On the phase of light diffracted by gratings

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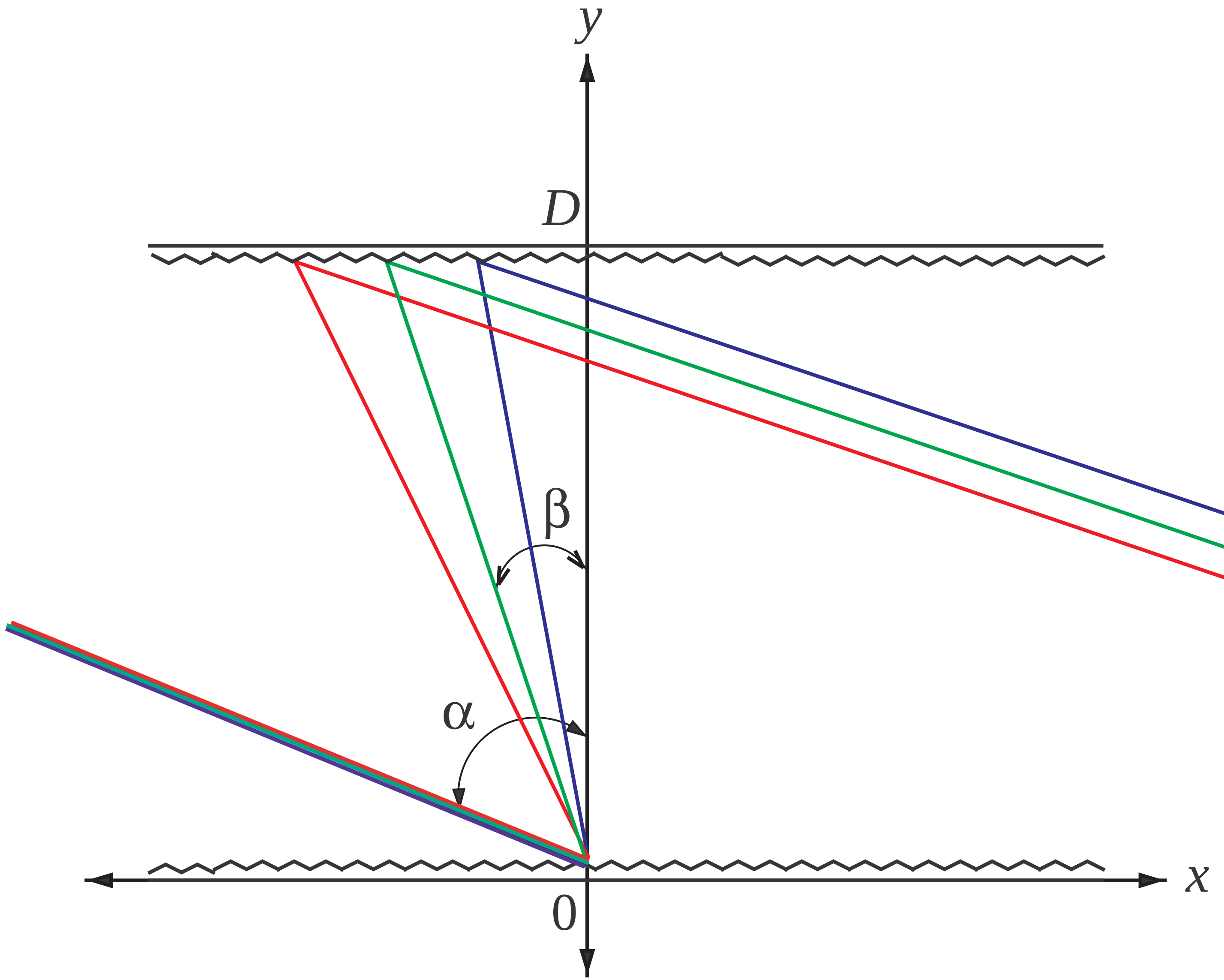
- The grating pulse compressor/expander
- Idea: white light cavities from two parallel gratings
- It doesn't work!
- Phase shift by gratings

Question 1: What is the (wavelength dependent) phase change arising from diffraction by a grating?

