Hough Statistics

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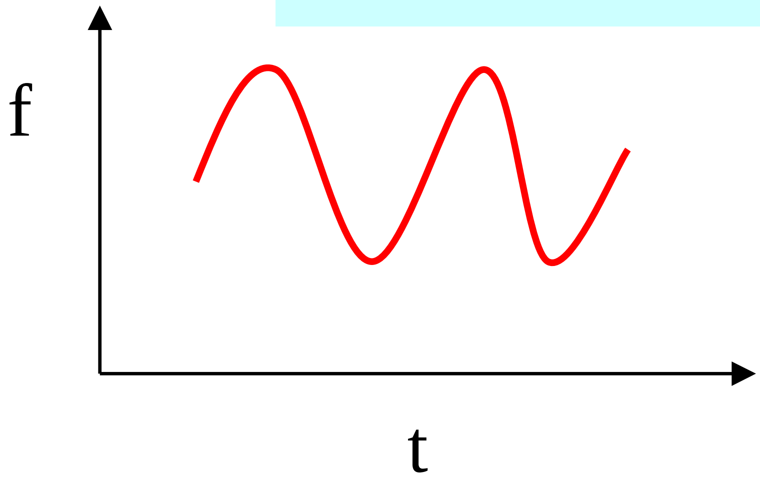
The Hough Transform

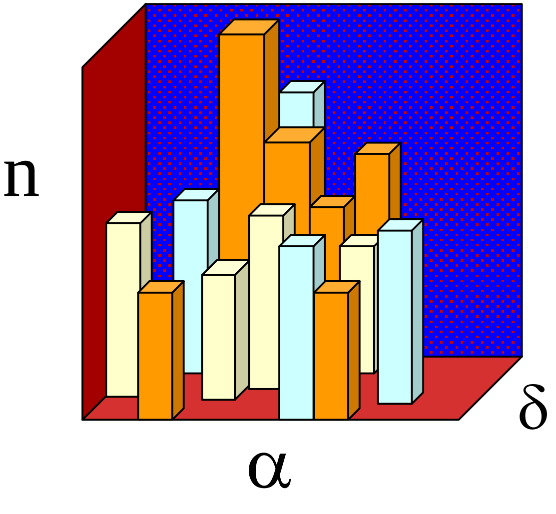
- Looks for patterns in frequency-time plane

  Expected pattern depends on $\{\alpha,\delta,f_0,f_i\}$

  Assume data is non-demodulated

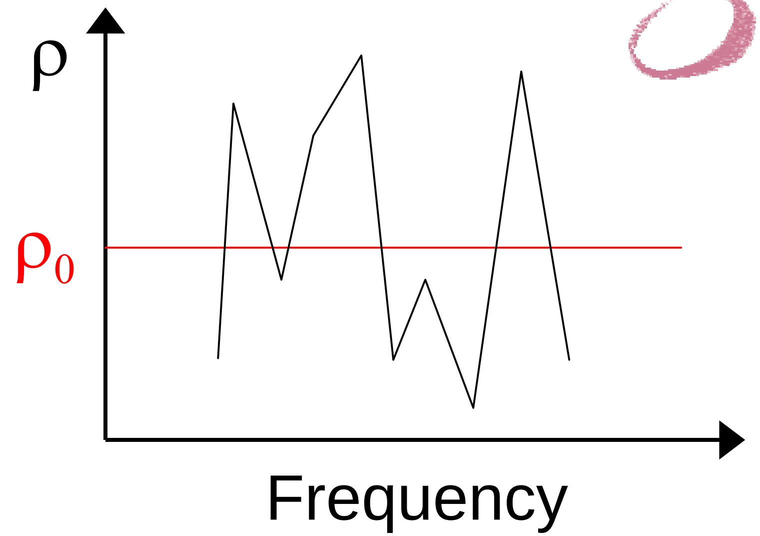
$$f(t) - f_0(t) = f_0(t)\frac{\vec{v}(t)}{c}\cdot\hat{n}$$



