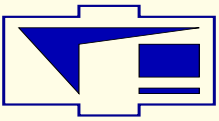


**Optical rigidity in signal-recycled
gravitational-wave detectors
with optical losses**

**F. Ya. Khalili, V. I. Lazebny and
S. P. Vyatchanin**

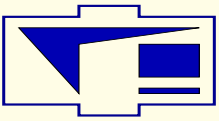
*Physics Department, Moscow State University,
Moscow 119992 Russia*

August 2005

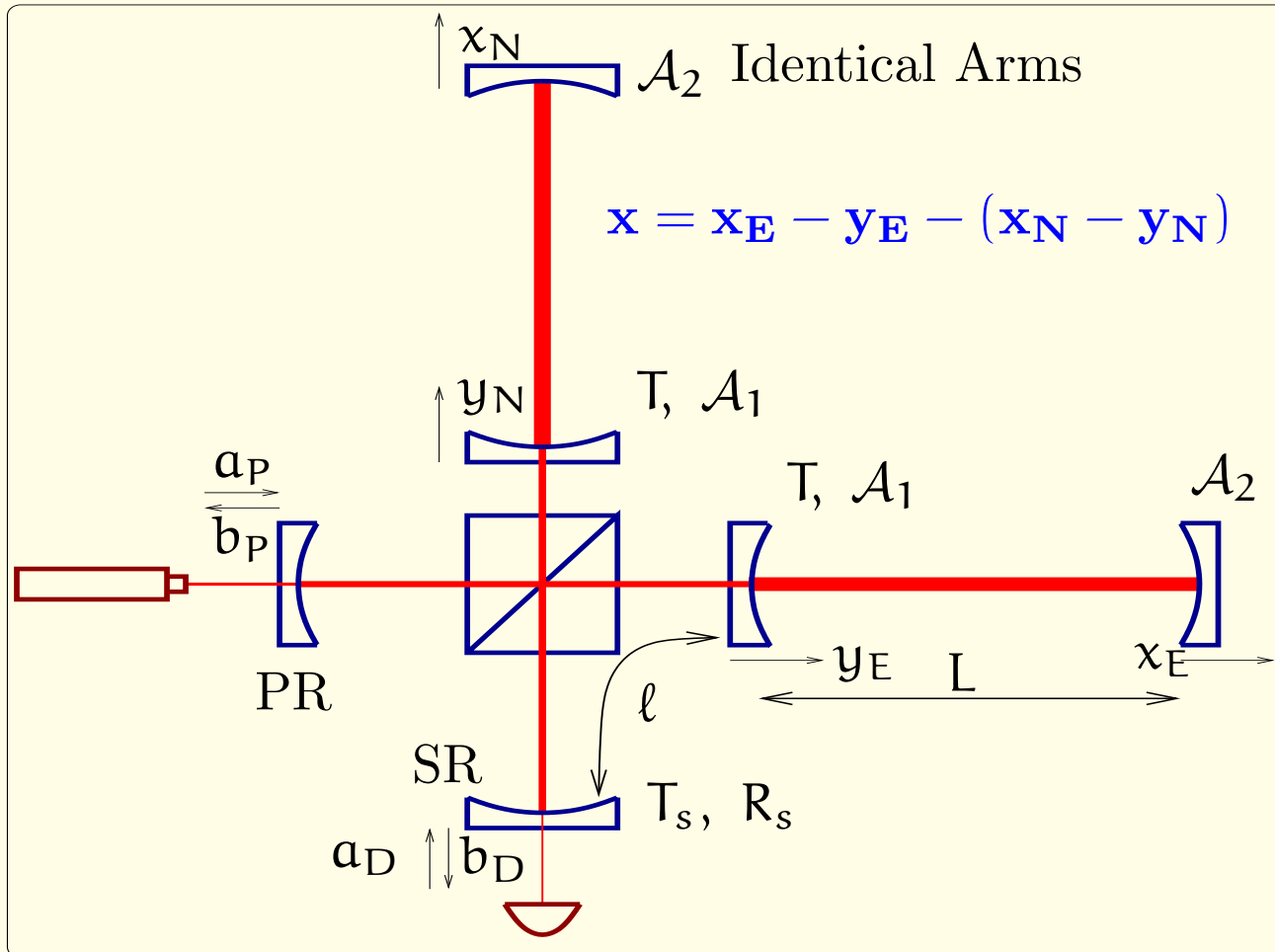


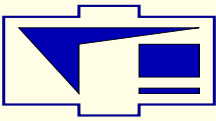
Summary

- The usage of the optical rigidity allows to obtain **better than Standard Quantum Limit (SQL)** sensitivity without increasing laser pumping.
- The gain in sensitivity is inversely proportional to the bandwidth where this gain is achieved. **One can modify “on line” the sensitivity dependence** on frequency from a single minimum to a double minimum dependence.
- **The optical losses degrade sensitivity** of measurements with usage of optical rigidity **by relatively small value** in contrast with other measurements beating SQL which degradation due to losses is considerable.
- This “narrow band” regimes provides the **gain in signal to noise ratio** even for signals with **wide spectrum** and the **losses decreases this gain by relatively small** factor.



Advanced LIGO inteferometer with losses in arms





Introduction

The sensitivity of interferometer $\xi(\Omega)$ consists of two terms:

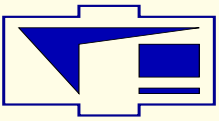
$$\xi^2(\Omega) = \frac{S_h(\Omega)}{\hbar_{\text{SQL}}^2(\Omega)} = \frac{2}{\hbar m \Omega^2} \left(S_F(\Omega) + |Z(\Omega)|^2 S_x(\Omega) \right), \quad (1)$$

$$Z(\Omega) = \frac{F(\Omega)}{x(\Omega)} \quad (2)$$

If S_x and S_F do not correlate: $S_F(\Omega) S_x(\Omega) \geq \hbar^2/4$. Then

$$\xi_{\min}^2(\Omega) = \frac{|Z(\Omega)|}{m_0 \Omega^2} < 1, \quad \text{or} \quad \xi^2 = \frac{\Delta\Omega}{\Omega} < 1 \quad (3)$$

in the bandwidth $\Delta\Omega$ where $|Z(\Omega)| < m_0 \Omega^2$
as for conventional oscillator.



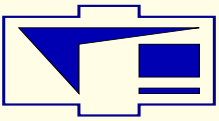
The sophisticated frequency dependence

of optical rigidity provides sensitivity:

$$\xi^2 = \frac{S_h}{h_{\text{SQL}}^2} = \left(\frac{\Delta\Omega}{\Omega} \right)^2, \quad (4)$$

For conventional oscillator sensitivity is worse:

$$\xi_{\text{conv}}^2 = \frac{\Delta\Omega}{\Omega}$$



Approximations and notations for difference mode

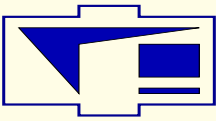
$$T, T_s, r_1, r_2 \ll 1, \quad \Omega \ll \omega_o, \quad \frac{L\Omega}{c} \ll 1, \quad \ell \ll L$$

$$\delta_0 = \frac{cT^2 R_s \sin 2\phi}{2L|1 + R_s e^{2i\phi}|^2} \quad \text{— detuning}$$

$$\gamma_0 = \gamma_0^{\text{load}} + \gamma_0^{\text{loss}}, \quad \text{— relaxation rate of difference mode}$$

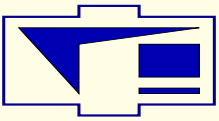
$$\gamma_0^{\text{load}} = \frac{cT^2 T_s^2}{4L|1 + R_s e^{2i\phi}|^2}, \quad \gamma_0^{\text{loss}} = \frac{c(\mathcal{A}_1^2 + \mathcal{A}_2^2)}{4L}. \quad \mathcal{A}_0^2 = \frac{\gamma_0^{\text{loss}}}{\gamma_0^{\text{load}}}$$

“loaded” γ_0^{load} is produced by transparencies of SR and input mirrors and “intrinsic” γ_0^{loss} is produced exclusively the intrinsic losses in mirrors. $\gamma_0 \ll \Omega_o$



Parameters planned to use in Advanced LIGO

Transmittivity of SR mirror	$T_s^2 = 0.05$
Transmittivity of input mirrors in arms	$T^2 = 0.005$
Loss coefficient of each mirror in arms	$\mathcal{A}_1^2 = \mathcal{A}_2^2 = 1.5 \times 10^{-5}$
Length of interferometer arm	$L = 4 \text{ km}$
Effective loss factor	$ \mathcal{A}_0 ^2 = 0.24$
Relaxation rate of difference mode	$\gamma_0 = 2.9 \text{ s}^{-1}$
“Intrinsic” relaxation rate	$\gamma_0^{\text{loss}} = 0.56 \text{ s}^{-1}$
Mean frequency of gravitational wave range	$\Omega_0 = 2\pi \times 100 \text{ s}^{-1}$



