Extended hierarchical search (EHS) for inspiraling compact binaries

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LIGO-G040556-00-R

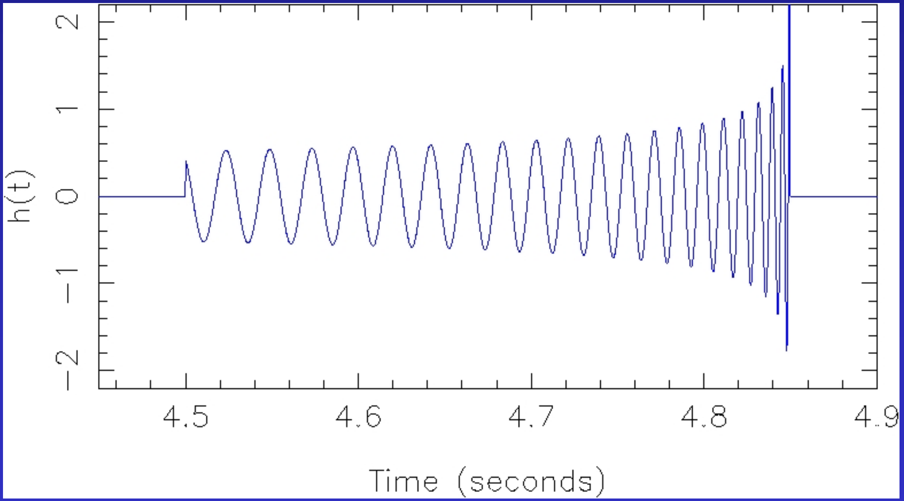
Plan of the Talk

- The 1-step (flat) search with 2PN templates
- The 2-step search with hierarchy in masses and decimation in time – Extended Hierarchical Search

  EHS
- Implementation and performance

The inspiraling compact binary

- Two compact objects: Neutron stars /Blackholes spiraling in together
- Waveform cleanly modeled



$$h \sim 2.5 \times 10^{-23} \left[\frac{M}{M_{sun}}\right]^{5/3} \left[\frac{r}{100\ Mpc}\right]^{-1} \left[\frac{f_a}{100\ Hz}\right]^{2/3}$$

The restricted PN waveform in the SPA

$$\tilde{h}(f) = \mathcal{N} f^{-7/6} \exp i\psi(f;\lambda^\alpha) + 2\pi i t_c f + i\phi_0$$

$$\psi(f;\lambda^\alpha) = \Sigma\, \theta^i(\lambda^\alpha)\zeta_i(f)$$

2PN and spinless templates: $\zeta_1(f) = f^{-5/3}, \zeta_2(f) = f^{-1}, \zeta_3(f) = f^{-2/3}, \zeta_4(f) = f^{-1/3}$

$$\begin{aligned}
\theta_1 &= \frac{3}{128\eta}(\pi M)^{-5/3} \\
\theta_2 &= \frac{1}{384\eta}\left(\frac{3715}{84} + 55\eta\right)(\pi M)^{-1} \\
\theta_3 &= -\frac{3\pi}{8\eta}(\pi M)^{-2/3} \\
\theta_4 &= \frac{3}{128\eta}\left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2\right)
\end{aligned}$$

The Parameter Space:

Parameters in which the ambiguity function is almost independent of location

$$\tau_0 = \frac{5}{256\mu M^{2/3}}(\pi f_a)^{-8/3}$$

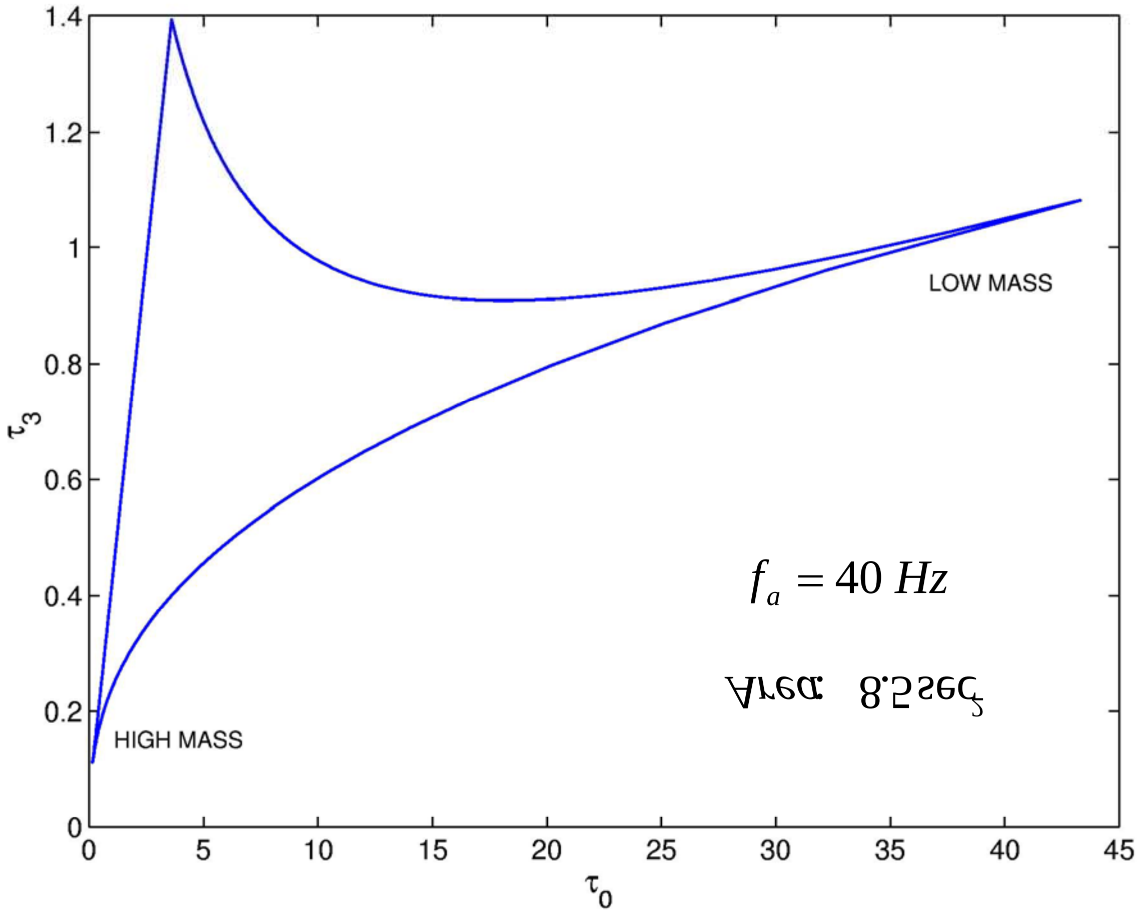
$$\tau_3 = \frac{1}{8\mu}\left(\frac{M}{\pi^2 f_a^5}\right)^{1/3}$$

$f_a = 40$ Hz.

