Frequency corrections to antenna-patterns: forward detector transfer function

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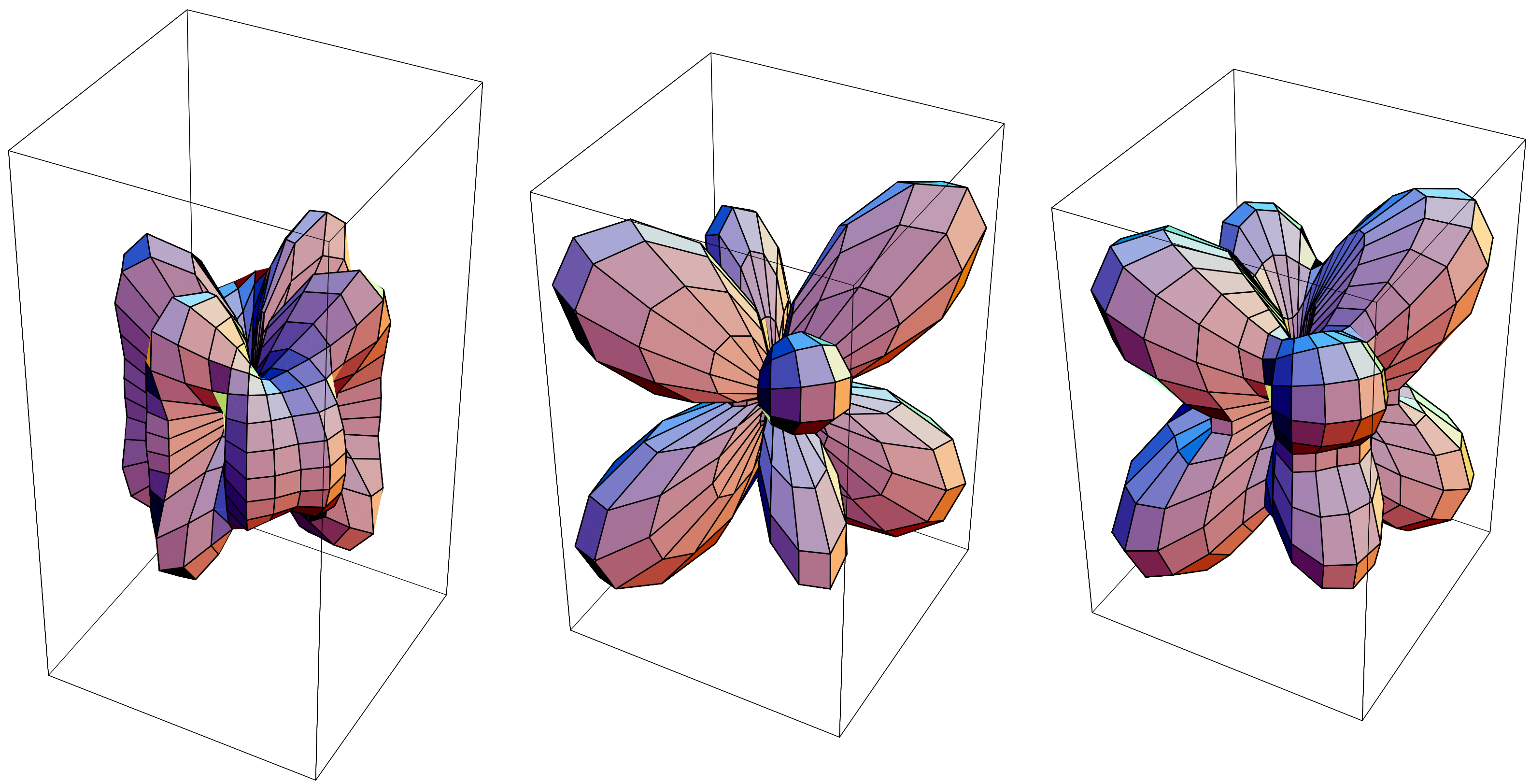
LSC documents regarding the frequency dependence of the antenna patterns and its implication for calibration:

- T970101-B, D. Sigg, *Strain calibration in LIGO*,
- T030296, D. Sigg and R. Savage, *Analysis proposal to search for gravitational waves at multiples of the LIGO arm cavity free-spectral-range frequency*,
- T030186, J. Markowicz, R.L. Savage, and P. Schwinberg, *Development of a readout scheme for high-frequency gravitational waves*,
- G050205, M. Rakhmanov and R. Savage, *LIGO detector response at high frequencies and its implications for calibration above 1kHz*,
- T050136, Hunter Elliott, *Analysis of the frequency dependence of the LIGO directional sensitivity (Antenna Pattern) and implications for detector calibration*,
- T060xxx, Jeffrey Parker, *Development of a high-frequency burst pipeline.*

High-frequency antenna patterns

Antenna patterns at FSR: response to +polarization ($\psi = 0°$), response ×polarization ($\psi = 90°$), averaged response.



from T970101-B, D.Sigg, *Strain calibration in LIGO*.

Brief derivation of the detector response to GW

Polarization tensor in the wave frame E_{gw} and the vector pointing to the source $\vec{n}$:

$$E_{gw} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \begin{aligned} n_x &= \sin\theta\cos\phi \\ n_y &= \sin\theta\sin\phi \\ n_z &= \cos\theta. \end{aligned}$$

Transformation from the wave frame to the detector frame, $R = R_z(\psi)R_y(\theta)R_z(\phi)$, induces the transformation of the polarization tensor: $E_{det} = R^T\, E_{gw}\, R$.

$$A_i = \frac{1 - e^{-(1-n_i)sT}}{1 - n_i}, \qquad B_i = \frac{1 - e^{-(1+n_i)sT}}{1 + n_i}.$$

Introduce the equivalent phase response and the cavity field response:

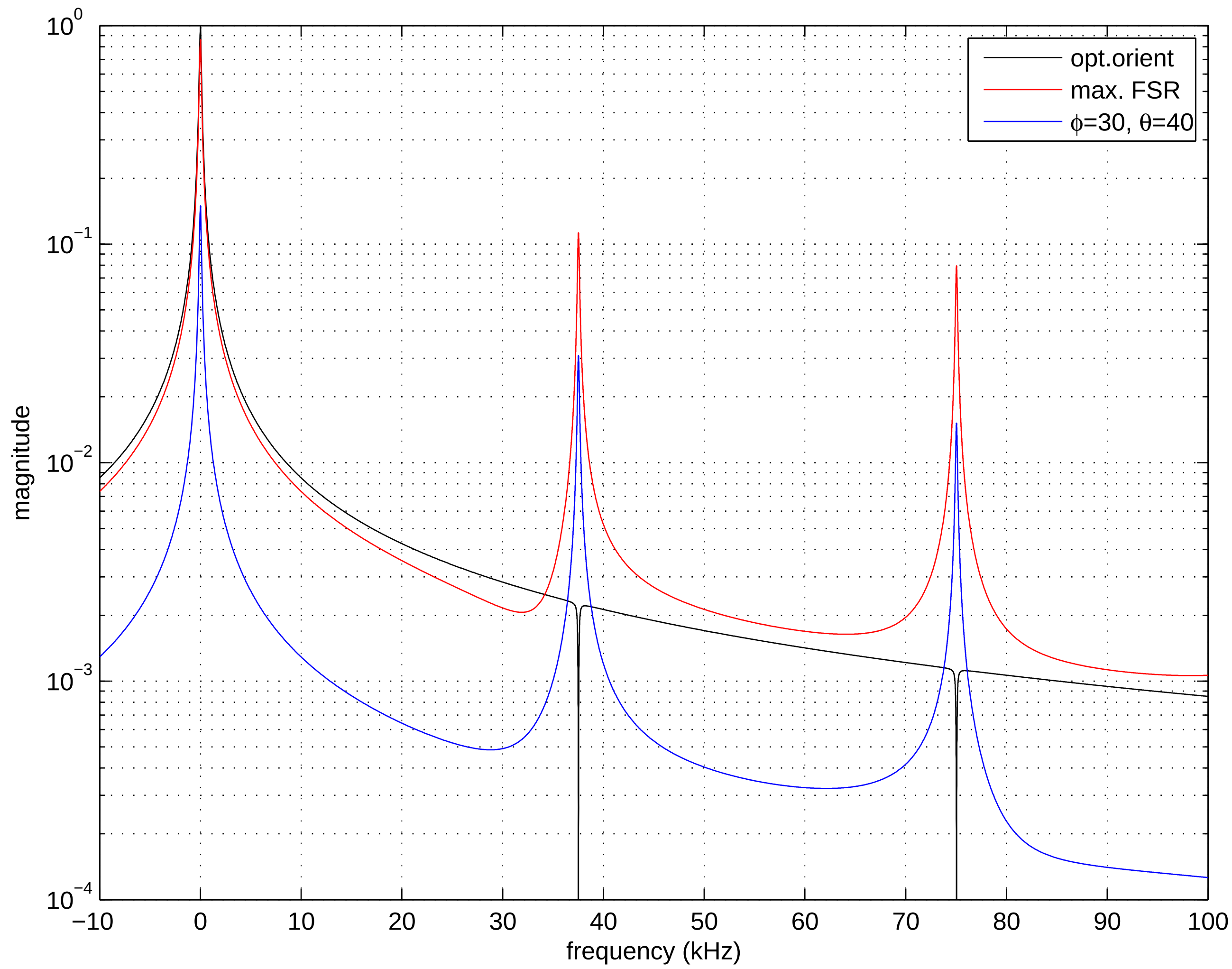
$$\phi_i = \frac{A_i - B_i e^{-2sT}}{2sT}, \qquad H_{cav}(s) = \frac{1 - r_a r_b}{1 - r_a r_b e^{-2sT}},$$