Optical rigidity

$$K(\Omega) \equiv \frac{-F_{\text{ponderomotive}}}{z} = \frac{8\omega_0 I_0 \delta_0}{cL \mathcal{D}}, \quad (5)$$

$$\mathcal{D} = [\gamma_0 - i\delta_0 - i\Omega][\gamma_0 + i\delta_0 - i\Omega], \quad (6)$$

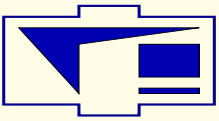
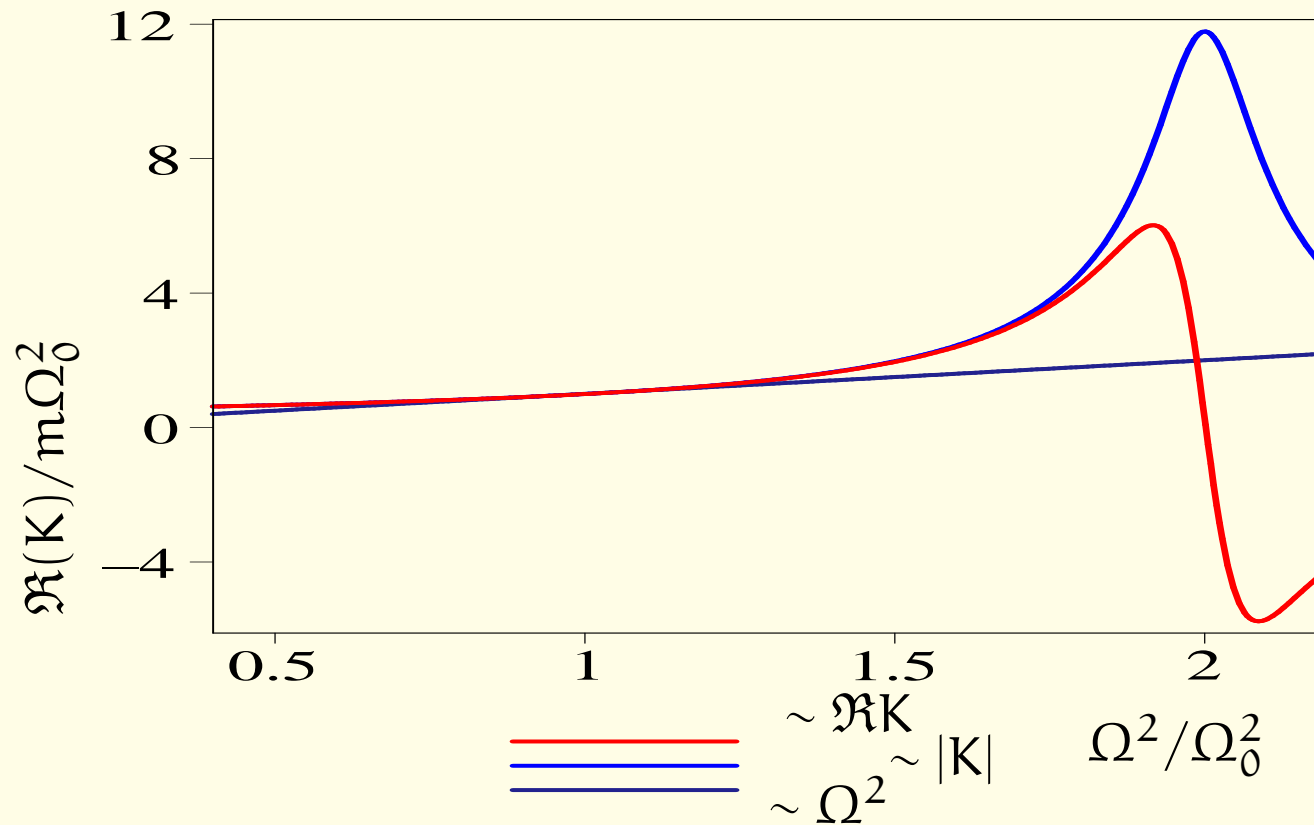
I_0 is the light power circulating inside each arm.

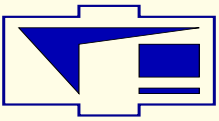
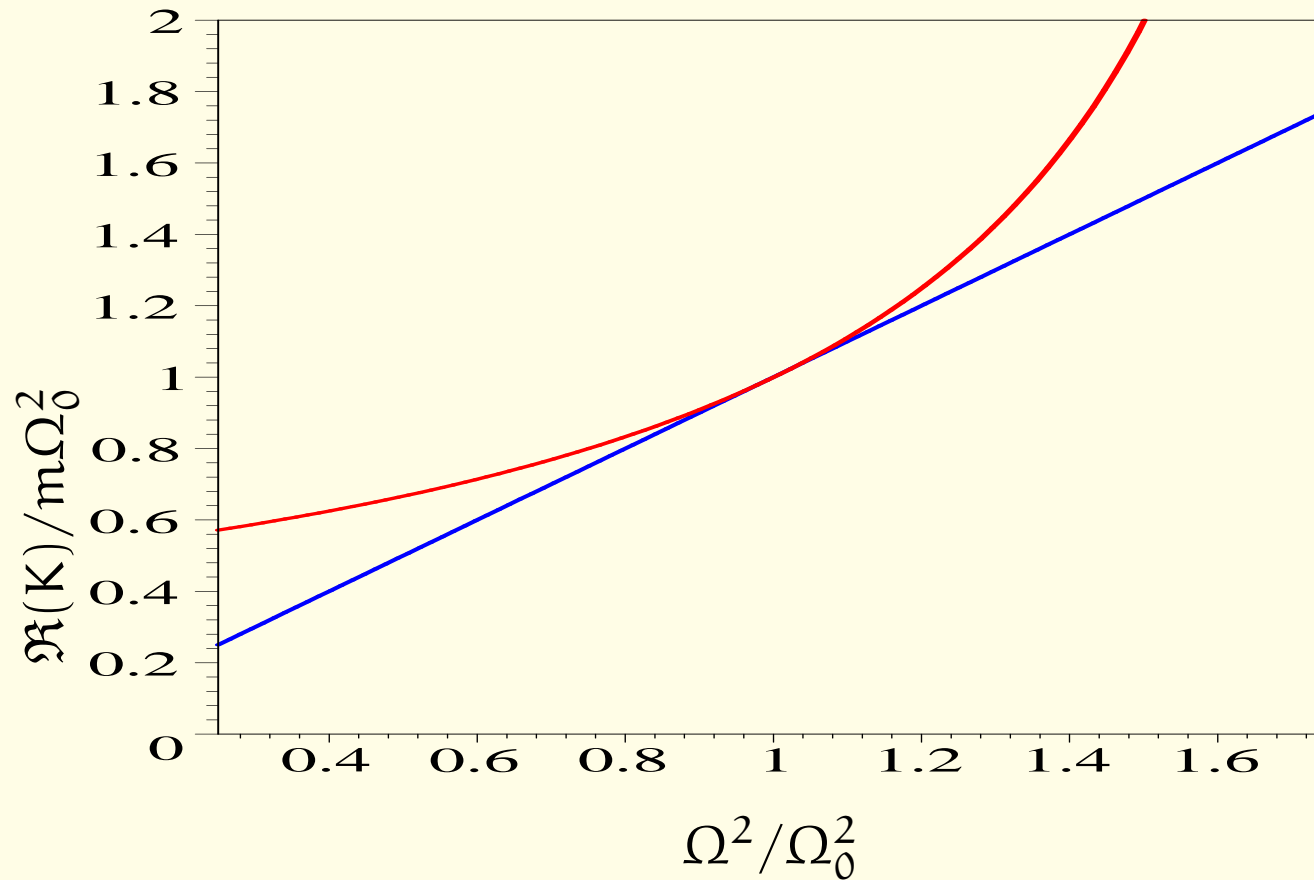
The **sophisticated** dependence $K(\Omega)$. Resonance:

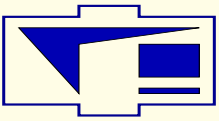
$$K(\Omega) - m\Omega^2 \simeq 0.$$

Conventional oscillator: $K - \text{const}$, one resonance frequency Ω_0 :