Mass Range: $1\ M_\odot \le m_1, m_2 \le 30\ M_\odot$

Area: $8.5\ \text{sec}^2$

Parameter Space for 1 – 30 solar masses



Detection Strategy

Sathyaprakash & SD 91, 94

Owen 96

Signal depends on many parameters:

Parameters: Amplitude, t_a , ϕ_a , m_1 , m_2

Strategy: Maximum likelihood method

- Amplitude: Use normalised templates
- Scanning over t_a : FFT
- Initial phase ϕ_a : Two templates for 0 and $\pi/2$ and add in quadrature
- masses ⟶ chirp times: τ_0 , τ_3 bank required

Detection

Data: $x(t)$

Templates: $h_0(t; \tau_0, \tau_3)$ and $h_{\pi/2}(t; \tau_0, \tau_3)$

Padding: about 4 times the longest signal

Fix τ_0, τ_3

Outputs: Use FFT to compute $c_0(\tau)$ and $c_{\pi/2}(\tau)$

τ is the time-lag: scan over t_a

Statistic: $c^2(\tau) = c_0^2(\tau) + c_{\pi/2}^2(\tau)$: Maximised over initial phase

Conventionally we use its square root $c(\tau)$

Compare $c(\tau)$ with the threshold η

Thresholding

Mohanty & SD 96,

Mohanty 98

In absence of the signal:

Each $c_0, c_{\pi/2}$ is Gaussian distributed with mean zero.

While $c = \sqrt{c_0^2 + c_{\pi/2}^2}$ is Raleigh distributed:

$R(c) = c\exp(-c^2/2)$

1 false alarm/yr, sample at 2 kHz, $n_t \sim 10^4$ at 3% mismatch

Range $1M_\odot \le m_1, m_2 \le 30M_\odot$ - LIGO I curve.

$\int_\eta^\infty R(c)dc = P_F$ gives $\eta \sim 8.2$

Detection probability Q_d

Defn: Signal Strength $S = c$ for perfect match of the parameters

c follows a Rician distribution

$\sim$ Gaussian with mean S, if $S >> 1$

With two adjacent templates:

For $Q_d = .95$ we get $S_{minmin} \sim \eta + 0.7$

$S_{min} \sim 8.2 + 0.7 + 3\% \sim 9.2$

FLAT SEARCH

Mismatch and Ambiguity Function

$$H(\lambda^\alpha, \Delta\lambda^\alpha) = \langle s(\lambda^\alpha), s(\lambda + \Delta\lambda^\alpha \rangle$$

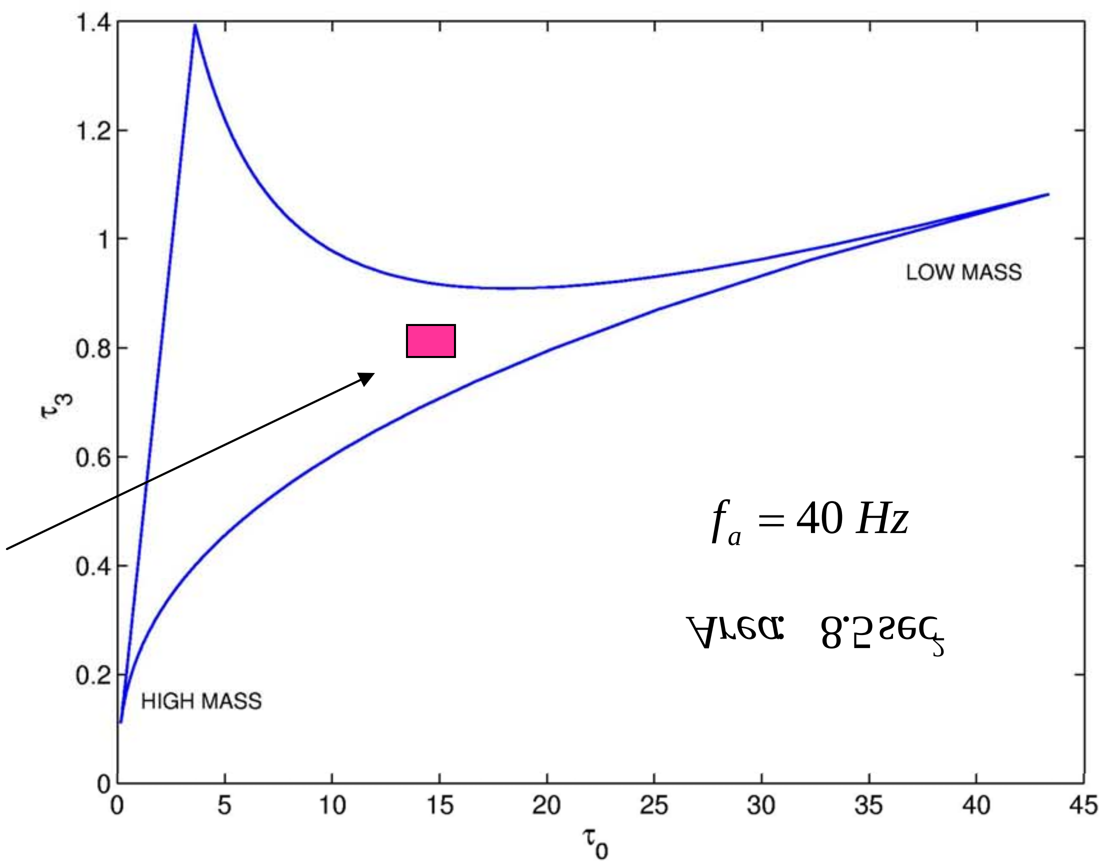
Intrinsic Ambiguity Function:

$$\mathcal{H}(\tau_0, \tau_3; \Delta\tau_0, \Delta\tau_3) = \max_{\Delta t_c, \Delta\phi_0} \; H(\lambda^\alpha, \Delta\lambda^\alpha)$$

