The "probability event horizon" and probing the astrophysical GW background

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Outline

We outline a model that describes the relationship between an observer-detector
and transient astrophysical events occurring throughout the Universe

We demonstrate how the signal from an astrophysical GW background evolves with observation time.

There may be a regime where the signature of an astrophysical population can be identified before a single local event occurs above the noise threshold.

Assumptions

Transient “events” are defined as cataclysmic astrophysical phenomena where the peak emission duration is much less than the observation time.

Supernovae (GWs and EM) GWs - milliseconds

GRBs (gamma rays) (seconds-minutes)

Double NS mergers (GWs) seconds ?

Events in a Euclidean Universe

$$\text{Cumulative event rate} = \frac{4}{3}\pi r^3 r_0$$

The events are independent of each other, so their distribution is a Poisson process in time: the probability for at least one event to occur in this volume during observation time *T* at a mean rate *R*(*r*) at constant probability is given by an exponential distribution:

$$p(n \geq 1; R(r), T) = 1 - e^{-R(r)T} = \varepsilon$$

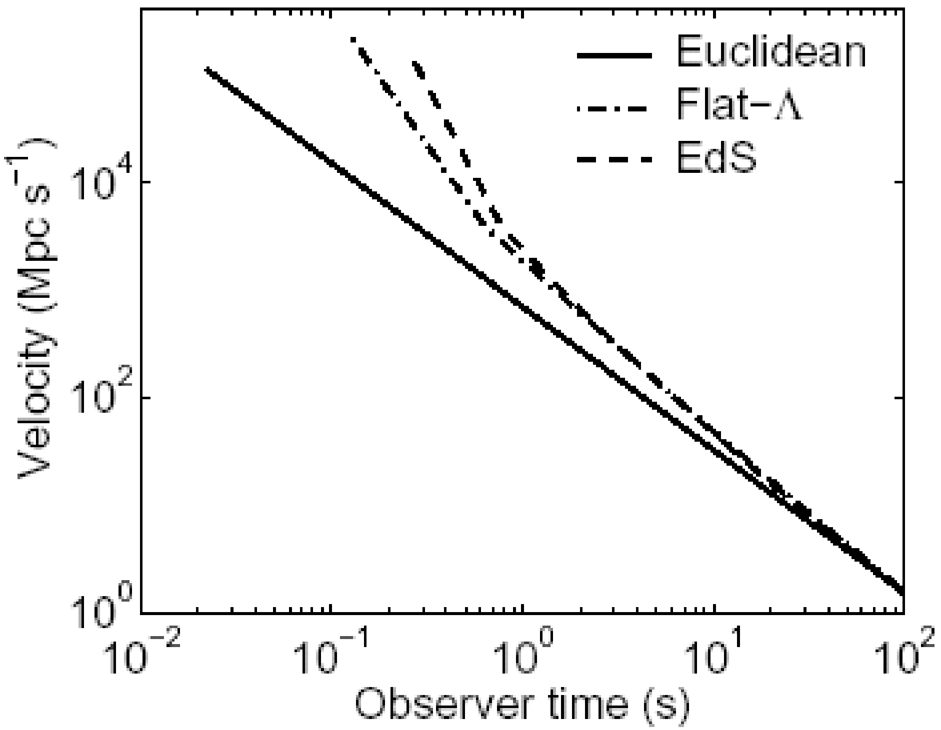
Probability with a speed?

The corresponding radial distance, a decreasing function of observation time, defines the PEH:

$$r_{\varepsilon}^{PEH}(T) = (3N_{\varepsilon} / 4\pi r_0)^{1/3} T^{-1/3}$$

The speed at which the horizon approaches the observer—the PEH velocity—is obtained by differentiating $r_{\varepsilon}(T)$ with respect to T:

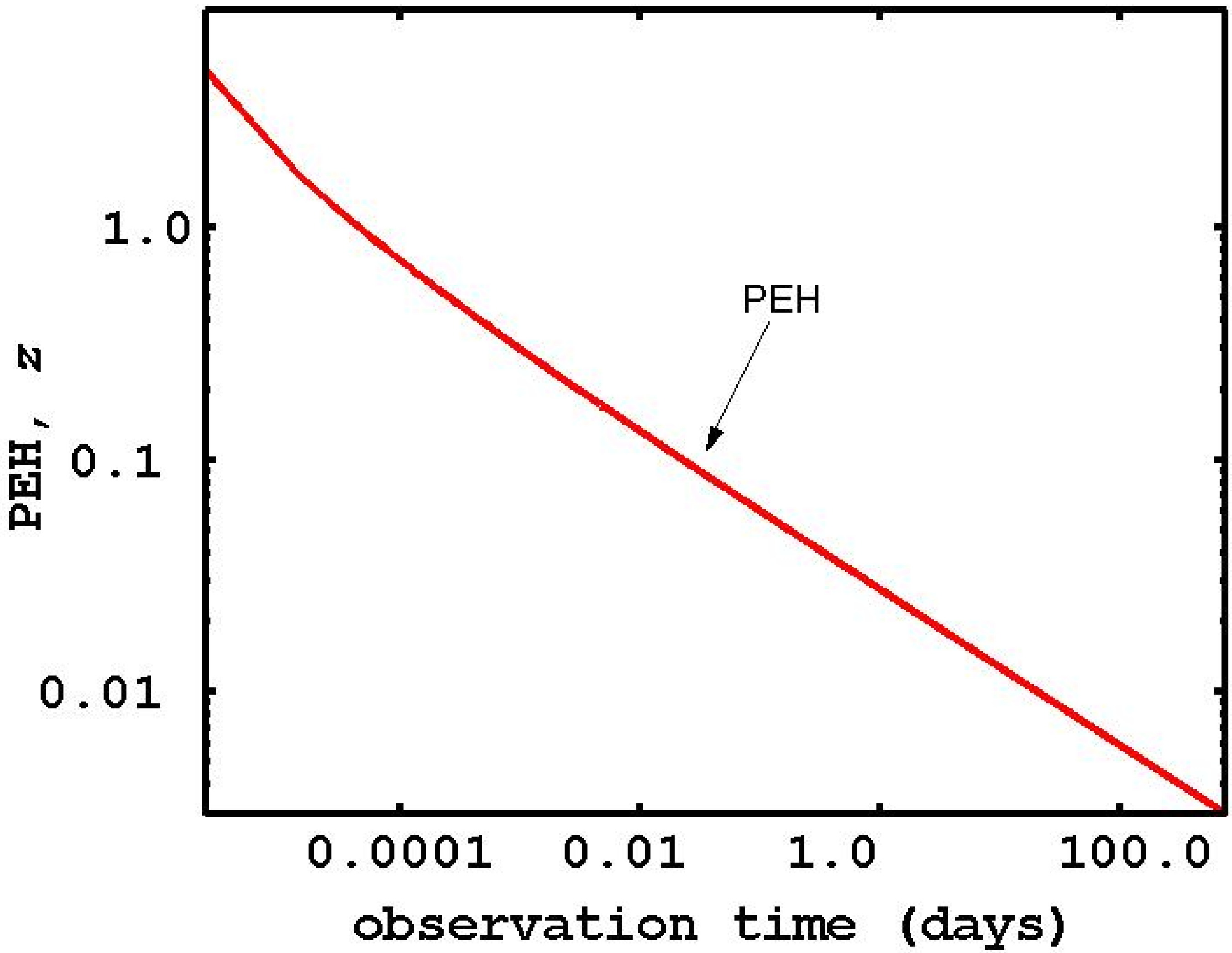
$$v_{\varepsilon}^{PEH}(T) = N_{\varepsilon} / 36\pi r_0 T^{-4/3}$$



PEH velocity for supernova for 3 cosmological models

Using Poisson statistics and knowledge of the event rate we can define a PEH :

“The min *z* (T_{obs}) for at least 1 event to occur at a 95 % confidence level”



(see D.M.Coward & R.R.Burman, accepted MNRAS, astro-ph0505181)

PEH and GRBs

- The probability “tail” of the GRB redshift distribution – physically it represents the local rate density of events (presently not well known).
- The PEH algorithm is very “sensitive” to the tail of the distribution because it includes the temporal evolution of the low probability events.
- By fitting a PEH model with the local rate density modeled as a free parameter, the method can be used to estimate the local rate density.

GRB	Instrument	IPN	XA	OT	RA	IAUC	z
970228	SAX/WFC	y	y	y	n	6572	0.695
970508	SAX/WFC		y	y	y	6649, 6654	0.835
970828	SAX/WFC	y	y	n	y	6726, 6728	0.9578
971214	SAX/WFC	y	y	y	n	6787, 6789	3.42
980425	SAX/WFC		y	SN	y	6884	0.0085
980613	SAX/WFC		y	y	n	6938	1.096
980703	RXTE/ASM		y	y	y	6966	0.966
990123	SAX/WFC	y	y	y	y	7095	1.6
990506	BAT/PCA	y	y		y	-	1.3
990510	SAX/WFC	y	y	y	y	7160	1.619
990705	SAX/WFC	y	y	y	n	7218	0.86
990712	SAX/WFC			y	n	7221	0.434
991208	Uly/KO/NE	y		y	y		0.706
000131	Uly/KO/NE	y		y			4.5
000210	SAX/WFC	y	y		y		0.846
000301C	ASM/Uly	y		y	y		2.03
000911	Uly/KO/NE	y		y	y		1.058
000926	Uly/KO/NE	y	y	y	y		2.066
010921	HE/Uly/SAX	y		y			0.45
011121	SAX/WFC	y	y	y	y		0.36
011211	SAX/WFC		y	y			2.14
020124	HETE	y		y			3.198
020405	Uly/MO/SAX	y		y	y		0.69
020813	HETE	y	y	y	y		1.25
020903X	HETE			y	y		0.25
021004	HETE		y	y	y		2.3
021211	HETE			y			1.01
030226	HETE		y	y			1.98
030323	HETE				y		3.372
030328	HETE		y	y			1.52
030329	HETE		y	y	y	8101	0.168
030429	HETE			y			2.65
031203	INTEGRAL		y	y	y	8250, 8308	0.105
040701X	HETE		y				0.2146