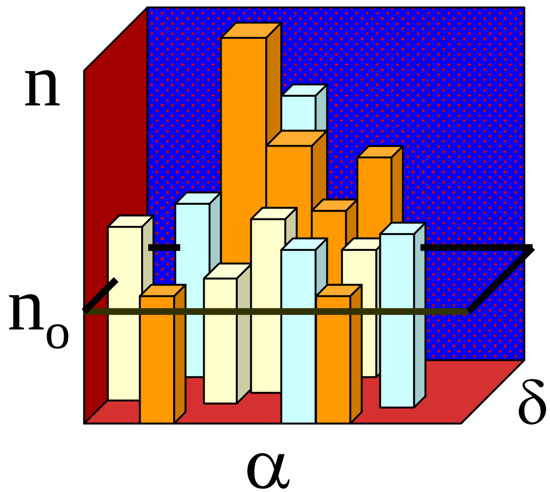


Basic pipeline for a single stage

Break up data into
N stacks and combine
Stacks incoherently



Need to optimize ρ_0 and n_0



$$n > n_0$$

Candidates
For next stage

Distribution of the power

- In absence of signal: χ^2 with 2-degrees of freedom

$$P(\rho) = \exp(-\rho)$$

$$\rho = \frac{|\tilde{x}|^2}{T_0 S_n}$$

T_0 : Time baseline of SFT

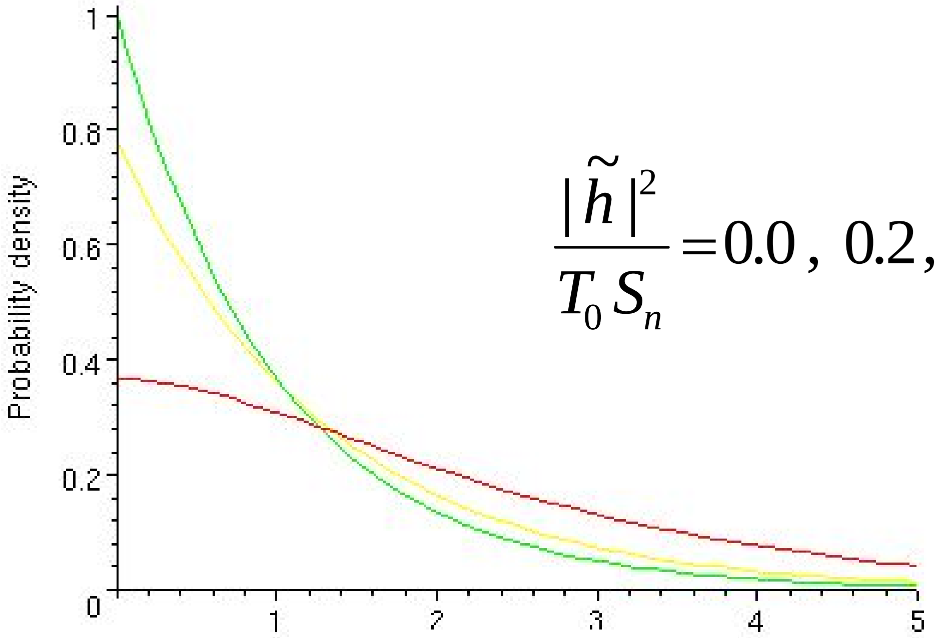
S_n: Power spectral density

$\tilde{x}$: Detector output Fourier transform

- In presence of signal : Has non-centrality parameter due to $|\tilde{h}(f)|^2$

$$P(\rho | h) = \exp\left(-\rho - \frac{|\tilde{h}|^2}{T_0 S_n}\right) I_0\left(\sqrt{\frac{4|\tilde{h}|^2 \rho}{T_0 S_n}}\right)$$

