

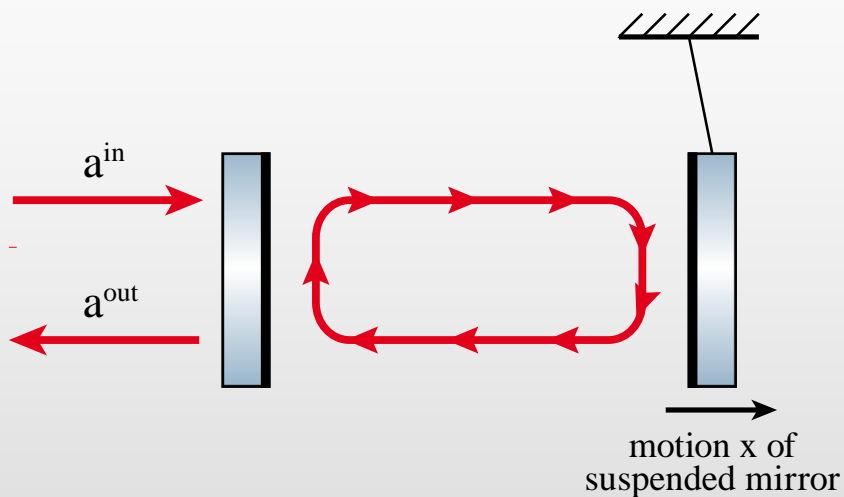
# Improving Advanced LIGO sensitivity using a local readout scheme

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# Dynamics of Single Cavity

## Equations of motion of suspended mirror



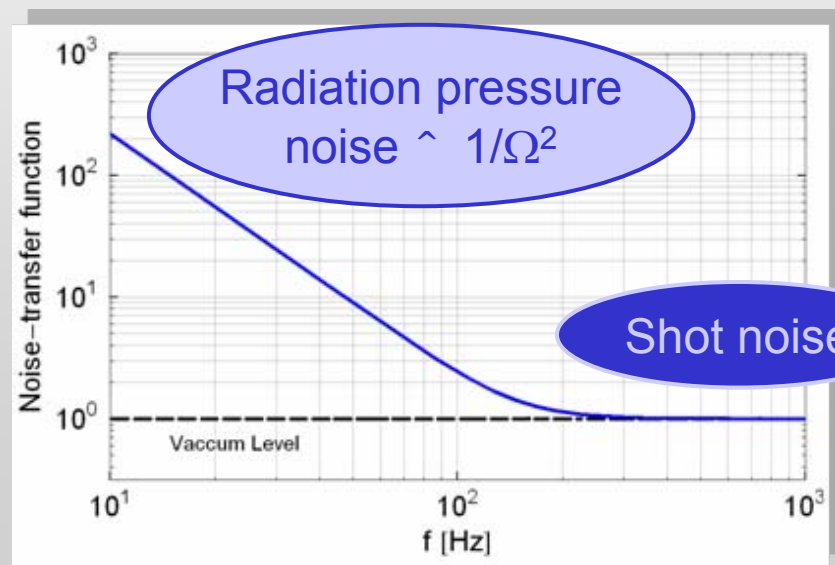
$$(m\ddot{x} = \text{Force})$$

Suspended mirror is subject to fluctuating radiation pressure force  $F_0(\Omega)$  and possible GW forces

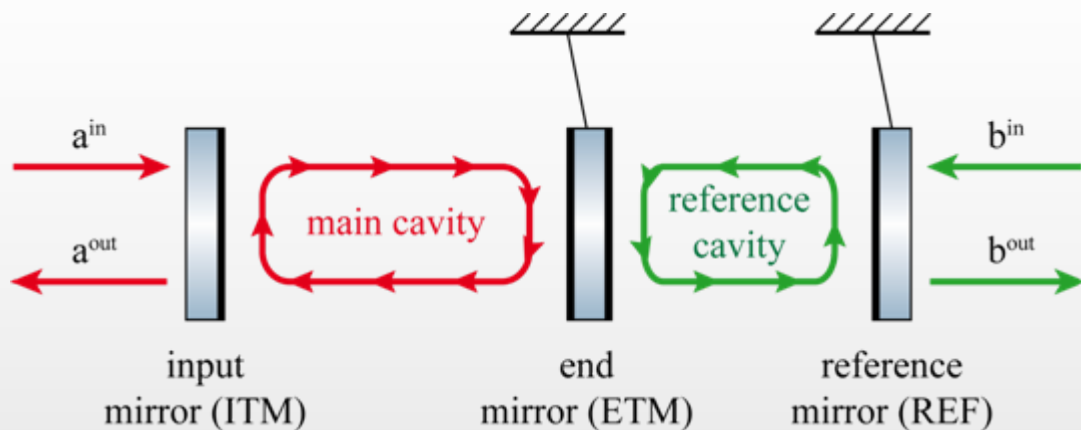
$$\hat{x}(\Omega) = R_{xx}(\Omega)\hat{F}_0(\Omega) + \text{GW Force}$$

$$R_{xx}(\Omega) \approx \frac{1}{m\Omega^2}$$

$$\hat{F}_0(\Omega) \sim \frac{1}{\gamma - i\Omega} \hat{a}_1^{\text{in}}$$



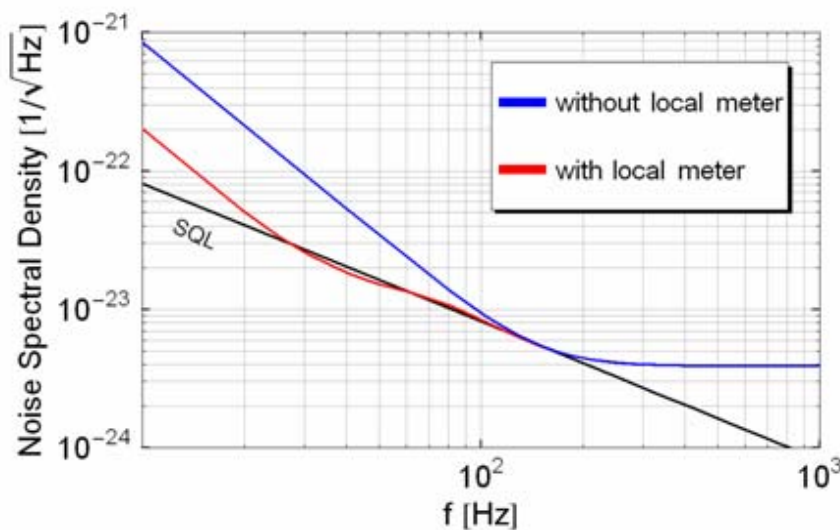
# Monitoring End-Mirror Displacement



Radiation pressure induced position fluctuations of the ETM caused by the main cavity field are measured by a reference cavity.

