Testing GR with Inspirals

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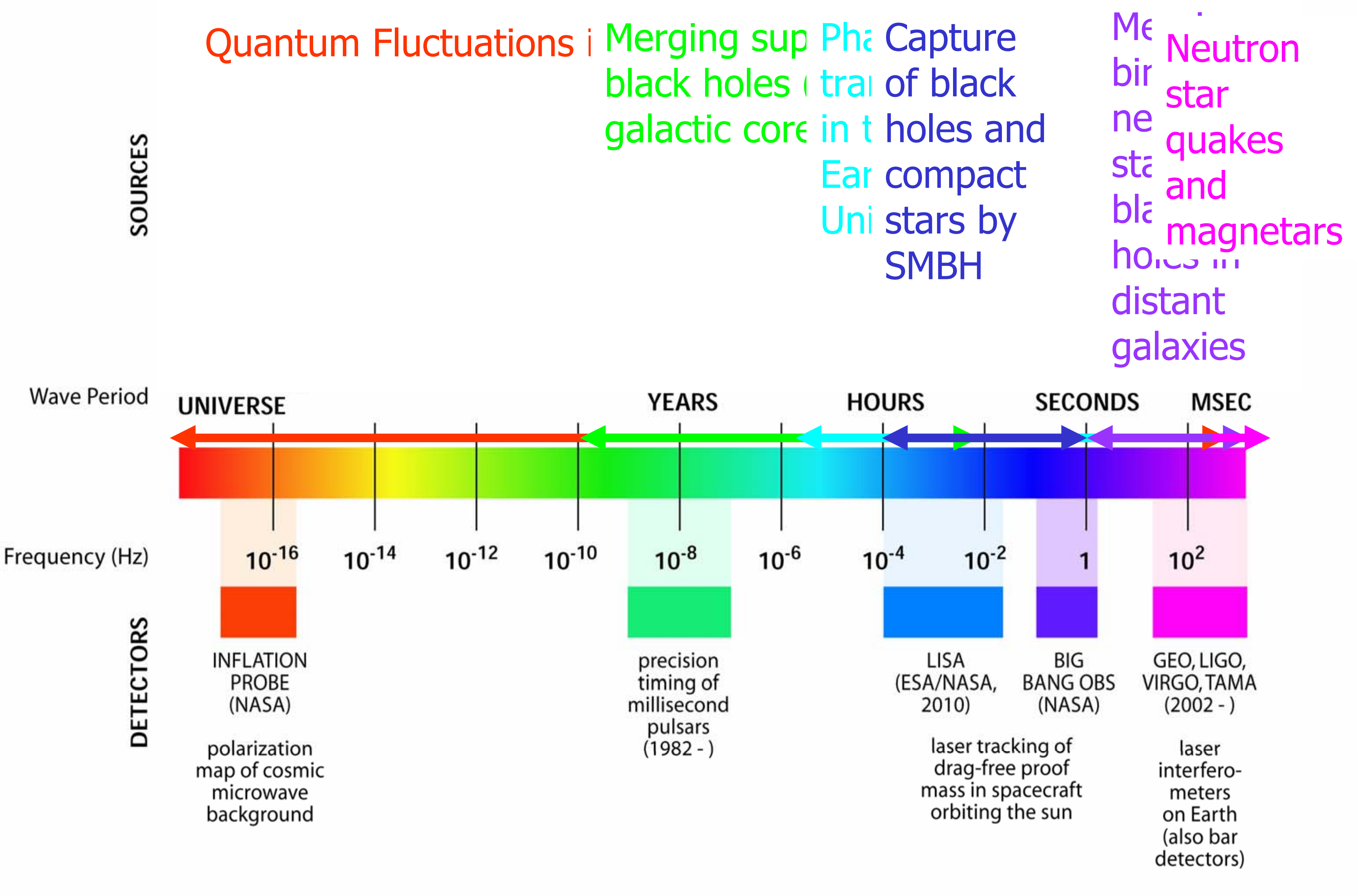
based on work with Arun, Iyer, Qusailah, Jones, Turner, Broeck, Sengupta

LIGO-G060681-00-Z

Plan

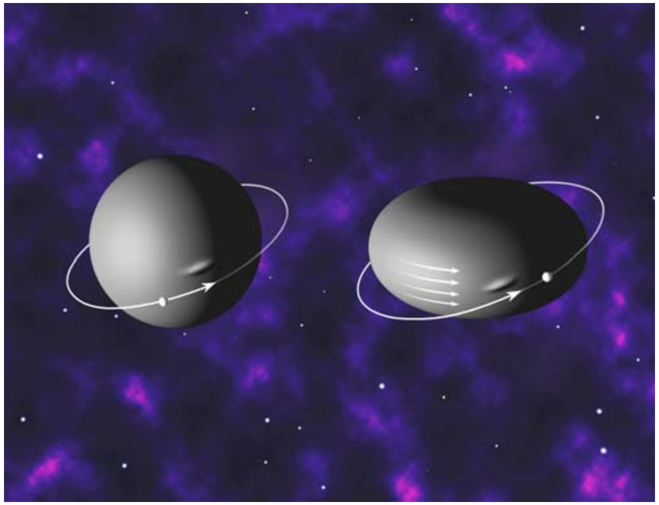
- Gravitational-wave spectrum
  - What might be observed from ground and space
- Gravitational-wave observables
  - amplitude, luminosity, frequency, chirp-rate
- Fundamental properties
  - speed, polarization, ...
- Strong field tests of general relativity
  - merger dynamics, QNM
- Predictions of PN gravity
  - presence of log-terms
- Cosmology

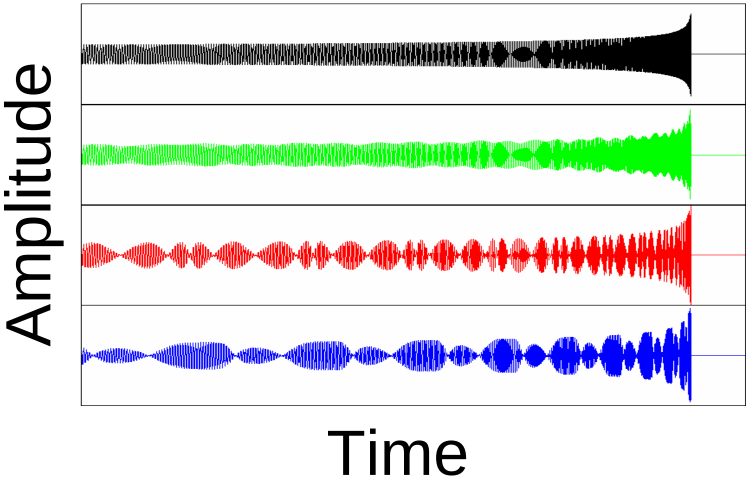
Gravitational Wave Spectrum



Compact Binary Inspirals

- Late-time dynamics of compact binaries is highly relativistic, dictated by non-linear general relativistic effects
- Post-Newtonian theory, which is used to model the evolution, is now known to $O(v^7)$
- The shape and strength of the emitted radiation depend on many parameters of binary system: masses, spins, distance, orientation, sky location, …
- Three archetypal systems
  - Double Neutron Stars (NS-NS)
  - Neutron Star-Black Hole (NS-BH)
  - Double Black Holes (BH-BH)





Gravitational Wave Observables

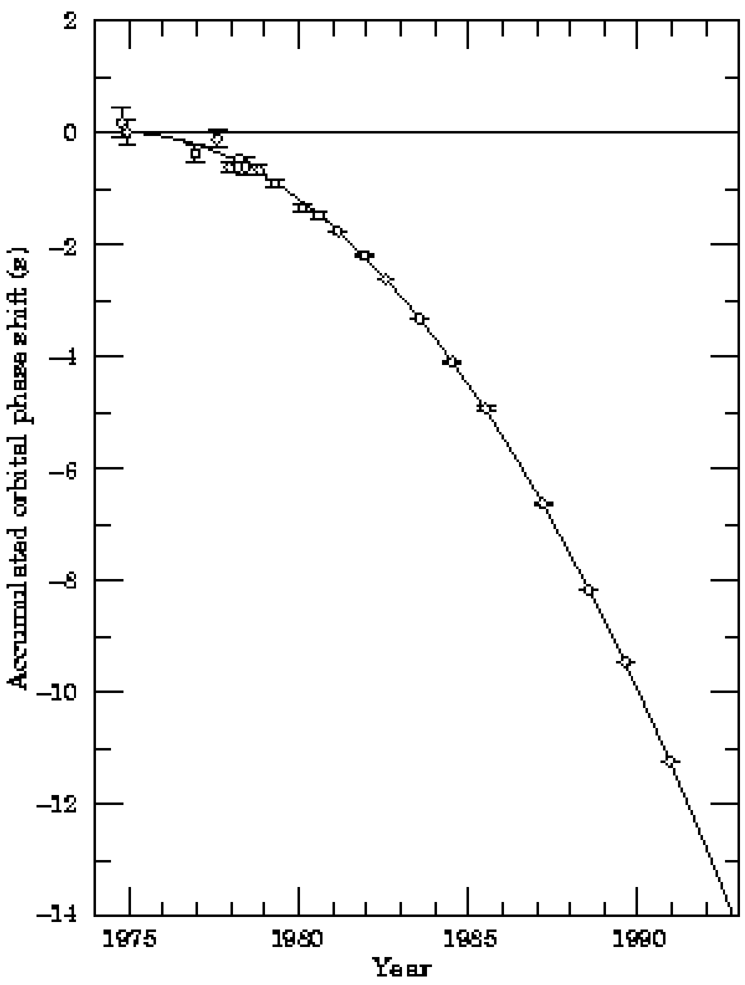
- Luminosity L = (Asymm.) v^{10}
  - Luminosity is a strong function of velocity: A black hole binary source brightens up a million times during merger
- Amplitude

  h = (Asymm.) (M/R) (M/r)
  - The amplitude gives strain caused in space as the wave propagates
  - For binaries the amplitude depends only on chirpmass$^{5/3}$/distance
- Frequency $f = \sqrt{\rho}$
  - Dynamical frequency in the system: twice the orb. freq.
- Binary chirp rate
  - Many sources chirp during observation: chirp rate depends only chirp mass
  - Chirping sources are standard candles
- Polarisation
  - In Einstein's theory two polarisations - plus and cross

Fundamental Measurements

Quadrupole formula

- Binary pulsars have already confirmed the quadrupole formula in weak-field regime
- GW observations will test the validity of the quadrupole formula in strong gravitational fields
- Gravitational potential $\Phi \sim 10^{-6}$ ($v \sim 10^{-3}$) n radio binary pulsars while $\Phi \sim 0.1$ ($v \sim 0.3$) in coalescing binaries
- PN effects at order v^7 are 10^{14} times more important in inpsiral observations than in radio pulsars



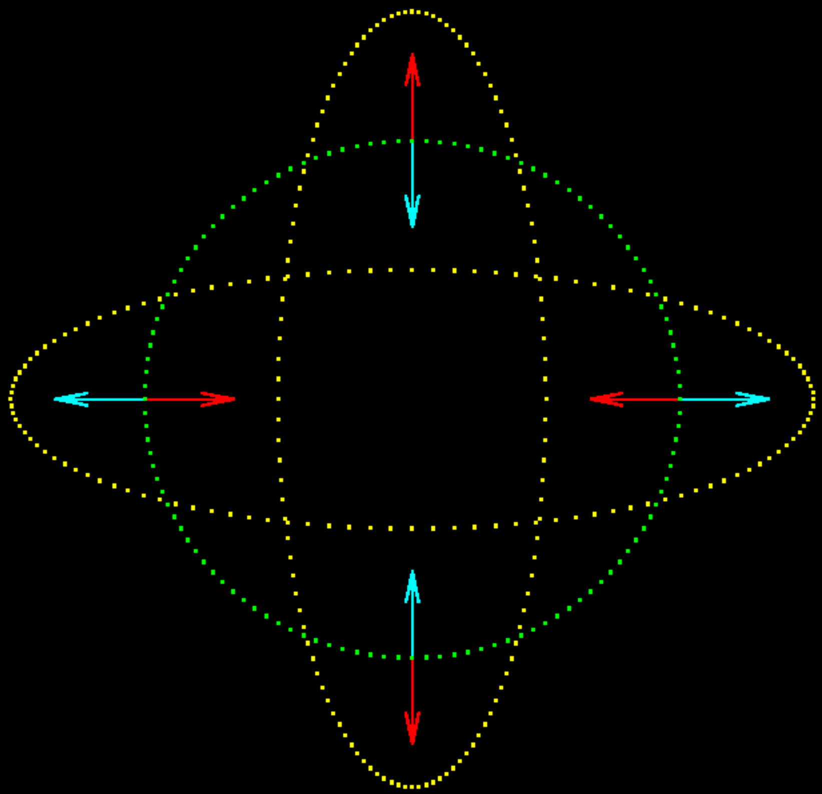
Speed of Gravitational Waves

- In general relativity gravitational waves travel on the light-cone
- How do we measure the speed of GW:
  - Coincident observation of gravitational waves and electromagnetic radiation from the same source
  - for a source at a distance *D* can test the speed of GW relative to EM to a relative accuracy of ~1/D