Distribution of the power + signal



ρ_0

$$\frac{|\tilde{h}|^2}{T_0 S_n} = 0.0,\ 0.2,\ 0.5$$

$$\langle \rho \rangle = 1 + \frac{|\tilde{h}|^2}{S_n T_0} \qquad \sigma_\rho{}^2 = 1 + \frac{2|\tilde{h}|^2}{S_n T_0}$$

Peak selection statistics

Basic definitions:

- False alarm probability :

$$\alpha(\rho_0) = \int_{\rho_0}^{\infty} P(\rho)\,d\rho = e^{-\rho_0}$$

- False dismissal probability :

$$\beta(\rho_0, h) = \int_{0}^{\rho_0} P(\rho | h)\,d\rho$$

- Detection probability :

$$\eta(\rho_0, h) = \int_{\rho_0}^{\infty} P(\rho | h)\,d\rho$$

Hough map statistics

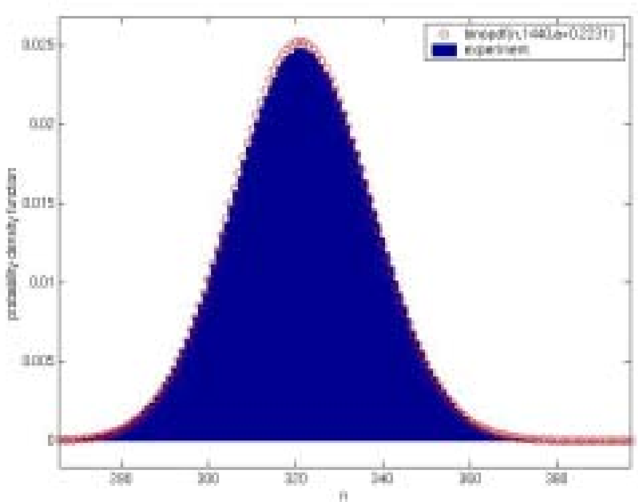
Distribution of the number count in each pixel:

$$P(n \mid p, h, N) = \binom{N}{n} p^{n} (1 - p)^{N-n}$$

$$\langle n \rangle = Np\,, \qquad \sigma_n^{\,2} = Np\,(1 - p)$$

$$p = \begin{cases} \alpha & \text{signal absent} \\ \eta & \text{signal present} \end{cases}$$

$$\eta \approx \alpha \left(1 + \frac{\rho_o\, |\tilde{h}|^2}{T_0\, S_n} \right)$$



$$\alpha_H(n_0, \rho_0, N) = \sum_{n=n_0}^{N} \binom{N}{n} \alpha^{n} (1-\alpha)^{N-n}$$

$$\beta_H(n_0, \rho_0, h, N) = \sum_{n=0}^{n_0-1} \binom{N}{n} \eta^{n} (1-\eta)^{N-n}$$

Setting upper limits

- For small signals: $$\eta(h) \approx \alpha\left(1+\frac{\rho_o\,|\tilde{h}|^2}{T_0\,S_n}\right)$$

- For a given loudest number count n_*, define the 95% upper limit h^{95} by $$0.95=\sum_{n=n_*}^{N} P\left(n \middle| \eta\left(h^{95}\right)\right)$$

- To calculate sensitivity: fix α_H and β_H, find corresponding $|h|^2$ and average over signal parameters to find h_0

