Flat Beam Profile to Depress Thermal Noise

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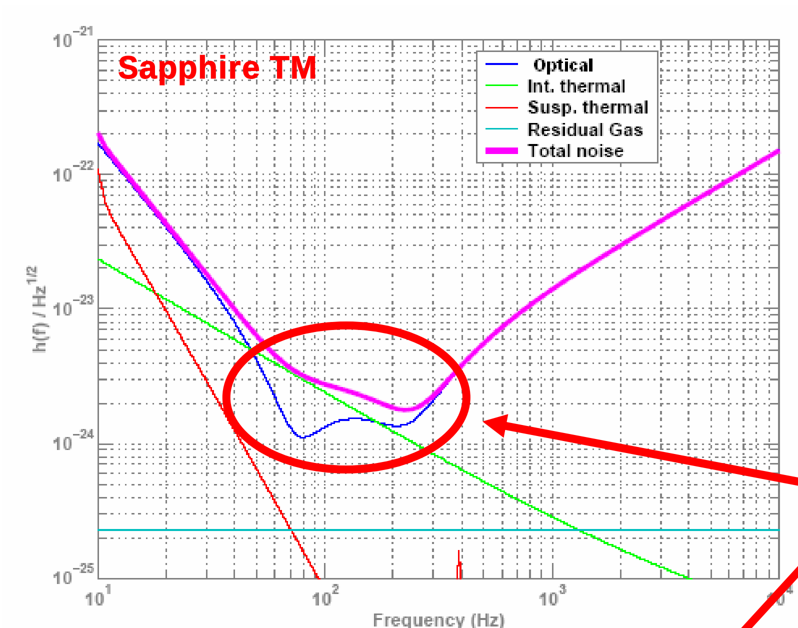
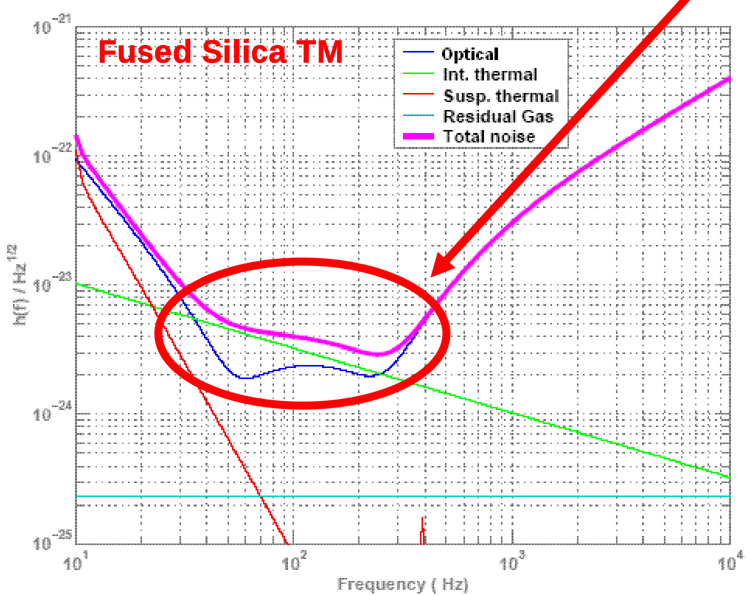
LIGO-G050041-00-Z

Mirror thermal noise problem:





Advanced-Ligo sensitivity

Dominated by test-masses thermoelastic (S-TM) or coating (FS-TM) thermal noises.

Can we reduce the influence of thermal noise on the sensitivity of the interferometer?

Without drastic design changes

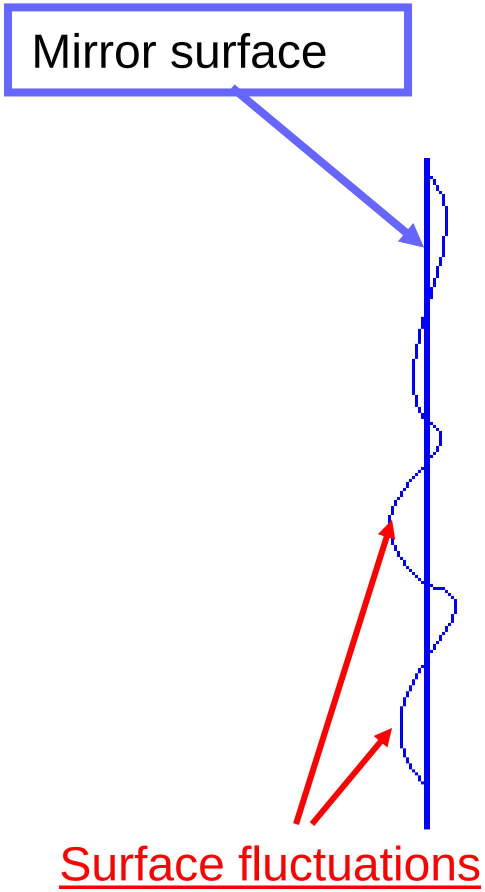
Mirror Thermal Noise:

