Test-mass state preparation and entanglement in laser interferometers

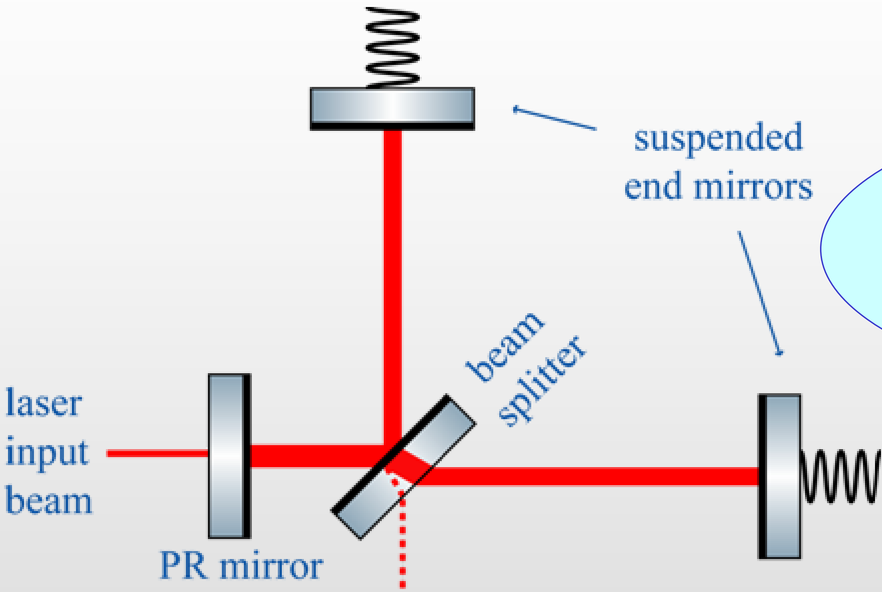
Helge Müller-Ebhardt, Henning Rehbein, Roman Schnabel,
Karsten Danzmann, Yanbei Chen
and the AEI-Caltech-MIT MQM discussion group

Max-Planck-Institut für Gravitationsphysik (AEI)
Institut für Gravitationsphysik, Leibniz Universität Hannover

Introduction

- Laser interferometer GW detectors are high-precision position-measurement devices.
- Noise level in currently operating first-generation GW detectors is a factor of » 10 in amplitude above the SQL.
- Second-generation interferometers are expected to be operative within » 5 years and may approach the SQL up to a factor of » 2 or even less.
- Future interferometers will have to surpass the SQL significantly.
- Lab-scale prototype interferometers with suspended test masses can reach and surpass the SQL before large-scale detectors.
- What about the test-masses’ state in such devices?

Model under consideration



Suspension thermal noise:

$$\langle \hat{\xi}_F(\Omega)\ \hat{\xi}_F^{\dagger}(\Omega')\rangle_{\mathrm{sym}} = 2\pi\ m\gamma_m\ \hbar\omega_m\ \coth\left(\frac{\hbar\omega_m}{2k_{\mathrm{B}}T}\right)\delta(\Omega-\Omega')$$

$$\hat{x}^{\mathrm{d}}(\Omega) = -\frac{2}{m(\Omega^2 + \mathrm{i}\gamma_m\Omega - \omega_m^2)}\left(\alpha\ \hat{a}_1(\Omega) + \hat{\xi}_F(\Omega)\right)$$

Differential motion between the end mirrors' centers-of-mass.

Back-action noise:

$$\alpha = \sqrt{\frac{8P\omega_0\hbar}{c^2\tau^2}}$$

Unconditional test-mass state

What is meant by preparing a quantum state? – Pure state!

(*Gaussian states* are pure iff *Heisenberg uncertainty* is minimal!)

$$V_{xx}V_{pp}-V_{xp}^2=\frac{\hbar^2}{4}\coth^2\left(\frac{\hbar\omega_m}{2k_\mathrm{B}T}\right)+\frac{\hbar\alpha^2}{4m\omega_m\gamma_m}\coth\left(\frac{\hbar\omega_m}{2k_\mathrm{B}T}\right)+\frac{\alpha^4}{16m^2\omega_m^2\gamma_m^2}$$

Usually highly thermal (mixed) state.

Even for T → 0 not in ground state because of coupling to the light – quantum back-action.

→ Need to reduce noise with e.g. feedback control. - But state is already conditioned on our measurement!

