Optical rigidity in signal-recycled gravitational-wave detectors with optical losses

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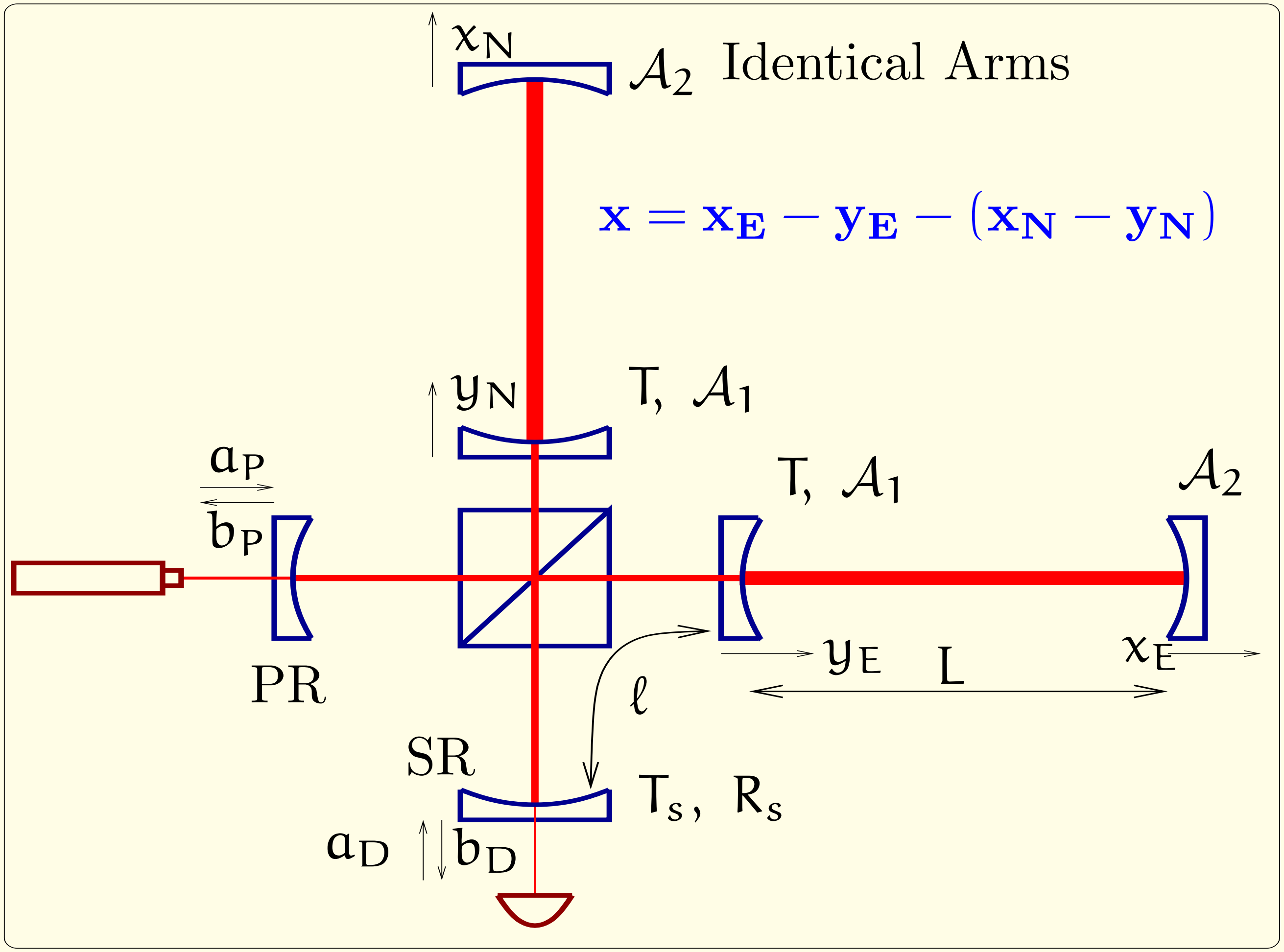
Summary

- The usage of the optical rigidity allows to obtain **better than Standard Quantum Limit (SQL)** sensitivity without increasing laser pumping.
- The gain in sensitivity is inversely proportional to the bandwidth where this gain is achieved. **One can modify "on line" the sensitivity dependence** on frequency from a single minimum to a double minimum dependence.
- **The optical losses degrade sensitivity** of measurements with usage of optical rigidity **by relatively small value** in contrast with other measurements beating SQL which degradation due to losses is considerable.
- This "narrow band" regimes provides the **gain in signal to noise ratio** even for signals with **wide spectrum** and the **losses decreases this gain by relatively small** factor.

Advanced LIGO inteferometer with losses in arms



Introduction

The sensitivity of interferometer $\xi(\Omega)$ consists of two terms:

$$\xi^2(\Omega) = \frac{S_h(\Omega)}{h^2_{\rm SQL}(\Omega)} = \frac{2}{\hbar m \Omega^2}\left(S_F(\Omega) + \left|Z(\Omega)\right|^2 S_x(\Omega)\right), \qquad (1)$$

$$Z(\Omega) = \frac{F(\Omega)}{x(\Omega)} \qquad (2)$$

If S_x and S_F do not correlate: $S_F(\Omega)\, S_x(\Omega) \geq \hbar^2/4$. Then

$$\xi^2_{\rm min}(\Omega) = \frac{|Z(\Omega)|}{m_0\Omega^2} < 1, \quad \text{or} \quad \xi^2 = \frac{\Delta\Omega}{\Omega} < 1 \qquad (3)$$

in the bandwidth $\Delta\Omega$ where $|Z(\Omega)| < m_0\Omega^2$
as for conventional oscillator.

The sophisticated frequency dependence

of optical rigidity provides sensitivty:

$$\xi^{\mathbf{2}} = \frac{\mathbf{S_h}}{\mathbf{h}^{\mathbf{2}}_{\mathrm{SQL}}} = \left(\frac{\Delta\Omega}{\Omega}\right)^{\mathbf{2}}, \qquad (4)$$

For conventional oscillator sensitivity is worse:

$$\xi^2_{\mathrm{conv}} = \frac{\Delta\Omega}{\Omega}$$

Approximations and notations for difference mode

$$T,\ T_s,\ r_1,\ r_2 \ll 1, \quad \Omega \ll \omega_o, \quad \frac{L\Omega}{c} \ll 1, \quad \ell \ll L$$

$$\delta_0 = \frac{cT^2 R_s \sin 2\phi}{2L\left|1 + R_s e^{2i\phi}\right|^2} \quad \text{— detuning}$$

$$\gamma_0 = \gamma_0^{\text{load}} + \gamma_0^{\text{loss}}, \quad \text{— relaxation rate of difference mode}$$

$$\gamma_0^{\text{load}} = \frac{c\,T^2 T_s^2}{4L\left|1 + R_s e^{2i\phi}\right|^2}, \quad \gamma_0^{\text{loss}} = \frac{c(\mathcal{A}_1^2 + \mathcal{A}_2^2)}{4L}. \quad \mathcal{A}_0^2 = \frac{\gamma_0^{\text{loss}}}{\gamma_0^{\text{load}}}$$

"loaded" γ_0^{load} is produced by transparencies of SR and input mirrors and "intrinsic" γ_0^{loss} is produced exclusively the intrinsic losses in mirrors. $\gamma_0 \ll \Omega_o$

Parameters planned to use in Advanced LIGO

Transmittivity of SR mirror	$T_s^2 = 0.05$
Transmittivity of input mirrors in arms	$T^2 = 0.005$
Loss coefficient of each mirror in arms	$\mathcal{A}_1^2 = \mathcal{A}_2^2 = 1.5 \times 10^{-5}$
Length of interferometer arm	$L = 4$ km
Effective loss factor	$\lvert\mathcal{A}_0\rvert^2 = 0.24$
Relaxation rate of difference mode	$\gamma_0 = 2.9\ \mathrm{s}^{-1}$
"Intrinsic" relaxation rate	$\gamma_0^{\mathrm{loss}} = 0.56\ \mathrm{s}^{-1}$
Mean frequency of gravitational wave range	$\Omega_0 = 2\pi \times 100\ \mathrm{s}^{-1}$

Optical rigidity

$$K(\Omega) \equiv \frac{-F_{\text{ponderomotive}}}{z} = \frac{8\omega_0 I_o \delta_0}{cL\mathcal{D}}, \tag{5}$$

$$\mathcal{D} = \left[\gamma_0 - i\delta_0 - i\Omega\right]\left[\gamma_0 + i\delta_0 - i\Omega\right], \tag{6}$$

I_o is the light power circulating inside each arm.