## Detection strategy:

- Output  $b^{\text{out}}$  can be feed back to ETM
- Optimal noise spectral density can be computed from outputs  $a^{\text{out}}$  and  $b^{\text{out}}$



Heidmann et. al. [quant-ph/0311167]



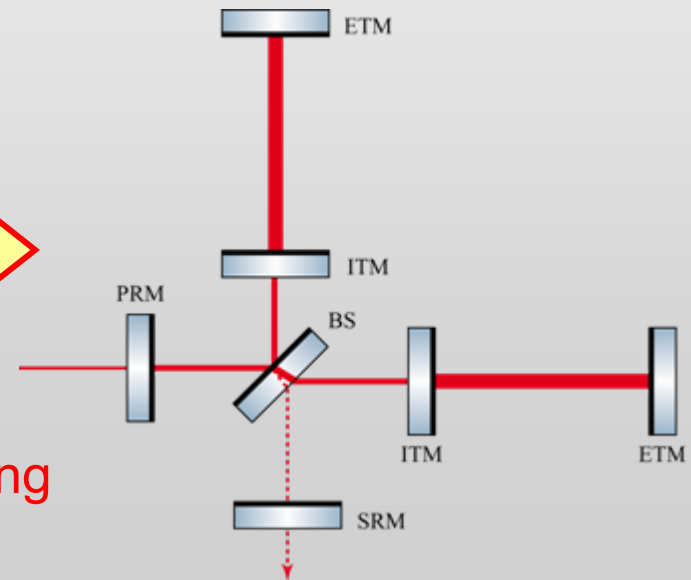
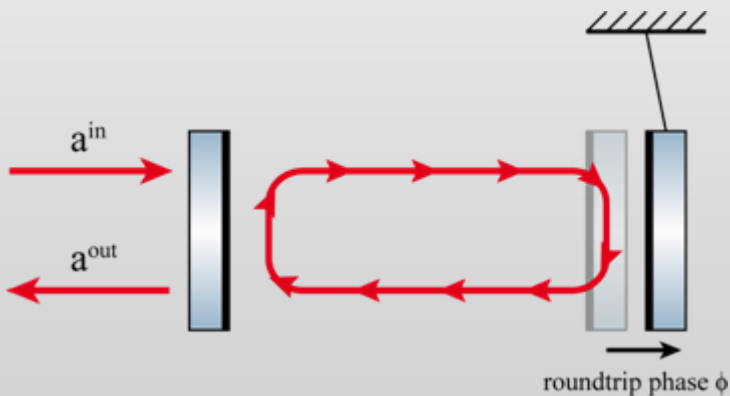
# Detuned Cavity

Detuning the cavity by  $\phi$  modifies equation of motion:

$$\hat{x}(\Omega) = R_{xx} \left( \hat{F}_0(\Omega) + R_{FF}(\Omega) \hat{x}(\Omega) \right) + \text{GW Force}$$

Radiation pressure

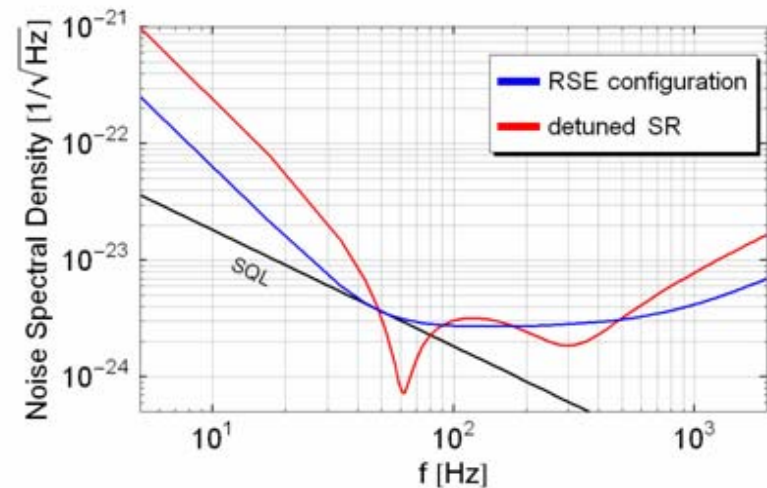
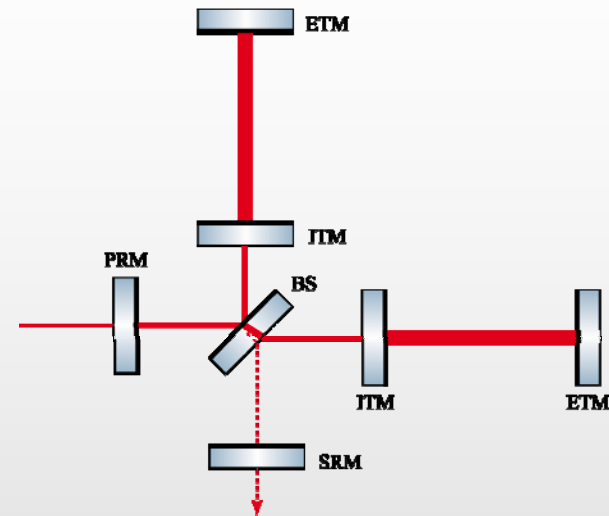
Harmonic restoring force (optical spring)



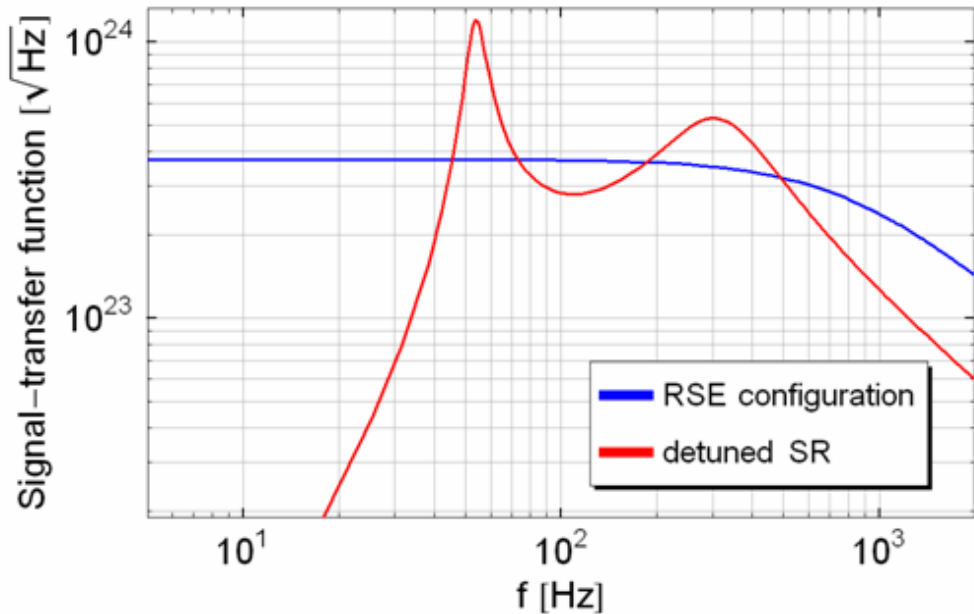
Is the local meter still useful for reducing radiation pressure effects?

