The SNR as a Predictor of Detectability in the S1 Bursts Search

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Motivation

- Want a simple, concise means to characterise detection efficiency of bursts pipeline for all types of simulated signals.
  - » S1: Tabulated lists of RSS, peak, char amplitudes for 50% efficiencies.
  - » What to do for 100s of simulated waveforms?

- Want method to estimate efficiency for waveforms which we have not tested with detailed simulations
  - » Needed for applying our upper limits to general astrophysical source models.

This Report

- **Procedure:** Study behaviour of three amplitude measures for simulated signals which have 50% efficiency in S1 TFClusters search:
  - » signal-to-noise ratio (SNR) ρ
  - » root-sum-square (RSS) amplitude h_{rss}
  - » peak amplitude h_{peak}

- **Conclusions:** SNR most promising for predicting efficiencies for any waveform.

- **Details:** See T040002.

Amplitude Measures

- SNR:

$$\rho = \left[4 \int_0^\infty df \, \frac{|\tilde{h}(f)|^2}{S(f)} \right]^{1/2}$$

S(f) - detector noise

- RSS:

$$h_{\rm rss} = \left[\int_{-\infty}^{\infty} dt \, h^2(t) \right]^{1/2} = \left[2 \int_0^\infty df \, |\tilde{h}(f)|^2 \right]^{1/2}$$

- Peak:

$$h_{\rm peak} = \max\left(|h(t)|\right)$$

independent of detector noise

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Frequency Measures

- Characteristic frequency f_c:

$$f_c \equiv \frac{\int_0^\infty df\, f \frac{|\tilde{h}(f)|^2}{S(f)}}{\int_0^\infty df \frac{|\tilde{h}(f)|^2}{S(f)}}$$

(Very close to central frequency for sine-Gaussians)

- Characteristic frequency f_w ignoring noise:

$$f_w \equiv f_c|_{S(f)=1} = \frac{\int_0^\infty df\, f |\tilde{h}(f)|^2}{\int_0^\infty df |\tilde{h}(f)|^2}$$

Procedure

- Plot each amplitude measure (ρ, h_{rss}, h_{peak}) versus characteristic frequency (f_c, f_w) for each waveform tested by simulations in S1.
  - » Use amplitudes for 50% triple-coincidence efficiency
  - » Restrict frequency integrals to >150Hz (high-pass filter in S1).
  - » For S(f) use envelope of official representative noise spectra:

$$S(f) = \max(S_{\mathrm{H1}}(f), S_{\mathrm{H2}}(f), S_{\mathrm{L1}}(f))$$

- Waveforms simulated in S1:
  - » 2 Gaussians (broadband, ad hoc)
  - » 7 sine-Gaussians (narrowband, ad hoc)
  - » 8 Zwerger-Mullers (broadband, astrophysical)

Quality of Measure "X"