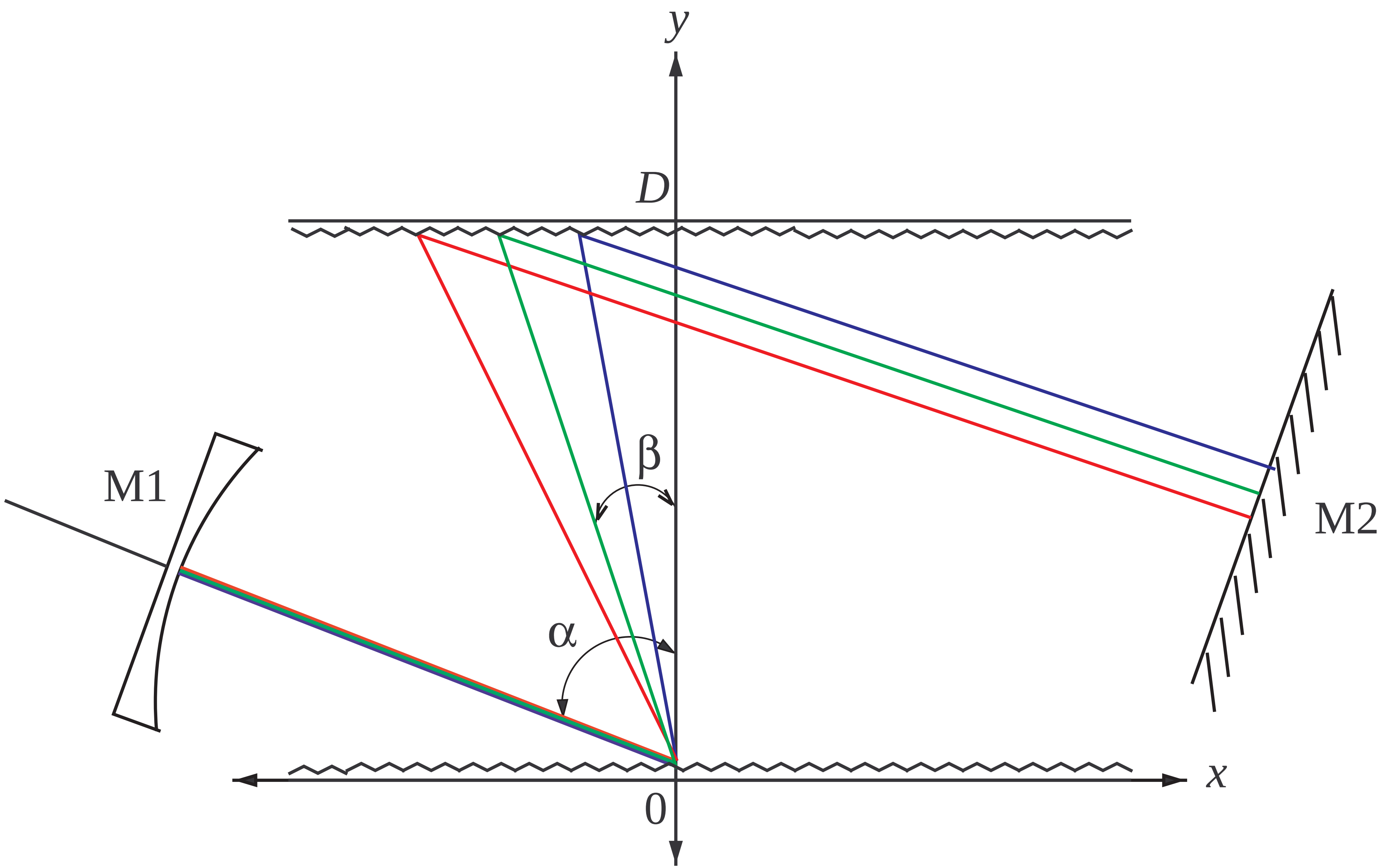
Question 2: What effect occurs when a grating is moved *parallel to its surface*?

Question 3: What are the implications for the use of gratings in some advanced GW detector?

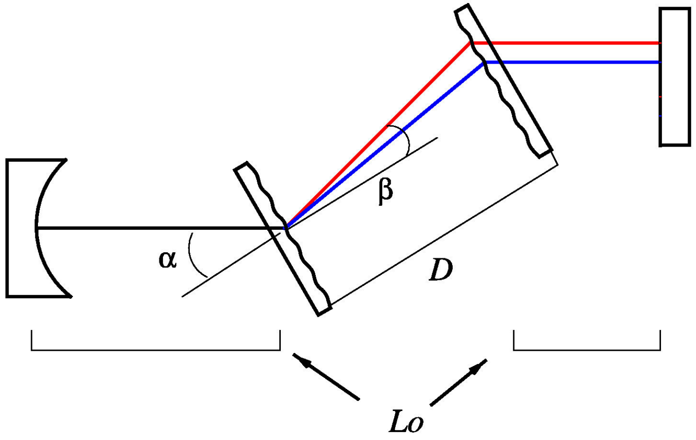
The grating pulse compressor/expander



Add mirrors at each end: Fabry-Perot cavity



Condition for equal geometrical paths



- We want (for a range of wavelengths λ)

$$\frac{\Phi(\lambda)}{2\pi} = M = \frac{L(\lambda)}{\lambda}.$$

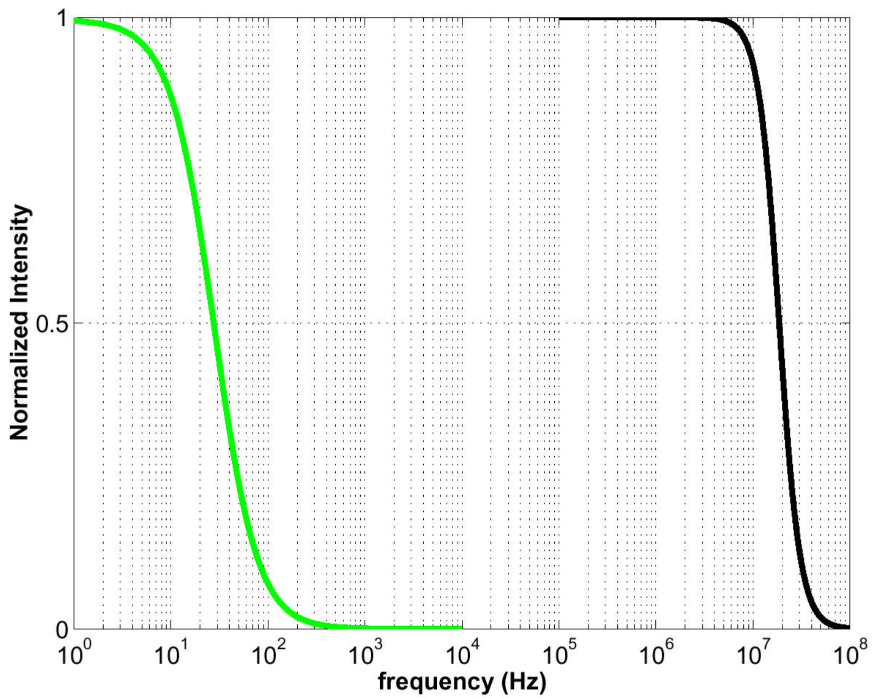
- This leads to the condition

$$\frac{d\Phi}{d\lambda} = 0 \quad \text{or} \quad \frac{dL(\lambda)}{d\lambda} = \frac{L(\lambda)}{\lambda}$$