# Detuned SR Interferometer

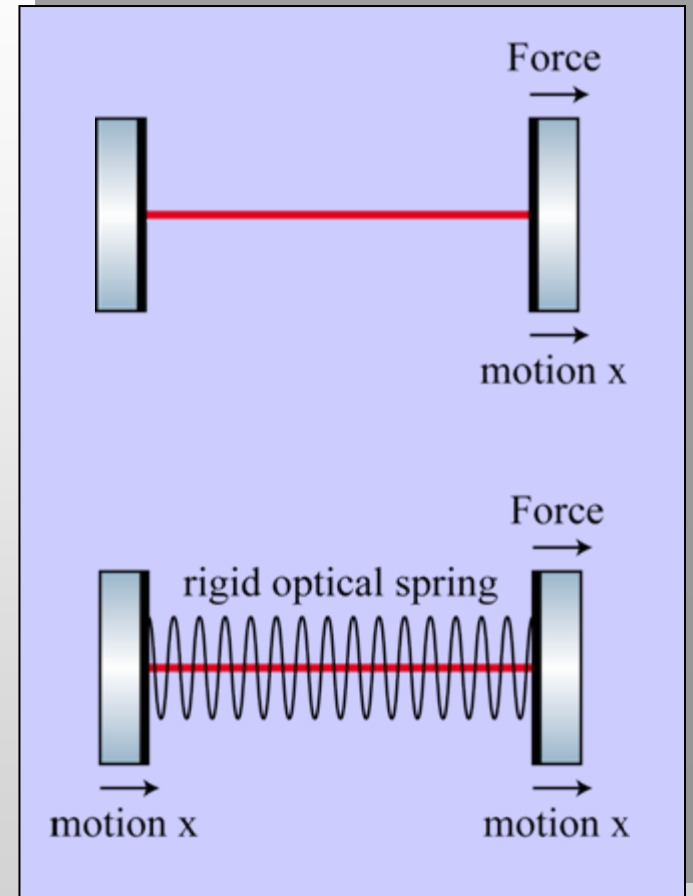
- **Remember: Detuned SR Interferometer is equivalent to single detuned cavity**
- Detuning is moved to signal recycling cavity
- Additional resonances:
  - optical resonance
  - optomechanical resonance
- Gain of sensitivity at the two resonances
- Loss of sensitivity below optomechanical resonance frequency



# Signal and Noise Transfer

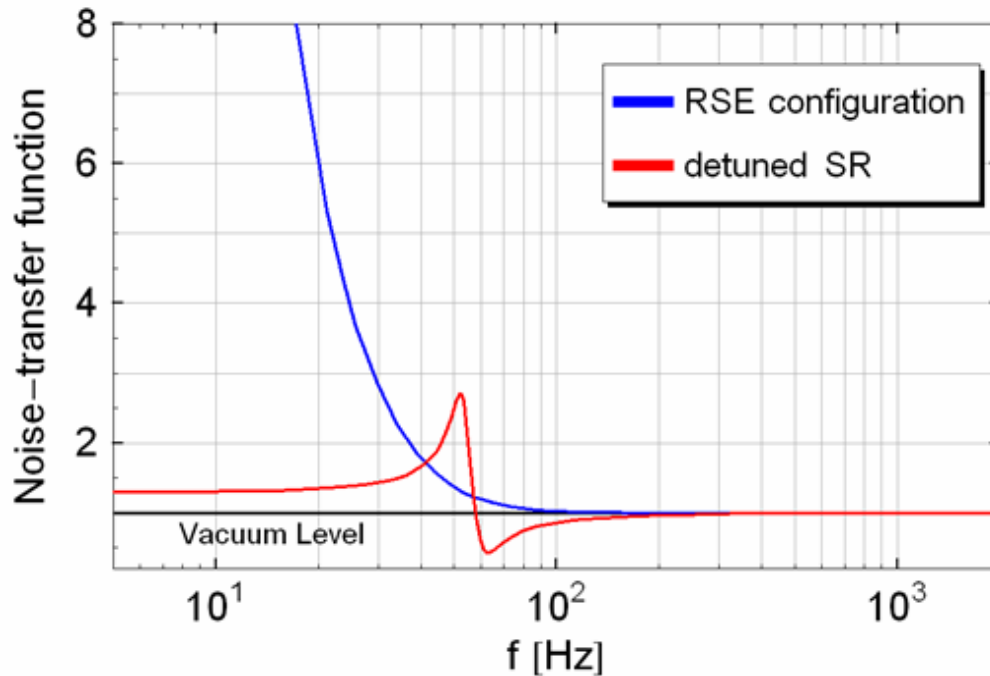


Mainly the rigid optical spring suppresses the response of the interferometer's differential mode to GW waves  $\rightarrow$  cf. *optical bar*



[Braginski, Gorodetsky & Khalili 1997;  
Braginski & Khalili 1999; Khalili 2002]

# Signal and Noise Transfer

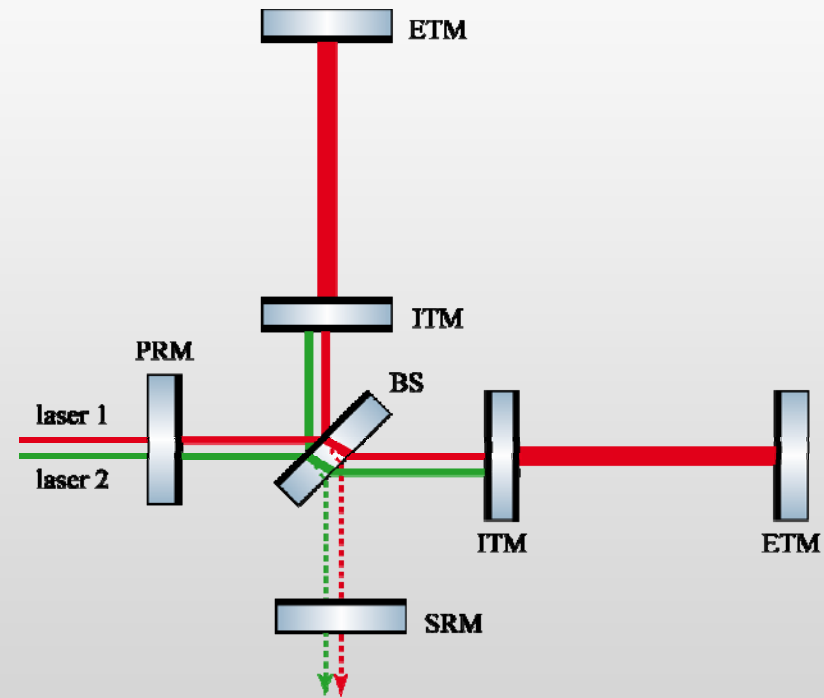


Radiation pressure noise of SR interferometer is also suppressed by the optical spring

➔ Further suppression of radiation pressure noise by a local meter is not necessary for detuned SR interferometers! But...

