Exogenous Gaussian Model of LIGO Noise Floor A Progress Report

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Goal, Motivation, Rationale

Goal : validating an exogenous-gaussian (EG) model for LIGO noise floor;

Motivation: once pre-conditioned (power lines, violin modes etc. removed), IFO noise is a kind of "breathing Gaussian"; preliminary work on TAMA DT-8 noise-floor consistent with EG model [R. Conte et al., 2005];

Rationale: possibly *simplest* (though fairly *general*) model capturing noise floor *non - gaussian non - stationary* features, while still being fairly manageable, as concerns :

i) detector optimization;
ii) noise simulation;
iii) identification of physical sources of specific noise-floor features.

Related LIGO-LSC Work on Noise-Floor

ARMA modeling of IFO noise-floor

[Mukherjee, CQG 21 (2004) S1783
Mukherjee LIGO G040361-00-Z (2004)];

Median based noise-floor tracker

[Mukherjee, CQG 20 (2003) S925];

Change-Point detection

[Mohanty, PRD 61 (2000) 122002
Mohanty & Mukherjee, CQG 19 (2002) 1471
McNabb et al., CQG 21 (2004) S1705
Mohanty & Jimenez, CQG 22 (2005) S1233].

Exogenous Gaussian Noise in a Nutshell

Compound gaussian (exogenous) RP

$$x(t) = s(t)\,g(t)$$

long-coherency RP ($s(t)$)

N(0,1) gaussian RP ($g(t)$)

- *"Long"* $x(t)$ time-series markedly *non - gaussian;*
- *"Short"* time-series *locally gaussian*, but with *different variance* in different time stretches (*nonstationary*)

To some extent, may trade gaussianity for stationarity…

[I.M. Pinto, LIGO-G-060473-00 for an introductory review]

LIGO Data & Pre-Processing

Results discussed below derived from:

H-H1_RDS_C02_LX-833000031-128.gwf
H-H1_RDS_C02_LX-833000159-128.gwf
H-H1_RDS_C02_LX-833000287-128.gwf
H-H1_RDS_C02_LX-833000415-128.gwf
H-H1_RDS_C02_LX-833000543-128.gwf
H-H1_RDS_C02_LX-833000671-128.gwf
H-H1_RDS_C02_LX-833000799-128.gwf
H-H1_RDS_C02_LX-833000927-128.gwf

Pre-Processing:

Passband filtering - (FIR, L=3518) chop outside band of interest (40-400 Hz);

Spectral equalization -divide spectral data by sqrt of smoothed PSD;

Narrowband features detection - Use Kay's $\chi 2$ test for tones ($\chi 2$ distribution assumed in each spectral bin) [S.Kay, *Modern Spectral Estimation*, Prentice-Hall, 1984];

Narrowband feature removal - Estimate amplitude/frequency/phase of detect-ed narrowband features using a sliding window. Subtract estimated tones in time domain from next window;

Subsampling – Whiten (decorrelate) process by subsampling

Local Gaussianity Tests, I

EDF based (generic)

Kolmogorov-Smirnov
Anderson-Darling

(*composite* hypothesis test, distribution parameters estimated *from data*)
[Dagostino & Stephens, *Goodness of Fit Techniques*, Dekker, 1986, ch.4]

Skewness/Kurtosis based (“omnibus”)

Jarque-Bera [Intl. Stat. Rev., 55 (1987) 163.]
Urzua [Economics Lett., 53 (1996) 247]

Rank based

Shapiro & Co-workers

[Biometrika, 52 (1965) 591; J. Am. Stat. Soc., 67 (1972) 215];

Royston [The Statistician, 42 (1993) 37]

Local Gaussianity Tests, II

Characteristic Function based

Koutrouvelis & Kellermeier [J. Roy. Stat. Soc. B43 (1981) 173]

Epps [Ann. Stat., 15 (1987) 1683]

HOS based

Giannakis & Tsatsanis [IEEE T-SP 32 (1994) 3460]

Hinich [J. Time Series Anal., 3 (1982) 169]

Nonlinear (memoryless) filtering based

Bussgang [Res. Lab. Elec. MIT, n. 216 (1952)]

Scarano et al., [IEEE-SP HOS-WS (1887) p.434]

LIGO-LSC

TF-Rayleigh Monitor [Finn et al. LIGO G020133-00-Z]