Conditioning on continuous measurement (1)

Conditional density matrix: projected onto subspace where the measurement-output operator takes measured value.

$$\hat{\rho}^{\text{cond}}(t) = \mathcal{P}_{[\hat{y}(t')=y(t'),t'<t]}\ \hat{\rho}(t)\ \mathcal{P}_{[\hat{y}(t')=y(t'),t'<t]}$$

Nano-mechanical oscillator people [Doherty, Habib, Hopkins, Jacobs, Milburne, Schwab, Wiseman…] like to use *SME*:

$$\begin{aligned} \mathrm{d}\hat{\rho} = & -\frac{\mathrm{i}}{\hbar}\left[\frac{\hat{p}^2}{2m}+\frac{m\omega_m^2\hat{x}}{2},\hat{\rho}\right]\,\mathrm{d}t - \frac{\mathrm{i}\gamma_m}{2\hbar}[\hat{x},\{\hat{p},\hat{\rho}\}]\,\mathrm{d}t - \frac{m\omega_m\gamma_m}{\hbar}\coth\left(\frac{\hbar\omega_m}{2k_\mathrm{B}T}\right)[\hat{x},[\hat{x},\hat{\rho}]]\,\mathrm{d}t \\ & -\frac{\alpha^2}{4\hbar^2}[\hat{x},[\hat{x},\hat{\rho}]]\,\mathrm{d}t + \frac{\alpha}{\sqrt{2}\hbar}\left(e^{\mathrm{i}\phi}\hat{x}\hat{\rho}+e^{-\mathrm{i}\phi}\hat{\rho}\hat{x}-2\hat{\rho}\,\mathrm{tr}(\hat{x}\hat{\rho})\cos\phi\right)\mathrm{d}W \end{aligned}$$

Back-action noise.

Stochastic term describing the conditioning on the measurement.

Conditioning on continuous measurements (2)

Wiener filter approach:

$$\hat{x}^{\mathrm{d}}(t) = \int_{-\infty}^{t} \mathrm{d}t' K_{x^{\mathrm{d}}}(t-t')\ \hat{y}(t') + \hat{R}_{x^{\mathrm{d}}}(t)$$

$$\langle \hat{R}_{x^{\mathrm{d}}}(t)\ \hat{y}(t')\rangle_{\mathrm{sym}} = 0$$

Classical causal Wiener filter.

Unknown part.

$$[\hat{y}(t), \hat{y}(t')] = 0$$

$$[\hat{x}^{\mathrm{d}}(t), \hat{y}(t')] = 0 \quad \forall t > t'$$

Conditional second-order moments:

$$V_{xx} = \langle \hat{R}_x^2 \rangle \qquad V_{pp} = \langle \hat{R}_p^2 \rangle \qquad V_{xp} = \langle \hat{R}_x \hat{R}_p \rangle_{\mathrm{sym}}$$

Insert spectral densities and integrate over all frequencies.

State preparation in laser interferometers

Homodyne detection gives measurement-output operator:

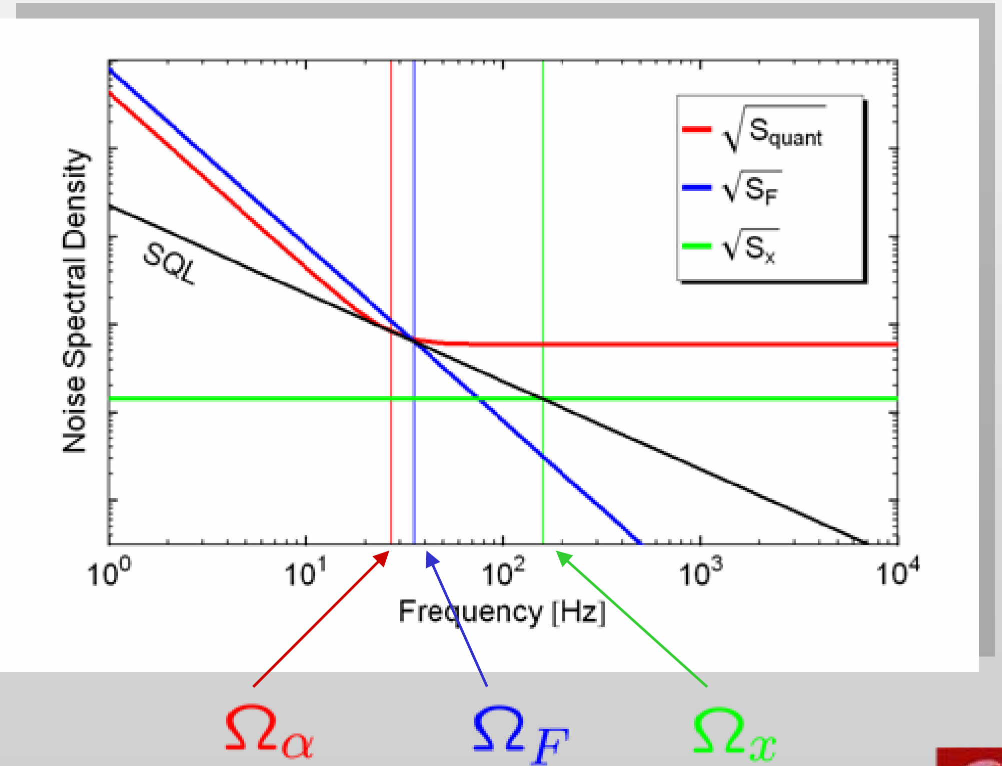
$$\hat{y}(\Omega) = \sin\phi\,\hat{a}_1(\Omega) + \cos\phi\,\left(\hat{a}_2(\Omega) + \frac{\alpha}{\hbar}\left(\hat{x}^{\mathrm{d}}(\Omega) + \hat{\xi}_x(\Omega)\right)\right)$$