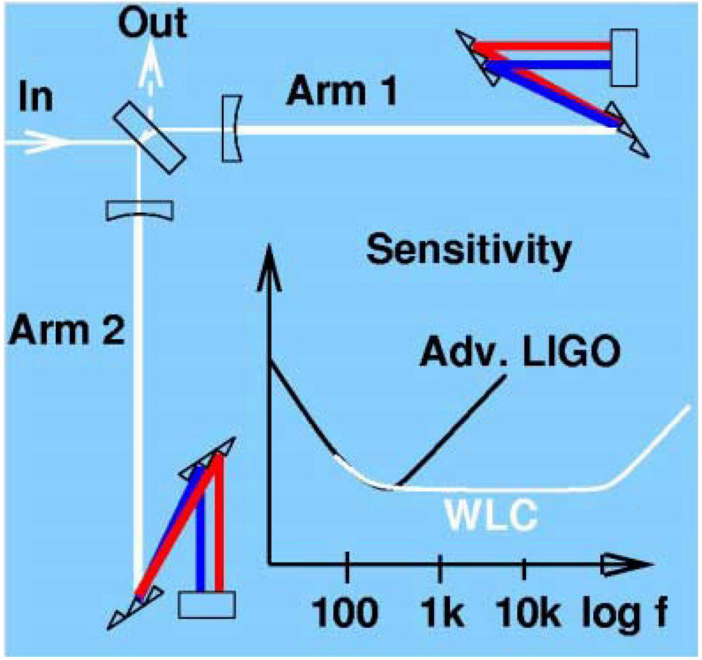
- The optimized length is

$$L(\lambda) = L_0 + \frac{D[1 + \sin\alpha \sin\beta(\lambda)]}{\cos\beta(\lambda)}$$

Were this true ...



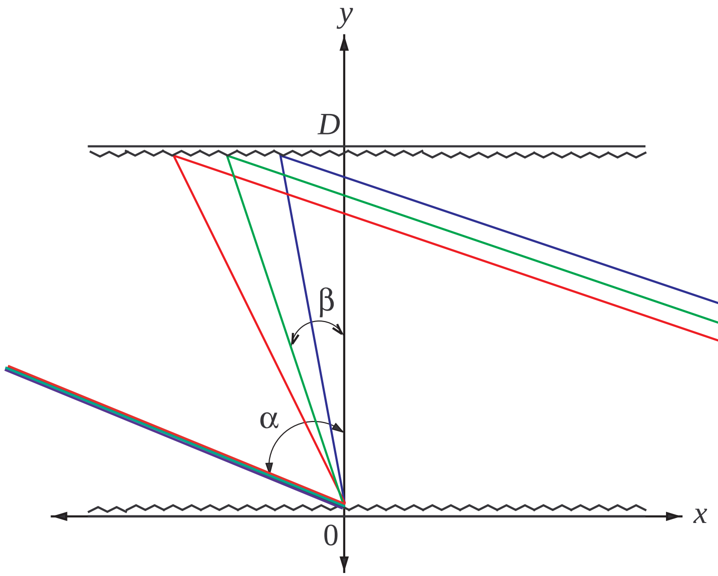
$T = 0.5\%$

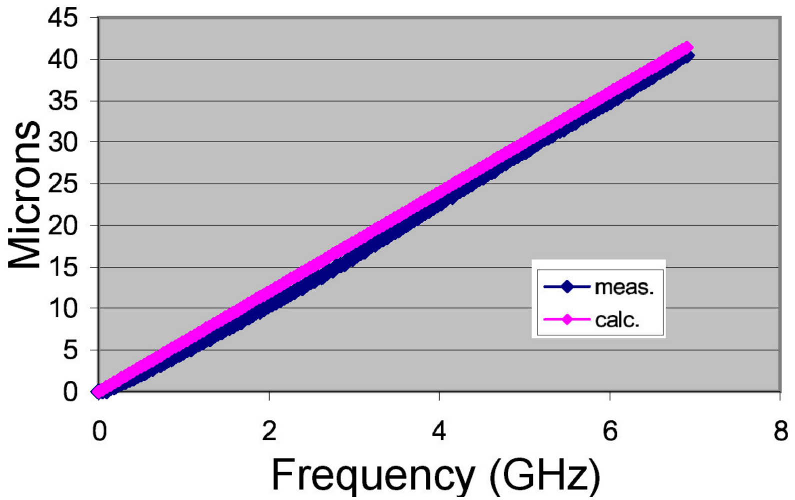


one could incorporate these gratings into the arms of a km-scale interferometer and get better high frequency performance (or turn up the finesse, and get greater sensitivity).

So we tried it

- Fabry Perot with 2 gratings: No significant gain in bandwidth.
- Were the gratings displacing the beams as expected? Yes.