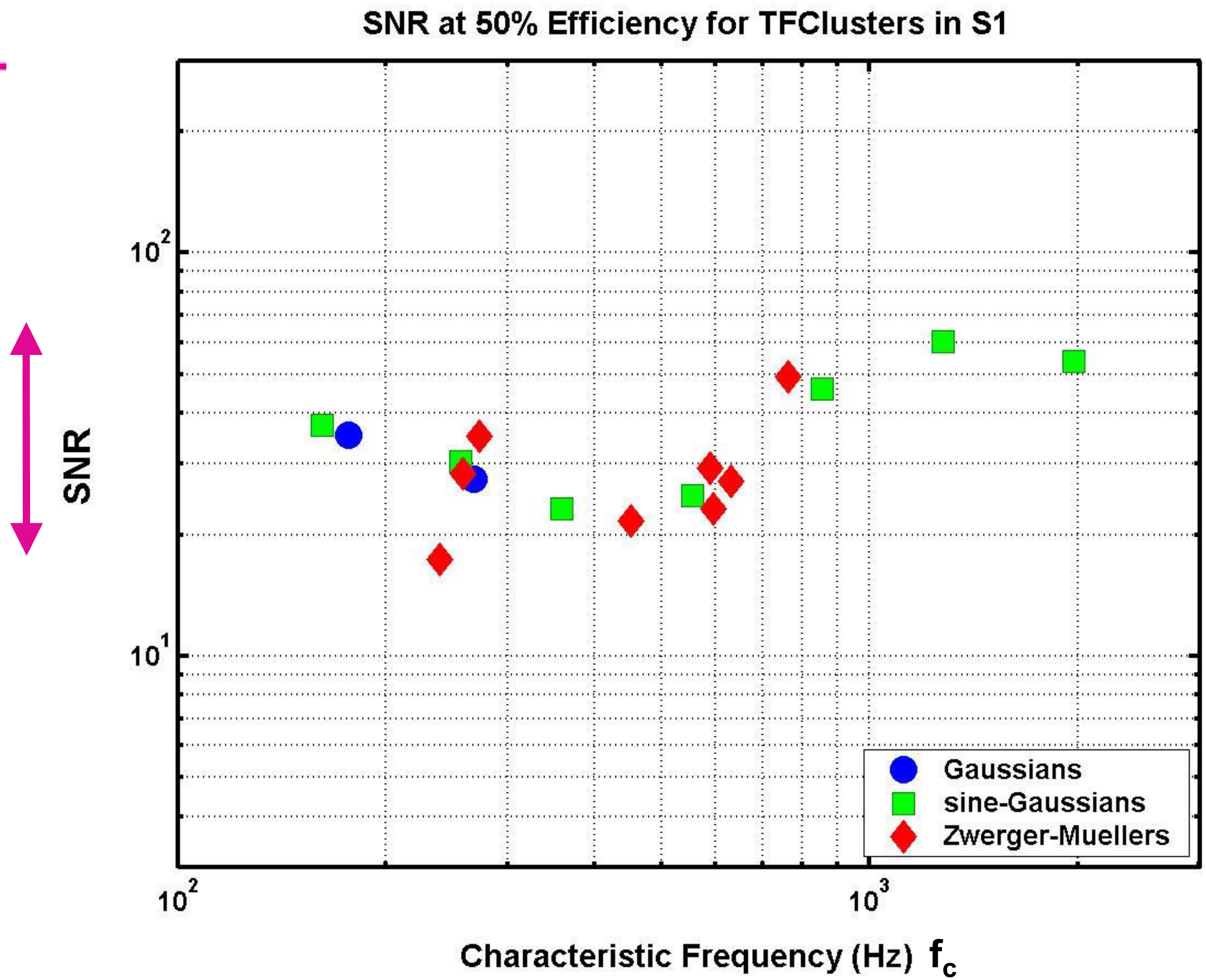
- **Ideal:** X(f) is universal constant X_0 for all waveforms (flat line). Can predict that a specific waveform is detectable with efficiency>50% if its measure >X_0.

- **Good:** Measure is smooth, sharply defined curve for all waveforms, varying over only small range. Can predict that a waveform is detectable with efficiency>50% if its measure >X (f).

- **Bad:** X(f) varies over large range and has different behaviour for different waveform types. Useless.