and two polarization components in detector frame: $E_{xx} = E_{det}(1,1), E_{yy} = E_{det}(2,2)$.

Then response to gravitational waves is

$$H_{gw}(s) = \frac{1}{2}\, H_{cav}(s)\, (E_{xx}\phi_x - E_{yy}\phi_y).$$

Magnitude of $H_{gw}(s)$



Long-wavelength approximation

Detector response = convolution:

$$x(t) = \int_0^T \left[H_+(t-t',\Omega)\; h_+(t') + H_\times(t-t',\Omega)\; h_\times(t') \right] dt'.$$

In Fourier domain

$$\tilde{x}(f) = H_+(f,\Omega)\; \tilde{h}_+(f) + H_\times(f,\Omega)\; \tilde{h}_\times(f).$$

The characteristic time scale $T = L/c$ (photon transit time) and the characteristic frequency scale: $\mathrm{FSR} = 1/(2T)$ (free spectral range) .

At low frequencies ($f \ll \mathrm{FSR}$) the response functions factorize:

$$H_i(f,\Omega) \approx F_i(\Omega) * C(f).$$

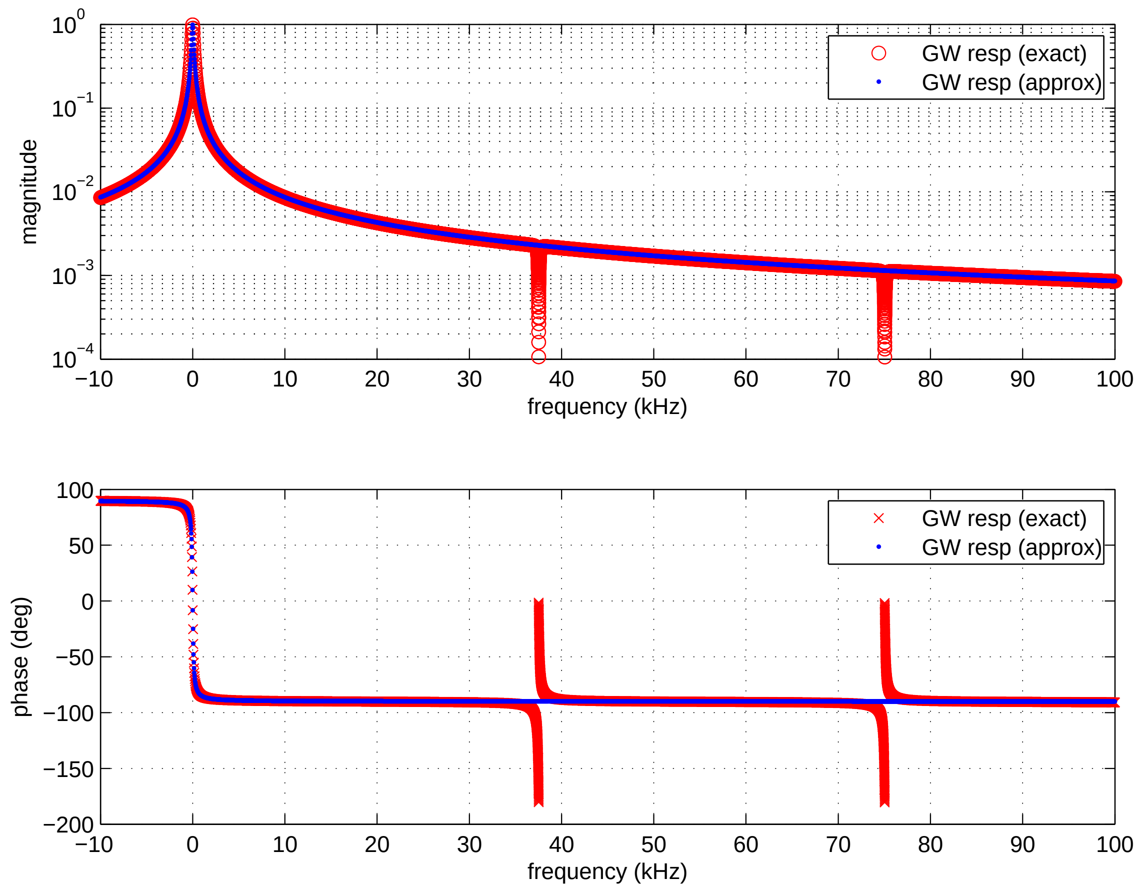
where $F_{+,\times}(\Omega)$ are static antenna-patterns, $\Omega = (\phi, \theta, \psi)$.

$C(f)$ is an approximate frequency response for optimal orientation:

$$C(f) = \frac{1}{1 + if/f_{cav}}, \qquad f_{cav} \approx 86\ \text{Hz}.$$

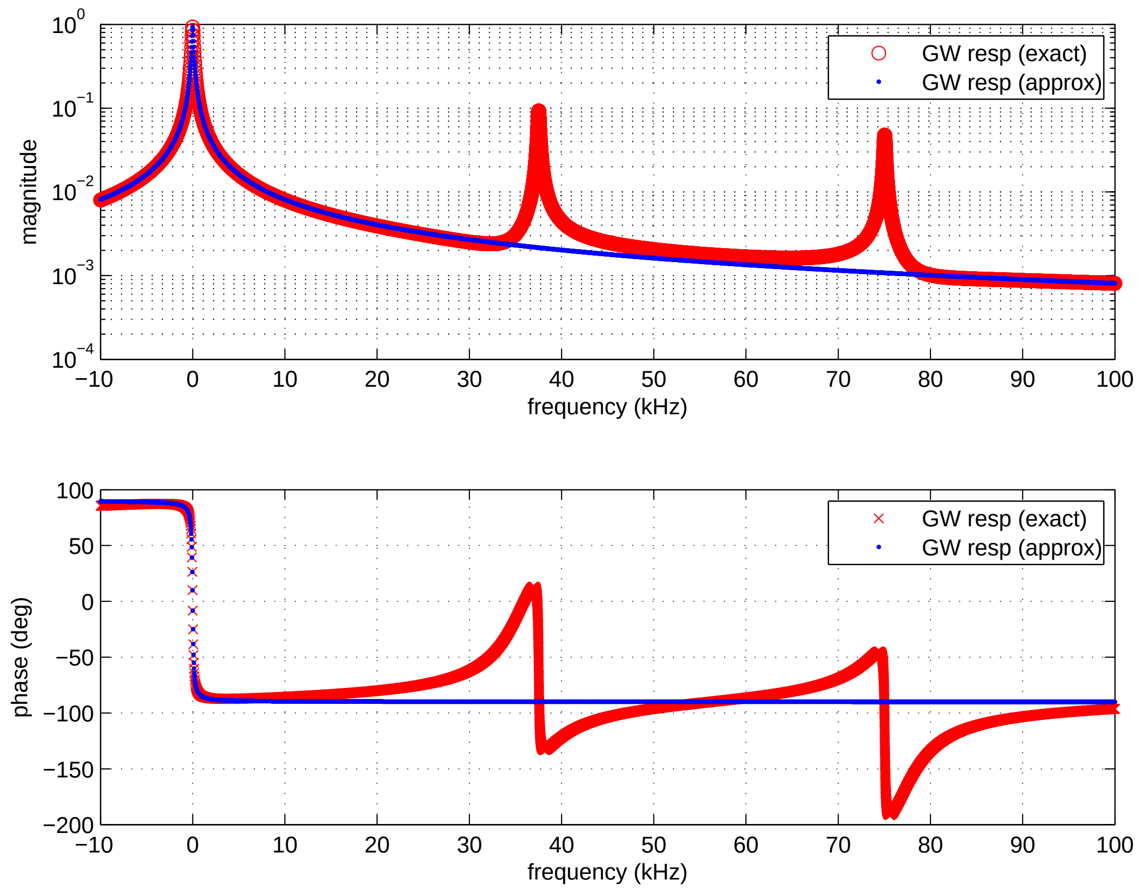
$H_{gw}(s)$: exact and approximate forms (1)

Source coordinates: $\phi = 0$, $\theta = 0$, $\psi = 0$.



