Data Analysis Techniques for LIGO

Laura Cadonati, M.I.T.

Trento, March 1-2, 2007

Lesson Plan

Yesterday:

1. Introducing the problem: GW and LIGO
2. Search for Continuous Waves
3. Search for Stochastic Background

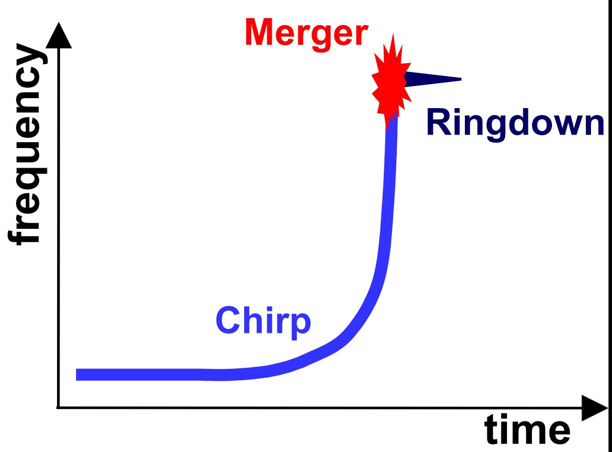
Today:

4. Search for Binary Inspirals
5. Search for Bursts
6. Network Analysis

Detecting GW from Inspirals

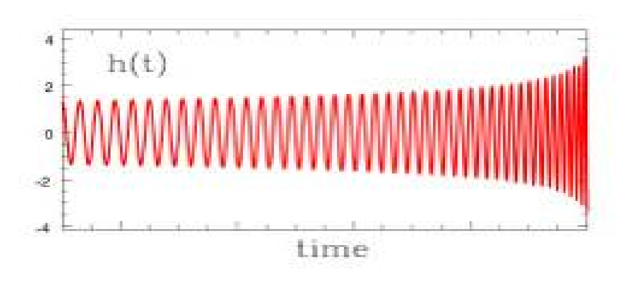
What does the signal look like?

Focus on inspiral phase at first: a chirp (frequency and amplitude increase in time). The duration is the time spent in the LIGO band, which depends on the two masses.



How do we quantify it?

"Triggers" when the matched filter SNR is above threshold.

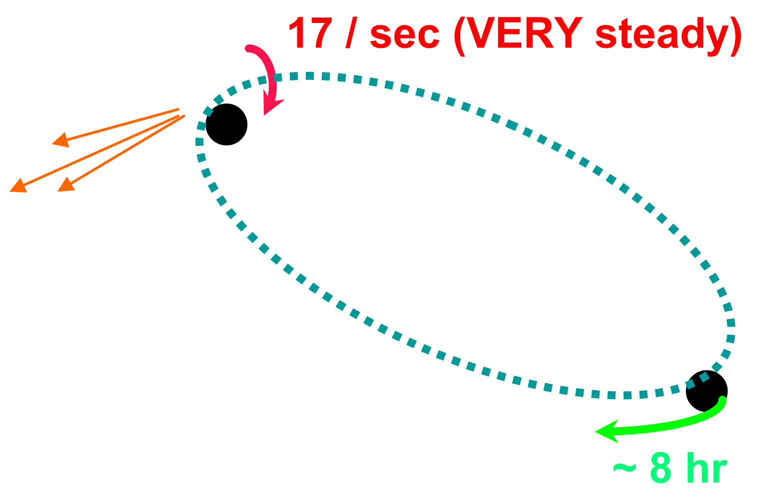


How do we look for it?

Matched filtering to assumed-known templates, followed by an event-like coincidence analysis between detectors.

Binary Systems

We know gravitational waves emitted from ***compact binary systems*** exist:

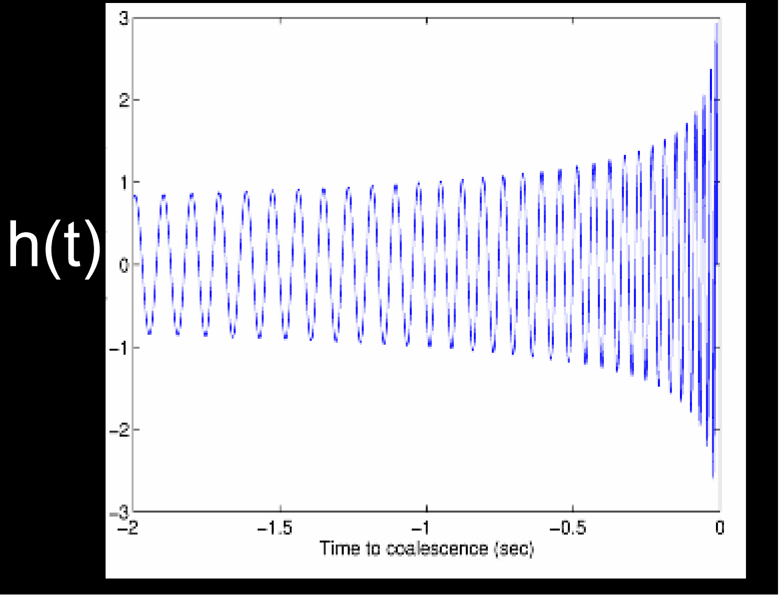


PSR1913+16 Hulse-Taylor

Neutron Star Binary System
- separated by 10^6 miles
- $m_1 = 1.4M_\odot$ $m_2 = 1.36M_\odot$ $\varepsilon = 0.617$

Exact match to general relativity
- spiral in by 3 mm/orbit
- shortening of orbital period

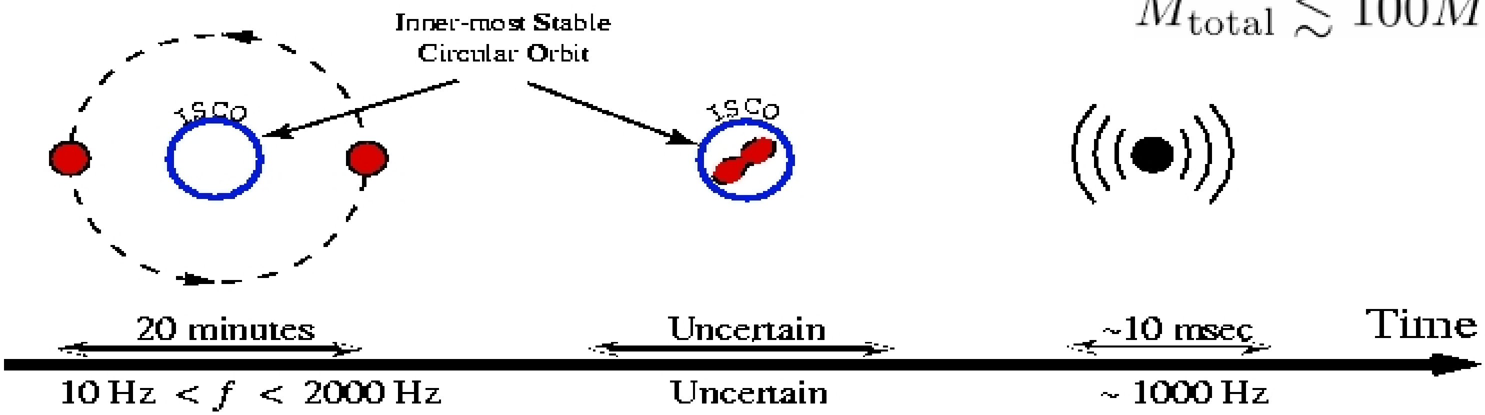
- Gravitational waves carry away energy and angular momentum. Orbit will continue decay
- In ~300 million years, the "inspiral" will accelerate, and the neutron stars coalesce
- Gravitational wave emission will be strongest near the end

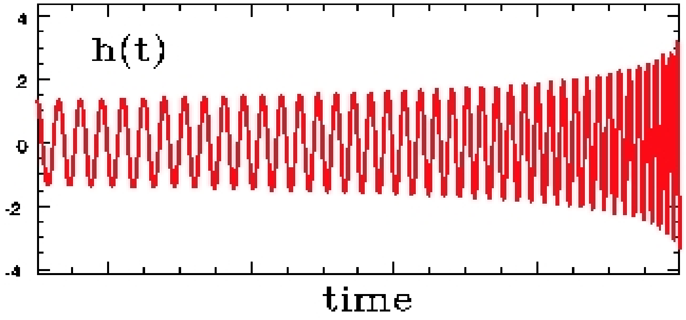


Evolution of Binary System

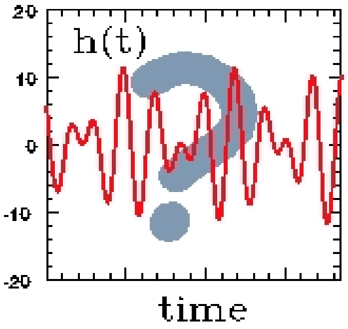
LIGO is sensitive to inspirals containing neutron stars and black holes

$M_{\mathrm{total}} \lesssim 100 M_{\odot}$

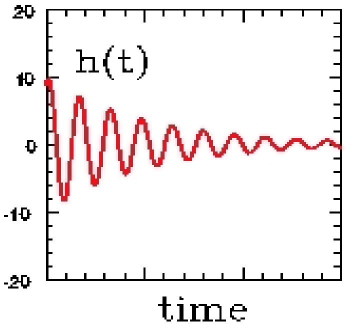




Matched filter



Template-less



Matched filter

Inspiral Chirp

To lowest order, as gravitational waves carry away energy:

“Newtonian” chirp:

Coalescence time

Frequency: $f(t) \propto (t-t_c)^{-3/8}$

Waveform: $h(t) = A(t) \cos(B\,(t-t_c)^{5/8} + \phi_c)$

$$= A'(f) \cos(\underbrace{B'\, f^{-5/3} + \phi_c}_{\Psi(f)})$$

Higher order, “Post-Newtonian” corrections change the phase evolution, function of total mass and reduced mass:

$$m = (m_1 + m_2)\,, \quad \eta = \frac{m_1\, m_2}{m^2}$$

Source Parameters vs. Signal Parameters

Source parameters:

- Masses (m1, m2)
- Spins → Assume negligible ***for now***
- Orbital phase at coalescence → Maximize analytically when filtering
- Inclination of orbital plane
- Sky location

Scale factor for a given detector

- Distance → Simply multiplicative

Signal at the Detector

$$h(t) = \frac{1\ \mathrm{Mpc}}{D_{\mathrm{eff}}}\left[\sin\alpha\, h_s^I(t-t_c) + \cos\alpha\, h_c^I(t-t_c)\right]$$

2 polarizations

α depends on orbital phase and orientation of the binary
t_c = time at the detector when the binary reaches ISCO
D_{eff} = effective distance, depends on true distance r and orientation, $D_{\mathrm{eff}}>r$