Recall:

$$\hat{x}^{\mathrm{d}}(\Omega) \propto \alpha\,\hat{a}_1(\Omega) + \hat{\xi}_F(\Omega)$$

$$\langle \hat{\xi}_F(\Omega)\,\hat{\xi}_F^\dagger(\Omega')\rangle_{\mathrm{sym}} = 2m\hbar\Omega_F^2\,\pi\,\delta(\Omega-\Omega')$$

$$\langle \hat{\xi}_x(\Omega)\,\hat{\xi}_x^\dagger(\Omega')\rangle_{\mathrm{sym}} = \frac{m\Omega_x^2}{2\hbar}\,\pi\,\delta(\Omega-\Omega')$$



Conditional test-mass state

Conditional Heisenberg uncertainty:

$$\omega_m,\ \gamma_m \ll \Omega_\alpha \equiv \frac{\alpha}{\sqrt{m\hbar}}$$

$$U \equiv V_{xx}V_{pp} - V_{xp}^2 = \frac{\hbar^2}{4} + \frac{\hbar^2}{\cos^2\phi}\left(\frac{\Omega_F^2}{2\Omega_\alpha^2} + \frac{\Omega_\alpha^2}{2\Omega_x^2} + \frac{\Omega_F^2}{\Omega_x^2}\right) \geq \frac{\hbar^2}{4}\left(1 + \frac{2\Omega_F}{\Omega_x}\right)^2$$

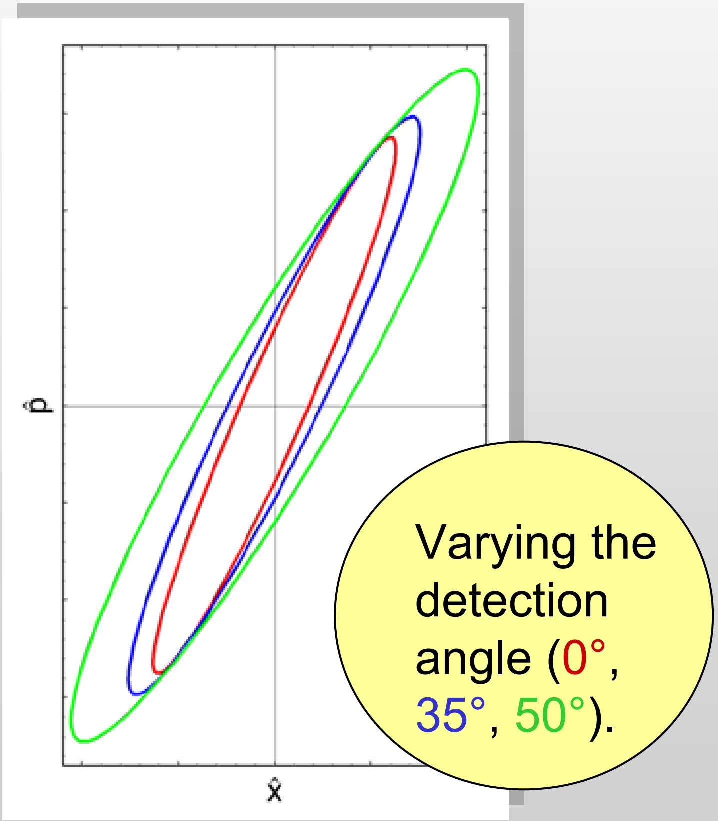
Recall: when $\Omega_x / \Omega_F > 2$, we have a non-zero frequency band in which the classical noise is completely below the SQL.