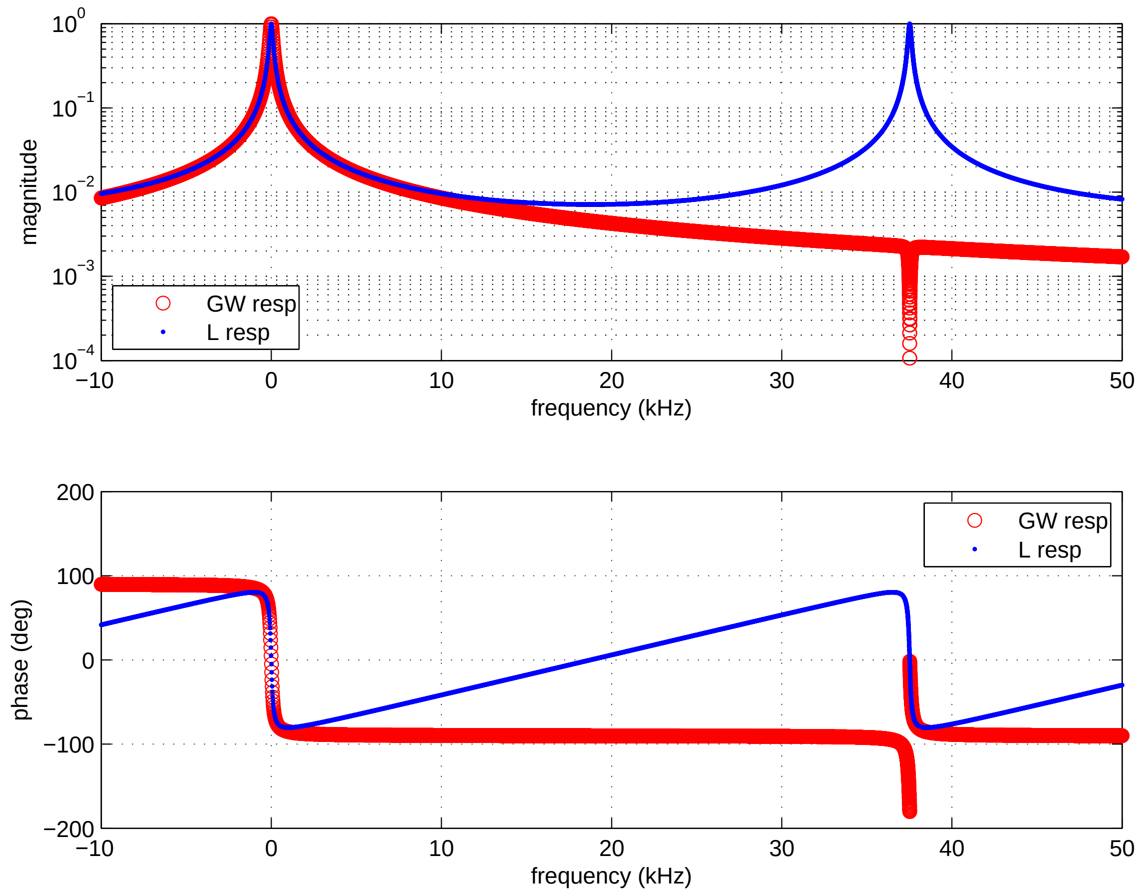
$H_{gw}(s)$: exact and approximate forms (2)

Source coordinates: $\phi = 