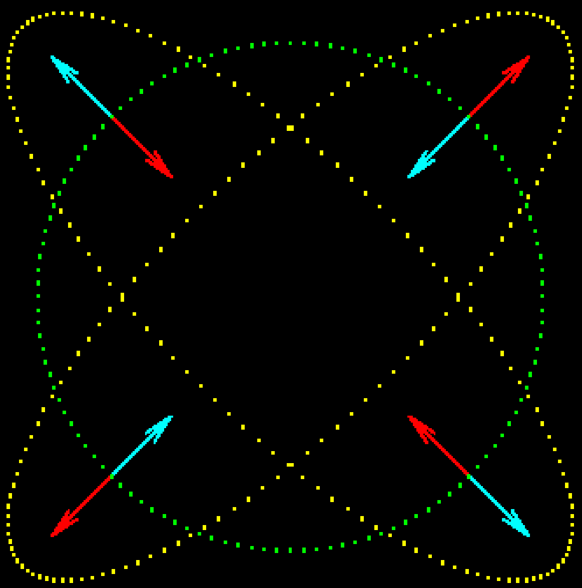
Constrain the mass of the graviton

- If graviton is massive then it will lead to dispersion of the waves (Cliff Will)
  - Different waves travel at different speeds
  - The phasing of the waves changes
  - The matched filter will have an additional parameter (mass of the graviton)
- Can constrain $\lambda g \sim 1.3 \times 10^{13}$ in EGO and 7×10^{16} km in LISA (Arun et al)

Polarisation of Gravitational Waves

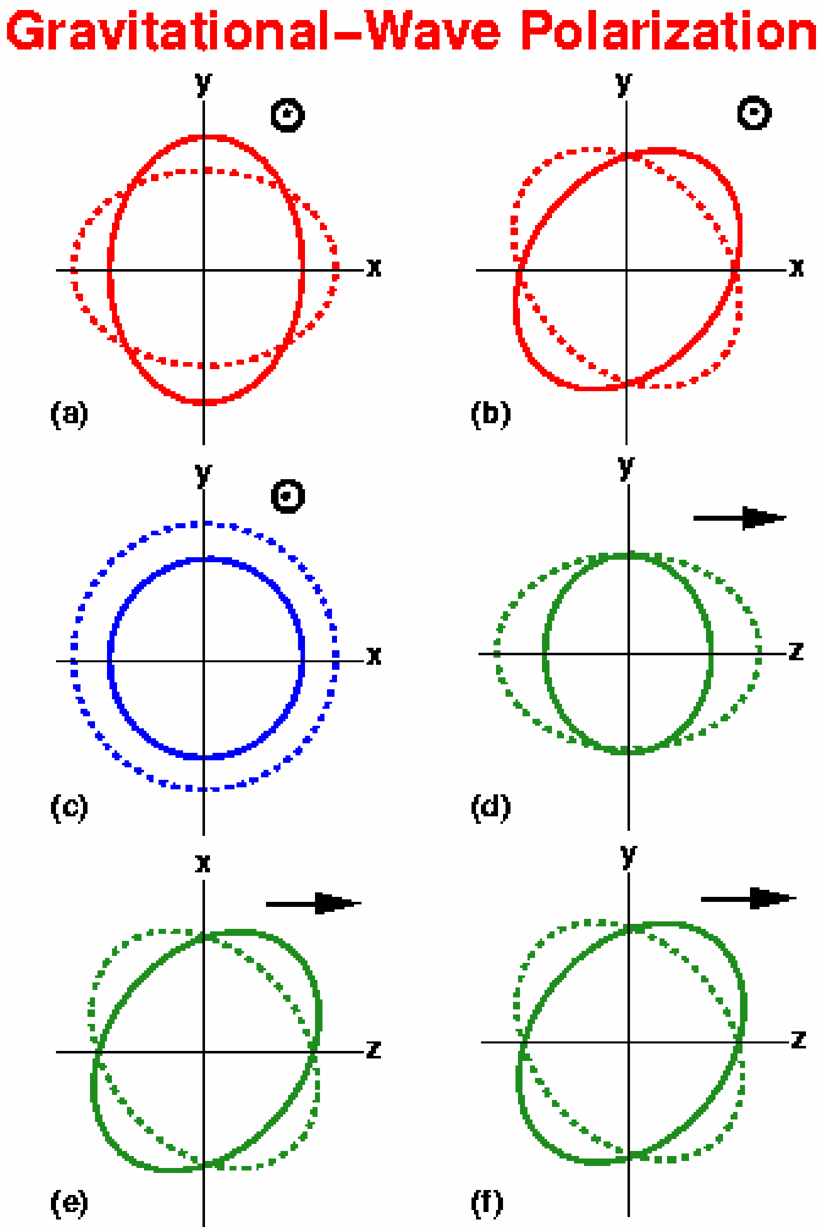


Plus polarization



Cross polarization

Cliff Will



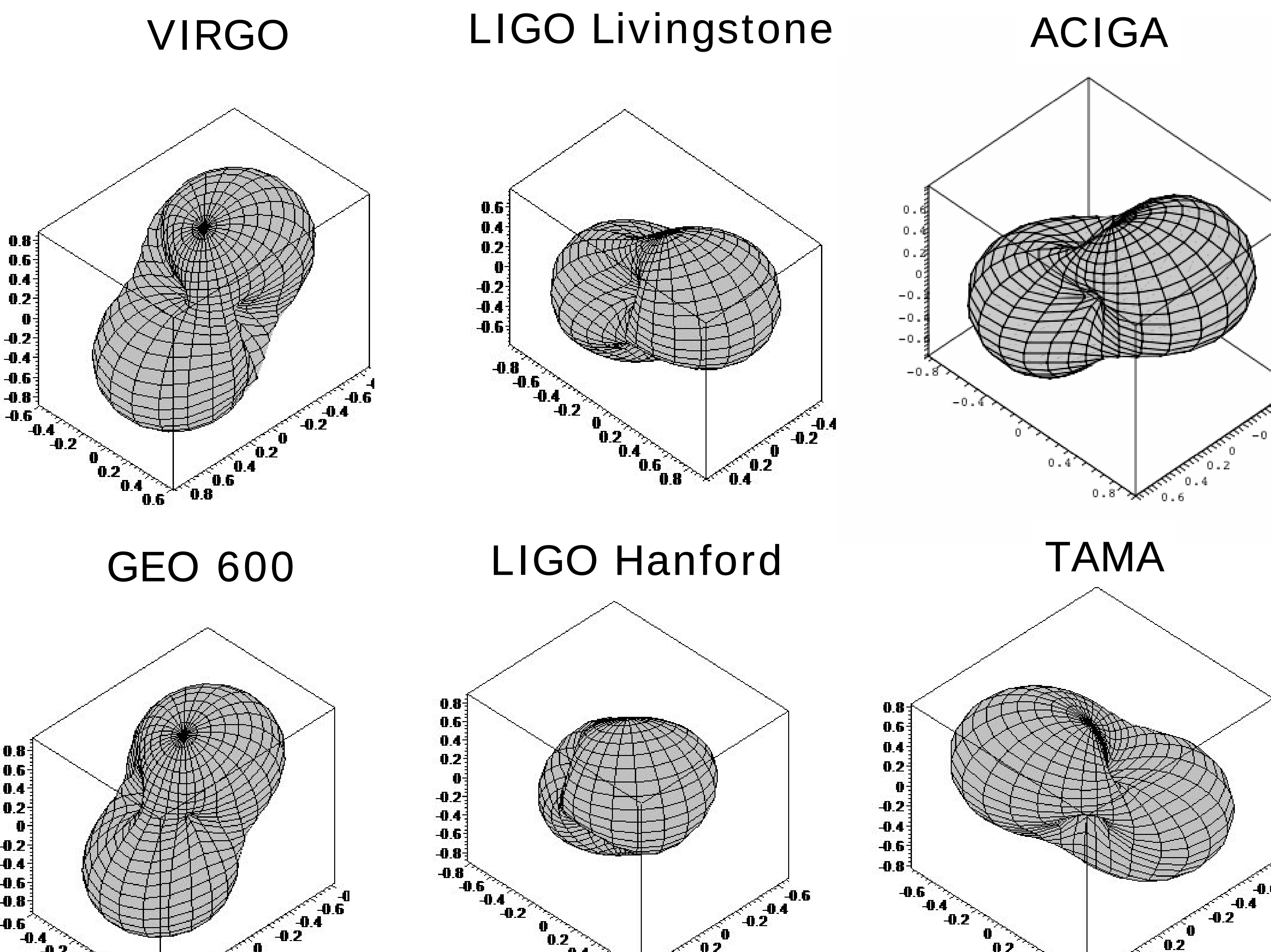
Response of a GW Detector

- $R(t,\theta,\phi,\psi) = F_+(\theta,\phi,\psi)\, h_+(t) + F_\times(\theta,\phi,\psi)\, h_\times(t)$
  - $h_+(t,\iota)$, $h_\times(t,\iota)$ – The two different polarisations of the gravitational wave in GR
  - $F_+(\theta,\phi,\psi)$, $F_\times(\theta,\phi,\psi)$ antenna response to the two different polarisations
  - θ, ϕ Direction to the source
  - Polarization angle ψ

Beam Pattern Function

$$F_i(\theta_i, \phi_i) = C_i \left[\left(\frac{1+\cos^2(\theta_i)}{2} \cos(2\phi_i) \right)^2 + \cos^2(\theta_i) \sin^2(2\phi_i) \right]^{1/2},$$

- Beam pattern of a detector is the sensitivity of an antenna to un-polarized radiation as a function of the direction of the incoming wave
- (θ_i , ϕ_i) source coordinates wrt with *i*-th detector, and the factor C_i is a constant used to mimic the difference in the strain sensitivity of different antennas.
- In order to compare different detectors it is necessary to choose a single coordinate system (Θ, Φ) with respect to which we shall consider the various detector responses


VIRGO
LIGO Livingstone
ACIGA
GEO 600
LIGO Hanford
TAMA

Extracting the Polarisation in GR

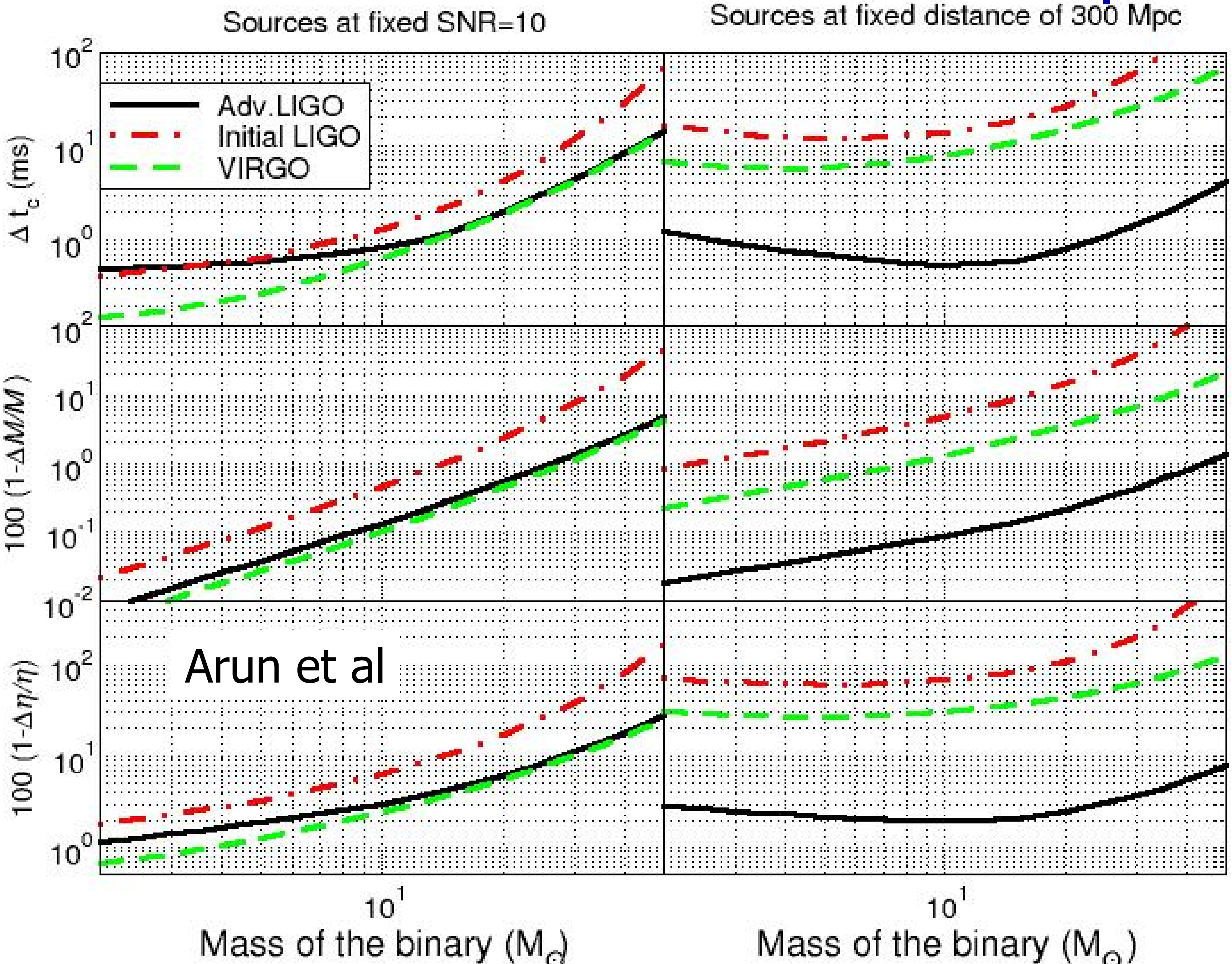
- Assuming that there are only two polarisations
  - We can extract the two polarizations using three or more detectors (three responses and two independent time delays to measure the fine unknowns)