Thermoelastic noise

Created by stochastic flow of heat within the test mass

Fluctuating hot spots and cold spots inside the mirror

Expansion in the hot spots and contraction in the cold spots creating fluctuating bumps and valleys on the mirror's surface



Brownian noise

Due to all forms of intrinsic dissipations within a material (impurities, dislocations of atoms, etc..)

Interferometer output: proportional to the test mass average surface position, sampled by to the beam's intensity profile.

Indicative thermal noise trends

Noise spectral densities in the **Gaussian beam** case

(infinite semi-space mirror)

$$S_h^{TE-s} \propto \frac{1}{r_0^3}$$

Substrate thermoelastic noise

$$S_h^{TE-c} \propto \frac{1}{r_0^2}$$

Coating thermoelastic noise

$$S_h^{B-s} \propto \frac{1}{r_0}$$

Substrate Brownian noise

$$S_h^{B-c} \propto \frac{1}{r_0^2}$$

Coating Brownian noise

Exact results require accurate information on material properties and finite size effects must be taken in account.

Mirror surface averaging

Gaussian beam

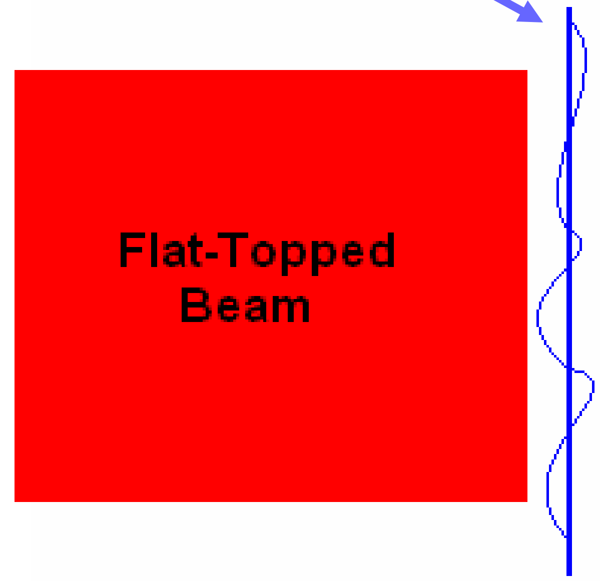
r_0 **As large as possible** (within diffraction loss constraint).

The sampling distribution changes rapidly following the beam power profile

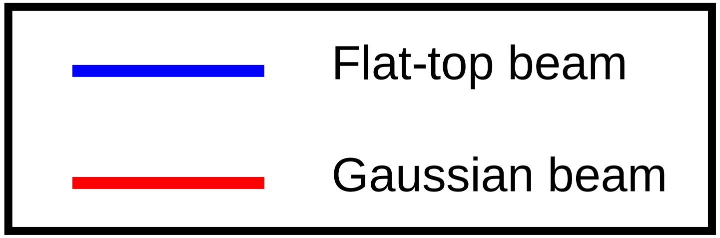
Flat Top beam

Larger-radius, **flat-top** **beam will better average over the mirror surface.**

Mirror surface fluctuations



Diffraction prevents the creation of a beam with a rectangular power profile…but we can build a nearly optimal flat-top beam:

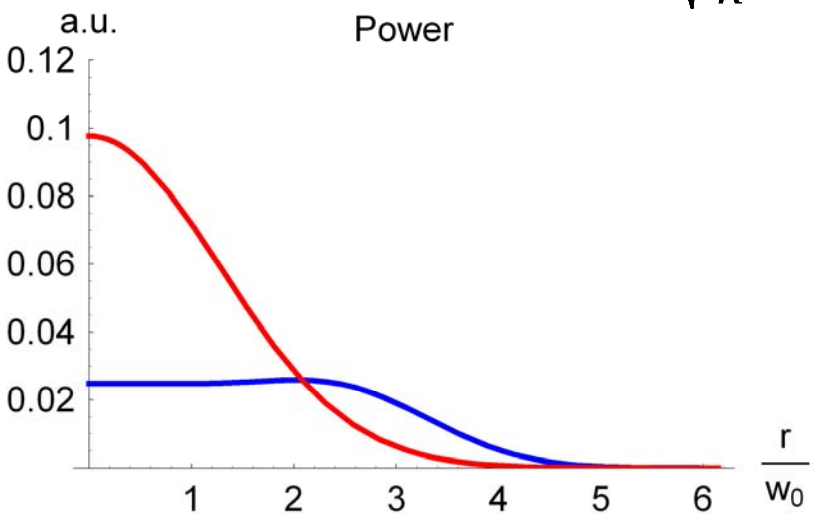


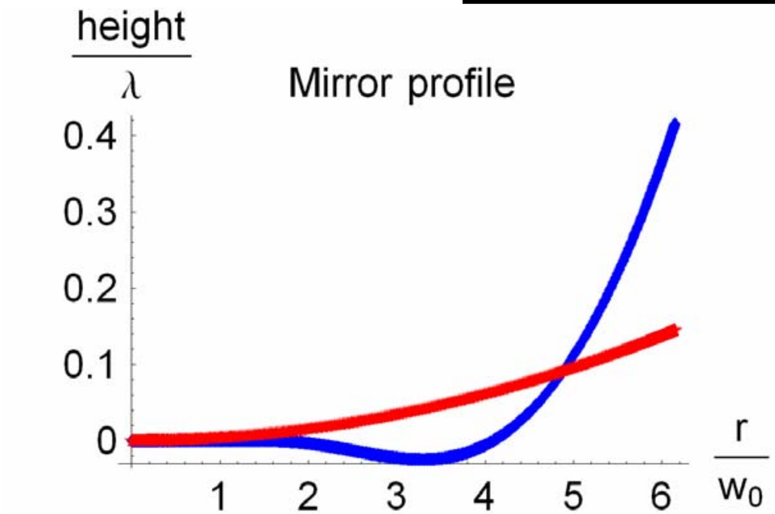
$$u_{FT}(r) \propto \int_{r' \leq D} d^2\vec{r}'\, e^{-\frac{(\vec{r}-\vec{r}')^2 (1-i)}{2w_0^2}}$$

$$u_G(r) \propto e^{-\frac{r^2}{2r_0^2}}$$

$$w_0 = \sqrt{\frac{L}{k}}$$

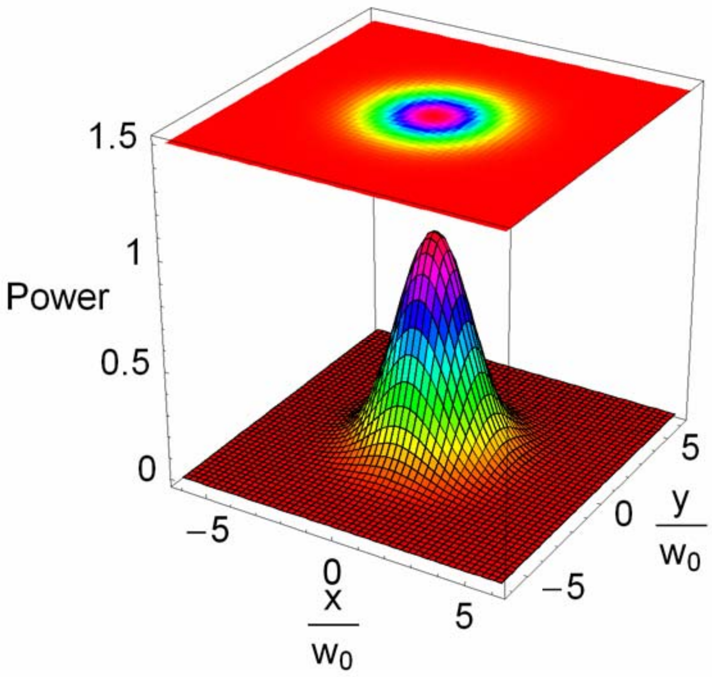
The mirror shapes match the phase front of the beams.

