

Notes about Noise in Gravitational Wave Antennae

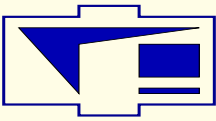
Created by Cosmic Rays

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We discussed three mechanical effects initiated by cosmic rays which may limit the sensitivity of gravitational wave antennae. Unsolved problems are formulated and several recommendations for the antennae designs are presented.

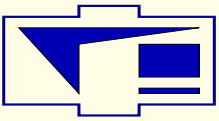


Introduction

It is reasonable to revise the contribution “of cosmic rays to the noise in gravitational wave antennae”.

We limit ourselves by only three possible mechanical “actions” on the rest masses (mirrors):

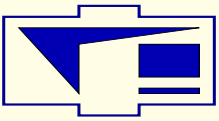
1. **Direct transfer of mechanical momentum** from cascade to the LIGO mirror.
2. **Distortion of mirror’s surface due to the heating by the cascade** and subsequent thermal expansion — thermoelastic effect.
3. **Fluctuating component of the Coulomb force** between electrically charged mirror and grounded metal elements located near the mirror’s surface.



Parameters of High Energy Cascades

\mathcal{E} is cascade energy, J_μ, J_h, J_e are the fluxes of cascades produced by muons, hadrons and by soft component, consequently, at the sea level; $N_{e, \max}$ is a number of electrons in the cascade maximum; $\Delta\mathcal{E}$ is energy lost by cascade in the 20 cm of SiO_2 ; N_{ev} is the expected number per year of events with energy losses higher than $\Delta\mathcal{E}$.

\mathcal{E} , TeV	0.5	1	2
J_μ 1/cm ² s	1.8×10^{-9}	2.8×10^{-10}	4.3×10^{-11}
J_h 1/cm ² s	2.5×10^{-9}	4.0×10^{-10}	7.2×10^{-11}
J_e 1/cm ² s	3×10^{-10}	8×10^{-11}	1.7×10^{-11}
$N_{e, \max}$	1000	2000	4000
$\Delta\mathcal{E}$, GeV	60	120	230
N_{ev}	~ 110	20	3 ÷ 4



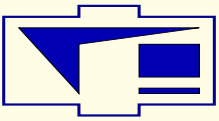
Direct transfer of mechanical momentum

$$\Delta P = \Delta \mathcal{E}/c, \quad \Rightarrow \Delta x \simeq \frac{\Delta \mathcal{E} \tau}{Mc}, \quad \tau \simeq \frac{1}{f} \simeq 0.01 \text{ s}$$

From Table: cascade with energy higher than 2 TeV created by muons, hadrons or soft component will be passing through the mirror approximately $2 \div 3$ times per year. The shower in thick 20 cm mirror will loose energy $\Delta \mathcal{E} \simeq 230 \text{ GeV}$. Hence

$$\Delta x \simeq (0.8 \div 3) \times 10^{-18} \text{ cm}, \quad \Delta L_{\text{Adv.LIGO}} \simeq hL/2 \simeq 2 \times 10^{-17} \text{ cm}$$

It is close to the amplitude of the sensitivity planned in Advanced LIGO: $\Delta L_{\text{Adv.LIGO}} \text{ cm}$.



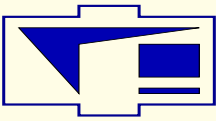
Distortion of mirror's surface

The cascade with energy 2 TeV loses the energy $\Delta\mathcal{E} \simeq 230$ GeV which mainly converts into heat on length $H = 20$ cm (thickness of mirror), The radius of cascade trace $R_c \simeq 1 \div 7$ cm is . Assuming that volume $\simeq R_c^3$ on the mirror surface can free expand we obtain:

$$\Delta H \simeq \frac{R_c}{H} \times \frac{\Delta\mathcal{E}_{\text{heat}}}{\rho C_V R_c^3} \times R_c \alpha \quad (1)$$