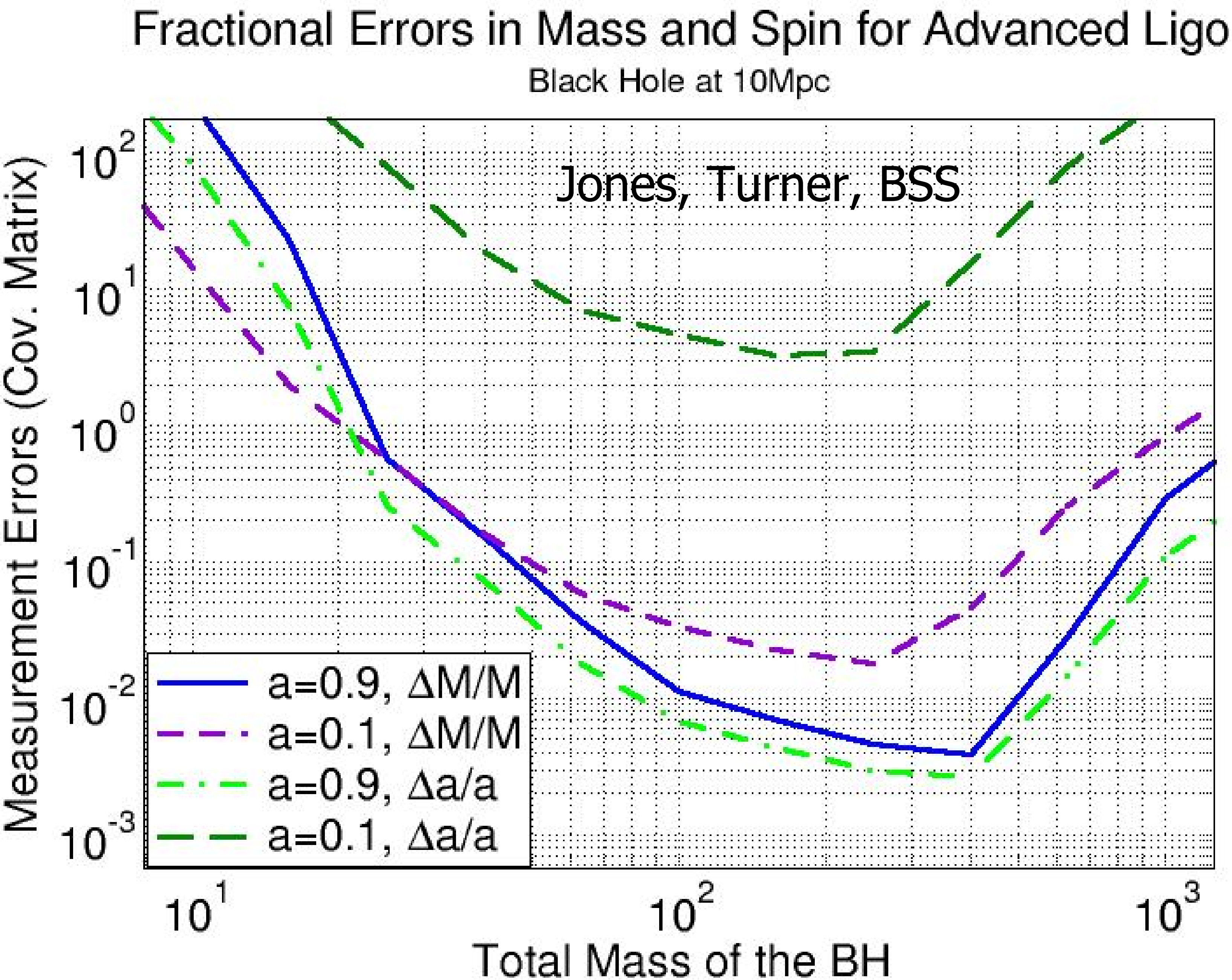
Strong field tests of relativity

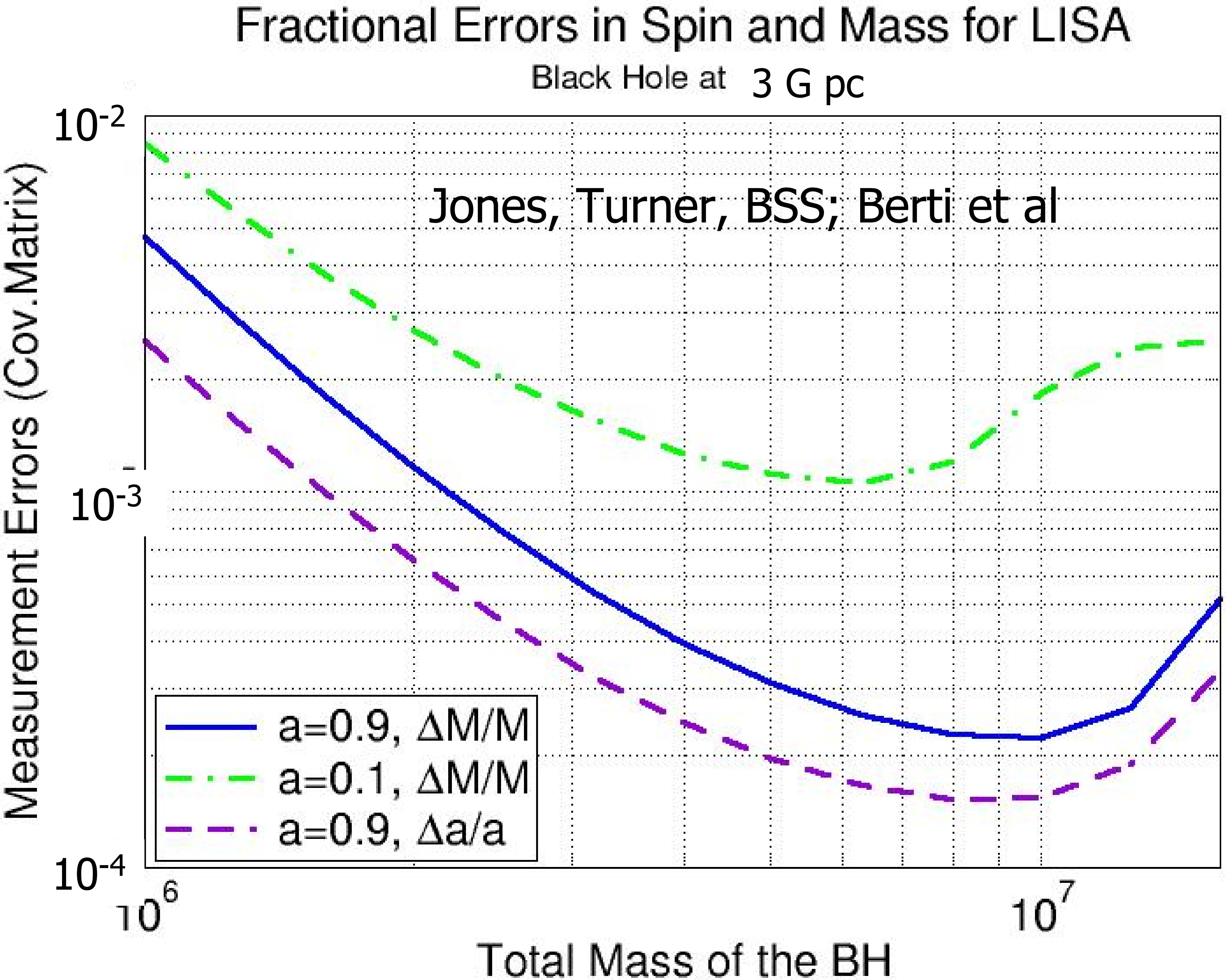
Fundamental questions on strong gravity and the nature of space-time

- From inspiral and ringdown signals
  - measure M and J before and after merger: test Hawking area theorem
  - Measure J/M^2. Is it less than 1?
  - Consistent with a central BH or Naked singularity or Soliton/Boson stars?

Accurate measurements from inspirals

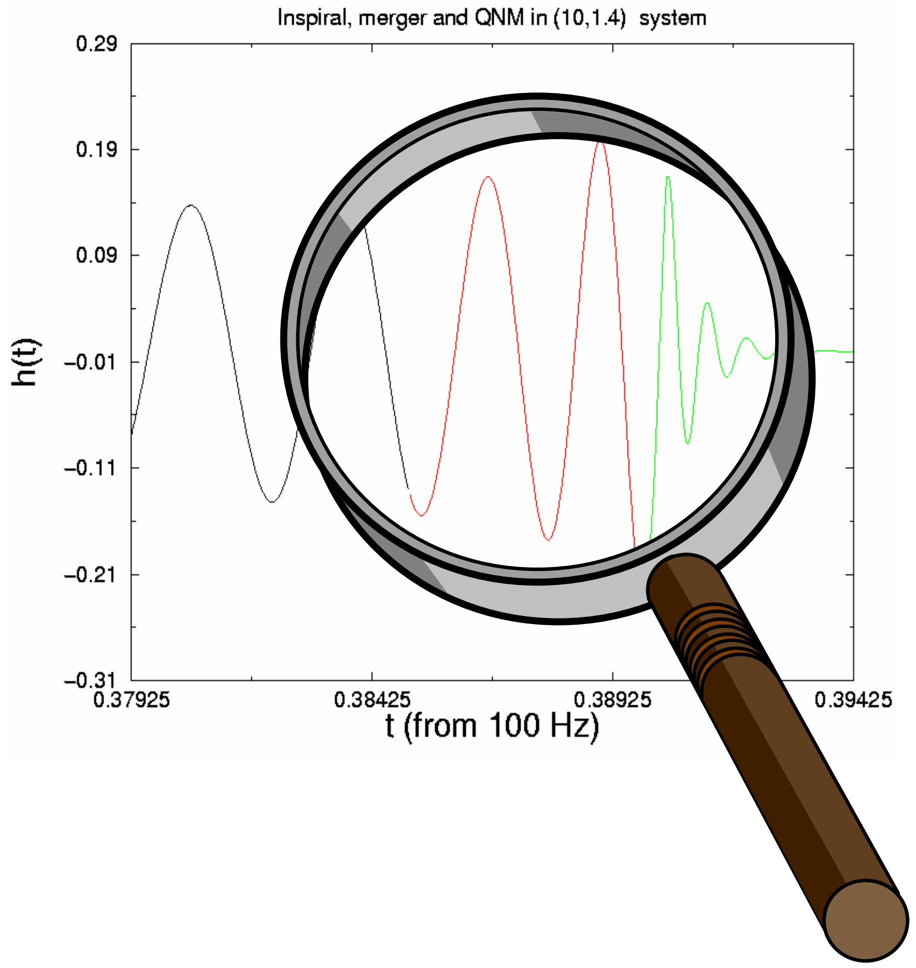



Fractional Errors in Mass and Spin for Advanced Ligo
Black Hole at 10Mpc
Jones, Turner, BSS
Measurement Errors (Cov. Matrix)
Total Mass of the BH
a=0.9, ΔM/M
a=0.1, ΔM/M
a=0.9, Δa/a
a=0.1, Δa/a
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^1
10^2
10^3