# Local Readout Scheme (1)

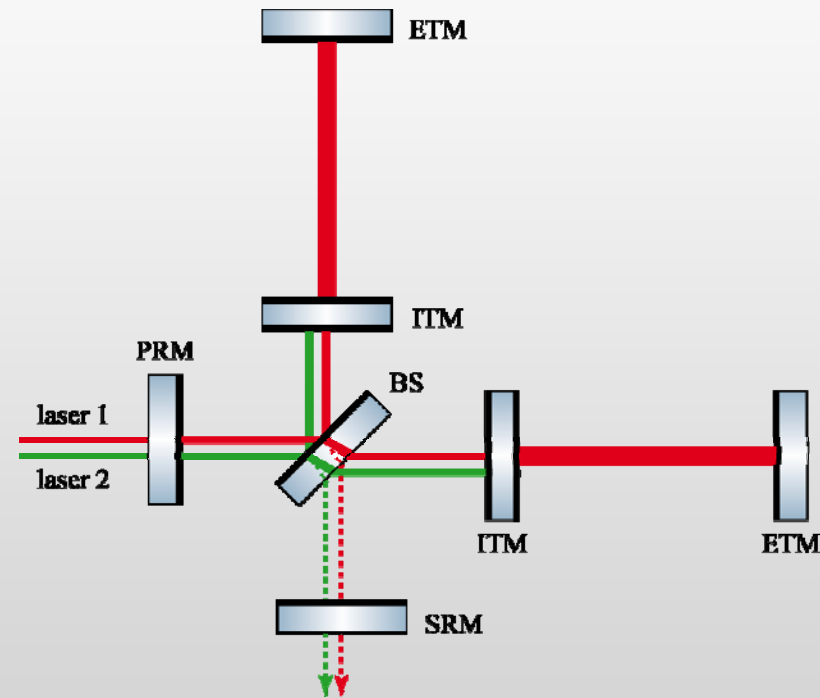
- Reading out the ITMs' motion by injection of second laser beam
- Additional laser beam should not resonate in arm cavities
- At frequencies below optomechanical resonance the local meter views bar formed by rigidly connected ITM and ETM
- Effective mass sensed by local meter is twice that of ITM or ETM (assuming equal masses)
- Recover low-frequency response of detuned SR interferometer
- Extension to stand-alone optical bar or SR schemes





# Local Readout Scheme (2)

| Symbol              | Physical meaning                              | Value                    |
|---------------------|---|--------------------------|
| $m$                 | Single mirror mass                            | 40 kg                    |
| $m_{BS}$            | BS mass                                       | 40 kg                    |
| $\omega_0^{(1)}$    | Light frequency 1 <sup>st</sup> laser         | $1.8 \times 10^{15}$ 1/s |
| $P^{(1)}$           | Circulating power 1st carrier                 | 100-800 kW               |
| $L^{(1)}$           | Large scale ifo arm length                    | 4 km                     |
| $\rho_{PR}$         | PRM reflectivity                              | $(0.5)^{1/2}$            |
| $\phi$              | Detuning for 1 <sup>st</sup> carrier          | $0-\pi$                  |
| $\rho_{SR}$         | SRM reflectivity                              | $(0.93)^{1/2}$           |
| $\gamma_0$          | Cavity half bandwidth 1st carrier             | $2\pi$ 15 Hz             |
| $\zeta^{(1)}$       | Det. angle for 1 <sup>st</sup> carrier        | $0-\pi$                  |
| $\omega_0^{(2)}$    | Light frequency 2 <sup>nd</sup> laser         | $1.8 \times 10^{15}$ 1/s |
| $P^{(2)}$           | Circulating power 2nd carrier                 | 0-16 kW                  |
| $L^{(2)}$           | Local meter arm length                        | 15 m                     |
| $\lambda^{(2)}$     | Detuning for 2 <sup>nd</sup> carrier          | 0 Hz                     |
| $\varepsilon^{(2)}$ | Cavity half bandwidth 2 <sup>nd</sup> carrier | $2\pi$ 4 kHz             |
| $\zeta^{(2)}$       | Det. angle for 2 <sup>nd</sup> carrier        | 0                        |



# Combined Sensitivity

The output can be written in the compact form

$$\hat{y}^{(1)} = \vec{n}_1^T \vec{v} + s_1 h \text{ and } \hat{y}^{(2)} = \vec{n}_2^T \vec{v} + s_2 h$$

and the combined output is given by

$$\hat{y} = K_1(\Omega) \hat{y}^{(1)} + K_2(\Omega) \hat{y}^{(2)}$$

with filter functions  $K_1(\Omega)$  and  $K_2(\Omega)$ . The resulting total noise spectral density

$$S_h(\Omega) = \frac{\begin{pmatrix} K_1(\Omega) & K_2(\Omega) \end{pmatrix} \begin{pmatrix} \vec{n}_1^T \vec{n}_1^* & \vec{n}_1^T \vec{n}_2^* \\ \vec{n}_2^T \vec{n}_1^* & \vec{n}_2^T \vec{n}_2^* \end{pmatrix} \begin{pmatrix} K_1^*(\Omega) \\ K_2^*(\Omega) \end{pmatrix}}{\begin{pmatrix} K_1(\Omega) & K_2(\Omega) \end{pmatrix} \begin{pmatrix} s_1 s_1^* & s_1 s_2^* \\ s_2 s_1^* & s_2 s_2^* \end{pmatrix} \begin{pmatrix} K_1^*(\Omega) \\ K_2^*(\Omega) \end{pmatrix}}$$

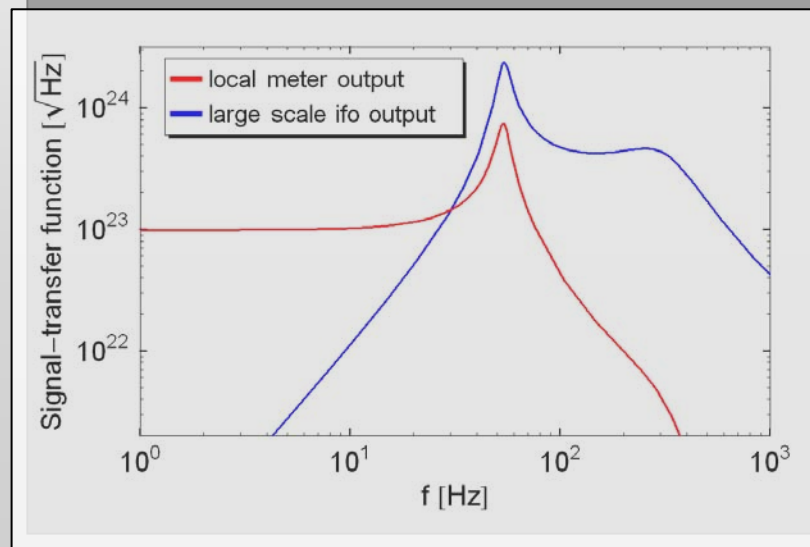
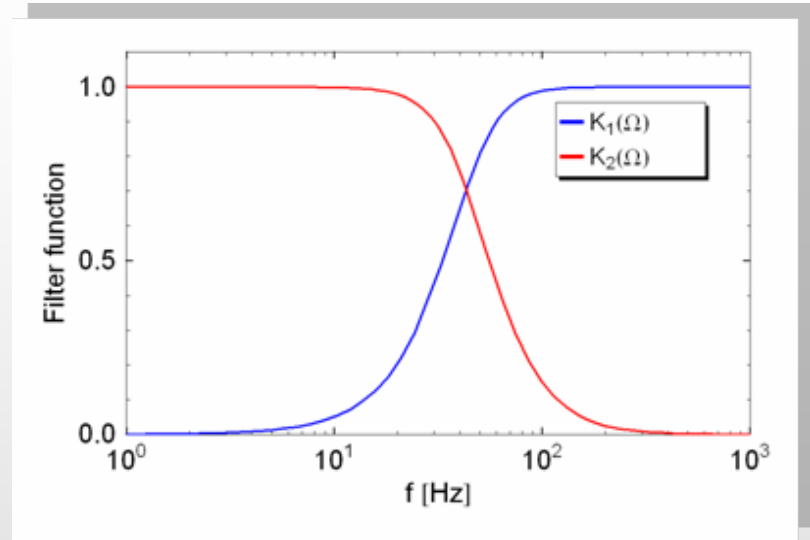
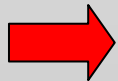
has to be minimized.