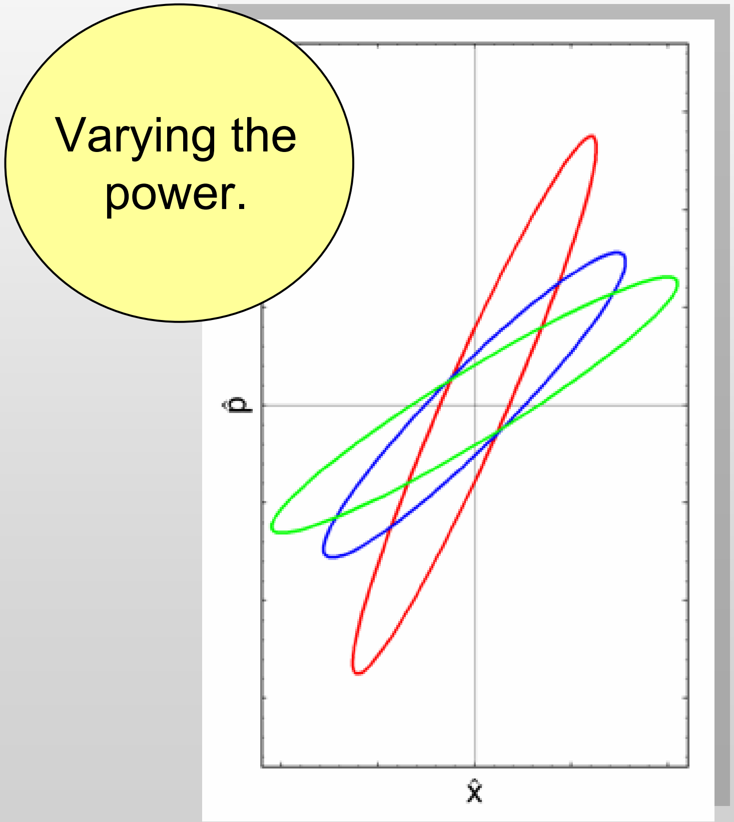
Equality for optimal power $\Omega_\alpha = \sqrt{\Omega_x \Omega_F}$ and for $\phi = 0$.

→ In absence of classical noise we have always a pure state! But even with classical noise we are able to get really close to the Heisenberg limit!!!

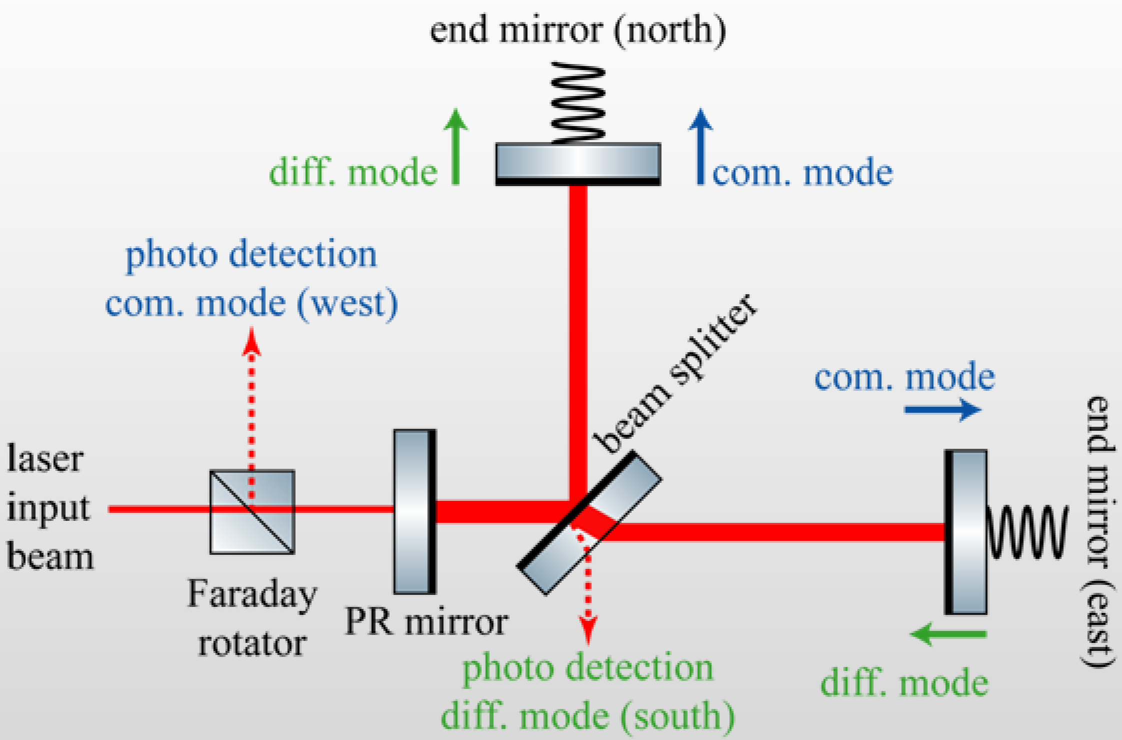
Squeezing

The conditional state of the two end mirrors' differential motion is usually highly squeezed in position with respect to the pendulum's ground state.