Minimal match $= 0.97$

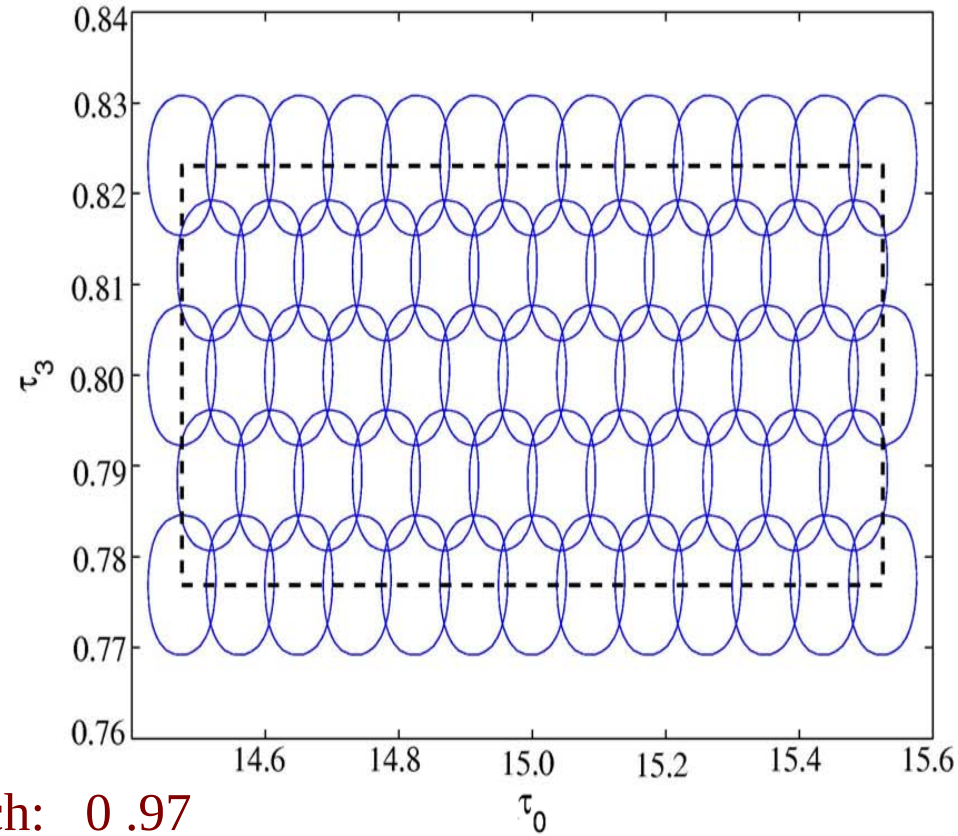
Parameter Space

Parameter space for the mass range 1 – 30 solar masses



Hexagonal tiling of the parameter space

LIGO I psd



Minimal match: 0 .97

Density of templates: ~ 1300/sec^2

Number of templates: ~ 11,050

Cost of the flat search

Longest chirp: 95 secs for lower cut-off of 30 Hz

Sampling rate: 2048 Hz

Length of data train (> x 4): 512 secs

Number of points in the data train: $N = 2^{20}$

Length of padding of zeros: 512 – 95 = 417 secs

This is also the processed length of data

Number of templates: $n_t = 11050$

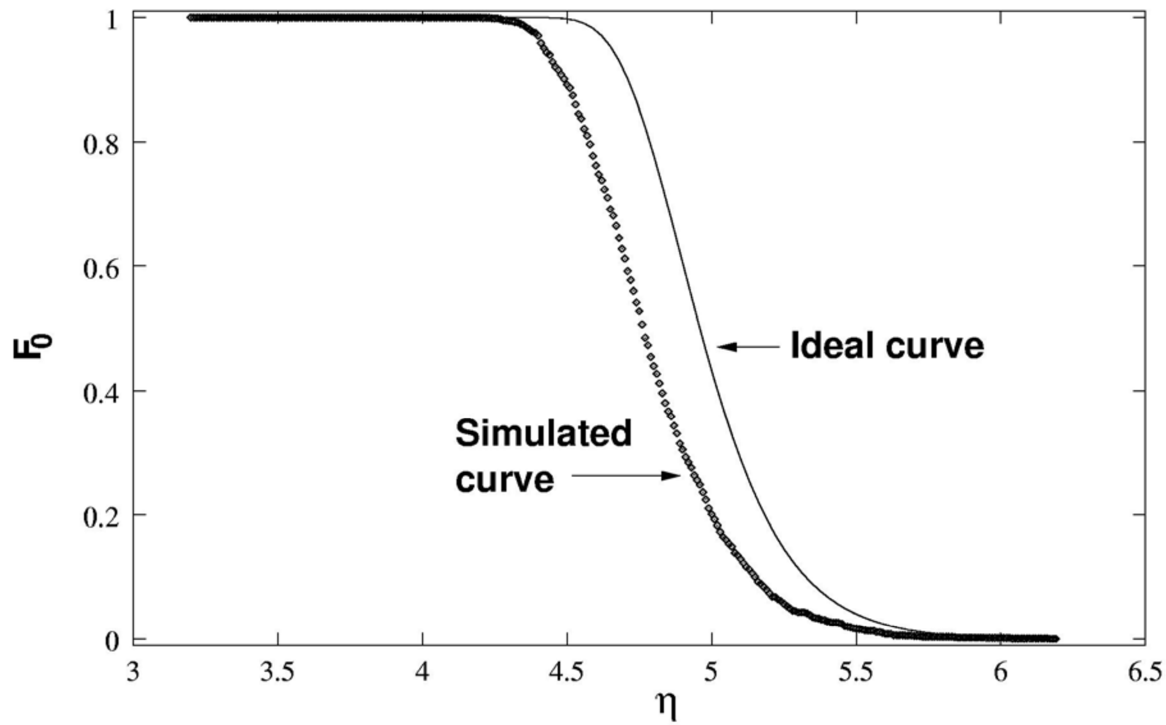
Online speed required: $n_t \times 6 N \log_2 N / 417$

$\sim$ 3.3 GFlop

The hierarchical search

The principle:

- Two thresholds and two banks of templates:
  - Lower threshold: η_1 and a coarse grid of templates
  - Higher threshold: $\eta_2 = \eta$ and the 1-step fine grid of templates
- η_1 sufficiently large - few false alarms - minimise cost of the fine search.
- η_1 small enough - coarse grid - minimise cost in the trigger stage.



The average number of crossings: $n_c = n_t^{(1)} \times F_0(\eta)$

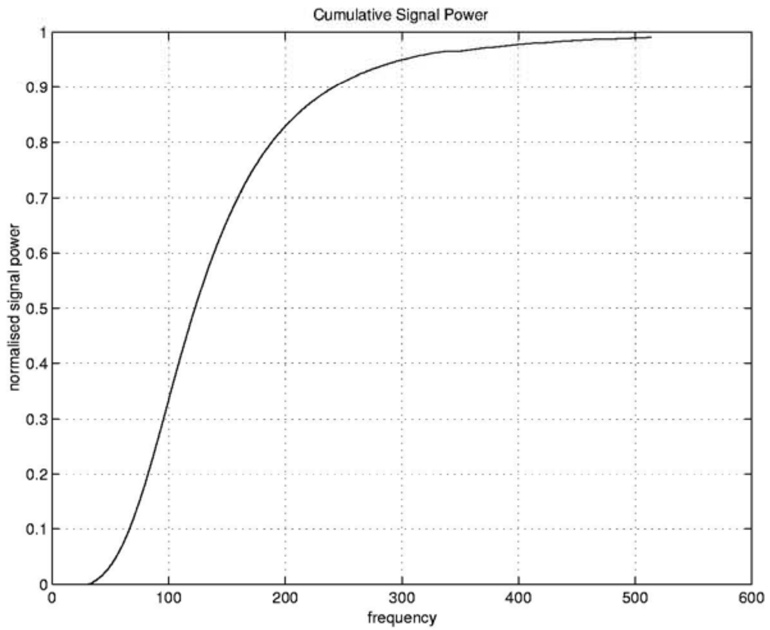
The extended hierarchical search

Include decimation in time in the previous hierarchical search

Cumulative Signal Power:

92% power at

f_c = 256 Hz



Tanaka & Tagoshi 96

Decimation in time

Use the fact that there is a lot of power at low frequencies in the chirp

At 256 Hz there is 92 % of the signal power

Sample at lower frequencies but lose little in power

Gain in reducing FFT operations in the trigger stage

Choice of the first stage threshold

- High enough to reduce false alarm rate

  -- reduce cost in the second stage

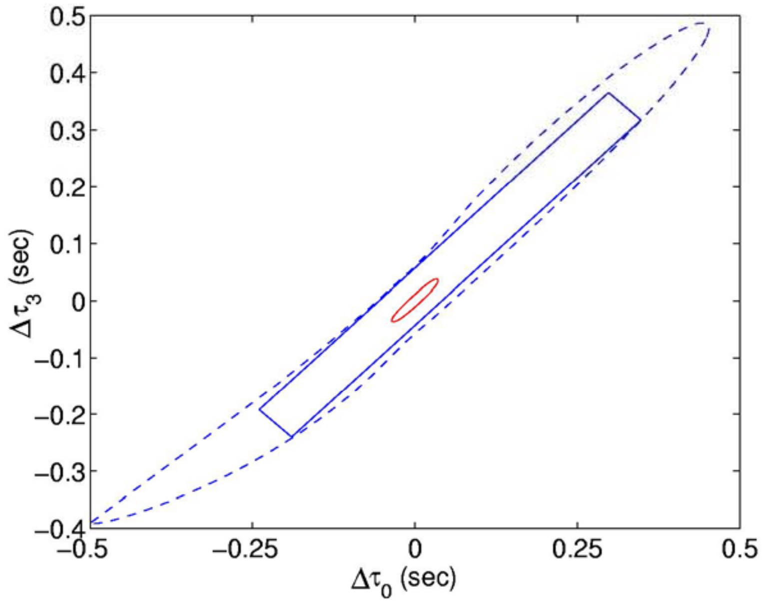
  -- for 2^{18} points in a data train $\eta_1 > 5$ or so

- Low enough to make the 1st stage bank as coarse as possible and hence reduce the cost in the 1st stage

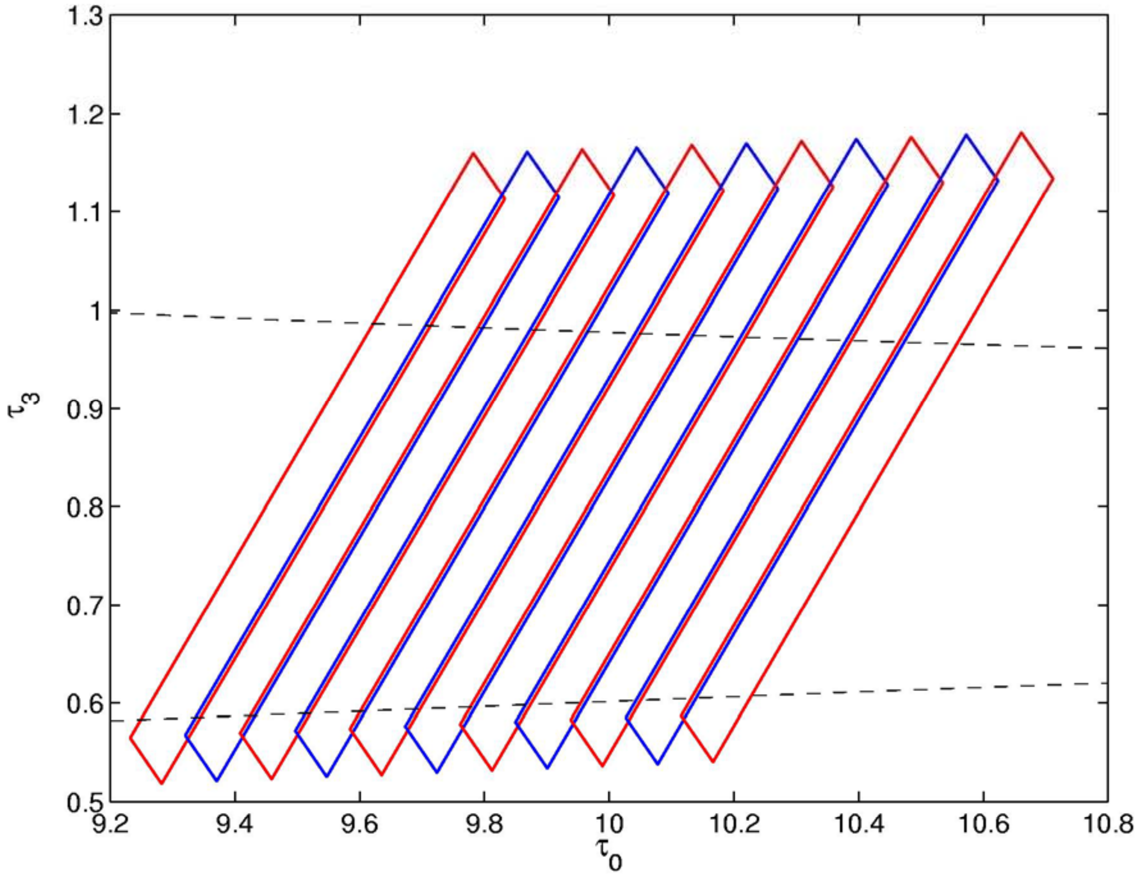
$$\eta_1 = (.92)(S_{min})\Gamma - \Delta S \sim 6$$