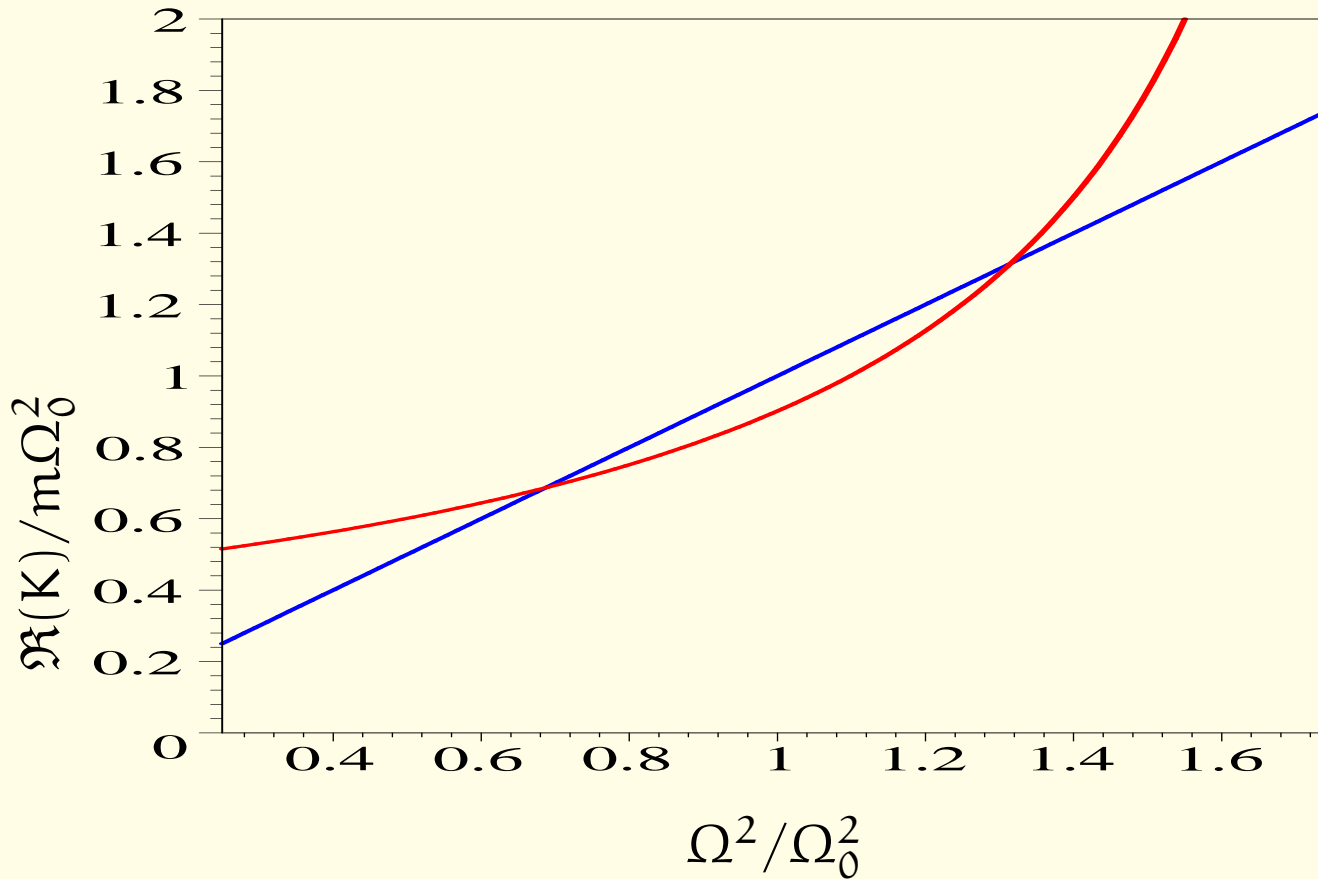
$$\Omega_0^2 = \frac{K}{m}$$

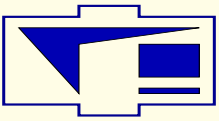
**Optical rigidity: sophisticated dependence on frequency**

**Double resonance (second order pole of susceptibility)**



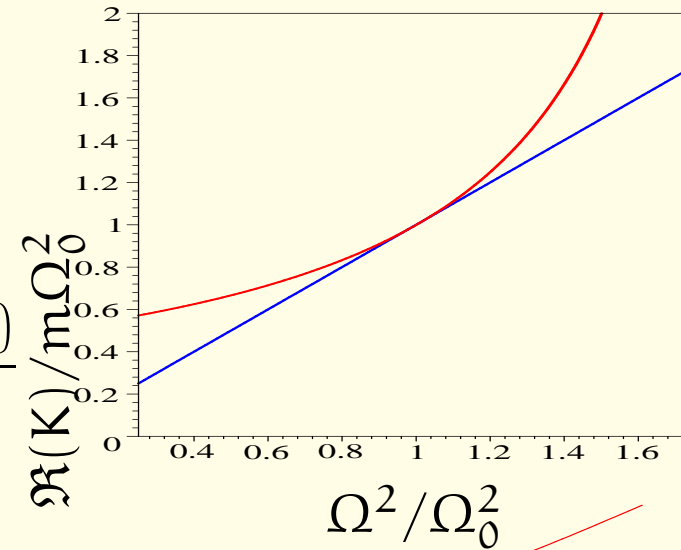
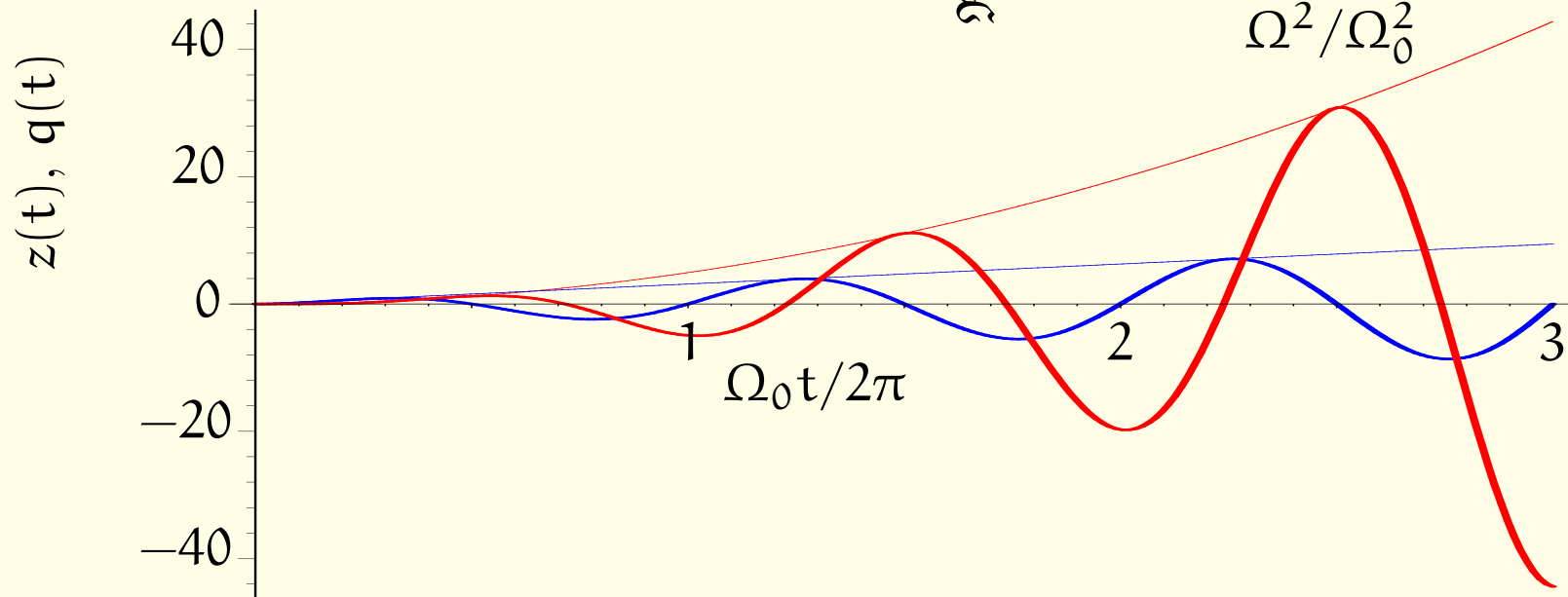
Two first order poles of susceptibility

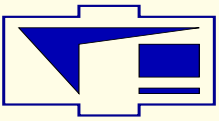


**“Double” resonance**

Conventional: $q(t) = \frac{f_0 t \sin(\Omega_0 t)}{2\Omega_0}$

“Double” res.: $z(t) = \frac{-f_0 t^2 \cos(\Omega_0 t)}{8}$





Quadrature Component Analysis

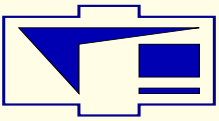
S_h is one-sided spectral density of noise of quadrature component b_ζ recalculated to variation of dimensionless metric h_s .
normalization to SQL sensitivity $h_{\text{SQL}}(\Omega_{\text{oo}})$:

$$b_\zeta = \frac{b_D e^{-i\zeta} + b_{D-}^+ e^{i\zeta}}{\sqrt{2}}, \quad \xi^2(\Omega) = \frac{S_h}{h_{\text{SQL}}(\Omega_{\text{oo}})^2} \quad (7)$$