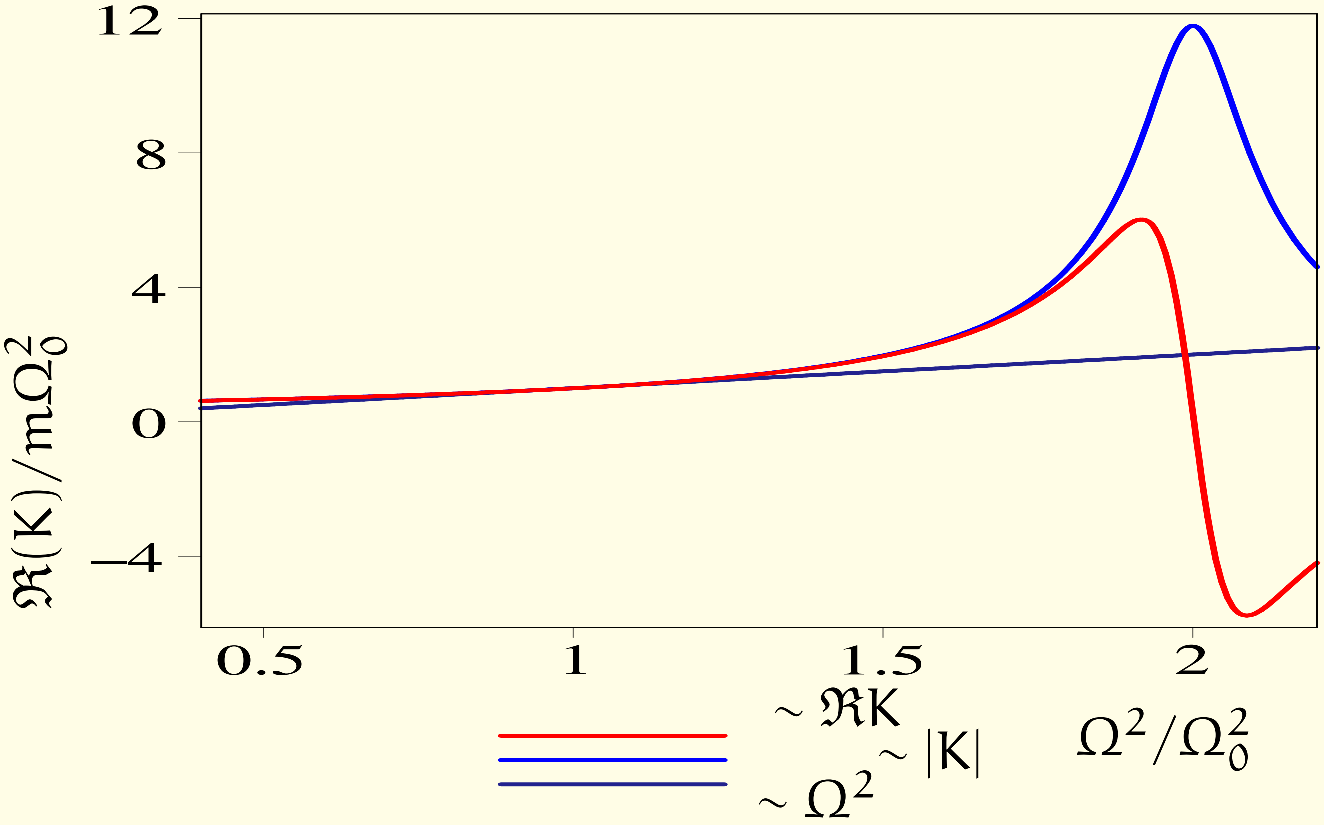
The **sofisticated** dependence $K(\Omega)$. Resonance:

$$K(\Omega) - m\Omega^2 \simeq 0.$$

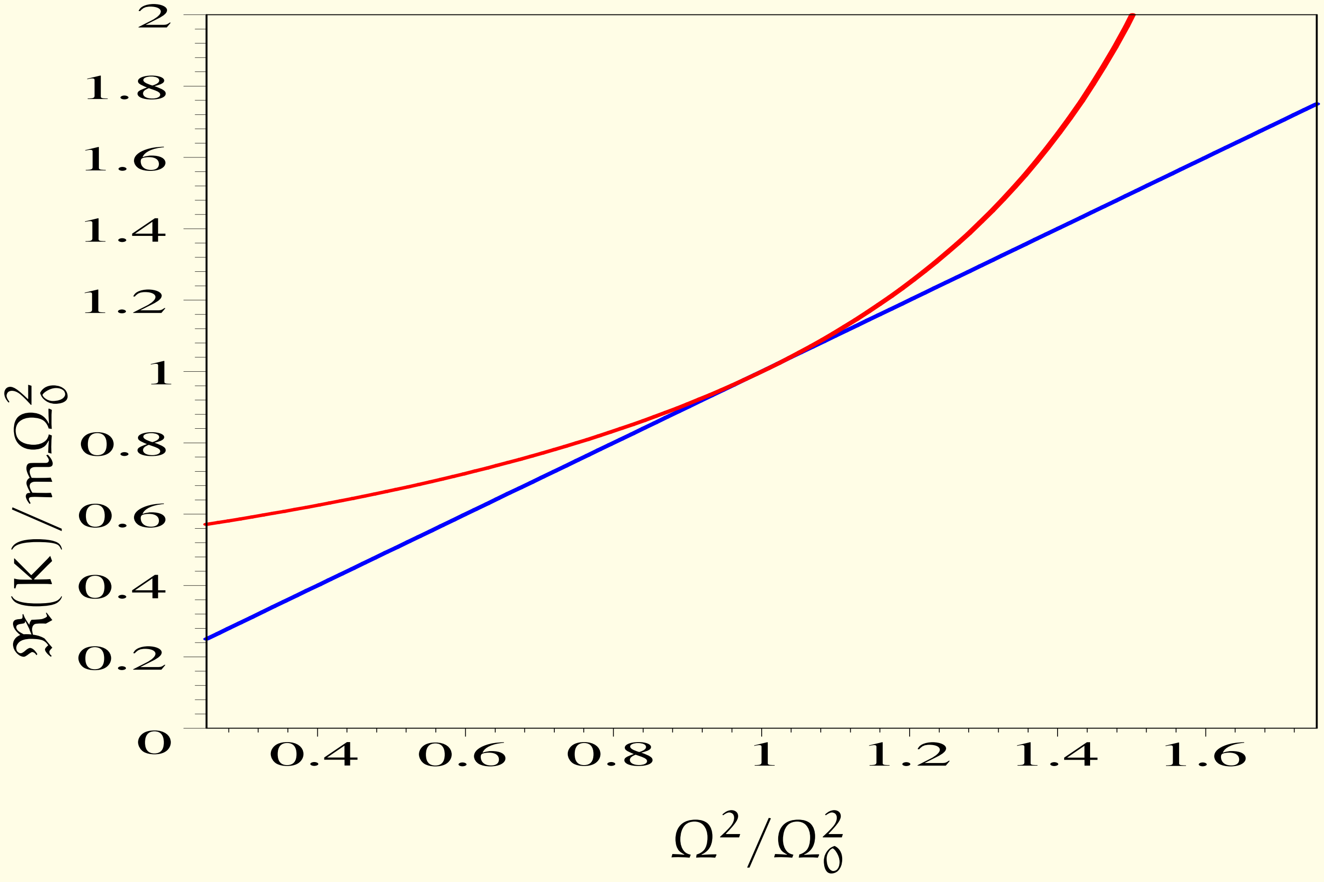
Coventional oscillator: K – const, one resonance frequency Ω_0:

$$\Omega_0^2 = \frac{K}{m}$$

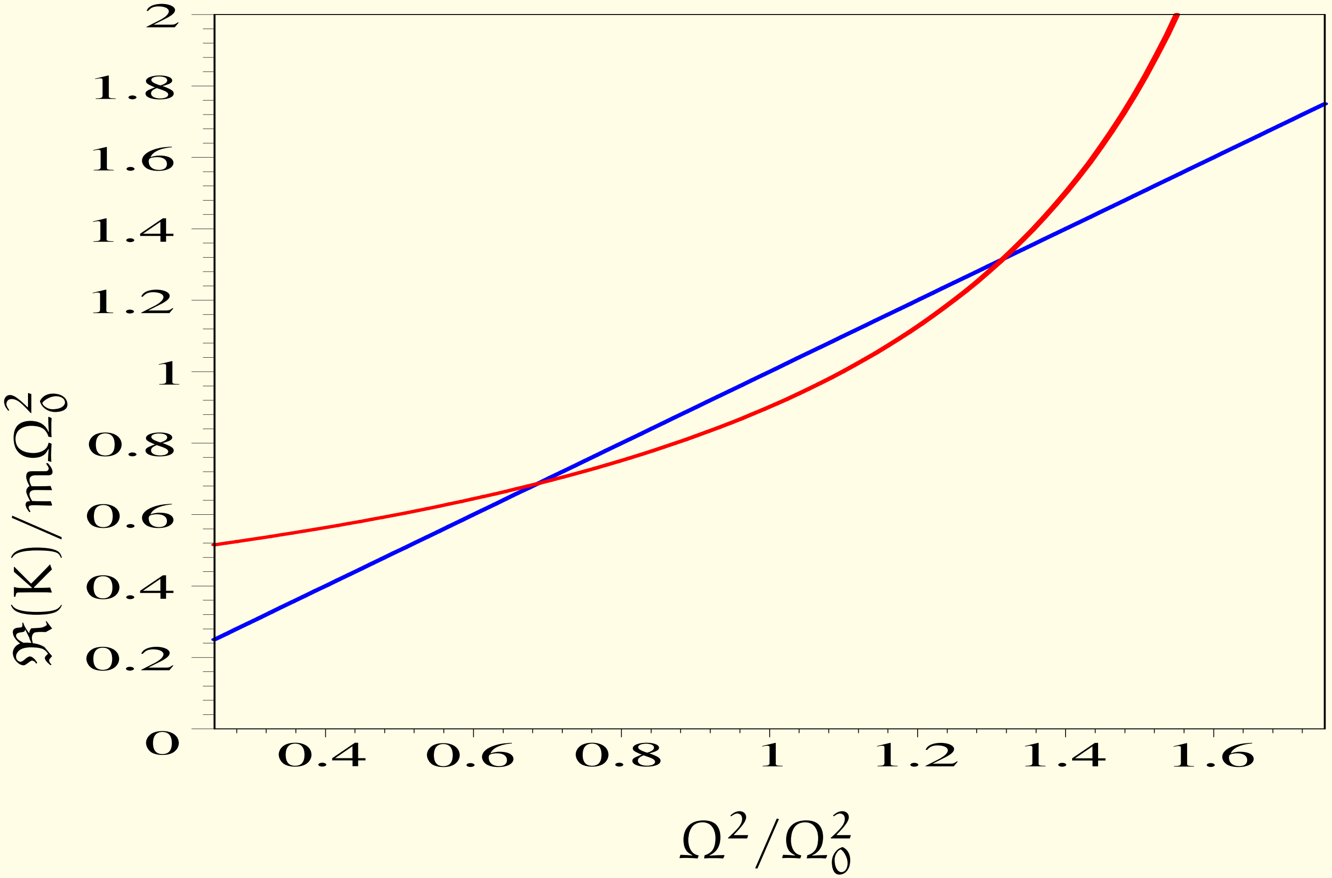
Optical rigidity: sophisticated dependence on frequency



Double resonance (second oder pole of susceptibility)



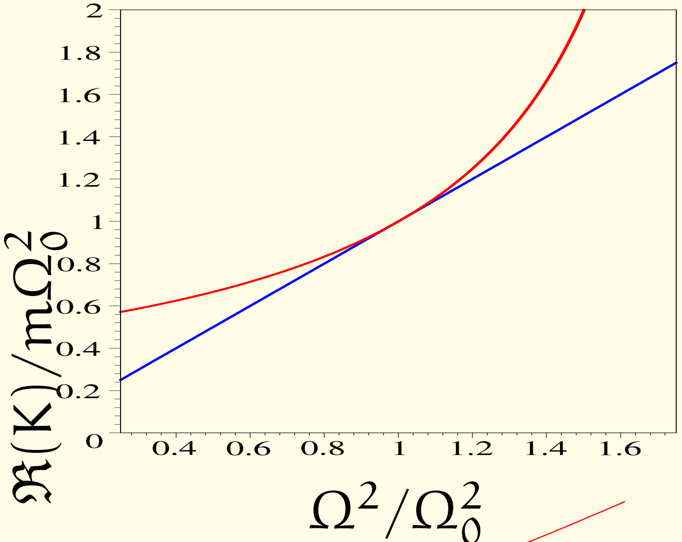
Two first order poles of susceptibility

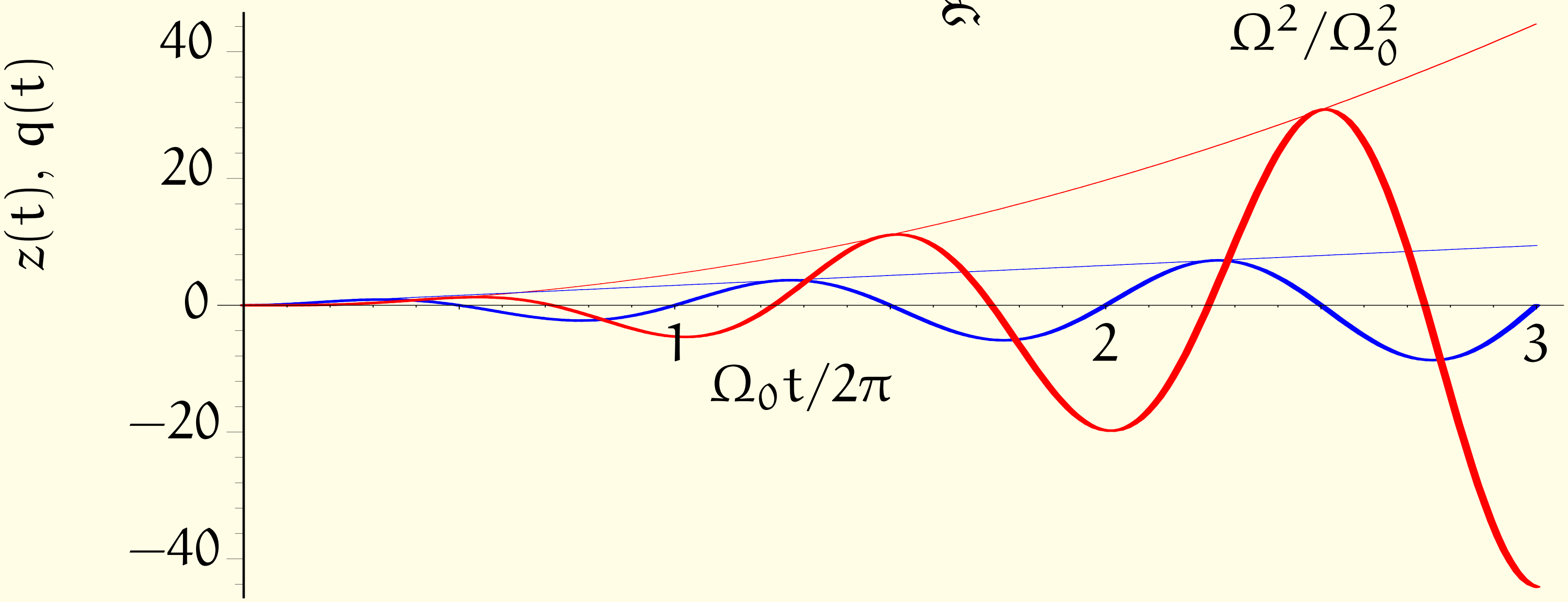


“Double” resonance

Conventional: $q(t) = \dfrac{f_0 t \sin(\Omega_0 t)}{2\Omega_0}$

“Double” res.: $z(t) = \dfrac{-f_0 t^2 \cos(\Omega_0 t)}{8}$





Quadrature Component Analysis

S_h is one-sided spectral density of noise of quadrature component b_ζ recalculated to variation of dimensionless metric h_s.
normalization to SQL sensitivity $h_{SQL}(\Omega_{oo})$:

$$b_\zeta = \frac{b_D e^{-i\zeta} + b_{D-}^{+} e^{i\zeta}}{\sqrt{2}}, \quad \xi^2(\Omega) = \frac{S_h}{h_{SQL}(\Omega_{oo})^2} \tag{7}$$

For $\gamma_0 \ll \Omega_0$ sensitivity ξ has minimum at frequency Ω_0 if
$I_o \simeq I_o^{crit}$

$$\boldsymbol{\Omega_0^2} \simeq \frac{\boldsymbol{\delta_0^2}}{\mathbf{2}}, \quad \mathbf{I_o^{crit}} = \frac{mL\Omega_0^4}{8k(\delta_0)} \simeq \frac{\mathbf{I_{SQL}}(\Omega_0)}{\sqrt{\mathbf{2}}},$$

$$\text{Then} \quad \xi^2 \sim (\Omega^2 - \Omega_0^2) + \eta^2\Omega_0^4 + \dots, \quad \eta = \sqrt{\mathbf{1} - \frac{\mathbf{I_o}}{\mathbf{I_o^{crit}}}}, \tag{8}$$

Single minimum case

$\eta = 0$ — power is equal to critical power: $I_o = I_o^{crit}$.