0$, $\theta = 20°$, $\psi = 0$.



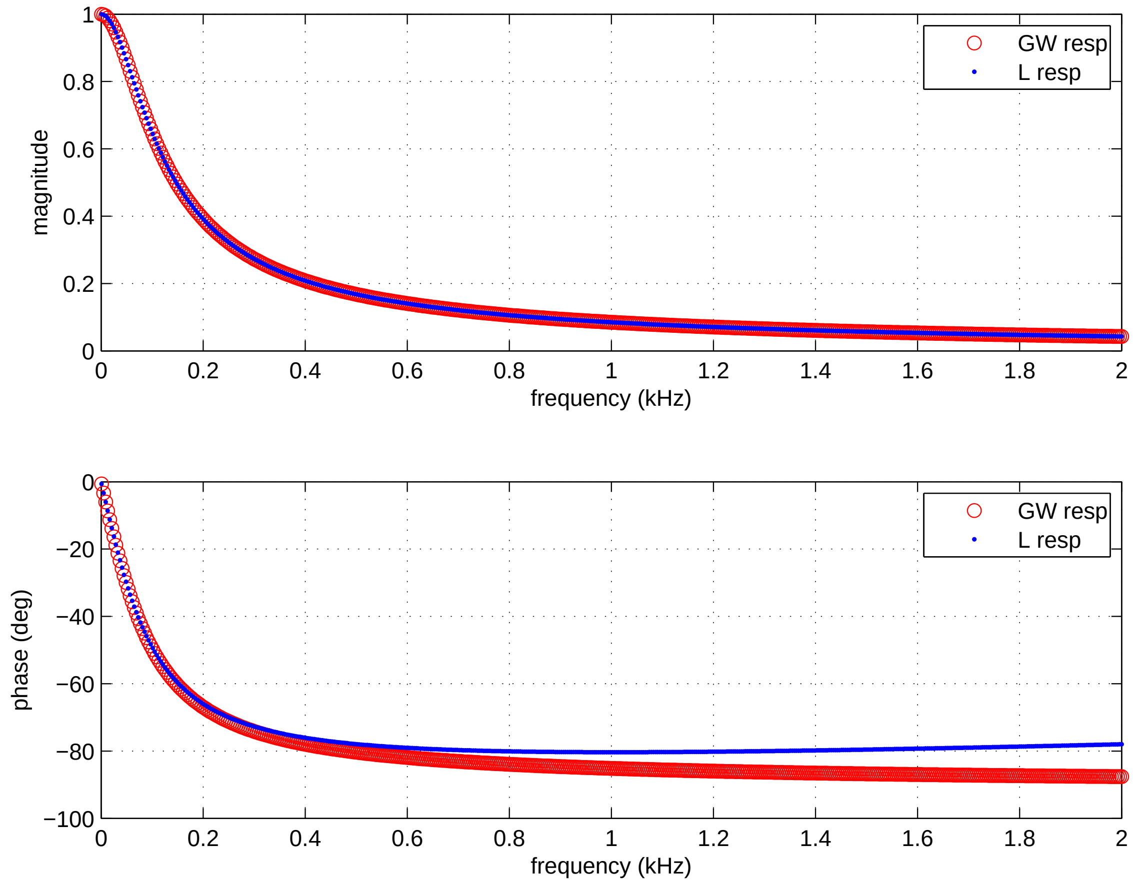
Comparison of $H_{gw}(s)$ and $H_L(s)$ (1)