Chirp cut at 256 Hz

Contour level $= (\eta_1 + \Delta S)/(.92 \times S_{min}) \sim 0.8$



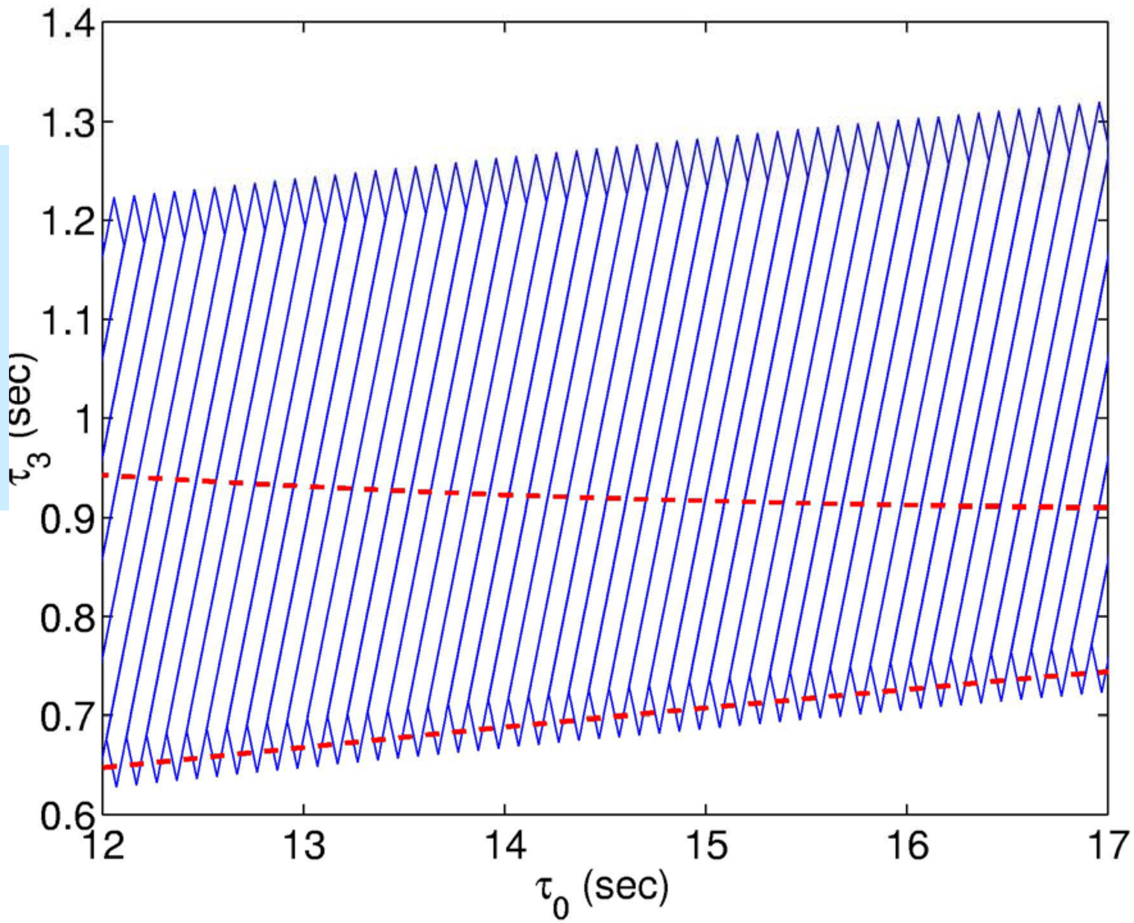
Relative size of boat/ellipse: .97 at 1 kHz and .8 at 256 Hz



1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
τ_3
9.2
9.4
9.6
9.8
10
10.2
10.4
10.6
10.8
τ_0

Zoomed view near low mass end

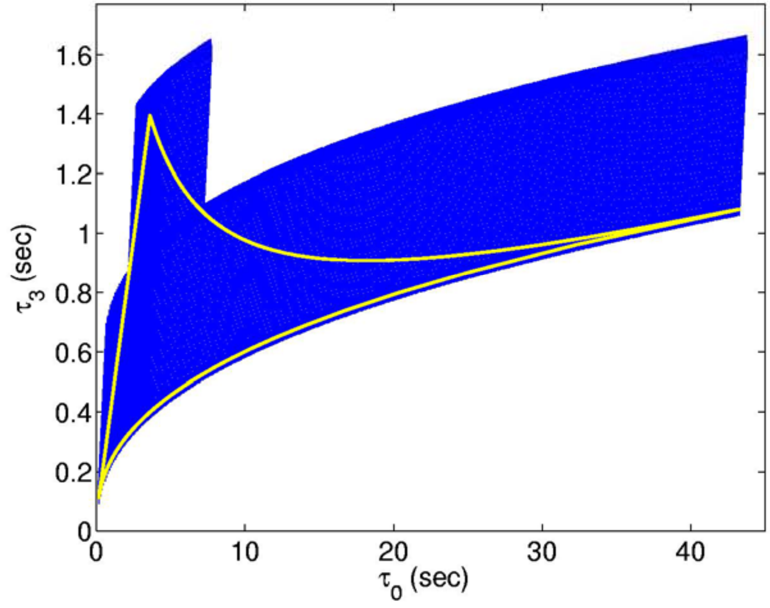
Templates flowing out of the deemed parameter space are not wasted



Boundary Effects

Inefficient tiling near the low mass end !!

For low mass cut-off of $1 M_{\odot}$: Need 535 templates



Cost of the EHS

Longest chirp ~ 95 secs T_{data} = 512 secs T_{pad} = 417 secs

f^{1}_{samp} = 512 Hz f^{2}_{samp} = 2048 Hz

$N^{1} = 2^{18}$ $N^{2} = 2^{20}$

$S_{min} \sim 9.2$

$\Gamma \sim 0.8$

$\eta_1 = .92\ S_{min}\ \Gamma - \Delta S \sim 6.05$

$\eta_2 = 8.2$

n_1 = 535 templates n_2 = 11050 templates

$n_{cross} \sim .82$

$\gamma \sim 20$

$\alpha \sim 2.5$

$S_1 \sim 6\, n_1 N^{1} \log_2 N^{1} / T_{pad} \sim 36$ Mflops

$S_2 \sim n_{cross}\, \gamma\, \alpha\ 6\ N^{2} \log_2 N^{2} / T_{pad} \sim 13$ Mflops

