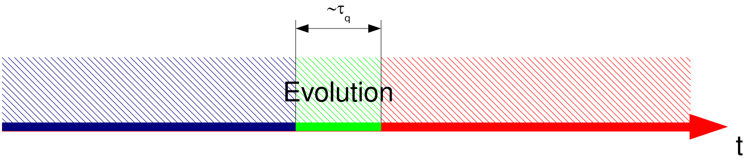

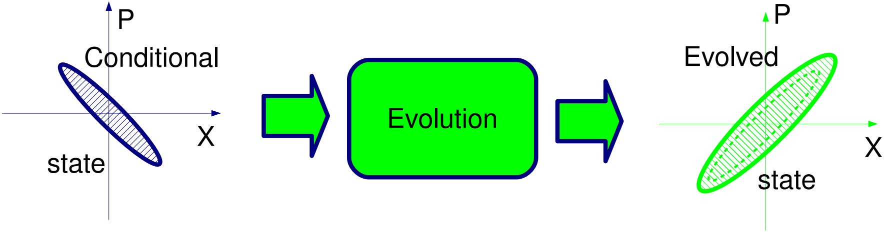

Timescales of verification and survival of test mass state

Covariance matrix of the test mass during verification has two parts

$$\mathbb{V}^{\mathrm{tot}} = \mathbb{V}^{\mathrm{cond}} + \mathbb{V}^{\mathrm{add}}$$

- $\mathbb{V}^{\mathrm{cond}}$ corresponds to the test mass conditional state after preparation,
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Order of magnitude estimate of entries of $\mathbb{V}^{\mathrm{cond}}$

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Variance of test mass displacement at measurement time τ:

$$\delta x^2 = V_{xx}^{\rm cond} = \frac{S_x^{\rm tot}}{\tau} + \frac{\tau^3 S_F^{\rm tot}}{m^2}$$

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Estimate using inequality between arithmetic and geometric mean:

$$\delta x^2 = V_{xx}^{\rm cond} = \frac{1}{3}\frac{S_x^{\rm tot}}{\tau} + \frac{1}{3}\frac{S_x^{\rm tot}}{\tau} + \frac{1}{3}\frac{S_x^{\rm tot}}{\tau} + \frac{\tau^3 S_F^{\rm tot}}{m^2} \geq \sqrt[4]{\frac{1}{3^3}\frac{(S_x^{\rm tot})^3 S_F^{\rm tot}}{m^2}}$$

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Minimal displacement variance takes place at $\tau = \tau_q$:

$$\delta x^2 = V_{xx}^{\rm cond} = \sqrt[4]{\frac{1}{3^3}\frac{(S_x^{\rm tot})^3 S_F^{\rm tot}}{m^2}} \simeq (1+2\zeta_x)^{3/4}(1+2\zeta_F)^{1/4}(\delta x^q)^2$$

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Similarly

$$V_{pp}^{\rm cond} = \delta p^2 \sim m^2 S_x^{\rm tot}/\tau^3 + \tau S_F^{\rm tot} \gtrsim (\delta p_q)^2 (1+2\zeta_x^2)^{\frac{1}{4}} (1+2\zeta_F^2)^{\frac{3}{4}}$$

and

$$V_{xp}^{\rm cond} \simeq \delta x \delta p \gtrsim \hbar/2 (1+2\zeta_x^2)^{\frac{1}{2}} (1+2\zeta_F^2)^{\frac{1}{2}} \,.$$

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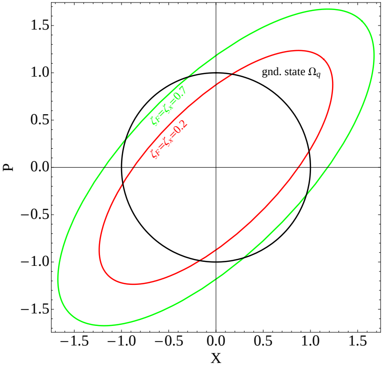