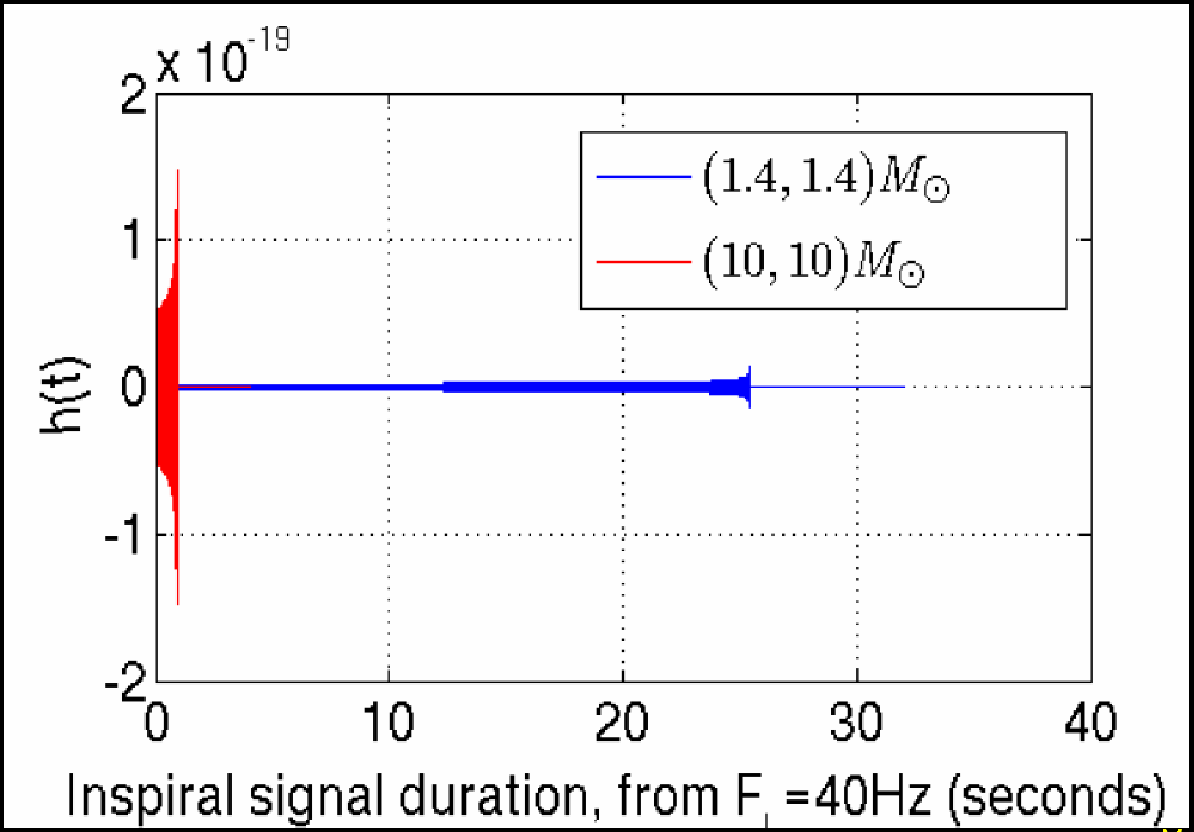
ISCO = Innermost Stable Circular Orbit

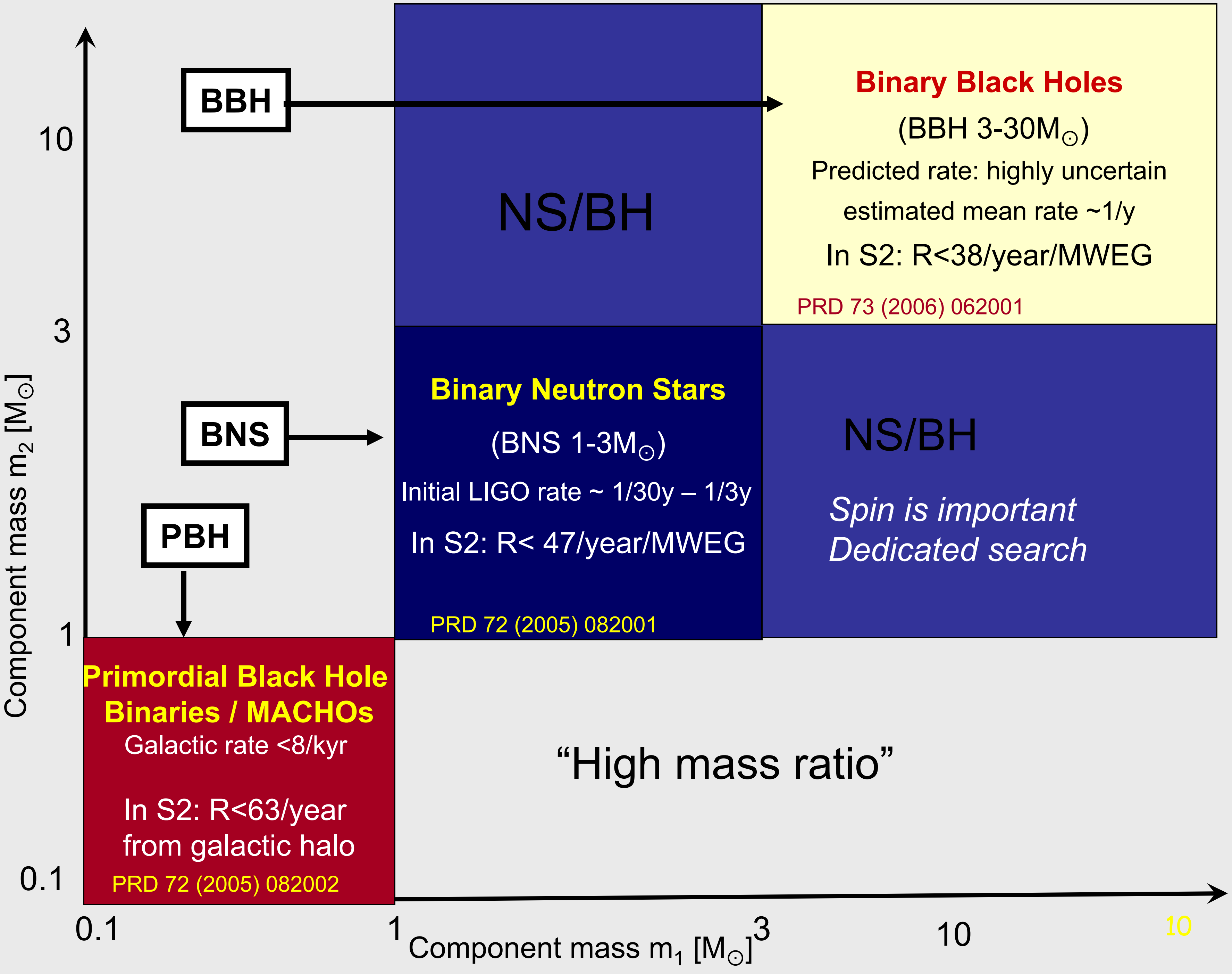
Stationary phase approximation: $\tilde{h}_c^I(f) = -i\tilde{h}_s^I(f)$ $\tilde{q}(f) = \int_{-\infty}^{\infty} e^{-2\pi i f t} q(t)\,dt$

Signal at the Detector

- Amplitude and duration only depend on the masses m_1 and m_2 and the lower cutoff frequency.
- D_{eff} depends on the physical distance and orientation of the binary system.

$$h(t) = \frac{1Mpc}{D_{\text{eff}}}\left[h_c(t)\cos\Phi + h_s(t)\sin\Phi\right]$$





Binary Black Holes

(BBH 3-30$M_\odot$)

Predicted rate: highly uncertain

estimated mean rate ~1/y

In S2: R<38/year/MWEG

PRD 73 (2006) 062001

Binary Neutron Stars

(BNS 1-3$M_\odot$)

Initial LIGO rate ~ 1/30y – 1/3y

In S2: R< 47/year/MWEG

PRD 72 (2005) 082001

Spin is important
Dedicated search

Primordial Black Hole Binaries / MACHOs

Galactic rate <8/kyr

In S2: R<63/year
from galactic halo

PRD 72 (2005) 082002

"High mass ratio"

Differences Between Searches

The BNS and PBH binary searches are very similar:

- Templates based on second order restricted to post-Newtonian waveforms, in the stationary phase approximation.
- Identical template bank placement.
- Identical filtering process
- Similar coincidence windows
- Hierarchical search
- Chi square

Final triggers associated to an effective SNR which combines the SNR and its Chi square value.

The BBH search used the same pipeline but :

- Target waveform non accurately known. We used templates based on phenomenological waveforms, which uses two phenomenological parameters. Consequences :
  1. Different template bank.
  2. Different filtering.
  3. Non physical mass parameters
- Coincidence in time, and the 2 phenomenological parameters.
- No chi square applied.

Final triggers associated to the classical SNR.

Matched Filtering

$$\rho(t) = \frac{|z(t)|}{\sigma}$$

Signal-to-Noise ratio

$$z(t) = x(t) + iy(t) = 4\int_0^\infty \frac{\tilde{h}_c^{I*}(f)\tilde{s}(f)}{S_n(f)} e^{2\pi i f t} df$$

$$\sigma^2 = \frac{1}{2}\langle|z(0)|^2\rangle = 4\int_0^\infty \frac{|\tilde{h}_c^I(f)|^2}{S_n(f)} df$$

one-sided noise PSD in the detector

In practice, the integral is limited between:

f_{low} set by the detector

f_{max} set by the template

If the template is normalized to strain at 1Mpc optimal orientation, the effective distance is:

$$D = \frac{\sigma}{\rho}\mathrm{Mpc}$$

Mismatch and Event Rate

- If the template we use does not exactly match the signal, we have a loss in signal-to-noise ratio
- Loss in signal-to-noise ratio is loss in detector range (the distance to which we can detect inspiral signals)
- Loss in event rate = (Loss in range)^3
- We must be careful that the mismatch between the signal and our templates does not unacceptably reduce our rate

$$\mathrm{match} = \max_{t_0,\phi_0,\mathcal{M},\eta,\ldots} \frac{\langle h | h_{\mathrm{true}} \rangle}{\sqrt{\langle h | h \rangle \langle h_{\mathrm{true}} | h_{\mathrm{true}} \rangle}}$$

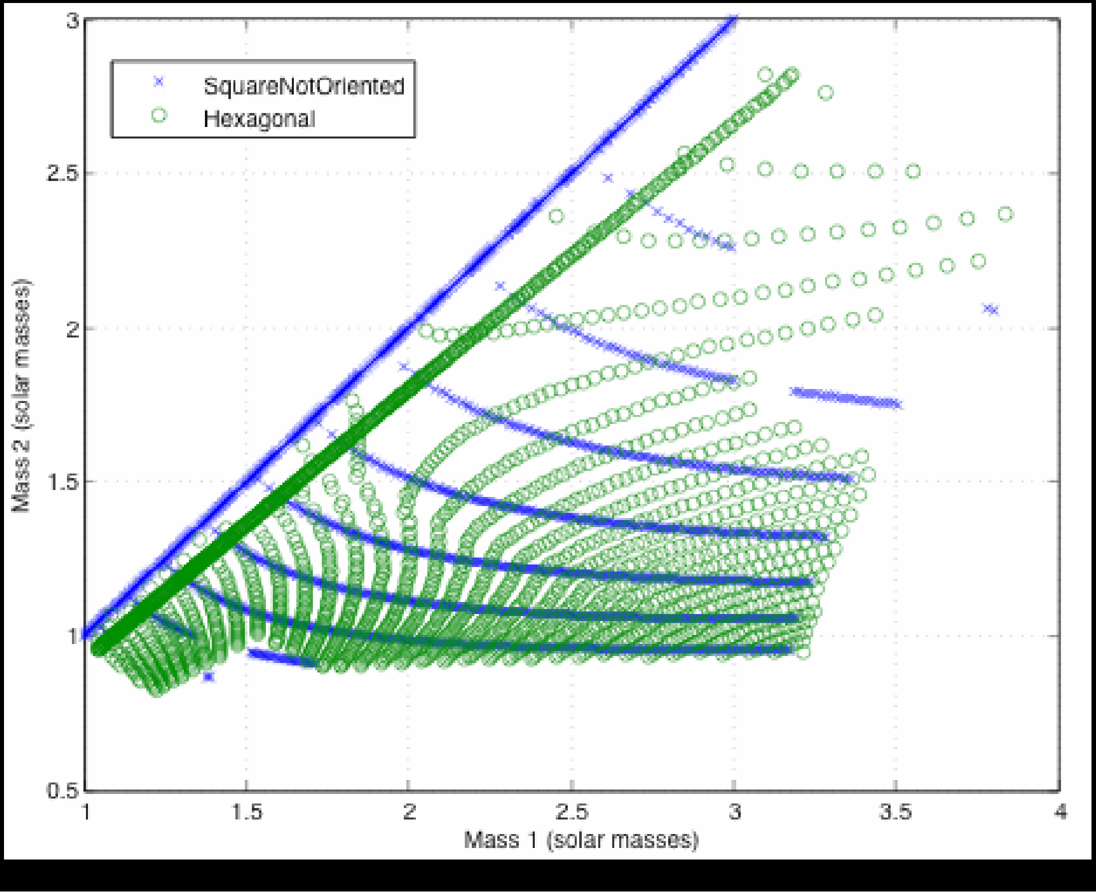
mismatch = 1 – match

$$\langle a | b \rangle = \int_{f_{\mathrm{low}}}^{f_{\mathrm{max}}} \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)}\, df$$