Extended model



Want also the common mode to be quantum.

↓

Need to additionally detect at the bright port.

↓

Have two independent systems of the same format with different parameters such as effective power (Ω_α) and homodyne detection angle ϕ.

It is impractical to entangle thermal states!!!

Mirror entanglement in absence of cl. noise

Recall: In absence of classical noise we have always a pure state!

The measurement processes for common and differential mode can be made different by choosing for both modes independently:

- effective power → Ω_α (due to power-recycling)
- homodyne detection angle ϕ.

Different due to measurement process

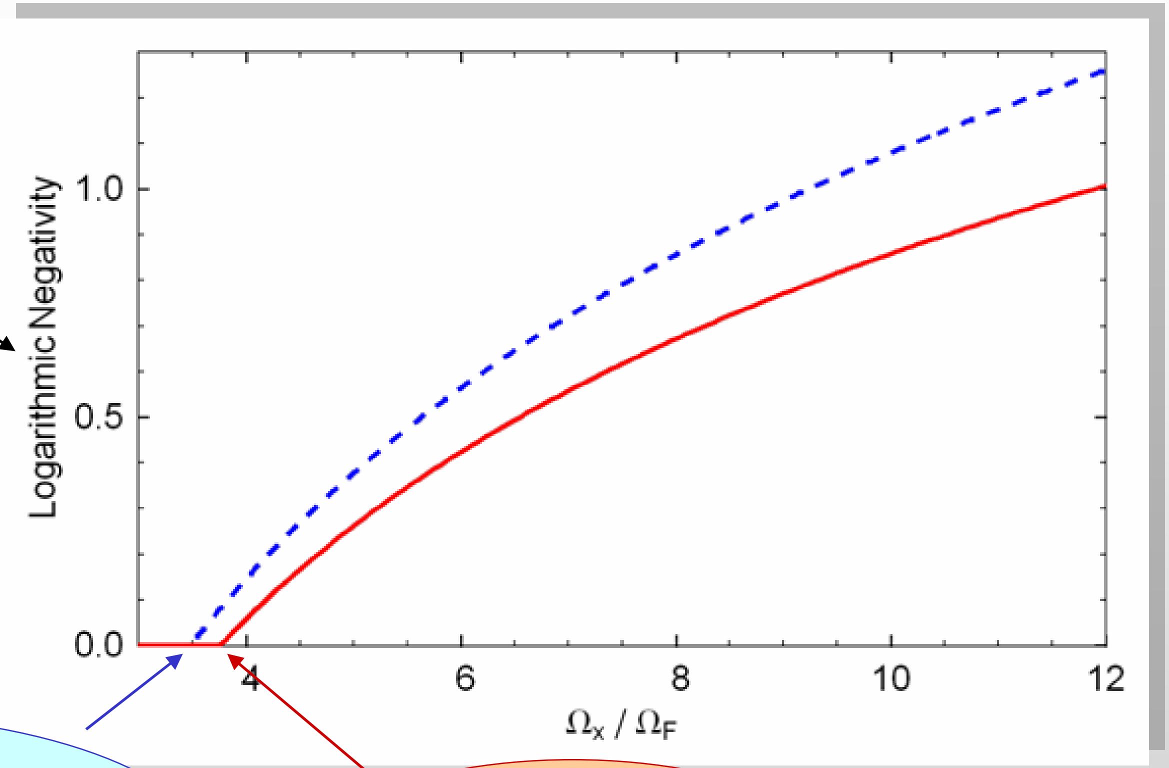
→ non-separable joint wave-function:

$$\Psi(x^{\mathrm{e}}, x^{\mathrm{n}}) = \psi^{\mathrm{c}}\left(\frac{x^{\mathrm{e}} + x^{\mathrm{n}}}{2}\right)\ \psi^{\mathrm{d}}\left(\frac{x^{\mathrm{e}} - x^{\mathrm{n}}}{2}\right)$$

Compare with creating entanglement by overlapping two differently squeezed beams on a beam splitter.

Mirror entanglement with cl. noise (1)

The *logarithmic negativity* [Vidal, Werner] is a quantitative measure of entanglement.