Purity of conditional state

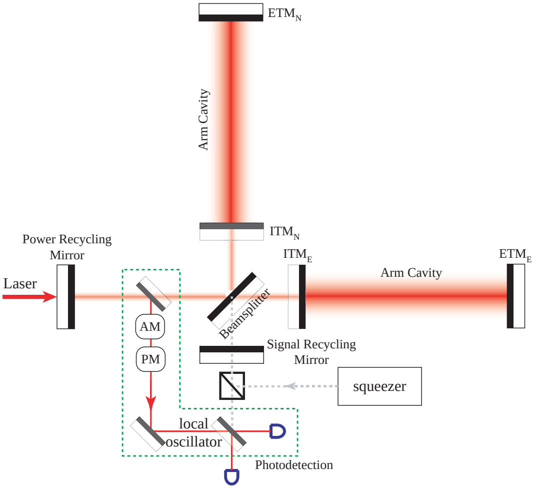
$$\det \mathbb{V}^{\rm cond} = V_{xx}^{\rm cond} V_{pp}^{\rm cond} - (V_{xp}^{\rm cond})^2 \simeq \frac{\hbar^2}{4}(1+2\zeta_x^2)(1+2\zeta_F^2) \geq \frac{\hbar^2}{4}$$

Parameters for left panel:

$\zeta_x = 0.2, \quad \zeta_F = 0.2$

$\zeta_x = 0.7, \quad \zeta_F = 0.7$

Ground state of oscillator with frequency Ω_q



Back action evasion for quantum state verification

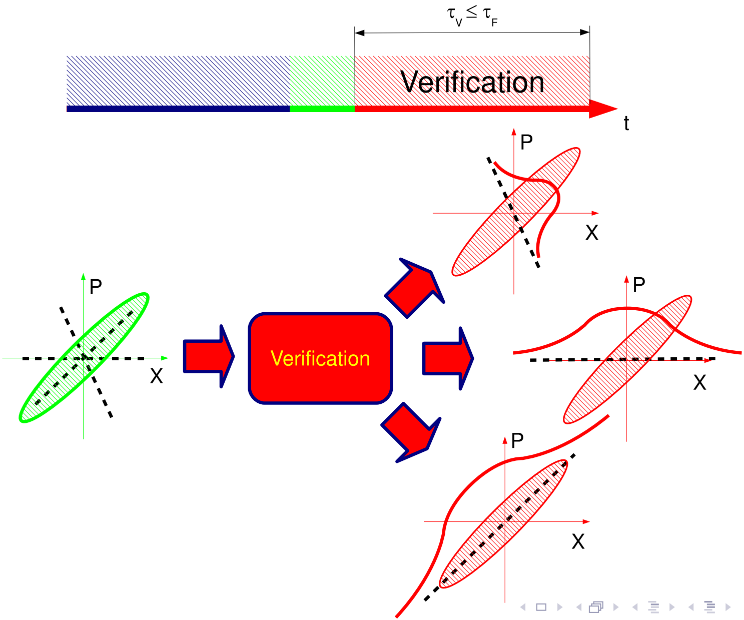
Use AM and FM of local oscillator light in homodyne scheme in the optimal way to eliminate radiation pressure of light from the measured output quadrature

All terms corresponding to S_F^q nullify:

$$(1 + 2\zeta_F^2) \Longrightarrow 2\zeta_F^2$$

S.P. Vyatchanin, E.A. Zubova, Phys. Lett. A ***201****, 265 (1995)*

QND methods in verification



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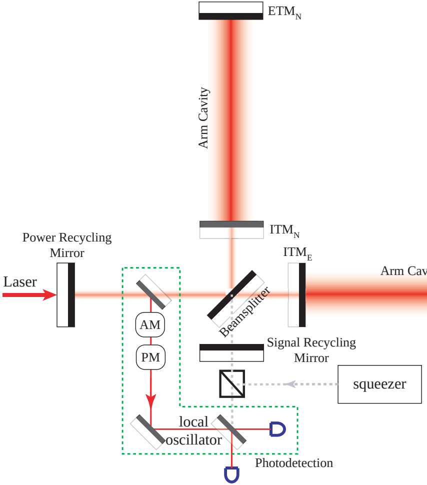
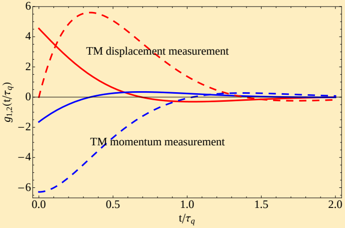