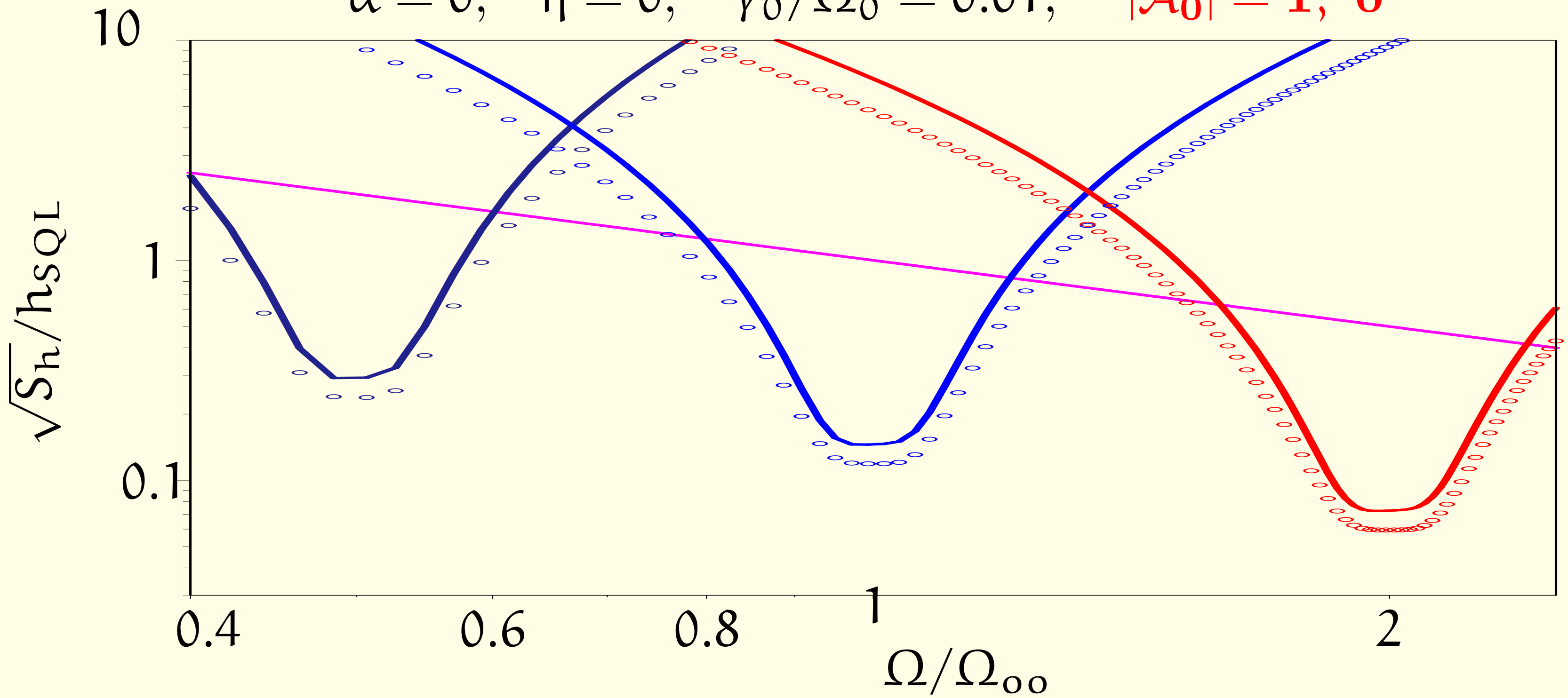
$$\xi_0 \equiv \xi(\Omega_0) \simeq \Omega_{oo}\sqrt{\frac{\tilde{\gamma}_0}{\sqrt{2}\,\Omega_0^3}}\,, \qquad (9)$$

$$\frac{\Delta\Omega}{\Omega_0} \simeq \sqrt{\frac{\tilde{\gamma}_0}{2\sqrt{2}\,\Omega_0\sqrt{1+|\mathcal{A}_0|^2}}}, \qquad (10)$$

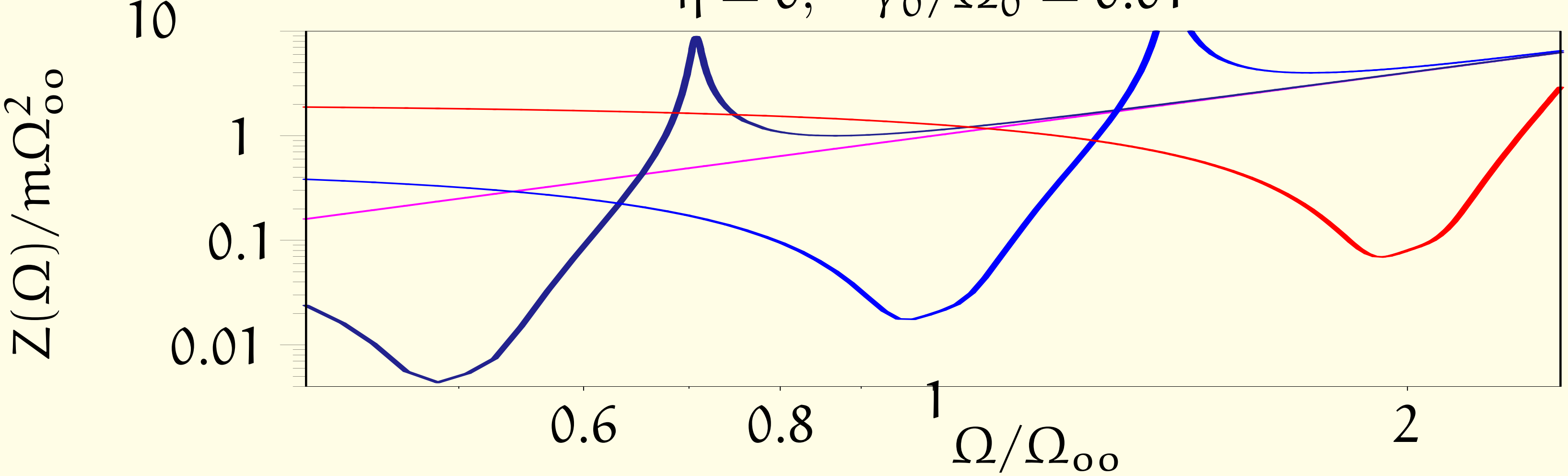
$$\tilde{\gamma}_0 = \gamma_0\sqrt{\left(2-\sin^2\alpha\right)^2 + 2|\mathcal{A}_0|^2}, \qquad (11)$$

$$|\mathcal{A}_0|^2 = \frac{\left(\mathcal{A}_1^2 + \mathcal{A}_2^2\right)\left|1 + R_s e^{i\phi}\right|^2}{T^2 T_s^2} < 1 \qquad (12)$$

