



# Data Analysis Techniques for LIGO

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LIGO-G070047-00

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# Lesson Plan

Today:

1. Introducing the problem: GW and LIGO
2. Search for Continuous Waves
3. Search for Stochastic Background

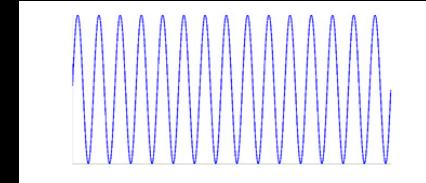
Tomorrow:

4. Search for Binary Inspirals
5. Search for Bursts
6. Network Analysis

# Detecting Continuous Waves

What does the signal look like?

a sine wave, with amplitude and frequency modulated by Earth's motion, and possibly spinning down



How do we quantify it?

Amplitude of gravitational wave:  $h_0$

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{I_{zz}\nu^2}{d} \epsilon$$

How do we look for it?

Bayesian analysis or matched filter search,  
coherent or semi-coherent methods



# Gravitational Waves from Spinning Neutron Stars



Non-axisymmetric distortions do not exist in perfect fluid stars, but in realistic neutron stars such deformations can be supported either by elastic stresses or by magnetic fields.

equatorial ellipticity

$I_{jj}$  are the three principal moments of inertia.

$$\epsilon \equiv \frac{I_{xx} - I_{yy}}{I_{zz}}$$

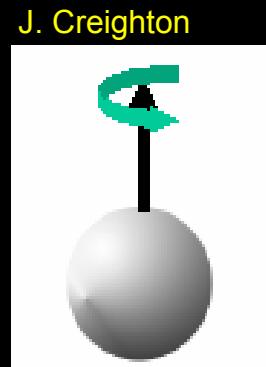
A non-axisymmetric neutron star at distance  $d$ , rotating with frequency  $\nu$  around the z-axis emits monochromatic GWs of frequency  $f = 2\nu$  and an amplitude  $h_0$

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{I_{zz}\nu^2}{d} \epsilon$$

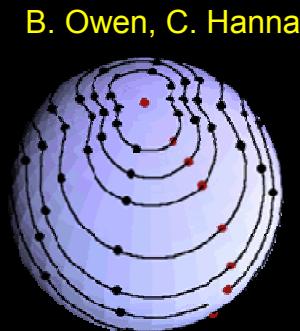
GW signal frequency is twice the rotation frequency



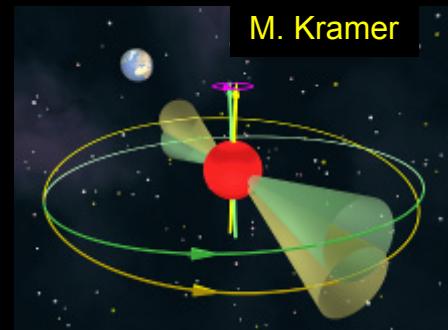
Accreting neutron stars



Pulsars with mountains



Oscillating neutron star



Wobbling neutron stars

J. Creighton

B. Owen, C. Hanna

M. Kramer



# Gravitational Waves from Spinning Neutron Stars



*Review article:* Reinhard Prix for the LIGO Scientific Collaboration, “The Search for Gravitational Waves from Spinning Neutron Stars”, to appear in a forthcoming volume in the Springer Lecture Notes Series. Available at [http://www.ligo.org/pdf\\_public/prix02.pdf](http://www.ligo.org/pdf_public/prix02.pdf)

GW energy emission rate from a spinning neutron star

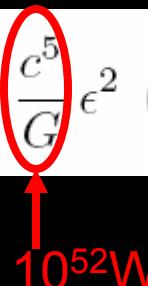
$$R_s = 2GM/c^2$$

Schwarzschild radius

$$V = 2\pi R\nu$$

Rotational velocity at the surface

$$L_{\text{GW}} \sim \frac{c^5}{G} \epsilon^2 \left(\frac{R_s}{R}\right)^2 \left(\frac{V}{c}\right)^6$$

 A red circle highlights the term  $c^5/G$  in the equation. A red arrow points from this circled term to the value  $10^{52} \text{W}$  located below the equation.

$$10^{52} \text{W}$$

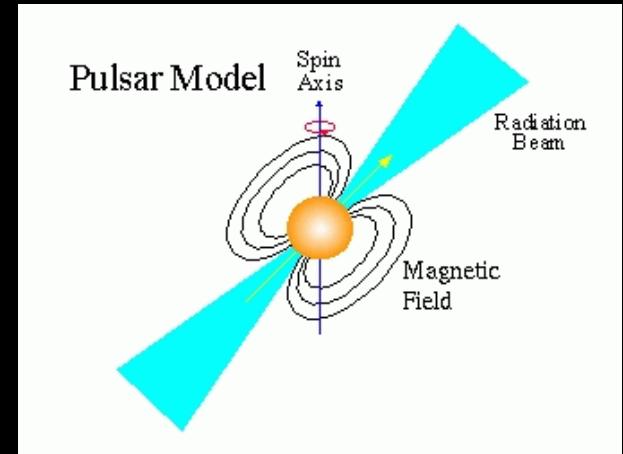
compact objects (i.e. with  $R_s \sim R$ ) in rapid rotation ( $V \sim c$ ), such as spinning neutron stars, can emit enormous GW luminosities even for small  $\epsilon$ . However, spacetime is a very “stiff” medium: large amounts of energy are carried by GWs of small amplitude.

$$h \sim 10^2 \frac{G}{c^4} \frac{\epsilon I_{zz} \nu^2}{d} \sim 3 \times 10^{-25} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{I_{zz}}{10^{38} \text{kg m}^2}\right) \left(\frac{\nu}{100 \text{ Hz}}\right)^2 \left(\frac{100 \text{ pc}}{d}\right)$$

# The Signal from a Neutron Star

The GW signal from a neutron star:

$$h(t) = h_0 A(t) e^{\Phi(t)}$$



Nearly-monochromatic continuous signal

- free precession (wobbling spin) at  $\sim f_{\text{rot}}$
- excited oscillatory modes such as the r-mode at  $4/3 * f_{\text{rot}}$
- non-axisymmetric distortion of crystalline structure, at  $2f_{\text{rot}}$

$$\text{(Signal-to-noise)}^2 \sim \int_0^T \frac{h^2(t)}{S_h(f_{gw})} dt$$



# Expected Signal on Earth (1)

Spin-down  $\Rightarrow$  phase evolution

the phase of the received signal depends on the initial phase, the frequency evolution of the signal and on the instantaneous relative velocity between source and detector.

$$\Phi(t) = \phi_0 + 2\pi \sum_{n=0}^{\infty} \frac{f_{(n)}}{(n+1)!} (T(t) - T(t_0))^{n+1}$$

$T(t)$  is the time of arrival of a signal at the Solar System Barycentre (SSB)  
 $t$  is the time at the detector

# Expected Signal on Earth (2)

Relative motion detector/source  $\Rightarrow$  frequency modulation

Doppler shift:

$$\frac{\Delta f}{f} = \frac{\mathbf{v} \cdot \mathbf{n}}{c}$$

Antenna sensitivity of the detector  $\Rightarrow$  amplitude modulation

$$h(t) = F_+(t; \psi) h_+(t) + F_\times(t; \psi) h_\times(t)$$

$$h_+ = A_+ \cos \Phi(t)$$
$$h_\times = A_\times \sin \Phi(t)$$

$$\left. \begin{aligned} &F_+(t, \psi) \\ &F_\times(t, \psi) \end{aligned} \right\}$$

strain antenna patterns of the detector to plus and cross polarization, bounded between -1 and 1.

They depend on the orientation of detector and source and on the waves polarization.



# Expected Signal on Earth (3)



For a tri-axial neutron star  
emitting at twice its rotational frequency:

$$A_+ = \frac{1}{2} h_0 (1 + \cos^2 \iota)$$
$$A_\times = h_0 \cos \iota$$

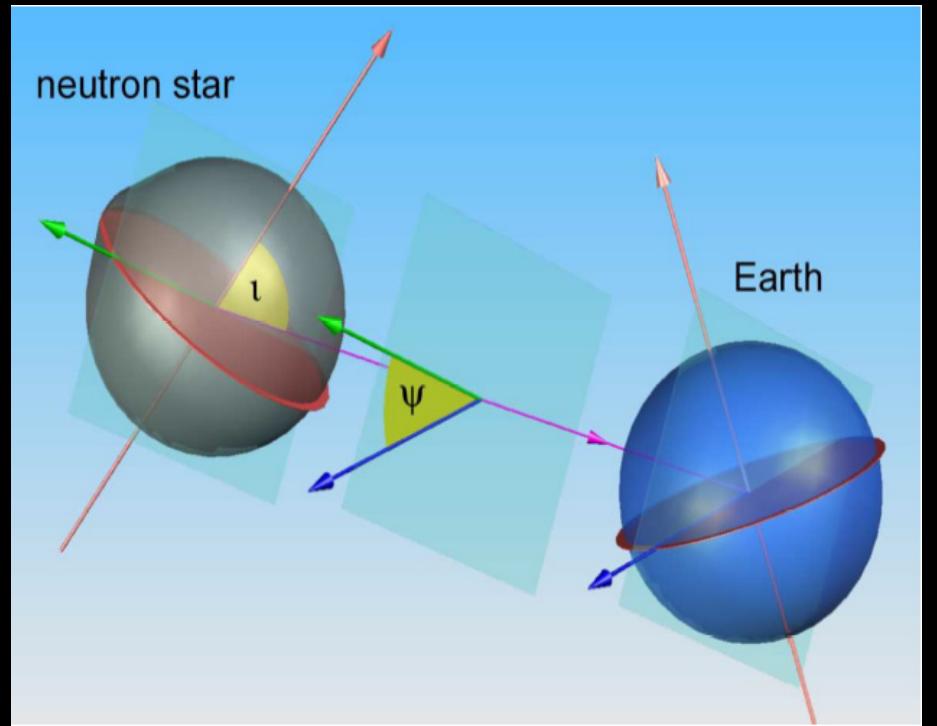
$$h(t) = F_+(t; \psi) h_0 \left( \frac{1 + \cos^2 \iota}{2} \right) \cos \Phi(t) - F_\times(t; \psi) h_0 \cos \iota \sin \Phi(t)$$