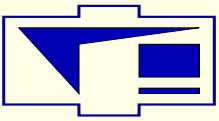
The height ΔH_{av} averaged over the square of laser beam spot with radius $r \simeq 10$ cm:

$$\Delta H_{\text{av}} \simeq \Delta H \times \frac{R_c^2}{r^2} \simeq \begin{cases} 2 \times 10^{-18} \text{ cm} & \text{if } \Delta\mathcal{E} = 60 \text{ GeV, } R_c = 1 \text{ cm} \\ \mathbf{8 \times 10^{-17} \text{ cm}} & \text{if } \Delta\mathcal{E} = 230 \text{ GeV, } R_c = 7 \text{ cm} \end{cases}$$

**“Parallel” event:**

$$\Delta H_{\text{av,parall}} \simeq \frac{\Delta \mathcal{E}}{\rho C_V R_c^2 2r} \times R_c \alpha \times \frac{R_c}{r} \simeq$$
$$\simeq \begin{cases} 2 \times 10^{-17} \text{ cm} & \text{if } \Delta \mathcal{E} = 60 \text{ GeV} \\ 6 \times 10^{-17} \text{ cm} & \text{if } \Delta \mathcal{E} = 230 \text{ GeV} \end{cases}$$

Such events are more rare than “perpendicular” ones by a factor of about $R_c^2/r^2 \simeq 0.01 \div 0.5$.



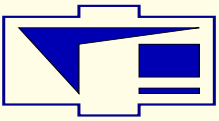
Fluctuating Coulomb force

The cosmic rays are very sensitive even to the thin layers of matter. If cascade developed in the heavy matter is coming to the light one, it brings to the light material more electrons than takes away. An example: number of electrons produced in iron is about 3 times larger than number produced in fused silica:

$$\frac{N_{e,Fe}}{N_{e,SiO_2}} \approx 3$$

This electron excess will stay near the surface of the mirror and will give an additional charge to it.

\mathcal{E} , TeV	0.5	1	2
$N(\mathcal{E}_e < 1\text{MeV})$	450	900	1700



The initial design of the “entourage” of the suspended mirror includes several parts which are planned to be made of metal (“cradles”, “stoppers” etc). Coulomb d.c. force:

$$F_{dc} \simeq 2\pi S\sigma^2 \simeq 1.5 \times 10^{-2} \text{ dyn}, \quad \text{if } S = 10^2 \text{ cm}^2$$

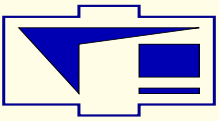
More important is the a.c. component of the Coulomb force may mimic the force F_{grav} which antenna has to register:

$$F_{grav} \simeq \frac{hLM\omega_{grav}^2}{2} \simeq 3 \times 10^{-7} \text{ dyn} \quad (2)$$

One can estimate from Table the relative fluctuations of charge density $\Delta\sigma/\sigma$ caused by a single cascade with energy $\mathcal{E} = 2 \text{ MeV}$:

$$\frac{\Delta\sigma}{\sigma} \simeq 10^{-6} \div 2 \times 10^{-5}$$

We see that it is enough to produce an a.c. component of Coulomb force larger than F_{grav} if relatively large surface of grounded metal plate will be located near the mirror.



Conclusion

There are two evident recommendations for the consequent measurement and analysis:

1. To measure bursts of electrons which appears on the mirror's surface with resolution better then 10^2 e/cm² and time shorter than 10^{-2} sec.
2. To analyze the possibility to cover the mirror's surface over the coating with a transparent few nanometers thick layer whith substantial conductivity to reduce the d.c. component of electrical charge.

In all three effects considered above the mechanical action on the mirror produces a step like displacement (either of the mirror's center of mass or of its surface). This type of response is similar to the one predicted for shape of gravitational wave bursts created in the process of supernova explosion predicted by V.Imshennik.