For $\gamma_0 \ll \Omega_0$ sensitivity ξ has minimum at frequency Ω_0 if $I_o \simeq I_o^{\text{crit}}$

$$\Omega_0^2 \simeq \frac{\delta_0^2}{2}, \quad I_o^{\text{crit}} = \frac{mL\Omega_0^4}{8k(\delta_0)} \simeq \frac{I_{\text{SQL}}(\Omega_0)}{\sqrt{2}},$$

$$\text{Then } \xi^2 \sim (\Omega^2 - \Omega_0^2) + \eta^2 \Omega_0^4 + \dots, \quad \eta = \sqrt{1 - \frac{I_o}{I_o^{\text{crit}}}}, \quad (8)$$



Single minimum case

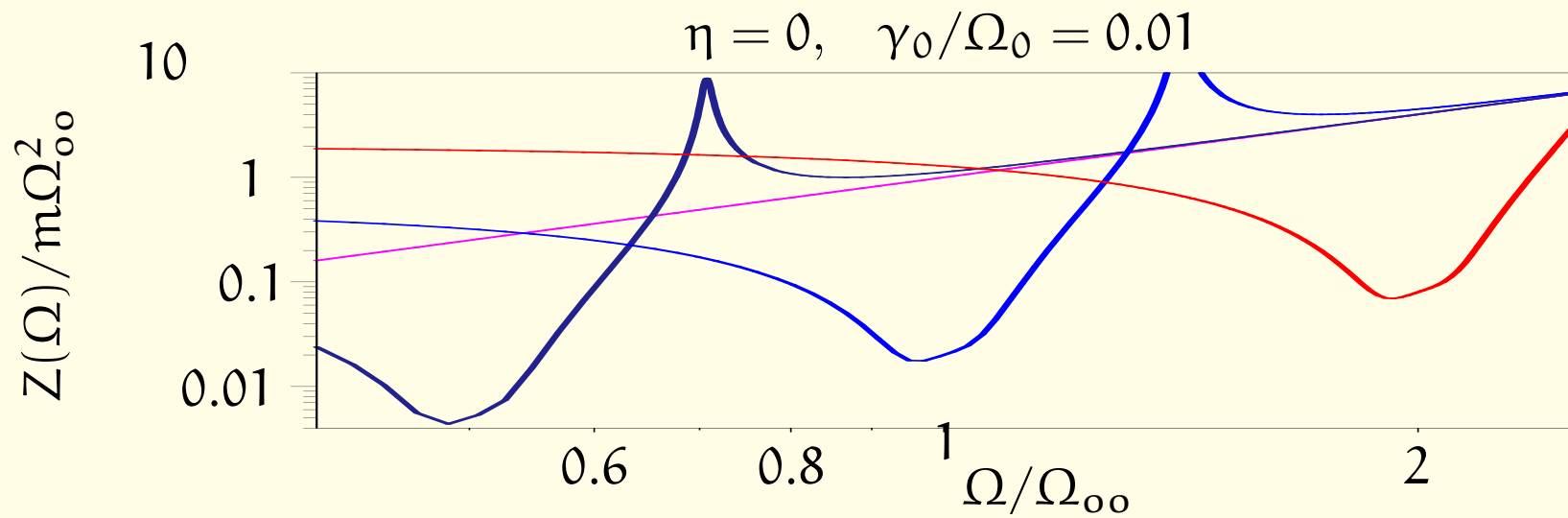
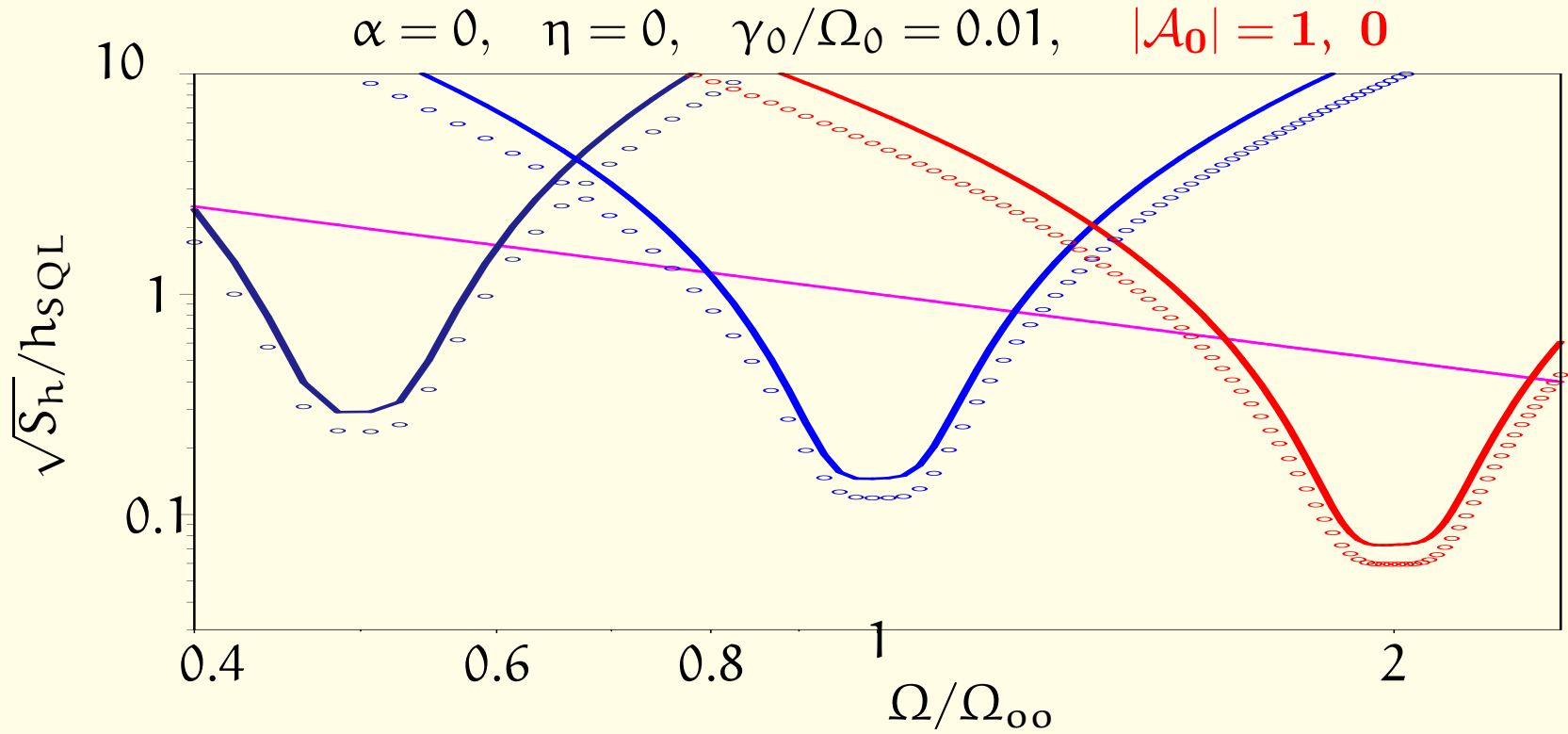
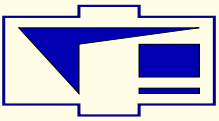
$\eta = 0$ — power is equal to critical power: $I_o = I_o^{\text{crit}}$.

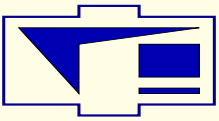
$$\xi_0 \equiv \xi(\Omega_0) \simeq \Omega_{oo} \sqrt{\frac{\tilde{\gamma}_0}{\sqrt{2} \Omega_0^3}}, \quad (9)$$

$$\frac{\Delta\Omega}{\Omega_0} \simeq \sqrt{\frac{\tilde{\gamma}_0}{2\sqrt{2} \Omega_0 \sqrt{1 + |\mathcal{A}_0|^2}}}, \quad (10)$$