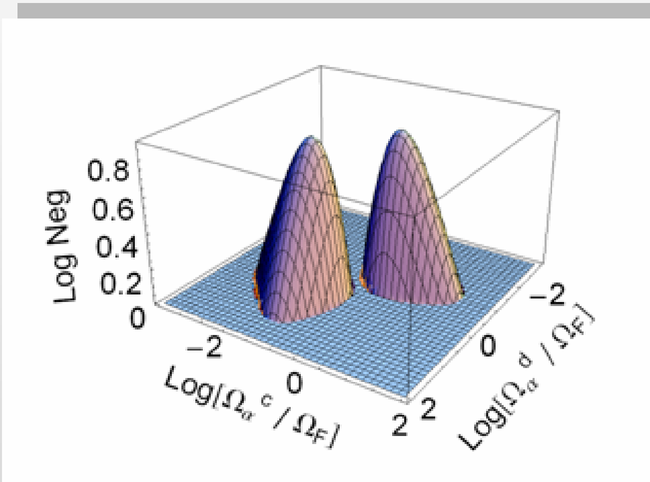


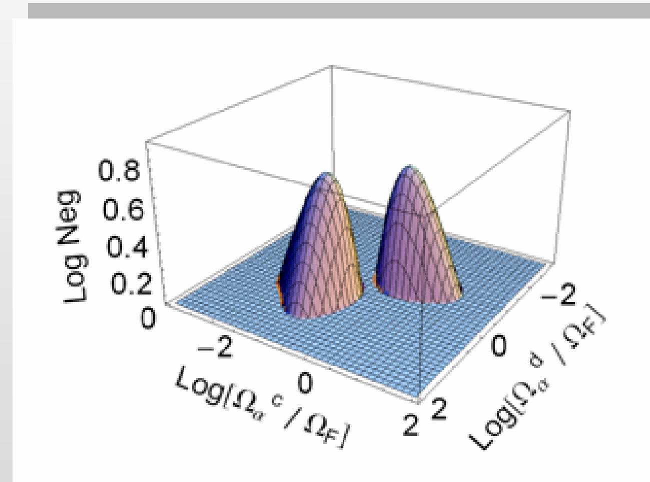
Threshold for measurement with changing detection angle: » 3.5!

Threshold for phase quadrature (ϕ = 0) measurement: » 3.8!

Mirror entanglement with cl. noise (2)

For Ω_x / Ω_F above the threshold, one can vary ($\Omega_\alpha{}^c / \Omega_F$, $\Omega_\alpha{}^d / \Omega_F$) in a certain range while maintaining entanglement.



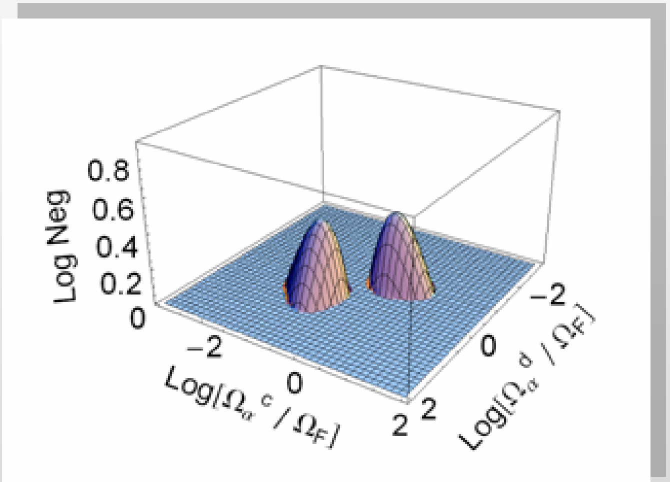


$\Omega_x / \Omega_F = 10$ corresponds to a window of Δf ¼ 1.63 f in which cl. noise is a factor of · 5 in power below the SQL.

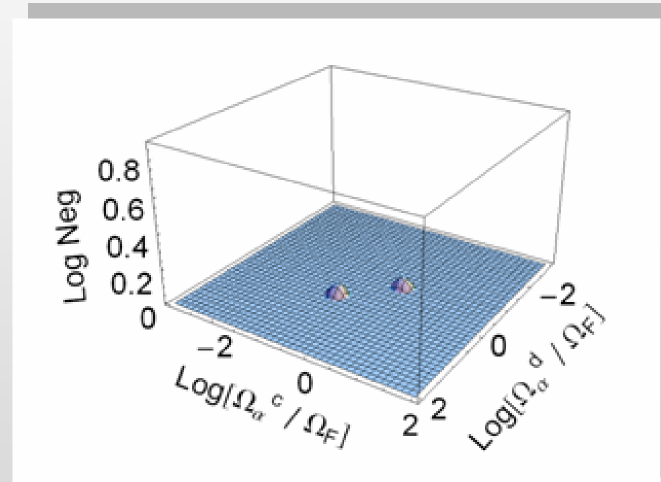
$\Omega_x / \Omega_F = 8$ corresponds to a window of Δf ¼ 1.55 f in which cl. noise is a factor of · 4 in power below the SQL.