Signal parameters:

- $\psi$  = polarization angle
- $\iota$  = inclination angle of source with respect to line of sight
- $\Phi(t) = \phi(t) + \phi_0$
- $\phi_0$  = initial phase of pulsar
- $h_0$  = amplitude of the GW signal

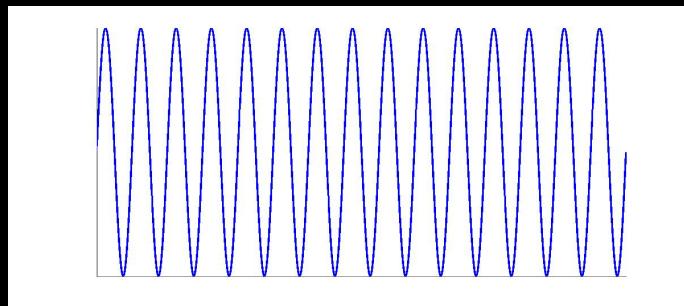
$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} \varepsilon f_{gw}^2}{d}$$
$$\varepsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

equatorial ellipticity

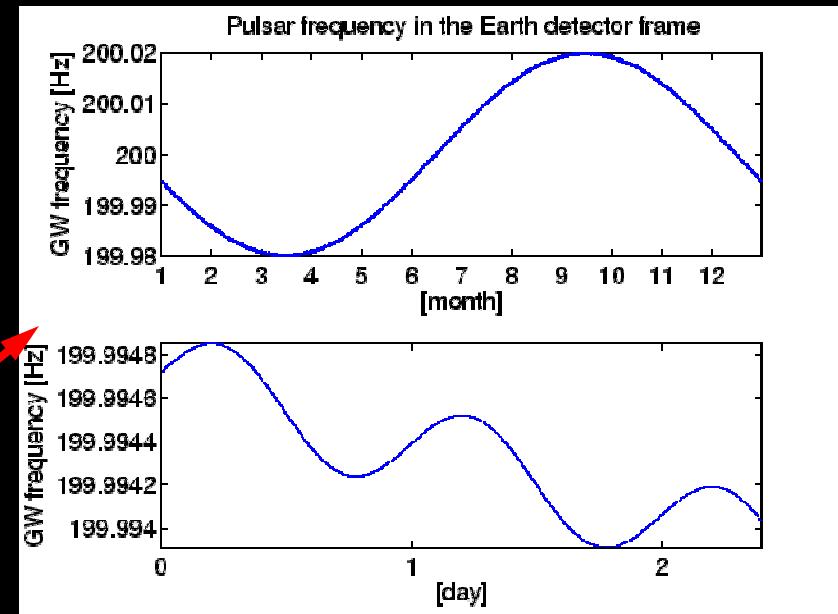


# The Signal

At the source



At the detector



- Annual variation: up to  $\sim 10^{-4}$
- Daily variation: up to  $\sim 10^{-6}$

... more complications for GW signals from pulsars in binary systems

Additional Doppler shift due to orbital motion of neutron star

Varying gravitational red-shift if orbit is elliptical

Shapiro time delay if GW passes near companion

# The Searches

- Most sensitive method: **coherently correlate** the data with the expected signal (template) and inverse weights with the noise. If the signal were really monochromatic, this would be equivalent to a Fourier Transform.
  - Templates: we assume various sets of unknown parameters and correlate the data against these different wave-forms.
  - we do not have to search explicitly over polarization, inclination, initial phase and amplitude.
- We have no pointing control. Because of the antenna pattern, our data stream has signals from all over the sky all at once. However: low signal-to-noise is expected. Hence confusion from many sources overlapping is not a concern.
- Input data to our analyses:
  - A calibrated data stream which with a better than 10% accuracy. Sampling rate 16kHz, but since the high sensitivity range is 40-1500 Hz we can down-sample at 3 kHz.



# Continuous Waves Searches (1)

These searches have several parameters:

*May be known, used in the search:*

position  
frequency,  
frequency derivatives

*Not explicitly searched for:*

initial phase  
inclination angle  
polarization  
amplitude

**Coherent methods:**

Require prediction of the signal phase evolution

**Semi-coherent methods:**

Require prediction of the signal frequency evolution

# Continuous Waves Searches (2)

## Searches

- Known pulsars
  - Coherent, time-domain
  - fine-tuned over a narrow parameter space
  - Use catalog of known pulsars and ephemeris
- Wide-area
  - Coherent matched filtering in frequency domain
  - All-sky, wide frequency range: computationally expensive
  - Hierarchical search under development
- All-sky semi-coherent
  - Fast, robust wide parameter search
  - Piece together *incoherently* result from shorter segments

The choice of a method is driven by the computational cost



# Coherent Detection Methods

## Time domain

process signal to remove frequency variations due to Earth's motion around Sun and spindown

- **Standard Bayesian analysis,** as fast numerically but provides natural parameter estimation
- **Best suited to target known objects, even if phase evolution is complicated**
- Efficiently handles missing data
- Upper limits interpretation:  
**Bayesian approach**

## Frequency domain

Conceived as a module in a hierarchical search

- **Matched filtering techniques.** Aimed at computing a **detection statistic.**  
These methods have been implemented in the frequency domain (although this is not necessary) and are very computationally efficient.
- **Best suited for large parameter space searches**  
(when signal characteristics are uncertain)
- **Frequentist approach** used to cast upper limits.



# Time Domain Search for GWs from Known Pulsars

Sky position and spin frequency known accurately

Method: heterodyne time-domain data using the known spin phase of the pulsar

- Requires precise timing data from radio or X-ray observations
- Include binary systems in search when orbits known accurately
- Exclude pulsars with significant timing uncertainties
- Special treatment for the Crab and other pulsars with glitches, timing noise

Heterodyne, i.e. multiply by:  $e^{-i\phi(t)}$

Known phase evolution of the signal:  
account for both spin-down rate and  
Doppler shift.

so that the expected demodulated signal is then:

$$y(t_k; \mathbf{a}) = \frac{1}{4} F_+(t_k; \psi) h_0 (1 + \cos^2 \iota) e^{i\phi_0} - \frac{i}{2} F_x(t_k; \psi) h_0 (\cos \iota) e^{i\phi_0}$$

Here,  $\mathbf{a} = \mathbf{a}(h_0, \psi, \iota, \phi_0)$ , a vector of the signal parameters.

The only  
remaining time  
dependence is  
that of the  
antenna pattern



# Search for Gravitational Waves from Known Pulsars

- Heterodyne IFO data at the instantaneous GW frequency (2\*radio rotation frequency)
- Low-pass at 0.5Hz
- Down-sample from 16384Hz to 1/60Hz, or 1/minute  $\Rightarrow B_k$
- Additional parameters  $a = a(h_0, \phi_0, \psi, \iota)$ , are inferred from their Bayesian posterior probability distribution, assuming Gaussian noise
  - The data are broken up into M time segments over which the noise can be assumed stationary. Marginalize over noise floor.

$$p(\{B_k\}|\mathbf{a}) \propto \prod_j^M \left( \sum_{k=1+\sum_{i=1}^{j-1} m_i}^{\sum_{i=1}^j m_i} |B_k - y_k|^2 \right)^{-m_j}$$

Number of data points  
in the j-th segment  
(5-30min to maximize  
analyzed data)

$$y_k = \frac{1}{4} F_+(t_k; \psi) h_0 (1 + \cos^2 \iota) e^{i 2 \phi_0} - \frac{i}{2} F_\times(t_k; \psi) h_0 \cos \iota e^{i 2 \phi_0}$$

# The Bayesian Approach

Bayesian statistics: quantifying the degree of certainty (or “degree of belief”) of a statement being true

$$\text{posterior } p(\vec{a} | \{B_k\}) \propto \text{prior } p(\vec{a}) p(\{B_k\} | \vec{a}) \text{ likelihood}$$

Assume uniform priors on all parameters

Combine posterior pdfs from different detectors

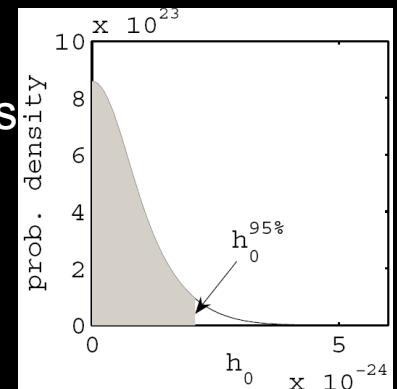
$$p(B_k | \mathbf{a})_{\text{Joint}} = p(B_k | \mathbf{a})_{\text{H1}} \cdot p(B_k | \mathbf{a})_{\text{H2}} \cdot p(B_k | \mathbf{a})_{\text{L1}}$$

Marginalize the posterior over *nuisance* parameters ( $\phi_0, \psi, \iota$ )  
 (i.e. integrate over possible values)