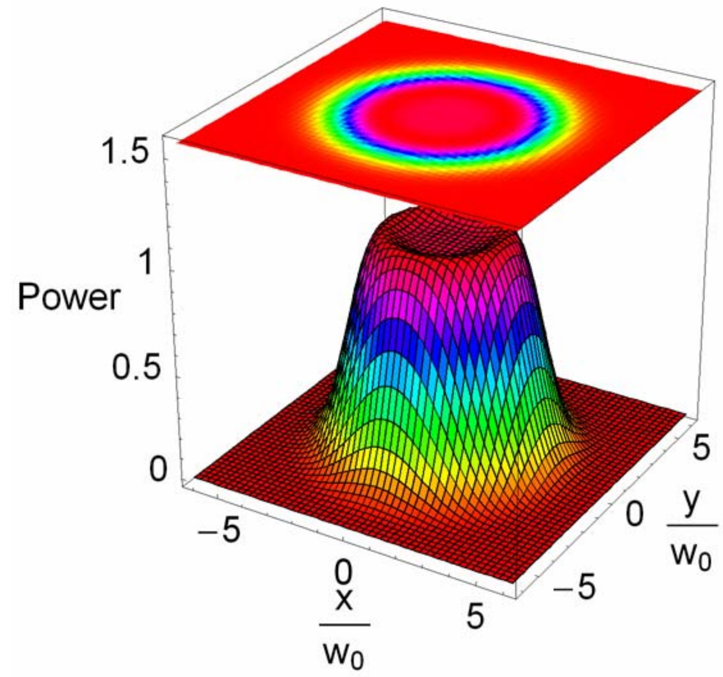




Sampling ability comparison between the two beams

(same diffraction losses, Adv-LIGO mirror size)





Sampled area

$$S(r_0) \approx 0.09 S_{mir}$$

$$S(r_{90\%}) \approx 0.01 S_{mir}$$

Advantage Ratio

$$R_{Flat-top/Gaussian} = 4$$

$$R_{Flat-top/Gaussian} = 20$$

Sampled area

$$S(r_0) \approx 0.36 S_{mir}$$

$$S(r_{90\%}) \approx 0.20 S_{mir}$$

Thermal noise for finite sized mirrors:

1. **Precise comparative estimation of the various thermal noise contributions for finite test masses (design optimization).**
2. **Noise suppression using Flat-Top beam.**

Thermal noise calculations

Interferometer is sensitive to the test mass surface displacement

$$q(t) = \int_{Mirror} d^2\vec{r}\, u_z(\vec{r},t)\, f(\vec{r})$$

Levin's approach to Fluctation Dissipation Theorem

$$S_q(\omega) = \frac{8\, k_B\, T}{\omega^2} \frac{W_{diss}}{F_0^2}$$

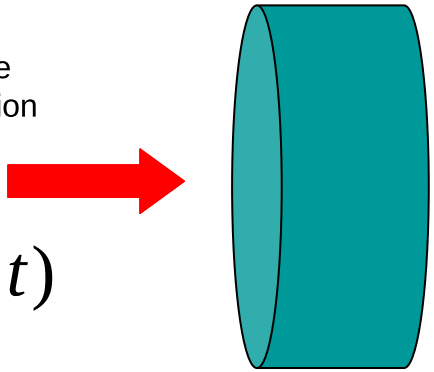
W_{diss} Is the energy dissipated by the mirror in responce to the oscillating pressure

$$P(\vec{r},t) = F_0\, f(\vec{r}) \cos(\omega t)$$

Assumptions in our analysis

Liu-Thorne (accurate) approximate analyical solution of elasicity equations for a cylindrical test mass

Pressure distribution



$$P(r,t)$$

Quasistatic approximation for the oscillations of stress and strain induced by P.

$$\tau_{sound} << \tau_{GW}$$

Adiabatic approximation for the thermoelastic problem (negligible heat flow during elastic deformation).

$$r_{heat} << r_{beam}$$

Material properties independent from frequency.