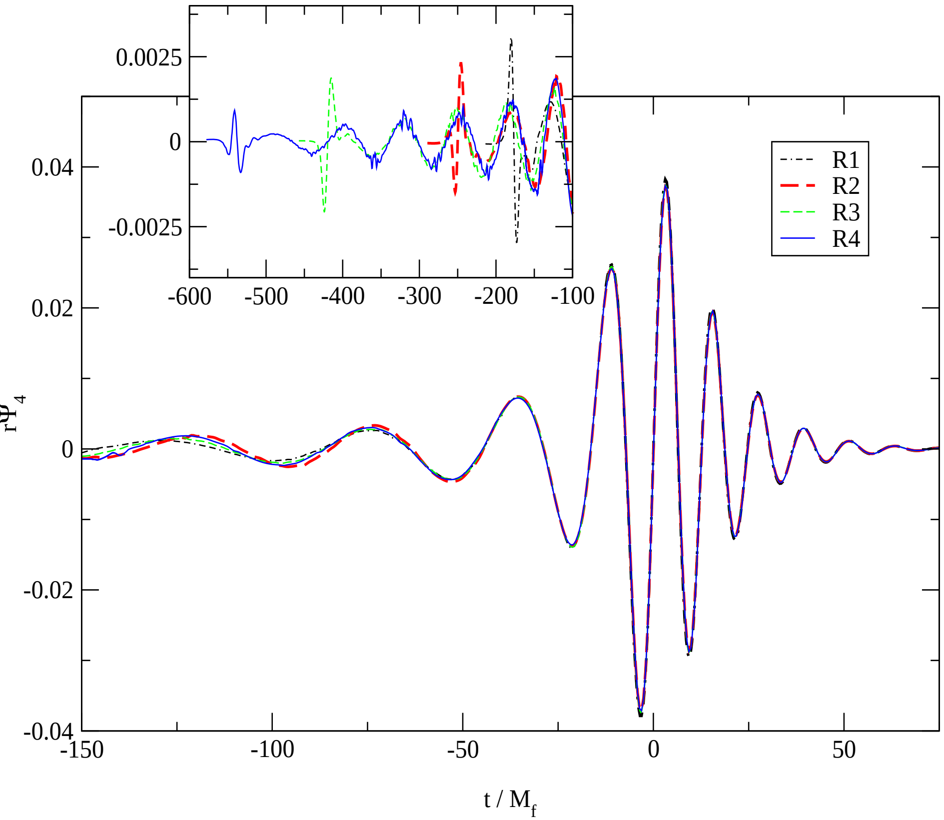

Fractional Errors in Spin and Mass for LISA
Black Hole at 3 G pc
Jones, Turner, BSS; Berti et al
Measurement Errors (Cov.Matrix)
10-2
10-3
10-4
10⁶
10⁷
Total Mass of the BH
a=0.9, ΔM/M
a=0.1, ΔM/M
a=0.9, Δa/a

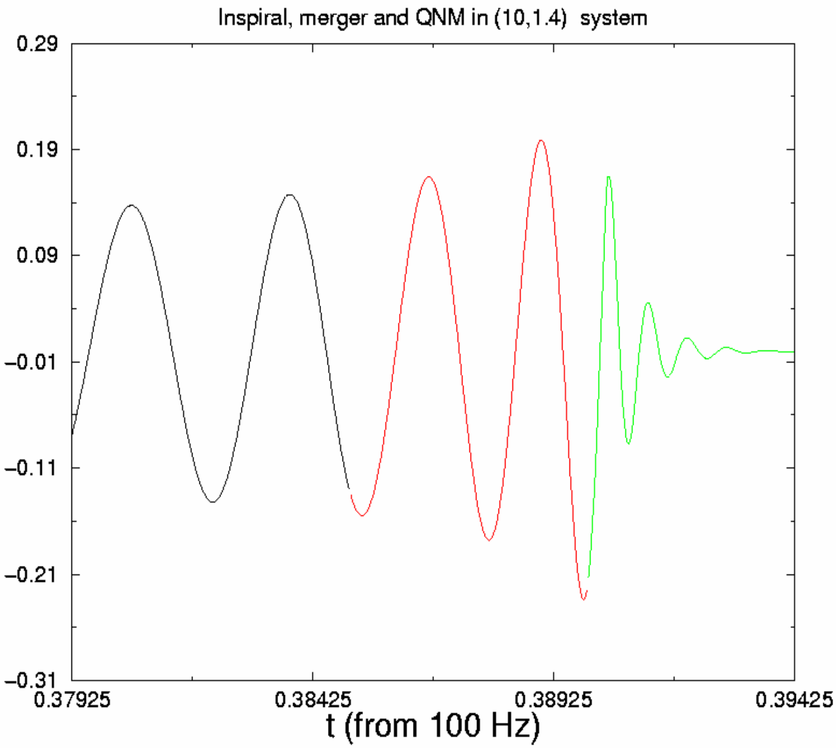
Testing the Merger Dynamics

- From inspiral, merger and quasi-normal modes
  - Test analytical models of merger and numerical relativity simulations
- Effective one-body (Buonanno and Damour)
  - 0.7% of total mass in GW
- Numerical relativity (Baker et al, AEI, Jena, PSU, UTB)
  - 1-3% of total mass in GW