$$\vec{v}^T = (a_1^{(1)}, a_2^{(1)}, a_1^{(2)}, a_2^{(2)}, \hat{\xi}_{\text{ITM}}^{\text{cl}}, \hat{\xi}_{\text{ETM}}^{\text{cl}}, \hat{\xi}_{\text{BS}}^{\text{cl}})$$

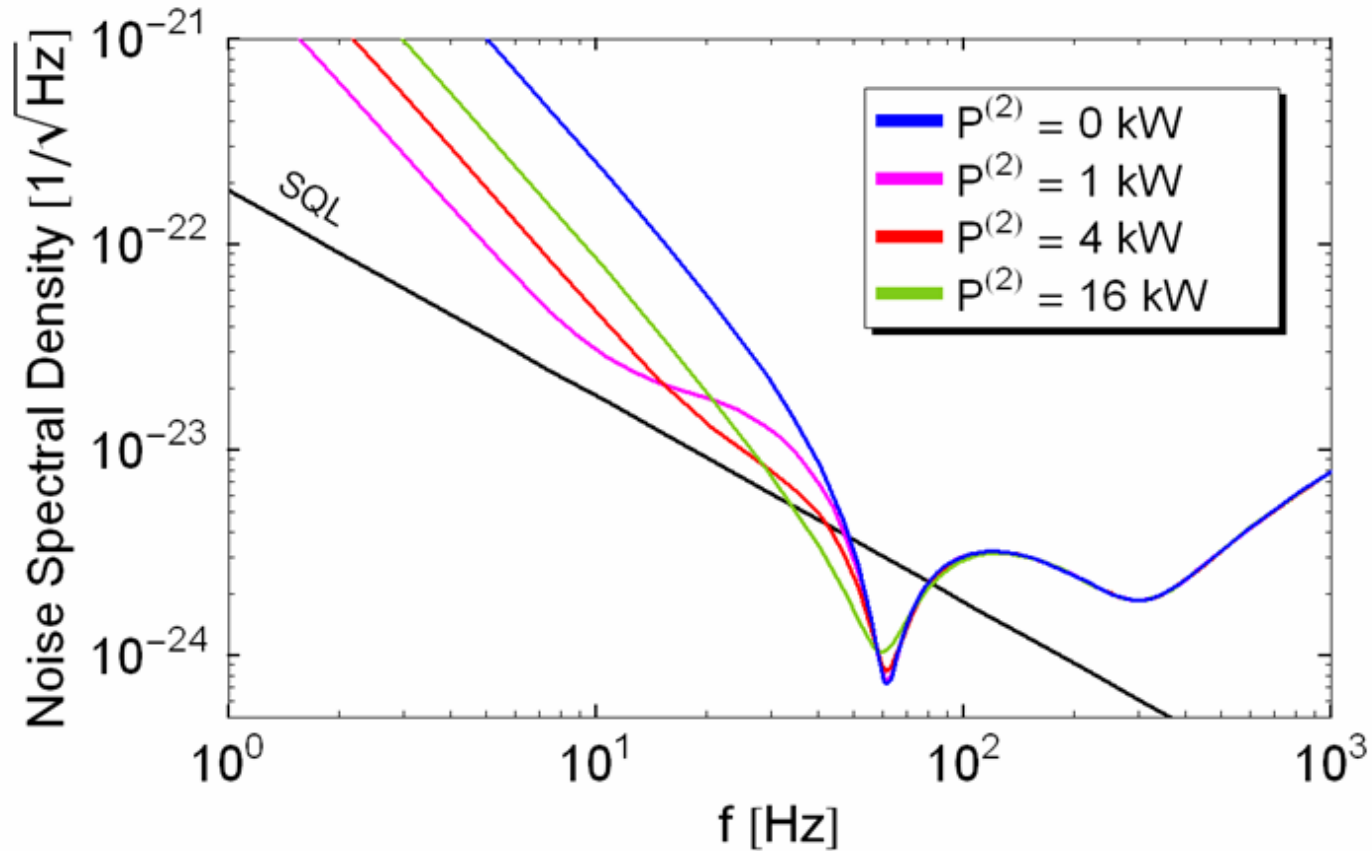
# Filter Functions

- Optimal filter functions
  - minimize noise spectral density
  - are close to step functions
- Signal transfer function of large scale interferometer below optomechanical resonance decreases rapidly but...
- ...signal transfer of local meter stays constant

Combination of locally sensed optical bar scheme and SR interferometer



# Noise Spectral Density



# Astrophysical Optimization

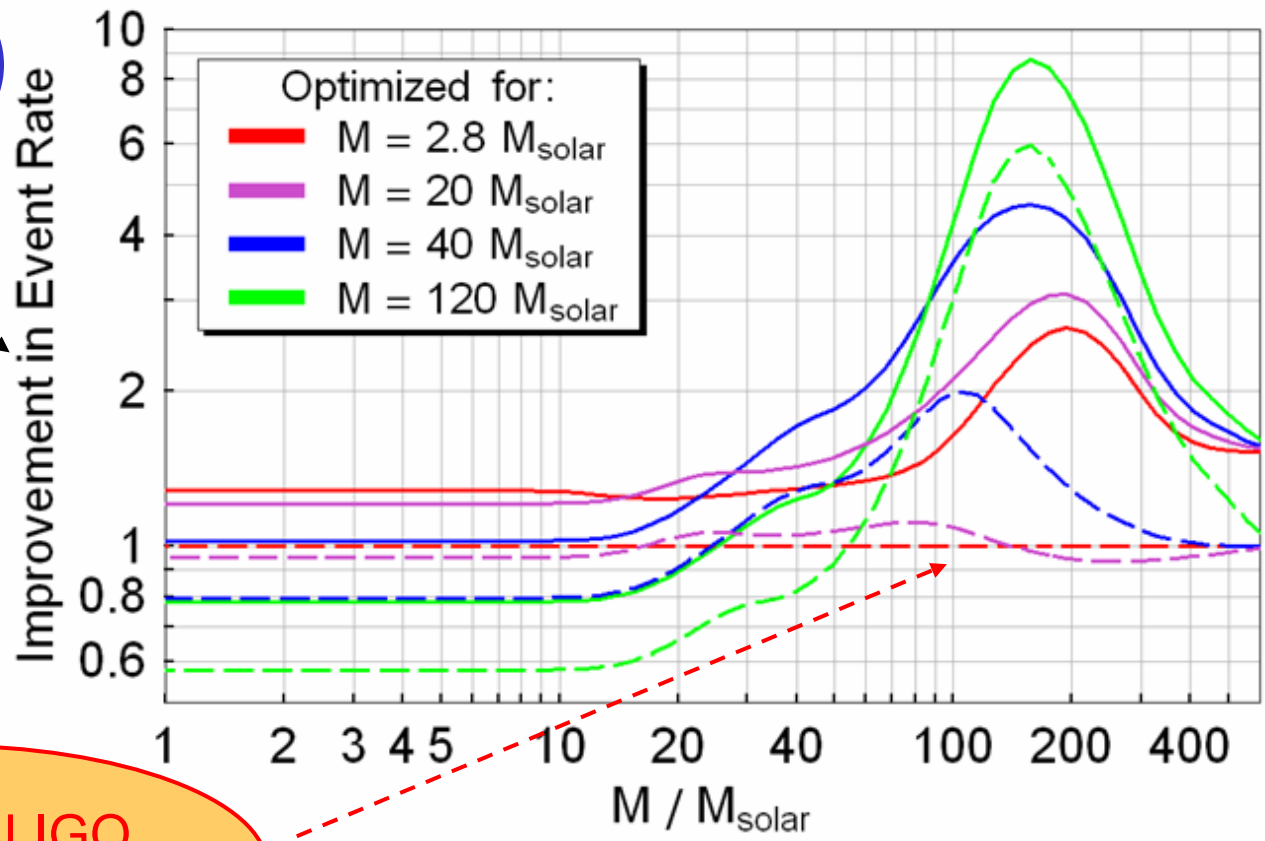
- Optimization with respect to binary systems with total mass  $M$  and reduced mass  $\mu$
- Observable distance for a given SNR  $\rho_0$  and averaged binary orientation has to be maximized:

$$D = \sqrt{\frac{2}{15} \frac{G^{5/6} \mu^{1/2} M^{1/3}}{\pi^{2/3} c^{3/2} \rho_0}} \sqrt{\int_{f_{\min}}^{f_{\max}} df \frac{f^{-7/6}}{S_h(f)}}$$

- Classical noise cutoff:  $f_{\min} \sim 7$  Hz
- Upper cutoff given by GW frequency at last stable circular orbit of the binary:  $f_{\max} \sim 4400 \text{ Hz } (M_{\odot} / M)$

# Comparison of Event Rates

Compared to  
Advanced LIGO

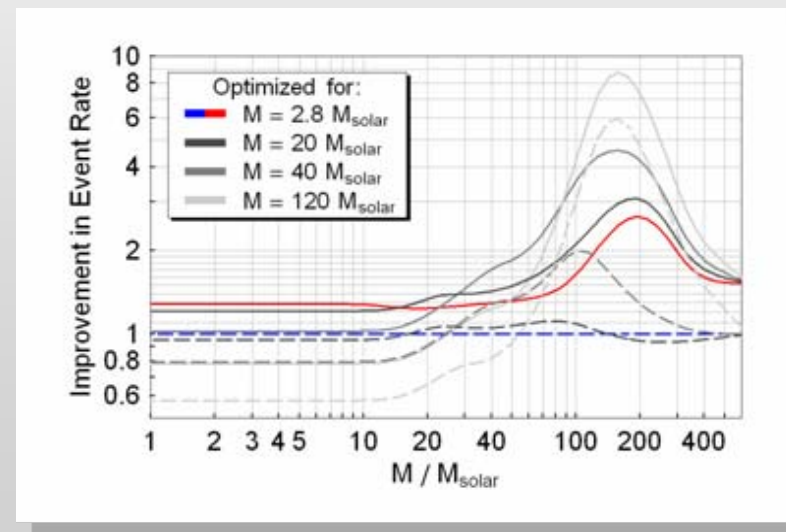
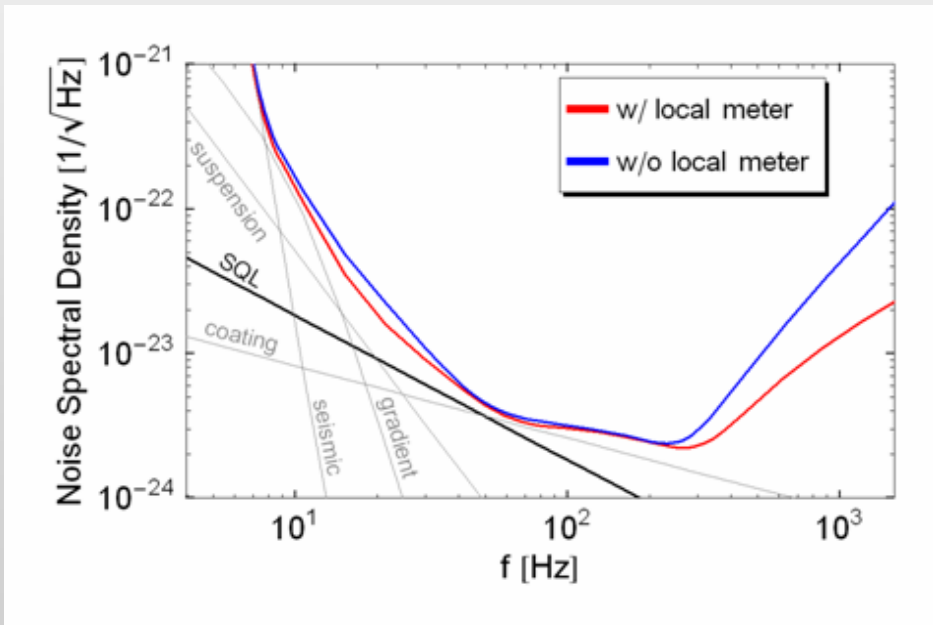


Advanced LIGO  
is optimized for 2.8 M.



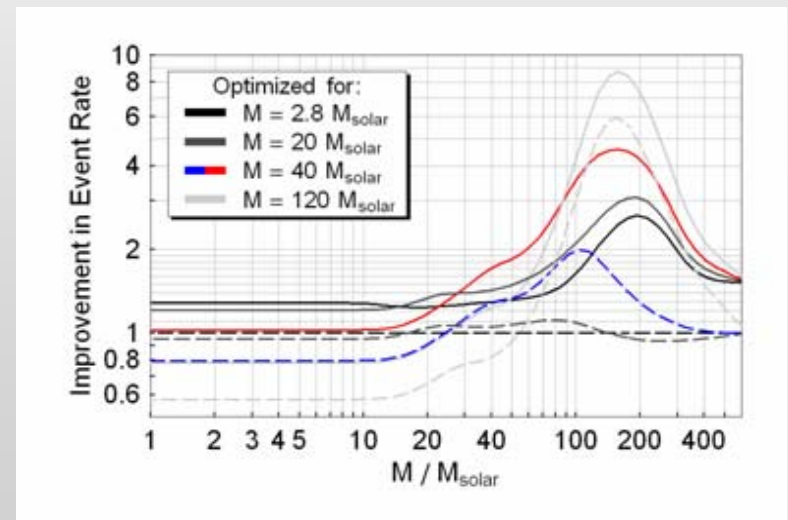
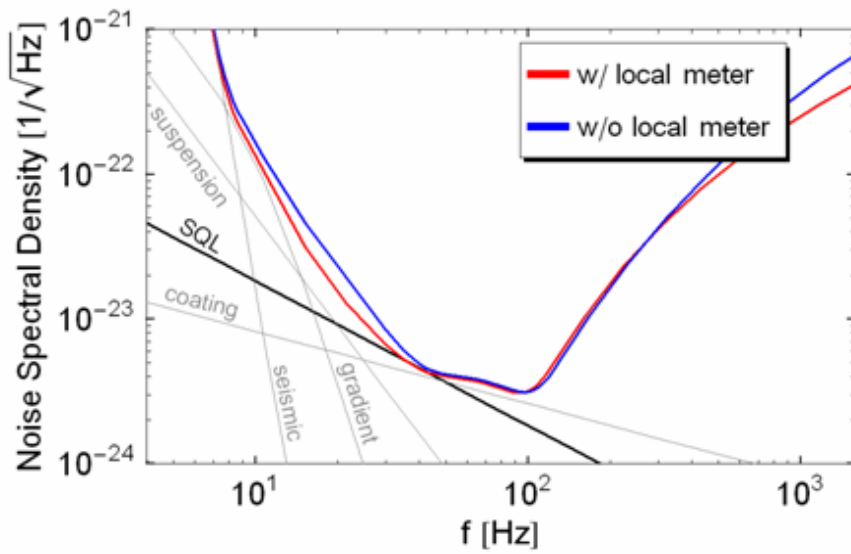
# Noise Spectral Densities with Classical Noise