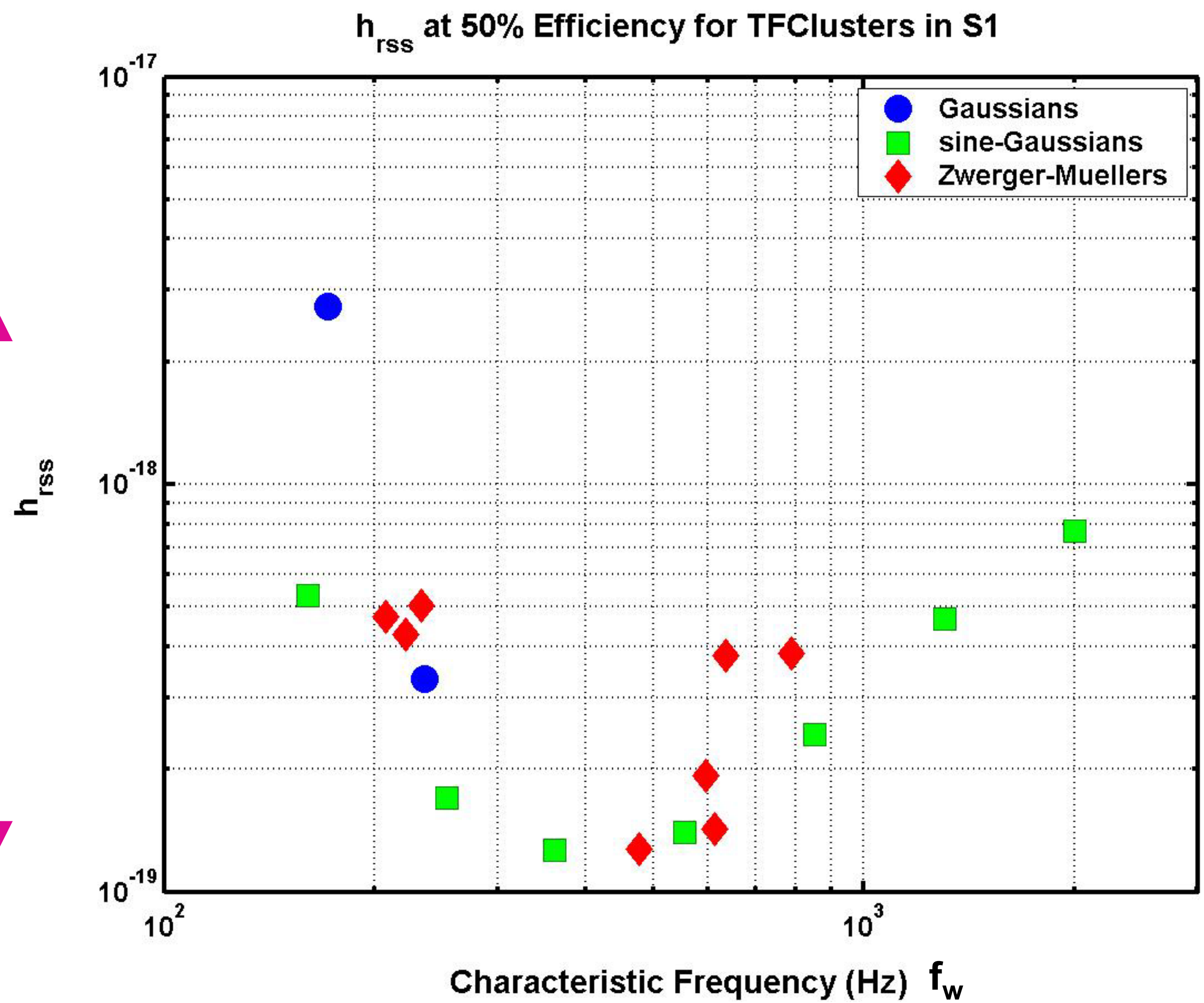
50% SNR vs f_c

Best-to-worst factor ~ 3.5



All waveforms fall on common curve

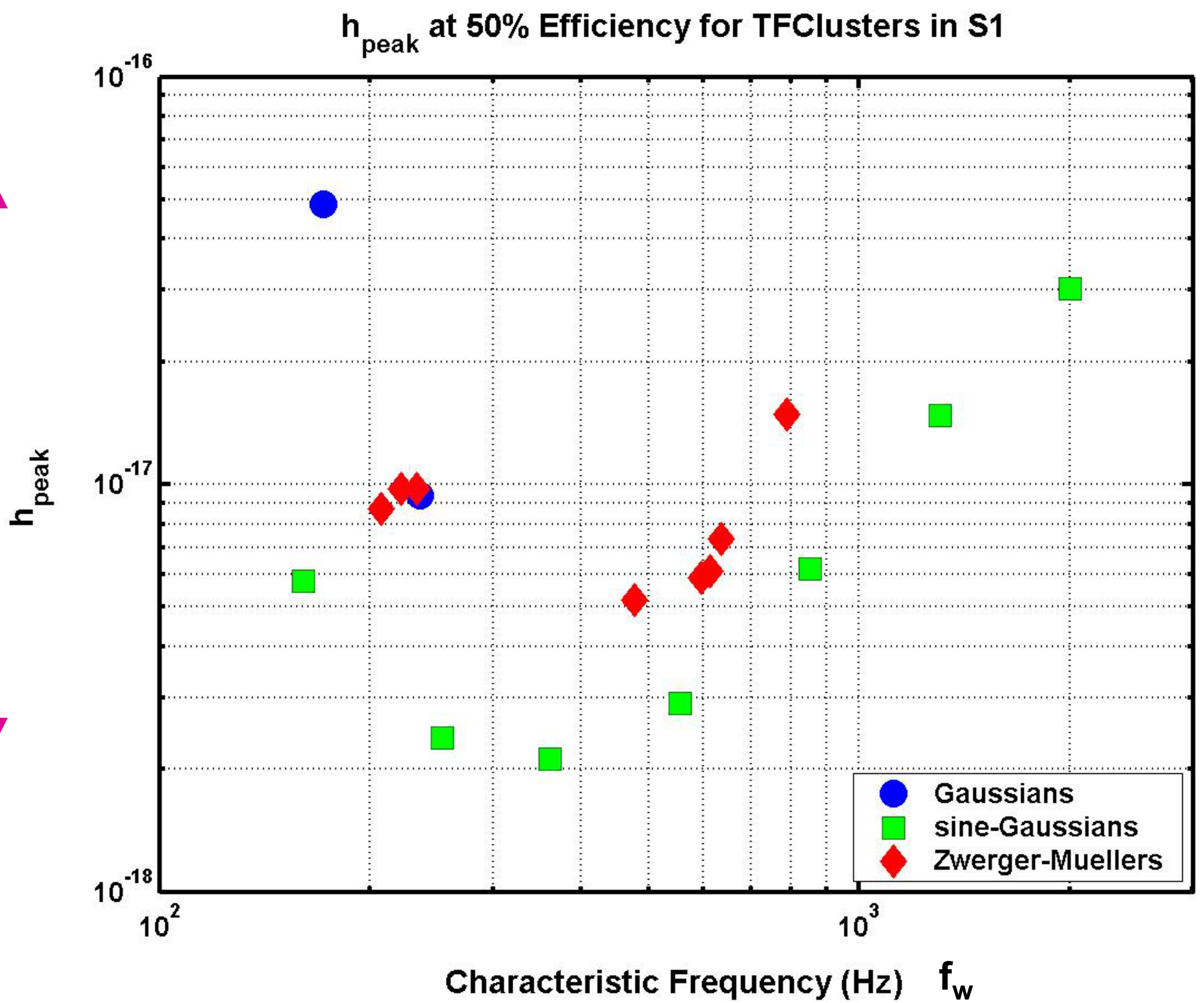
50% h_{rss} vs f_w



Best-to-worst factor: ~ 22

Narrowband and broadband waveforms fall on separate curves