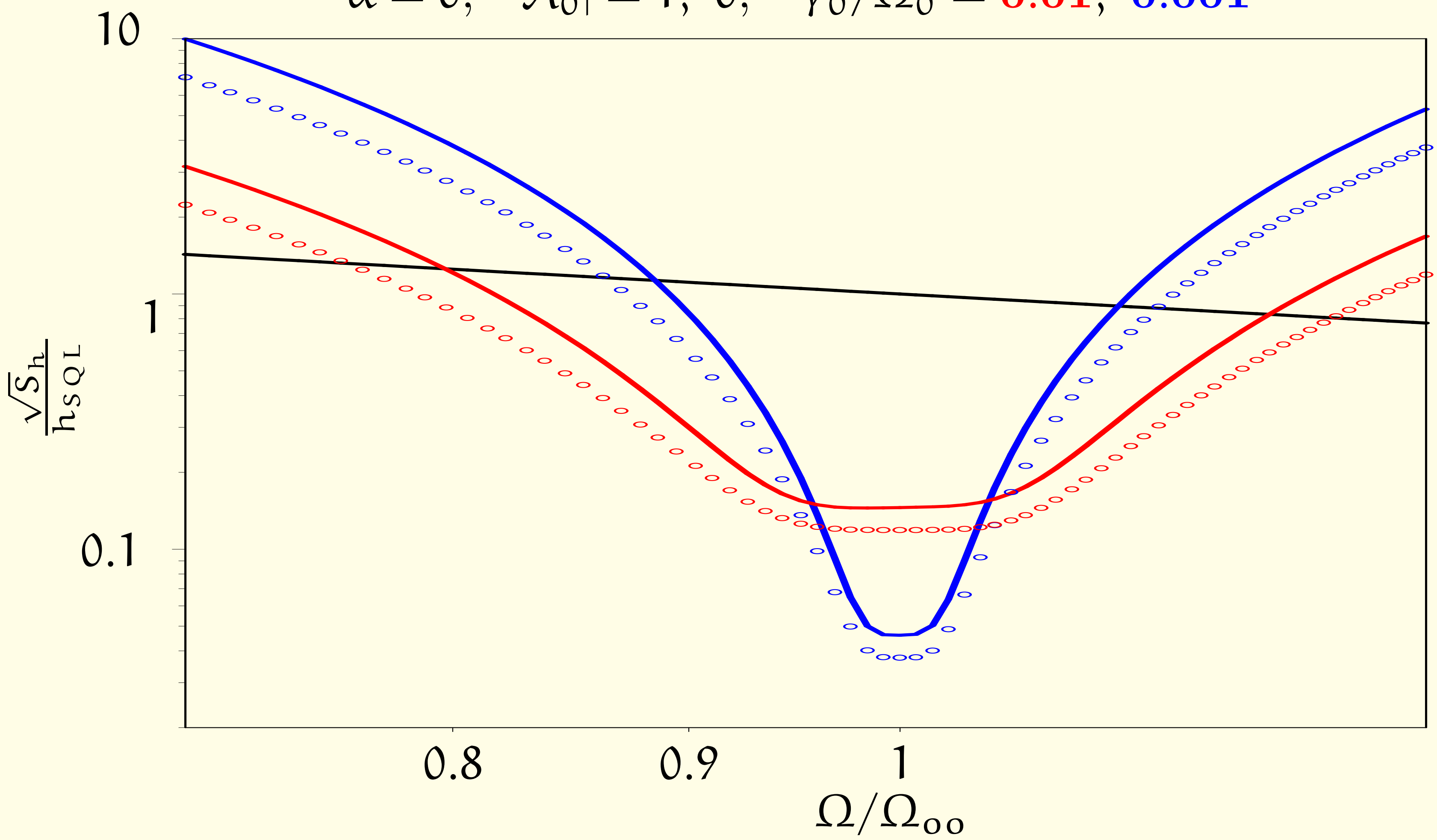
$\alpha = 0, \quad \eta = 0, \quad \gamma_0/\Omega_0 = 0.01, \quad |\mathcal{A}_0| = 1,\ 0$



$\eta = 0, \quad \gamma_0/\Omega_0 = 0.01$



$\alpha = 0, \quad \mathcal{A}_0| = 1,\ 0, \quad \gamma_0/\Omega_0 =$ **0.01**, **0.001**



Minimum of ξ

Note: $\gamma_0^{\rm loss}$ is fixed (it depends on mirror's absorption only) whereas effective loss factor $|\mathcal{A}_0|^2$ can be modified by variation of mirrors transmttivities.

Optimal $|\mathcal{A}_0|^2$:

$$|\mathcal{A}_0^{\rm opt}|^2 = \frac{1+\cos^2\alpha}{\sqrt{2}},$$

$$\xi^2(\Omega_0) \ge \xi^2_{\rm min} = \frac{\gamma_0^{\rm loss}}{\Omega_0}\left(2+\frac{3}{\sqrt{2}}\right), \quad \xi_{\rm min} \simeq 0.06,$$

$$\frac{\Delta\Omega_{\rm min}}{\Omega_0} = \frac{2\gamma_0^{\rm loss}}{\Omega_0}\sqrt{2+\frac{3}{\sqrt{2}}}\,.$$

Change of parameters planned for Adv.LIGO

We have restriction

$$\delta_0 < \frac{2\gamma_0^{\rm load}}{T_s^2},. \qquad (13)$$

$\gamma_0^{\rm load}/T_s^2$ is the relaxation rate of alone FP cavuity in one arm. In the double resonance regime we have to have

$$\delta_0 \approx \sqrt{2}\Omega_0 \approx 10^3 \ {\rm s}^{-1}$$

However, for the Advanced LIGO parameters

$$\frac{2\gamma_0^{\rm load}}{T_s^2} \simeq 100 \ {\rm s}^{-1} \qquad (14)$$

It is less by order than required. The problem can be solved by modification of the Advanced LIGO parameters:

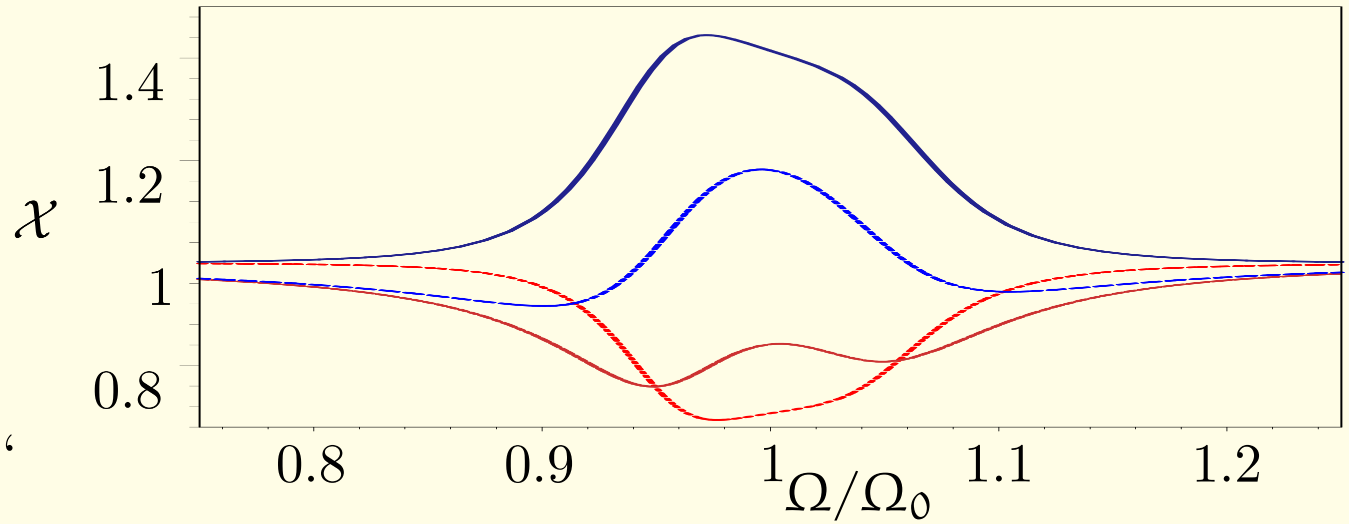
$$T_s^2 \to 0.005, \quad T^2 \to 0.05$$

Squeezing factor $\mathcal{X}$

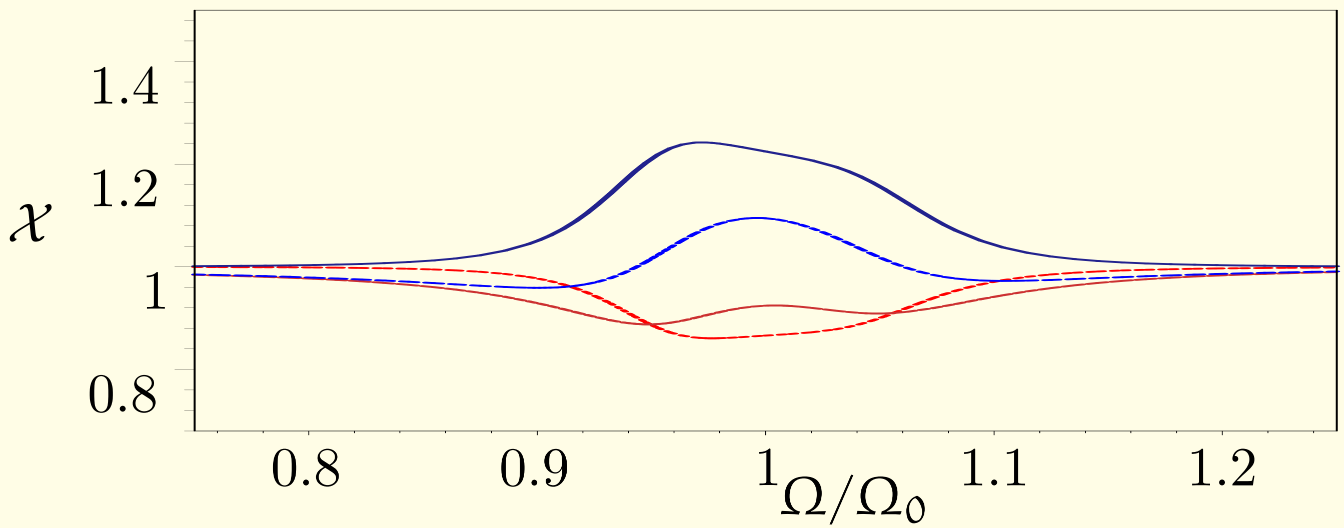
$$\mathcal{X}(\Omega) \equiv \sqrt{\frac{\langle b_\zeta b_\zeta^+ + b_\zeta^+ b_\zeta \rangle}{\langle a_{\rm vac} a_{\rm vac}^+ + a_{\rm vac}^+ a_{\rm vac} \rangle}} \tag{15}$$

For the coherent quantum state, $\mathcal{X} = 1$.

No losses: $|\mathcal{A}_0| = 0, \quad \eta = 0, \quad \gamma_0^{\text{no losses}}/\Omega_0 = 0.01$



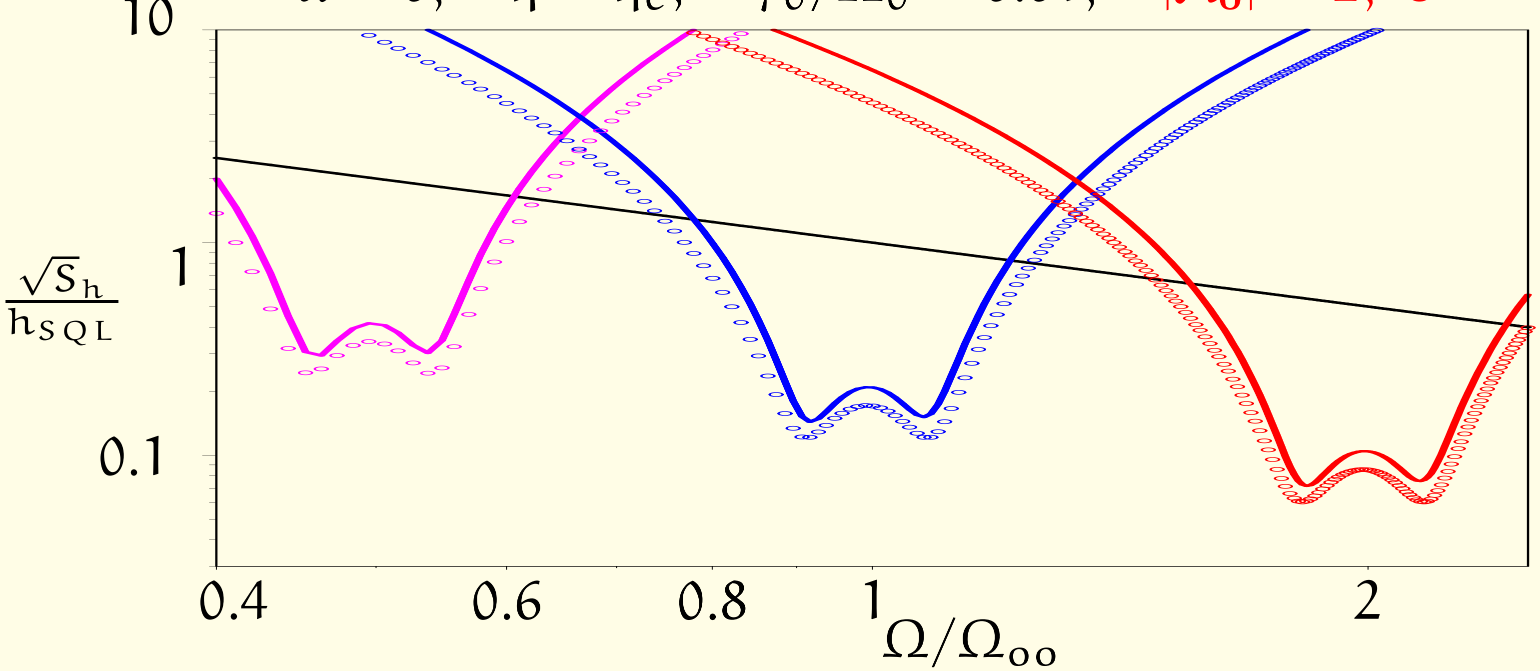
Optical losses: $|\mathcal{A}_0| = 1, \quad \eta = 0, \quad \gamma_0/\Omega_0 = 0.01$