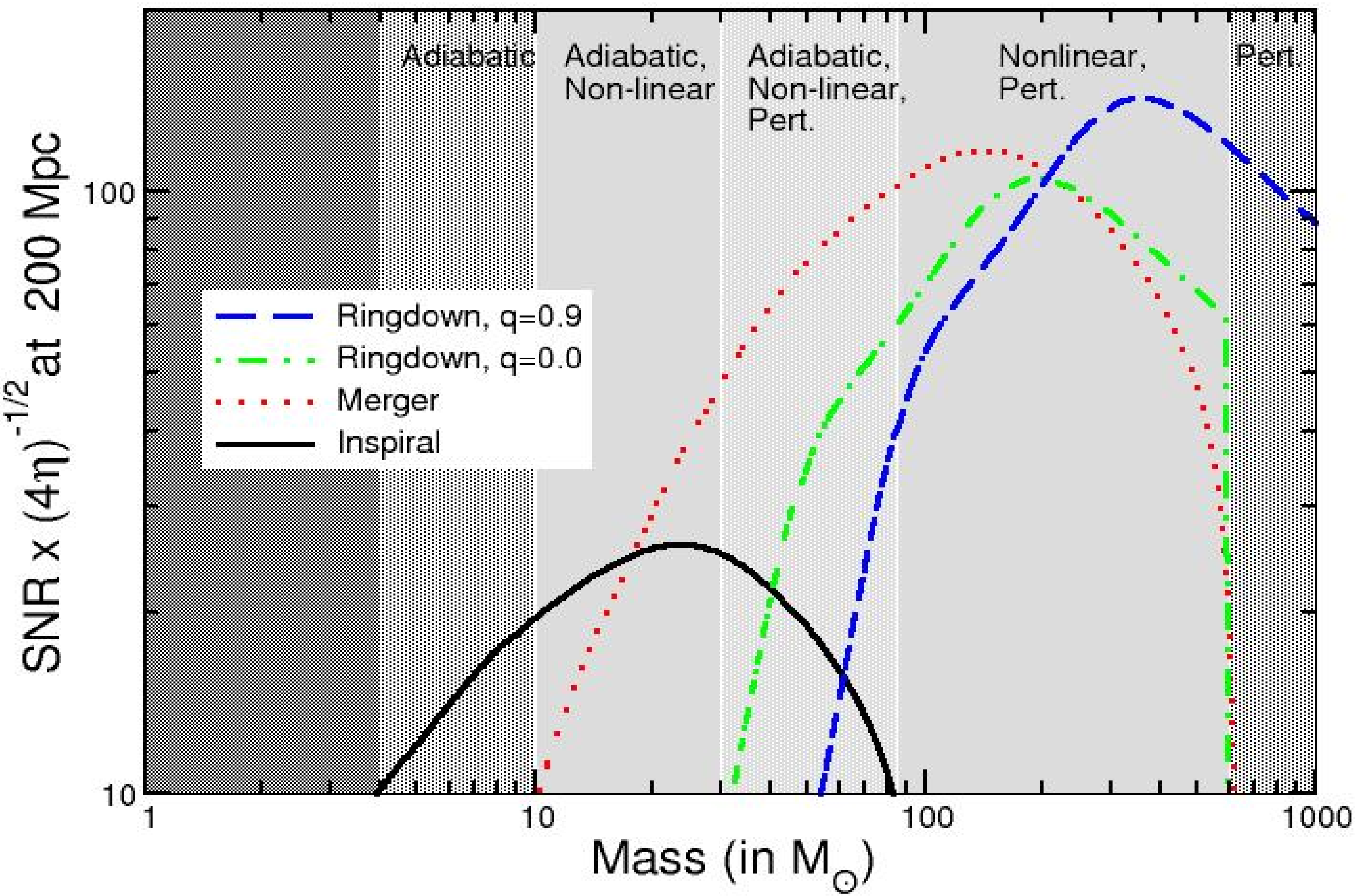


Analytical Vs Numerical Relativity

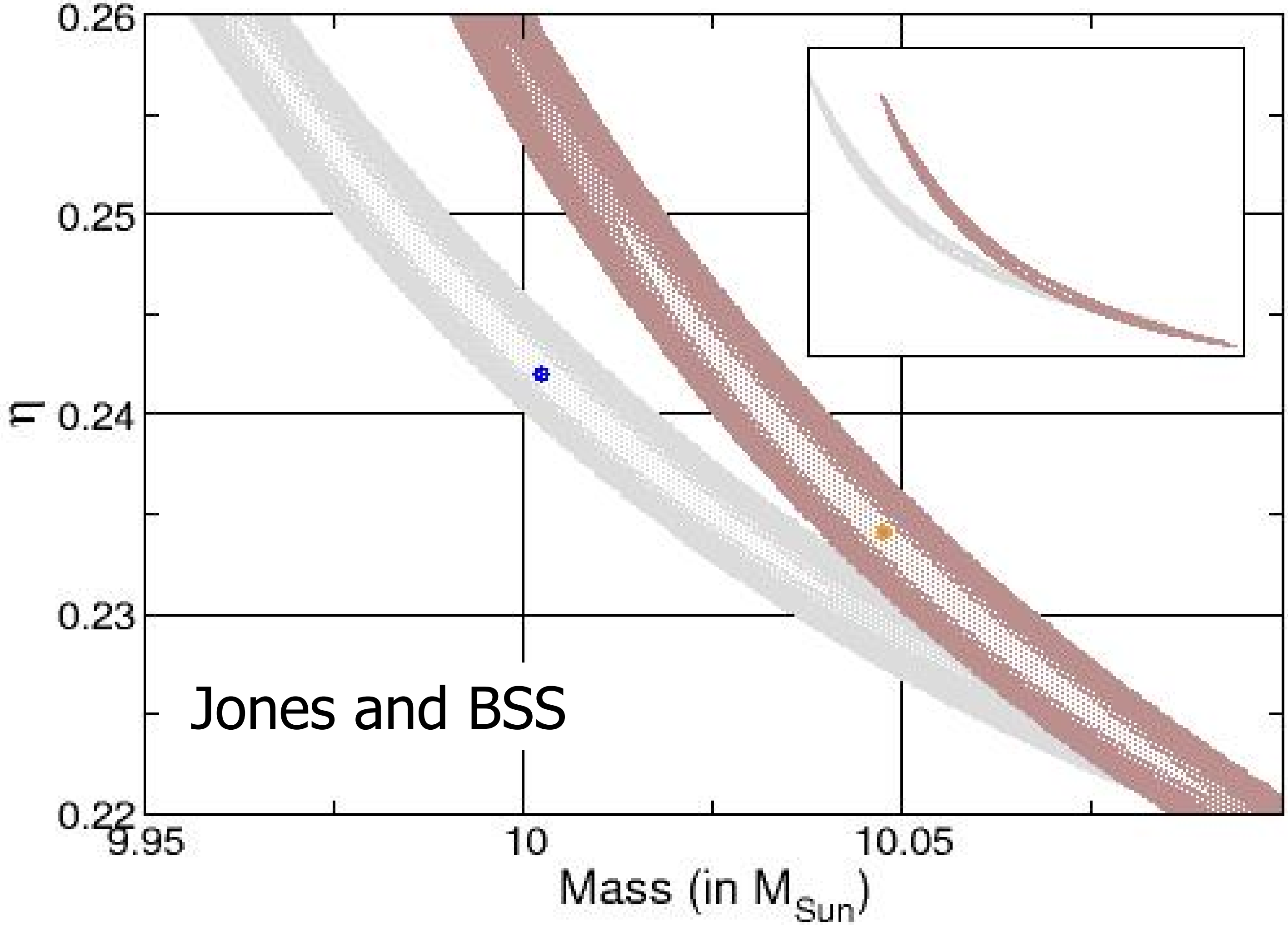




Adv LIGO Sensitivity to Inspirals



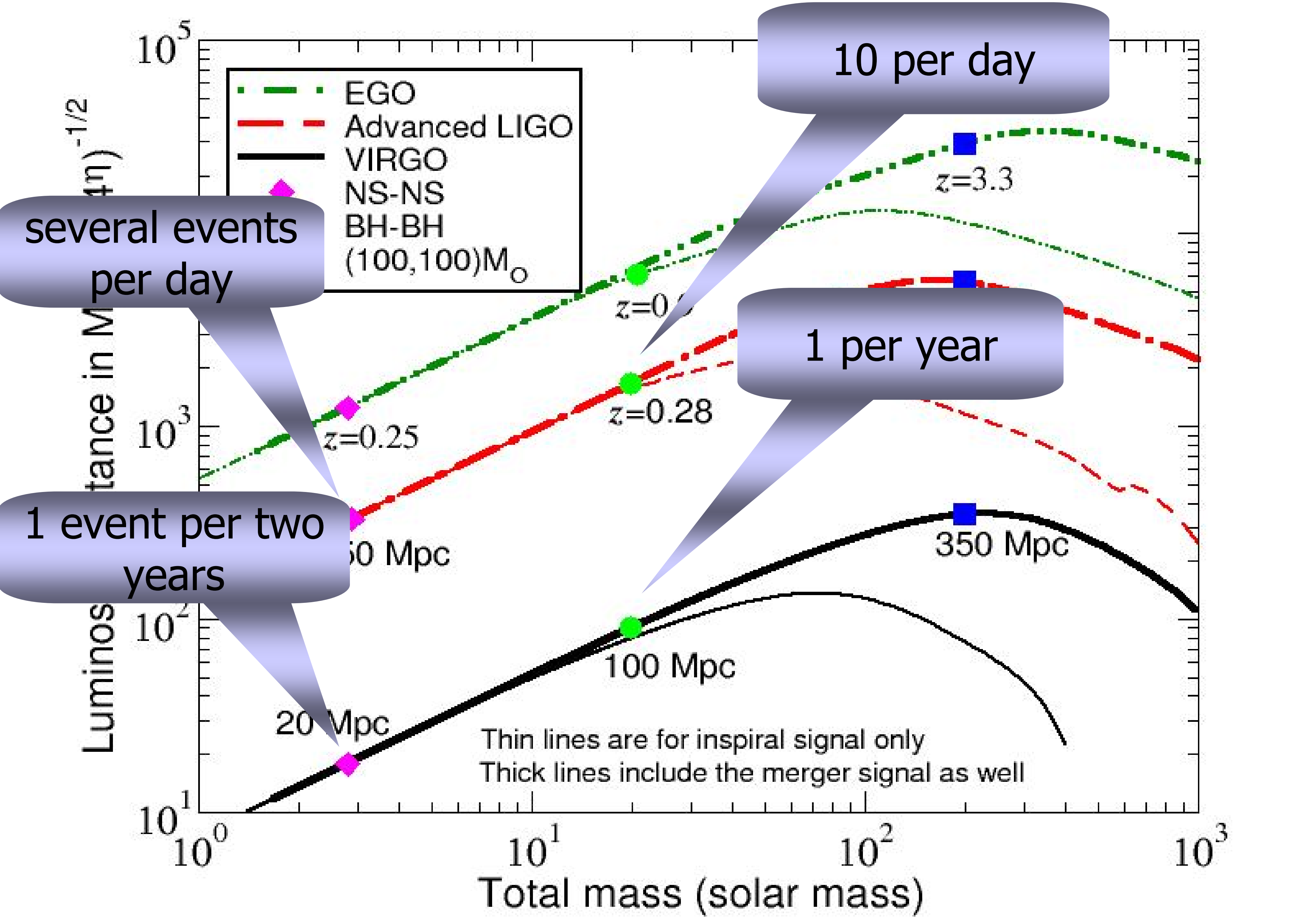
Strong field tests of gravity Consistency of Parameters



Testing Post-Newtonian Gravity

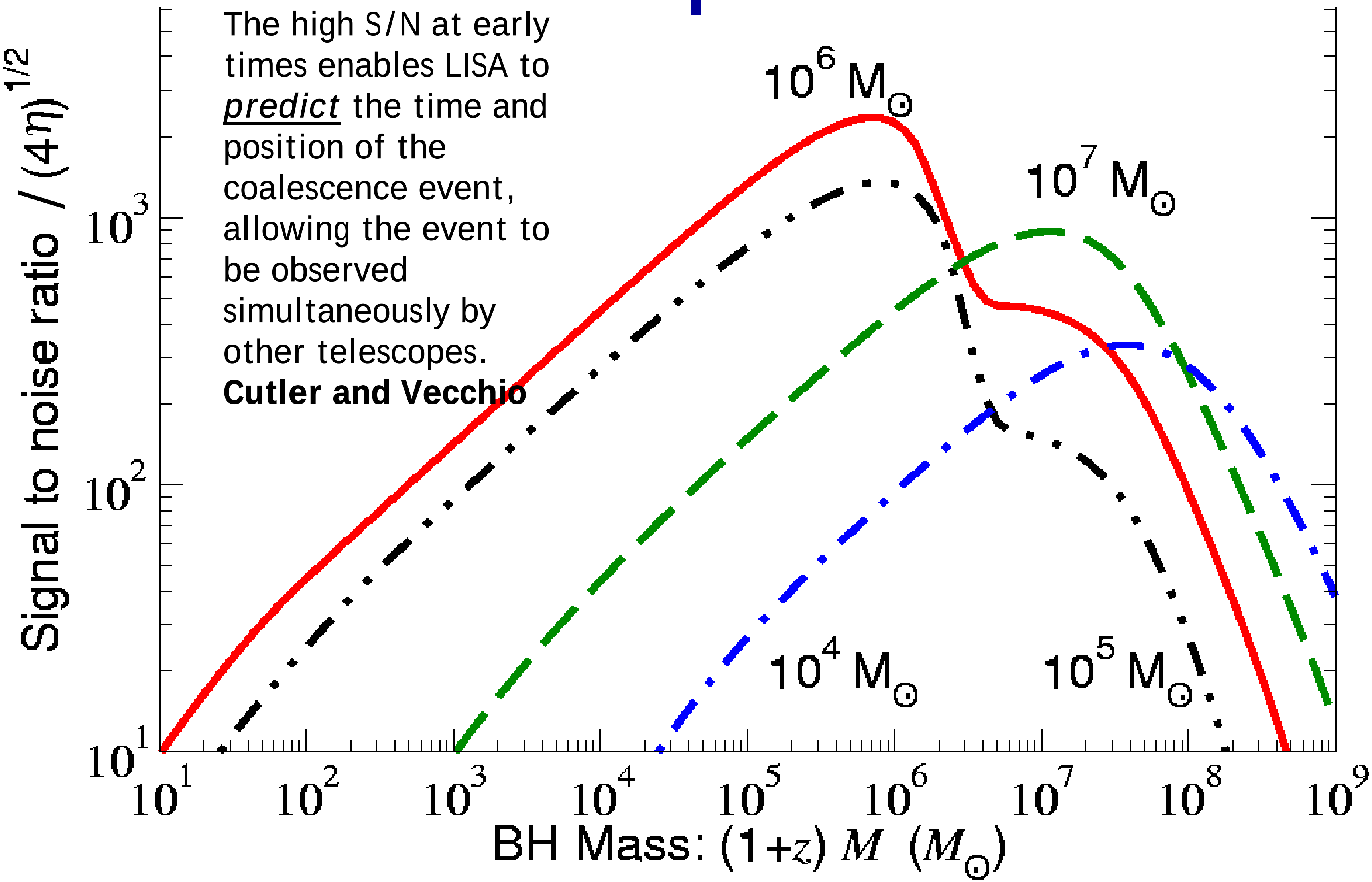
GR two-body problem is ill-posed

- GW detectors are a tool to explore the two-body problem and tests the various predictions of general relativity



10 per day
several events per day
1 per year
1 event per two years
EGO
Advanced LIGO
VIRGO
NS-NS
BH-BH
(100,100)M$_O$
z=3.3
z=0.25
z=0.28
350 Mpc
100 Mpc
20 Mpc
Thin lines are for inspiral signal only
Thick lines include the merger signal as well
10^5
10^3
10^2
10^1
10^0
10^1
10^2
10^3
Total mass (solar mass)

Merger of supermassive black holes - no templates needed!

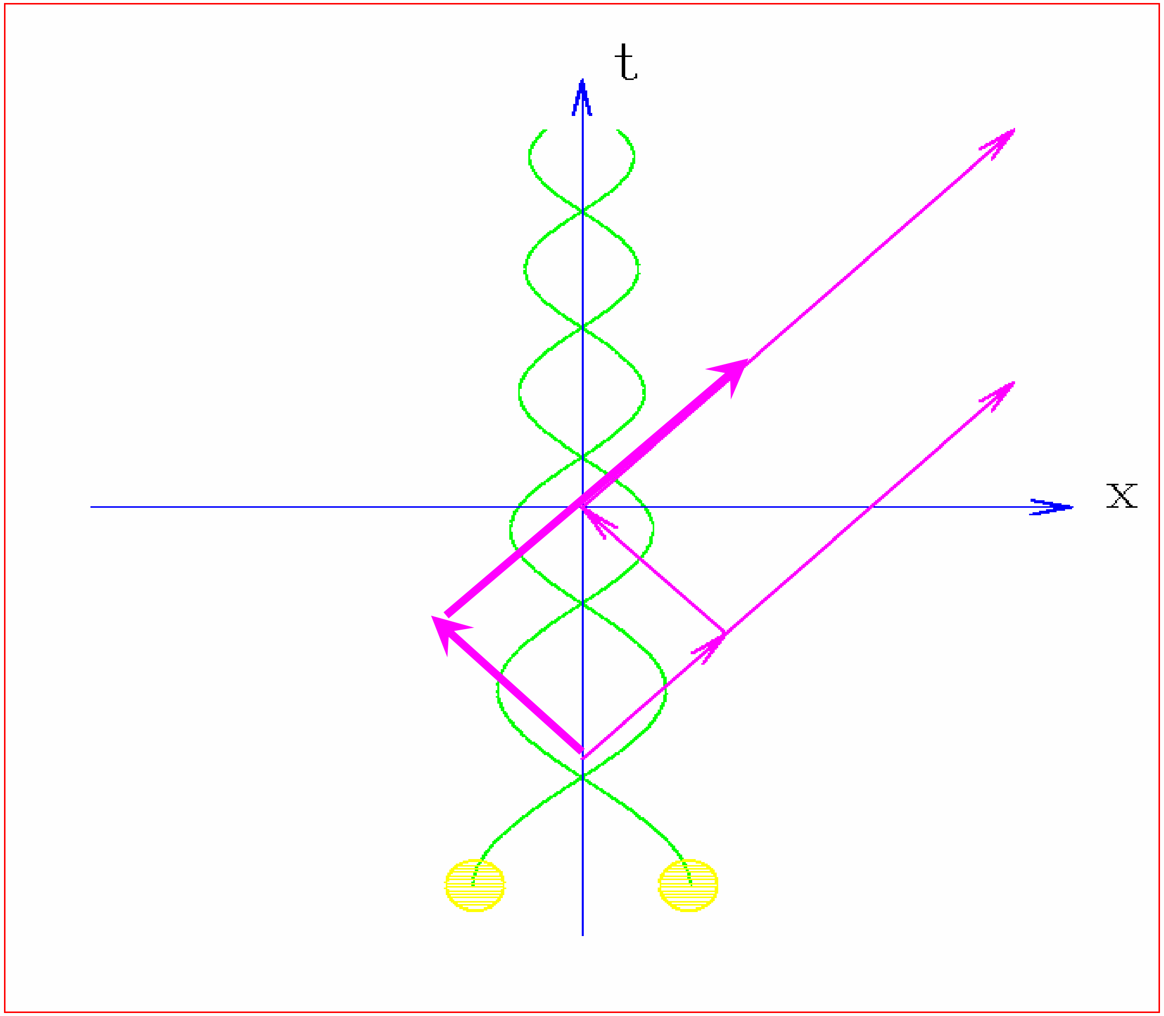


Phasing Formula for GW akin to Timing Formula for Binary PSRs

Blanchet
Damour
Faye
Farase
Iyer
Jaranowski
Schaeffer
Will
Wiseman
…

$$
\begin{aligned}
\Phi(t) = & -\frac{1}{\eta\tau^5}\Big\{1 \quad \text{Newtonian} \\
& + \left(\frac{3715}{8064} + \frac{55}{96}\eta\right)\tau^2 \quad \text{1PNterm} \\
& - \frac{3}{4}\pi\tau^3 \quad \text{Tails of GW} \\
& + \left(\frac{9275495}{14450688} + \frac{284875}{258048}\eta + \frac{1855}{2048}\eta^2\right)\tau^4 \\
& + \left(-\frac{38645}{172032} - \frac{15}{2048}\eta\right)\pi \ln\left(\frac{\tau}{\tau_0}\right)\tau^5 \\
& + \Big[\frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}C + \\
& \left(-\frac{123292747421}{4161798144} + \frac{2255}{2048}\pi^2 - \frac{11429}{14784}\right)\eta + \\
& \frac{154565}{1835008}\eta^2 - \frac{1179625}{1769472}\eta^3 - \frac{107}{56}\ln\left(\frac{1}{2\tau}\right)\Big]\tau^6 \\
& + \left(\frac{188516689}{1835008} + \frac{140495}{114688}\eta - \frac{122659}{516096}\eta^2\right)\pi\tau^7\Big\}
\end{aligned}
$$