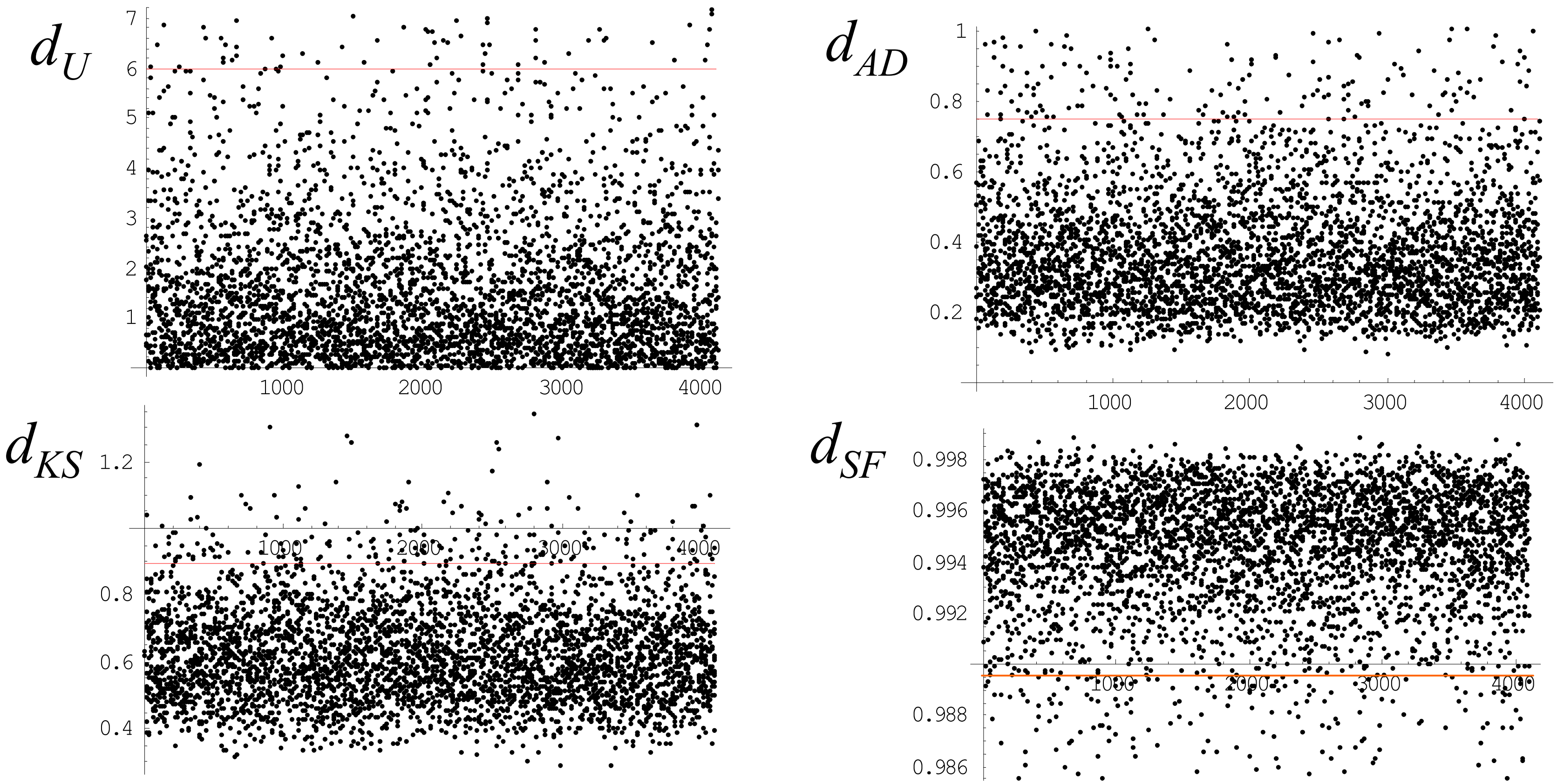
GaussMon (Hinich test) [Penn]