S ~ 49 Mflops S_{flat} ~ 3.3 GFlops

Gain ~ 68

Performance of the code on real data

Work in Progress

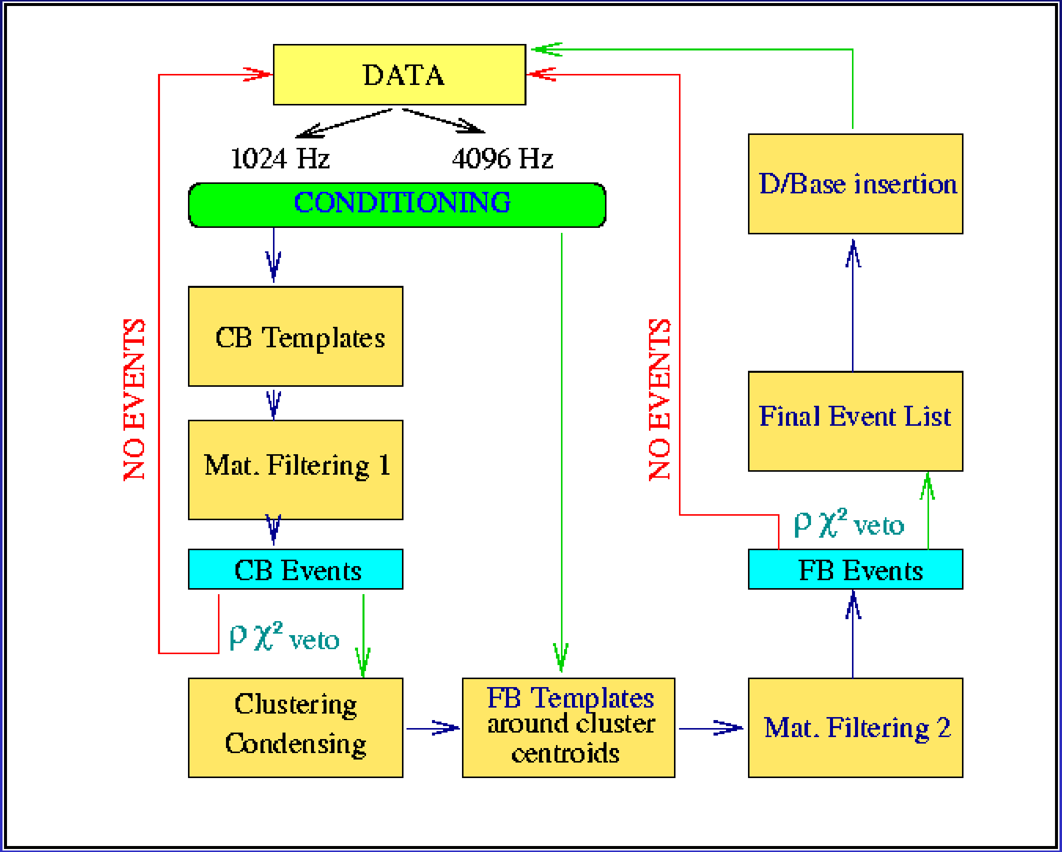
- S2 - L1 playground data was used – 256 seconds chunks, segwizard – data quality flags
- 18 hardware injections - recovered impulse time and SNR – matched filtering part of code was validated
- Monte-Carlo chirp injections with constant SNR - 679 chunks - test the running of the pipeline

EHS pipeline ran successfully

- The coarse bank level generates many triggers which pass the ρ and χ^2 thresholds

Clustering algorithm

The EHS pipeline



Can run in standalone mode (local Beowulf) or in LDAS

Data conditioning

Preliminary data conditioning: LDAS datacondAPI

- Downsamples the data at 1024 and 4096 Hz
- Welch psd estimate
- Reference calibration is provided

Code data conditioning:

- Calculate response function, psd, h(f)
- Inject a signal

Template banks and matched filtering

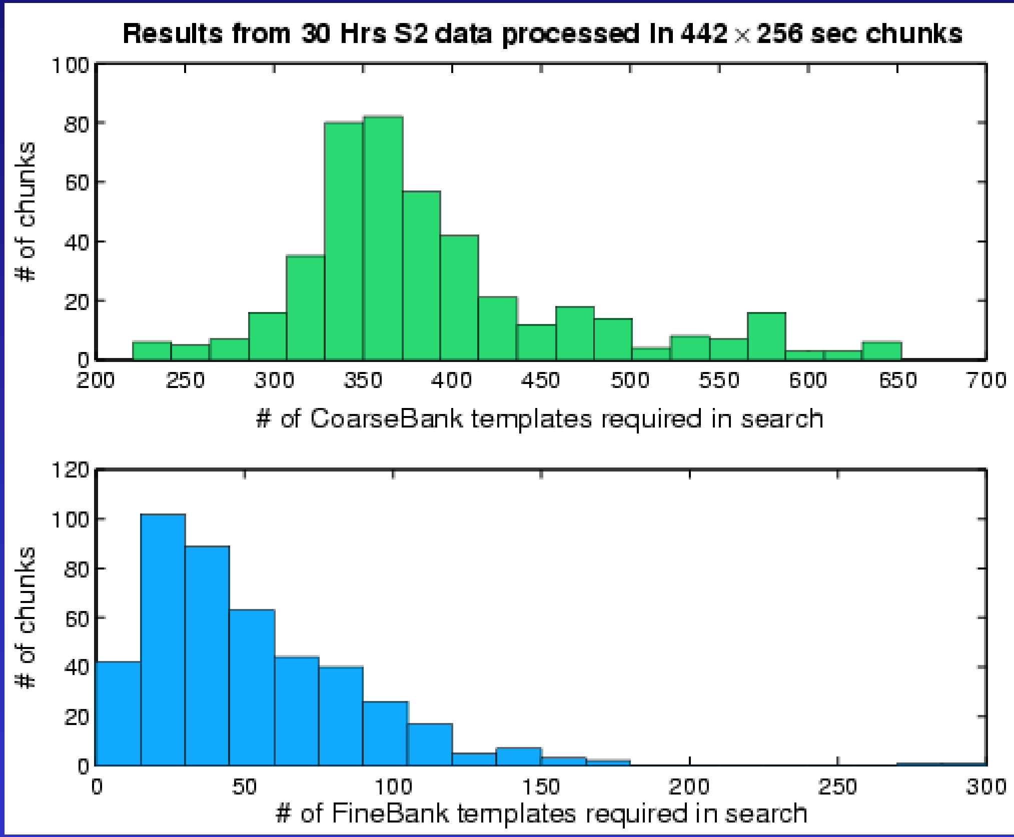
- LAL routines generate fine and the coarse bank

$MM_{fine} = 0.97$ $MM_{coarse} = 0.9$

- Compute χ^2 for templates with initial phases of 0 and $\pi/2$

- Hundreds of events are generated and hence a clustering algorithm is needed

Number of Templates statistics
S2 playground data



Indicative of non-stationarity of noise

Total number of data chunks: 442

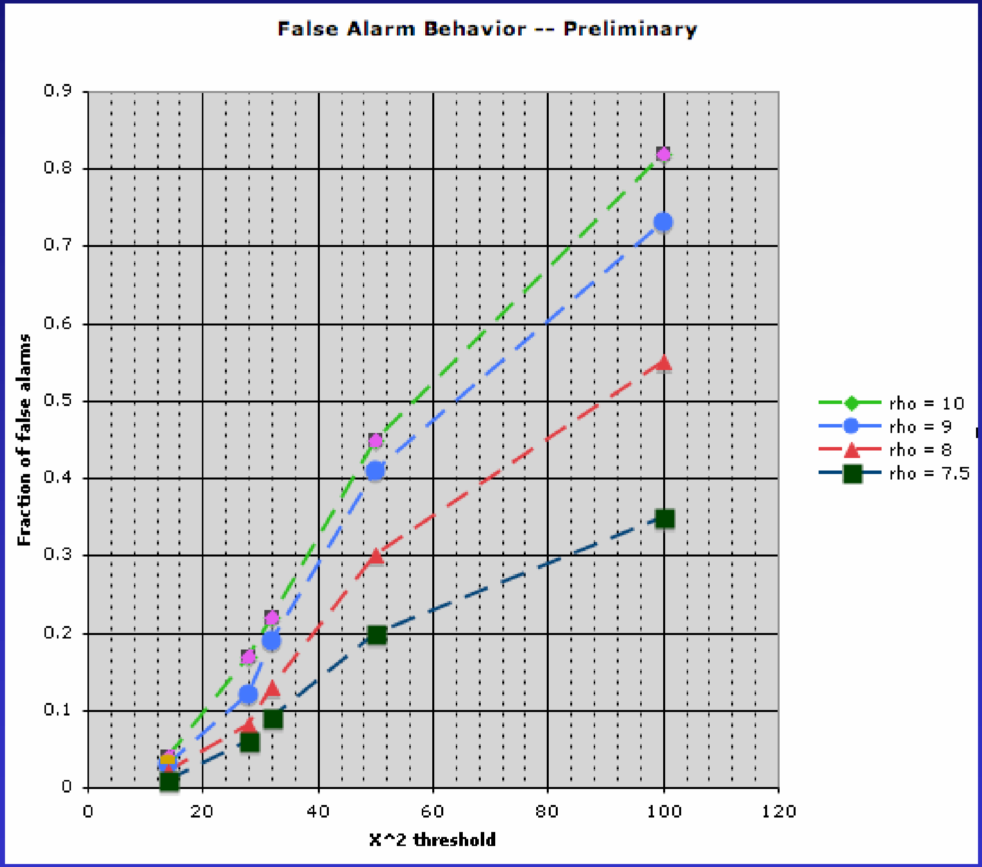
False alarm analysis ($\rho-\chi^2$)
30 hours of S2 playground data