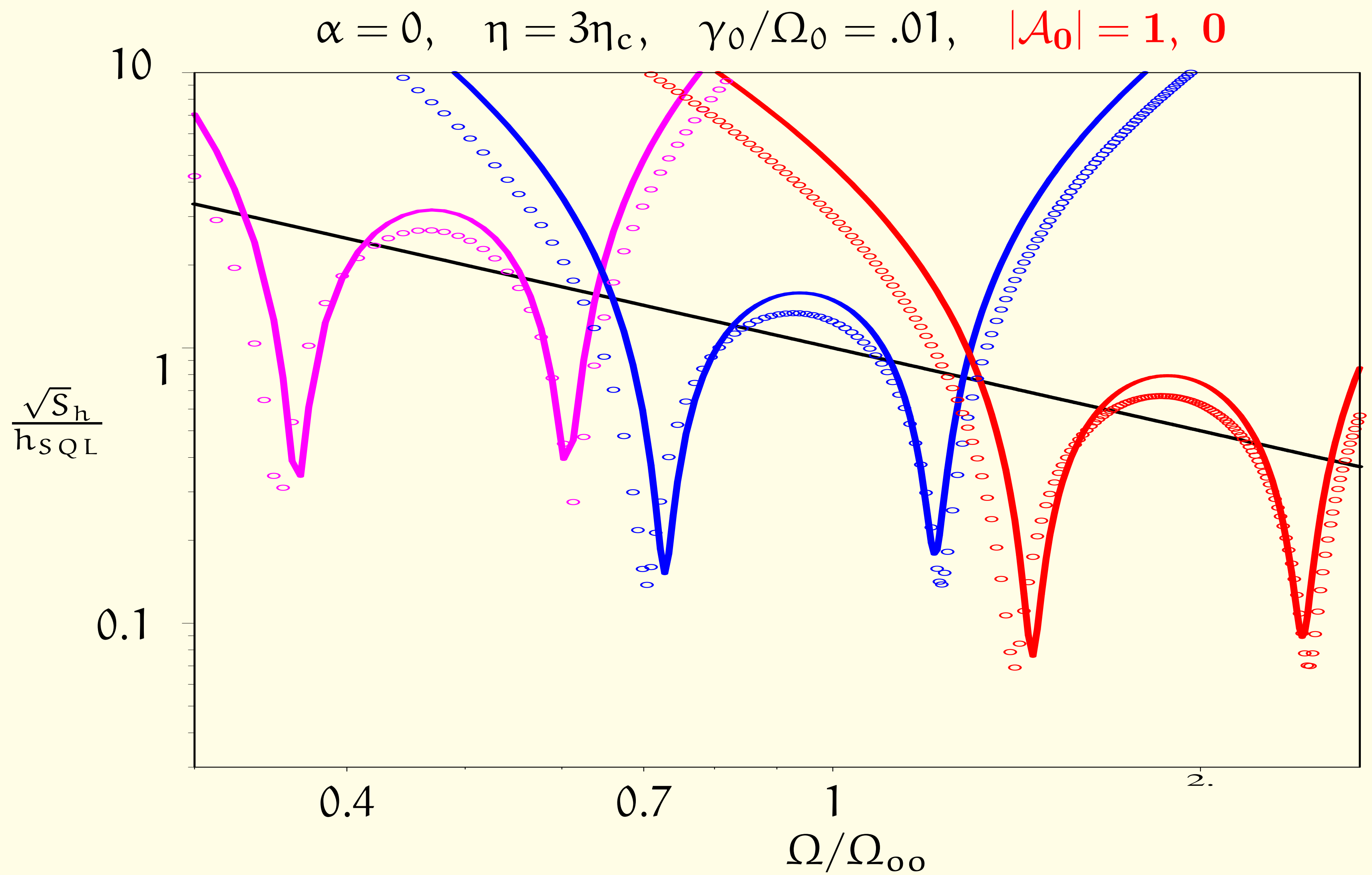
Double minima. $I_o < I_{crit}$

$$\eta = \eta_c, \quad \Leftrightarrow \ \sqrt{2}\xi(\Omega_\pm) = \xi(\Omega_0), \quad \eta_c \simeq \sqrt{\frac{\sqrt{2}\,\tilde{\gamma}_0}{\Omega_0\sqrt{1+|\mathcal{A}_0|^2}}}\,.$$

$\alpha = 0, \quad \eta = \eta_c, \quad \gamma_0/\Omega_0 = 0.01, \quad |\mathcal{A}_0| = 1,\ 0$

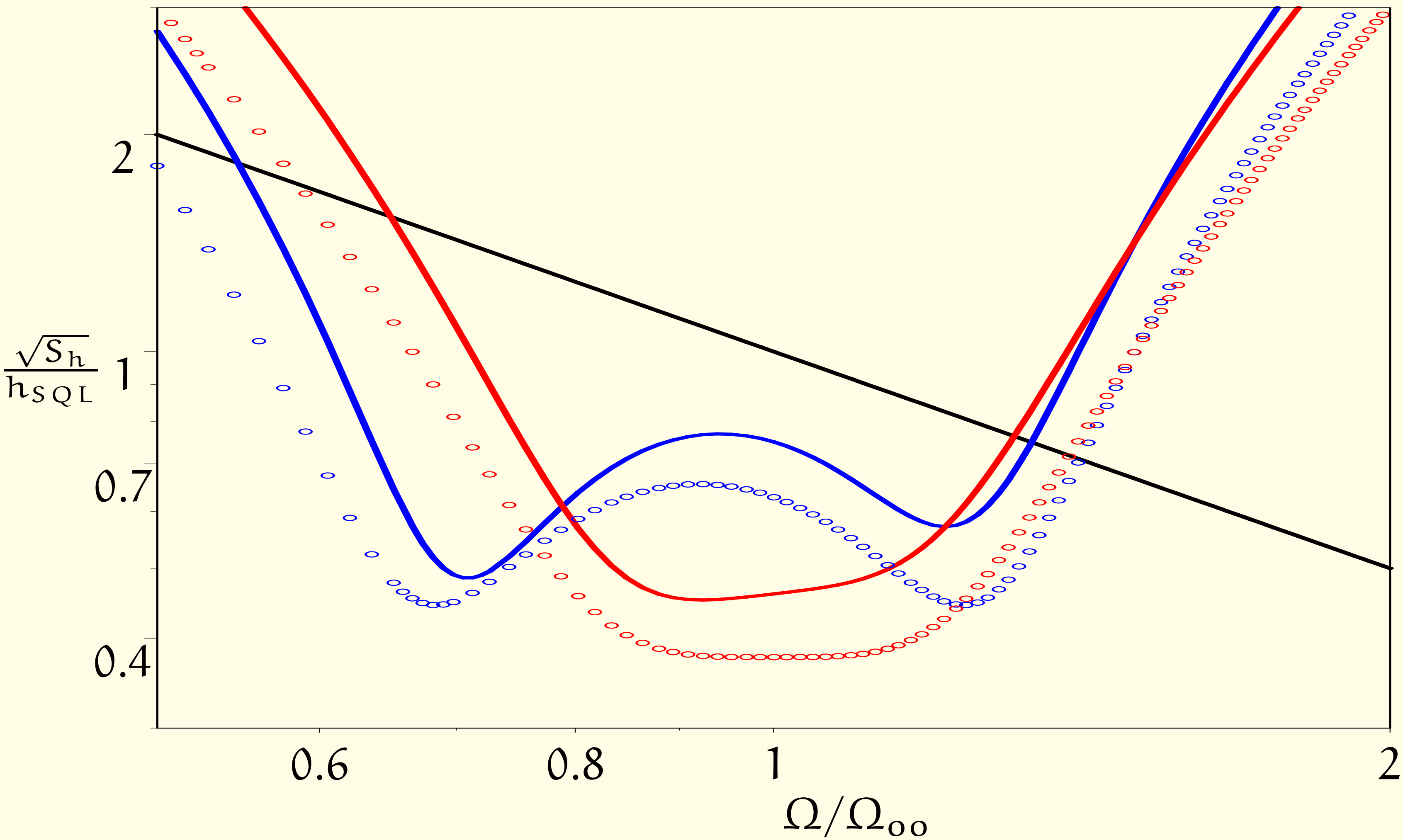


Two separate minima. $\eta = 3\eta_c$



Wide band regime

$\alpha = 0, \quad \gamma_0/\Omega_0 = 0.1, \quad \eta = 0,\ \eta_c, \quad |\mathcal{A}_0| = 1,\ 0$



Gain in signal to noise ratio

The gain in sensitivity even for wide spectrum signals.

$$s_{SR} = \frac{2}{\pi}\int \frac{|h_s(\Omega)|^2}{S_h(\Omega)}\, d\Omega, \quad \mathcal{P} = \frac{s_{SR}}{s_{conv}}, \quad h_s(\Omega) = \frac{\text{const}}{\Omega}. \qquad (16)$$