| M/M <sub>⊙</sub> | opt. parameter with local meter |         |                        | opt. parameter without local meter |         |                        | Improv. |
|------------------|---------------------------------|---------|------------------------|------------------------------------|---------|------------------------|---------|
|                  | P <sup>(1)</sup> [kW]           | ϕ [rad] | ζ <sup>(1)</sup> [rad] | P <sup>(1)</sup> [kW]              | ϕ [rad] | ζ <sup>(1)</sup> [rad] |         |
| 2.8              | 800                             | 0.48 π  | 0.7 π                  | 800                                | 0.48 π  | 0.49 π                 | 29%     |
| 20               | 450                             | 0.47 π  | 0.58 π                 | 500                                | 0.48 π  | 0.48 π                 | 28%     |
| 40               | 150                             | 0.45 π  | 0.43 π                 | 150                                | 0.45 π  | 0.46 π                 | 33%     |
| 120              | 100                             | 0.46 π  | 0.32 π                 | 100                                | 0.47 π  | 0.41 π                 | 42%     |



# Noise Spectral Densities with Classical Noise

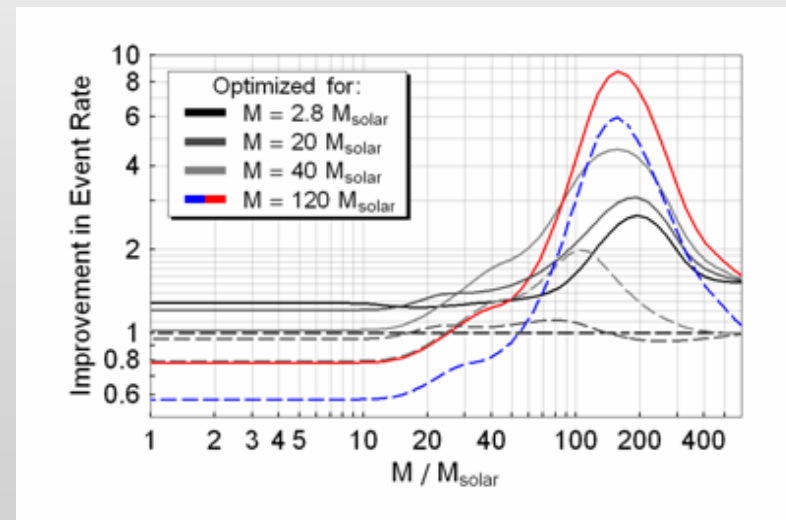
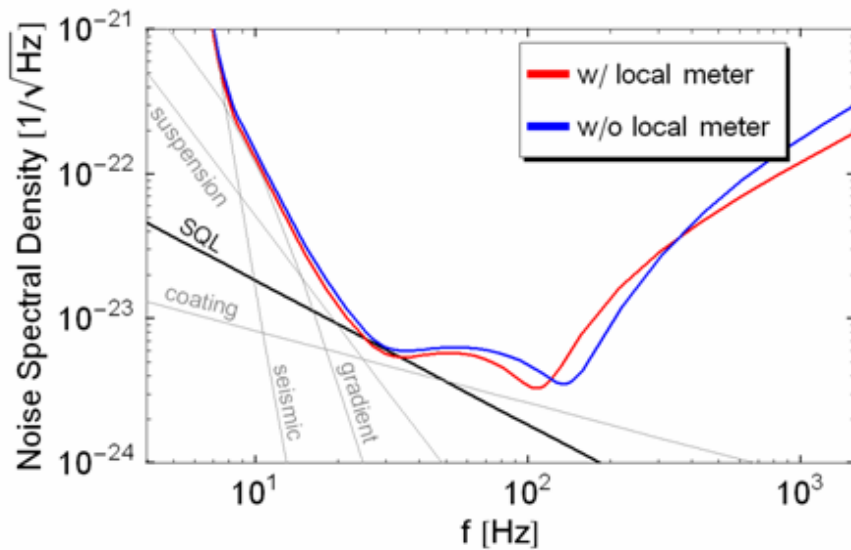
| M/M <sub>⊙</sub> | opt. parameter with local meter |         |                        | opt. parameter without local meter |         |                        | Improv. |
|------------------|---------------------------------|---------|------------------------|------------------------------------|---------|------------------------|---------|
|                  | P <sup>(1)</sup> [kW]           | ϕ [rad] | ζ <sup>(1)</sup> [rad] | P <sup>(1)</sup> [kW]              | ϕ [rad] | ζ <sup>(1)</sup> [rad] |         |
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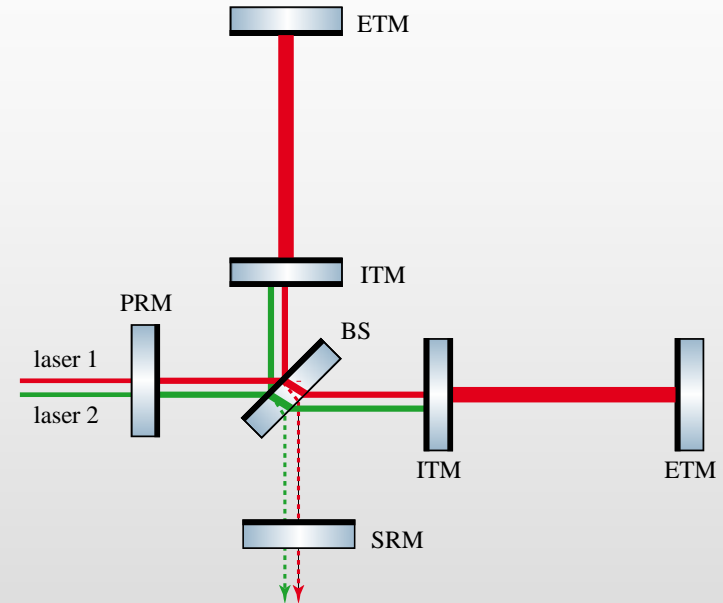
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| 120              | 100                             | 0.46 π  | 0.32 π                 | 100                                | 0.47 π  | 0.41 π                 | 42%     |



