Recent Parametric Instability Modeling

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Caltech LIGO

PI "*R*" calculation from entire cavity field

- Previous: model arm cavity field as discreet SHOs

$$R = \frac{4PQ_m}{mL{\omega_m}^2 c}\left(\sum_i \frac{Q_i^s \Lambda_i^s}{1+(\Delta\omega_i^s/\delta_i^s)^2} - \sum_j \frac{Q_j^{as}\Lambda_j^{as}}{1+(\Delta\omega_j^{as}/\delta_j^{as})^2}\right)$$

Stokes *a-Stokes* ***Each*** acoustic {m}

Braginsky, Vyatchanin

- Cavity specific modes {j}, Q_j, Λ_{jm} need accurate specification
  - » Proves difficult for modest order HTMs: high loss/distortion
  - » Is a significant "background" PI missed (K. Thorne, W. Kells: aS ~cancels S)?

- Now: build on FFT tool, ideally suited to net SS field from <<λ *static* distortions (acoustic surface amplitude from FEA)

$$R = \frac{4PQ_m}{m{\omega_m}^2 c}\left(\frac{V\int|E_0^s|Im(E_{bk}^s)u_z dA}{\int|E_0^s|^2 dA\int|\vec{u}|^2 dV} - \frac{V\int|E_0^{as}|Im(E_{bk}^{as})u_z dA}{\int|E_0^{as}|^2 dA\int|\vec{u}|^2 dV}\right)$$

- Instead of SB(+/- f_m) modes: FFT cavity length shifted by $\pm\frac{\lambda}{2}\frac{f_m}{FSR}$ for proper Stokes (a-Stokes) simulation.