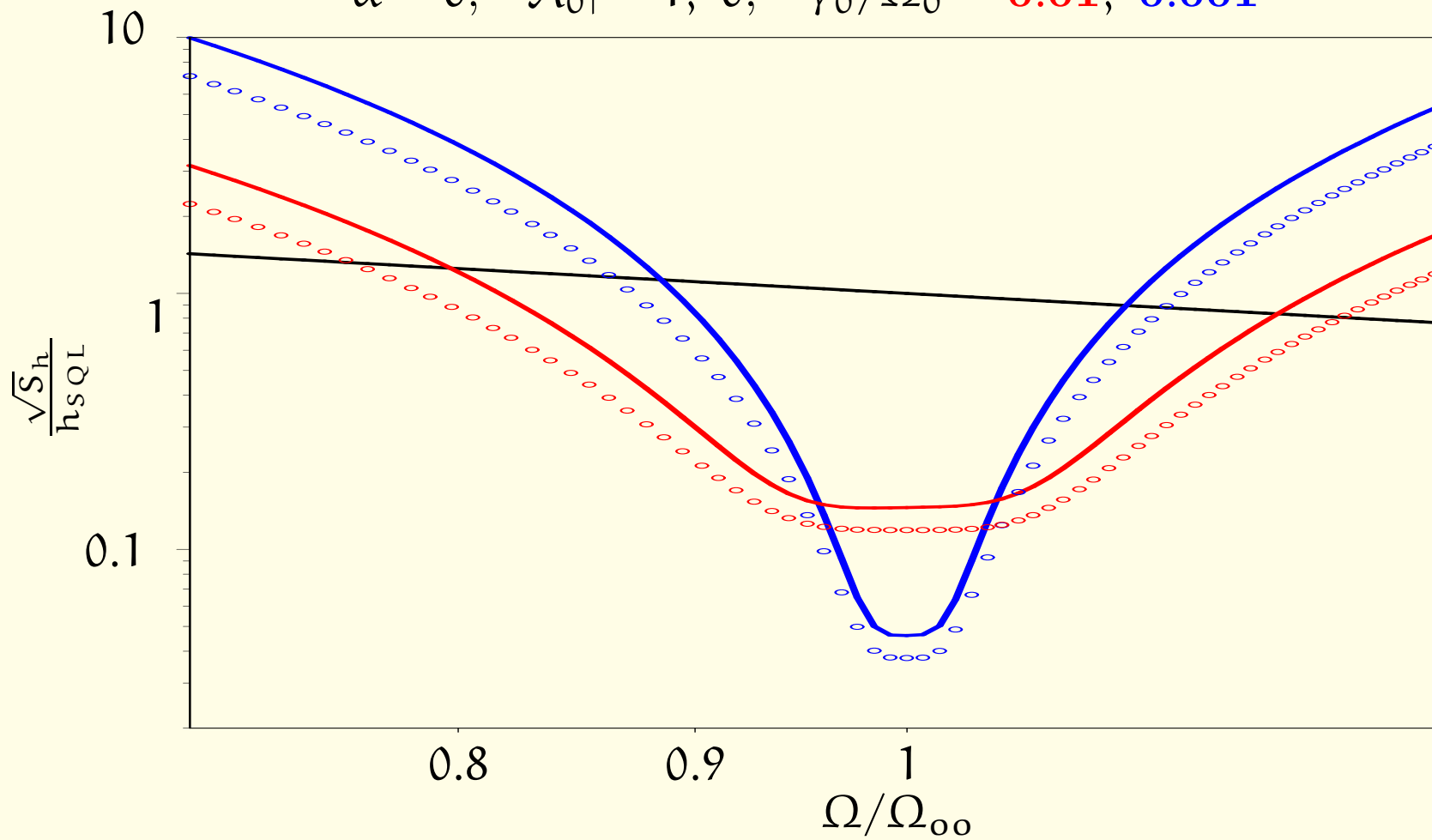
$$\tilde{\gamma}_0 = \gamma_0 \sqrt{(2 - \sin^2 \alpha)^2 + 2|\mathcal{A}_0|^2}, \quad (11)$$

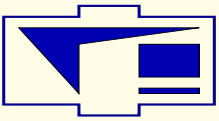
$$|\mathcal{A}_0|^2 = \frac{(\mathcal{A}_1^2 + \mathcal{A}_2^2) |1 + R_s e^{i\phi}|^2}{T^2 T_s^2} < 1 \quad (12)$$





$$\alpha = 0, \quad \mathcal{A}_0| = 1, \quad 0, \quad \gamma_0/\Omega_0 = \mathbf{0.01}, \quad \mathbf{0.001}$$





Minimum of ξ

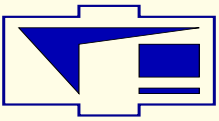
Note: γ_0^{loss} is fixed (it depends on mirror's absorption only) whereas effective loss factor $|\mathcal{A}_0|^2$ can be modified by variation of mirrors transmittivities.

Optimal $|\mathcal{A}_0|^2$:

$$|\mathcal{A}_0^{\text{opt}}|^2 = \frac{1 + \cos^2 \alpha}{\sqrt{2}},$$

$$\xi^2(\Omega_0) \geq \xi_{\min}^2 = \frac{\gamma_0^{\text{loss}}}{\Omega_0} \left(2 + \frac{3}{\sqrt{2}} \right), \quad \xi_{\min} \simeq 0.06,$$

$$\frac{\Delta\Omega_{\min}}{\Omega_0} = \frac{2\gamma_0^{\text{loss}}}{\Omega_0} \sqrt{2 + \frac{3}{\sqrt{2}}}.$$



Change of parameters planned for Adv.LIGO

We have restriction

$$\delta_0 < \frac{2\gamma_0^{\text{load}}}{T_s^2}, \quad (13)$$

$\gamma_0^{\text{load}}/T_s^2$ is the relaxation rate of alone FP cavity in one arm. In the double resonance regime we have to have

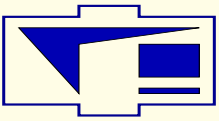
$$\delta_0 \approx \sqrt{2}\Omega_0 \approx 10^3 \text{ s}^{-1}$$

However, for the Advanced LIGO parameters

$$\frac{2\gamma_0^{\text{load}}}{T_s^2} \simeq 100 \text{ s}^{-1} \quad (14)$$

It is less by order than required. The problem can be solved by modification of the Advanced LIGO parameters:

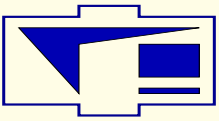
$$T_s^2 \rightarrow 0.005, \quad T^2 \rightarrow 0.05$$



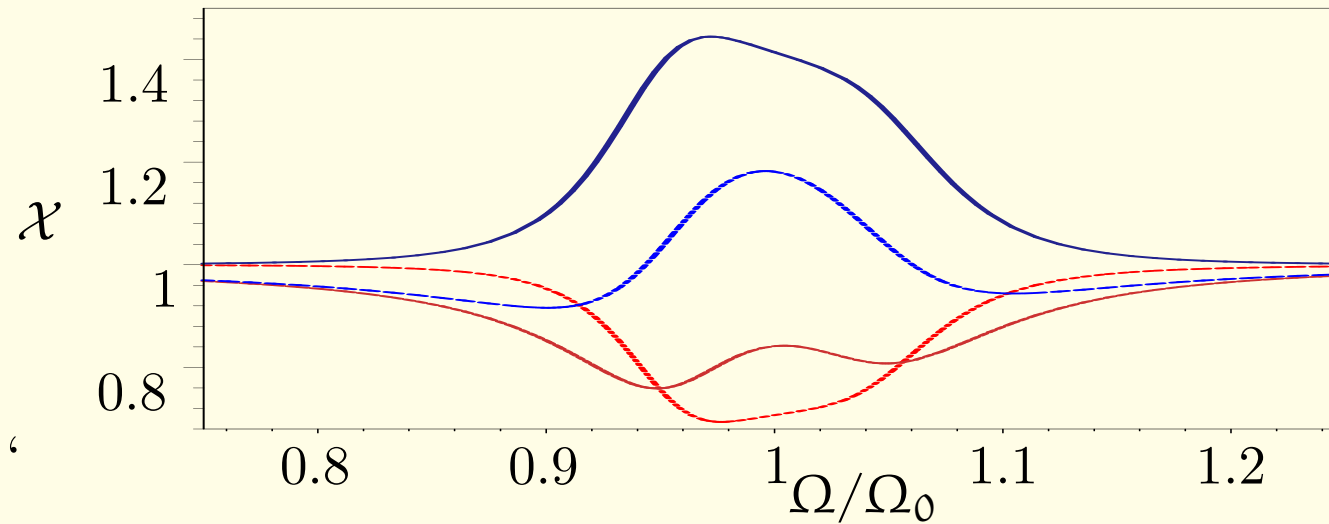
Squeezing factor \mathcal{X}

$$\mathcal{X}(\Omega) \equiv \sqrt{\frac{\langle \mathbf{b}_\zeta \mathbf{b}_\zeta^+ + \mathbf{b}_\zeta^+ \mathbf{b}_\zeta \rangle}{\langle \mathbf{a}_{\text{vac}} \mathbf{a}_{\text{vac}}^+ + \mathbf{a}_{\text{vac}}^+ \mathbf{a}_{\text{vac}} \rangle}} \quad (15)$$