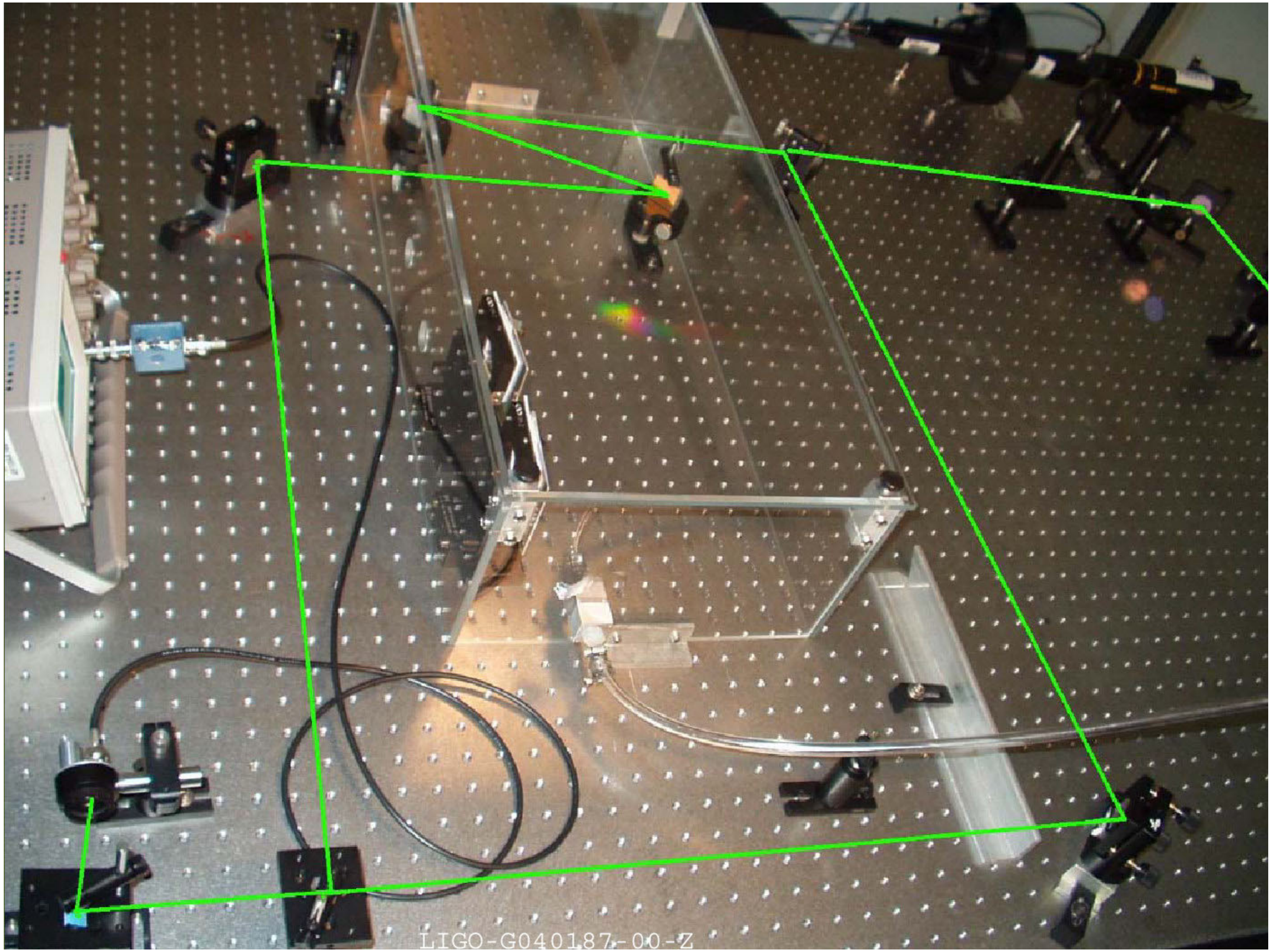


- Near equal-arm Michelson with gratings in one arm.

  Expected: rapid change of interference with laser wavelength.

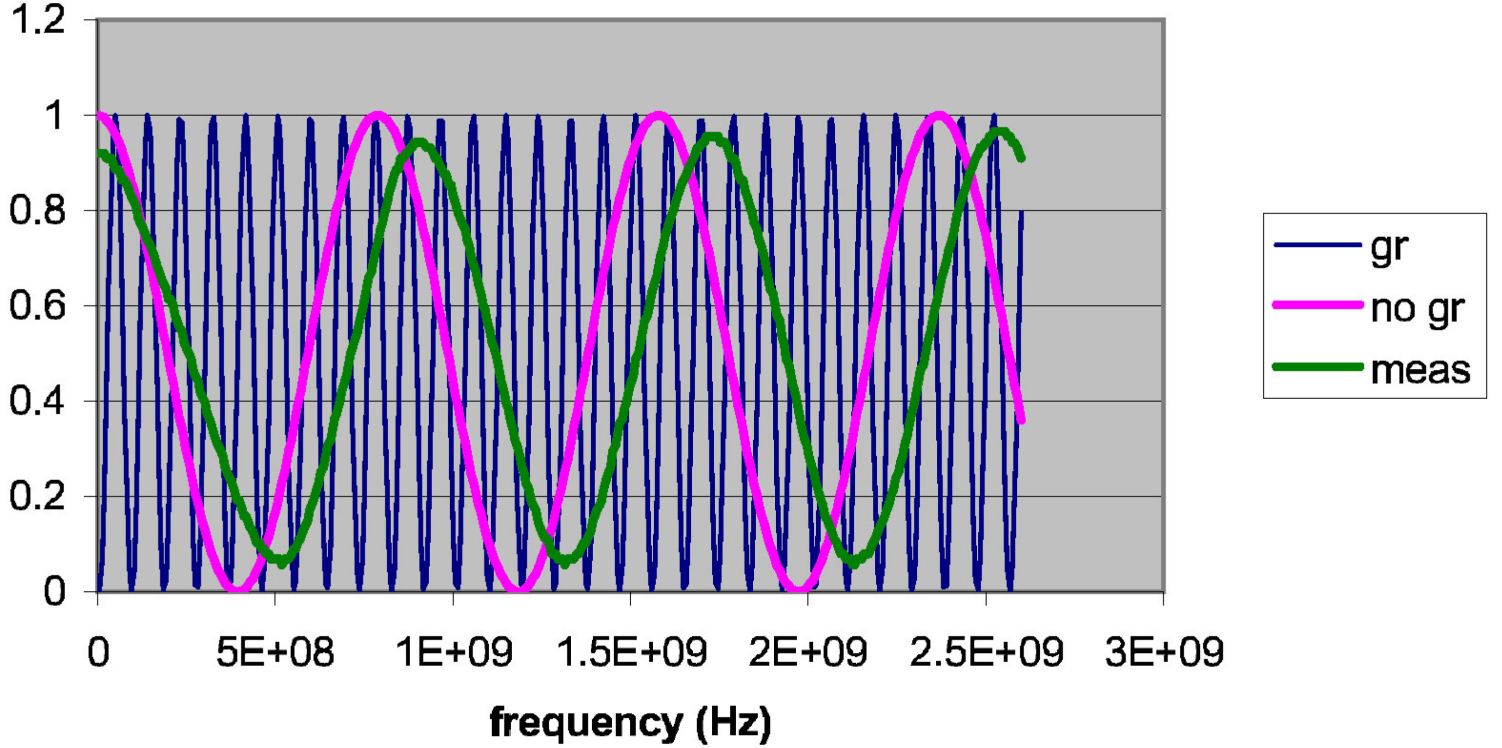
  Observed: none.

