The Ins and Outs of Inspiral Searches

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A Perfect Gravitational Wave Source

To lowest order, gravitational wave emission is determined by second time derivative of mass quadrupole moment tensor

$$h_{ij}^{\mathrm{TT}} = \frac{2G}{c^4 R}\mathcal{P}_{ijab}(\mathrm{N})\left\{\frac{d^2\mathrm{Q}_{ab}}{dT^2}(T - R/c) + \mathcal{O}\left(\frac{1}{c}\right)\right\} + \mathcal{O}\left(\frac{1}{R^2}\right)$$

Projection depending on wave direction

See: Luc Blanchet, “Gravitational Radiation from Post-Newtonian Sources and Inspiralling Compact Binaries”, *Living Rev. Relativity* **5**, (2002), 3. http://www.livingreviews.org/lrr-2002-3

- A compact binary system is *all* quadrupole moment!
- Power emitted in gravitational waves:

$$\mathcal{L} = \frac{G}{5c^5}\left\{\frac{d^3\mathrm{Q}_{ab}}{dT^3}\frac{d^3\mathrm{Q}_{ab}}{dT^3} + \mathcal{O}\left(\frac{1}{c^2}\right)\right\}$$

“Newtonian” Inspiral Chirp

To lowest order, as gravitational waves carry away energy:

Frequency: $f(t) \propto (t - t_c)^{-3/8}$ ← Coalescence time (t_c)

Waveform: $h(t) = A(t) \cos\left(B\,(t - t_c)^{5/8} + \phi_c \right)$

$$= A'(f) \cos\left(\underbrace{B' f^{-5/3} + \phi_c}_{\Psi(f)} \right)$$

Chirp Waveform to Higher Order

"Post-Newtonian" corrections change phase evolution:

$$\Psi(f) = 2\pi f t_c + \frac{3}{128\eta}(\pi m f)^{-5/3}$$

$$+ \frac{5}{96\eta}\left(\frac{743}{336} + \frac{11}{4}\eta\right)(\pi m f)^{-1}$$ 1PN

$$- \frac{3\pi}{8\eta}(\pi m f)^{-2/3}$$ 1.5PN

$$+ \frac{15}{64\eta}\left(\frac{3058673}{1016064} + \frac{5429}{1008}\eta + \frac{617}{144}\eta^2\right)(\pi m f)^{-1/3}$$ 2PN

$$+ \cdots$$

where

$$m = (m_1 + m_2)\,, \quad \eta = \frac{m_1\, m_2}{m^2}$$

Source Parameters vs. Signal Parameters

Source parameters

Masses (m1, m2)

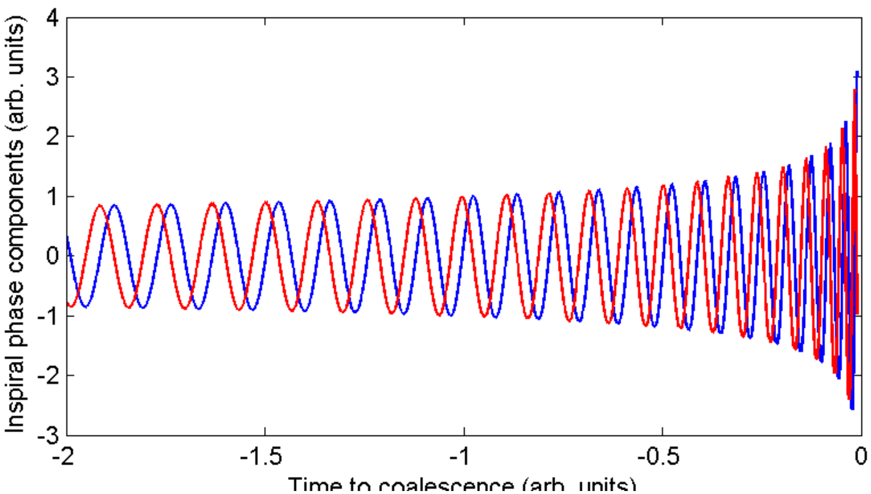
Spins → Assume negligible for now

Orbital phase at coalescence → Maximize analytically when filtering

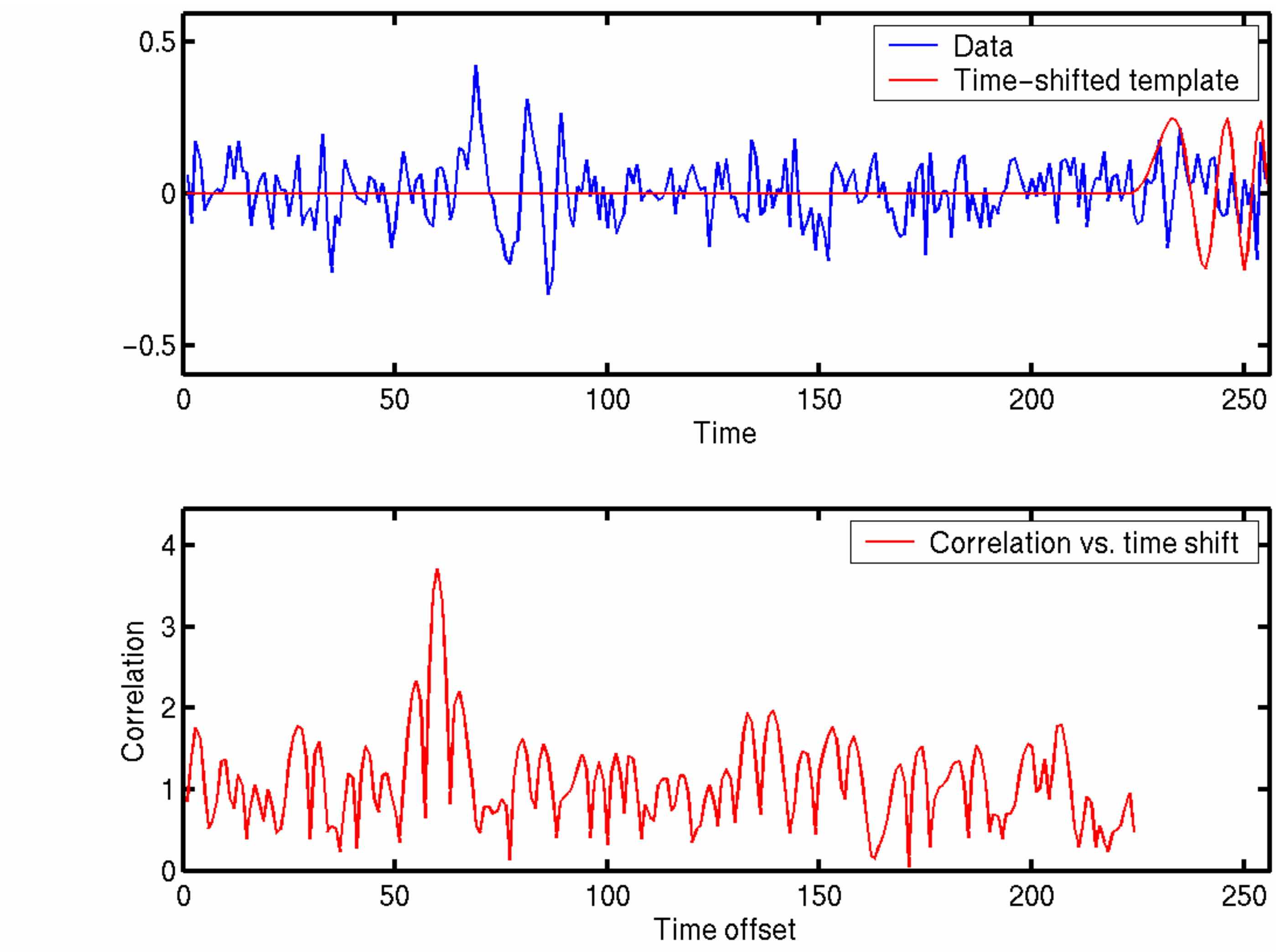
Inclination of orbital plane
Sky location → Simply multiplicative for a given detector