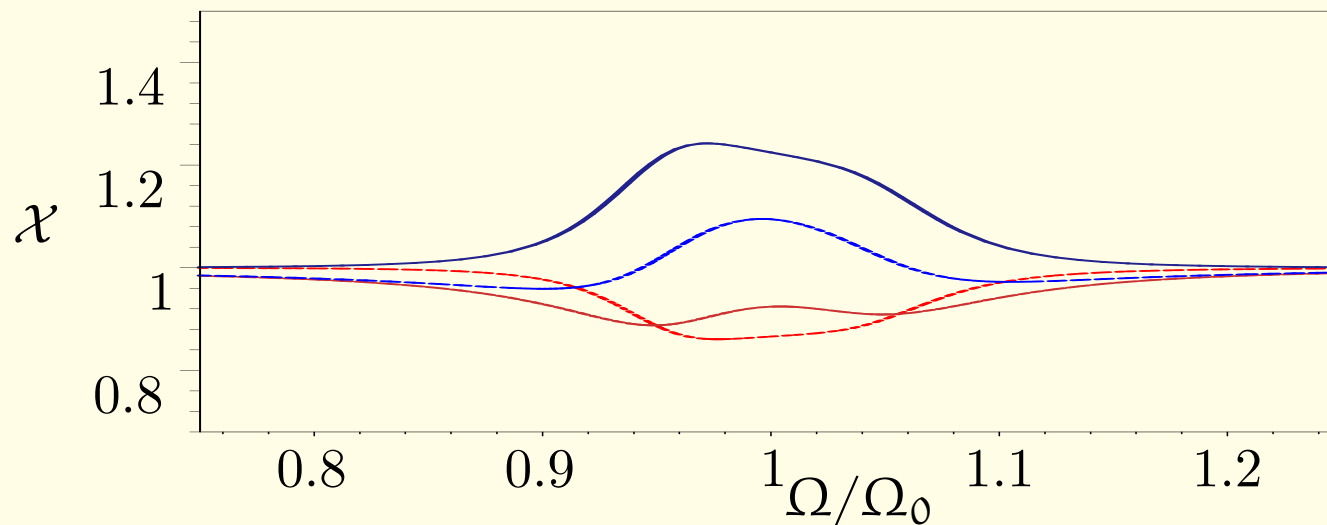
For the coherent quantum state, $\mathcal{X} = 1$.

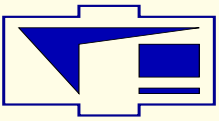


No losses: $|\mathcal{A}_0| = 0$, $\eta = 0$, $\gamma_0^{\text{no losses}}/\Omega_0 = 0.01$



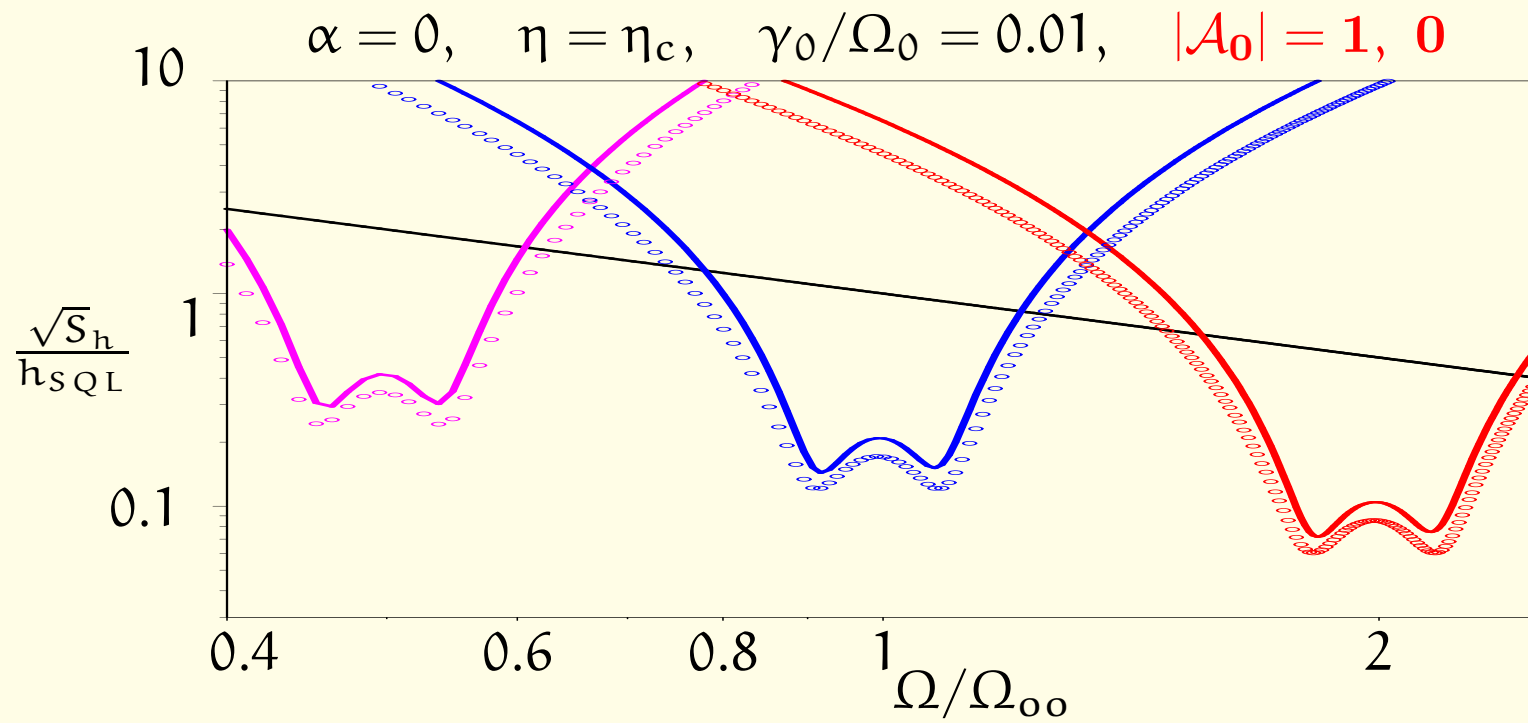
Optical losses: $|\mathcal{A}_0| = 1$, $\eta = 0$, $\gamma_0/\Omega_0 = 0.01$

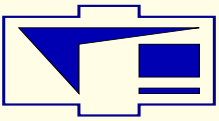




Double minima. $I_o < I_{\text{crit}}$

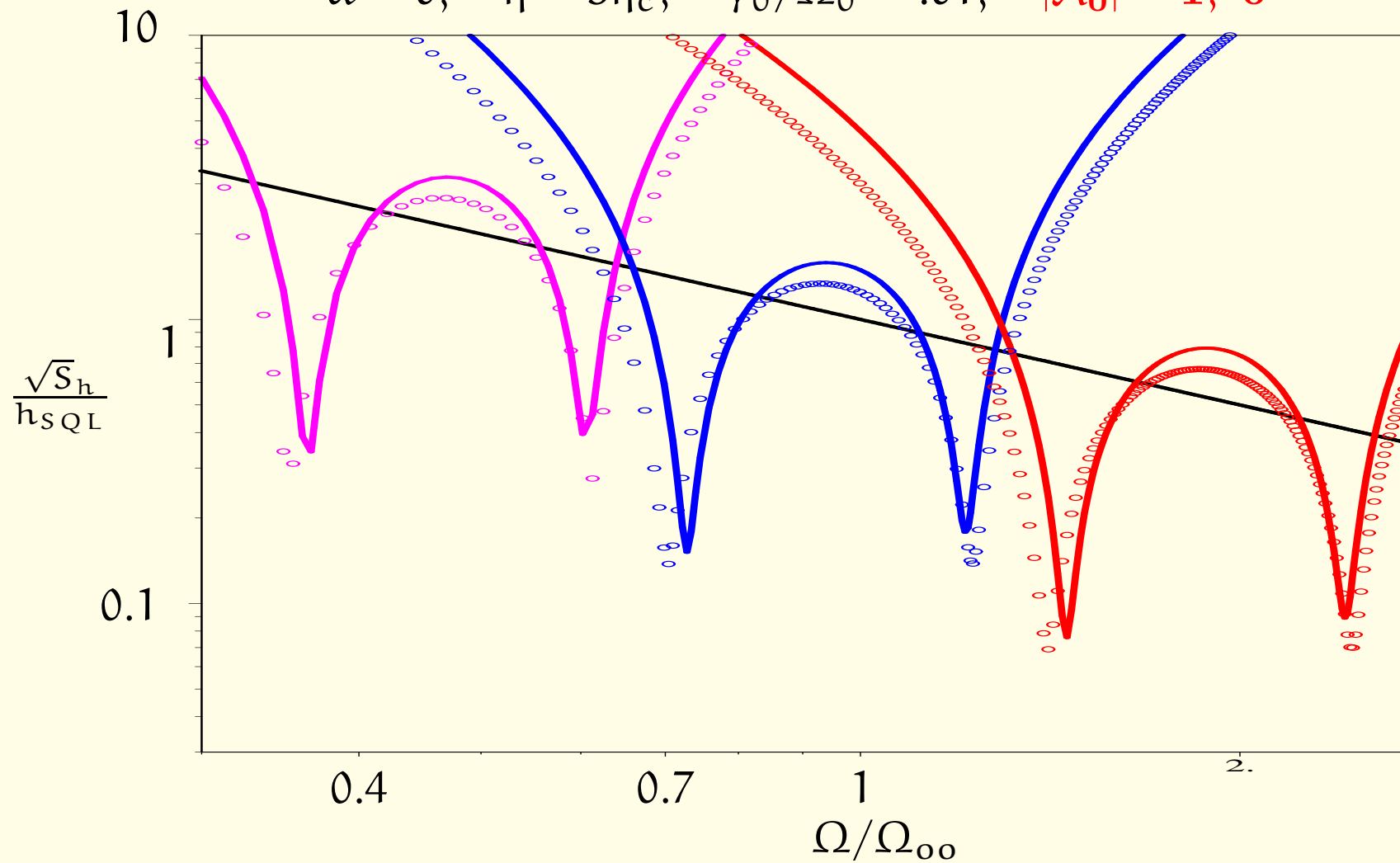
$$\eta = \eta_c, \quad \Leftrightarrow \quad \sqrt{2}\xi(\Omega_{\pm}) = \xi(\Omega_0), \quad \eta_c \simeq \sqrt{\frac{\sqrt{2}\tilde{\gamma}_0}{\Omega_0\sqrt{1+|\mathcal{A}_0|^2}}}.$$