Gravitational wave tails



Blanchet and Schaefer 95, Blanchet and Sathyaprakash 96

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&+ \left(\frac{188516689}{1835008}+\frac{140495}{114688}\eta-\frac{122659}{516096}\eta^2\right)\pi\tau^7\Big\}
\end{aligned}
$$

Signal in the Fourier Domain

$$\tilde{h}(f) \equiv \int_{-\infty}^{\infty} h(t) \exp(2\pi i f t) dt$$

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\Psi(f)}$$

$$\mathcal{A} = \frac{\mathcal{C}}{r\pi^{2/3}} \sqrt{\frac{3}{384}} \mathcal{M}^{5/6}$$

$$\Psi(f) = 2\pi f t_C + \Phi_C + \sum_k \psi_k f^{k-5/3}.$$

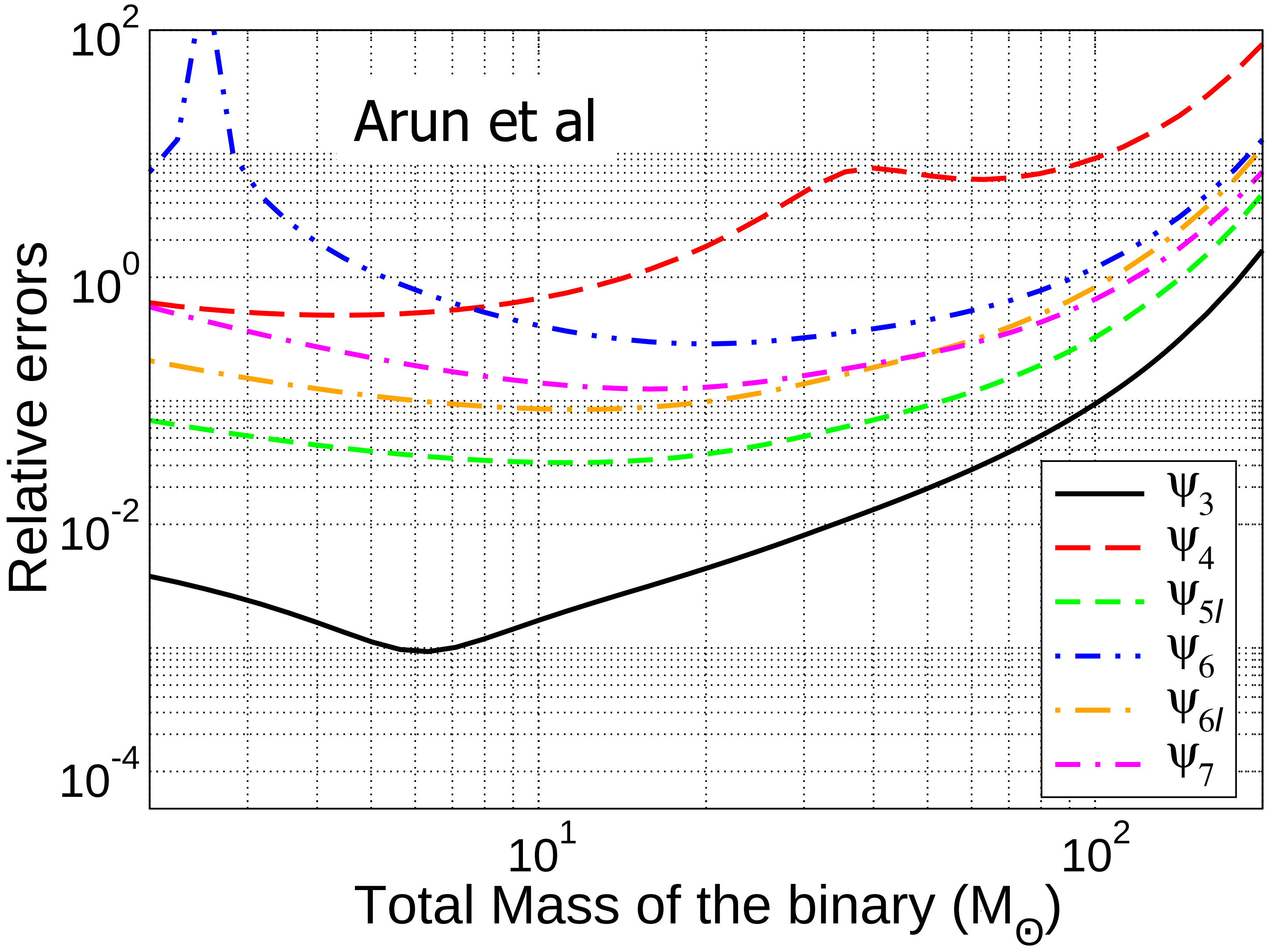
Here t_C and Φ_C are the fiducial time- and phase-offsets of of the signal.

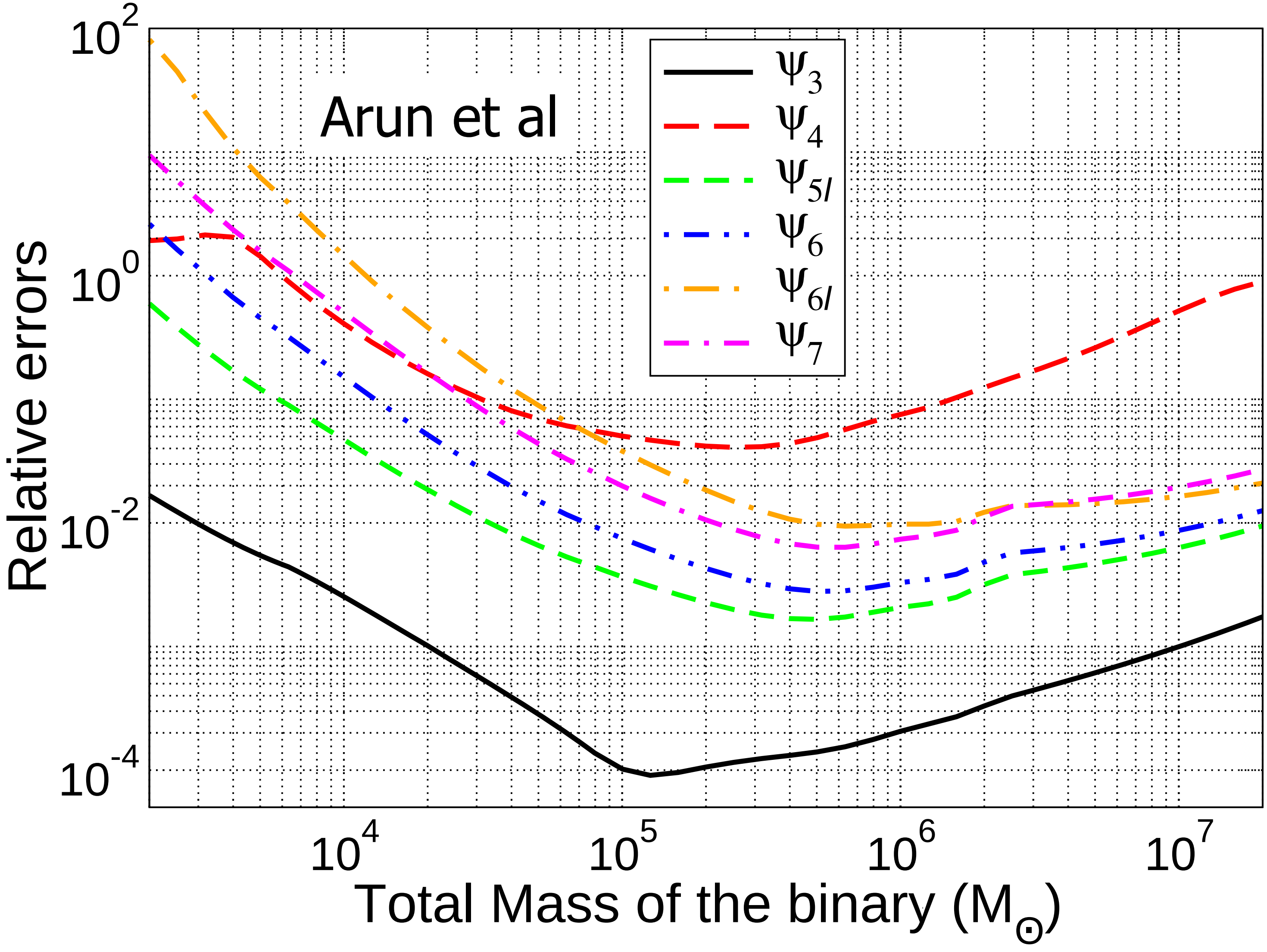
$$\psi_k = \frac{3}{128\,\eta}(\pi M)^{(k-5)/3}\alpha_k$$

post-Newtonian parameters

$$\begin{aligned}
\alpha_0 &= 1 \\
\alpha_1 &= 0 \\
\alpha_2 &= \frac{3715}{756} + \frac{55}{9}\eta \\
\alpha_3 &= -16\pi \\
\alpha_4 &= \frac{3085}{72}\eta^2 + \frac{27145}{504}\eta + \frac{15293365}{508032} \\
\alpha_5 &= \pi\left(\frac{38645}{756} - \frac{65}{9}\eta\right)\left[1 + \ln\left(6^{3/2}\pi M f\right)\right] \\
\alpha_6 &= \frac{11583231236531}{4694215680} - \frac{640}{3}\pi^2 - \frac{6848}{21}\gamma \\
&+ \left(-\frac{15335597827}{3048192} + \frac{2255}{12}\pi^2 - \frac{91432}{693}\right)\eta \\
&+ \frac{76055}{1728}\eta^2 - \frac{127825}{1296}\eta^3 - \frac{6848}{63}\ln(64\pi M f) \\
\alpha_7 &= \pi\left(\frac{77096675}{254016} + \frac{378515}{1512}\eta - \frac{74045}{756}\eta^2\right).
\end{aligned}$$

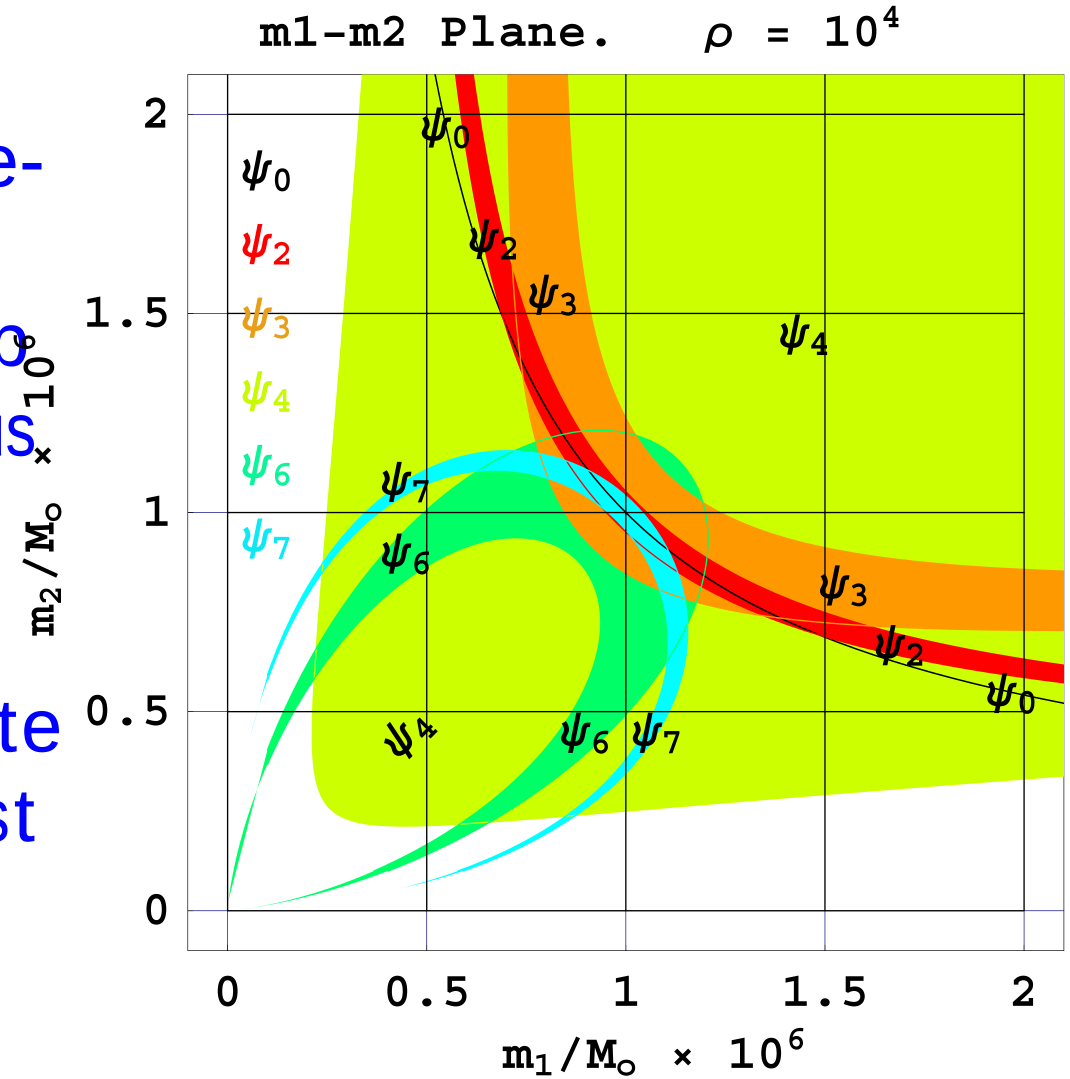
ψ_4 and ψ_6 involve $\log(f)$ terms.