Local Gaussianity Tests Compared

Chunk fraction for which Gaussianity is rejected at 5% error level				
Chunk size	Kolmogorov.Smirnov (composite hyp.)	Anderson-Darling (composite hyp.)	Urzua (mod. Barque-Jera)	Shapiro-Francia
256	0.0510	0.0488	0.0427	0.0434
1024	0.0566	0.0517	0.0889	0.0839
4096	0.0781	0.0898	0.1367	0.1328
8192	0.0547	0.1092	0.2896	0.2657
32768	0.1875	0.2812	0.5937	0.6250
131072	0.625	0.875	1.000	1.000

At $N_C=2^{17}$, Urzua & Shapiro-Francia (*gaussianity- specific tests*) reject gaussianity for *all* chunks; Kolmogorov-Smirnov and Anderson-Darling (*generic tests*) reject 62.5% and 87.5% of chunks

Local Gaussianity Tests at α=0.05, $N_C = 2^8$



Fraction of chunks for which Gaussian assumption is rejected

KS=0.0510, AD=0.049, UBJ=0.0427, SF=0.0435

all consistent with test error level ⟶ data *locally gaussian*

“Short” yet Non-Gaussian Chunks

-A fraction of “short” chunks still identified as non-gaussian found to contain *true* non gaussian features, (e.g., abrupt (*non adiabatic*) jumps in exogenous factor;

-Jumps can be finely located using *change-point* detection algos (e.g., blocknormal, or akin);

-It makes sense to look for *correlations* between these abrupt-changes, and data from IFO aux-channels;

-Exogenous models apply on the left/right of the above non-adiabatic changes *only*.

SIRP Property of Exogenous-Gaussian Models

- Form triplets out of time series $\vec{\tau}_j = \{x_j, x_{j+1}, x_{j+2}\}$;
- View those triplets as cartesian coords. in R^3, and transform to spherical (polar) coords. $\vec{\tau}_j = \{R_j, \theta_j, \varphi_j\}$;
- Check that the $\{\vartheta_j \mid j = 1,2,\ldots\}, \{\varphi_j \mid j = 1,2,\ldots\}$ are distributed as follows

$$\left.\begin{aligned} f_\vartheta(\vartheta) &= \sin\vartheta \\ f_\varphi(\varphi) &= (2\pi)^{-1} \end{aligned}\right\} \Longleftrightarrow$$

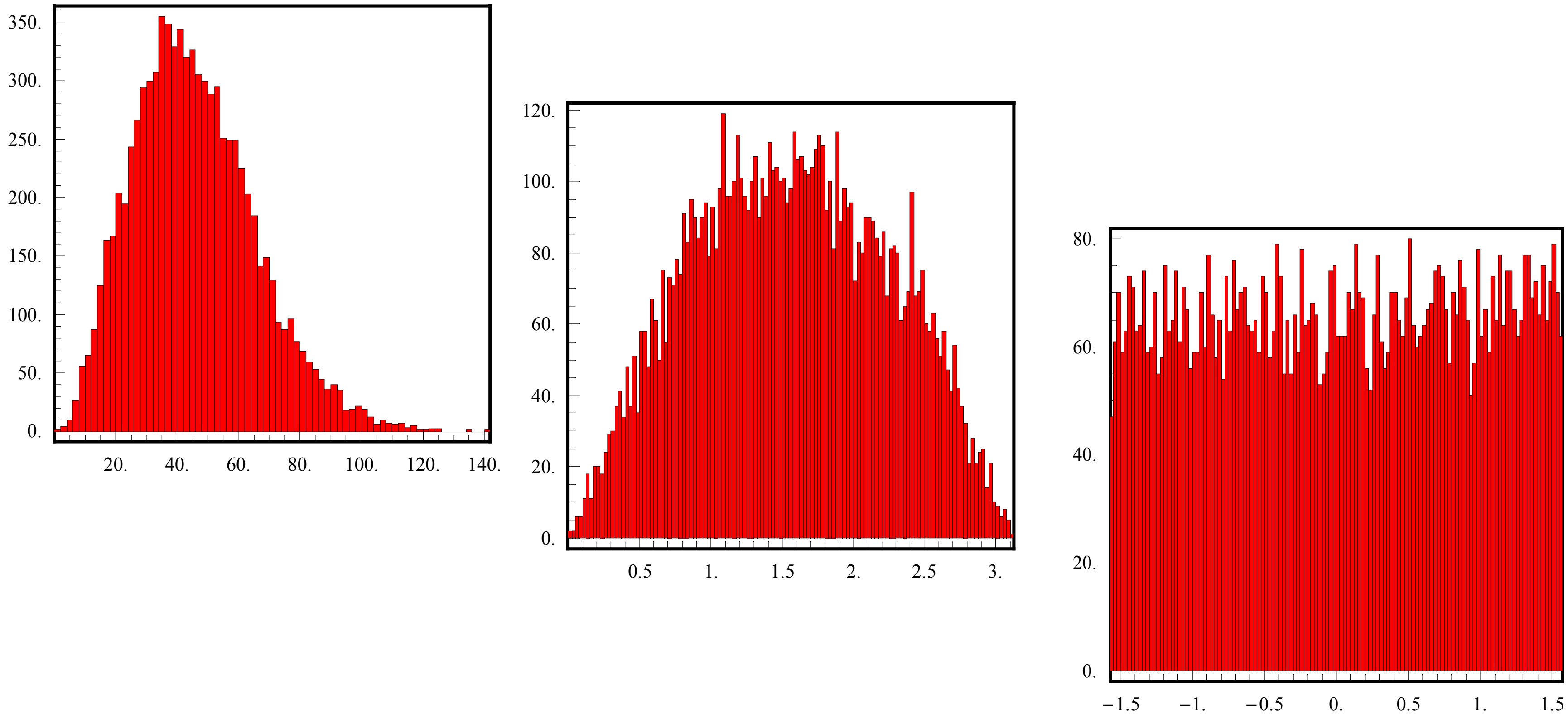
The tips of the random (unit) vectors $\vec{\tau}_j / |\vec{\tau}_j|$ fill the unit sphere *uniformly.*