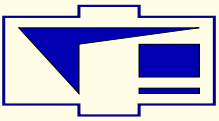




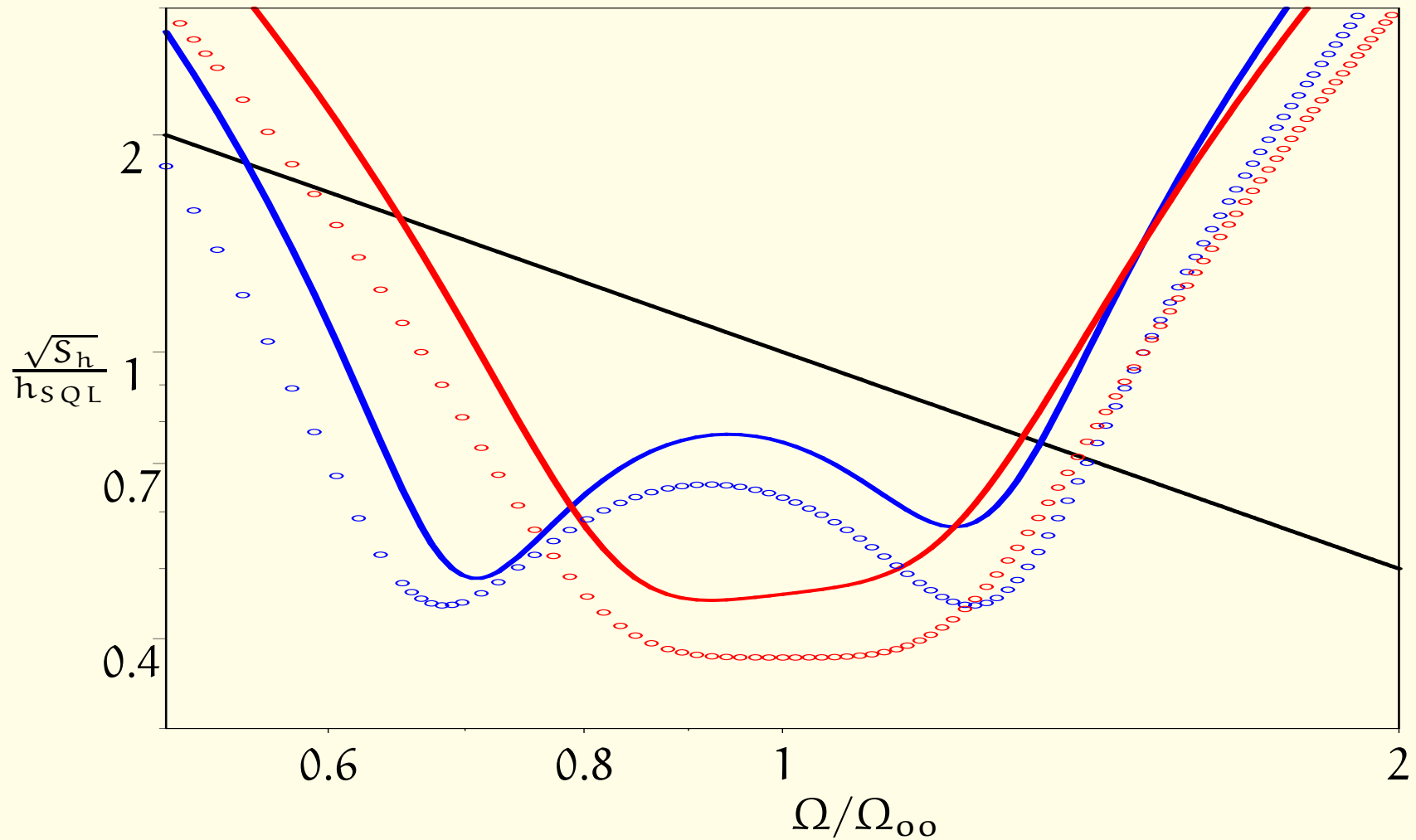
Two separate minima. $\eta = 3\eta_c$

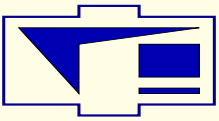
$\alpha = 0, \quad \eta = 3\eta_c, \quad \gamma_0/\Omega_0 = .01, \quad |\mathcal{A}_0| = 1, 0$



**Wide band regime**

$$\alpha = 0, \quad \gamma_0/\Omega_0 = 0.1, \quad \eta = 0, \quad \eta_c, \quad |\mathcal{A}_0| = 1, \quad 0$$





Gain in signal to noise ratio

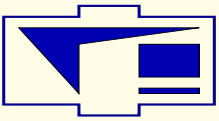
The gain in sensitivity even for wide spectrum signals.

$$s_{\text{SR}} = \frac{2}{\pi} \int \frac{|h_s(\Omega)|^2}{S_h(\Omega)} d\Omega, \quad \mathcal{P} = \frac{s_{\text{SR}}}{s_{\text{conv}}}, \quad h_s(\Omega) = \frac{\text{const}}{\Omega}. \quad (16)$$

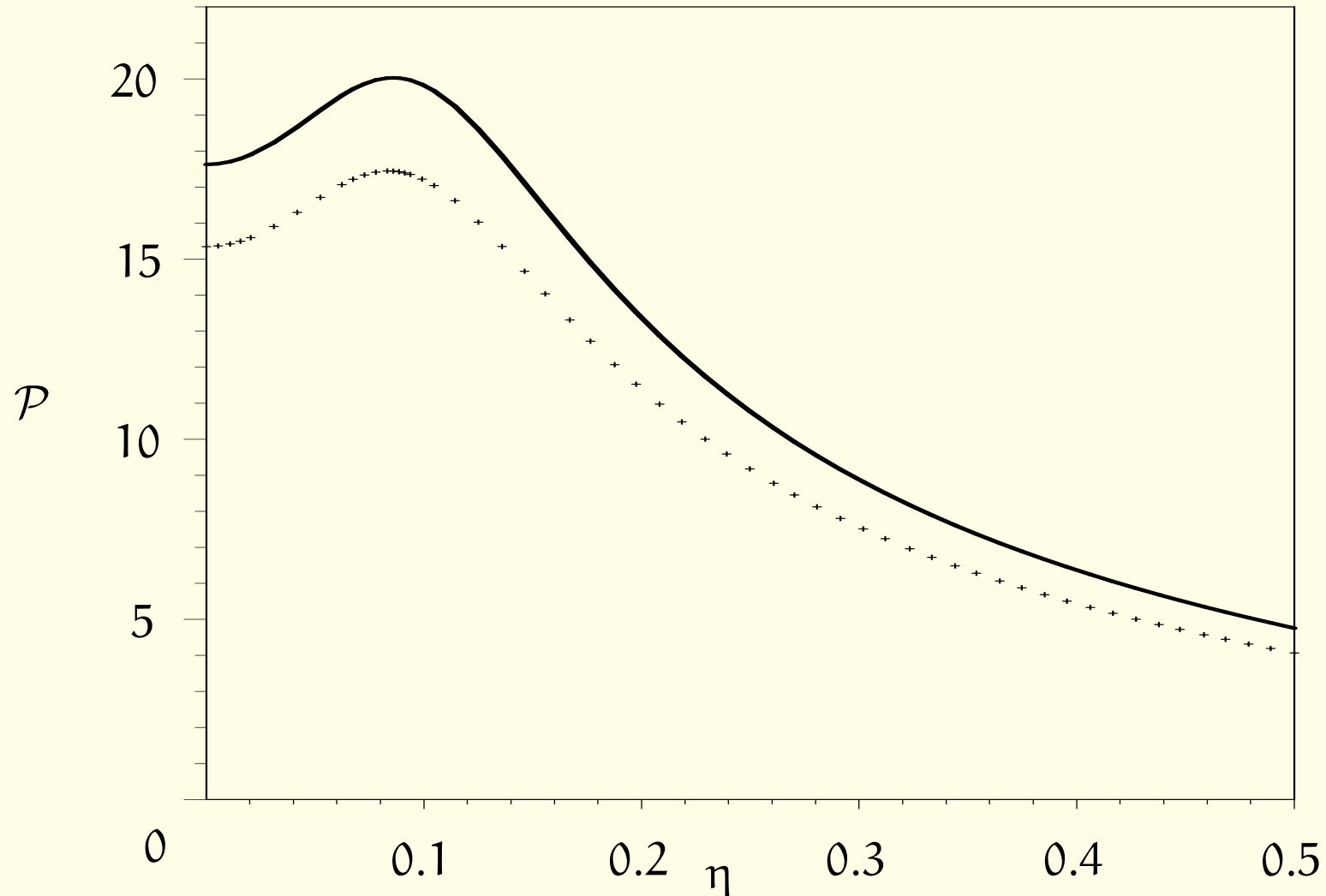
One can estimate $s_{\text{SR}} \sim \Delta\Omega/\xi^2$