Mach-Zehnder experiments



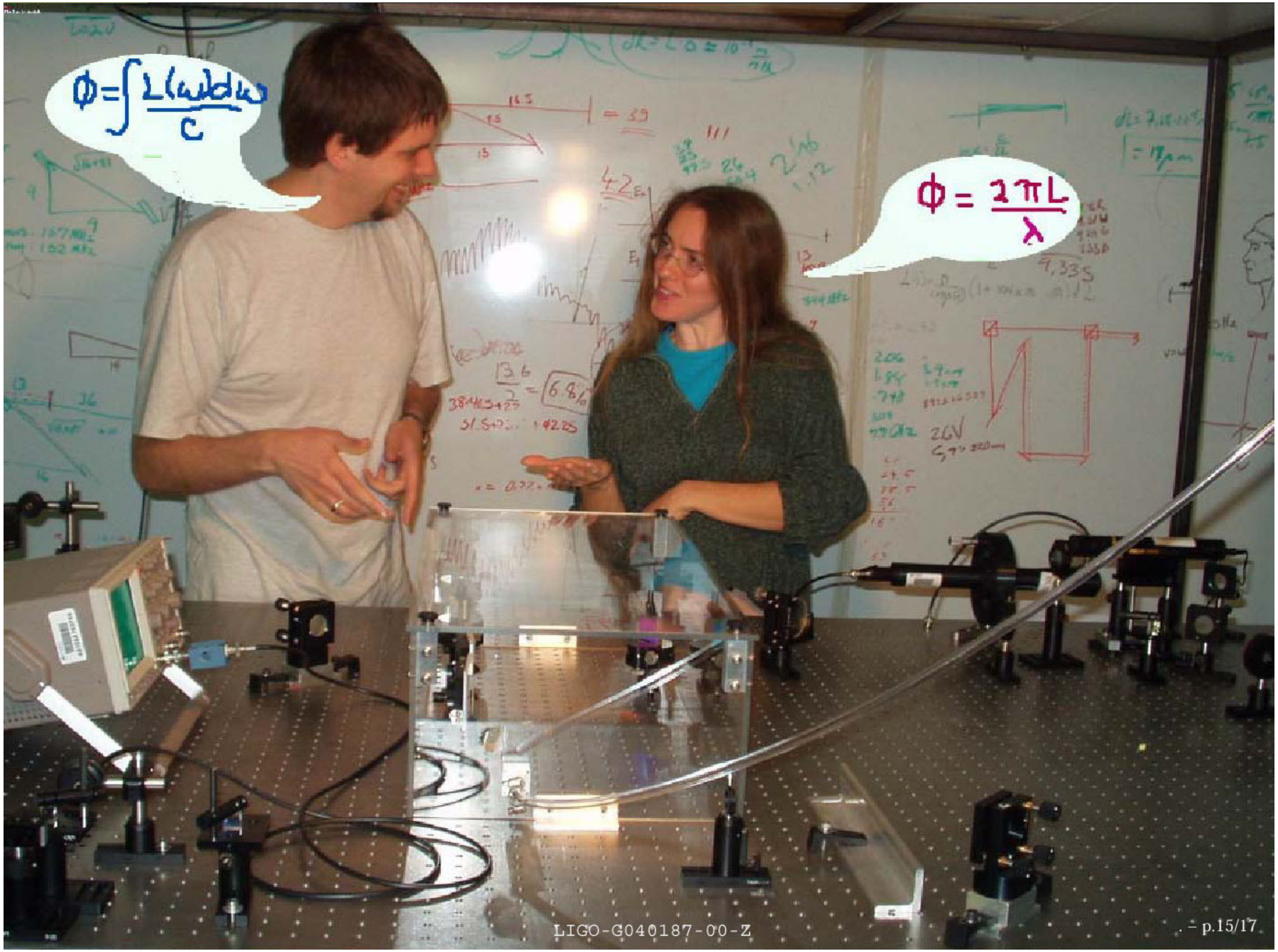
Laser is a 1.06 μm diode laser, with $\sim$ 7 GHz tunability.

Wavelength dependence of Mach Zehnder Signal



- "gr" and "no gr" — calculations based on ruler measurements of the optical paths in the MZ, and the computed $L(\lambda)$ for these dimensions.
- For "no gr" we considered the gratings to be replaced by mirrors oriented to reflect the beam in the correct direction.
- "meas" — observed for a 2.5 GHz sweep of the laser wavelength.

Phased and confused



Yanbei's solution

- Gratings bestow a phase factor on the light of

$$e^{ikG(x)} = \sum_m C_m e^{imgx} \approx e^{-igx} \quad \text{and} \quad e^{-ig(x-x_o)}$$

where $g = 2\pi/d$, $m = -1$, $C_{-1} = 1$, and x_o is the offset of the second grating wrt the first.