First application of the PEH concept to GRB redshift data

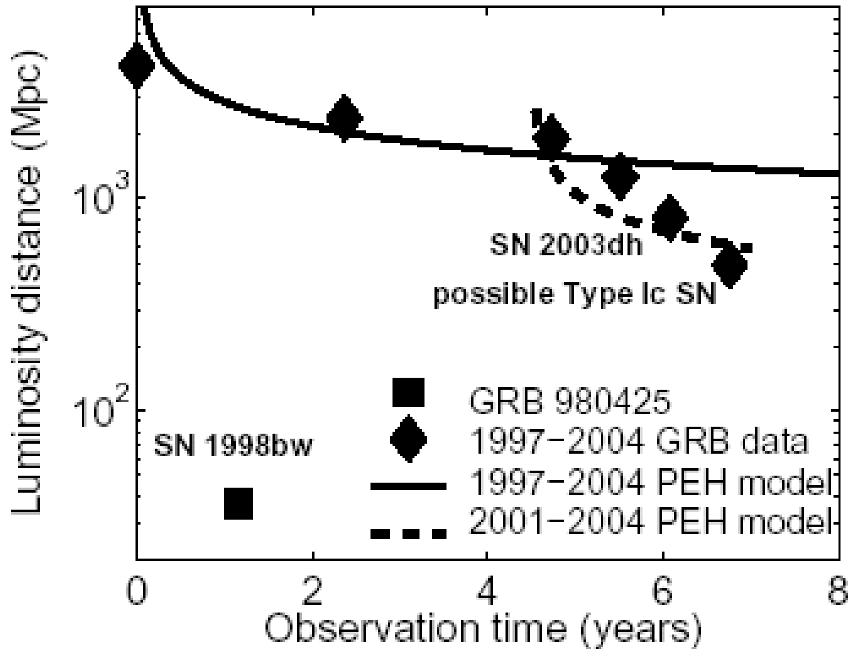
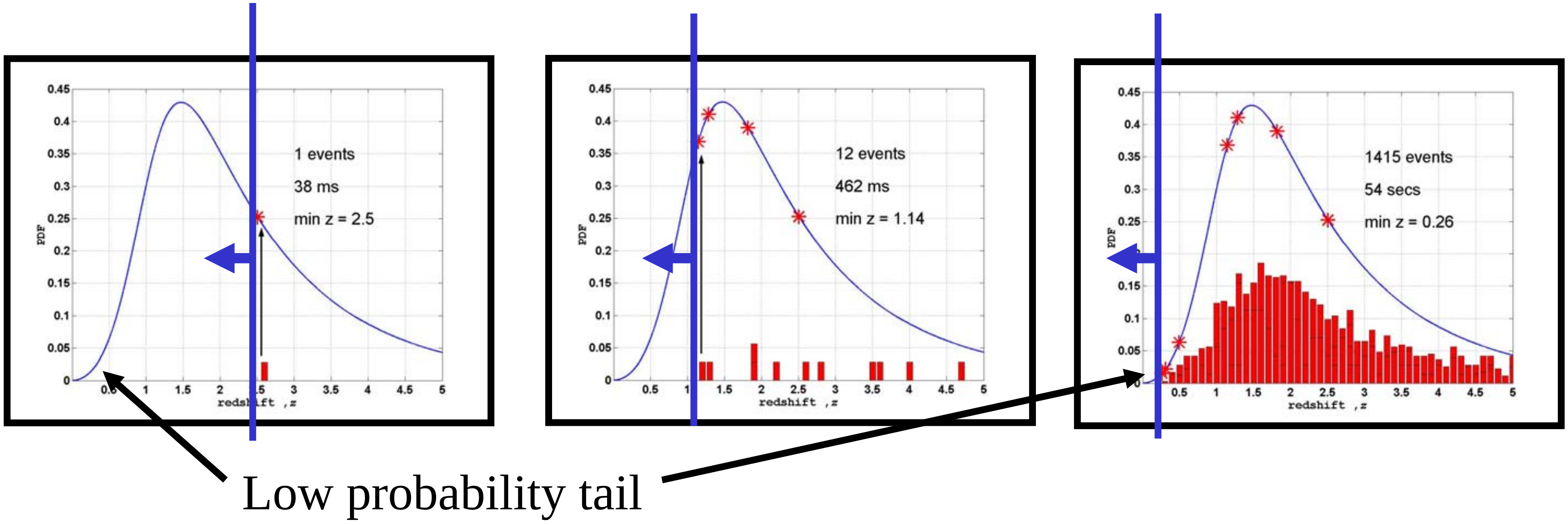


Figure 8. Filled diamonds show the closest events as a function of observation time for the 34 GRBs for which the redshift is known with reasonable confidence; the solid square is the highly under-luminous GRB 980425. The observations span a 7-yr period, starting from the first redshift measurement in 1997 February and continuing to 2004 July. The solid curve shows the PEH, excluding the outlier GRB980425, assuming a flat-Λ (0.3, 0.7) cosmology and a source rate evolution based on the star formation rate model labelled SF1 in Porciani & Madau (2001). A local rate density $r_0 \approx 0.8\ \mathrm{yr}^{-1}\ \mathrm{Gpc}^{-3}$ is used to fit to the redshift data for the classical GRBs for the first 4–5 yr of observation. (The

- Running minima of z (T_{obs}) define a horizon approaching the detector
- The horizon's initial approach is rapid for around the first 100 events
- The horizon slows down as a function of T_{obs}



Poisson process - probability distribution of the number of occurrences of an an event that happens rarely but has many opportunities to happen