Set 95% upper limits on GW amplitude emitted by each pulsar  
 solving:

$$0.95 = \int_0^{h_0^{95\%}} p(h_0 | \{B_k\}) dh_0$$

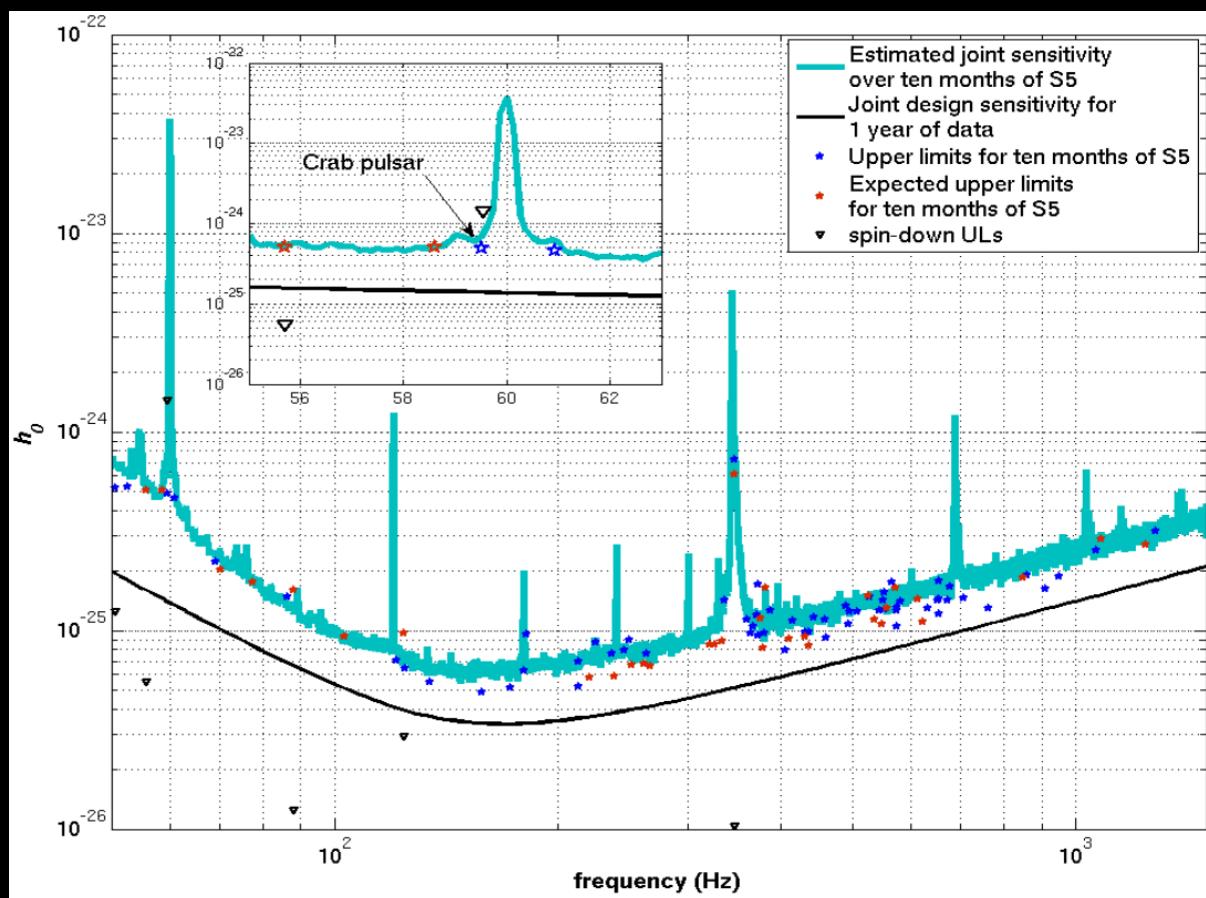
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# Preliminary S5 Results

$$S(f) = \left( \frac{T_{\text{obs H1}}}{S_h(f)_{\text{H1}}} + \frac{T_{\text{obs H2}}}{S_h(f)_{\text{H2}}} + \frac{T_{\text{obs L1}}}{S_h(f)_{\text{L1}}} \right)^{-1}$$
$$h_0^{95\%} = 10.8\sqrt{S(f)},$$

Combined sensitivity  
 $S_h(f)$ =single-sided PSD



Ref: LIGO-G060623-00

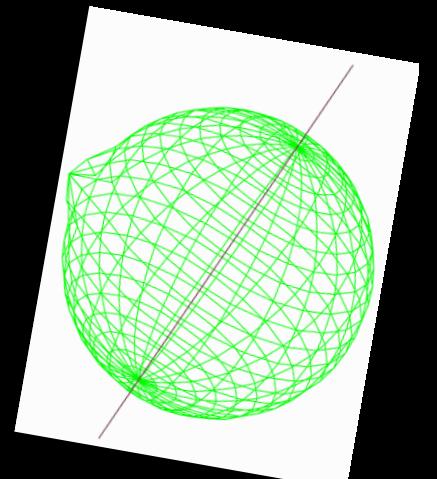
# Connection with Neutron Star Properties

- Infer limits on equatorial ellipticity  $\varepsilon$  :

$$\varepsilon = \frac{I_{xx} - I_{yy}}{I_{zz}},$$

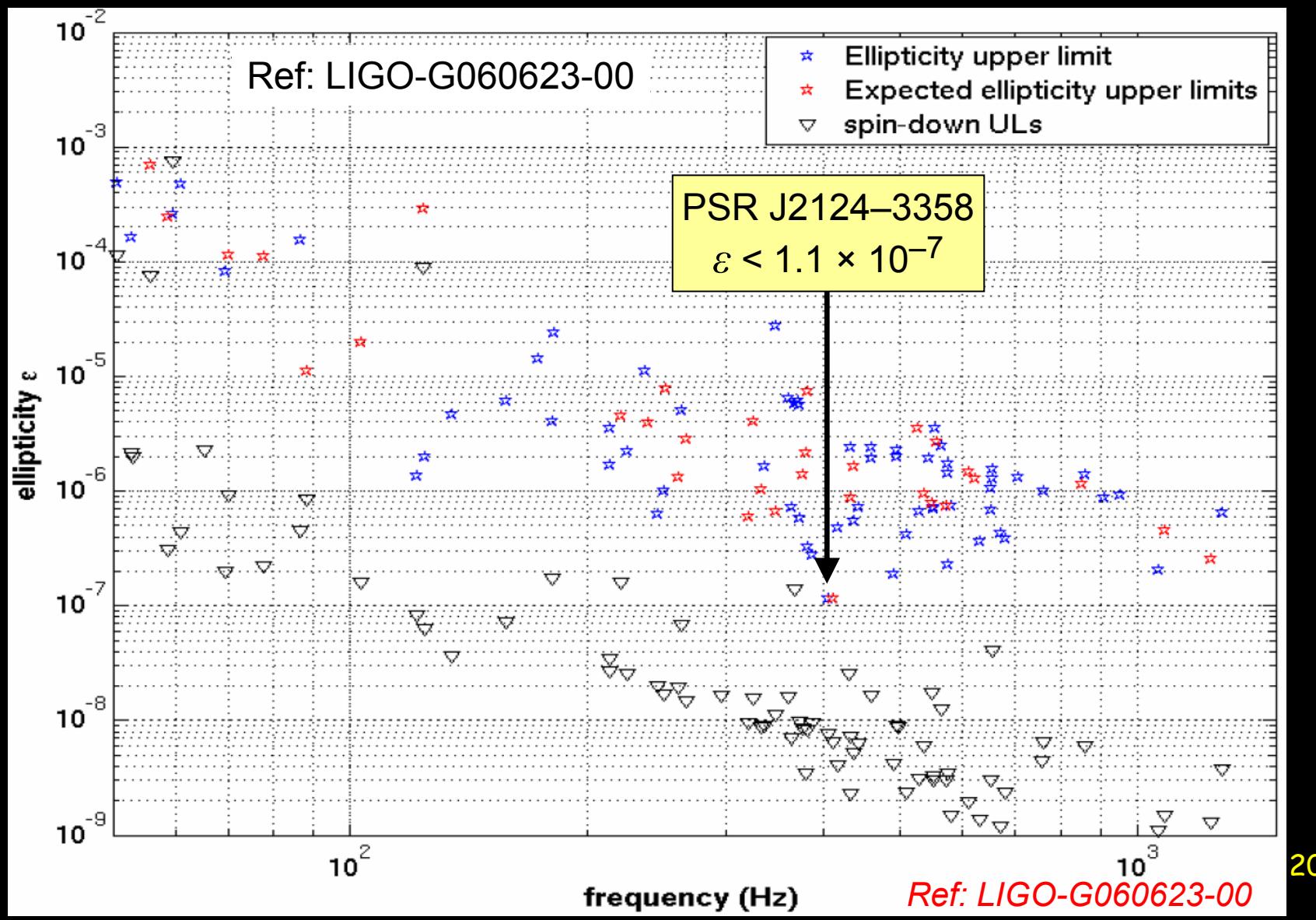
$$\varepsilon = 10^{-6} \times \left( \frac{h_0}{10^{-26}} \right) \times \left( \frac{10^{45} \text{ g cm}^2}{I_{zz}} \right) \times \left( \frac{100 \text{ Hz}}{f} \right)^2 \times \left( \frac{r}{1 \text{ kpc}} \right)$$

- $I_{zz}$  probably is in the range  $(1 \sim 3) \times 10^{45} \text{ g cm}^2$
- Physically plausible *maximum* values of  $\varepsilon$  :
  - $\sim 10^{-6}$  for ordinary neutron star matter, supported by crust or by internal magnetic fields
  - $\sim 10^{-5}$  for a hybrid star (ordinary neutron star matter on the outside, with a core of mixed quark & baryon matter)
  - $\sim \text{few} \times 10^{-4}$  for a solid strange quark star





# Preliminary S5 Results: Ellipticity Limits





# Frequency Domain Wide Parameter-Space Search

## Coherent search for unknown isolated pulsars



Method: apply a bank of matched filters for specific signal models. Unknown params:

- Sky position
- Spin axis inclination and azimuthal angle
- Frequency, spindown, initial phase
- Binary orbit parameters (if in a binary system)

Detection statistic:  $\mathcal{F}$  = log of the likelihood maximized over the nuisance params

*Jaranowski, Krolak and Schutz, PRD58 (1998) 063001;  
Jaranowski and Krolak, PRD59 (1999) 063003*

analytically maximizes over spin axis inclination & azimuthal angle and initial phase.