FFT "*R*" values: AdvLIGO arm

- Complete R^S-R^{aS} for AdvLIGO arm

  FEA generated {m}: f_m to few %

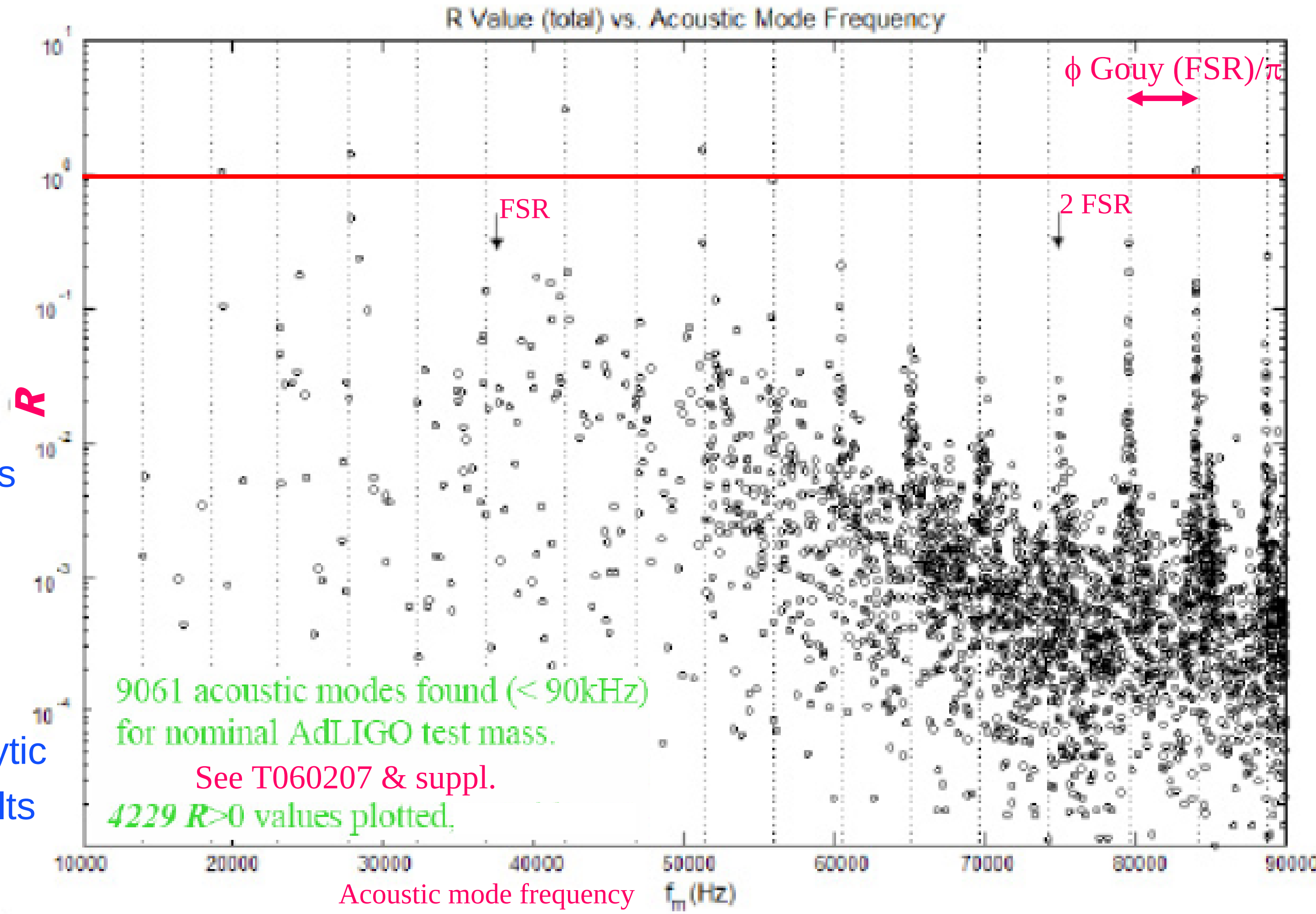
  No flats, ears, wedge

  ~only 6 R>1

  No anomalous "background"