Calibration: provides $H_L(f)$ not $H_{gw}(f)$. The sensing function in the inverse calibration, $C(f)$, is the response to length. This is transferred to the $h(t)$-channel.



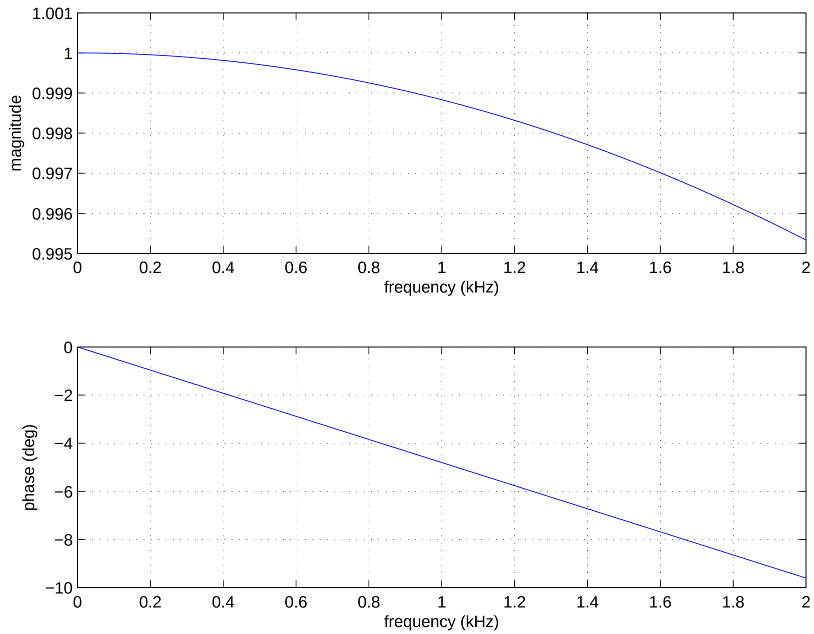
Comparison of $H_{gw}(s)$ and $H_L(s)$ (2)

At low frequencies the magnitude of the length response and that of the gravitational-wave response are almost the same. The phase is slightly different though.



Difference between $H_{gw}(s)$ and $H_L(s)$

Plot the ratio: $H_{gw}(s)/H_L(s)$: error in the magnitude $< 0.5\%$, error in the phase < 10 degrees.



Conclusions:

- The frequency dependence of the antenna patterns does not introduce a significant error.
- The small difference in the phase (for $f \leq 2000$ Hz) due to the approximation of the GW-response with the length response needs to be taken into account.
- Static antenna patterns combined with the single-pole well approximate the true response.