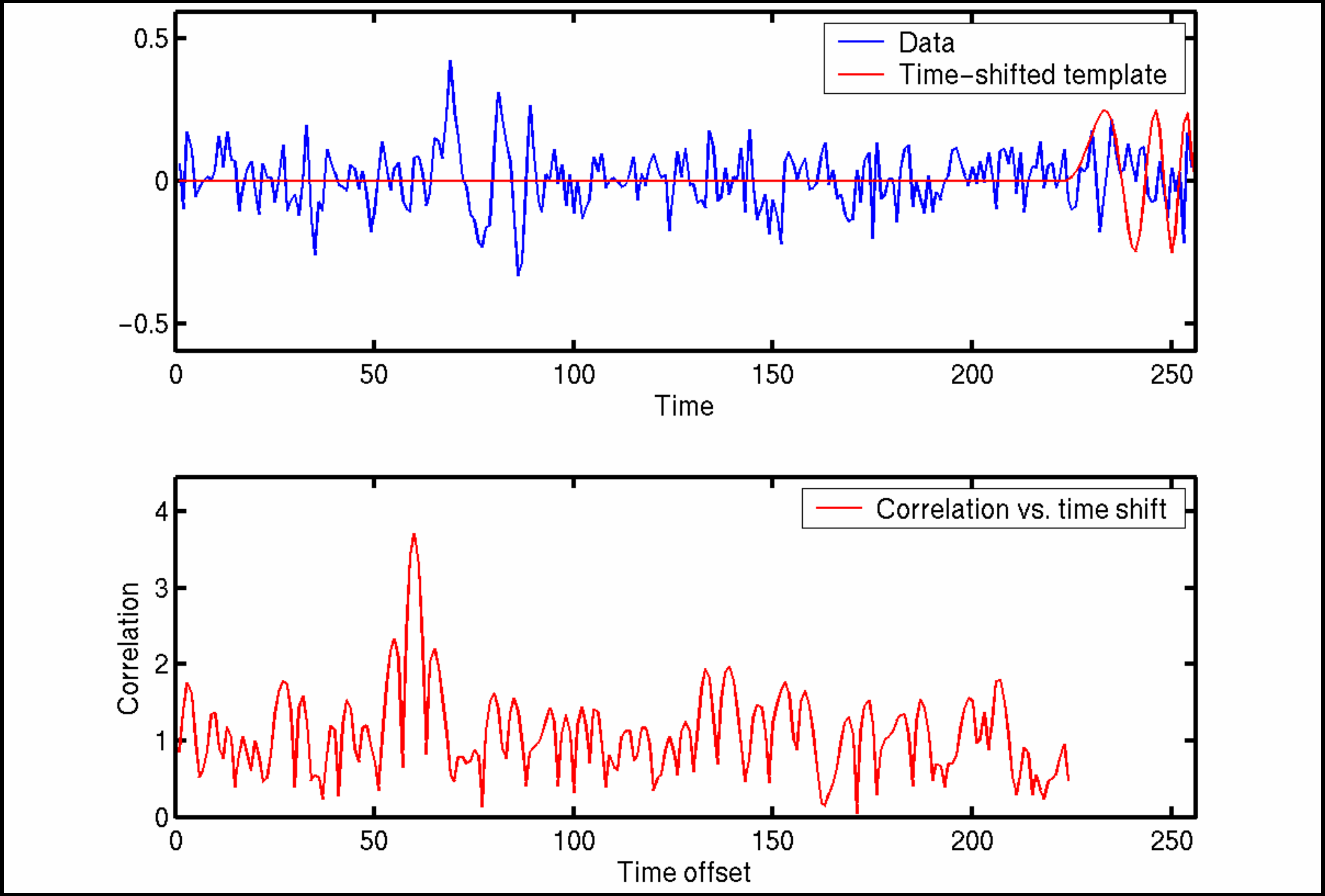
Inspiral Template Banks

- To search for signals in the mass region of interest, we must construct a template bank
- Lay grid of templates so that loss in SNR between signal in space and nearest template is no greater than ~ 3%

Different possible ways to lay the template bank



Finding "Triggers"



Finding “Triggers”

Data after FFT

Template, generated in freq. domain using stationary phase approx.

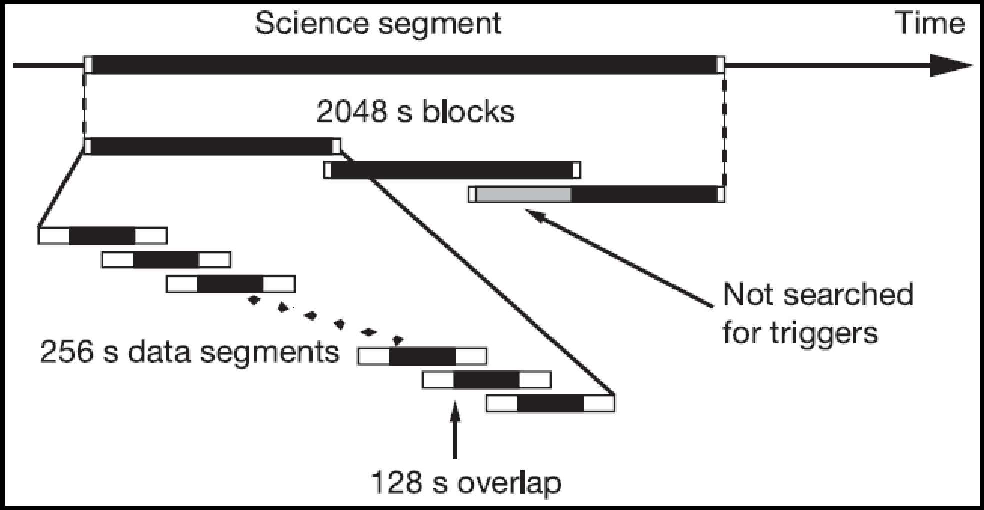
$$z(t) = 4\int_{0}^{\infty} \frac{\tilde{s}(f)\ \tilde{h}^{*}(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

One-sided noise power spectral density

- Look for maximum of $|z(t)|$ above some threshold ➔ trigger

- Search overlapping intervals to cover science segment, avoid wrap-around effects

- Estimate power spectrum from bin-by-bin median of fifteen 256-sec data segments

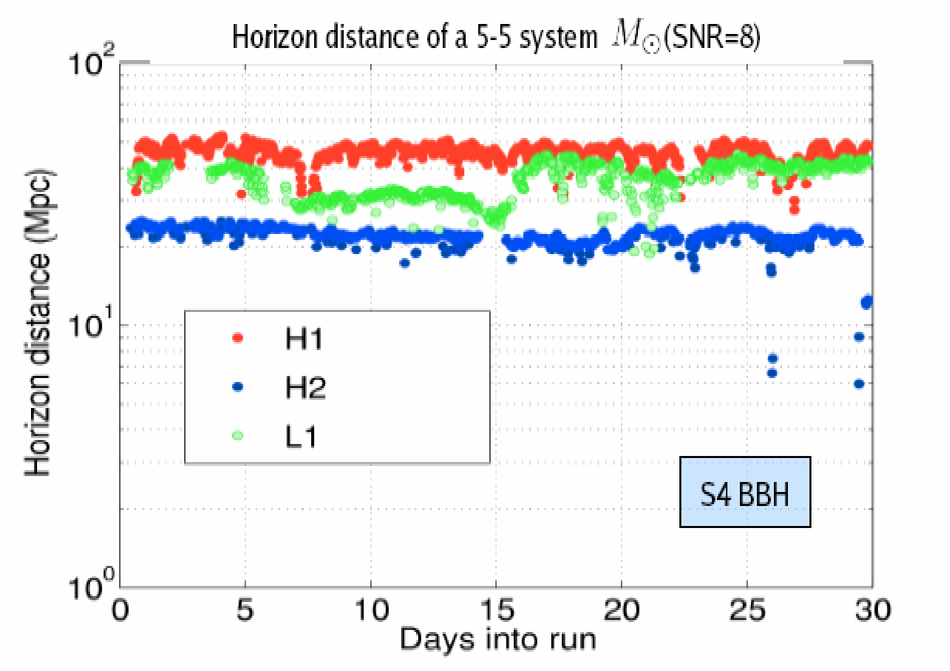


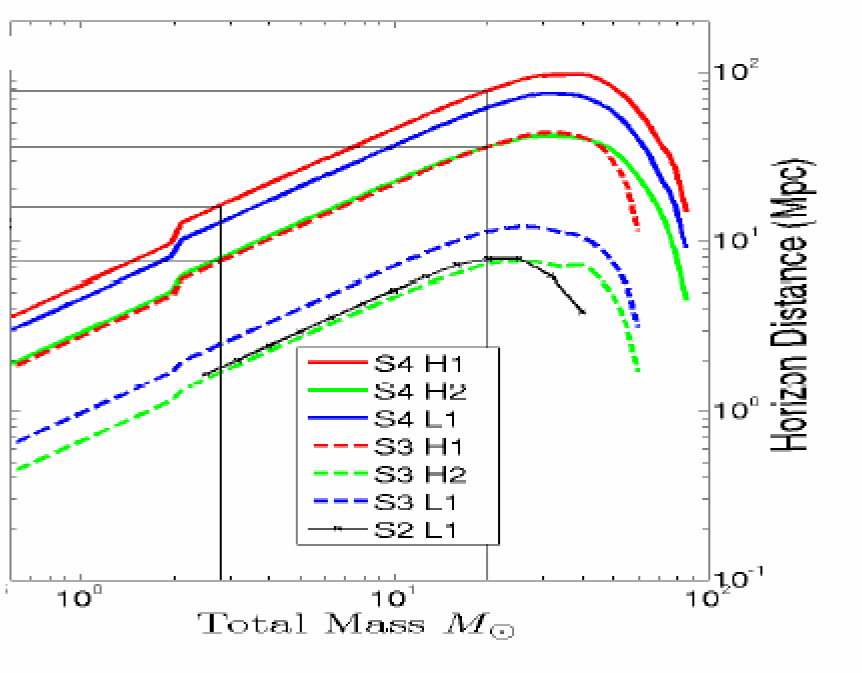
Horizon Distance

- Distance at which an optimally oriented and located binary system can be seen with signal-to-noise ratio ρ=8
- It is computed from the noise PSD every 2048 sec, to track non-stationarities

$$D_\rho(Mpc) = \frac{A}{1\mathrm{Mpc}\times\rho}\times f(m_1,m_2)\times\int_{F_L}^{f_{cut}}\frac{f^{-7/3}}{S_h(f)}df$$

(horizon=range at peak of antenna pattern; ~2.3 x antenna pattern average)

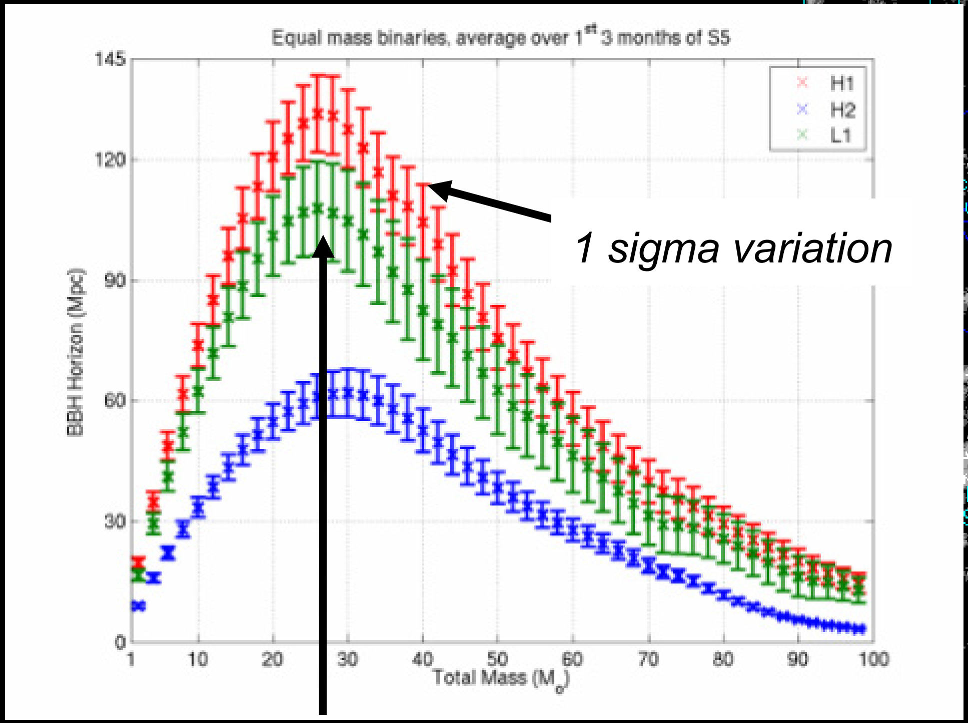




Ref: LIGO-G060630-00

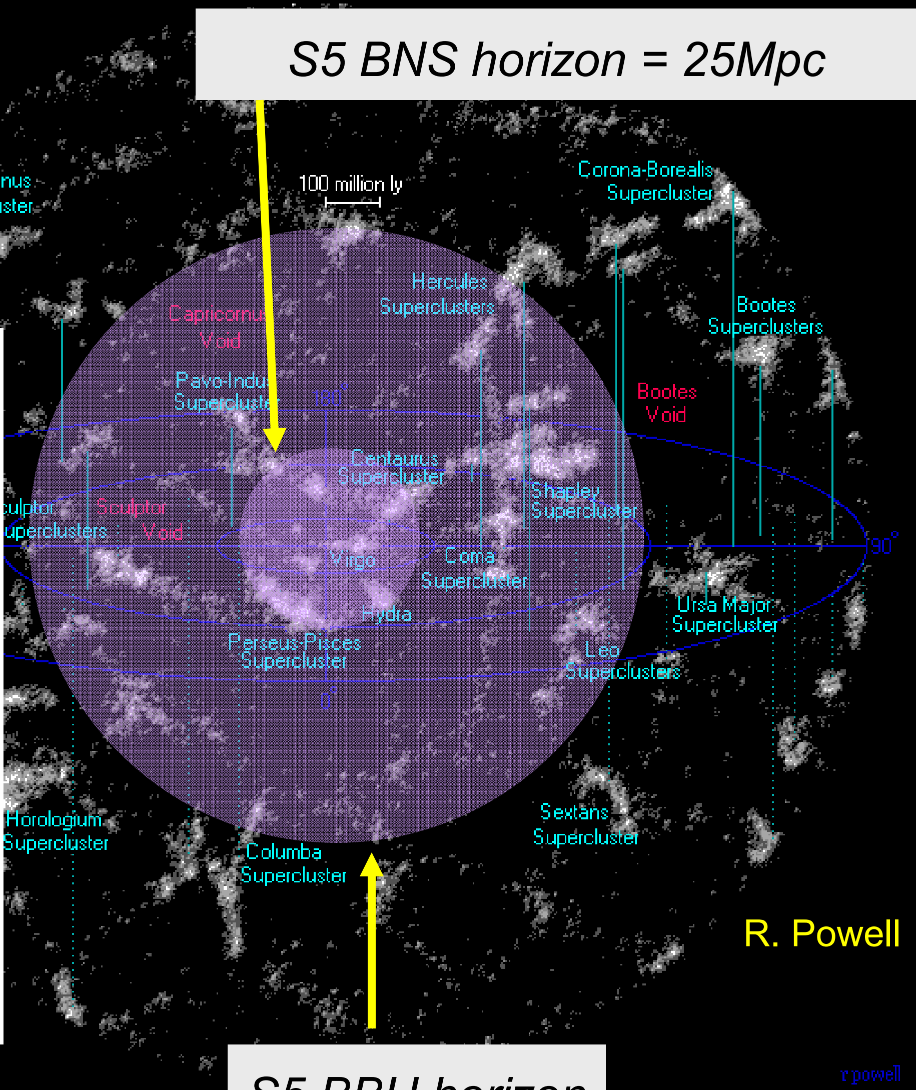
Horizon as Measure of Sensitivity

For 1.4-1.4 M_o binaries, ~ 200 MWEGs in range
For 5-5 M_o binaries, ~ 1000 MWEGs in range