$$p(r) = \frac{M^r e^{-M}}{r!}$$, where $M = R(z)\,T$

M = mean number of events

R(z) = rate of events throughout the Universe

T = observation time

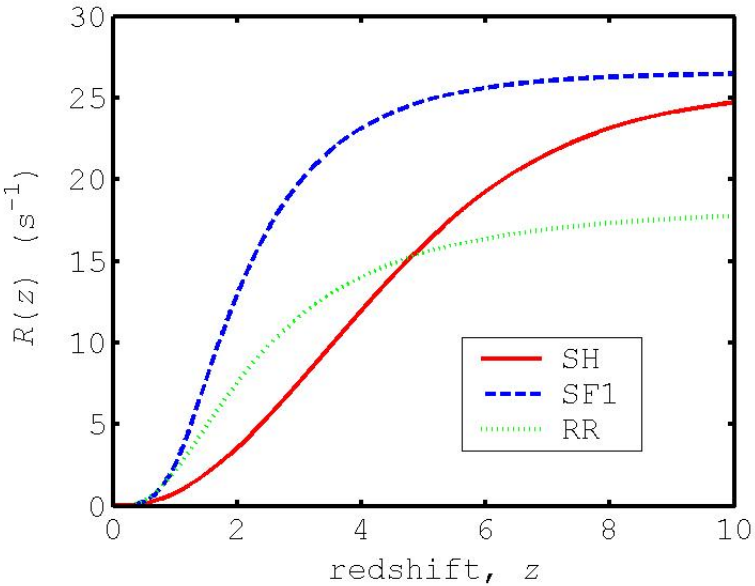
$P(\text{at least 1 event}) = 1 - e^{-M}$

If we set a 95% confidence level

$$0.95 = 1 - e^{-R(z)T}$$

$$-R(z)T = \ln(0.05)$$

$$R(z)T = 3$$



additionally $P(0) = e^{-M}$

Simulating the PEH

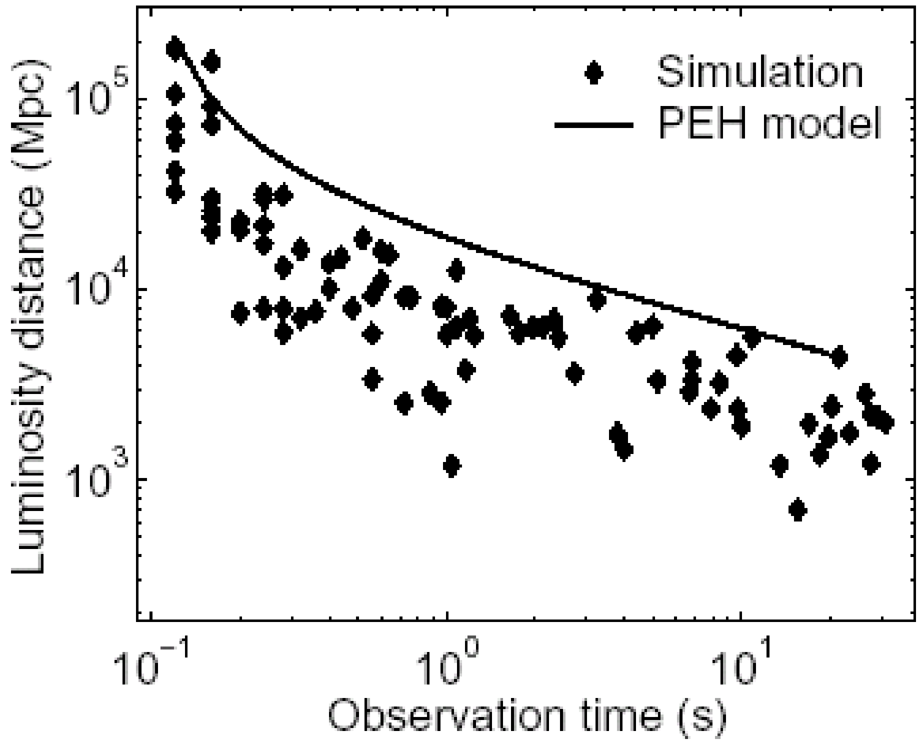
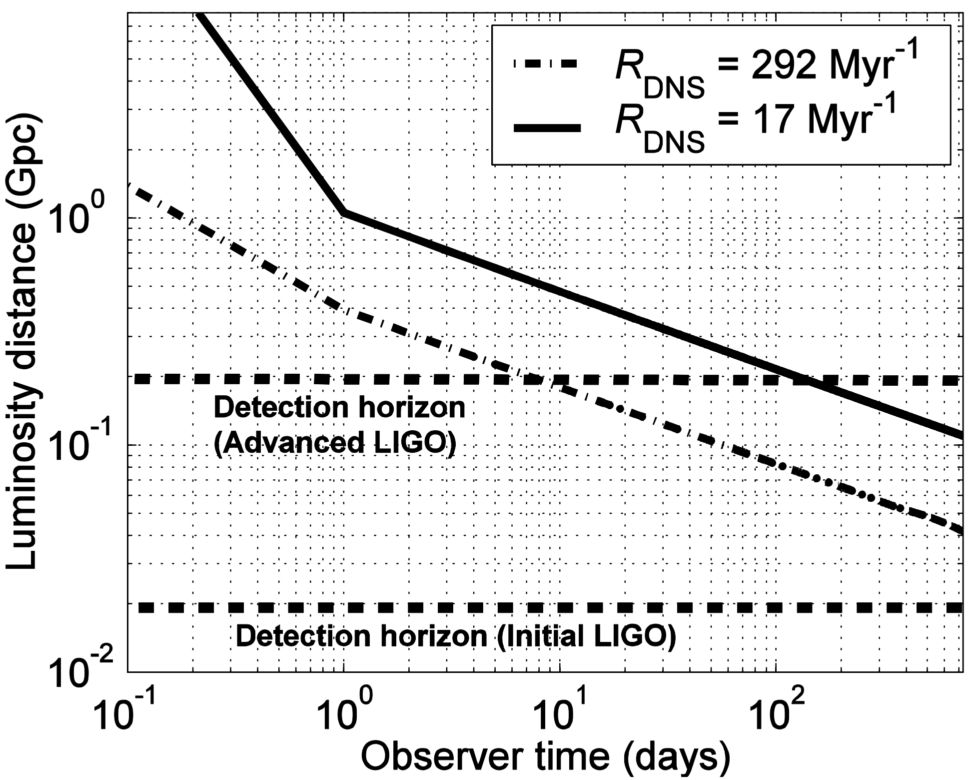