Distance → Simply multiplicative

Filter with orthogonal templates, take quadrature sum



Basic Matched Filtering



Optimal Matched Filtering in Frequency Domain

Data after FFT

Template, generated in freq. domain using stationary phase approx.

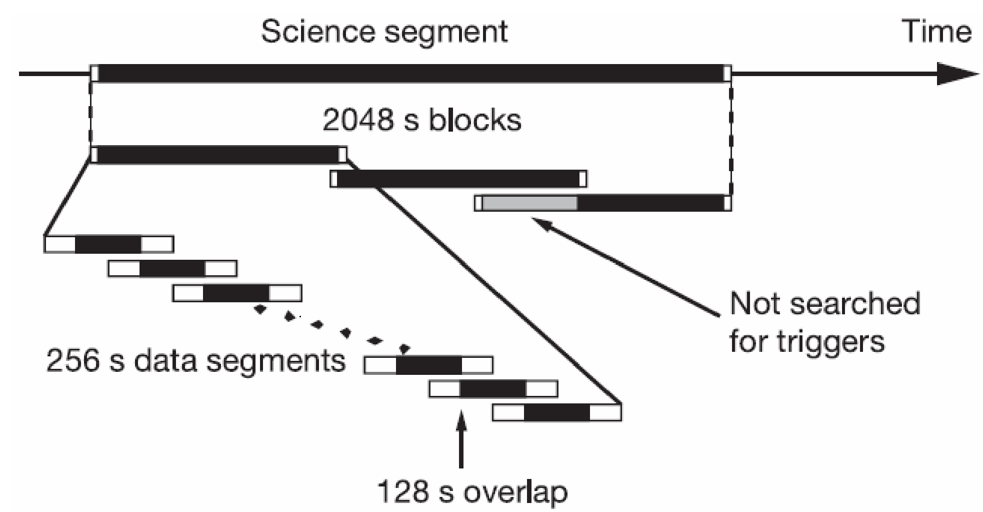
$$z(t) = 4 \int_0^\infty \frac{\tilde{s}(f)\ \tilde{h}^*(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

Noise power spectral density

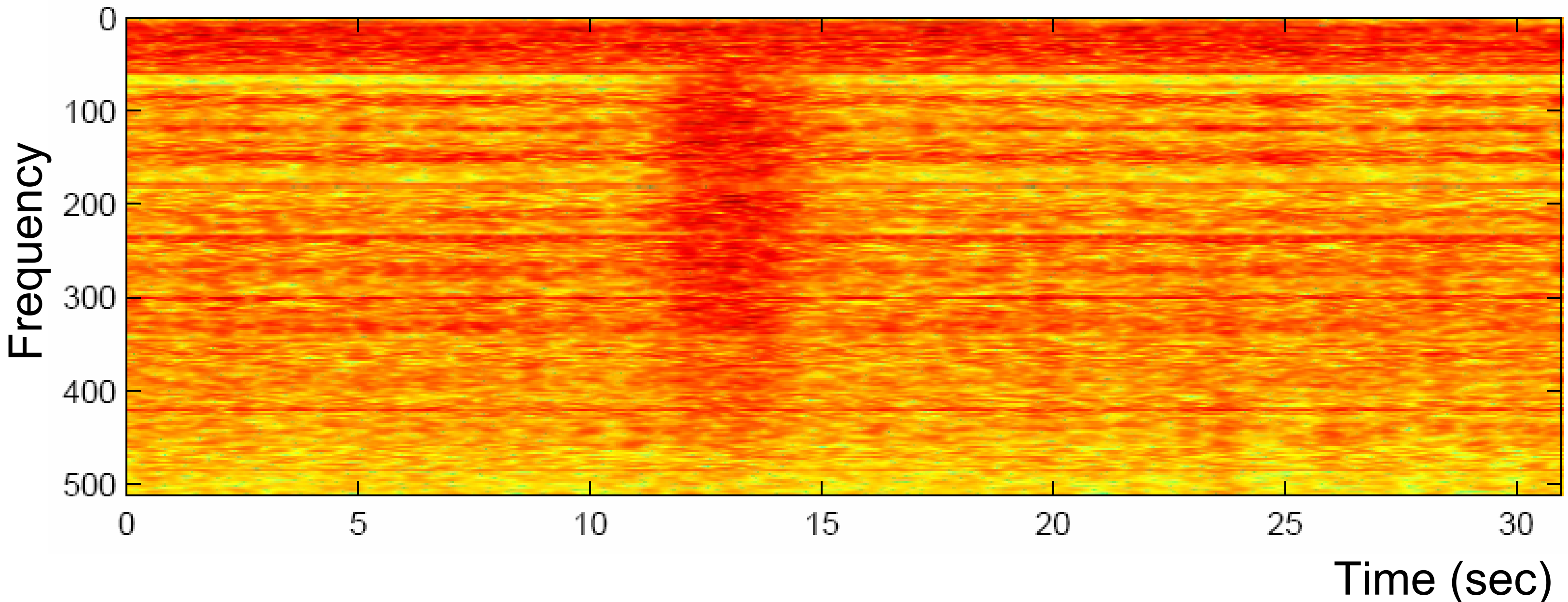
Look for maximum of $|z(t)|$ above some threshold ➔ trigger