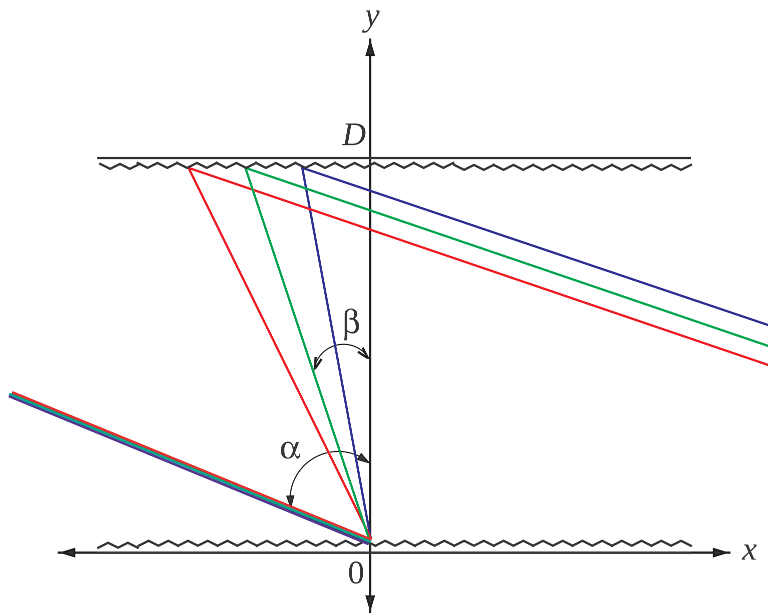
- Then

$$E_{1,in} = E_o e^{ik(x \sin\alpha - y \cos\alpha)} \qquad (1)$$

$$E_{1,out} = E_o e^{i[(k \sin\alpha - g)x + ky \cos\beta]} \qquad (2)$$

$$E_{2,out} = E_o e^{i[k(x \sin\alpha + D \cos\beta) + g x_o]} \qquad (3)$$

$$E_{em} = E_o e^{i[k\{x \sin\alpha - (y-D)\cos\alpha + D\cos\beta\} + g x_o]}. \qquad (4)$$



Phase

- The phase $\Phi(\omega, x, y)$ is

$$\Phi = \frac{\omega}{c}\left[x \sin\alpha - (y - D)\cos\alpha + D\cos\beta\right] + g x_o$$

so that

$$\frac{\partial \Phi}{\partial \omega} = \frac{1}{c}\left[x \sin\alpha - (y - D)\cos\alpha\right] + \frac{D}{c}\left(\cos\beta - \omega \frac{\partial \beta}{\partial \omega}\sin\beta\right).$$

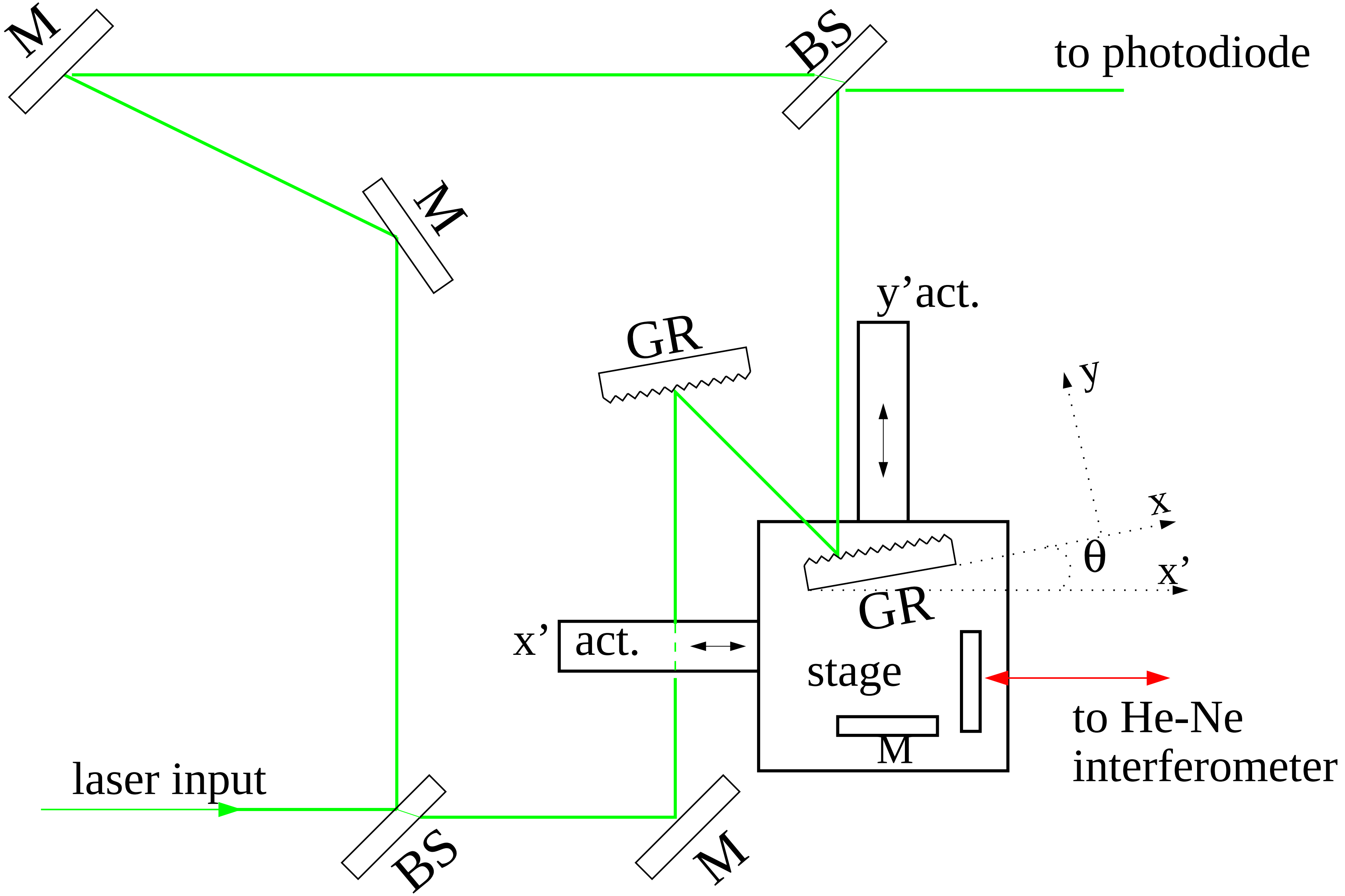
- Using $\frac{\partial \beta}{\partial \omega}$ from the grating equation and the (wavelength-dependent) geometric path length $L(\omega)$ from the first grating (at the origin) to the end mirror, we find