Optimization strategy for setting thresholds

Choose thresholds which minimize false dismissal rate for a given false alarm

Choose α_H

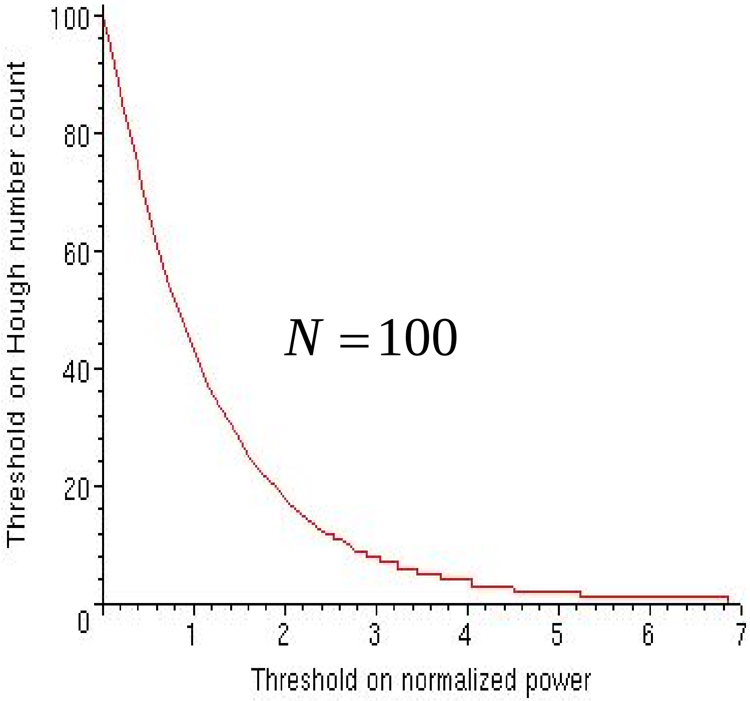
$\left(\alpha_H(n_0, \rho_0, N) = 0.01\right)$

$$n_0 = n_0(\rho_0, N)$$

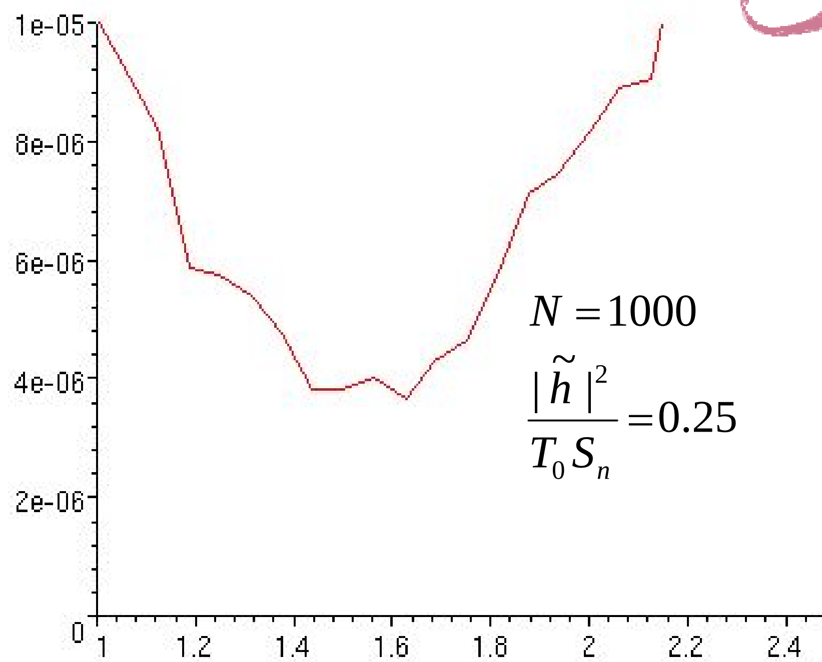
$$\beta_H = \beta_H\left(n_0(\rho_0), \rho_0, h, N\right)$$

Choose ρ_0 which minimizes β_H

Plot of n_0 vs. ρ_0 :



Plot of β_H vs. ρ_0 :



- Optimal value of $\rho_0 = 1.5 — 1.6$
- Corresponds to $\alpha \sim 22.3\,\% - 20.2\%$ and $n_0 \sim N\alpha$
- Independent of |h| for small signals

Conclusions and to do list

- Have optimized the code parameters using expected statistics
- In actual practice, will use Monte Carlo simulations to determine distributions and upper-limits
- Repeat with demodulated data: F statistic
- Make sure that parameter space grid is not too coarse