  All high R values correspond to known HTMs

  Simple cases agree with analytic & previous results



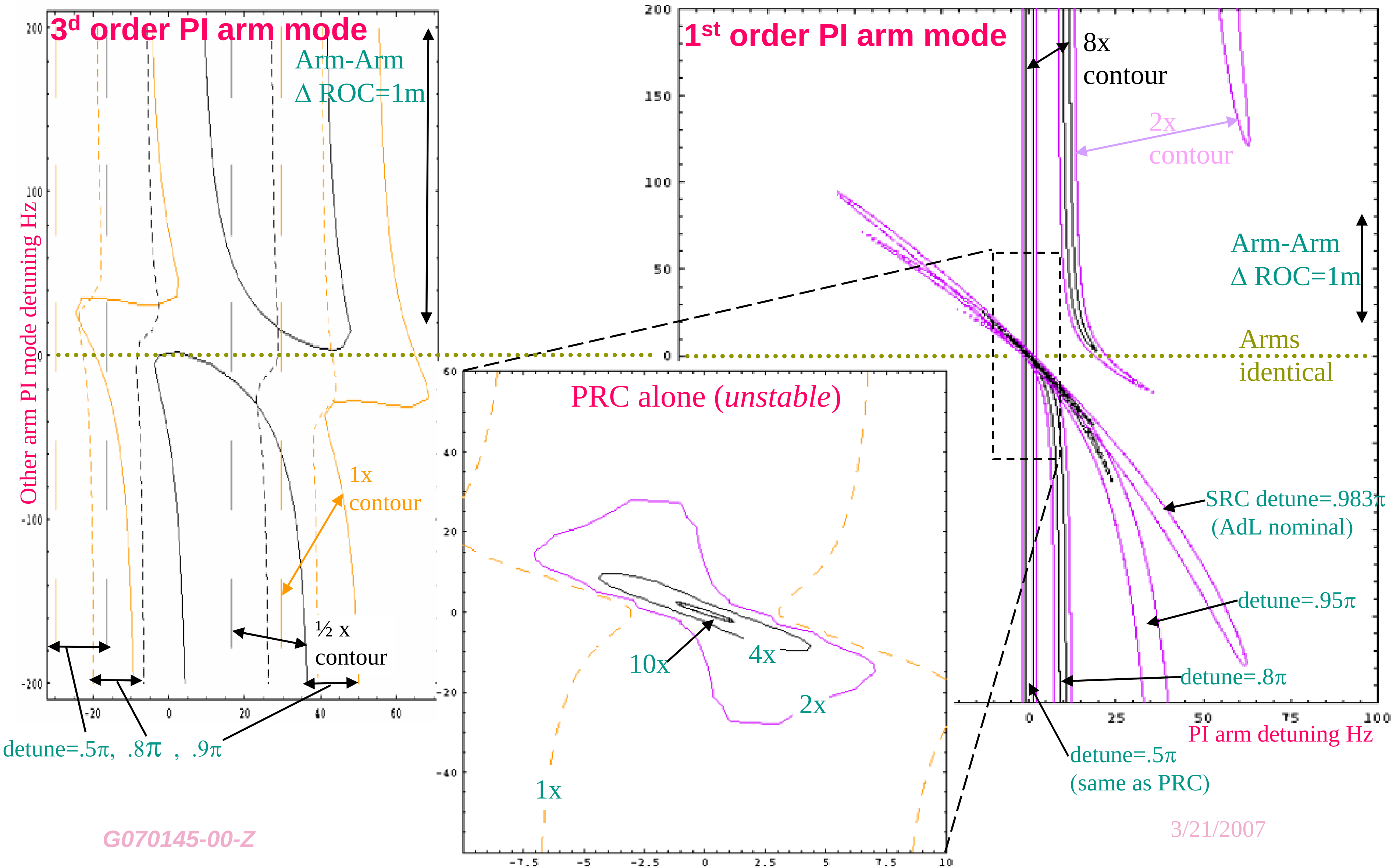
Full Ifo: essential Features

- Single arm only
  - » PI gain strongly depends on Q_m and Q_j
    - – For AdL this limits plausible {m} to f_m<90kHz, ~3-4000 Stokes dominated modes
    - – Above at least 6^{th} order {j} cavity loss too high
    - – 1^{st} order (tilt) and perhaps 2^{nd} order (curvature) special (control possible).

⇨ Complete survey Via FFT simulation

- PRC only (AdL parameters)
  - » ~10x recycling enhancement of PI gain. However:
    - – This peaking is ~10x narrower (+/-2 Hz tune of PI mode from arm res.)
    - – Arms must have nearly same Gouy phase (Δ ROC <1-2m)
    - – No enhancement (over arm alone PI gain) for > 3d order Pi modes (too lossy)
- PRC + SRC (AdL parameters) LIGO-T060159 & G060475 (Braginsky, Vyatchanin, et al TBP)
  - » Peak gain enhancements (& widths) < PRC alone.
  - » Significant “pulling” of peaks with SRC phasing (GW detuning + Gouy multiplet)
  - » Certain phases are dangerous: Δ ROC_{arm} “protection” obviated