Even so, the computing cost scales as  $\sim T^6$

Detection threshold also must increase with number of filters

Check for signal consistency in multiple detectors

# The $\mathcal{F}$ statistics

The  $\mathcal{F}$  statistic derives from the method of maximum likelihood. The log-likelihood function  $\ln \Lambda$  is, for Gaussian noise,

$$\ln \Lambda = (s|h) - \frac{1}{2}(h|h), \quad (4.3)$$

where

$$(s|h) = 4\mathcal{R} \int_0^{\infty} \frac{\tilde{s}(f)\tilde{y}^*(f)}{S_n(f)} df, \quad (4.4)$$

$s$  is the calibrated detector output time series,  $h$  is the target signal (commonly referred to as the template), the tilde is the Fourier transform operator, and  $S_n(f)$  is the one-sided power spectral density of the noise. The  $\mathcal{F}$  statistic is the maximum value of  $\ln \Lambda$  with respect to all unknown signals parameters, given our data and a set of known template parameters.

- Noise only:  $2\mathcal{F}$  is a random variable that follows a  $\chi^2$  distribution with 4 degrees of freedom
- Noise+Signal: distribution has a non-centrality parameter

$$\lambda \propto \int dt h^2(t)$$



In the absence of signal,  $2\mathcal{F}$  is distributed according to a (central)  $\chi^2$  distribution with 4 degrees of freedom and the relevant probability density function is given by

$$p_0(2\mathcal{F}) = \frac{2\mathcal{F}}{4} e^{-\frac{2\mathcal{F}}{2}}. \quad (32)$$

We define the false alarm probability of  $2\mathcal{F}$  as

$$P_0(2\mathcal{F}) = \int_{2\mathcal{F}}^{\infty} p_0(2\mathcal{F}') d(2\mathcal{F}'). \quad (33)$$

Ref: gr-qc/0605028v2

In the presence of a signal,  $2\mathcal{F}$  follows a non-central  $\chi^2$  distribution with 4 degrees of freedom and non-centrality parameter  $\rho^2$ ; the associated probability density function is

$$p_1(2\mathcal{F}) = \frac{1}{2} e^{-(2\mathcal{F}+\rho^2)/2} \sqrt{\frac{2\mathcal{F}}{\rho^2}} I_1(\sqrt{2\mathcal{F}\rho^2}), \quad (34)$$

where  $I_1$  is the modified Bessel function of the first kind of order one and

$$\rho^2 = \frac{2}{S_h(f)} \int_0^{T_{\text{obs}}} h^2(t) dt. \quad (35)$$

The expected value of  $2\mathcal{F}$  is  $4 + \rho^2$

The expected value of  $2\mathcal{F}$  is  $4 + \rho^2$ . From Eq. (35) it is clear that the detection statistic is proportional to the square of the amplitude of the gravitational wave signal,  $h_0^2$ , given by Eq. (2).

# Expected Sensitivity

Typical noise levels of LIGO during the S2 run were approximately  $[S_h(f)]^{1/2} \approx 3 \times 10^{-22} \text{ Hz}^{-1/2}$ , where  $S_h$  is the strain noise power spectral density, as shown in Fig. 1. Even for a *known* GW pulsar with an average sky position, inclination angle, polarization, and frequency, the amplitude of the signal that we could detect in Gaussian stationary noise with a false alarm rate of 1% and a false dismissal rate of 10% is [15]

$$\langle h_0(f) \rangle = 11.4 \sqrt{\frac{S_h(f)}{T_{\text{obs}}}}, \quad (17)$$

where  $T_{\text{obs}}$  is the integration time and the angled brackets indicate an average source. In all-sky searches for pulsars with *unknown* parameters, the amplitude  $h_0$  must be several times greater than this to rise convincingly above the background.

Ref: gr-qc/0605028v2



# For instance: the S2 analysis



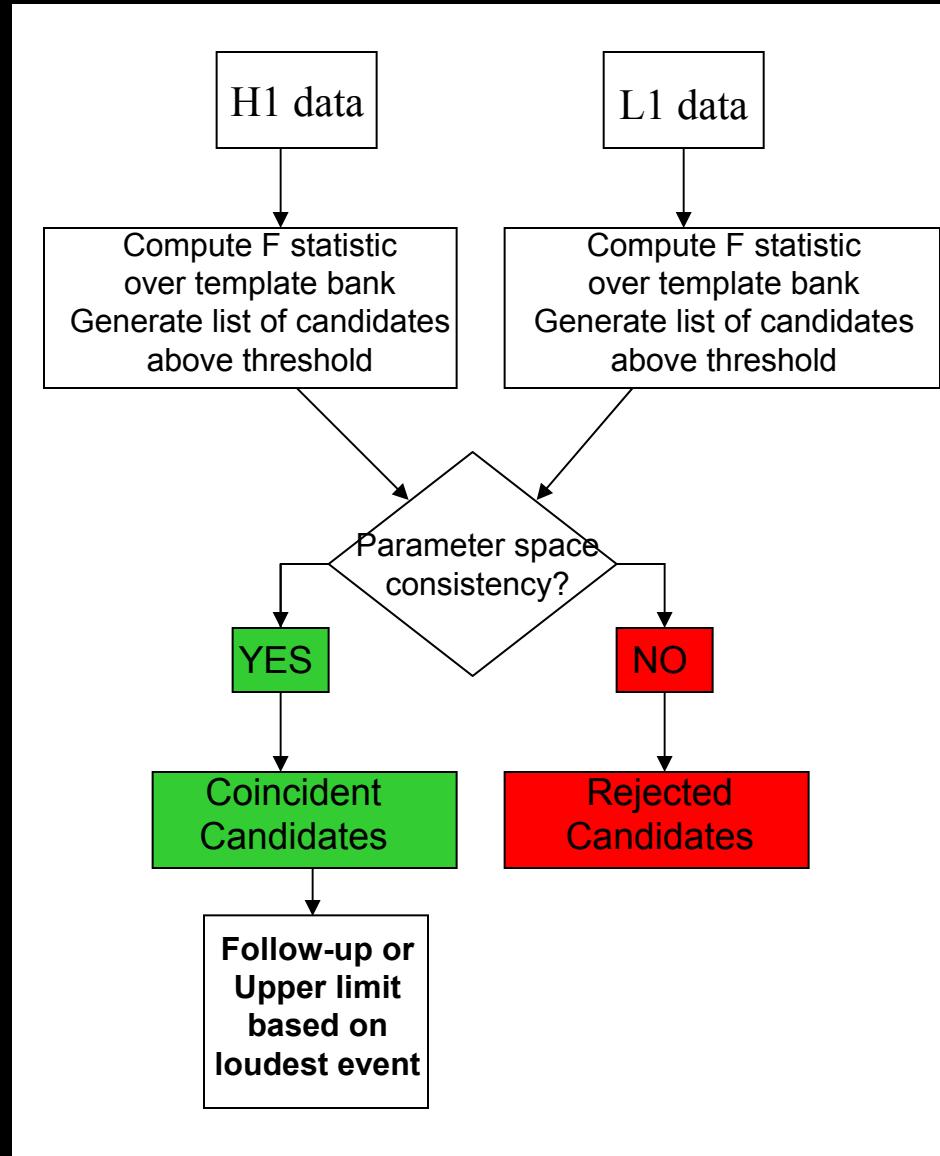
## Isolated neutron stars:

- **10 hours** most sensitive S2 data  
Hanford 4k (H1) and Livingston 4k (L1)  
IFOs
- All-sky search: ~ 3e4 sky position templates
- Wide frequency band: 160 Hz-728.8 Hz
- Coincidence between L1 and H1
- Templates without spin-down → maximum spin-down  
 $2.5 \times 10^{-11}$  Hz/s.
- Set upper limits in 1.2Hz bands

## Binary neutron stars:

- Dedicated Sco-X1 search
- 6 hour** coherent integration of best S2 data (set by computational resources)
- 2 frequency windows: 464 – 484 Hz (strong spectral features) and 604 – 624 Hz (reasonably clean)
- Search over uncertainties in orbital parameter space of Sco-X1
- Analyse L1 and H1 in coincidence
- Set upper limits in 1Hz bands