442 data chunks

x 256 seconds

$\rho_{threshold}$ > 7 :

Total of 21,357 events seen



Making a choice of ρ , χ^2 threshold values

The Clustering Algorithm

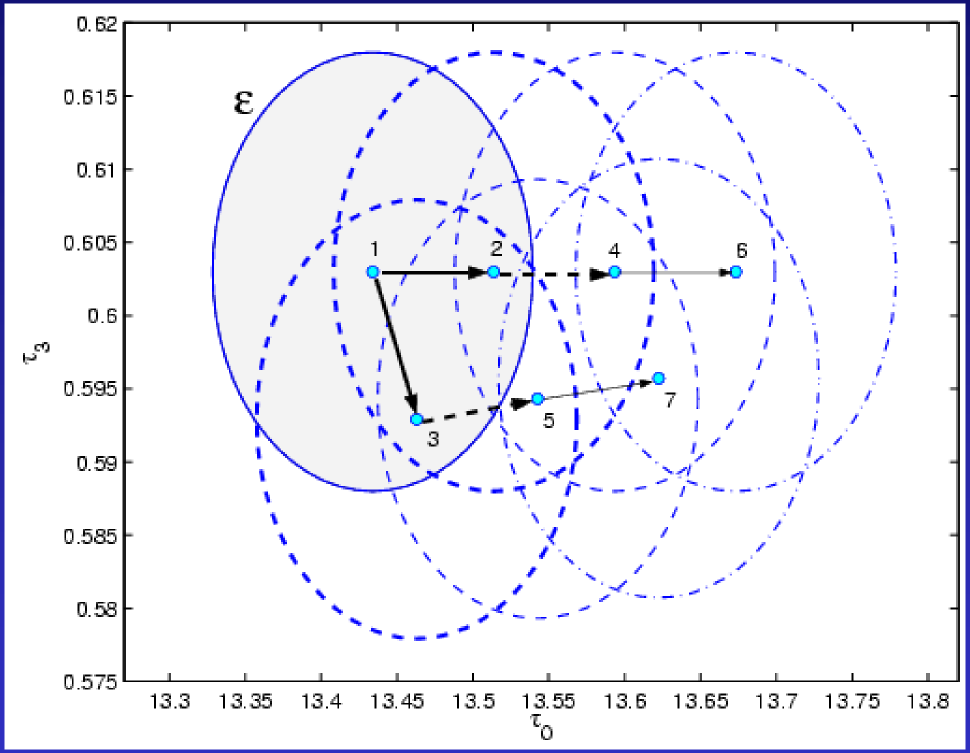
Step 1: DBScan algorithm is used to identify clusters/islands

Step 2: The cluster is condensed into an event which is followed up by a local fine bank search

Order of N ln N operations

DBScan (Ester et al 1996)

1. For every point p in C there is a q in C such that p is in the ε-nbhd of q

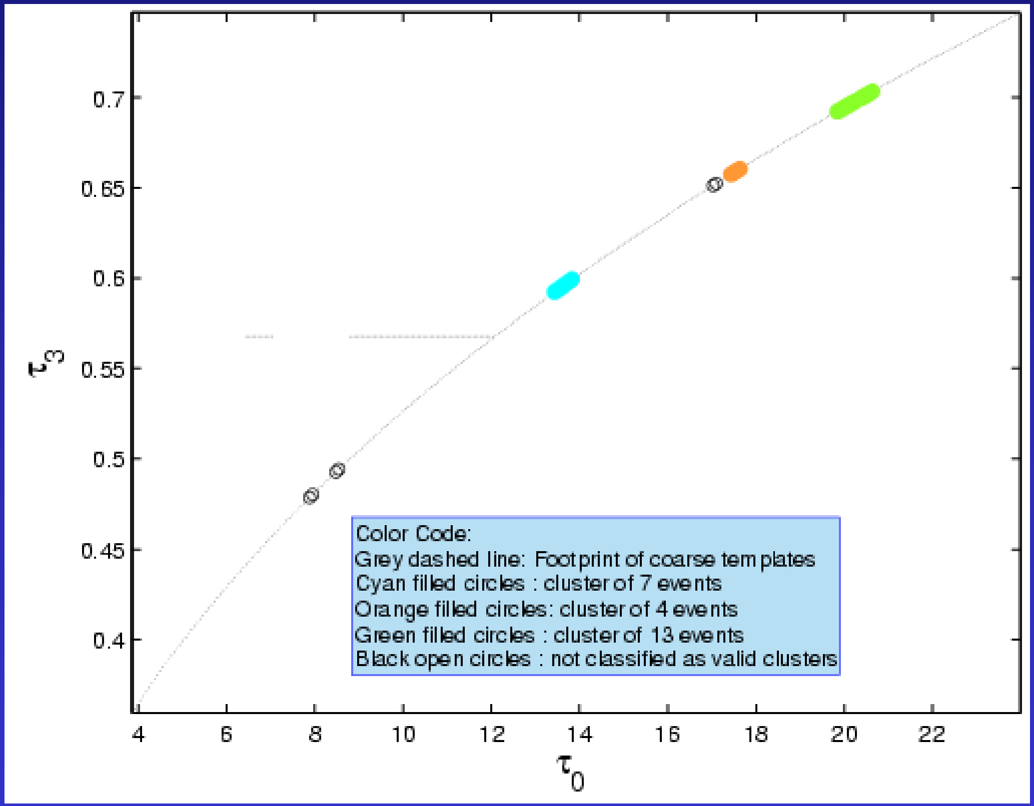


2. This breaks up the set of triggers into connected components

A cluster must have at least N_{min} number of points – otherwise it is an outlier