Figure 7. Simulation illustrating the rapid motion of the PEH for the first few tens of seconds of observation time, and a model curve with a probability threshold $\epsilon = 0.95$ for at least one event to occur at a distance less than that of the PEH. We assume a universal cumulative event rate of about 25 s^{-1} as seen in our frame, comparable to the NS birth rate integrated throughout the Universe, a flat-Λ (0.3, 0.7) cosmology and a source rate evolution locked to the SFR model labelled SF2 in Porciani & Madau (2001)

The “probability event horizon” for DNS mergers



DNS mergers are key targets for LIGO and Advanced LIGO

The horizon represents the radial boundary for at least one DNS merger to occur with 95% confidence.

Upper and lower uncertainties in the local DNS rate densities respectively are taken from (Kalogera et al. 2004).

Suboptimal filtering methods in time domain

a) Norm Filter

$$y_k = \sum_{i=k}^{k+N-1} x_i^2$$

Determines signal energy in a moving Window of size N

b) Mean Filter

$$y_k = \frac{1}{N}\sum_{i=k}^{k+N-1} x_i$$

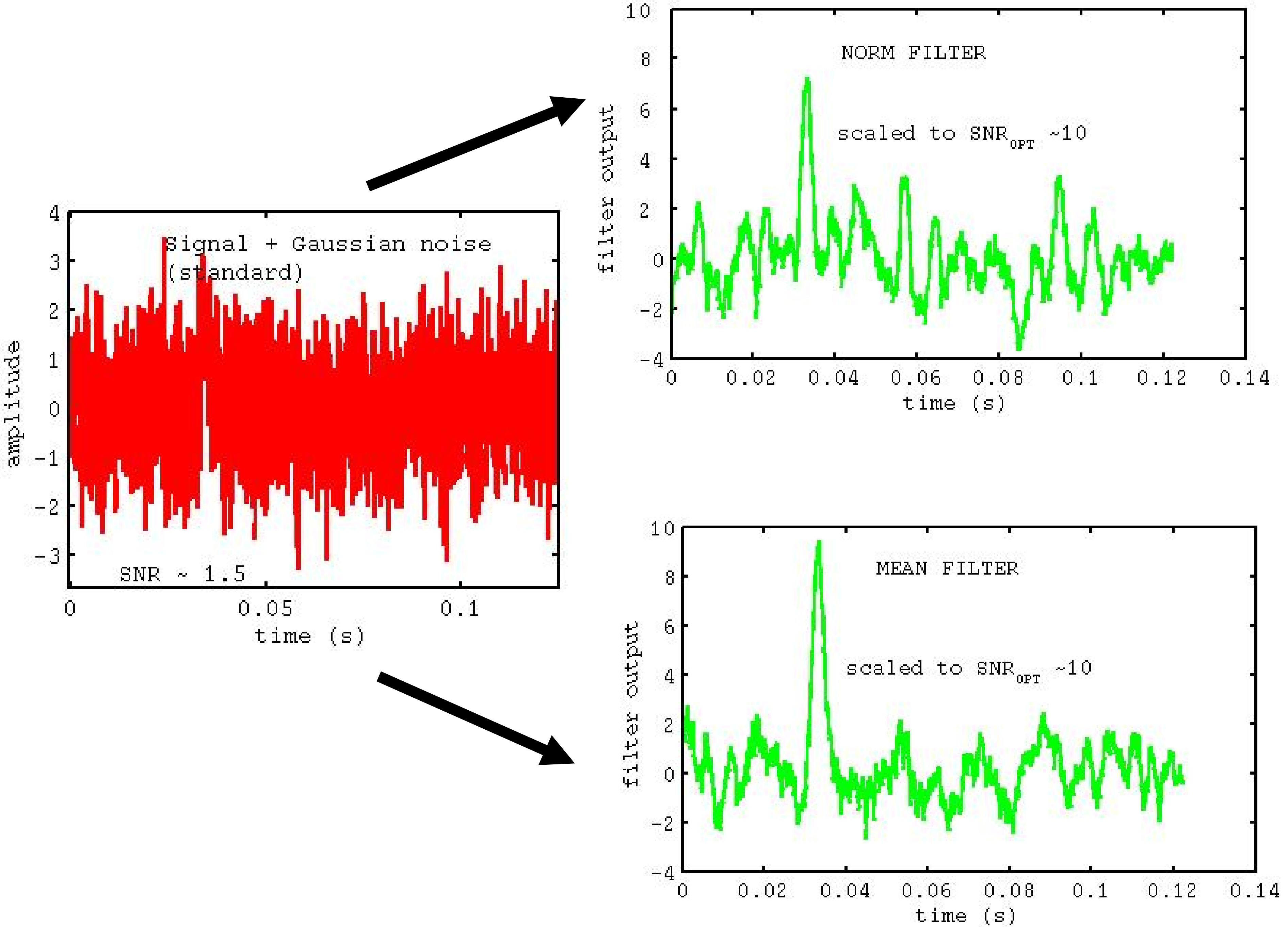
Determines mean of data in a moving window of size N