# Coincidence



## Coincidence

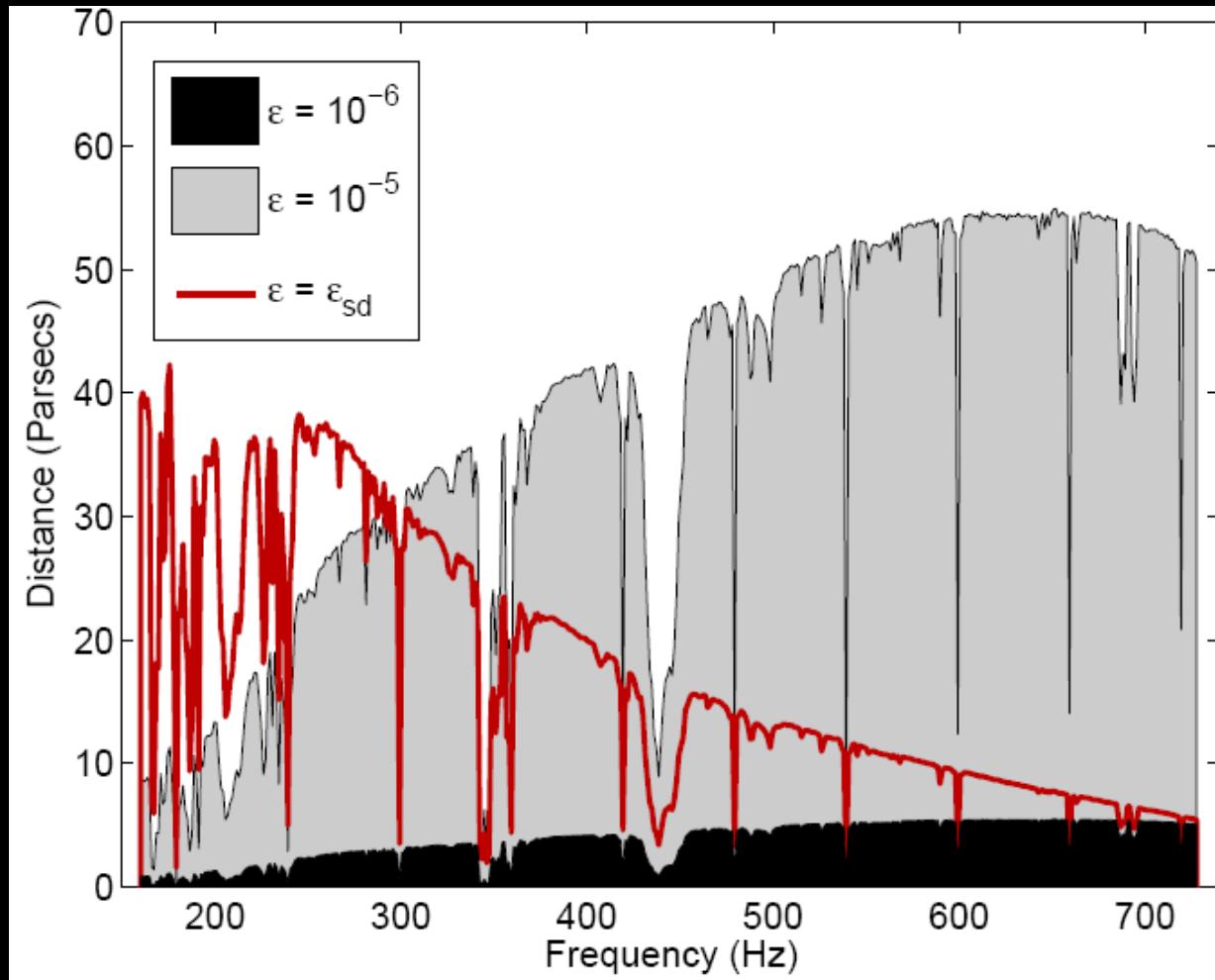
The coincident templates are then sorted in order of descending *joint significance*. If we indicate with  $2\mathcal{F}_{L1}$  and  $2\mathcal{F}_{H1}$  the values of the detection statistic for a pair of templates in coincidence, we define their joint significance as:

$$s(2\mathcal{F}_{L1}, 2\mathcal{F}_{H1}) = 1 - P_0(2\mathcal{F}_{L1}) P_0(2\mathcal{F}_{H1}), \quad (43)$$

where  $P_0(2\mathcal{F})$ , defined in Eq. (33), is the single detector false alarm probability for  $2\mathcal{F}$ , under the assumption that the noise is Gaussian and stationary. We consider the *loudest* coincident template pair as that yielding the largest value of joint significance. In practice, in the numerical implementation we rank events according to  $-\{\log [P_0(\mathcal{F}_{L1})] + \log [P_0(\mathcal{F}_{H1})]\}$  with  $\log [P_0(\mathcal{F})] = \log (1 + \mathcal{F}) - \mathcal{F}$ .

→ See slide 23

# Range of the S2 All-Sky Search



$$h_0 = \frac{16\pi^2 G}{c^4} \frac{I_{zz}\nu^2}{d} \epsilon$$

Effective average range of the search as a function of frequency for three ellipticities:  $10^{-6}$  (maximum for a normal neutron star),  $10^{-5}$  (maximum for a more optimistic object), and  $\epsilon_{\text{sd}}$ , the spin-down limit.

For sources above 300 Hz the reach of the search is limited by the maximum spin-down value of a signal that may be detected without loss of sensitivity.

Ref: [gr-qc/0605028v2](https://arxiv.org/abs/gr-qc/0605028v2)

# Setting Upper Limits

- Frequentist approach, loudest event statistics

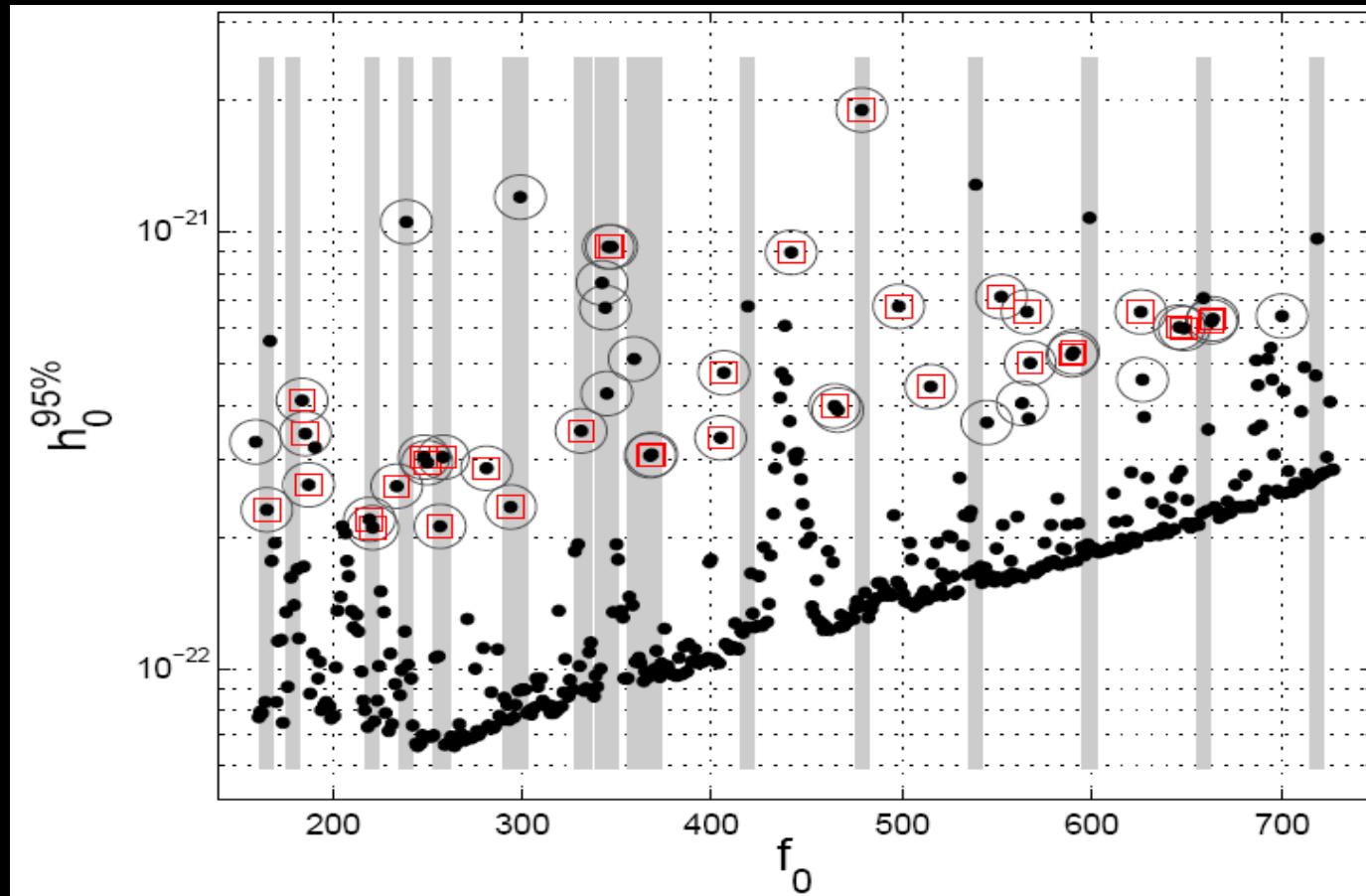
Let  $s^*(f_0)$  indicate the measured value of the joint significance of the loudest coincident event in the sub-band beginning at frequency  $f_0$ . For every sub-band a set of  $N$  injections of fake signals in the real data is performed at fixed amplitude  $h_0$ . Each injection is searched for in the data and, if detected as a coincident event, its significance is computed. A confidence  $C(h_0)$  is assigned to this set of injections

$$C(h_0) = n(h_0)/N \quad (44)$$

with  $n(h_0)$  being the number of trials out of  $N$  in which the measured joint significance of the injected signal is greater than or equal to  $s^*$ . Eq. (44) defines the  $h_0$  upper limit value as a function of the confidence  $C$ .