Material properties:

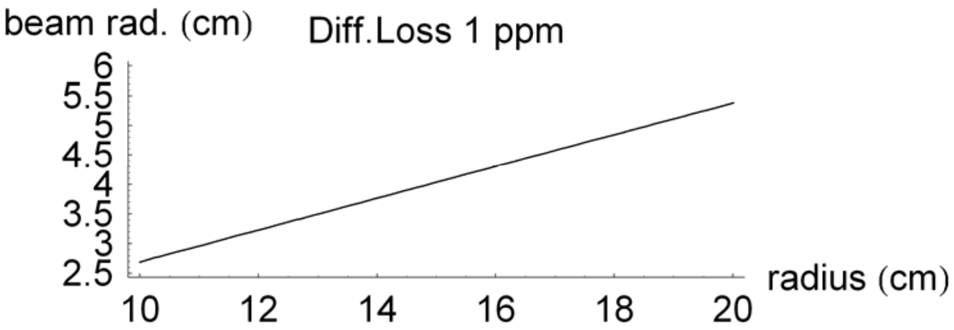
Parameters : (c.g.s. units)	**Fused Silica:**	**Sapphire:**	**Coating (Ta2O5 + SiO2):**
Density	2.2	4	
Young modulus	$7.2\ 10^{11}$	$4\ 10^{12}$	$1.1\ 10^{12}$
Poisson ratio	0.17	0.29	0.2
Loss angle	$3\ 10^{-9}$	$5\ 10^{-9}$	10^{-4}
Linear thermal expansion coeff.	$5.\ 5\ 10^{-7}$	$5\ 10^{-6}$	
Specific heat per unit mass (V const.)	$6.7\ 10^{6}$	$7.9\ 10^{6}$	
Thermal conductivity	$1.4\ 10^{5}$	$4\ 10^{6}$	
Total thickness	variable	variable	$6\ 10^{-4}$

Ideas behind calculations

- **Fixed total mirror mass = 40 Kg.**
- **The Gaussian beam radius is dynamically adjusted to maintain a fixed diffraction loss = 1ppm (clipping approximation).**
- **The mirror thickness is also dynamically adjusted as a function of the mirror radius in order to maintain the total 40 Kg mass fixed.**
- **Calculation at the frequency 100 Hz**

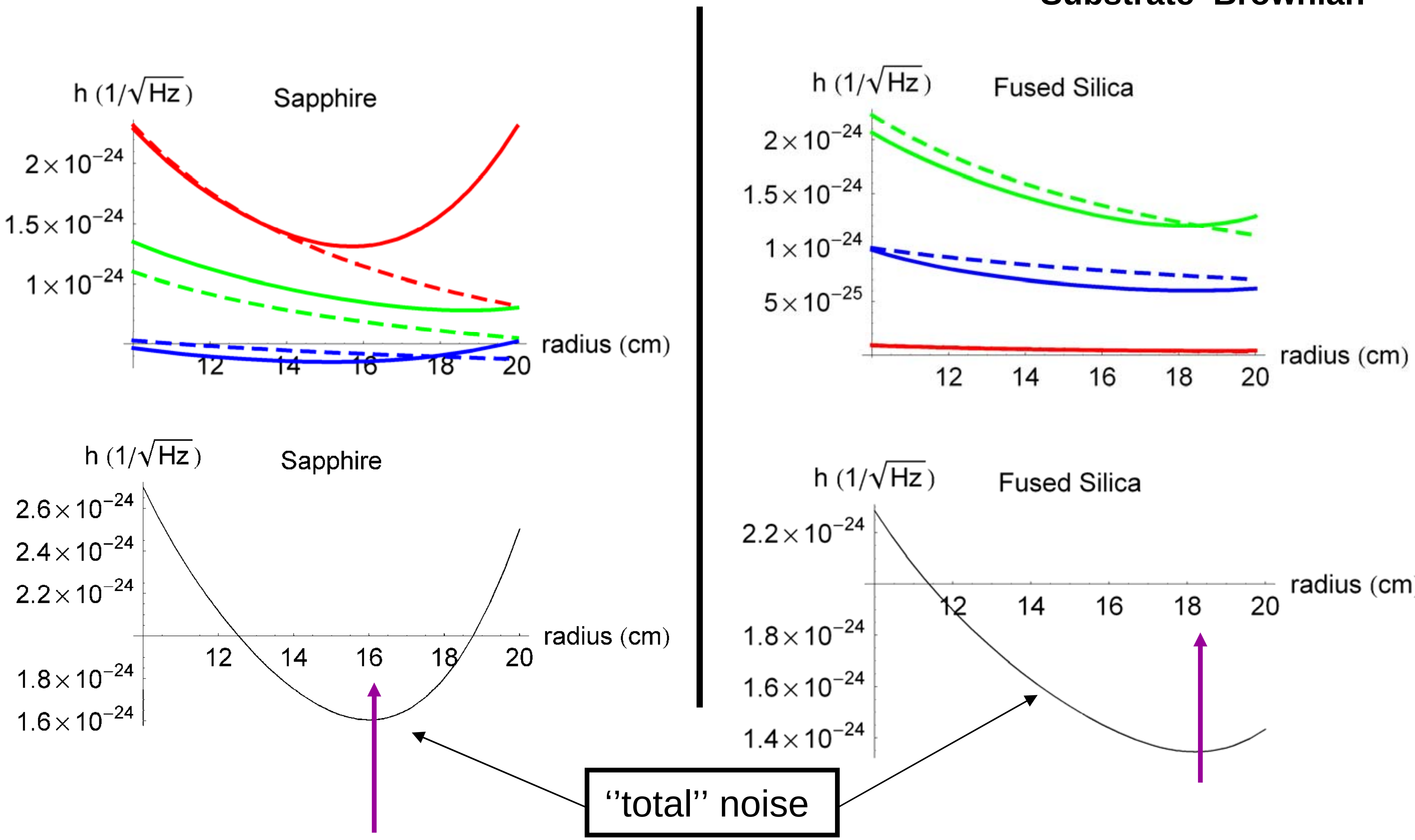
$$\text{Noise}_{\text{TE-s}} \propto 1/f$$