Arun et al
Relative errors
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{1}
10^{2}
Total Mass of the binary ($M_{\odot}$)
ψ_3
ψ_4
ψ_{5l}
ψ_6
ψ_{6l}
ψ_7


Arun et al
Ψ_3
Ψ_4
Ψ_{5l}
Ψ_6
Ψ_{6l}
Ψ_7
Relative errors
10^2
10^0
10^{-2}
10^{-4}
10^4
10^5
10^6
10^7
Total Mass of the binary ($M_\odot$)

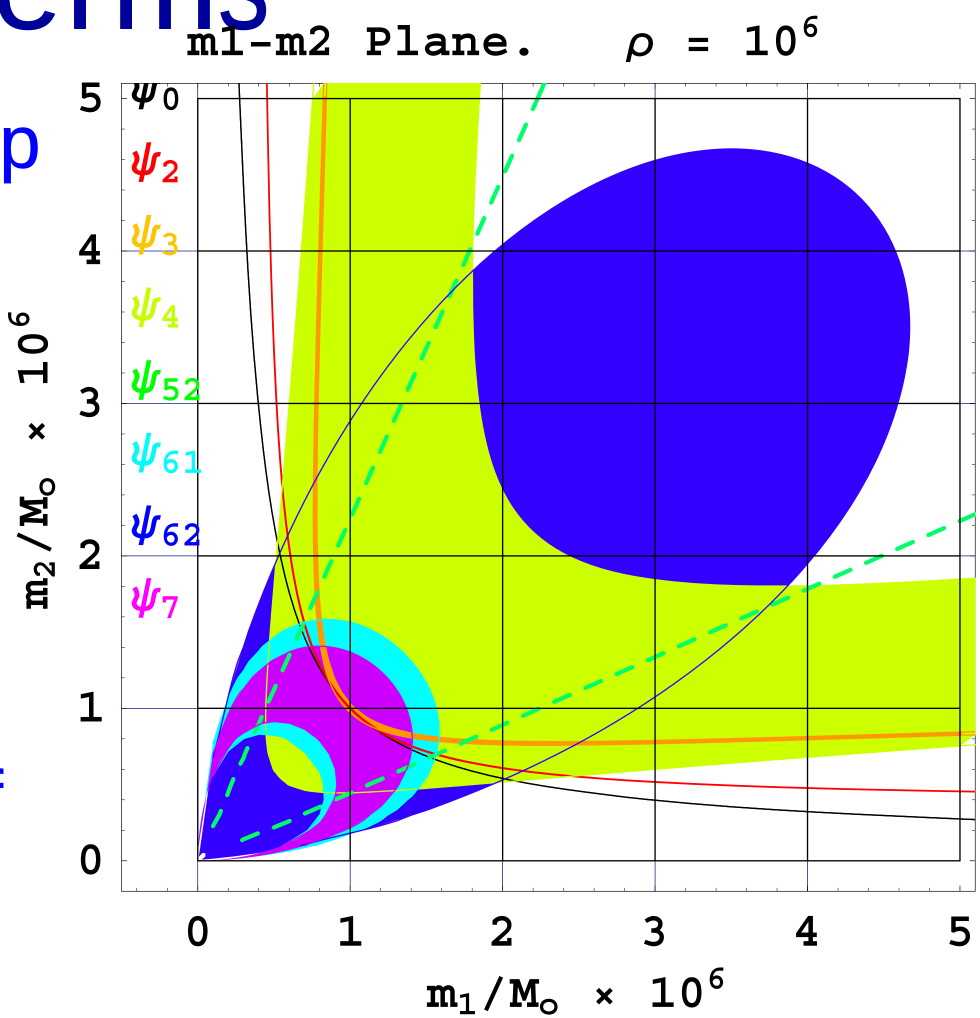
Testing other PN effects in LISA

- In this test we re-expand the log-terms and absorb them into various post-Newtonian orders
- The test can quite reliably test most PN parameters except ψ_4

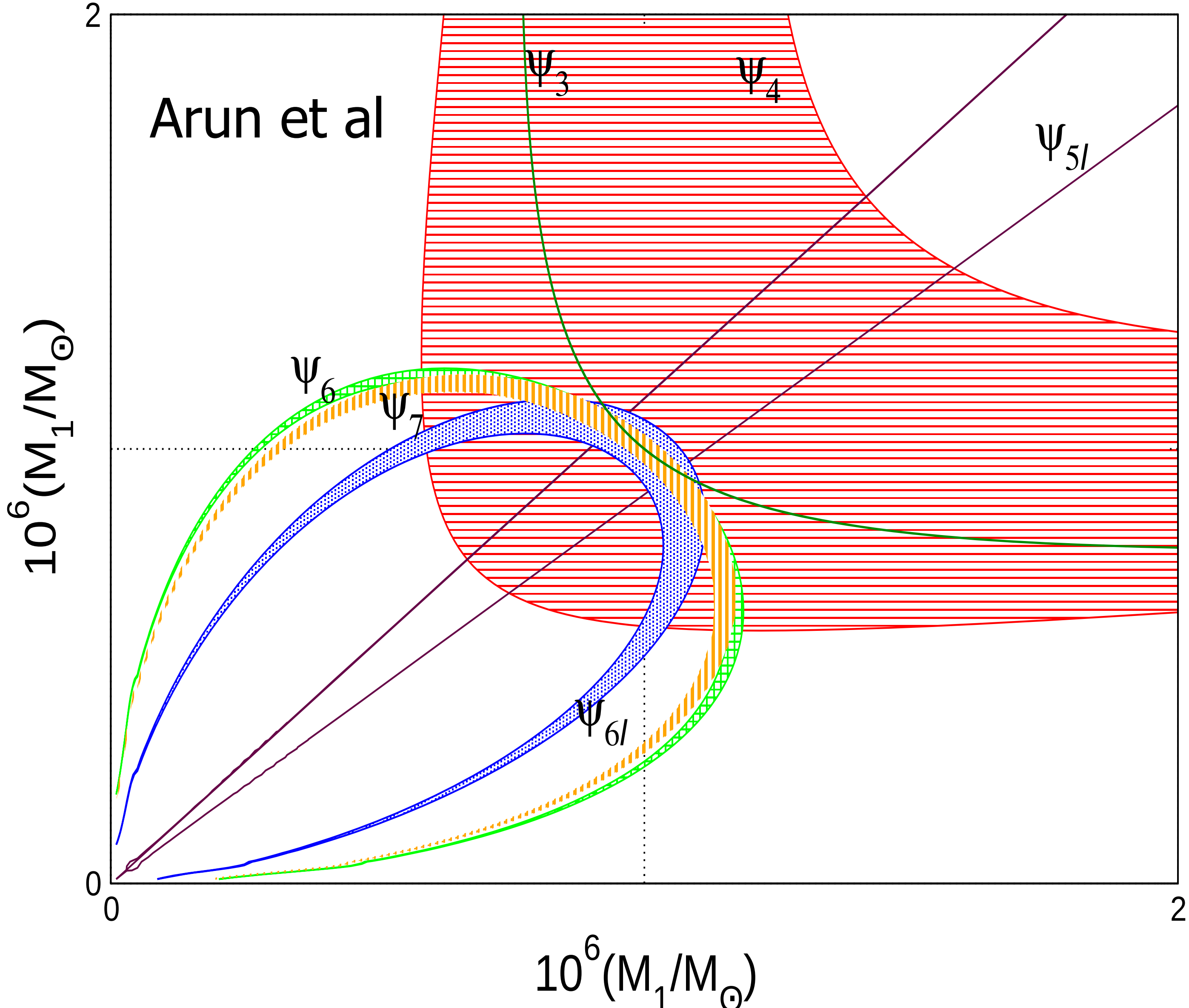


Testing the presence of log terms

- In this test we keep the log-terms as they appear but introduce new parameters corresponding to the log-terms
- Greater number of parameters means that we have a weaker test

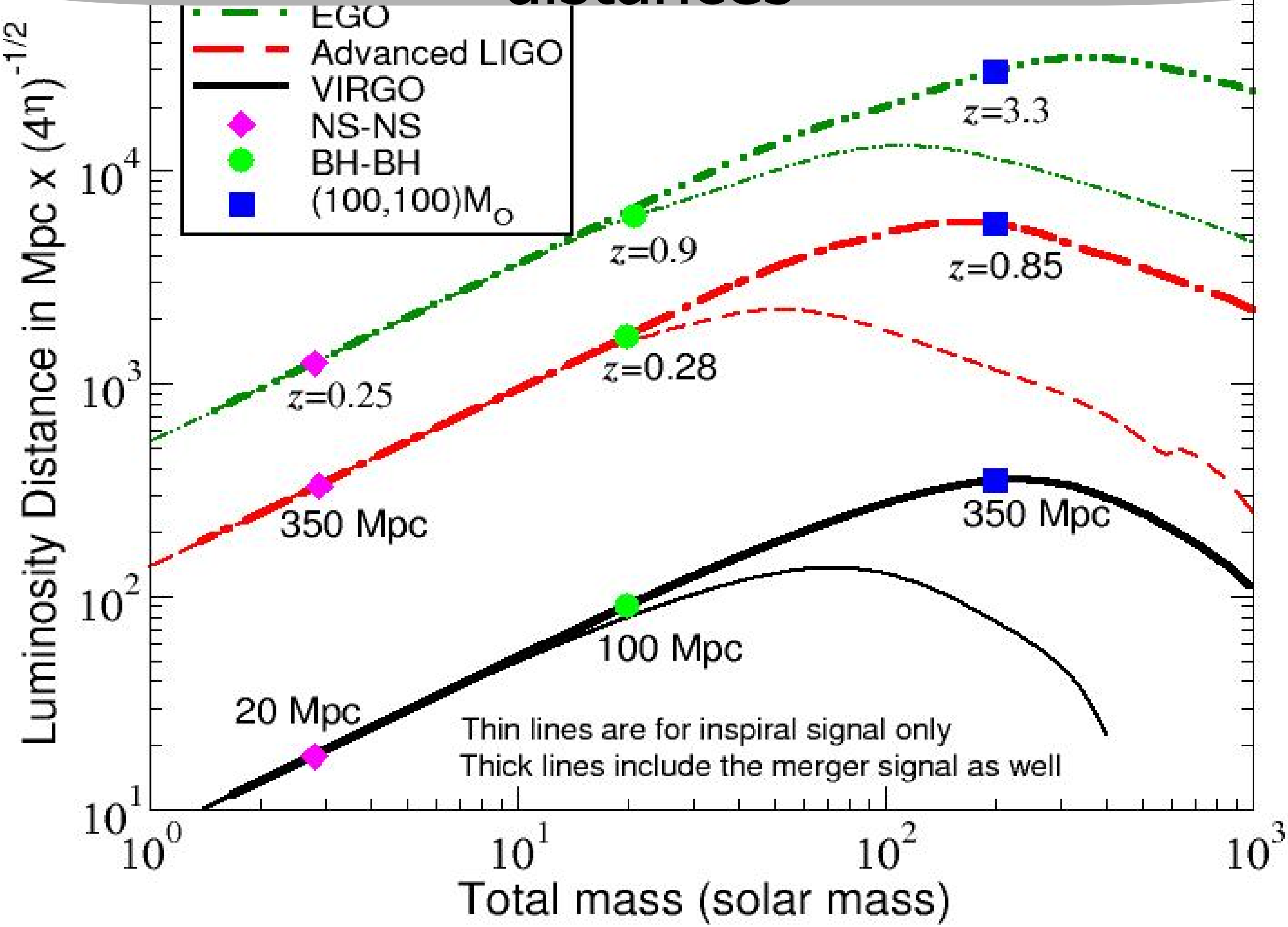


Consistency of PN Coefficients including log-terms



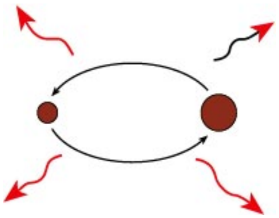
Cosmology

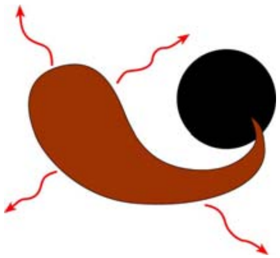
Inspirals can be seen to cosmological distances