1 sigma variation

Peak 130Mpc at total mass ~ 25M_{sun}

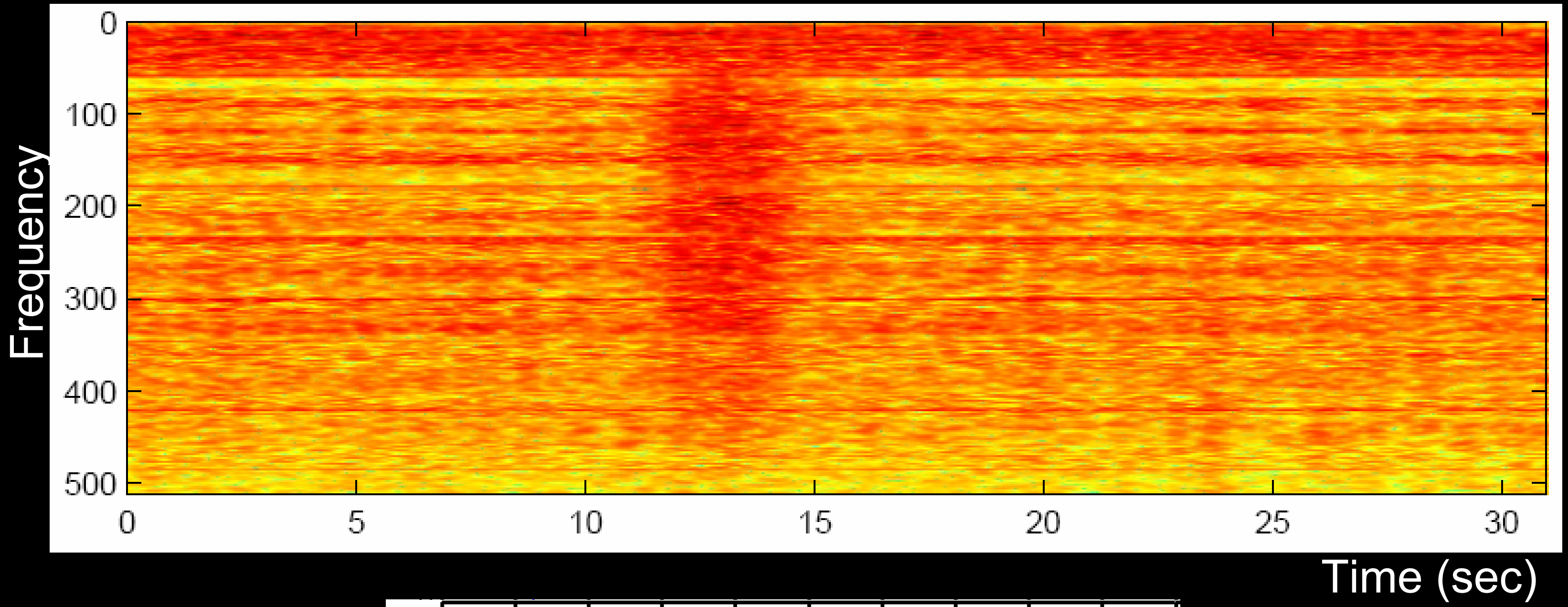


S5 BNS horizon = 25Mpc

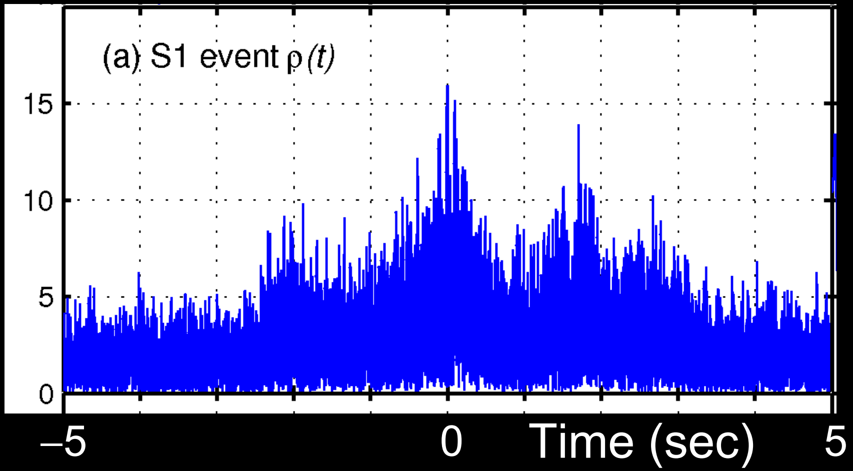
S5 BBH horizon

R. Powell

Dealing with Non-Stationary Noise



Inspiral filter output:



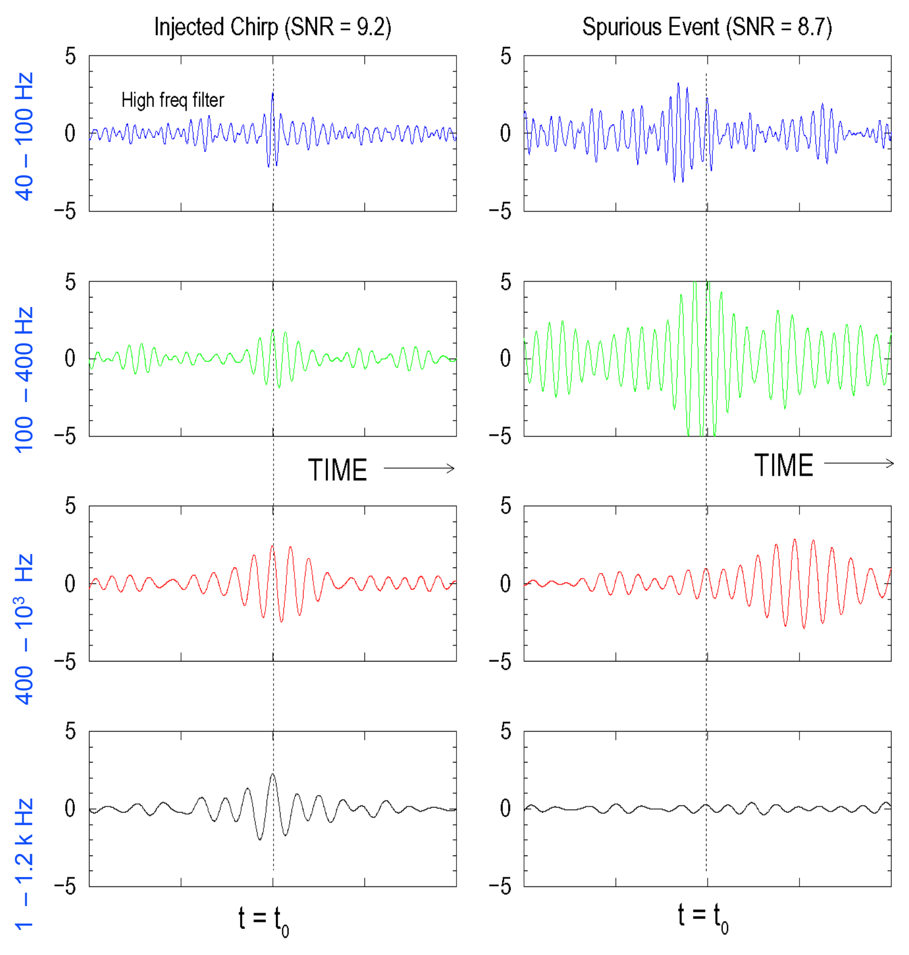
χ^2 Signal Based Veto

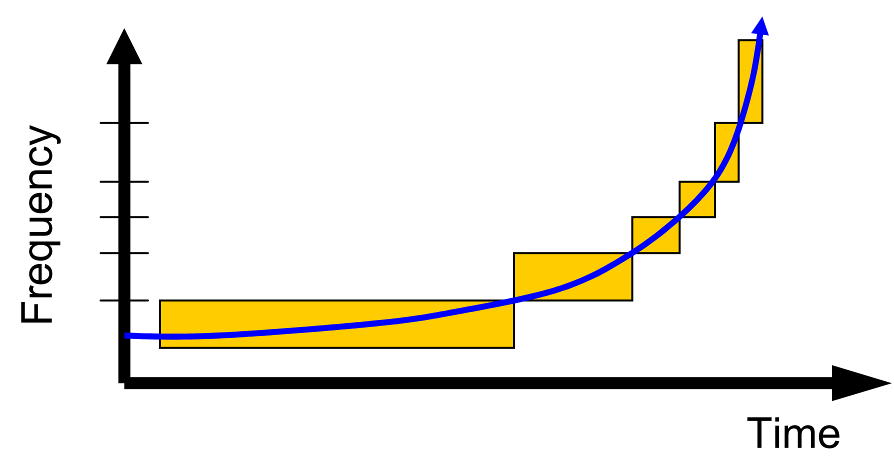
Divide template into p bands, compute $z_l(t)$ in each band