$$\text{Noise}_{\text{B-s / B-c}} \propto 1/\sqrt{f}$$



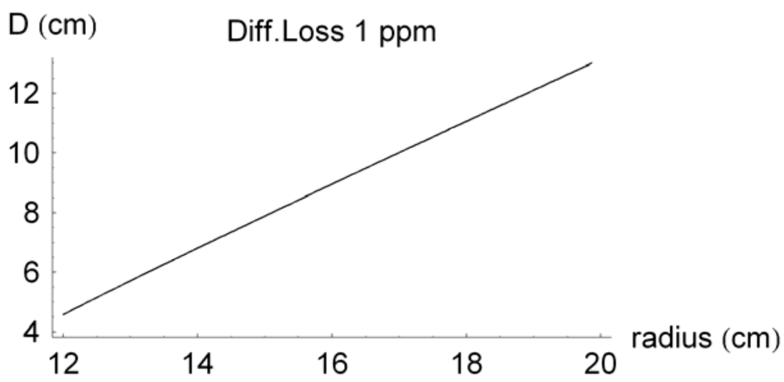
Results for Gaussian beam

Substrate Thermoelastic
Coating Brownian
Substrate Brownian



Results for Flat Top beam

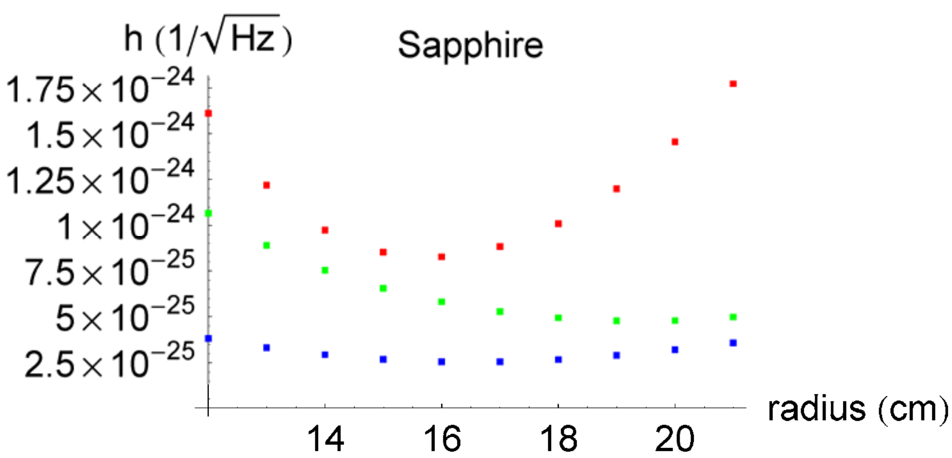
Same procedures for calculations...but more computational time.