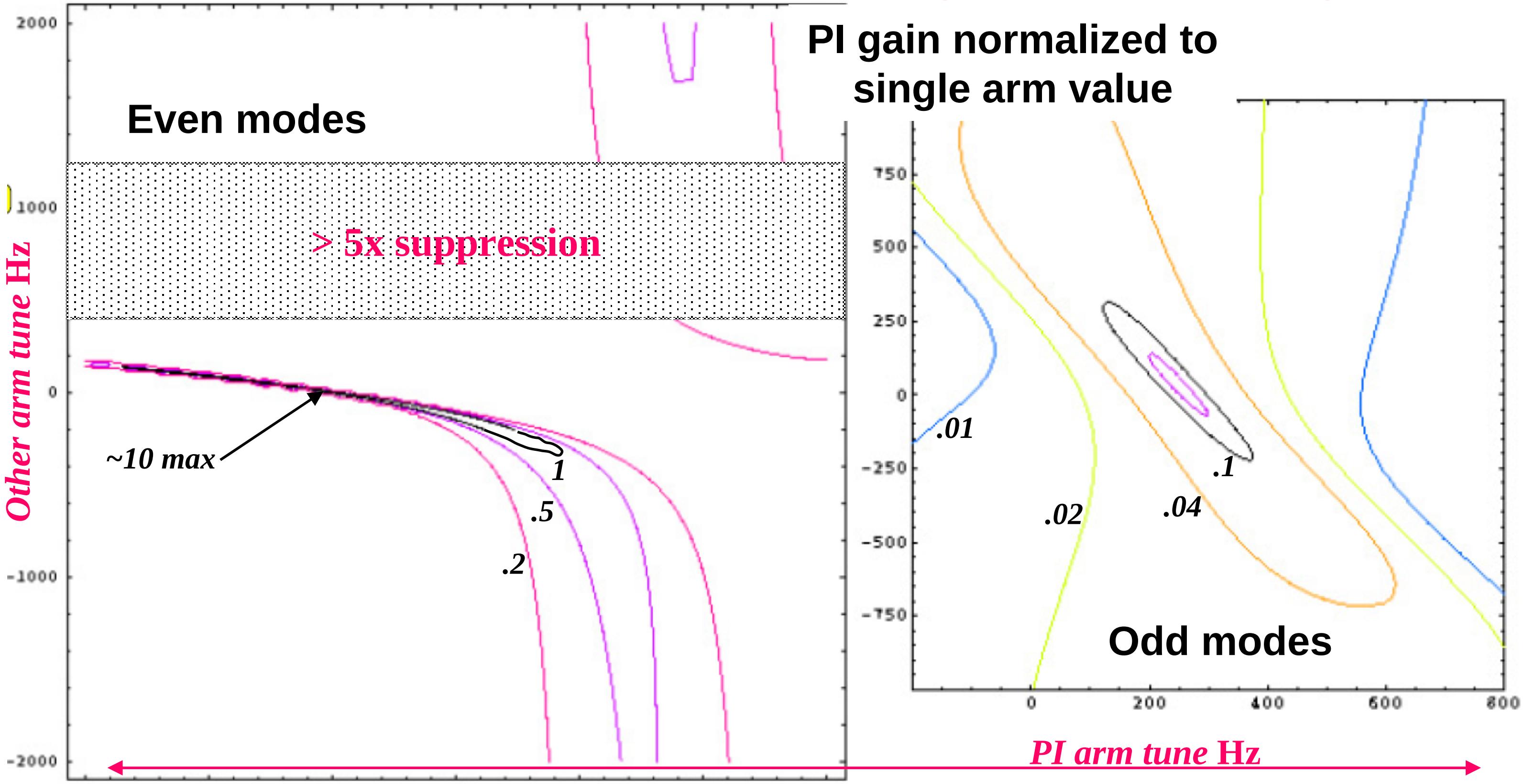
Recycled tuning landscape



LIGO

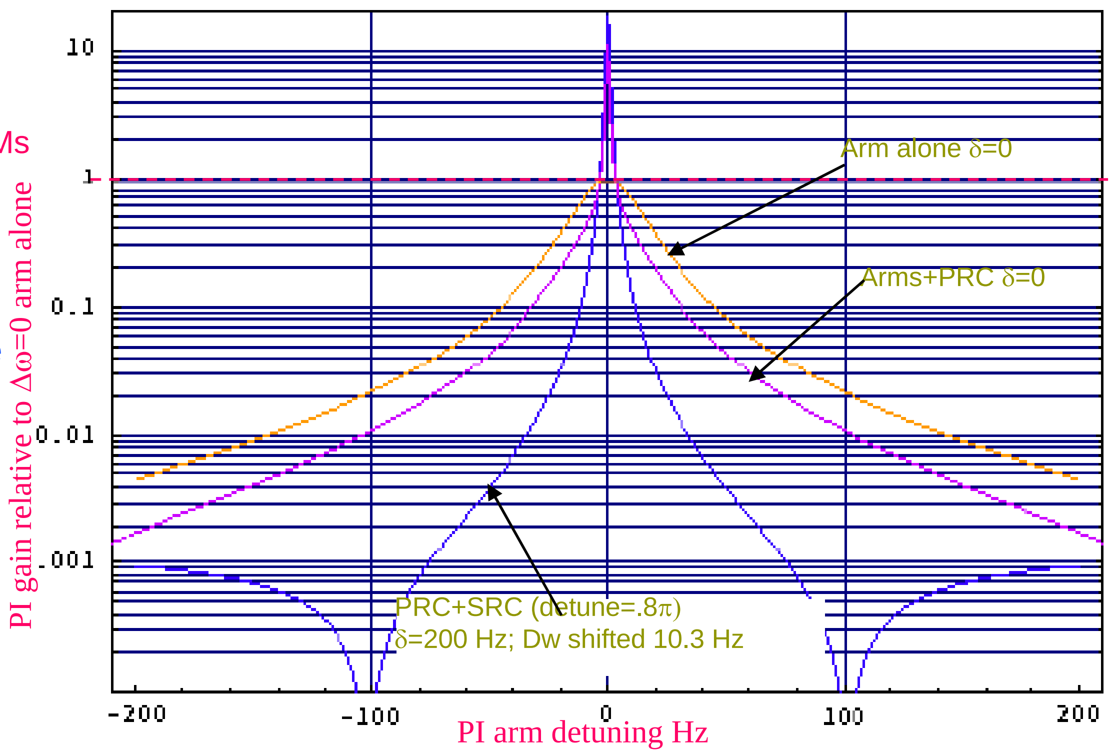
Fine Tuning !?