$$\chi^2(t) = p \sum_{l=1}^{p} \| z_l(t) - z(t)/p \|^2$$

$$\xi*^2 = \frac{\chi^2}{p(1+\delta^2\rho^2)} < \textbf{thr}$$

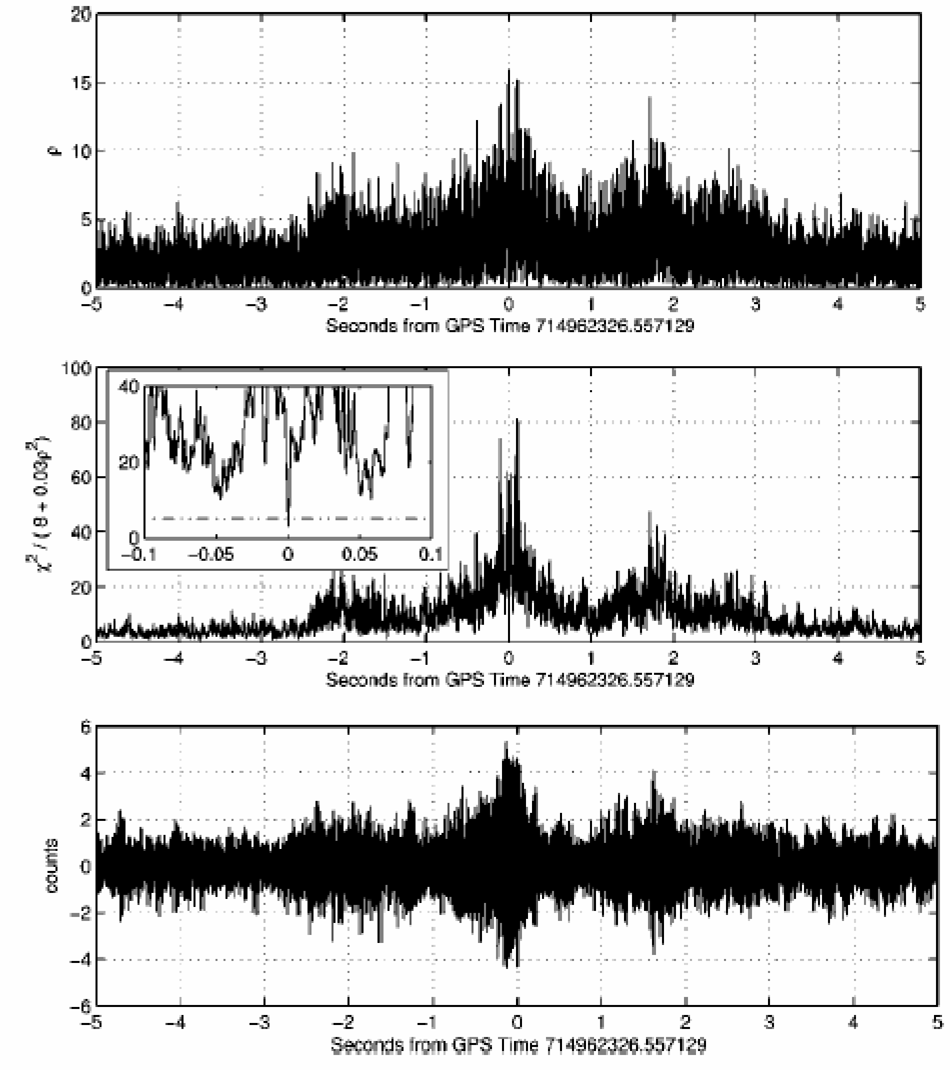
Account for template mismatch δ=0.03



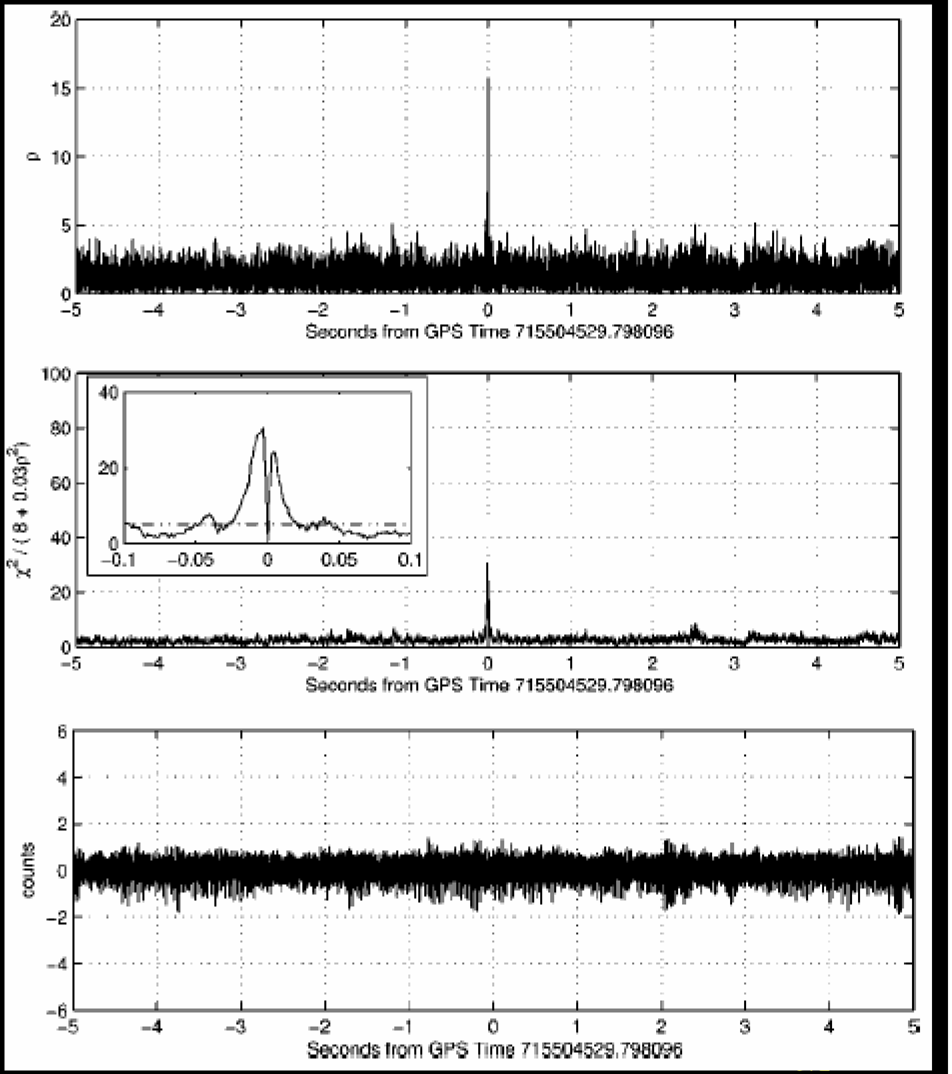


χ^2 Veto Sometimes is Not Enough

L1 loudest event in the S1 BNS search



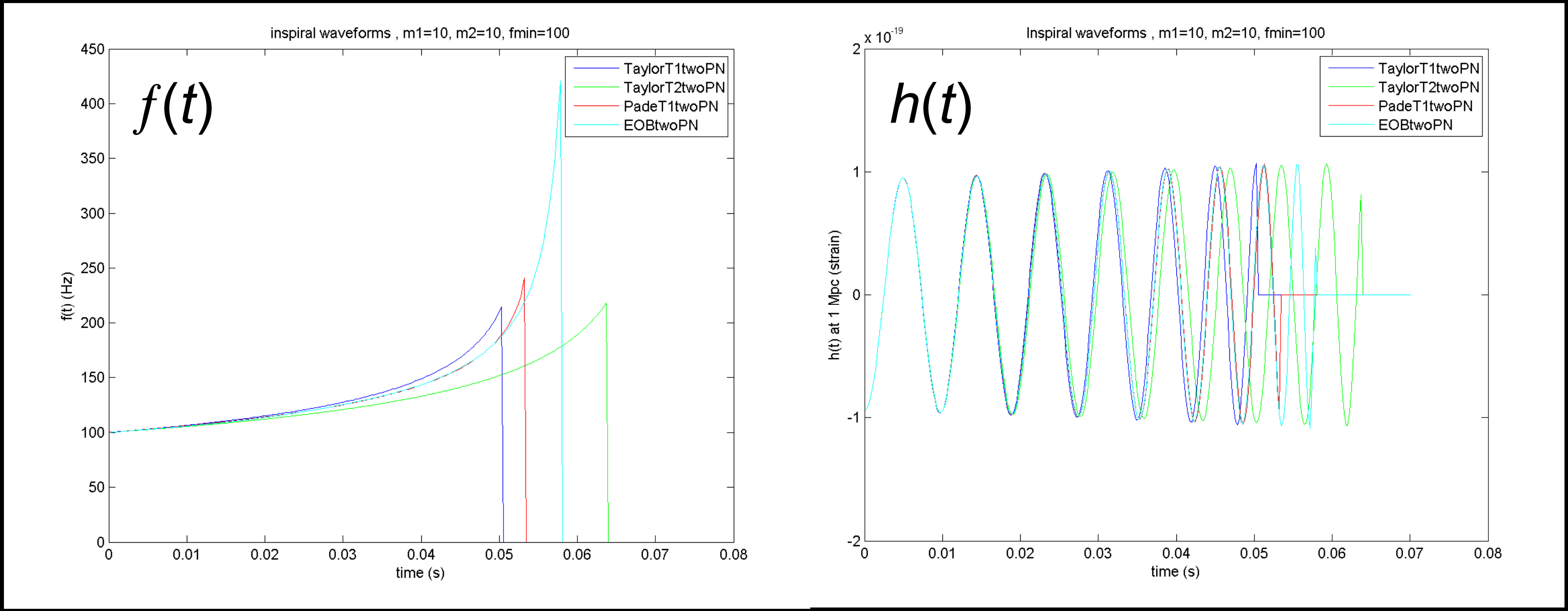
Simulated signal added to the data



Ref: PRD 69, 122001 (2004)

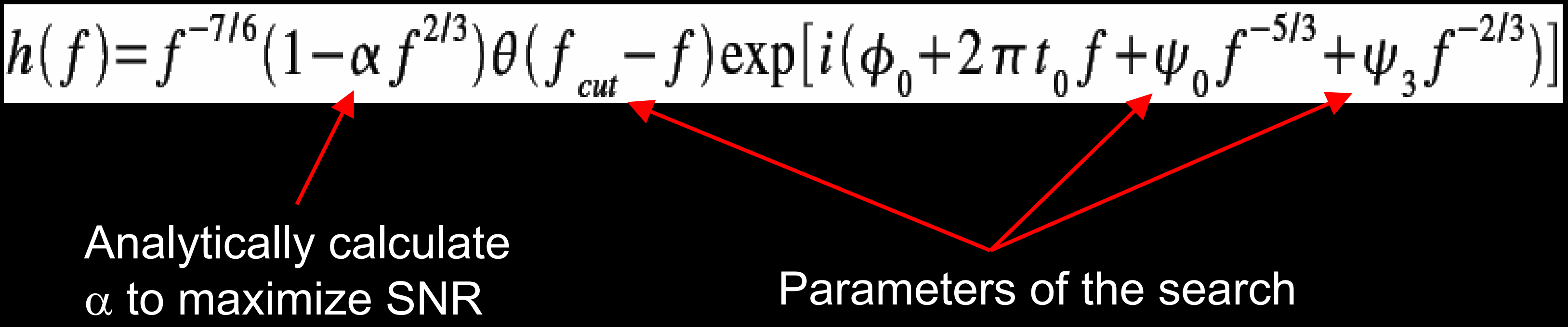
Uncertain Waveforms for High-Mass Inspirals

Different models for 10+10 $M_{\odot}$ black hole binary inspiral



BCV Detection Template Family

- Buonanno, Chen, and Vallisneri, Phys. Rev. D 67, 104025 (2003)



$$h(f) = f^{-7/6}(1 - \alpha f^{2/3})\theta(f_{cut} - f)\exp[i(\phi_0 + 2\pi t_0 f + \psi_0 f^{-5/3} + \psi_3 f^{-2/3})]$$