Mirror entanglement with cl. noise (3)

For Ω_x / Ω_F above the threshold, one can vary $(\Omega_\alpha^c / \Omega_F, \Omega_\alpha^d / \Omega_F)$ in a certain range while maintaining entanglement.



Ω_x / Ω_F = 6 corresponds to a window of Δf ¼ 1.41 f in which cl. noise is a factor of · 3 in power below the SQL.



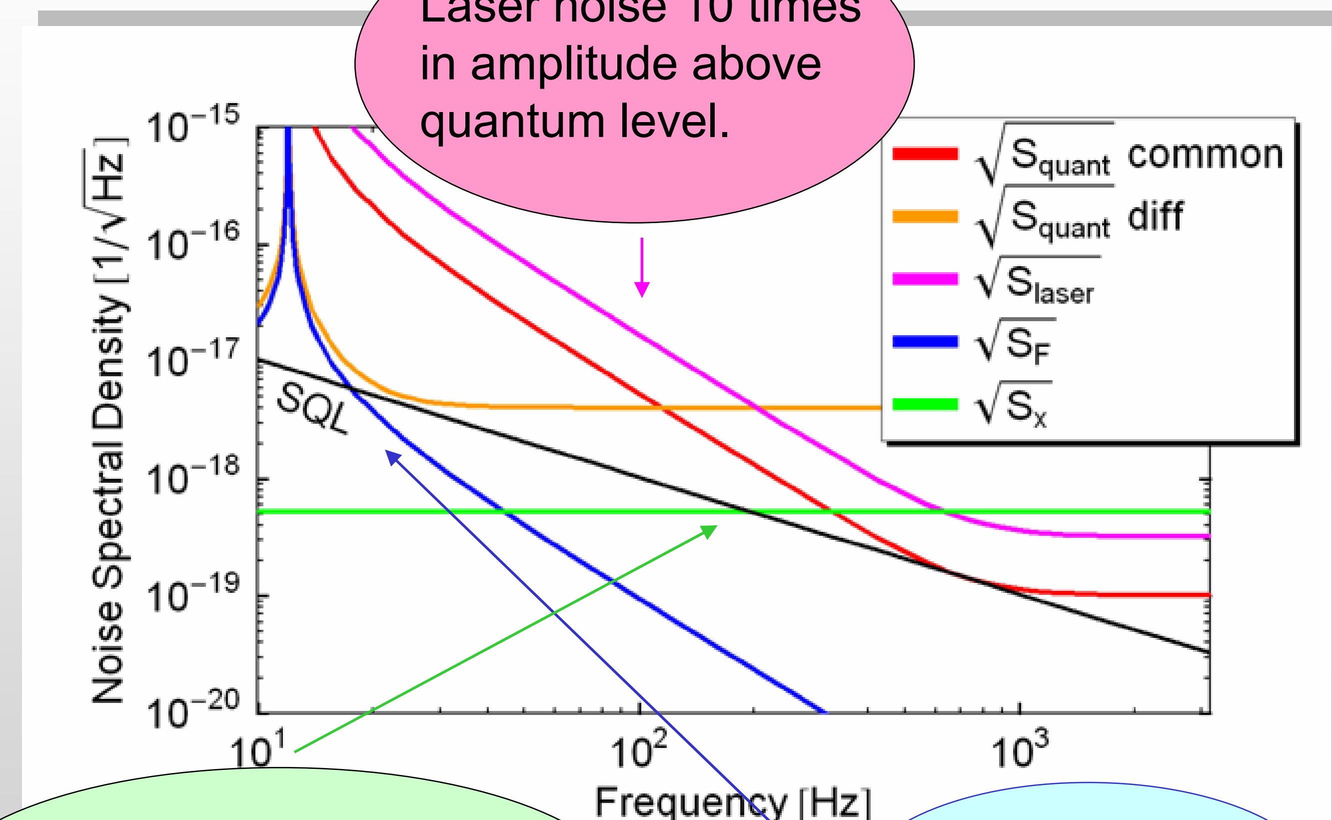
Ω_x / Ω_F = 4 corresponds to a window of Δf ¼ 1.15 f in which cl. noise is a factor of · 2 in power below the SQL.

Concrete example for mirror entanglement (1)

P	0.1 W
λ	1064 nm
τ	0.05
L	1 m
m	1 g
ω_m	2π 12 Hz
γ_m	$2\pi\ 10^{-10}$ Hz
T	10 K
Y	$7.3\ 10^{10}$ N/m^2
Y'	$1.1\ 10^{11}$ N/m^2
d	10 μm
r_0	1 mm
$\phi_{\|\|}$	$4\ 10^{-4}$
$\phi_?$	$4\ 10^{-4}$

[Corbitt et al.]