# Setting Upper Limits

No convincing signal detected; set upper limits in freq. bands



Ref:  
[gr-qc/0605028v2](https://arxiv.org/abs/gr-qc/0605028v2)

FIG. 24: Upper limits based on the loudest template over the whole sky in 1.2 Hz sub-bands. The vertical stripes mark the sub-bands containing known spectral disturbances. The circles mark the 90th percentile most significant results. The squares indicate that the values of the detection statistic in the two detectors are not consistent with what one would expect from an astrophysical signal.

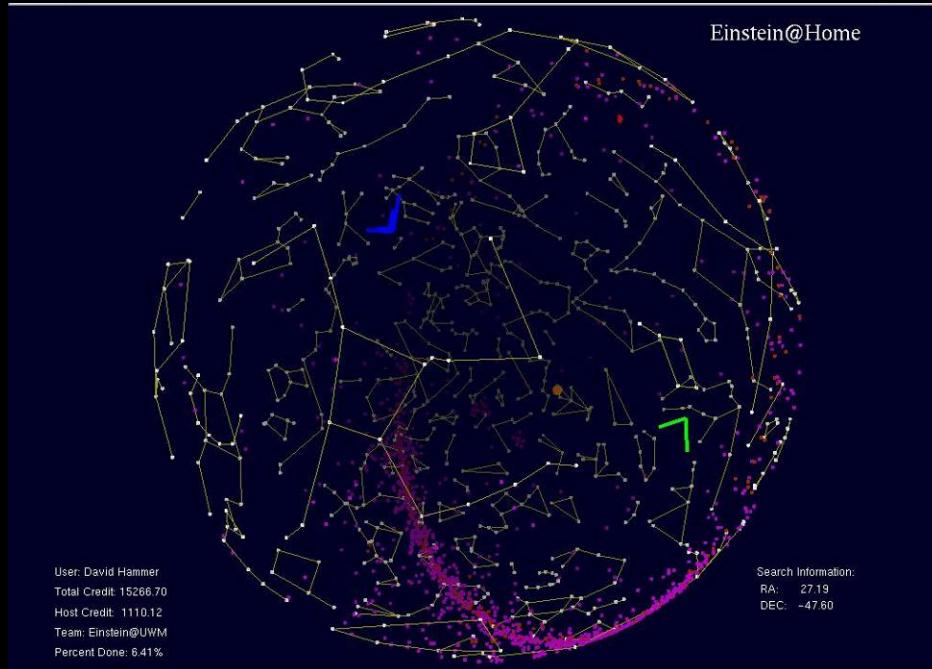


# Computing Cost of this Search

- The coherent all-sky search is computationally intensive:
  - searching 1 year of data: 3 billion frequencies in a 1000Hz band
  - For each frequency: 100 million independent sky positions
  - pulsars spin down, approximately one billion times more templates
  - Number of templates for each frequency:  $\sim 10^{23}$
- The closest implementation to date: S3 Frequency-domain (“F-statistic”) all-sky search
  - The F-Statistic uses a matched filter technique, minimizing chisquare (maximizing likelihood) when comparing a template to the data
  - $\sim 10^{15}$  templates search over frequency (50Hz-1500Hz) and sky position
  - For S3 we are using the 600 most sensitive hours of data
  - We are combining the results of multiple stages of the search incoherently using a coincidence scheme

# ...we are doing this with a Little Help...

- Public distributed computing project: Einstein@Home
- Small bits of data distributed for processing; results collected, verified, and post-processed



Screen saver  
graphics

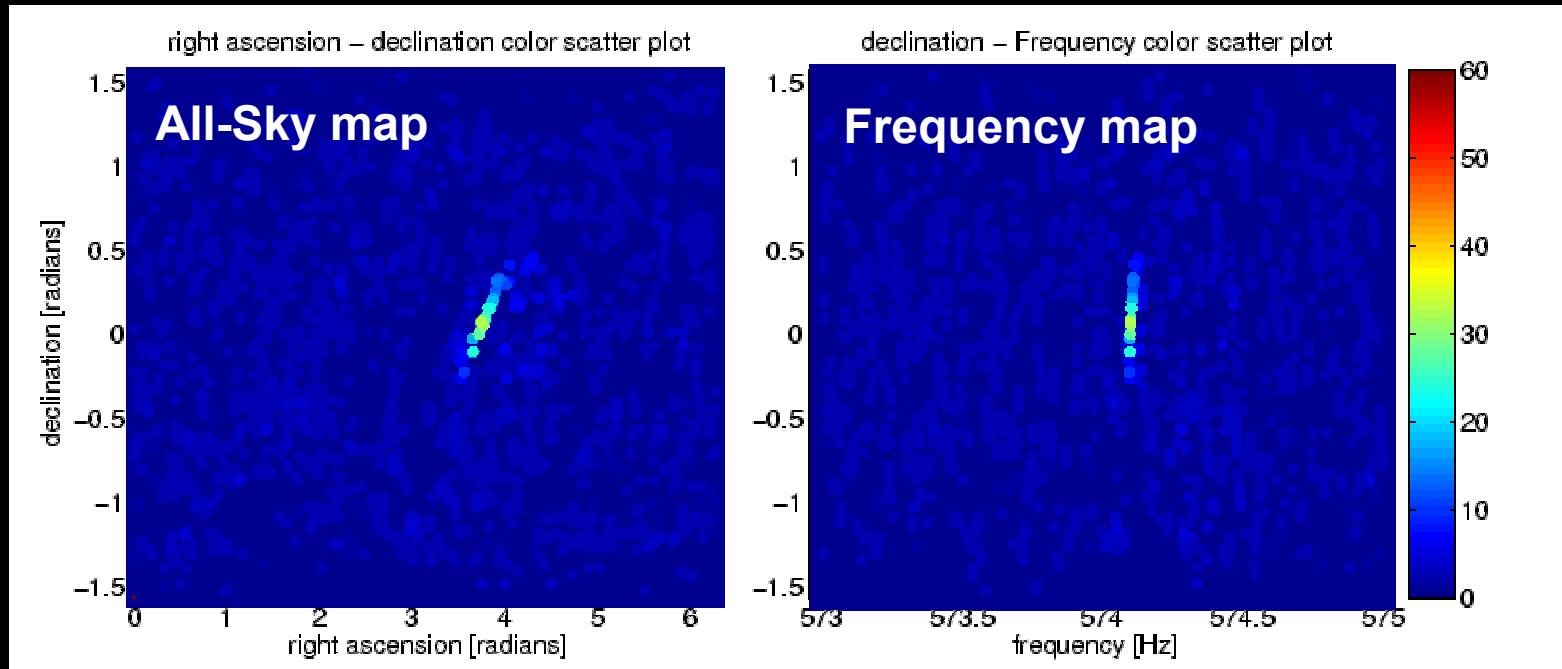
<http://einstein.phys.uwm.edu/>

So far 156,000 users, currently providing ~77 Tflops

LIGO-G070047-00

# What would a pulsar look like?

- Post-processing step: find points on the sky and in frequency that exceeded threshold in many of the ten-hour segments
- Software-injected fake pulsar signal is recovered below



Simulated (software) pulsar signal in S3 data

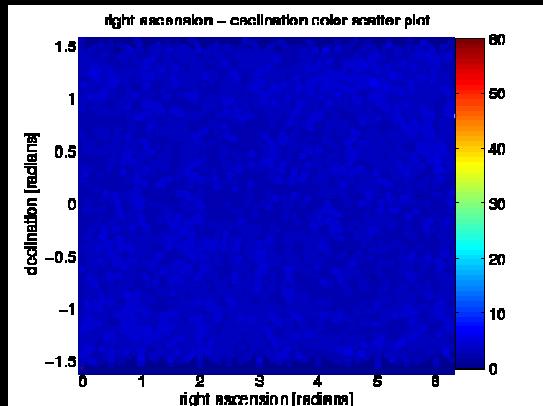
# S3 results

<http://einstein.phys.uwm.edu/PartialS3Results/>

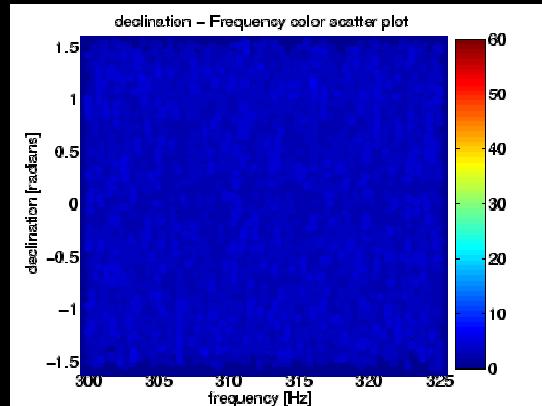
Analyzed 60 10-hour stretches of the best H1 data

Post-processing step on centralized server: find points in sky and frequency that exceed threshold in many of the sixty ten-hour segments analyzed

All-Sky map

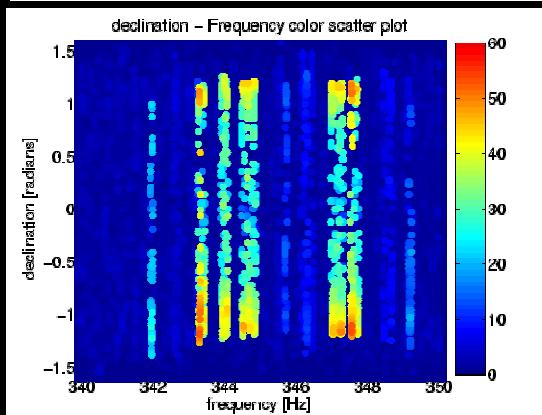
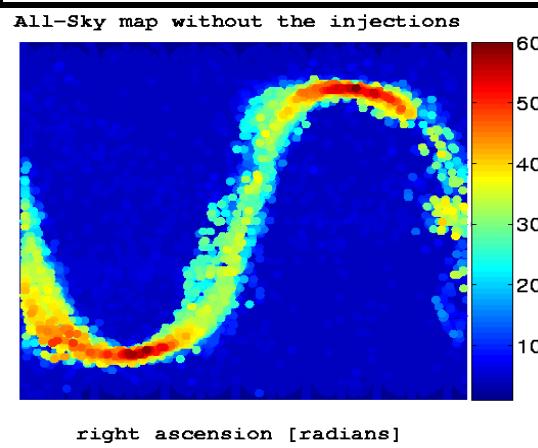