Analytically calculate
α to maximize SNR

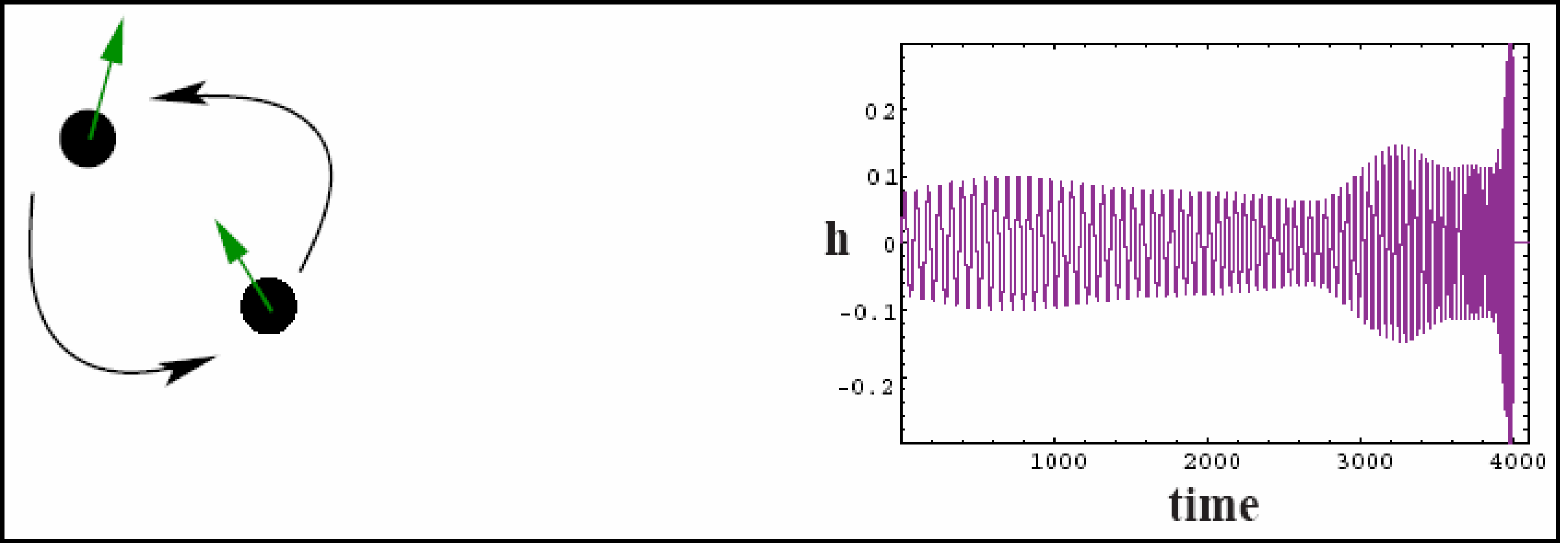
Parameters of the search

Can match the various waveform models rather well

This is intended for binary components with negligible spin

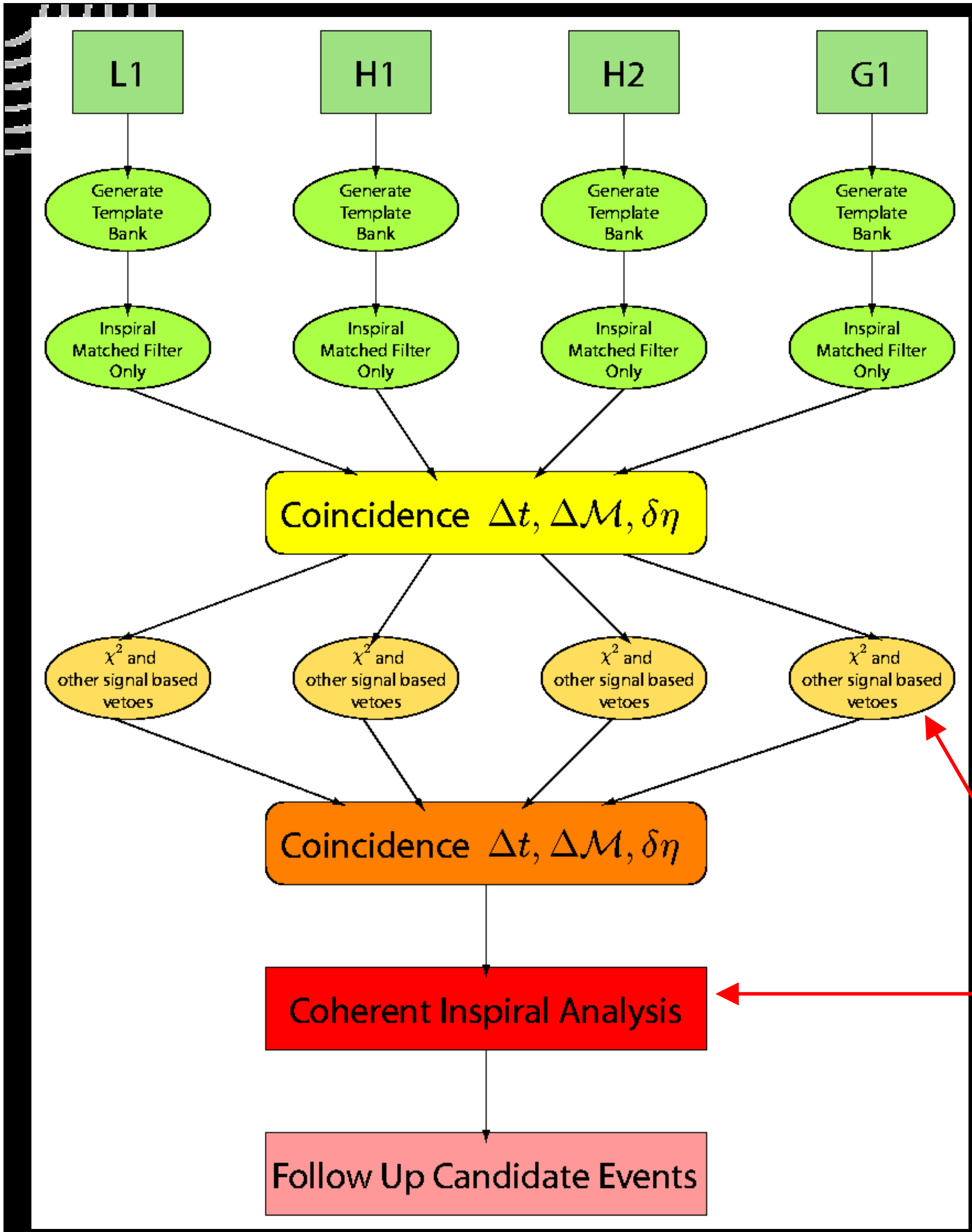
Binary Systems with Spin

- Waveform can be much more complicated !



- Another BCV detection template family for systems with spin
  - Six more analytically calculated parameters
  - One more search parameter ⇒ 4-dimensional parameter space

Coincident Analysis “Pipeline”



L1

H1

H2

G1

Generate Template Bank

Inspiral Matched Filter Only

Coincidence $\Delta t, \Delta\mathcal{M}, \delta\eta$

χ^2 and other signal based vetoes

Coincidence $\Delta t, \Delta\mathcal{M}, \delta\eta$

Coherent Inspiral Analysis

Follow Up Candidate Events

Single-detector SNR is not enough to establish confidence in an event.

We require coincident detection in at least two detectors (factor sensitivity in)

Computationally expensive tasks

Coincidence Requirements

- Require a coincident trigger between at least two detectors.

- Coincidence parameters are tuned by software "injection" of simulated signals in the data stream
  - Mass -- particularly chirp mass
  - End time -- also used for estimation of sky location
  - Distance -- only important for co-located Hanford instruments

- Competing considerations:
  - Windows must be wide enough not to miss potential signals
  - Tighter coincidence windows yield a lower false alarm rate

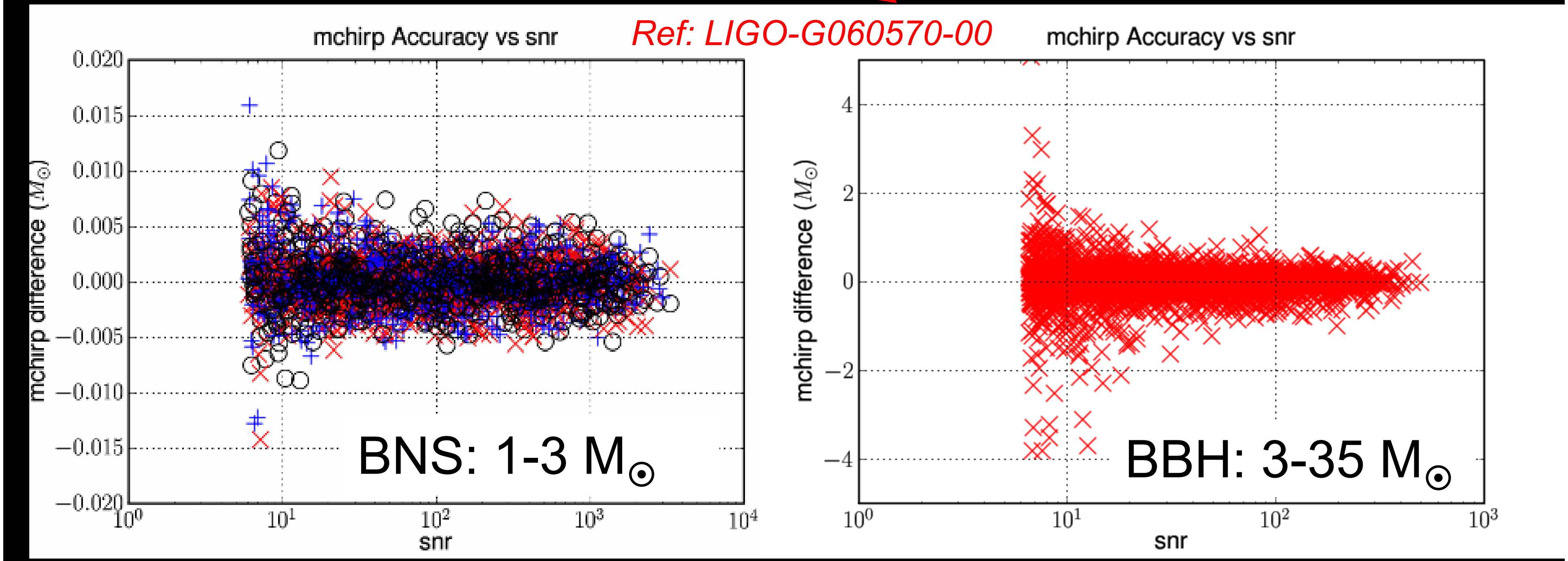
It all boils down to our ability to determine the waveform parameters, which improves for longer waveforms and larger SNR

Chirp Mass Accuracy

$$\mathcal{M} = M\eta^{3/5} \qquad \eta = \frac{m_1 m_2}{M^2}$$

Significantly decreases for higher mass

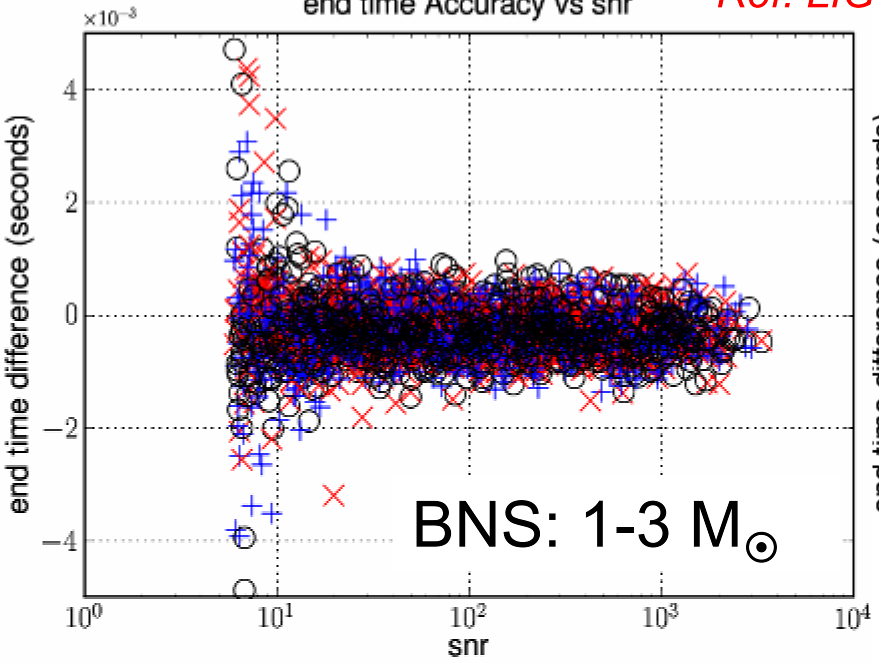
Difference between simulated and measured, vs SNR

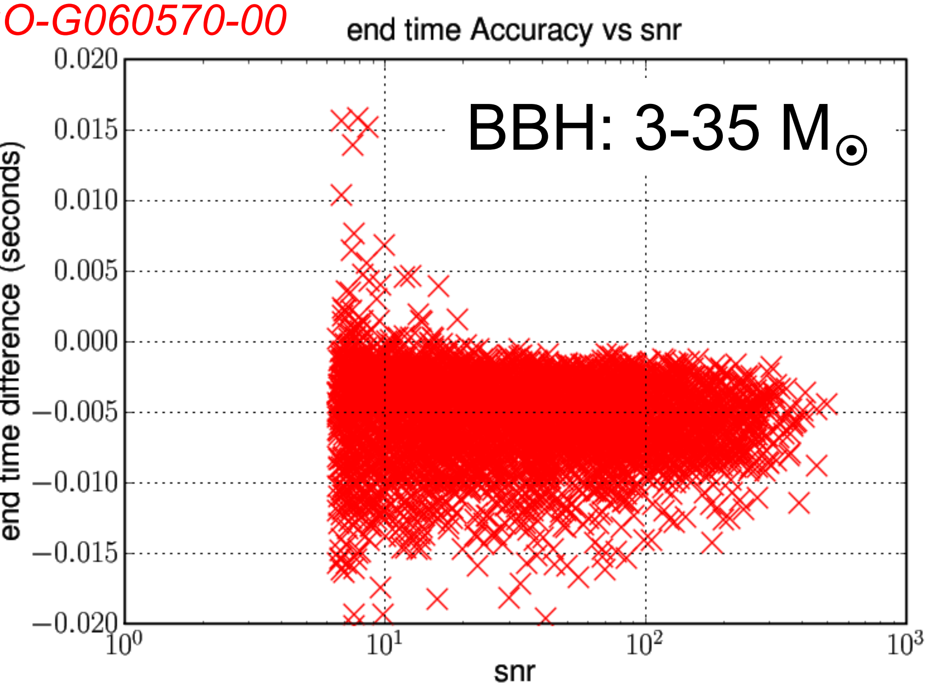

Ref: LIGO-G060570-00

Timing Accuracy

- As before, parameter accuracy better for longer templates.
- Timing accuracy determines ability to recover sky location
- Timing systematic is due to injecting TD, recovering FD.
  - Overall systematic (same at all sites) does not affect sky location.