Search overlapping intervals to cover science segment, avoid wrap-around effects

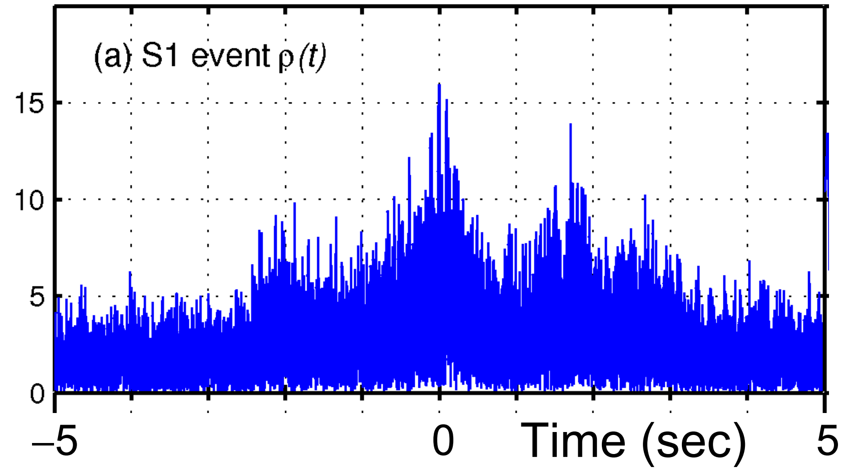
Estimate power spectrum from bin-by-bin median of fifteen 256-sec data segments



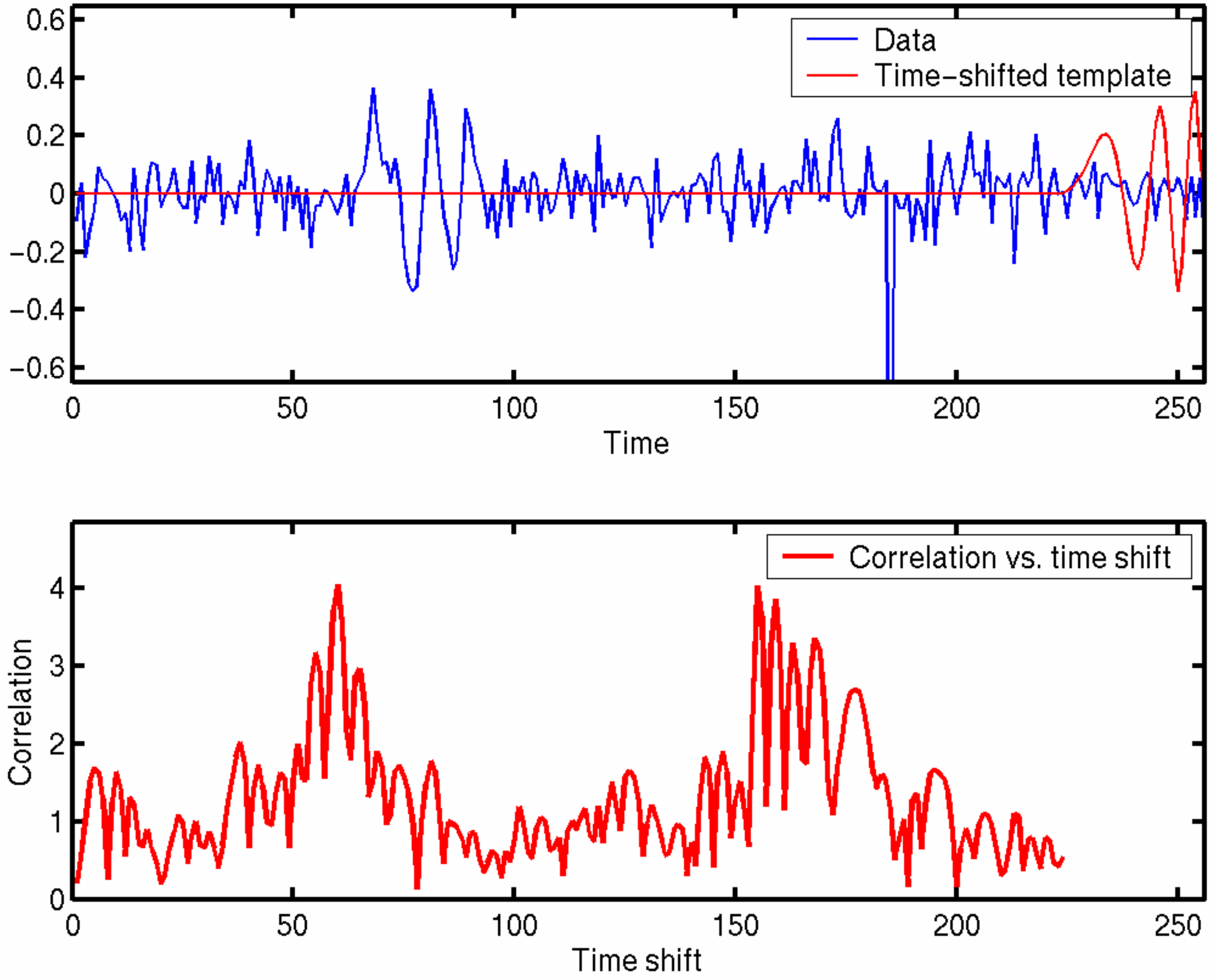
Dealing with Non-Stationary Noise



Inspiral filter output:



Matched Filtering Susceptibility to Glitches

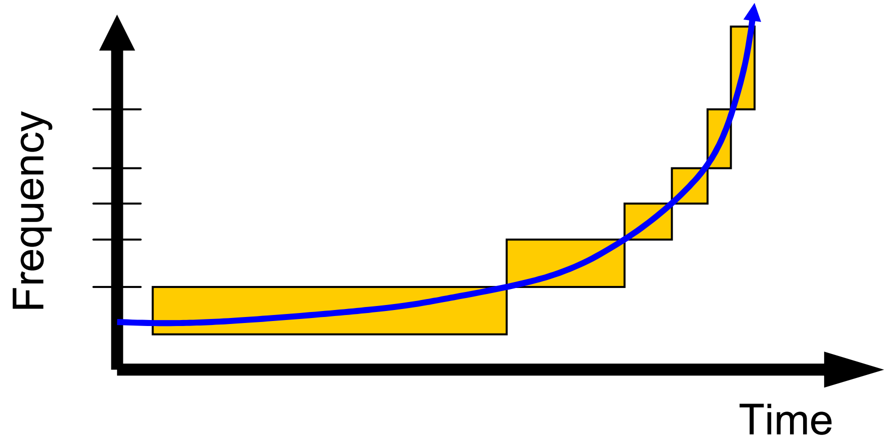


Waveform Consistency Tests

Chi-squared test

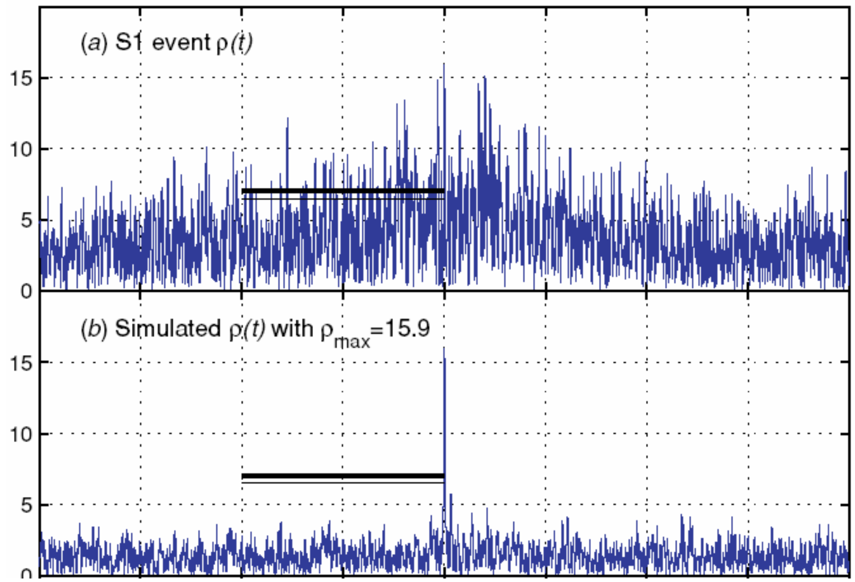
Divide template into p parts, calculate

$$\chi^2(t) = p \sum_{l=1}^{p} \| z_l(t) - z(t)/p \|^2$$