# Conclusion and Outlook

- Locally reading out ITMs' motion by second laser beam improves sensitivity significantly
- Maintaining or improving sensitivity of Advanced LIGO even for reduced power in arm cavities
- Combinable with other QND schemes, e.g. injection of squeezed vacuum
- Our proposed upgrade for Advanced LIGO should be realizable with low effort



# Equations of Motion (1)

$$\hat{x}_{\text{ITM}} = -R_{xx}(\Omega) \left( \hat{F}^{(1)}(\Omega) + R_{FF}^{(1)}(\Omega) (\hat{x}_{\text{ETM}} - \hat{x}_{\text{ITM}}) \right) + R_{xx}(\Omega) \left( \hat{F}^{(2)}(\Omega) - R_{FF}^{(2)}(\Omega) (\hat{x}_{\text{ITM}} + \sqrt{2} \hat{x}_{\text{BS}}) \right) + \hat{\xi}_{\text{ITM}}^{\text{cl}}$$

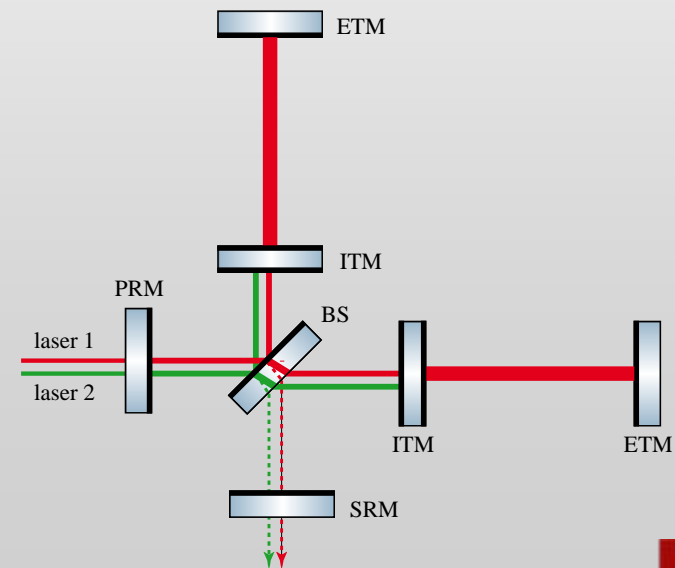
$$\hat{x}_{\text{ETM}} = R_{xx}(\Omega) \left( \hat{F}^{(1)}(\Omega) + R_{FF}^{(1)}(\Omega) (\hat{x}_{\text{ETM}} - \hat{x}_{\text{ITM}}) \right) + L h + \hat{\xi}_{\text{ETM}}^{\text{cl}}$$

$$\hat{x}_{\text{BS}} = R_{xx}^{\text{BS}}(\Omega) \left( \hat{F}^{(2)}(\Omega) + R_{FF}^{(2)}(\Omega) (\hat{x}_{\text{ITM}} + \sqrt{2} \hat{x}_{\text{BS}}) \right) + R_{xx}^{\text{BS}}(\Omega) \hat{F}_{\text{BP}}(\Omega) + \hat{\xi}_{\text{BS}}^{\text{cl}}$$

BS motion is

- only important in the local meter
- negligible for the large scale interferometer due to arm cavities
- also driven by bright port laser input fields

[Harms et. al. 2002]



# Equations of Motion (2)

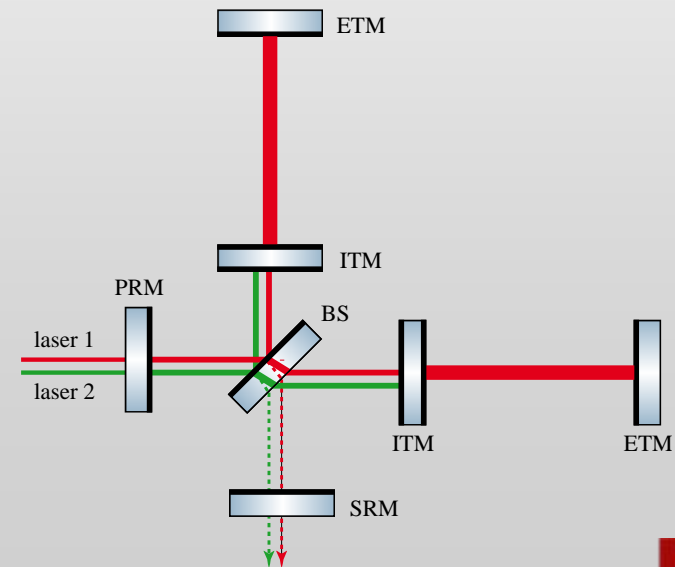
$$\hat{x}_{\text{ITM}} = -R_{xx}(\Omega) \left( \hat{F}^{(1)}(\Omega) + R_{FF}^{(1)}(\Omega) (\hat{x}_{\text{ETM}} - \hat{x}_{\text{ITM}}) \right) + R_{xx}(\Omega) \left( \hat{F}^{(2)}(\Omega) - R_{FF}^{(2)}(\Omega) (\hat{x}_{\text{ITM}} + \sqrt{2} \hat{x}_{\text{BS}}) \right) + \hat{\xi}_{\text{ITM}}^{\text{cl}}$$

$$\hat{x}_{\text{ETM}} = R_{xx}(\Omega) \left( \hat{F}^{(1)}(\Omega) + R_{FF}^{(1)}(\Omega) (\hat{x}_{\text{ETM}} - \hat{x}_{\text{ITM}}) \right) + L h + \hat{\xi}_{\text{ETM}}^{\text{cl}}$$

$$\hat{x}_{\text{BS}} = R_{xx}^{\text{BS}}(\Omega) \left( \hat{F}^{(2)}(\Omega) + R_{FF}^{(2)}(\Omega) (\hat{x}_{\text{ITM}} + \sqrt{2} \hat{x}_{\text{BS}}) \right) + R_{xx}^{\text{BS}}(\Omega) \hat{F}_{\text{BP}}(\Omega) + \hat{\xi}_{\text{BS}}^{\text{cl}}$$

## Classical noise

- modeled by operators  $\xi_i^{\text{cl}}, i \in \{\text{ITM}, \text{ETM}, \text{BS}\}$
- contributions are uncorrelated
- each mirror is subject to fourth of the spectrum generally expected for whole differential mode



# Implementation

- RF sidebands (1% of carrier light) already probe ITM in planned Advanced LIGO
- For our scheme a second laser beam is needed with the same power as the first laser (125 W).
- Polarized light or different frequencies can be used
- Frequency of second laser must be chosen such that it does not resonate in the arm cavities
- Circumvent shot-noise-limited sensitivity of large scale interferometer since sensitivity in the detection band cannot be lower than shot-noise level of local measurement of the ITMs in the control bandwidth

# Control Filter

- Instability induced by optical spring requires feedback control
- In general local meter and large scale interferometer could be detuned and therefore show instabilities
- It can be shown that an appropriate control feedback leaves noise spectral density unchanged