Ref: LIGO-G060570-00





Coincidence Windows

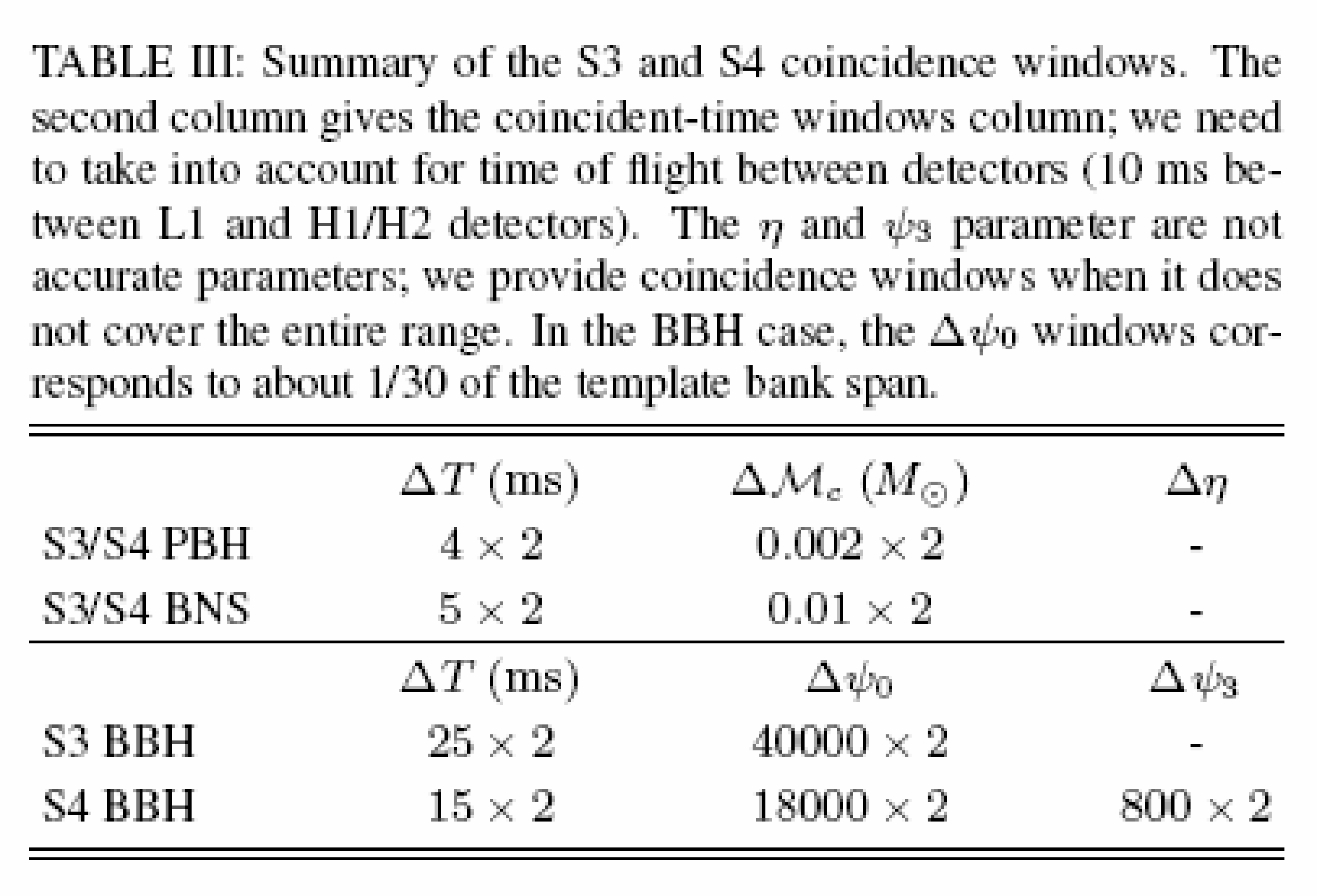
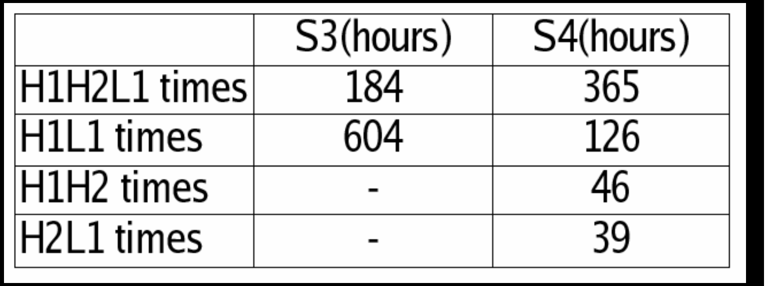


TABLE III: Summary of the S3 and S4 coincidence windows. The second column gives the coincident-time windows column; we need to take into account for time of flight between detectors (10 ms between L1 and H1/H2 detectors). The η and ψ_3 parameter are not accurate parameters; we provide coincidence windows when it does not cover the entire range. In the BBH case, the $\Delta\psi_0$ windows corresponds to about 1/30 of the template bank span.

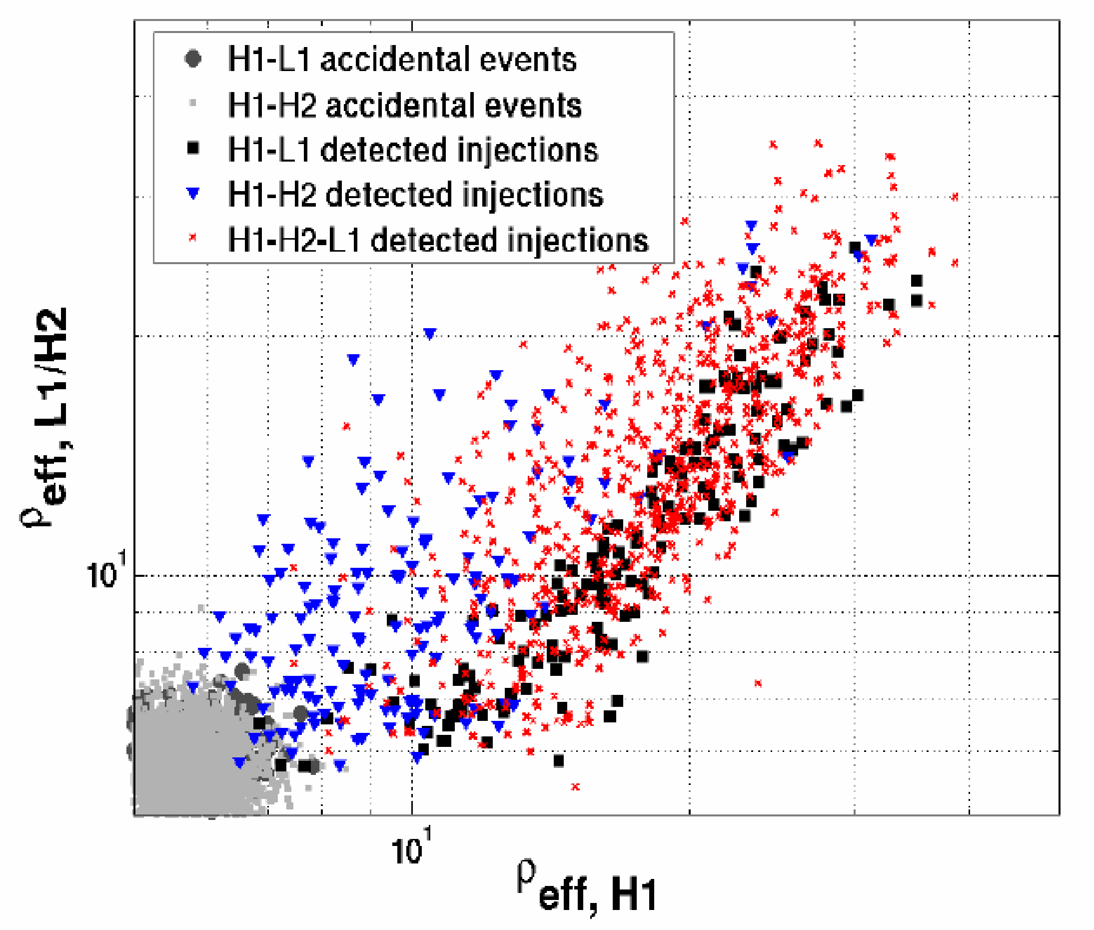
	ΔT (ms)	$\Delta \mathcal{M}_c$ ($M_\odot$)	$\Delta\eta$
S3/S4 PBH	4×2	0.002×2	-
S3/S4 BNS	5×2	0.01×2	-
	ΔT (ms)	$\Delta\psi_0$	$\Delta\psi_3$
S3 BBH	25×2	40000×2	-
S4 BBH	15×2	18000×2	800×2

Tuning Coincident Searches

- software simulations for efficiency estimate
- 100 time slides for background estimate



	S3(hours)	S4(hours)
H1H2L1 times	184	365
H1L1 times	604	126
H1H2 times	-	46
H2L1 times	-	39



For BNS and PBH:

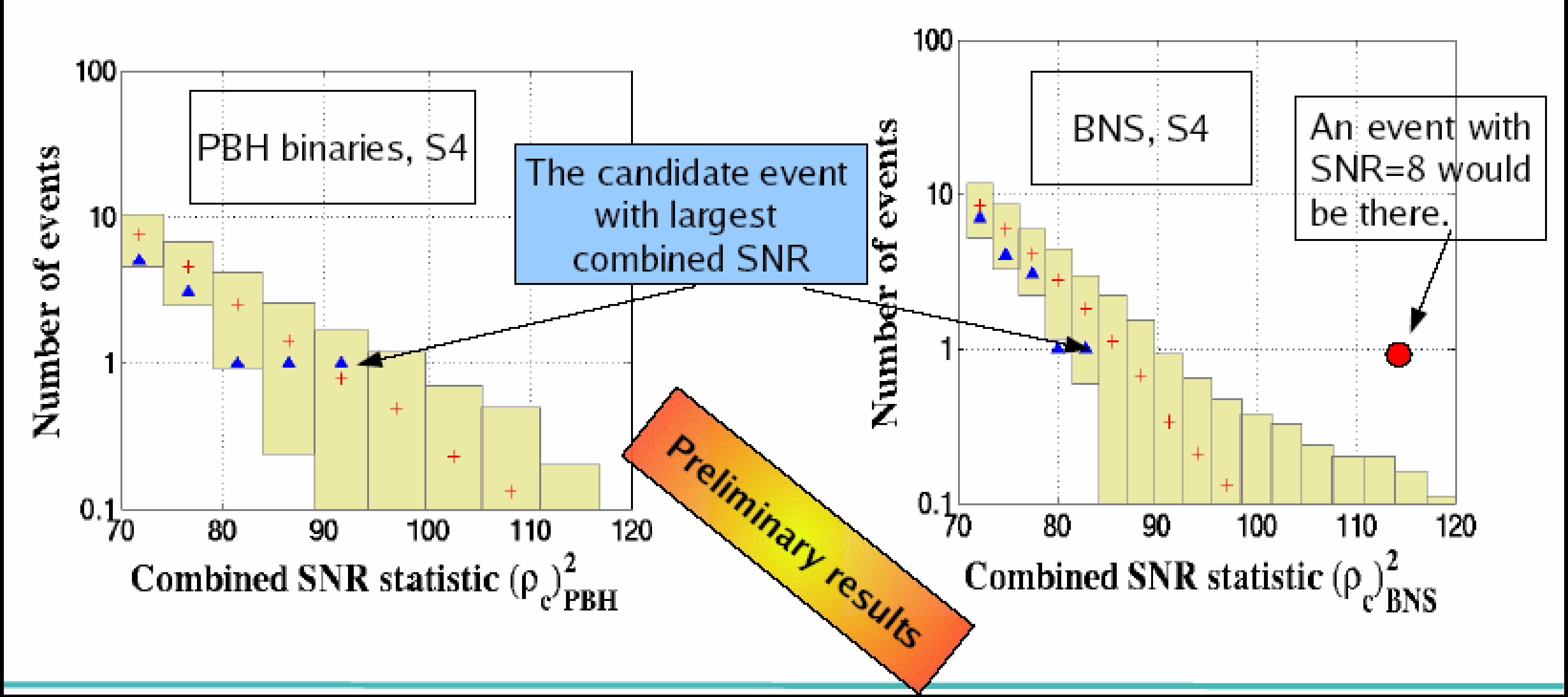
$$\rho^2_{\mathrm{effective}} = \rho^2 \Bigg/ \sqrt{\left(\frac{\chi^2}{2p-2}\right)\left(1+\frac{\rho^2}{250}\right)}$$

Ref: LIGO-G060630-00

Comparing Coincidences with Background

$$(\rho_c)^2_{\mathrm{BNS,PBH}} = \sum_i^N \rho^2_{\mathrm{eff,i}}$$

Combined effective SNR (sum over detectors)



PBH binaries, S4

The candidate event with largest combined SNR

BNS, S4

An event with SNR=8 would be there.

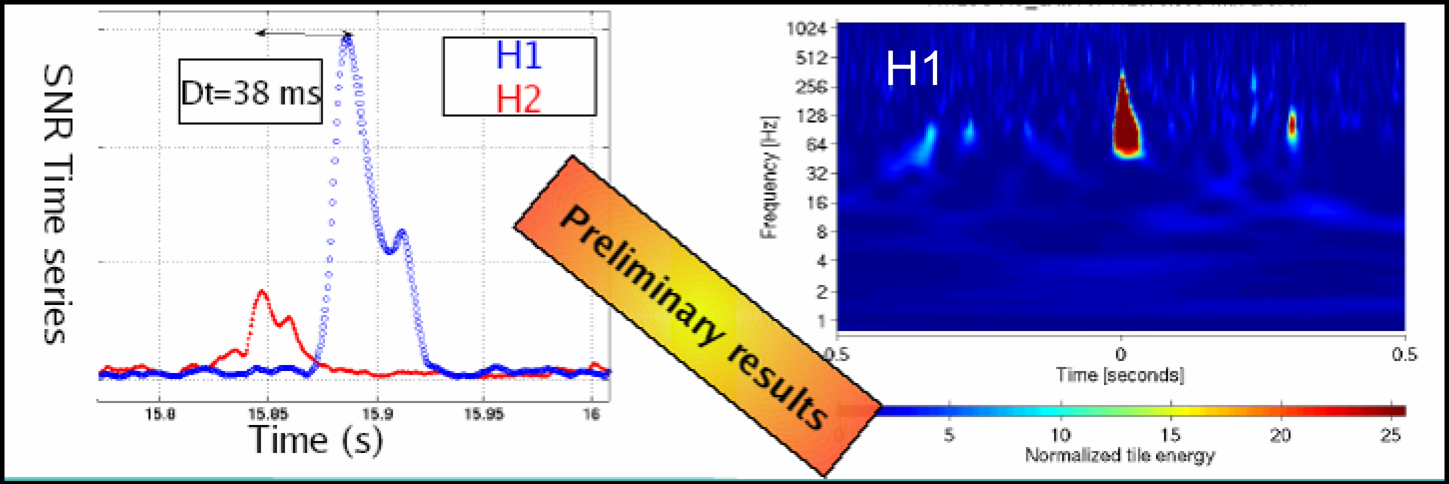
Preliminary results

Ref: LIGO-G060630-00

Follow-up of Loudest Candidate

All loudest events are scrutinized: status of the detector, spectrograms, a closer look at parameters…