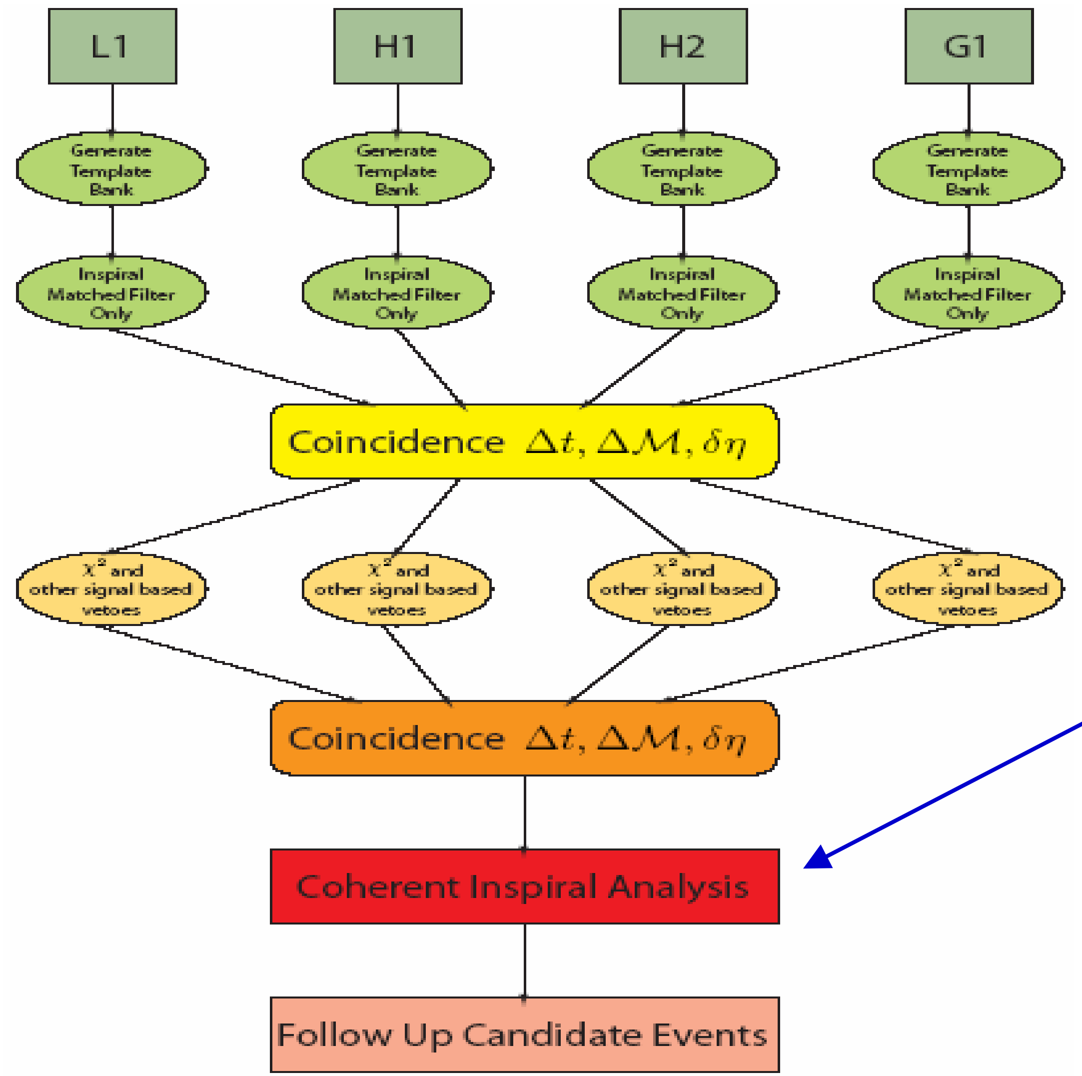


Tests using filter output

e.g. time above threshold

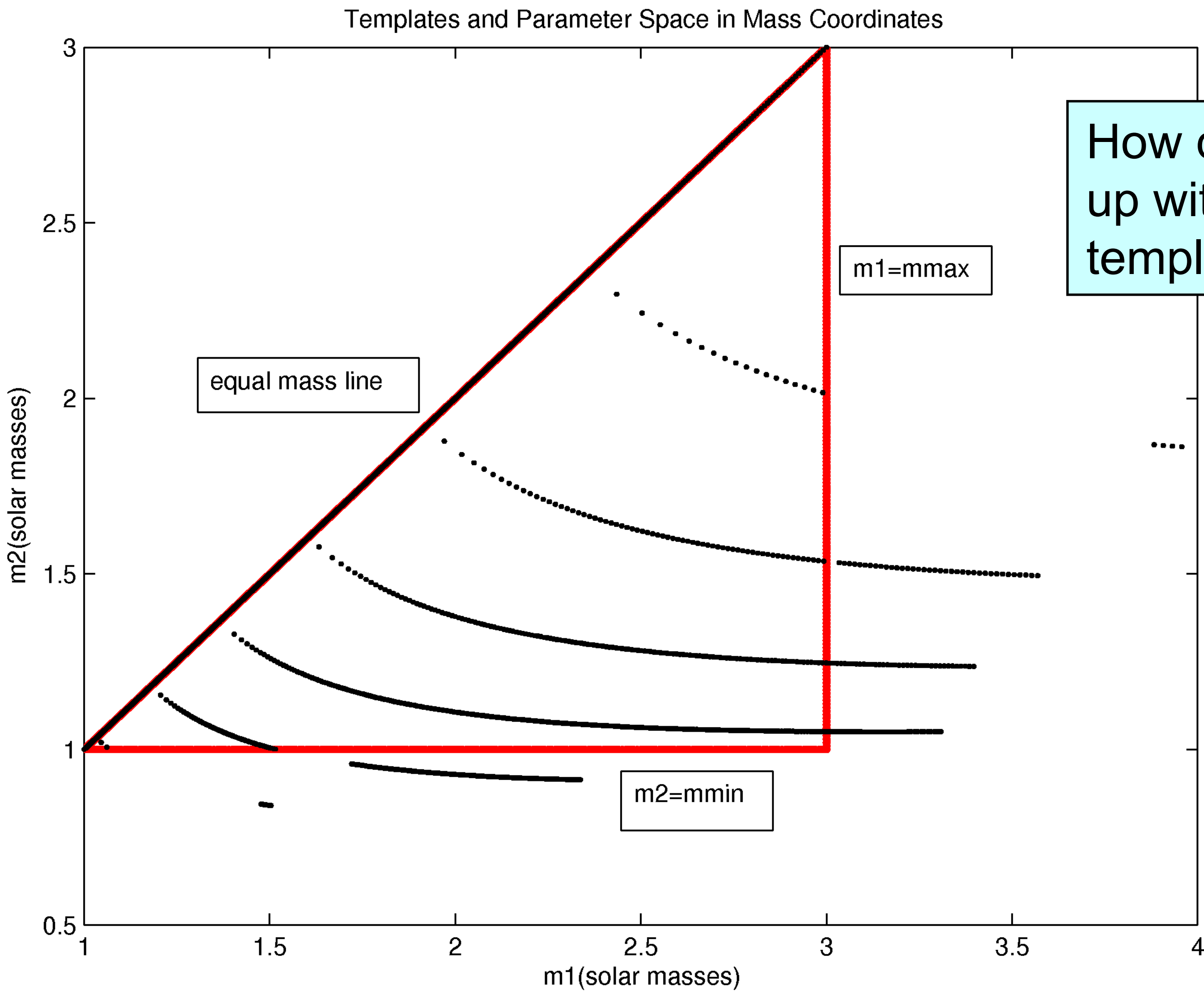


Analysis “Pipeline” for Computational Efficiency



Computationally expensive tasks

Template Bank Construction



How did we come up with this set of templates???