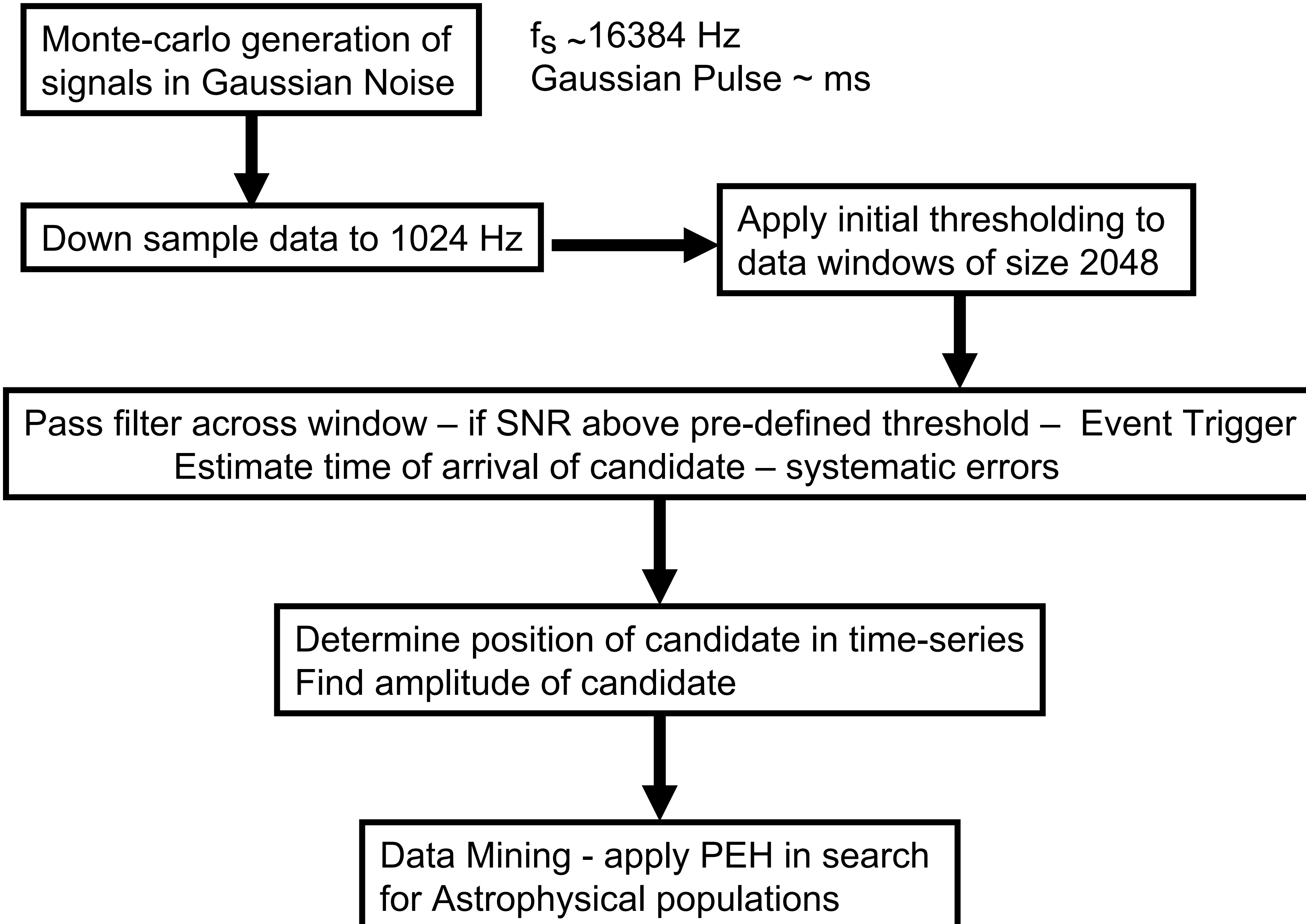
ROBUST – similar efficiencies for different signals – e.g. Gaussian pulses, DFM

- Important as waveforms not known accurately
- Only *a priori* knowledge – short durations the order of ms