$$\frac{\partial \Phi}{\partial \omega} = \frac{L(\omega)}{c},$$

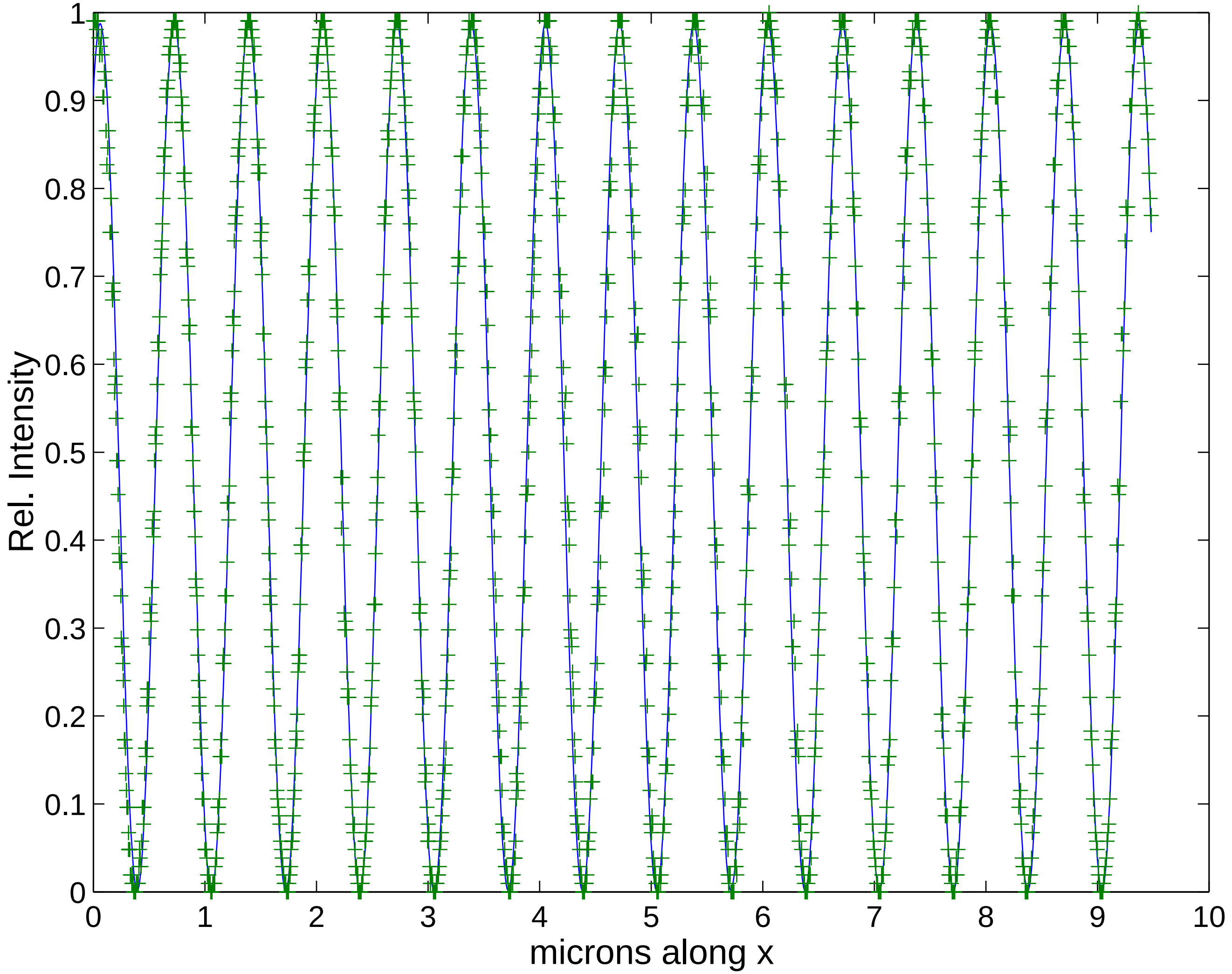
making it clear that the variation of phase with frequency cannot be set to zero.

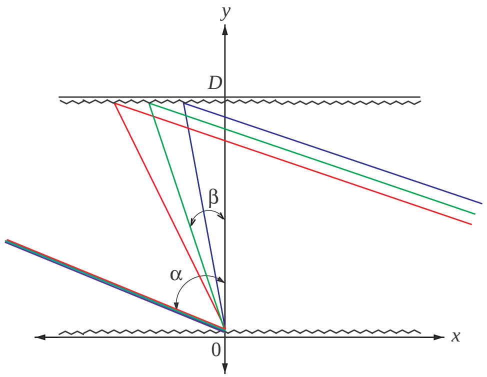
- This forumla for $\frac{\partial \Phi}{\partial \omega}$ is familiar to short-pulse laser physicists as the group delay.