Laser noise 10 times in amplitude above quantum level.

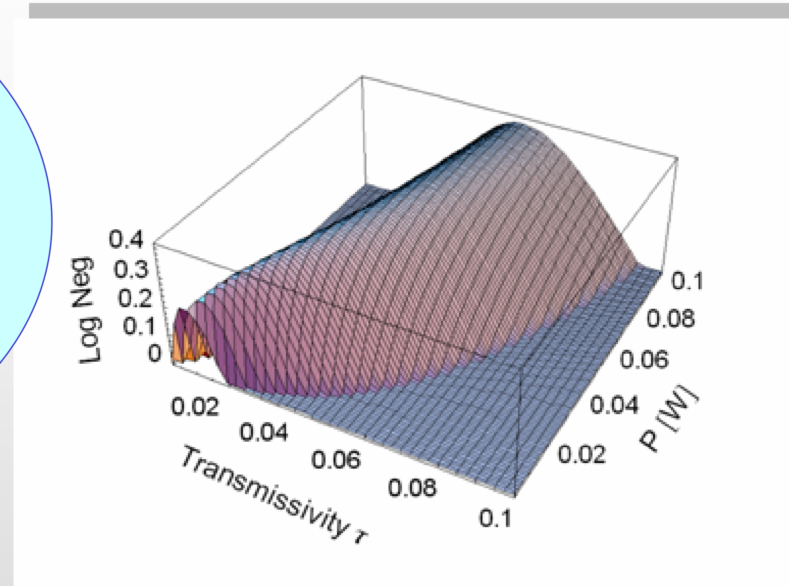


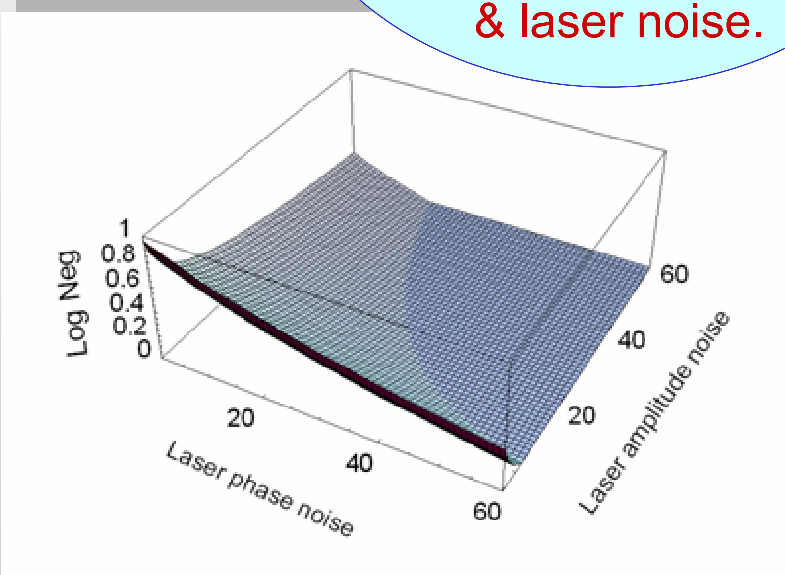
$$\Omega_x = \sqrt{\frac{2\hbar}{m\frac{4k_BT}{\pi\omega_m}\frac{d}{r_0^2Y}\left(\frac{Y'}{Y}\phi_{||} + \frac{Y}{Y'}\phi_{\perp}\right)}}$$

$$\Omega_F = \sqrt{\frac{2\gamma_m k_B T}{\hbar}}$$

Concrete example for mirror entanglement (2)

Recall that we have around f = 73 Hz a window of Δf = 132 Hz in which cl. noise is a factor of · 10 in power below the SQL & laser noise.





→ Considering suspension thermal noise (Q » 10^{10}) and coating thermal noise both at T = 10 K as well as moderate laser noise entanglement can survives.

Conclusion and discussion

- Conditioning on continuous measurement helps with preparing a (pure) quantum state (very close to Heisenberg limit).
- Devices need to be sub-SQL in a certain frequency band.
- Such sub-SQL devices with double detection are very likely to show entanglement of their end mirrors.
- Need to consider a complete noise model.
- Laser noise problem could be solved by using short arms and long cavities.
- Squeezing of both, vacuum at dark port and laser at bright port, will also help.
- In order to do without double readout the arms of our device could be replaced by the dark ports of a pair of coherently operated interferometers.