ASSUMPTION – data is whitened by some suitable filter

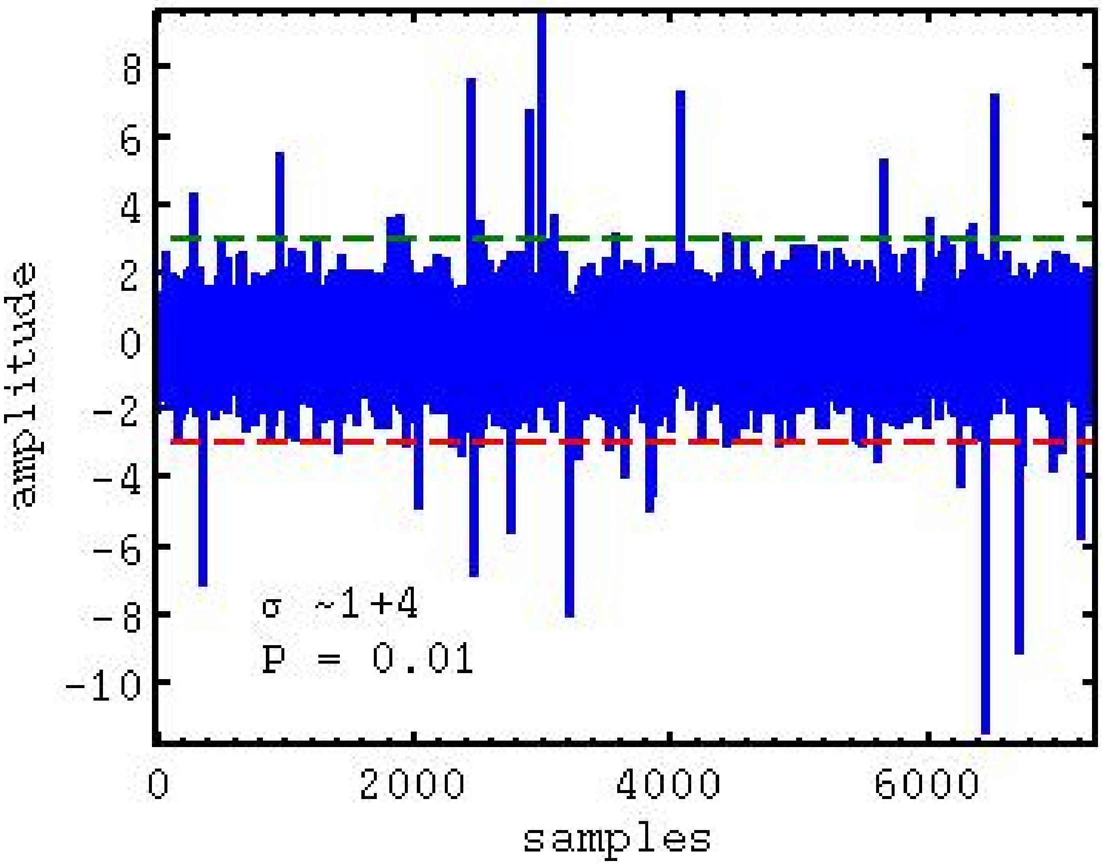
Arnaud at al. PRD 67 062004 2003, Arnaud at al. PRD 59 082002 1999


Signal + Gaussian noise
(standard)
SNR ~ 1.5
amplitude
time (s)
NORM FILTER
scaled to SNR$_{OPT}$ ~10
filter output
time (s)
MEAN FILTER
scaled to SNR$_{OPT}$ ~10
filter output
time (s)


Monte-carlo generation of signals in Gaussian Noise
f_S ~16384 Hz
Gaussian Pulse ~ ms
Down sample data to 1024 Hz
Apply initial thresholding to data windows of size 2048
Pass filter across window – if SNR above pre-defined threshold – Event Trigger
Estimate time of arrival of candidate – systematic errors
Determine position of candidate in time-series
Find amplitude of candidate
Data Mining - apply PEH in search for Astrophysical populations