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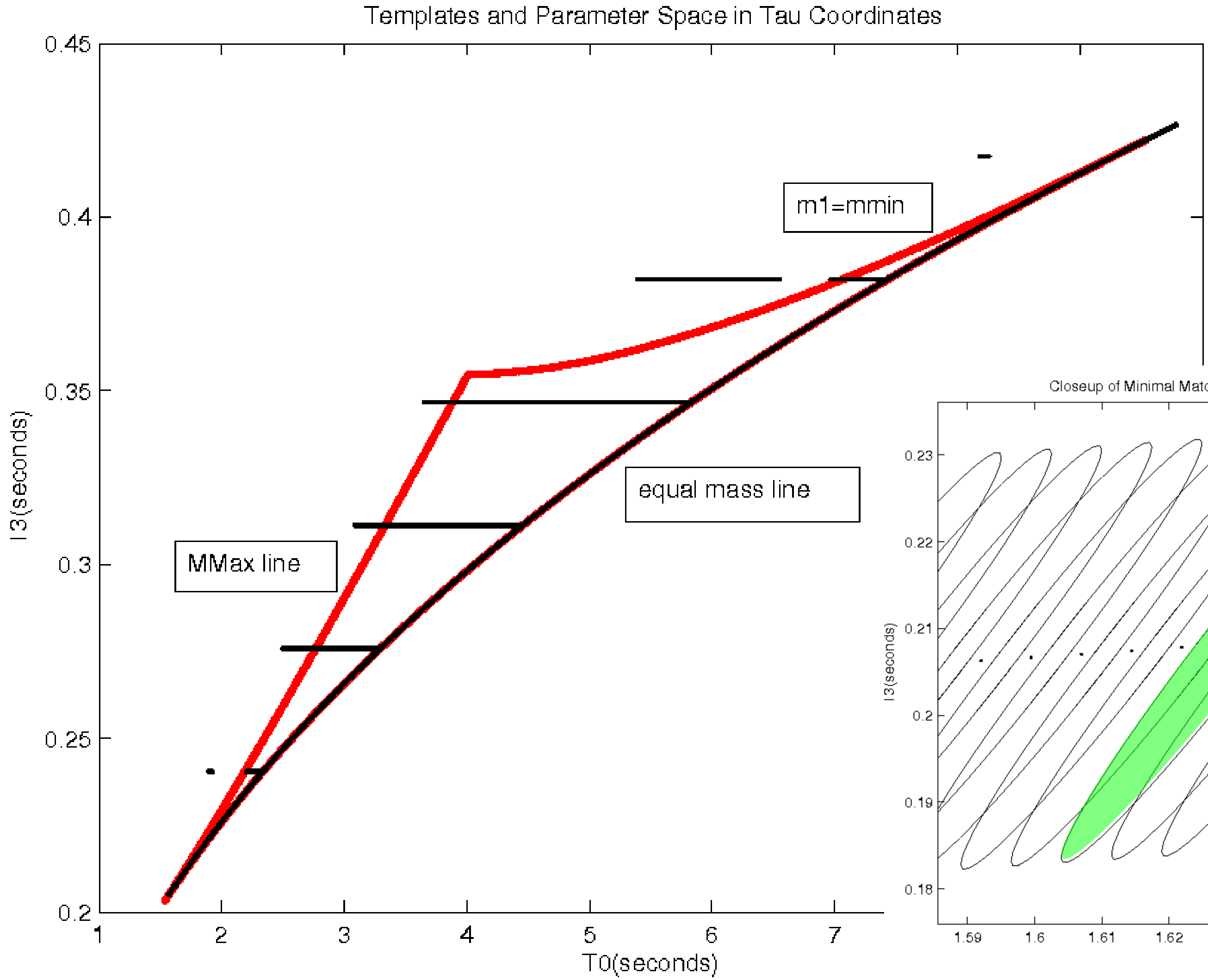
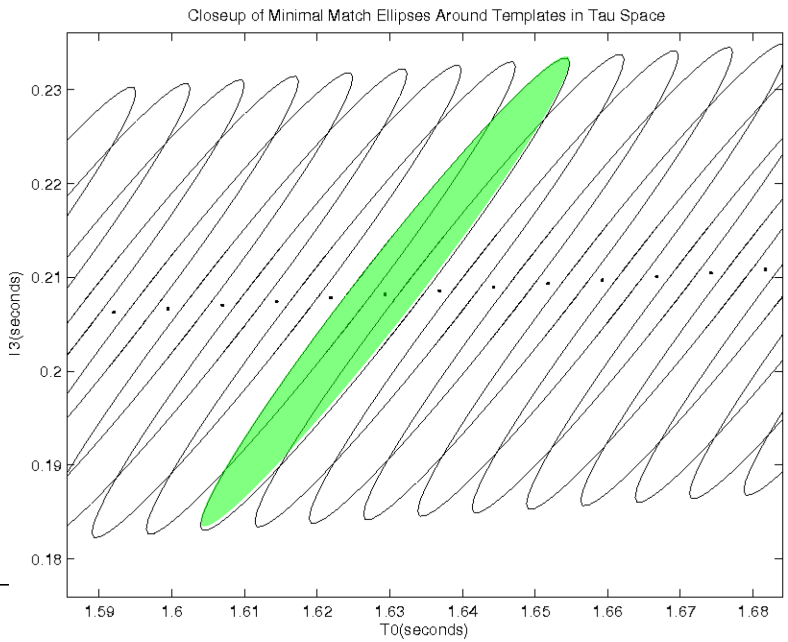
Template Bank Construction in (τ_0,τ_3) space