Measuring the phase

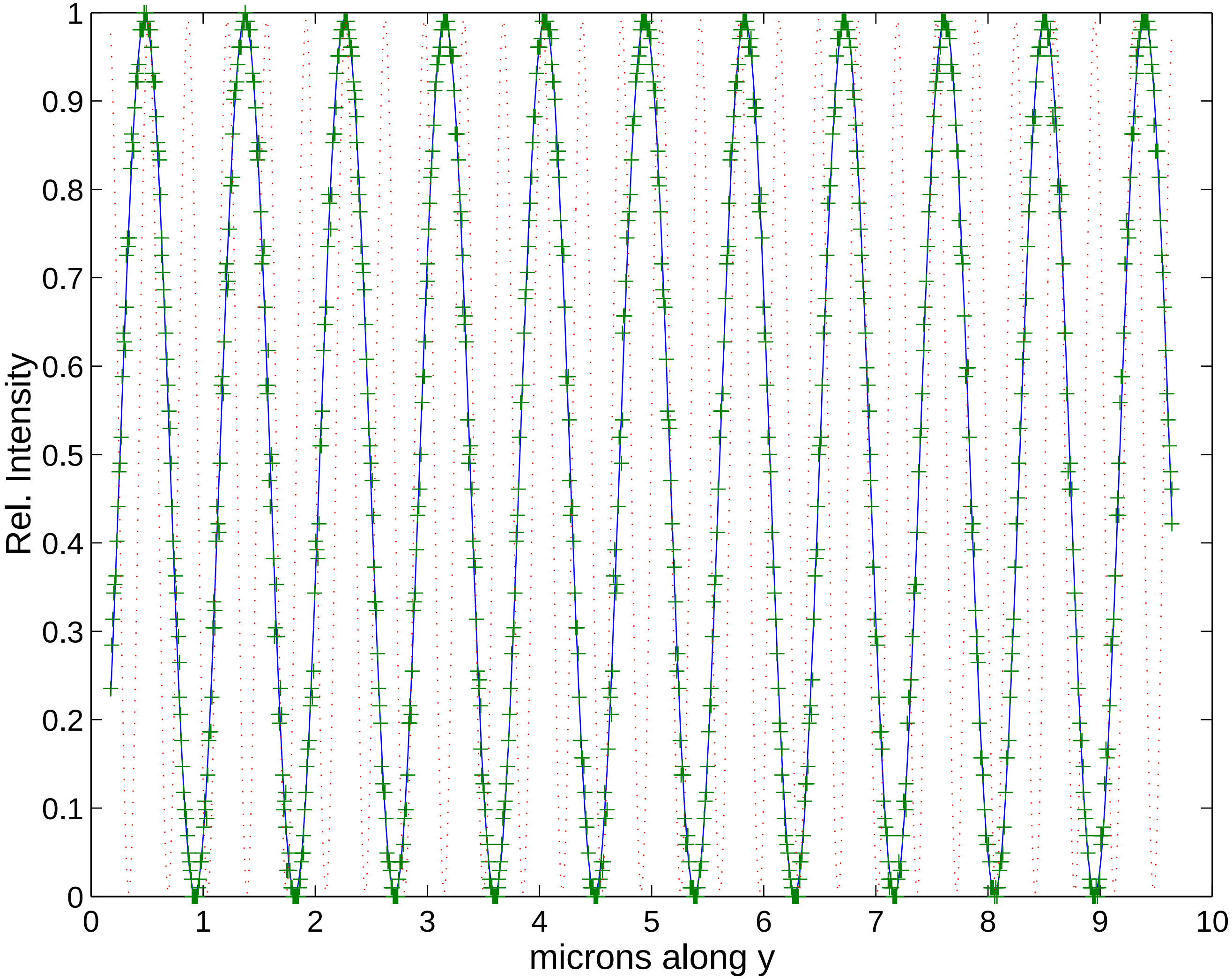


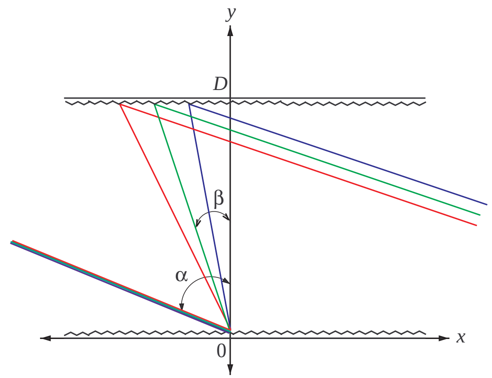
Motion of the grating parallel to its surface





Motion of the grating perpendicular to its surface





Implications for km-length interferometers

- Diffraction gratings are being considered for use as beamsplitters in future detectors.
- Scanning the beam across the grating $\Leftrightarrow$ scanning the grating across the beam.
- Motion of $\sim 1\ \mu$m $\rightarrow 2\pi$ phase shift.
- For a mirror 4 km from the grating, angular motion of 1×10^{-10} radians will cause 2π phase shift.
- To split a fringe by $10^{-10}/\sqrt{\mathrm{Hz}}$, requires 1×10^{-22} rad$/\sqrt{\mathrm{Hz}}$ in in-band angular stability. (!)
- To make the power (in a simple Michelson) at the dark port less than 200 mW* requires 6×10^{-12} rad in angular stability.

*1% of 200 W input

Summary

- The phase of light reflected by or transmitted through a diffraction grating cannot be deduced from Bragg's law and geometry alone.
- That derivation neglects the curious result that the absolute phase is proportional to the distance along the grating face at which the light strikes.

Angular requirements

- Angular motion of mirror directly translates into the phase of the beam.
- SRD says:

$$dx = 10^{-20}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

- This can be written as a phase

$$d\phi = dx\frac{2\pi}{\lambda}$$

- The allowed shift of the beam spot on the grating is then

$$dh = d\phi\frac{d}{2\pi}$$

- The requirement on the angular beam stability (taking into account that the arm cavities dewiggle the beam by the finesse F) is

$$d\Theta = dh\cos\alpha\frac{F}{L}$$

or

$$d\Theta = 1 \times 10^{-22}\ \mathrm{rad}/\sqrt{\mathrm{Hz}}\left(\frac{d}{0.8\ \mu}\right)\left(\frac{1\mu}{\lambda}\right)\left(\frac{\cos\alpha}{0.5}\right)\left(\frac{4\ \mathrm{km}}{L}\right)\left(\frac{F}{100}\right)$$

Note that for a 40 cm mirror, 1×10^{-22} rad corresponds to displacements of the mirror edges by 2×10^{-23} m.