These guys do not depend on breathing factor *s* …

➡ **The uniform-sphere-filling property holds also for plain gaussian noise …**

…but *the radial distribution* is peculiar **(3 d.o.f. χ only for Gaussian !)**

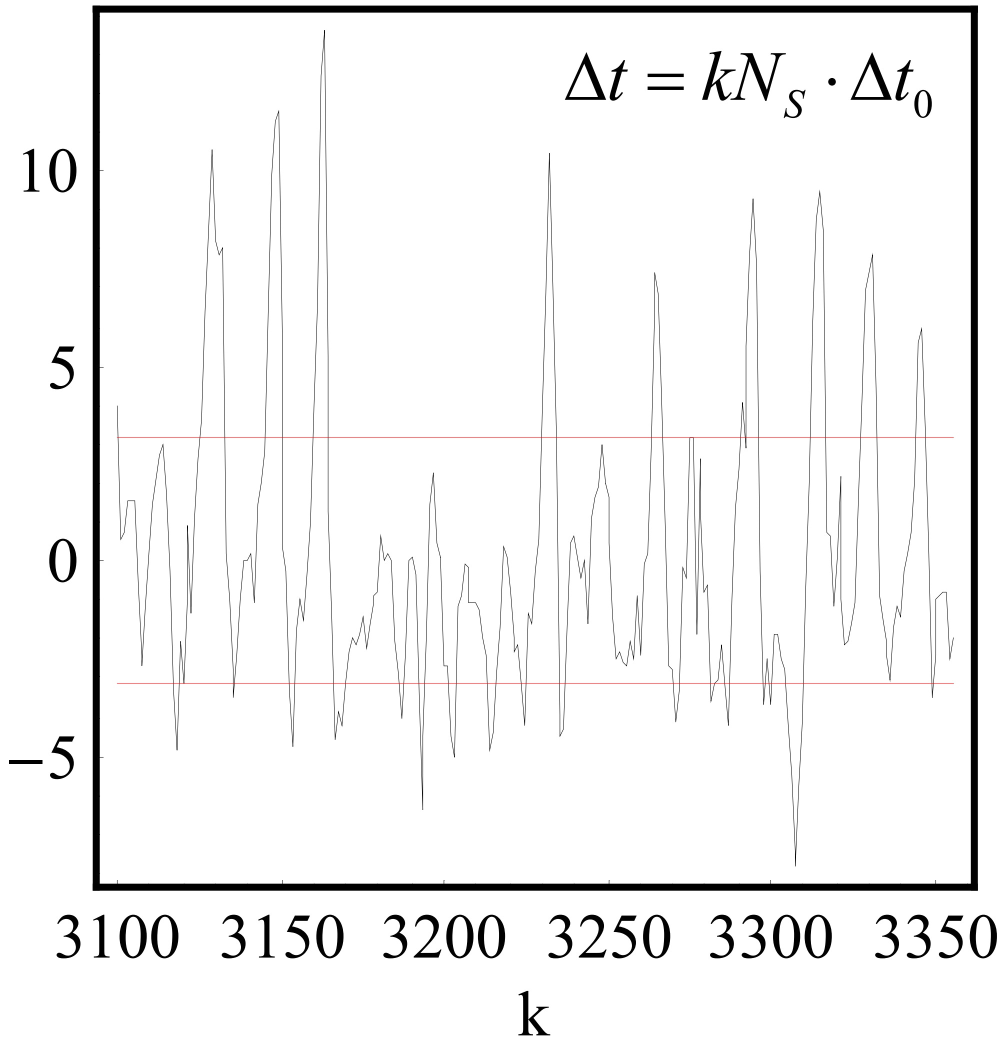
3D SIRP Behaviour of H1 Noise Floor

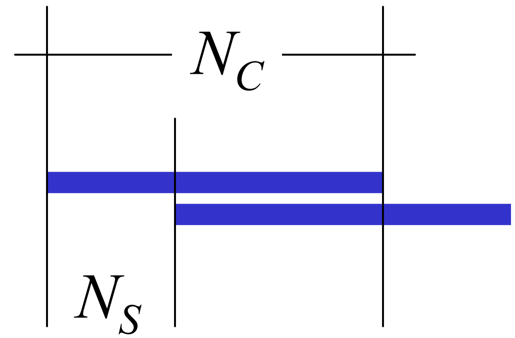


Triplets of LIGO (H1) noise-floor samples as points in R^3
distributions (histograms) of ρ, θ, φ.

LIGO-G060627-00-Z

The Exogenous Factor (Robust Estimator)