Non-Gaussian noise model – Mixture Gaussian

$$\Psi(m,\sigma) = (1-P)\Psi_1(0,\sigma_1) + P\Psi_2(0,\sigma_2)$$



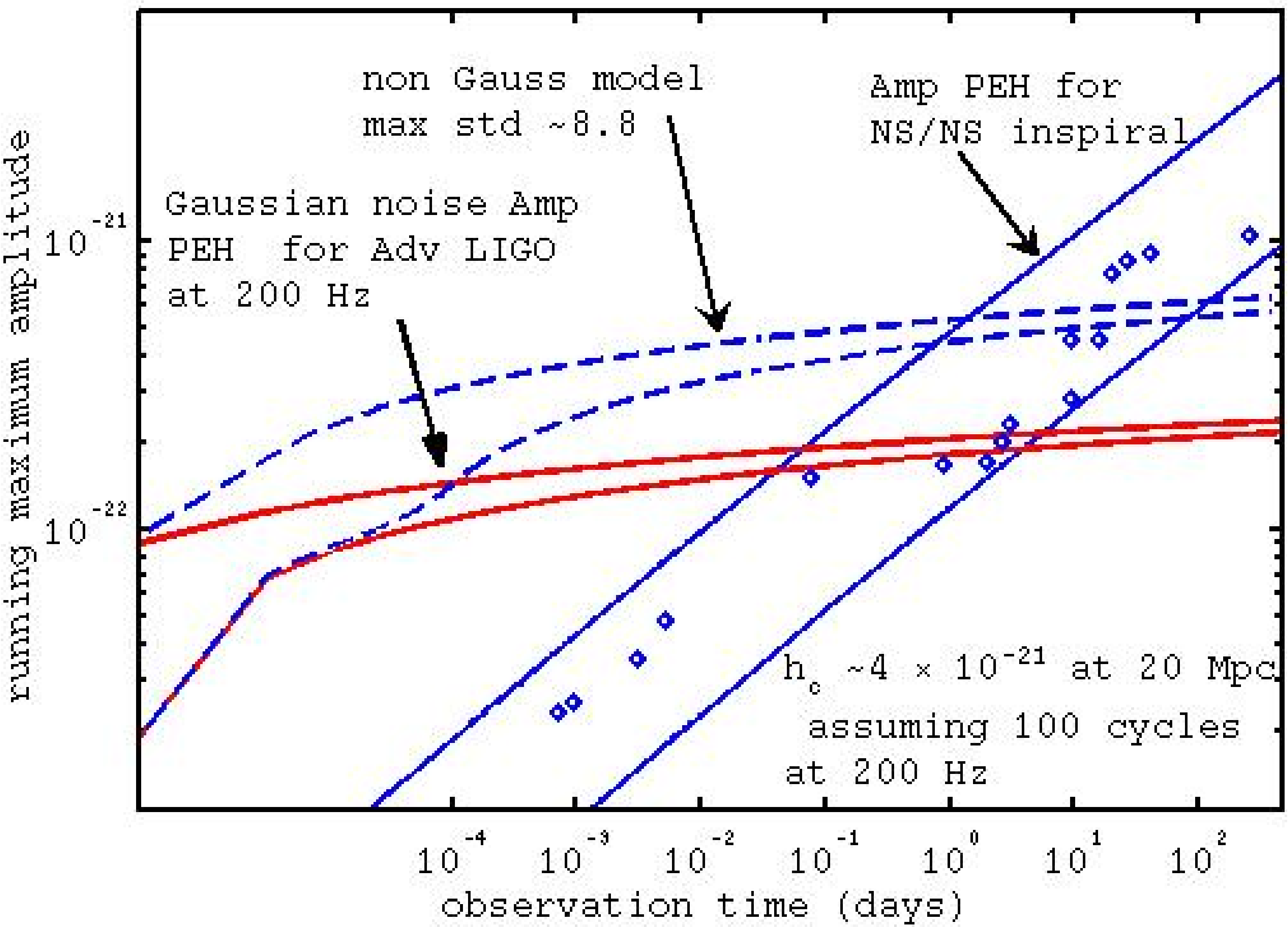
Ψ_1 and Ψ_2 are Gaussian distributions

Weighting factor: $P << 1$

$P \in (1,0)$

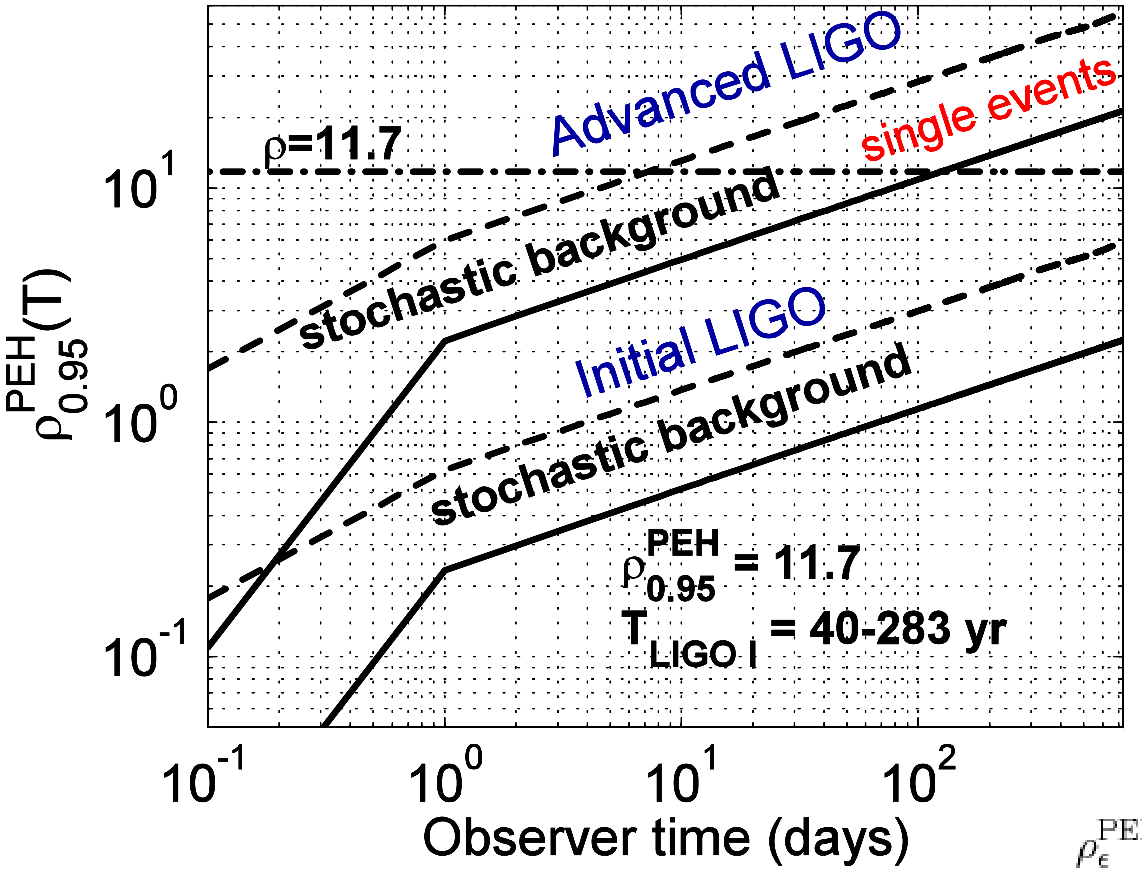
$\sigma_1 = 1$ and $\sigma_2 = 4$

Allen at al. PRD 65122002 2002, Finn S. PRD 63 102001 2001



non Gauss model
max std ~8.8
Amp PEH for
NS/NS inspiral
Gaussian noise Amp
PEH for Adv LIGO
at 200 Hz
$h_c \sim 4 \times 10^{-21}$ at 20 Mpc
assuming 100 cycles
at 200 Hz
running maximum amplitude
10^{-21}
10^{-22}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
observation time (days)

PEH for the signal-noise-ratio for LIGO



This analysis shows that the detectability of a GW stochastic background and individual DNS mergers are inexorably linked to the observer and to the sensitivity of the detector

$$\rho_\epsilon^{\mathrm{PEH}}(T) = 11.7\frac{|\Psi|^2}{2.56}\left(\frac{r_{\mathrm{L}}}{r_\epsilon^{\mathrm{PEH}}(T)}\mathrm{Mpc}\right)$$

Future Work

- We plan to test the PEH concept by developing a PEH filter and applying it to simulated data

- Inject simulated signals in real data to test filter performance.

- Utilize the PEH filter in the frequency domain.

THE END