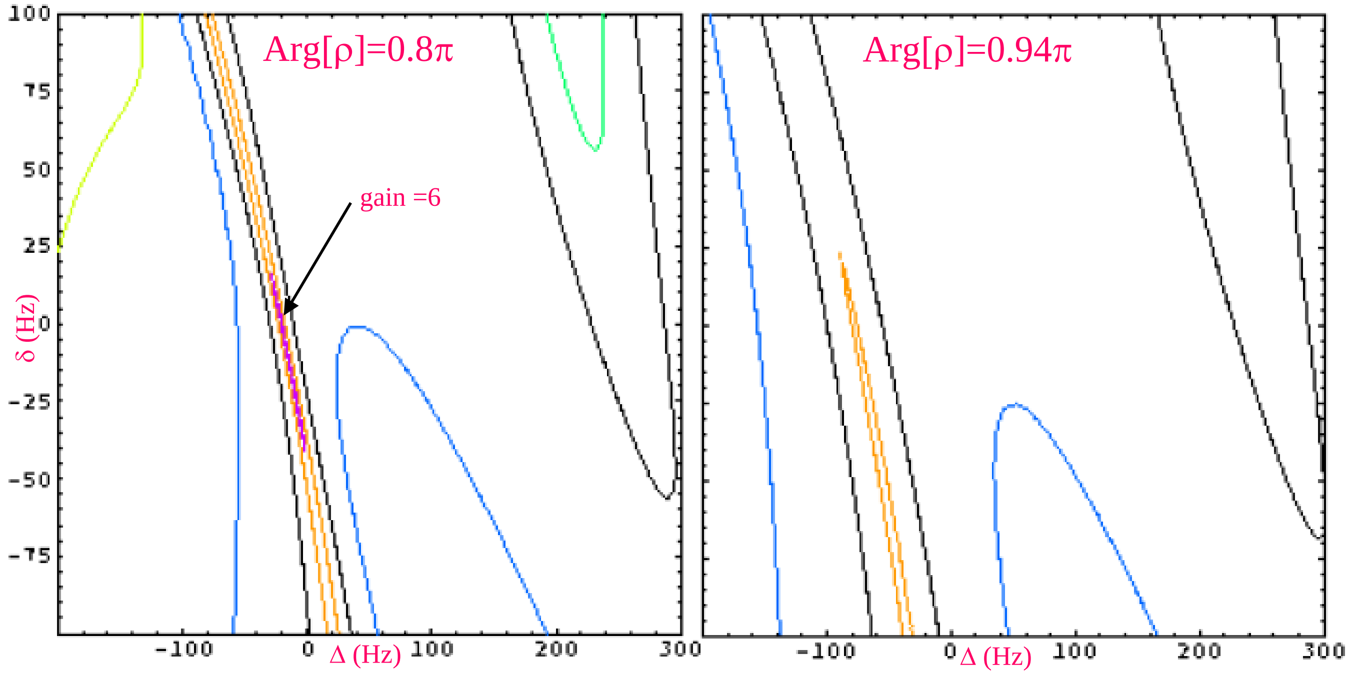
- Incorporating SRC/PRC stability DOF, can AdLIGO optics configuration be “tuned” to mitigate PI?
- Generally: no. Special case: confocal PRC + ΔROC large (D. Ottaway) can suppress all PI mode gains of concern large factor below single arm value.