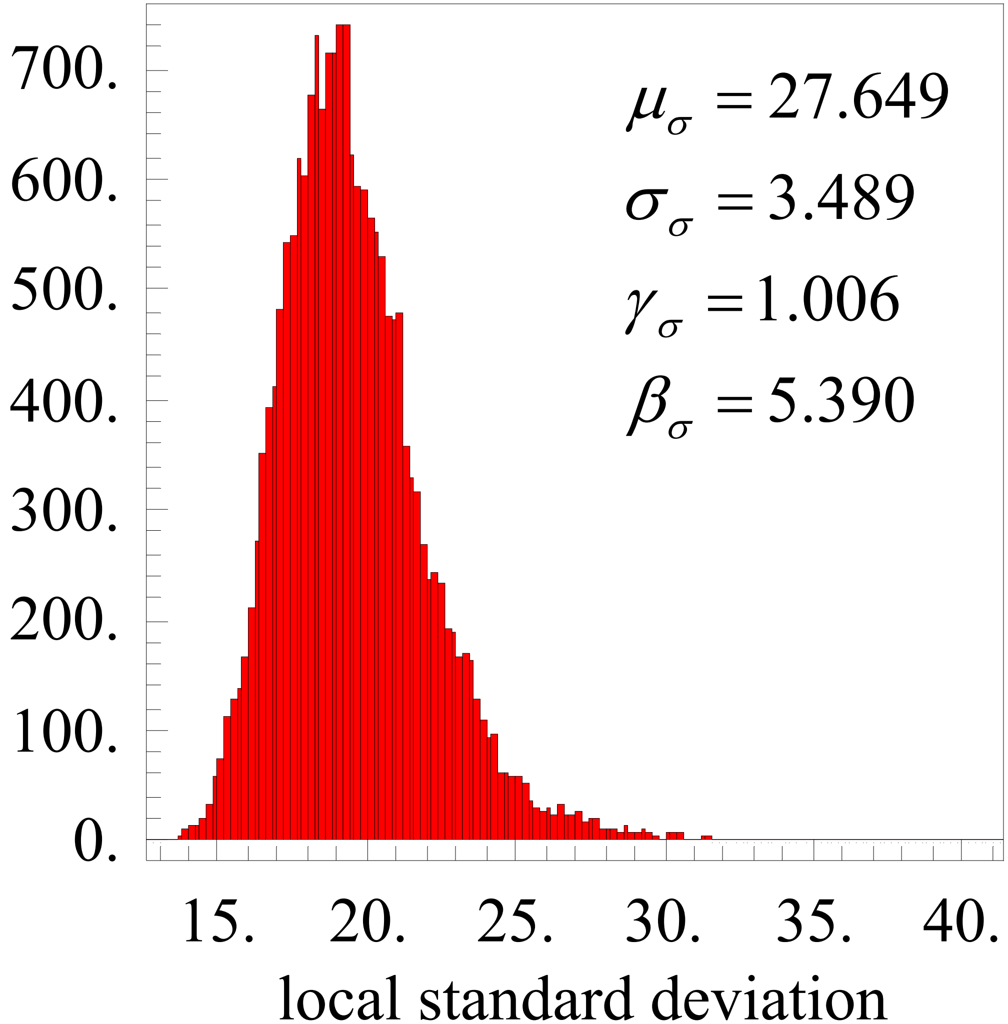




$N_C = 256$

$N_S = 64$

The Exogenous Factor: Distribution

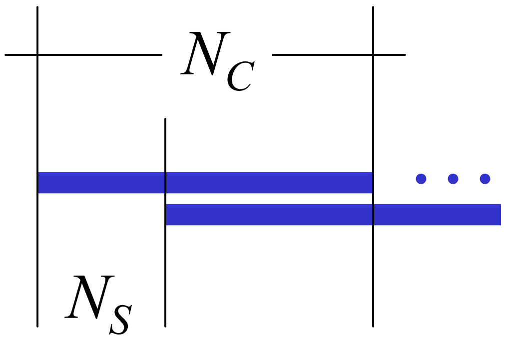


$$\mu_\sigma = 27.649$$

$$\sigma_\sigma = 3.489$$

$$\gamma_\sigma = 1.006$$

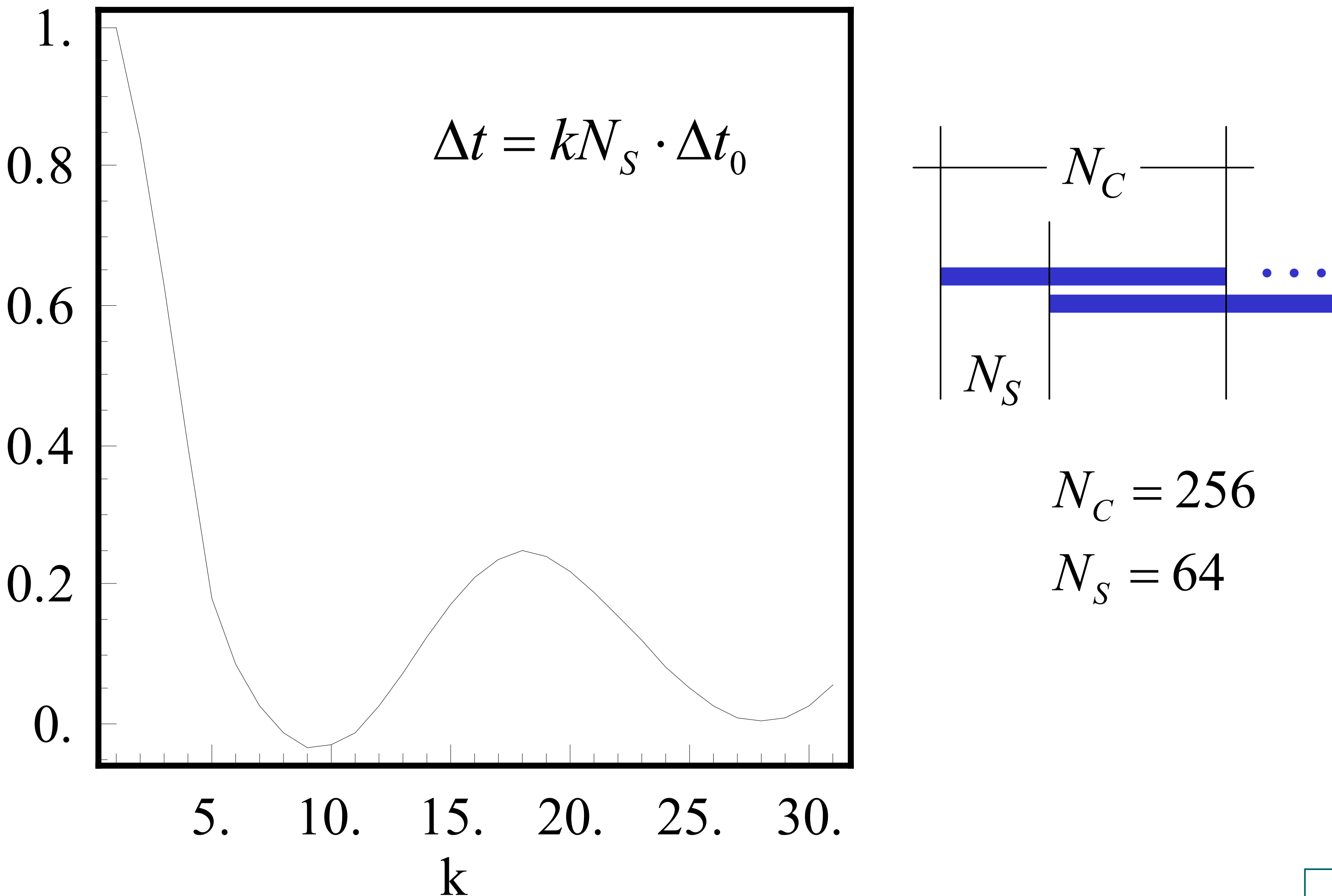
$$\beta_\sigma = 5.390$$



$$N_C = 256$$

$$N_S = 64$$

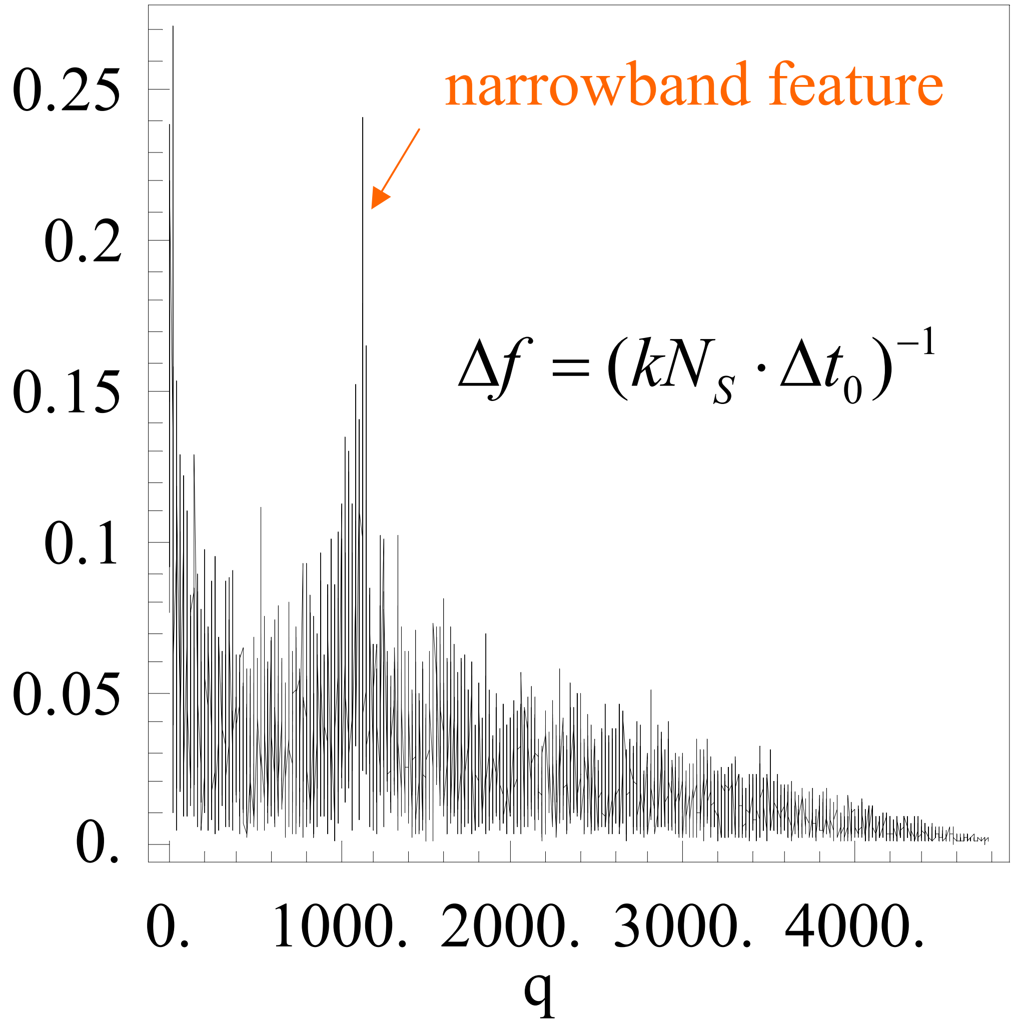