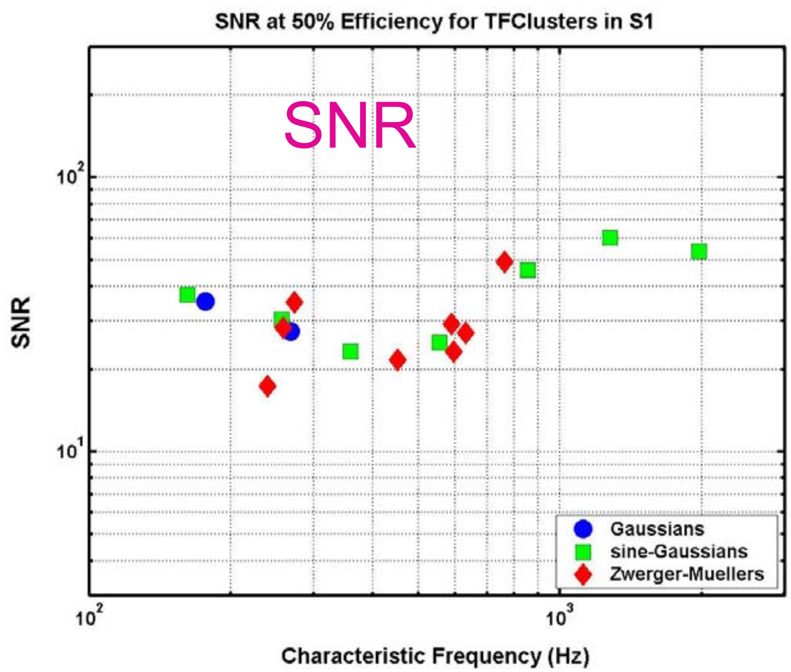
50% h_{peak} vs f_w

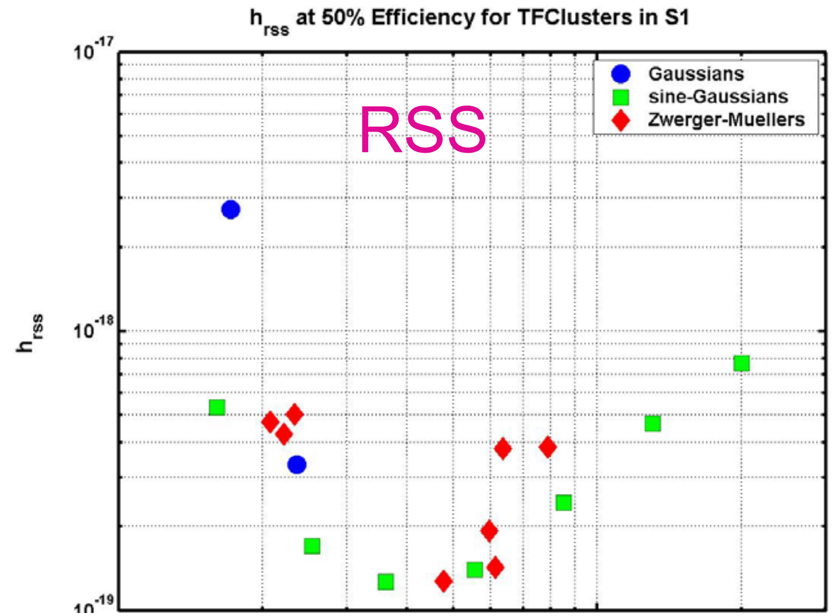


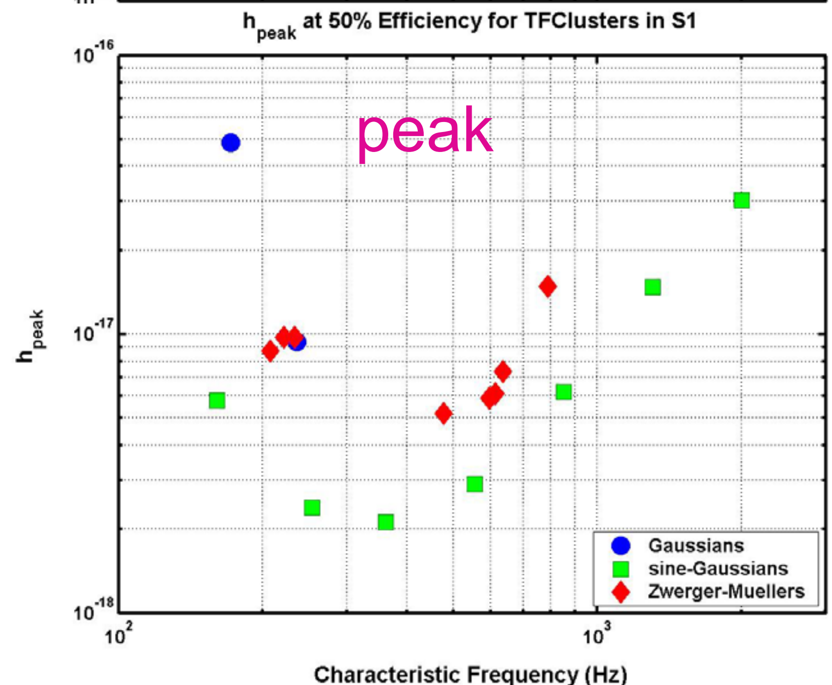
Best-to-worst factor: ~ 23

Narrowband and broadband waveforms fall on separate curves

all together…







Summary

- SNR at which signal becomes detectable (eff=50%) shows common behaviour for all three types of waveforms tested:
  - » simple ad-hoc and complicated ZM waveforms
  - » narrow- and broad-band
- Implies SNR could be good predictor of detectability of general waveforms.
- Needs more study!