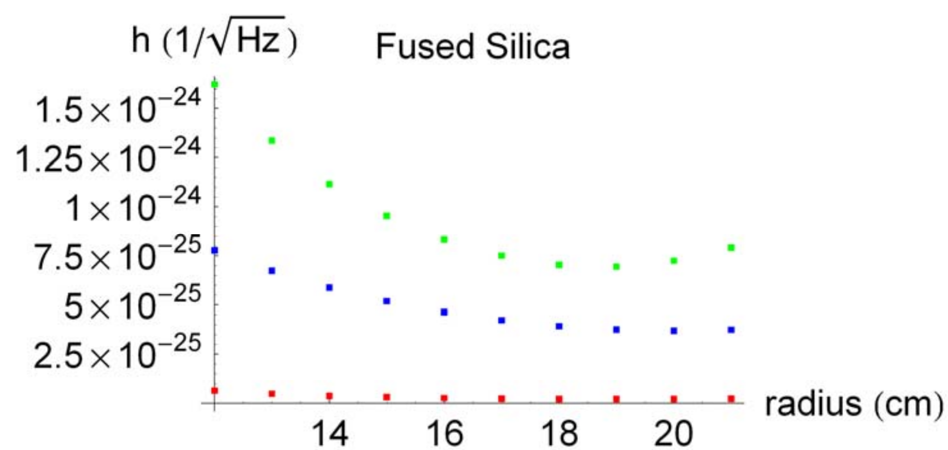


Substrate Thermoelastic (red)

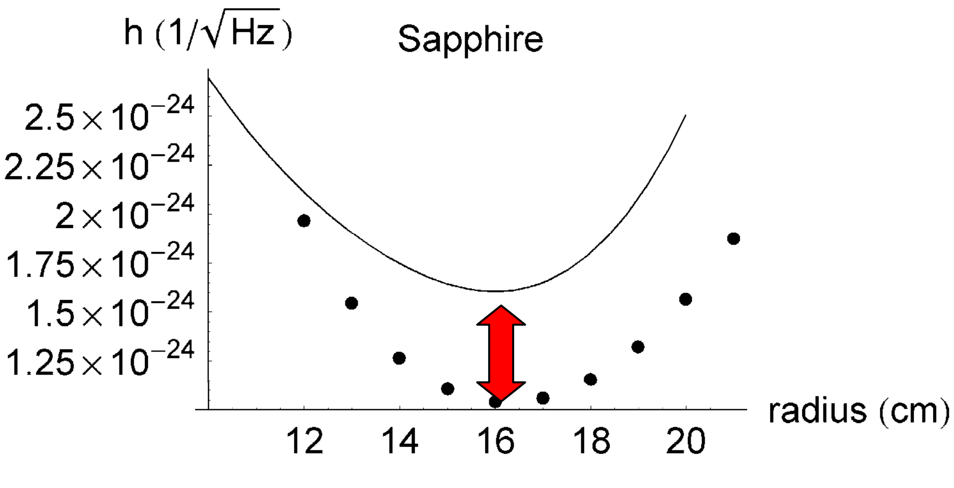
Coating Brownian (green)

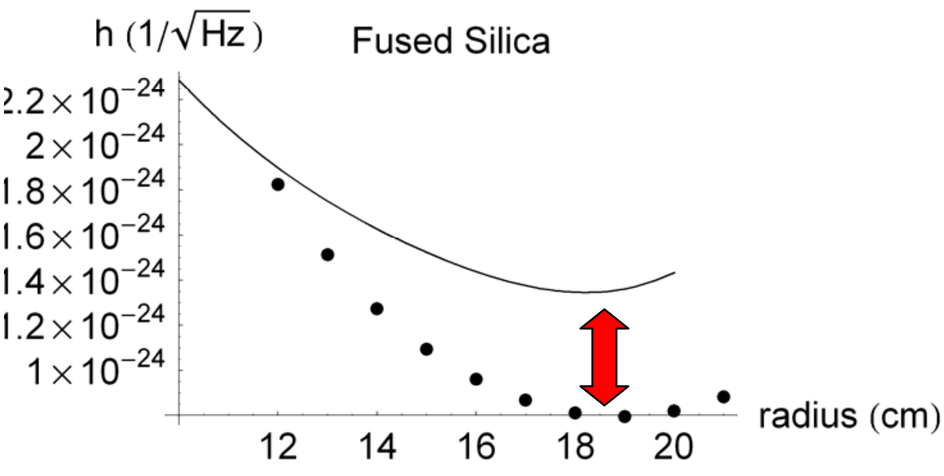
Substrate Brownian (blue)