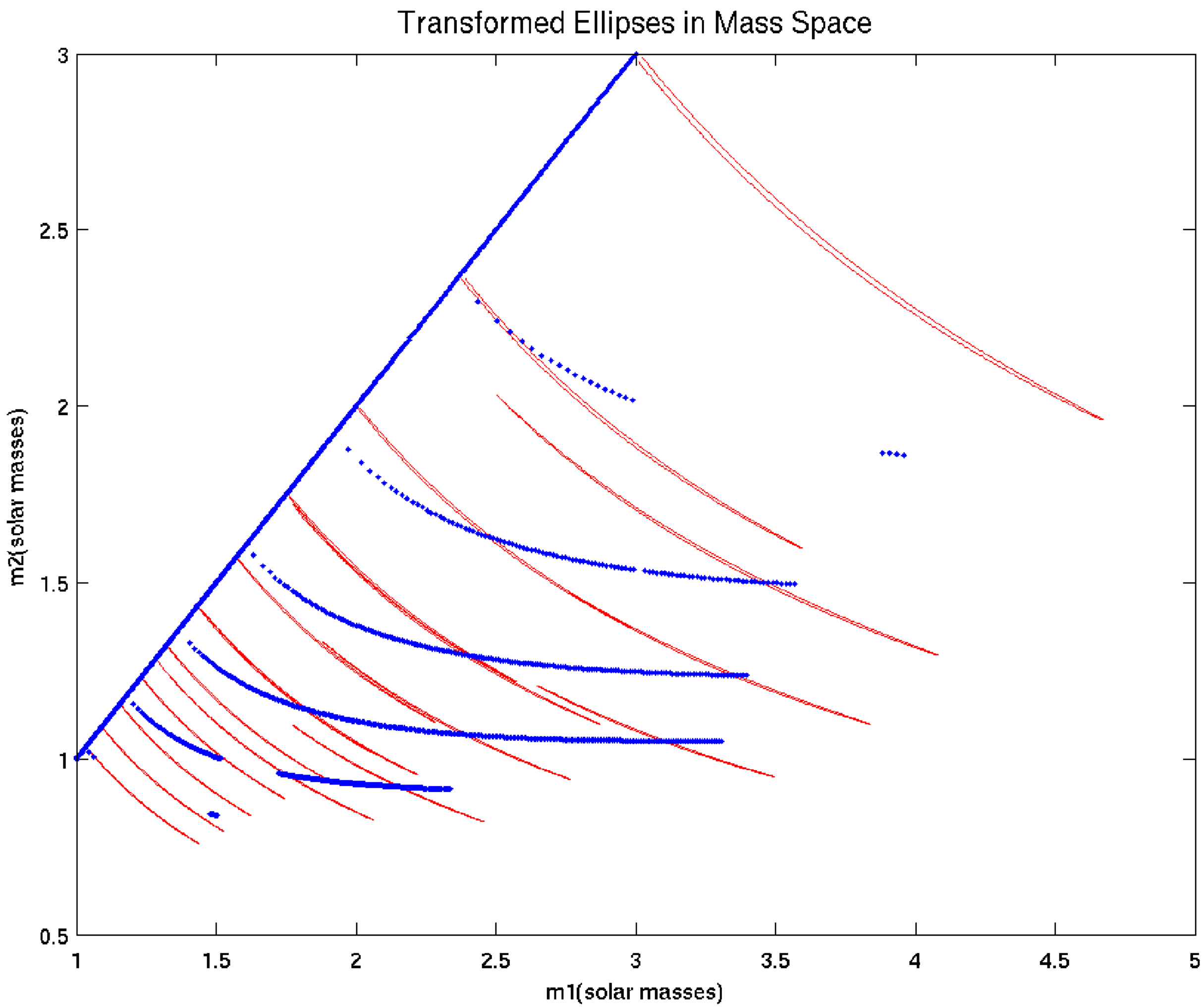




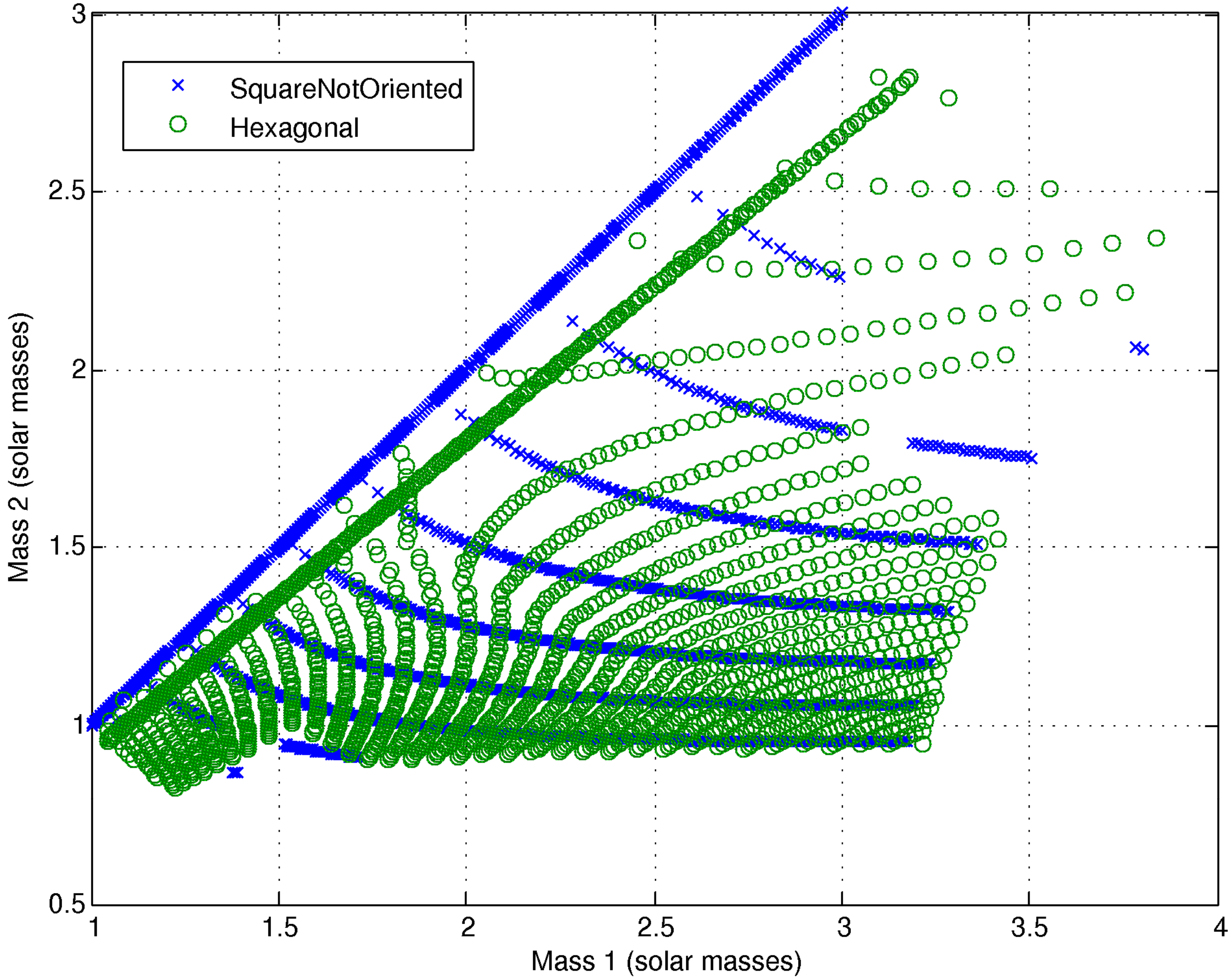
$$\tau_0 \propto m^{-5/3} / \eta$$

$$\tau_3 \propto m^{-2/3} / \eta$$

Ellipses in Mass Space

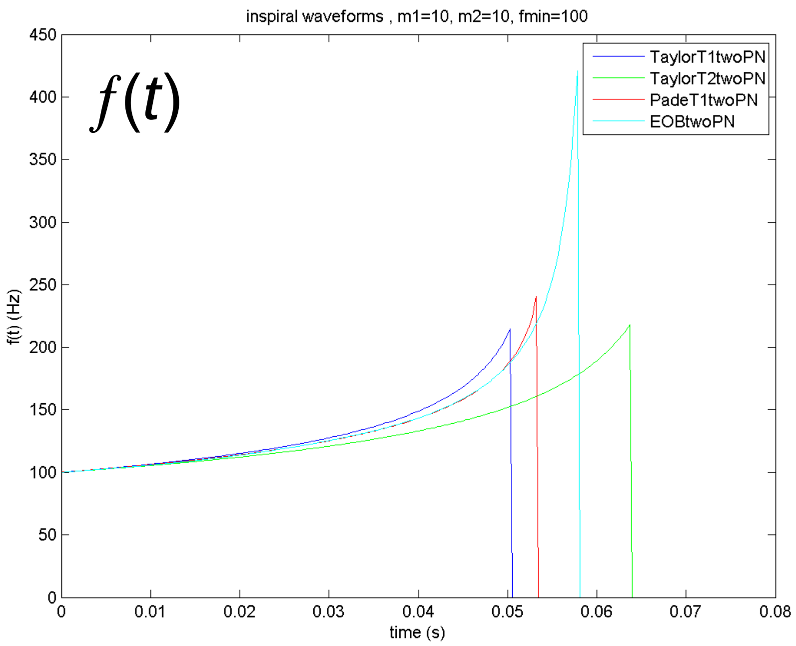


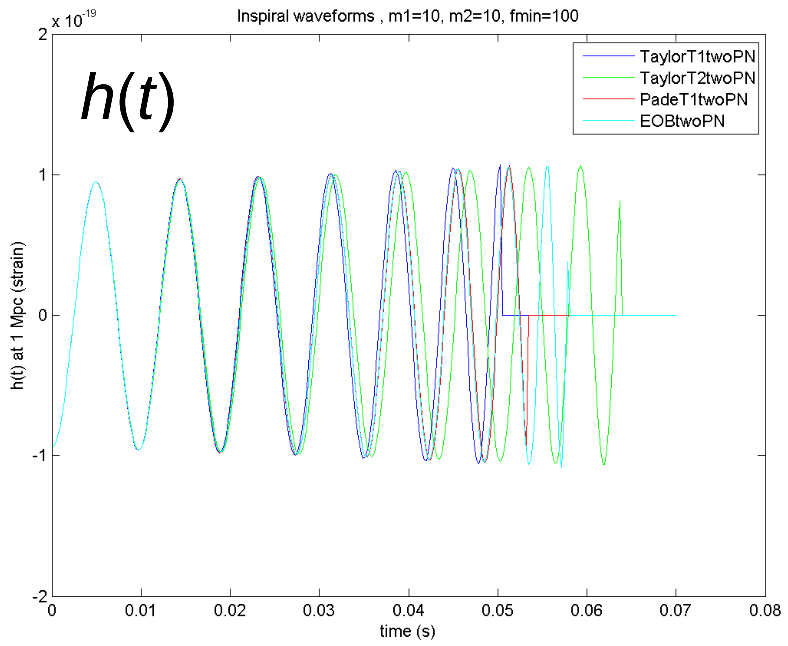
Different Bank Layout Methods



Uncertain Waveforms for High-Mass Inspirals

Different models for 10+10 M_{sun} black hole binary inspiral





BCV Detection Template Family

Buonanno, Chen, and Vallisneri, Phys. Rev. D 67, 104025 (2003)

$$h(f) = f^{-7/6}(1 - \alpha f^{2/3})\theta(f_{cut} - f)\exp[i(\phi_0 + 2\pi t_0 f + \psi_0 f^{-5/3} + \psi_3 f^{-2/3})]$$

Analytically calculate α to maximize SNR

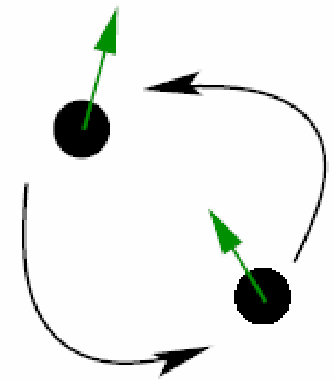
Parameters of the search

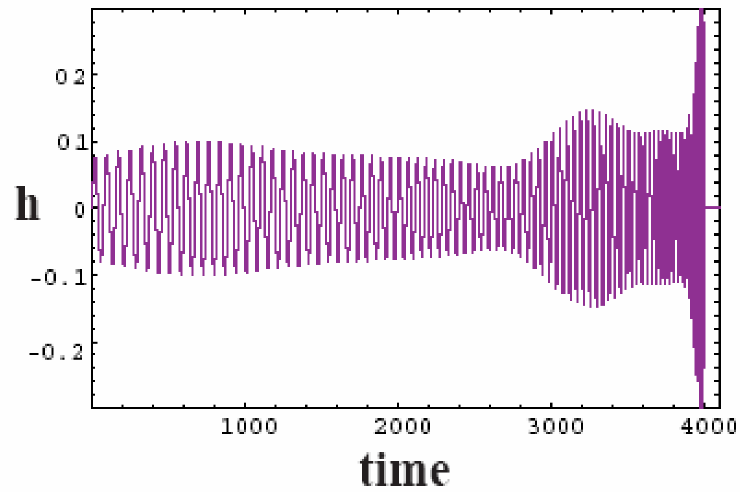
Can match the various waveform models rather well

This is intended for binary components with negligible spin

Binary Systems with Spin

Waveform can be much more complicated !





Another BCV detection template family for systems with spin

- Six more analytically calculated parameters
- One more search parameter $\Rightarrow$ 4-dimensional parameter space

Issues of Astrophysical Interpretation

What population characteristics do we expect ?

Neutron star binaries

- Mass distribution from population synthesis simulations
- Spatial distribution following blue light luminosity?

} Not certain

- Have placed limits on rate per Milky Way equivalent galaxy

Primordial binary black holes in the galactic halo

- Can make a reasonable spatial model
- Don’t know mass distribution

BH+BH and BH+NS binaries

- Don’t have a handle on mass and spatial distributions

Summary

Searching for inspiral signals is simple in principle but fairly complicated in practice

Have to deal with non-stationary noise

Have to use a multi-stage pipeline to keep computational costs under control

Astrophysical interpretation is nontrivial – but that's where the excitement will be, eventually!