One can estimates $s_{SR} \sim \Delta\Omega/\xi^2$

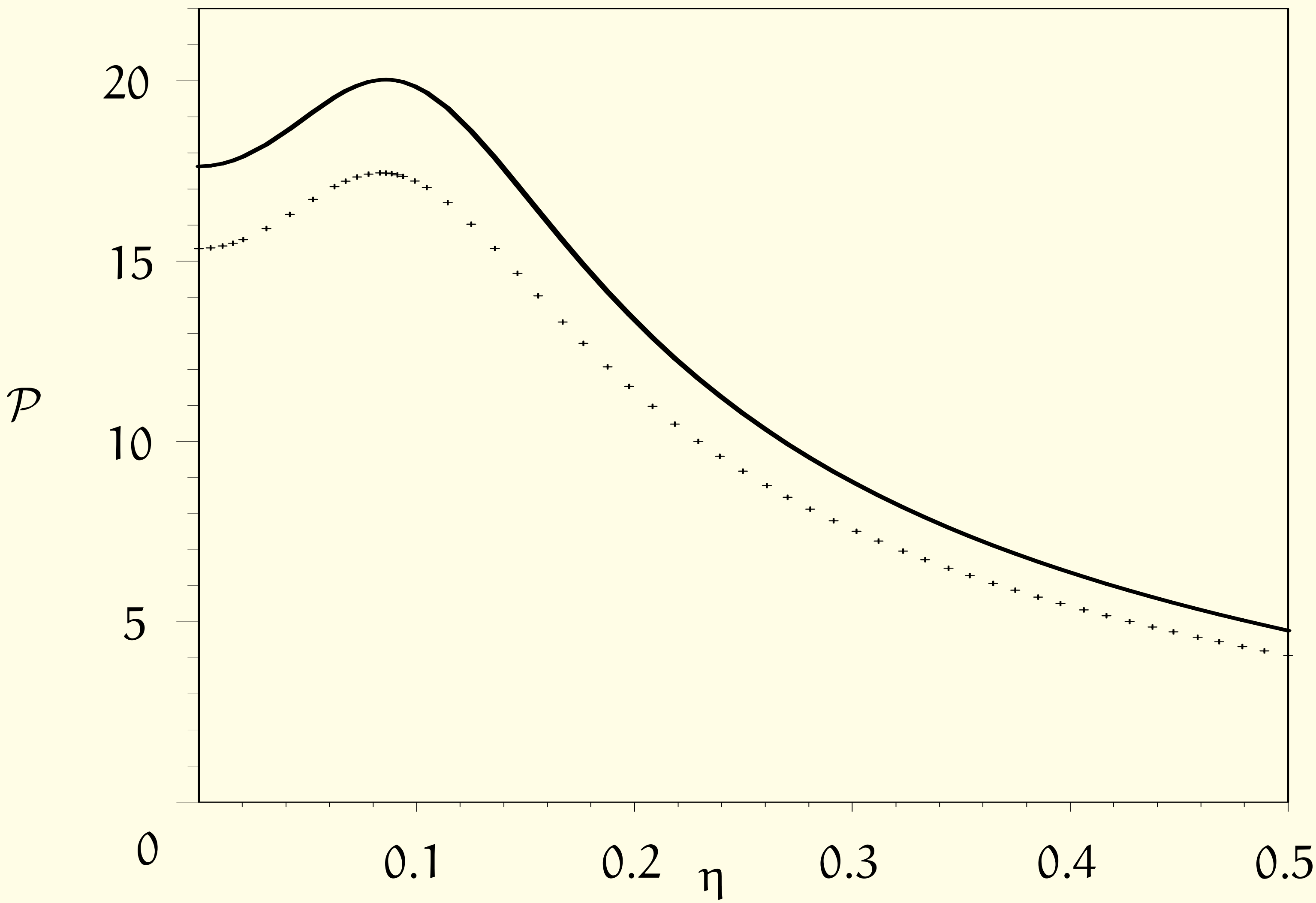
$$\xi_0 \equiv \xi(\Omega_0) \simeq \Omega_{oo}\sqrt{\frac{\tilde{\gamma}_0}{\sqrt{2}\,\Omega_0^3}}, \quad \frac{\Delta\Omega}{\Omega_0} \simeq \sqrt{\frac{\tilde{\gamma}_0}{2\sqrt{2}\,\Omega_0\sqrt{1+|\mathcal{A}_0|^2}}},$$

$$s_{SR} \sim \Delta\Omega/\xi^2 \sim \frac{1}{\sqrt{\tilde{\gamma}_0}}, \quad s_{SR}^{\text{no losses}} \sim \frac{1}{\sqrt{\gamma_0^{\text{no losses}}}}$$

For no losses case the gain in signal to noise ratio is not restricted formally if $\gamma_0^{\text{no losses}} \to 0$. **The optical losses restricts this gain.**

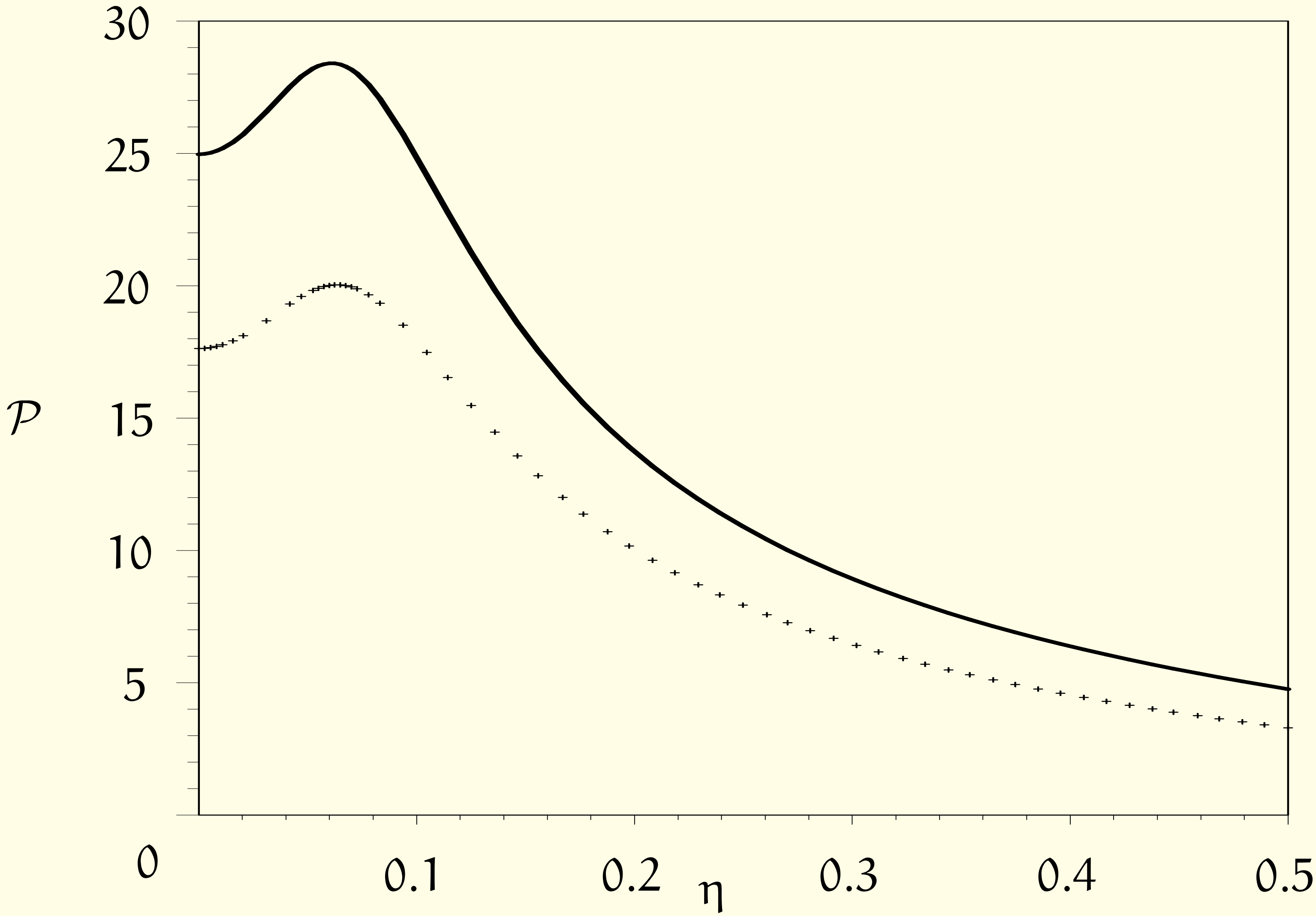
Numerically calculated plots of signal to noise ratio gain $\mathcal{P}(\eta)$

$\alpha = 0, \quad \gamma_0/\Omega_0 = 0.0046, \quad \mathcal{A}_0^2 = 0.24$



Numerically calculated plots of signal to noise ratio gain $\mathcal{P}(\eta)$

$\alpha = \pi/2, \quad \gamma_0/\Omega_0 = 0.0046, \quad \mathcal{A}_0^2 = 0.24$



Maximization of gain $\mathcal{P}$ at fixed optical losses

Recall: $\gamma_0^{\rm loss}$ is fixed (it depends on mirror's absorption only) whereas effective loss factor $|\mathcal{A}_0|^2$ can be modified by variation of mirrors transmittivities.

$$\eta_{\rm opt} = \frac{\eta_c}{\sqrt[4]{3}},$$

$$|\mathcal{A}_0^{\rm opt}|^2 \simeq \frac{\left(1+\cos^2\alpha\right)\left(\sqrt{73}-3\right)}{16}$$

$$\mathcal{P}_{\rm max} \simeq 0.61 \times \sqrt{\frac{\Omega_0}{\gamma_0^{\rm loss}}} \simeq 20.5$$

Conclusion

- We show that usage of optical rigidity allows to circumvent SQL sensitivity for Advanced LIGO topology with power in arms approximately **smaller** by $\sqrt{2}$ times than SQL power in conventional interferometer.
- Usually losses of any kind degrade considerably of quantum measurements — it is not the case for usage of optical rigidity. With value of optical losses planned in Advanced LIGO the degradation of sensitivity due to losses is **negligible**.
- Our "narrow band" regime produces the gain in signal to noise ratio even for signals with wide spectrum — it is a unique feature of the "double" resonance regime. We demonstrate that optical losses put the limit in this gain.

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