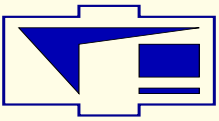
$$\xi_0 \equiv \xi(\Omega_0) \simeq \Omega_{00} \sqrt{\frac{\tilde{\gamma}_0}{\sqrt{2} \Omega_0^3}}, \quad \frac{\Delta\Omega}{\Omega_0} \simeq \sqrt{\frac{\tilde{\gamma}_0}{2\sqrt{2} \Omega_0 \sqrt{1 + |\mathcal{A}_0|^2}}},$$
$$s_{\text{SR}} \sim \Delta\Omega/\xi^2 \sim \frac{1}{\sqrt{\tilde{\gamma}_0}}, \quad s_{\text{SR}}^{\text{no losses}} \sim \frac{1}{\sqrt{\gamma_0^{\text{no losses}}}}$$

For no losses case the gain in signal to noise ratio is not restricted formally if $\gamma_0^{\text{no losses}} \rightarrow 0$. **The optical losses restricts this gain.**

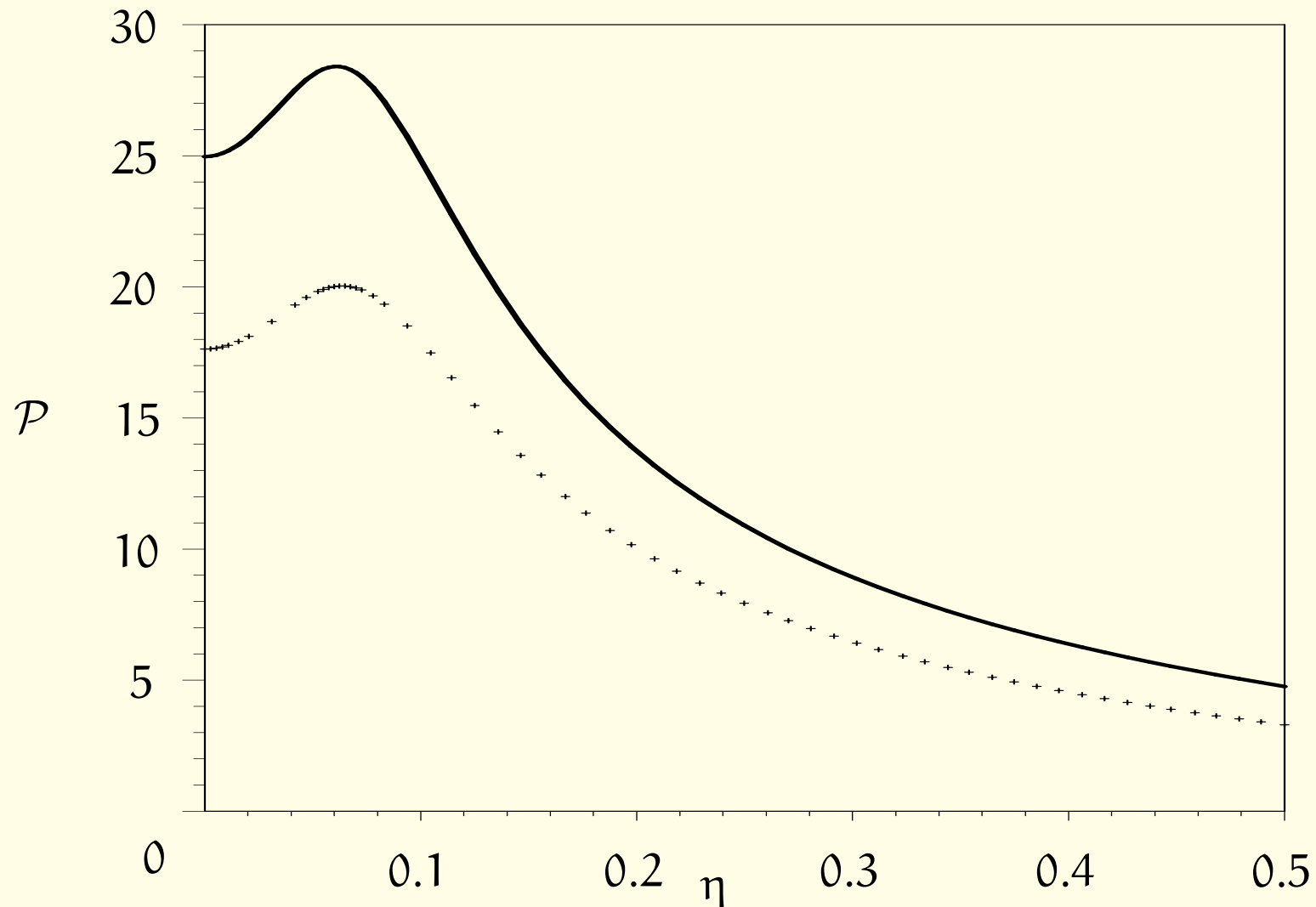
**Numerically calculated plots of signal to noise ratio gain $\mathcal{P}(\eta)$**

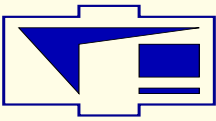
$$\alpha = 0, \quad \gamma_0/\Omega_0 = 0.0046, \quad \mathcal{A}_0^2 = 0.24$$



**Numerically calculated plots of signal to noise ratio gain $\mathcal{P}(\eta)$**

$$\alpha = \pi/2, \quad \gamma_0/\Omega_0 = 0.0046, \quad \mathcal{A}_0^2 = 0.24$$

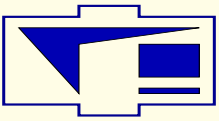




Maximization of gain \mathcal{P} at fixed optical losses

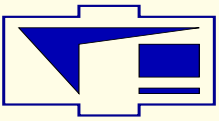
Recall: γ_0^{loss} is fixed (it depends on mirror's absorption only) whereas effective loss factor $|\mathcal{A}_0|^2$ can be modified by variation of mirrors transmittivities.

$$\eta_{\text{opt}} = \frac{\eta_c}{\sqrt[4]{3}},$$
$$|\mathcal{A}_0^{\text{opt}}|^2 \simeq \frac{(1 + \cos^2 \alpha)(\sqrt{73} - 3)}{16}$$
$$\mathcal{P}_{\text{max}} \simeq 0.61 \times \sqrt{\frac{\Omega_0}{\gamma_0^{\text{loss}}}} \simeq 20.5$$



Conclusion

- We show that usage of optical rigidity allows to circumvent SQL sensitivity for Advanced LIGO topology with power in arms approximately **smaller** by $\sqrt{2}$ times than SQL power in conventional interferometer.
- Usually losses of any kind degrade considerably of quantum measurements — it is not the case for usage of optical rigidity. With value of optical losses planned in Advanced LIGO the degradation of sensitivity due to losses is **negligible**.
- Our “narrow band” regime produces the gain in signal to noise ratio even for signals with wide spectrum — it is a unique feature of the “double” resonance regime. We demonstrate that optical losses put the limit in this gain.



Acknowledgements

This work was stimulated by series of articles by Alessandra Bounanno and Yanbei Chen. Many thanks to Yanbei for fruitful discussions.

We are very grateful to V.B. Braginsky for stimulating and very useful discussions.