Comparison between Gaussian and Flat Top beam



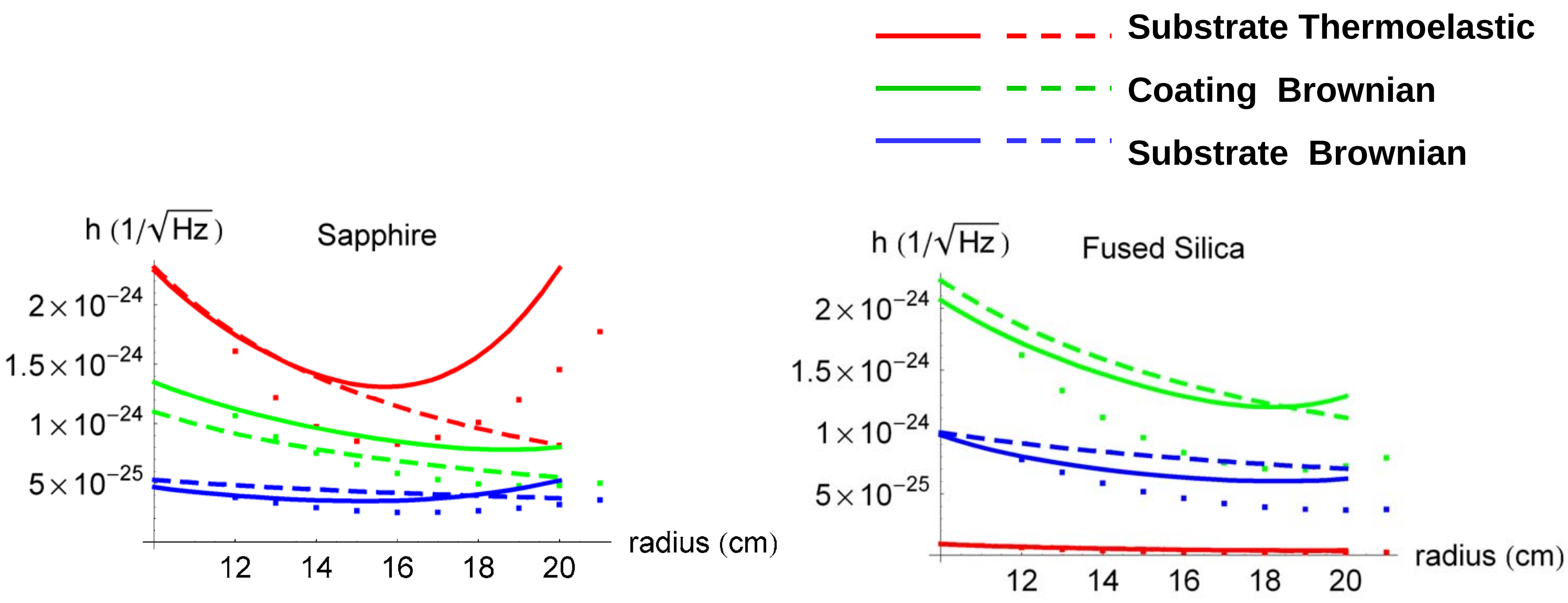


Gain factor ≈ 1.6

Gain factor ≈ 1.7

Beware of the clipping approximation!

Detailed Comparison:



Further analysis will include:

- Addition of the coating thermoelastic noise.
- Sensitivity optimization allowing larger diffraction losses (5-10 ppm).
- Non isotropic loss angle and elastic properties for the coating.
- Frequency dependence (beyond adiabatic approximation, etc..etc..).
- Thermal lensing effect.
- Comparison of these semi-analytical results with FEM analysis (collaboration with Enrico Campagna, VIRGO).

First next step...5ppm