Frequency map



300-325 Hz

50-1500 Hz band shows no evidence of strong pulsar signals in sensitive part of the sky



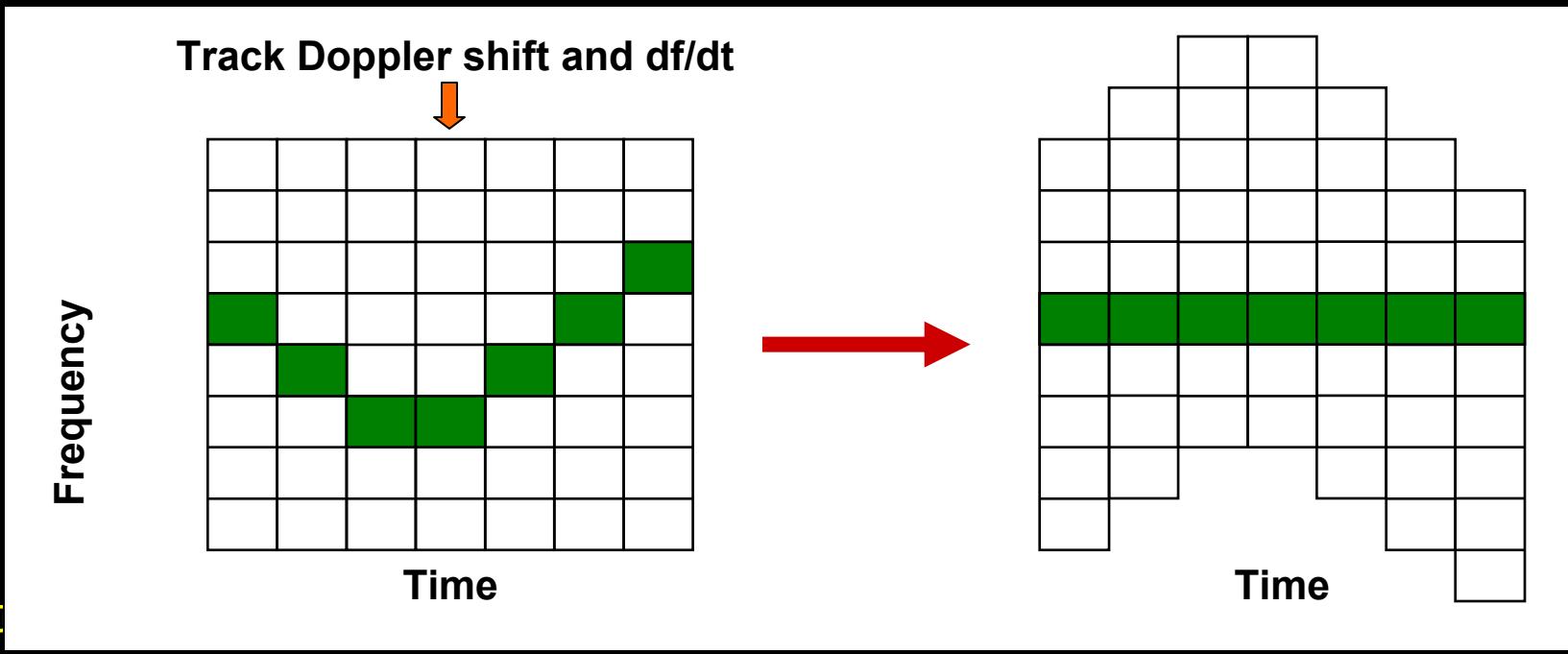
340-350 Hz: violin modes

Outliers consistent with instrumental lines

# Semi-coherent Searches

All-sky, all-time searches

- Semi-coherent methods are computationally efficient but less sensitive than the coherent methods, for a given observation time BUT we can use all data
- 3 methods, all break up the data into segments, FFT each, producing Short (30 min) Fourier Transforms (SFTs) from  $h(t)$ , [coherent step]; then track the frequency drifts due to Doppler modulations and  $df/dt$  [incoherent step].



# Semi-coherent Searches

- Data is calibrated and high-passed

- Discrete Fourier Transform

$$\tilde{x}_k = \Delta t \sum_{j=0}^{M-1} x_j e^{-2\pi i j k / M}$$

- Power associated to frequency bin  $k$  in segment  $i$

$$P_k^{(i)} = 2|\tilde{x}_k^{(i)}|^2$$

- Normalized power ( $T_{coh}=30m$ )

$T_{coh}$ =SFT time baseline

$S_n(f)$ =single-sided noise

$$\rho_k^{(i)} = \frac{P_k^{(i)}}{T_{coh} S_n^{(i)}(f_k)}$$

e.g., for StackSlide:

$$S_n \cong 2 < |\tilde{x}_k^{(i)}|^2 > / T_{coh}$$

# Semi-coherent Searches

- StackSlide sums normalized SFT power. Detection statistics is P

$$P = \frac{1}{N} \sum_{i=0}^{N-1} \rho_{k_i}^{(i)}$$

P has mean value 1 and s=1/sqrt(N) in gaussian noise

The simplest, does well with sensitivity and cost

- PowerFlux sums weighted SFT power (weights are proportional to SNR of a signal in a given SFT)

$$\rho = \sum_{i=1}^N w_i |\tilde{x}_k^{(i)}|^2$$

The most sensitive

- Hough sums weighted binary counts

$$n = \sum_{i=0}^{N-1} w_i n_k^{(i)}$$

$$n_k^{(i)} = \begin{cases} 1 & \text{if } \rho_k^{(i)} \geq \rho_{\text{th}} \\ 0 & \text{if } \rho_k^{(i)} < \rho_{\text{th}} \end{cases}$$

The most robust

$$w_i \propto \frac{F_+^2(t_i) + F_\times^2(t_i)}{S_n^{(i)}(f)}$$

# The StackSlide S4 pipeline

## Search

Read in command line argument,  
data, and initialize variables

Clean narrow instrument lines

For each point in the parameter  
space:

- Stack, slide, and sum the power
- Search for peaks above a  
threshold or the loudest power

Matlab code:

- Finds upper limits
- Makes plots

## Monte Carlo Simulation

Read in command line argument,  
data, and initialize variables

Generate fake signal and add  
it to the data.

Clean narrow instrument lines

For points surrounding the  
injection:

- Stack, slide, and sum the power
- Search for the loudest power

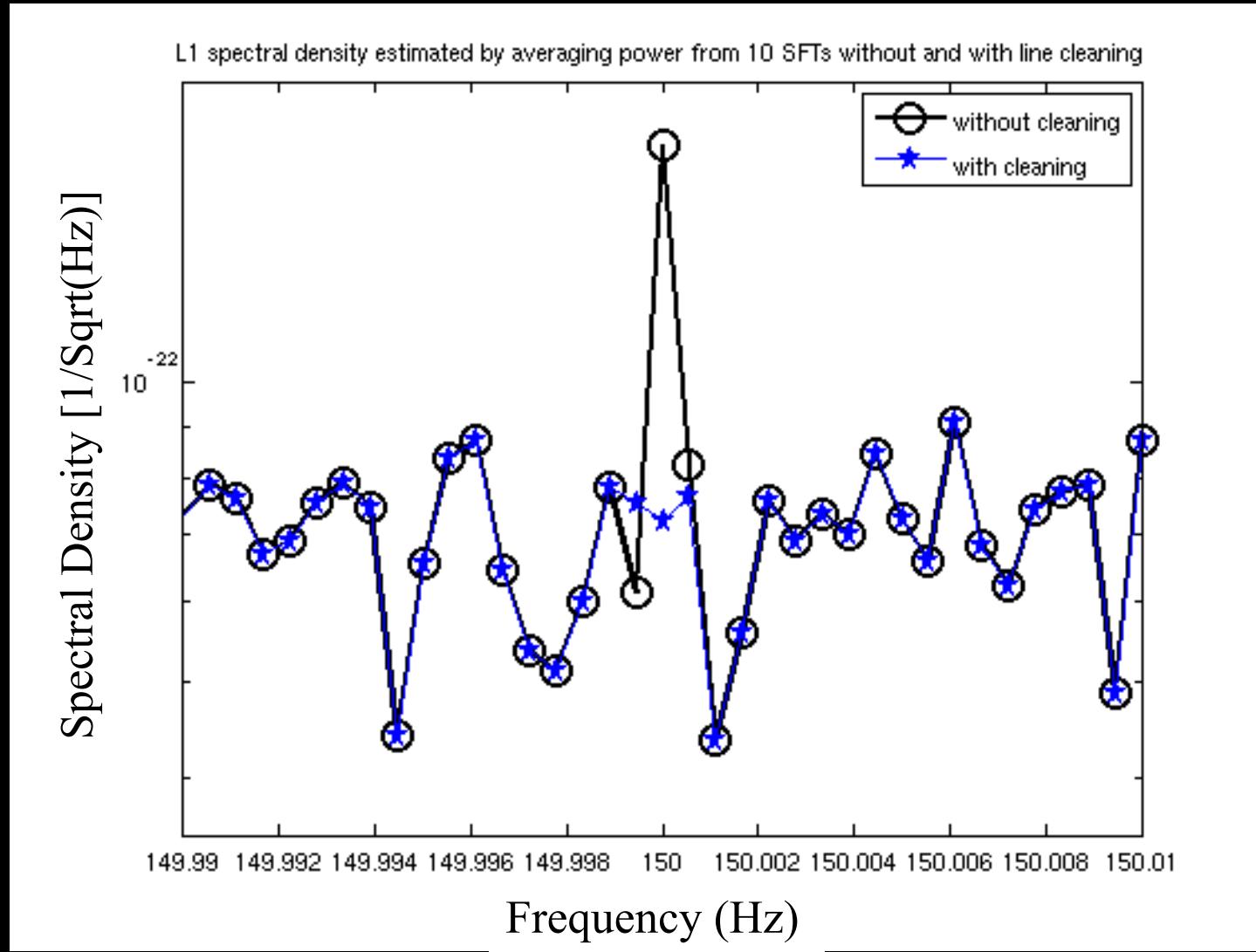
Yes

Done?

