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Equivalence relation between non spherical optical cavities and application to advanced G.W. interferometers.

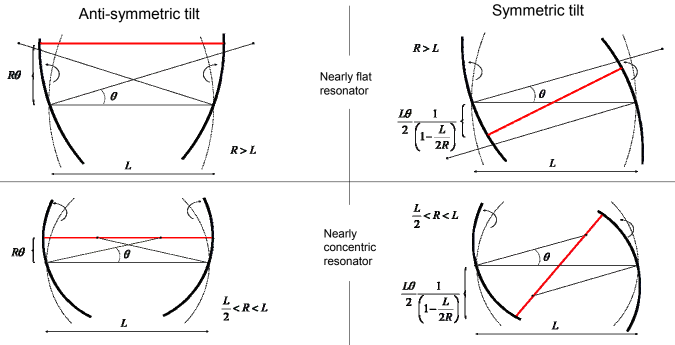
Juri Agresti and Erika D'Ambrosio

Aims of this study :

- Define an equivalence relation between optical cavities having non-spherical mirrors and supporting non-Gaussian Flat-Top beams.
- Application to misalignment instability in the case of Advanced LIGO.

Our analysis is motivated by two important issues

Nearly concentric cavities are more stable than nearly flat cavities for misalignments coupled to radiation pressure.


Anti-symmetric tilt

Symmetric tilt

Nearly flat resonator

Nearly concentric resonator

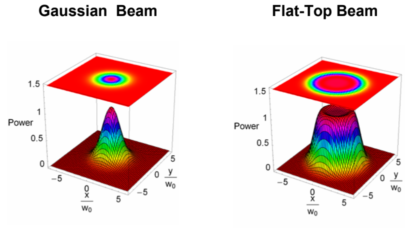
Radiation pressure makes the mirrors tilt further.

The optical torque is smaller for a nearly concentric cavity.

Radiation pressure provides a restoring torque

Concentric cavity benefits more from the restoring torque

Larger and flatter beam sampling the mirror surface provide larger suppression of thermal noises (thermoelastic, coating thermal noise, etc…) because of a better average of the surface fluctuations.


Gaussian Beam

Flat-Top Beam

$$w_0 = \sqrt{\frac{L}{k}}$$

We generalized the known equivalence between flat and concentric cavities to the case of non spherical mirrors supporting a Flat-Top Beam.

Integral equation for cavity modes $\longrightarrow$ $\gamma\, u(\vec{r}) = \int_{\substack{Mirror\\Surface}} K(\vec{r},\vec{r}')\, u(\vec{r}')\, d\vec{r}'$

$K(\vec{r},\vec{r}')$	Propagator from surface to surface
$u(\vec{r})$	Field distribution over mirror surface
γ	Eigenvalue

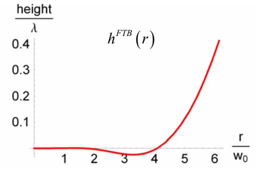
Equal mirrors and cylindrical symmetry

Nearly flat cavity

$$K_{flat}(\vec{r},\vec{r}') = \frac{ik}{2L\pi} Exp\left[-ikL + ikh(r) - \frac{ik}{2L}\left|\vec{r}-\vec{r}'\right|^2 + ikh(r')\right]$$

$h(r)$ Mirror profile of the nearly flat cavity

Mirror profile yielding Flat-Top Beam as fundamental mode in nearly flat mirror configuration. $\longrightarrow$



Equivalent nearly concentric cavity

$$K_{conc}(\vec{r},\vec{r}') = \frac{ik}{2L\pi} Exp\left[-ikL - ikh(r) + \frac{ik}{2L}\left|\vec{r}+\vec{r}'\right|^2 - ikh(r')\right]$$

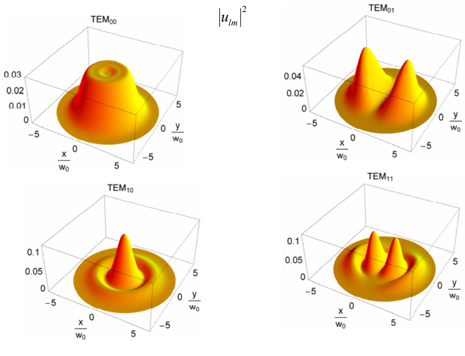
$\frac{r^2}{2R} - h(r)$ Mirror profile of the equivalent nearly concentric cavity configuration.

$$R = \frac{L}{2}$$

Notice: this equivalence between cavities with non spherical mirrors, applies, in general, to any mirror profile h(r) within the paraxial approximation. Here we focused on a particular mirror shape in order to obtain a FTB.

What this equivalence tell us:

The intensity distribution on the mirrors for the modes of the two resonators are the same (l and m are the radial and angular mode numbers).



Unique mapping between the eigenvalues of the nearly concentric and nearly flat cavity for all orders

$$e^{ikL}\gamma_{lm}^{conc} = (-1)^{m+1} e^{-ikL}\left(\gamma_{lm}^{flat}\right)^*$$

The two cavities have the same diffraction loss per bounce $\longrightarrow$ $1-\left|\gamma_{lm}\right|^2$

Evaluating the angular instability due to radiation pressure:

Anti-symmetric tilt cause this coupling between modes $u_{tilt} = u_{00} + \alpha\, u_{01}$

Radiation pressure torque T in equivalent configurations: $\frac{T_{conc}}{T_{flat}} \approx \frac{\alpha_{conc}}{\alpha_{flat}}$

Since the coupling parameter α depends on the eigenvalues and eigenmodes of the cavity, **we used our proven equivalence** to compare the sensitivity to misalignment of nearly concentric and nearly flat cavities supporting the same Flat-Top Beam.

$$\boxed{\frac{\alpha_{conc}^{FTB}}{\alpha_{flat}^{FTB}} \approx \frac{1}{247}}$$

Corresponding spherical cavities supporting Gaussian beams have a larger ratio. $\longrightarrow$ $\boxed{\frac{\alpha_{conc}^{G}}{\alpha_{flat}^{G}} \approx \frac{1}{40}}$ and

$$T_{conc}^{G} \approx 1.1\, T_{conc}^{FTB}$$

$$T_{flat}^{G} \approx 0.2\, T_{flat}^{FTB}$$

(Laser wavelength: 1.064 µm, cavity length ~ 4Km, mirror radius ~ 16 cm, diffraction losses ~ 20 ppm, spherical cavities g factor: g =±0.952)

Conclusions:

We pointed out an equivalence relation between nearly concentric and nearly flat cavities with non spherical mirrors.

In the case of non spherical mirrors supporting a Flat-Top Beam, both configurations provide the same suppression of thermal noises with respect to Gaussian beams because of the same power distribution over the mirror surface but the nearly concentric cavity is much less sensitive to angular instability due to radiation pressure.

Nearly concentric non spherical mirrors are proposed as an alternative to Advanced LIGO baseline.

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