Cosmology and Astronomy from Stellar Mass Binary Coalescences

- Cosmology
  - Measure luminosity distance to within 10% and, with the aid of EM observations of host galaxies, determine cosmological parameters; binary coalescences are standard candles, build a new distance ladder, measure $d_L(z)$; infer about dark matter/energy

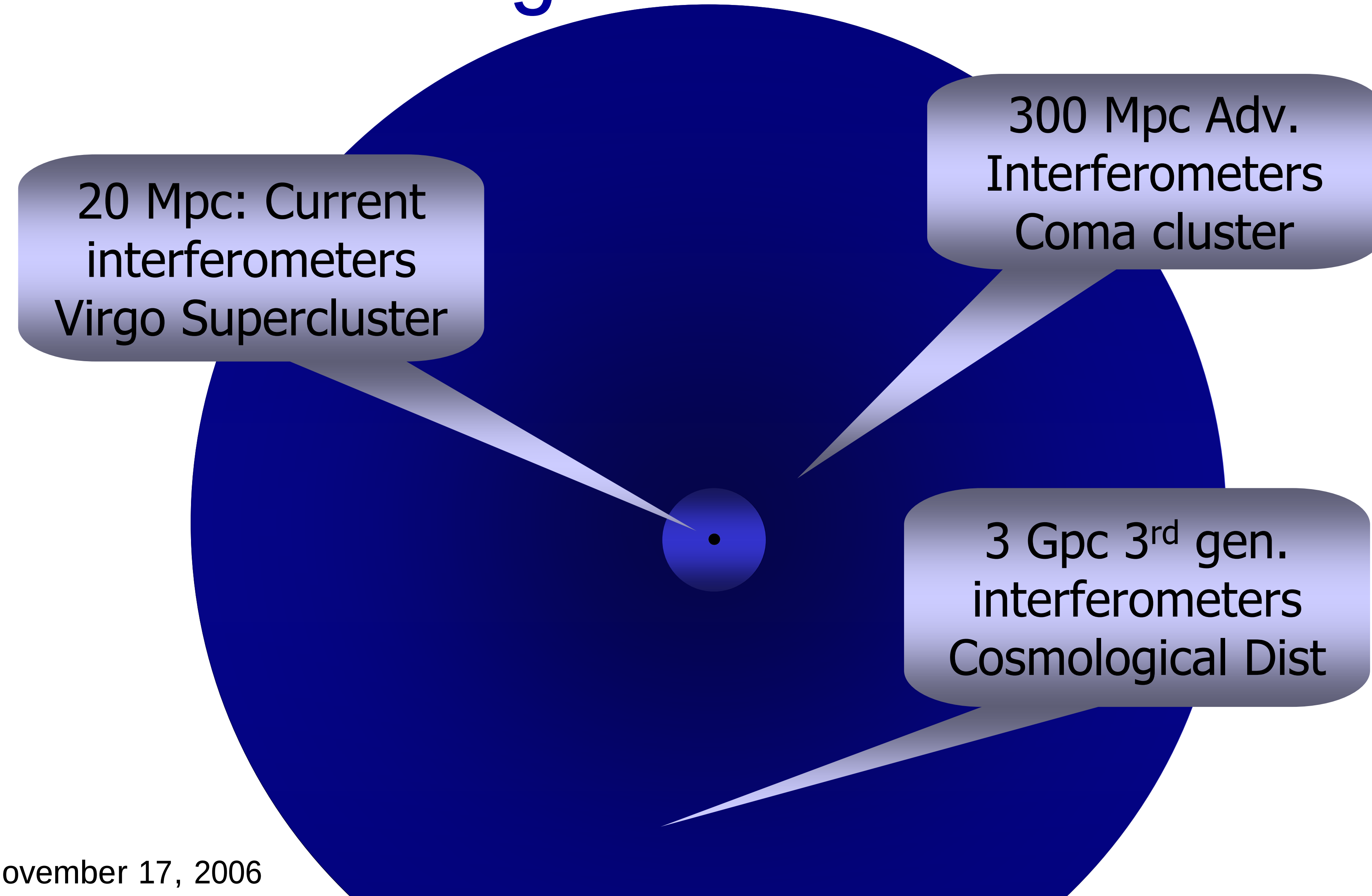




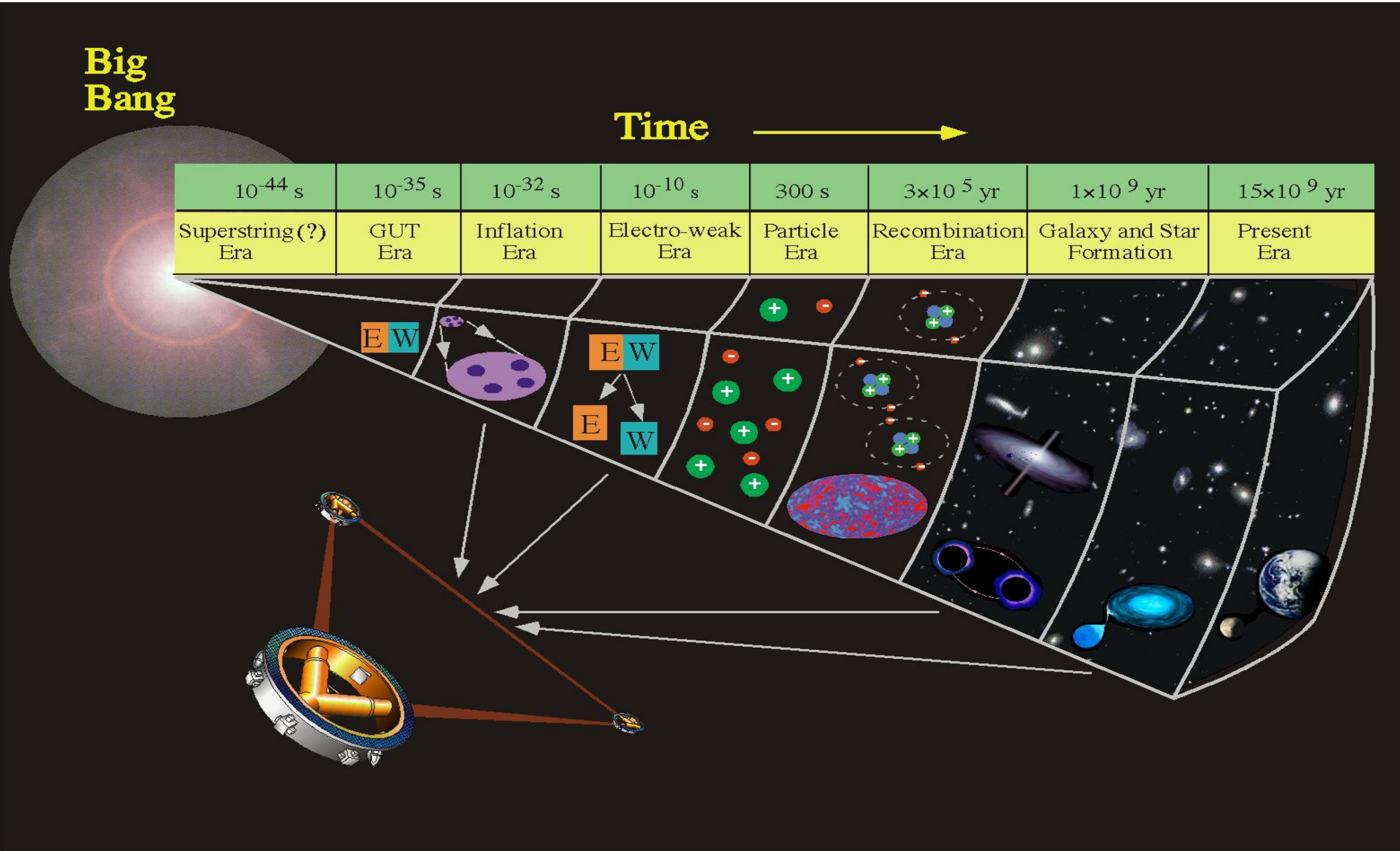
- Search for EM counterpart, e.g. γ-burst. If found:
  - Learn the nature of the trigger for that γ-burst, deduce relative speed of light and GW's: ~ 1 / 3x10^9 yrs ~ $\mathbf{10^{-17}}$
  - measure Neutron Star radius to 15% and deduce equation of state
- Deduce star formation rate from coalescence rates

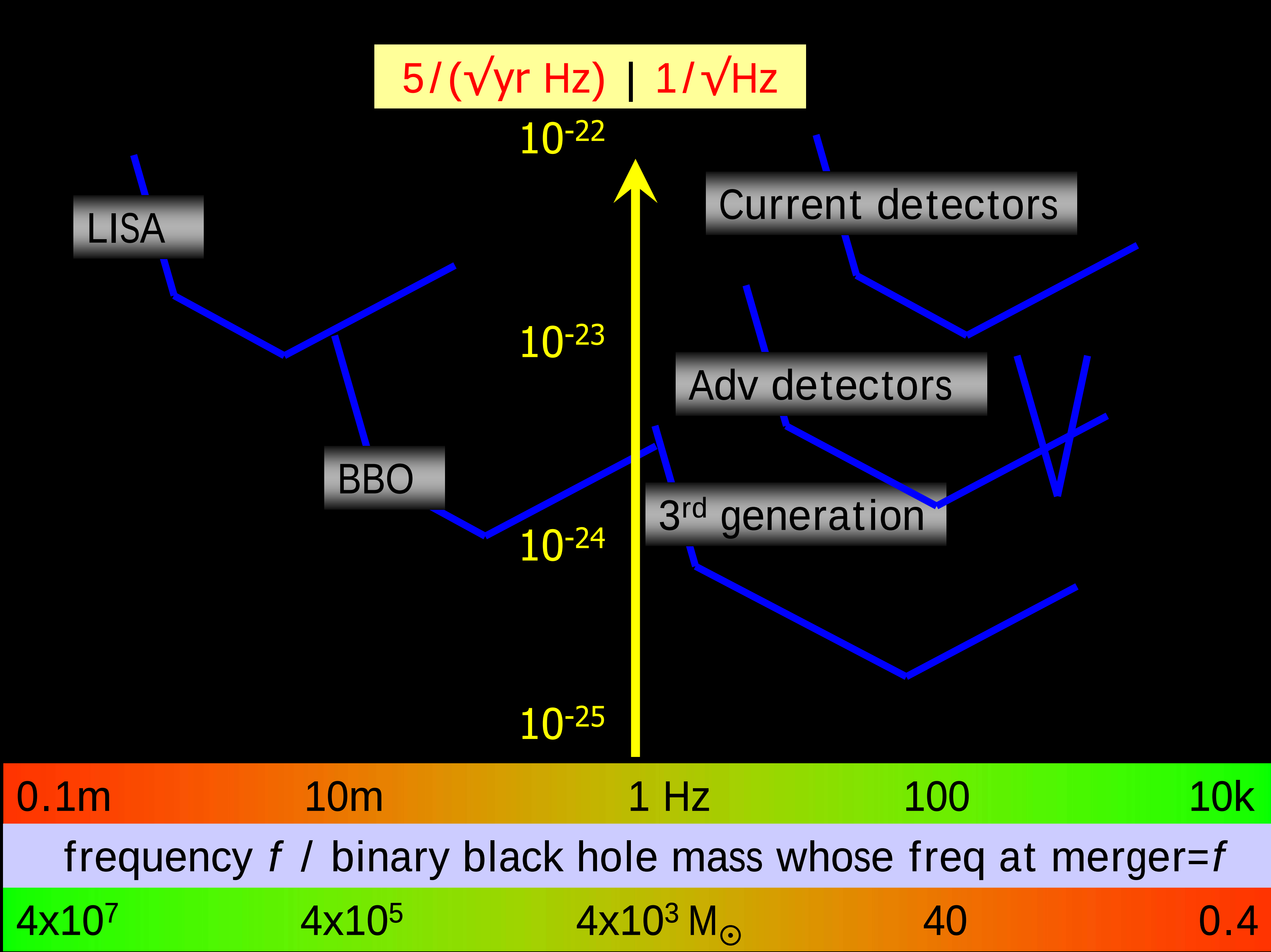
In conclusion

Ground-Based Detectors: Nearby to High-z Universe



LISA: Fundamental Physics, Astrophysics and Cosmology





5/(√yr Hz) | 1/√Hz
10-22
10-23
10-24
10-25
LISA
BBO
Current detectors
Adv detectors
3rd generation
0.1m
10m
1 Hz
100
10k
frequency f / binary black hole mass whose freq at merger=f
4x107
4x105
4x103 M⊙
40
0.4