No

# Line Cleaning Method

(Only clean known lines; those wider than 0.021 Hz are not cleaned.)



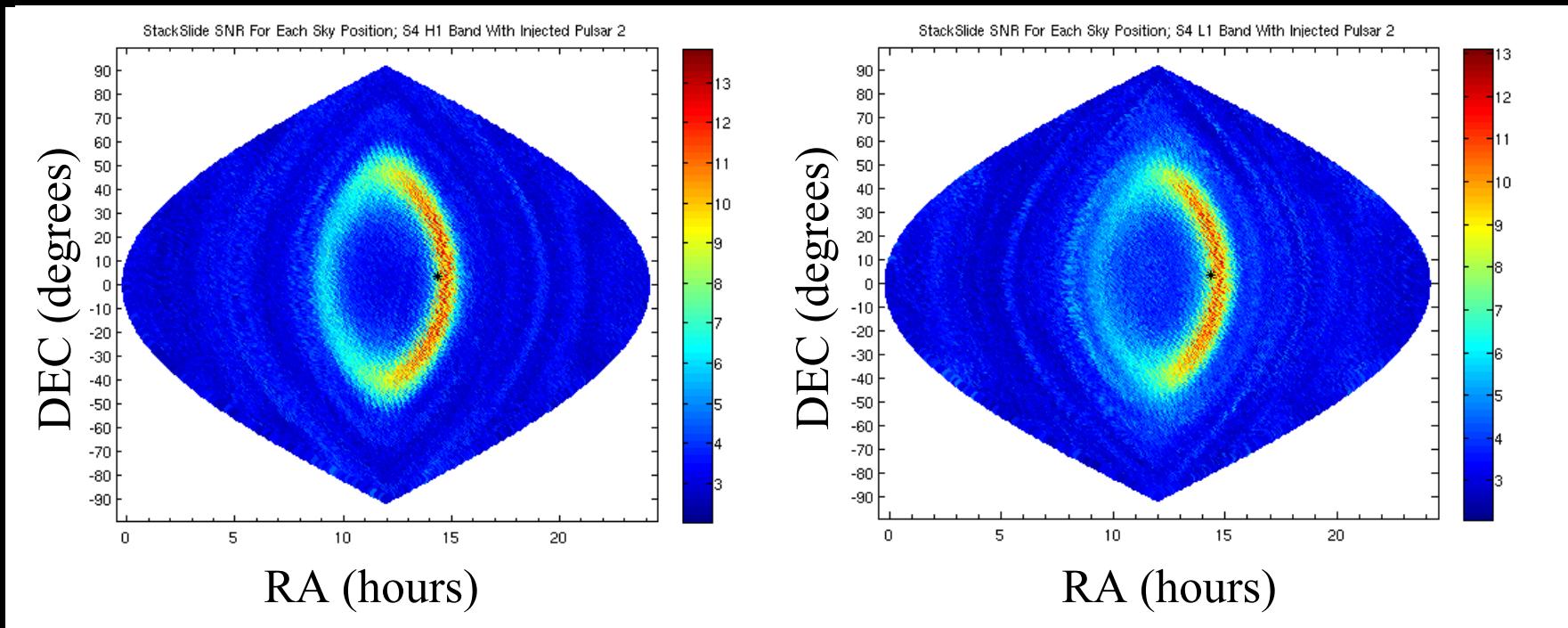
Ref:  
LIGO-G060177-00



# LIGO Analysis of Hardware Injections

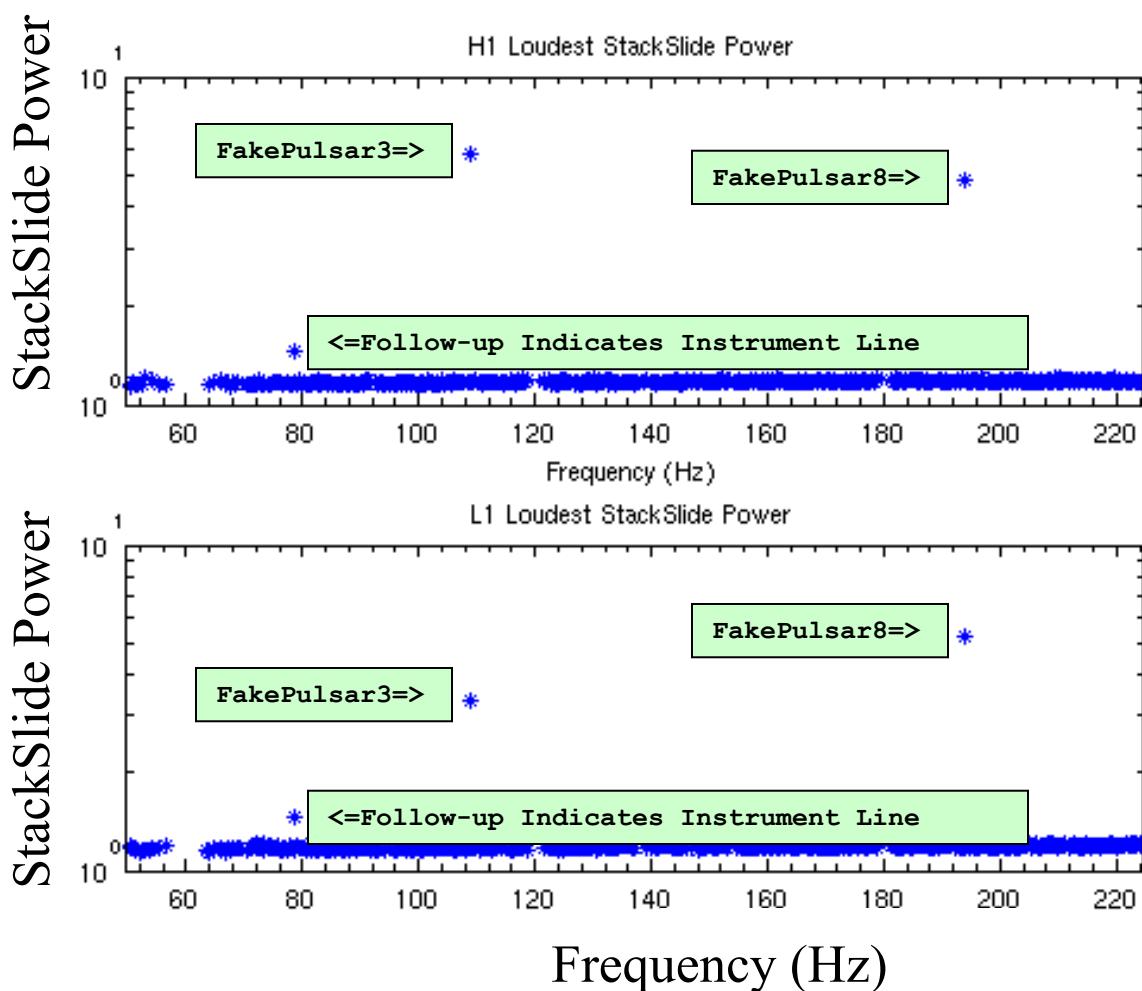


Fake gravitational-wave signals corresponding to rotating neutron stars with varying degrees of asymmetry were injected for parts of the S4 run by actuating on one end mirror. Sky maps for the search for an injected signal with  $h_0 \sim 7.5\text{e-}24$  are below. Black stars show the fake signal's sky position.



# "Loudest Events" 50-225 Hz

## PRELIMINARY



- Searched 450 freq. per .25 Hz band, 51 values of  $df/dt$ , between 0 &  $-1e-8$  Hz/s, up to 82,120 sky positions (up to  $2e9$  templates). The expected loudest StackSlide Power was  $\sim 1.22$  (SNR  $\sim 7$ )
- Veto bands affected by harmonics of 60 Hz.
- Simple cut: if SNR  $> 7$  in only one IFO veto; if in both IFOs, veto if  $\text{abs}(f_{H1} - f_{L1}) > 1.1e-4 * f_0$

Ref:  
LIGO-G060177-00

# Towards Hierarchical Searches

- The smallest signal detectable with a given confidence becomes larger as the parameter space increases. There is not much sense in using techniques that “dig out of the noise” signals that are too small to be significant.
- Instead, use methods that are less computationally intensive and not as sensitive to narrow down the parameter space to the final sensitivity level.
- This is a better use of computational resources because no calculations are lost to search regions of the parameter space where, if present, a signal would be too small to be confidently detected.
- Hierarchical searches alternate semi-coherent and fully coherent stages and get us closer to optimal sensitivity, at a manageable CPU cost