In the S3 BBH analysis, one candidate was above the estimated background in H1-H2 coincidence.
We know that H1-H2 background is underestimated (environmental correlations in co-located instruments).
Tighter constraints on H1-H2, null-stream, etc… reject this event



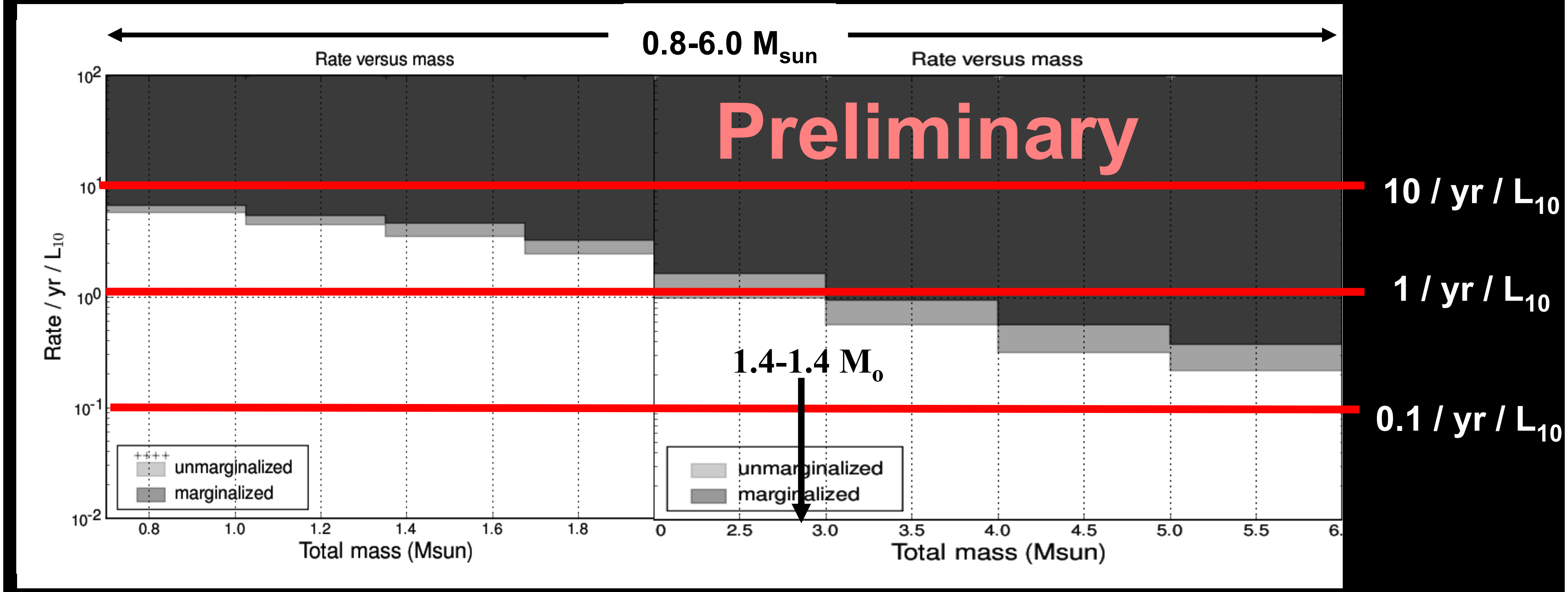
Ref: LIGO-G060630-00

If no Detection, Set an Upper Limits

The Bayesian upper limit calculation is based on the loudest event statistic *(Class.Quant.Grav. 21 (2004) S1775-S1782 or gr-qc/0405044)* which uses

- The detection efficiency at the loudest event (how many injections are found with combined SNR above the largest candidate event.
- The background triggers.
- Galaxy Population
- Time analysed
- systematics errors such as Monte-Carlo errors, waveform inaccuracy, calibration errors...

And here is what the end result looks like



Rate/year/L_{10} vs. binary total mass

$L_{10} = 10^{10} L_{sun,B}$ (1 Milky Way = 1.7 L_{10})

Dark region excluded at 90% confidence.

Issues of Astrophysical Interpretation

What population characteristics do we expect ?

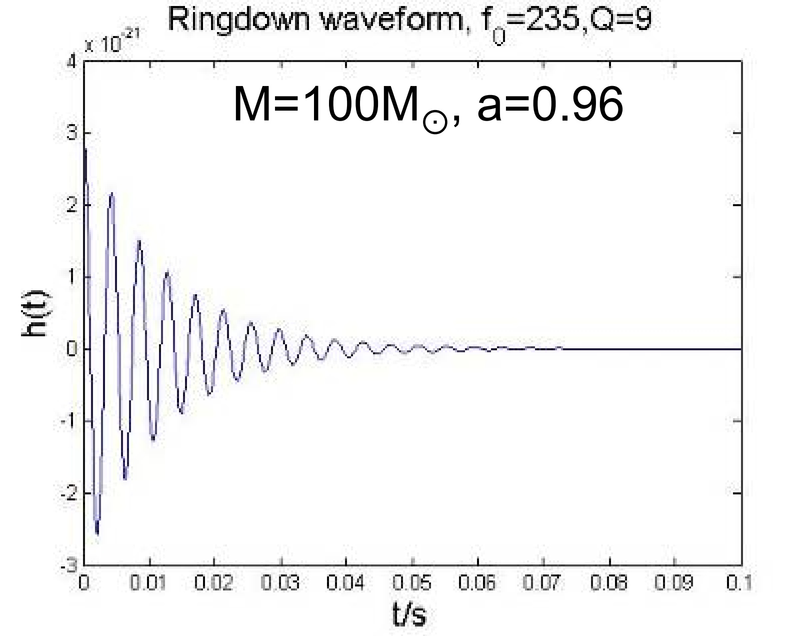
- Neutron star binaries
  - Mass distribution from population synthesis simulations
  - Spatial distribution following blue light luminosity?

  } Not certain
  - Have placed limits on rate per Milky Way equivalent galaxy
- Primordial binary black holes in the galactic halo
  - Can make a reasonable spatial model
  - Don’t know mass distribution
- BH+BH and BH+NS binaries
  - Don’t have a handle on mass and spatial distributions

Ringdown Waveforms

- If final product of inspiral is perturbed black hole, it will settle down to a Kerr black hole by quasinormal ringdown

- Waveforms are well modeled by black hole perturbation theory



Mass fraction emitted as GW's ε = 1%

$$h(t,\iota,\phi_0) = \frac{A(\varepsilon, f_0, Q)}{r} e^{-\frac{\pi f_0}{Q}t} \cos(2\pi f_0 t - \phi_0)$$

$$f_0 = \frac{c^3}{2\pi G M}\left[0.63(1-a)^{\frac{3}{10}}\right]$$

$$Q \approx 2(1-a)^{-\frac{9}{20}}$$

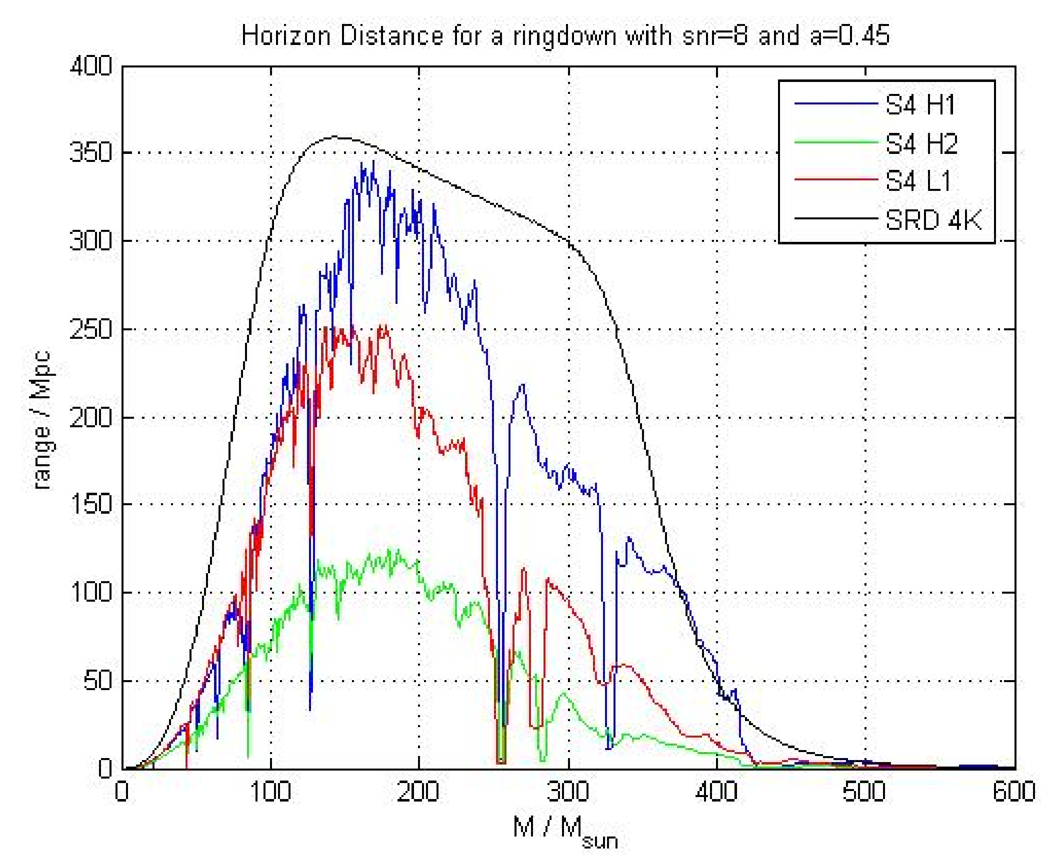
$$a = S\frac{c}{GM^2}$$

$$0 \le a \le 1$$

(Echeverria, 1989)

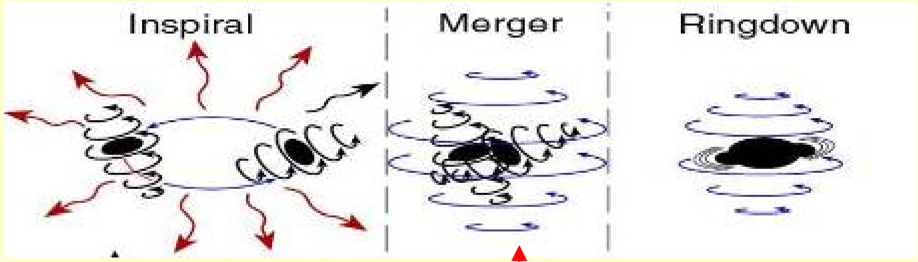
Ringdown Search

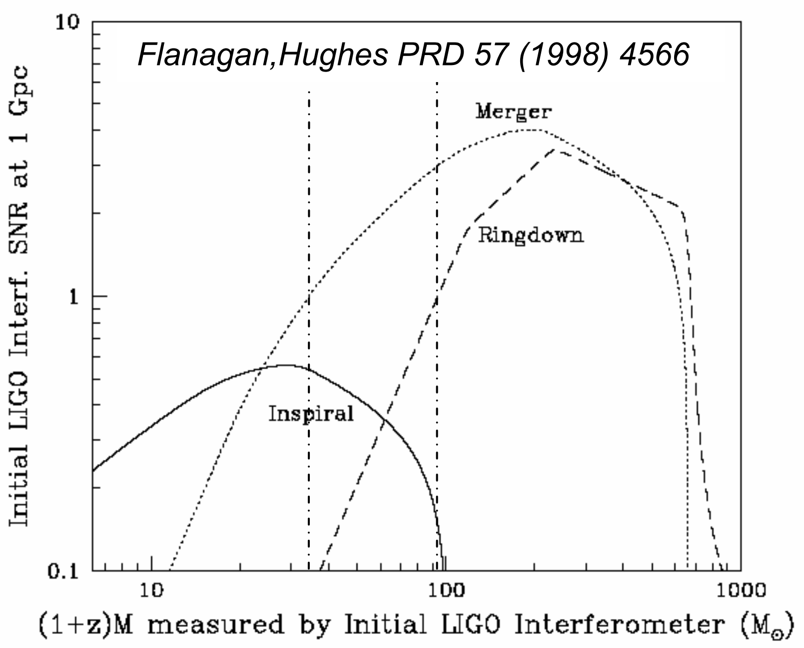
- There is a separate ringdown search
  - Search over frequency, f, and quality factor, Q, using a template bank.
- Use similar multi-IFO analysis pipeline as for inspiral.
- Systematic uncertainty
  - Unknown power contained in the ringdown.
  - Which modes are excited.
  - Assume 1% of final mass emitted in l=2, m=2 mode.



Under study: IBR follow-up

Inspiral and ringdown phases are being searched separately by the inspiral group with the matched filtering technique. The merger is addressed by the burst group, but results/resources are not combined (to date, only exploratory work).




Flanagan,Hughes PRD 57 (1998) 4566

Burst techniques… NEXT