Optimal modulation functions for BAE



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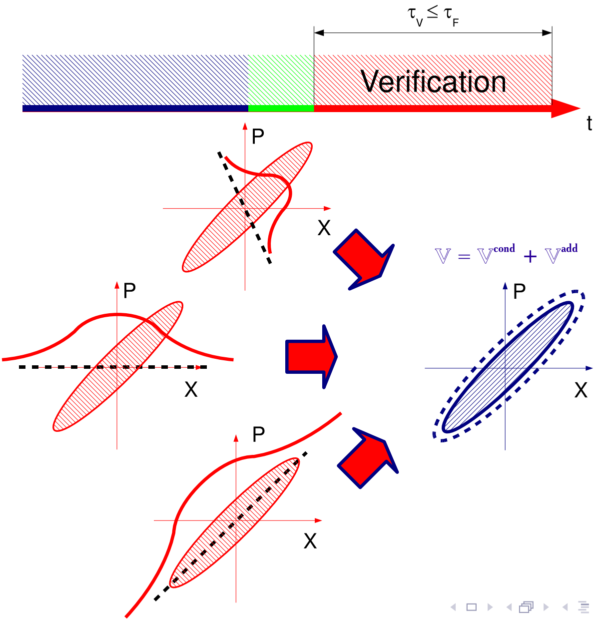
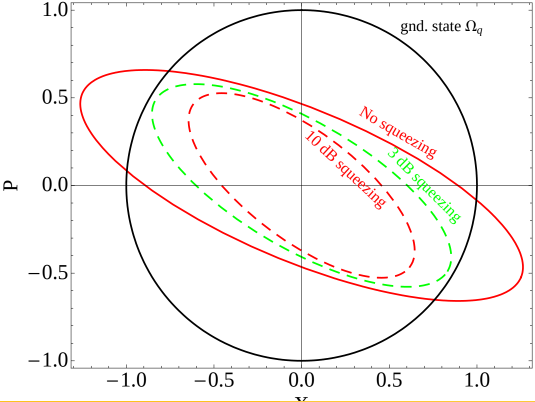
$$\zeta_F = \Omega_F / \Omega_q$$

QND methods in verification



Order of magnitude estimate of $\mathbb{V}^{\mathrm{add}}$

$$\delta x_V^2 \sim S_x^{\mathrm{tot}}/\tau + \tau^3 S_F^{\mathrm{th}}/m^2 \sim (1+2\zeta_x^2)^{3/4}\zeta_F^{1/2}\delta x_q^2$$
$$\delta p_V^2 \sim m^2 S_x^{\mathrm{tot}}/\tau^3 + \tau S_F^{\mathrm{th}} \sim (1+2\zeta_x^2)^{1/4}\zeta_F^{3/2}\delta p_q^2\,,$$



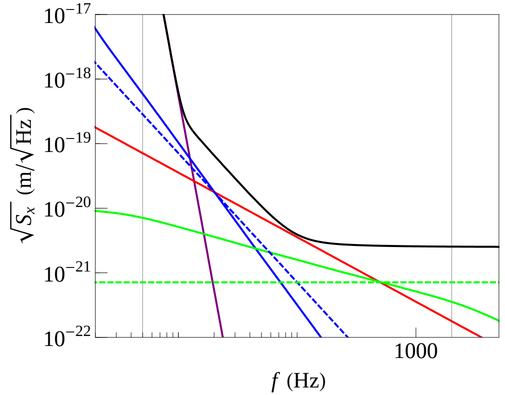
Precision of verification

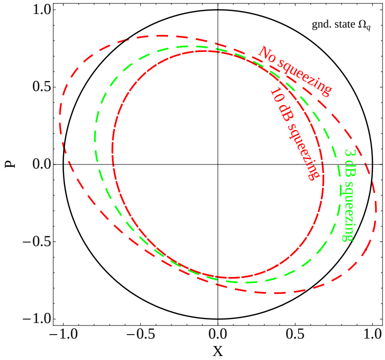
- Sub-Heisenberg sensitivity available with BAE
- Squeezing increase precision

Noise: $\zeta_x = \Omega_q/\Omega_x = 0.2$, $\zeta_F = \Omega_F/\Omega_q = 0.2$

Verification with non-“white” noise

Realistic noises $\Longrightarrow$ still sub-Heisenberg precision





Realistic noise budget for GW detector

Conclusion

1. GW detectors can be used to study quantum mechanics of truly macroscopic test masses, by using a two-staged process of preparation and verification;
2. Quantum state can be verified with sub-Heisenberg precision using plausible experimental technology;
3. Elaborated procedure can be also applied in small scale devices, with even more ease;

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THANK YOU

FOR YOUR ATTENTION!!!