DBScan implemented on triggers
S2 playground data

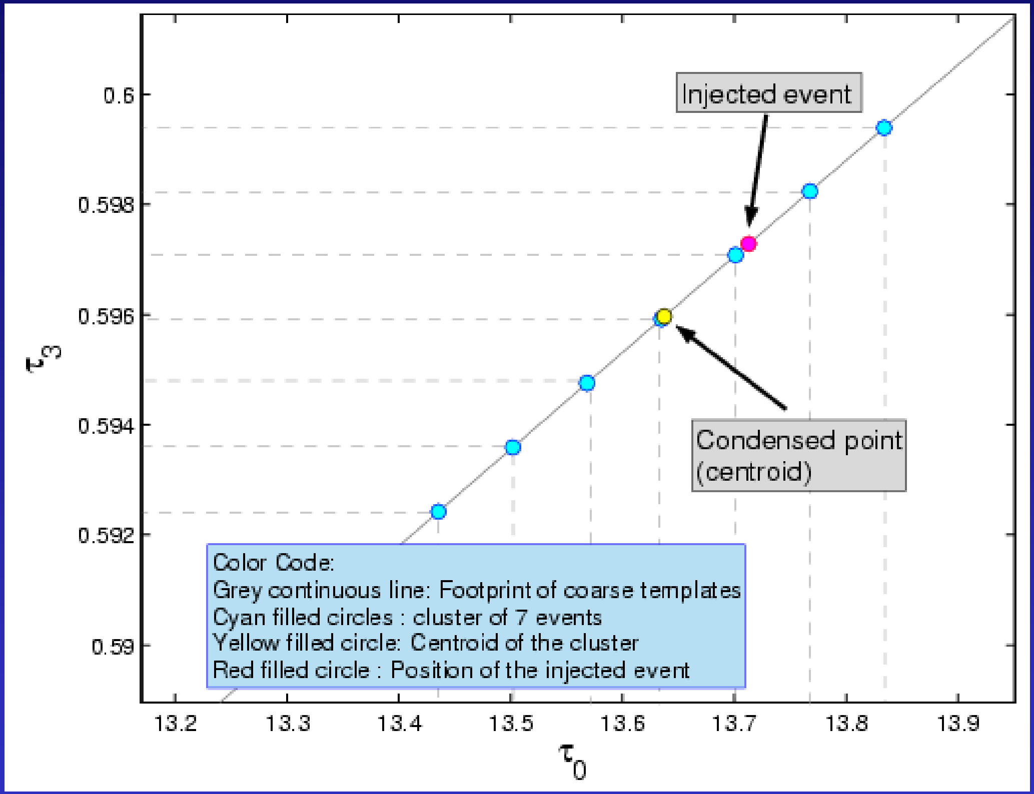
$\varepsilon \sim 3 \Delta \tau$ $N_{min} = 2$



Condensing a cluster

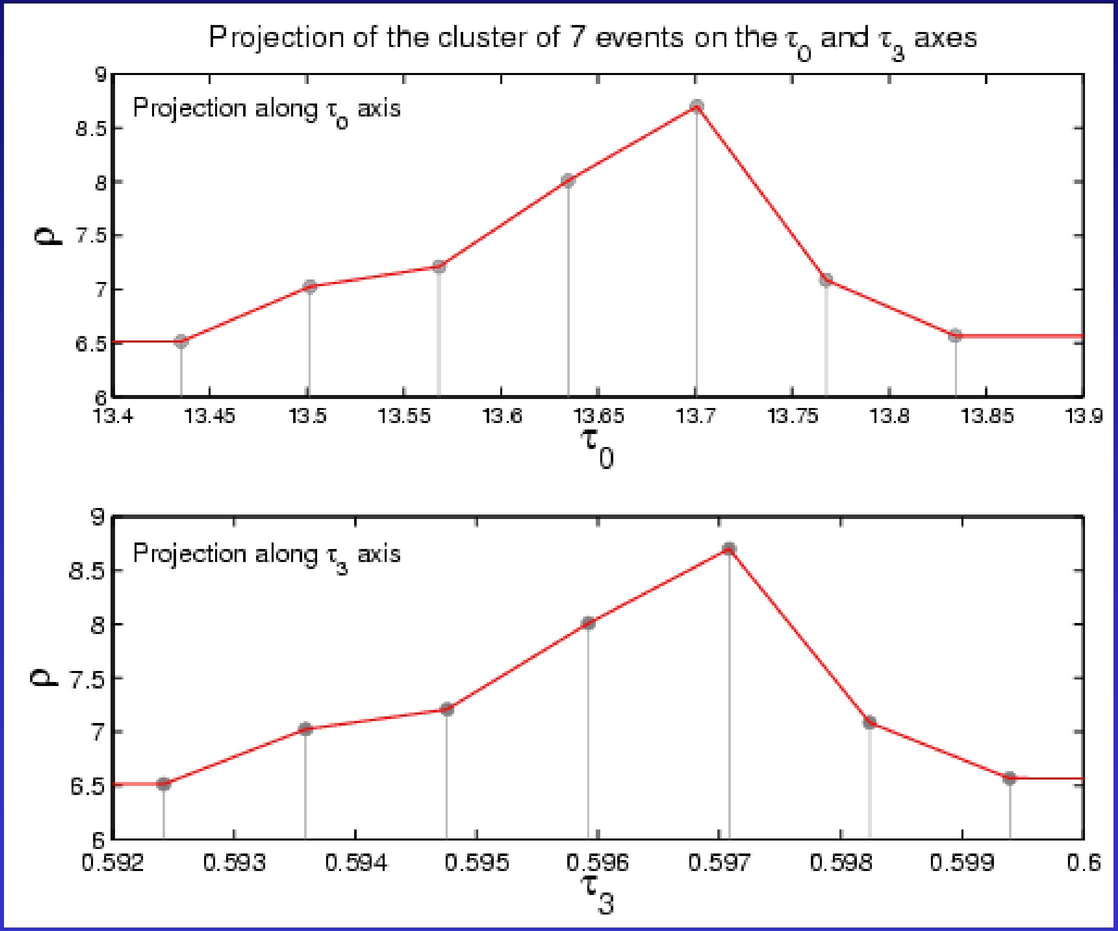
- Non uniform template bank
- Project events in the cluster to the tau 0 and tau 3 axes
- Compute the average signal position by the

Trapezoidal rule:



$$< x > = \frac{\sum (\rho_k + \rho_{k+1})(x_k + x_{k+1})(x_{k+1} - x_k)}{2\sum (\rho_k + \rho_{k+1})(x_{k+1} - x_k)}$$

Condensing a cluster: Trapezoidal rule



Conclusions

- Pipeline and code are in place but still need to be tuned/automated

- Gain factors between 7 and 10 on IUCAA cluster

- Galaxy based Monte Carlo simulation is required to figure out the efficiency of detection.

- Hierarchical search frees up CPU for additional parameters - spin