PI gain widths !

- SRC increases enhancement ridge "area" in 2 arm tuning
- Large SRC Gouy phase range leaves ridge unavoidable for allowed ΔROC
- However, width of ridge is also critical !
  Likelihood of "hitting" enhancement:
  ~(enhancement width)/FSR =4Hz/38kHz
  x ~6000/2 acoustic modes x 4 TM x 2-3 HTMs
  = order 3 probable cases
- Already PRC reduces width factor 2
- SRC narrows further, so may actually be Advantageous if a very few exact resonances can be avoided (e.g. thermal tuning)

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gain =6
δ (Hz)
Δ (Hz)
Arg[ρ]=0.94π
Δ (Hz)
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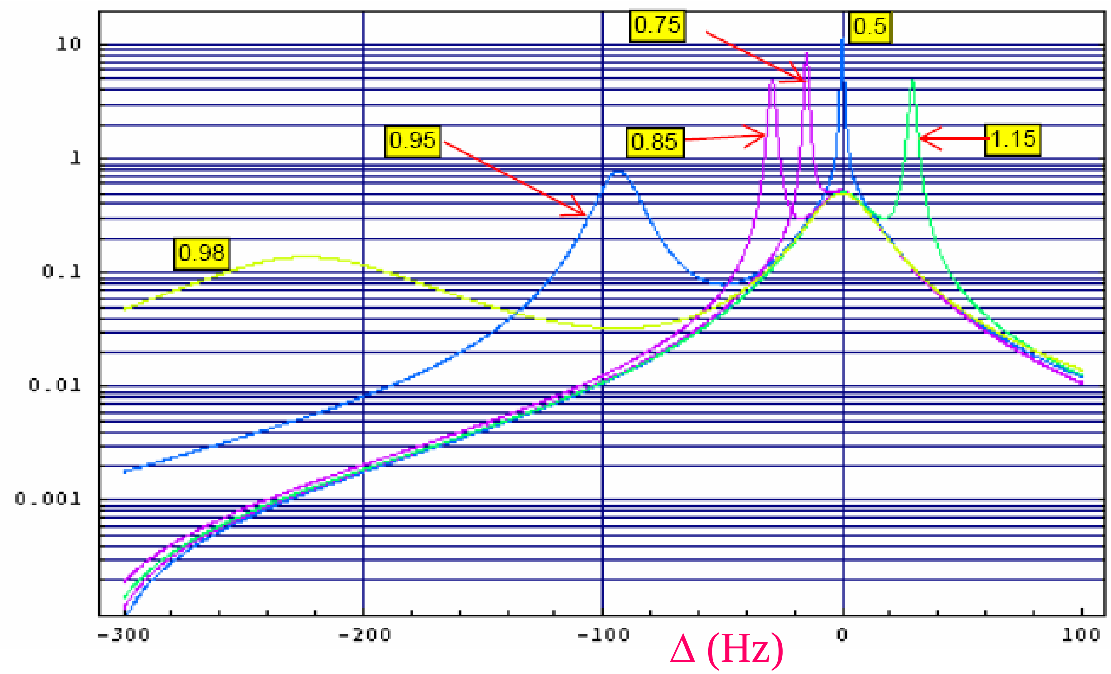
75
50
25
0
-25
-50
-75
-100
0
100
200
300


10
1
0.1
0.01
0.001
-300
-200
-100
0
100
0.75
0.5
0.95
0.85
1.15
0.98
Δ (Hz)