The Exogenous Factor: Correlation



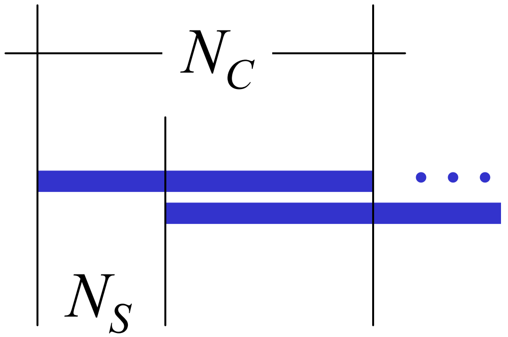
$N_C = 256$

$N_S = 64$

The Exogenous Factor: Spectrum



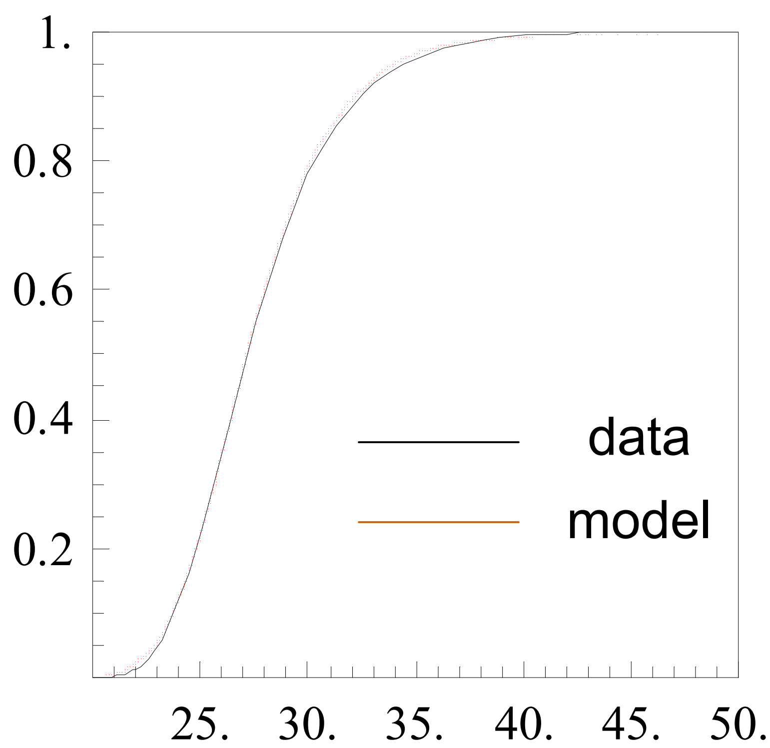
$$\Delta f = (kN_S \cdot \Delta t_0)^{-1}$$



$$N_C = 256$$

$$N_S = 64$$

The Exogenous Factor Distribution Model



Fisher-Tippett distribution model:

$$CDF(x) = Exp\left[-Exp\left(-\frac{x-\mu}{\beta}\right)\right]$$

$$\hat{\mu} = 26.116\,, \quad \hat{\beta} = 2.803 \quad [D_{KS} = 1.172]$$

(KS test passed at $\alpha \leq 0.05$)

Exogenous Factor CDF model + correlation & power spectrum provide sufficient information to build noise - floor generator (…and "optimum" detector…)

Work in Progress

-Implementation/testing (in terms of statistical departure from “real” LIGO noise) of exogenous gaussian noise-floor simulator

-Tools for investigating correlation between noise-floor features (change points, narrowband spectral features) and auxiliary IFO channels

-Implementation/testing of “generalized” matched filter appropriate to an exogenous/SIRP noise-floor [many results available in the Radar Literature !]