Resolving signals in the LISA data

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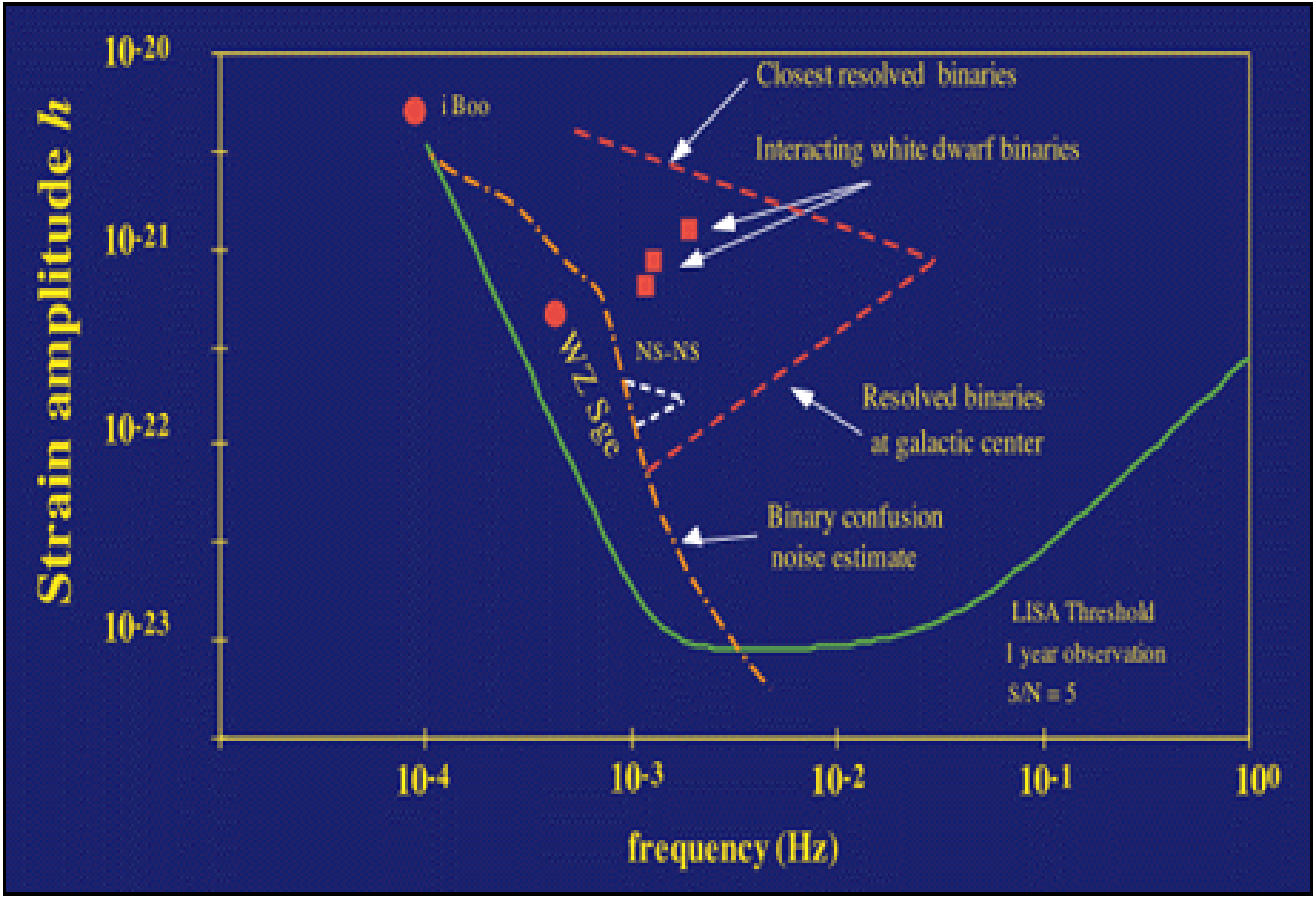
Gravitational waves from short-period binaries are signals guaranteed to be observed by LISA mission

- These sources will be dominated by detached white dwarf – white dwarf binaries (100 million of such binaries expected on our Galaxy).
- Population studies have shown that the number of such sources will be so large that they will constitute a stochastic background that will lie above LISA instrumental noises.
- It is important to calculate the background upper frequency cutoff above which one will be able to resolve individual signals.

We calculate the frequency assuming that as our data analysis method we use matched-filtering technique.



Strain amplitude h
10^{-20}
10^{-21}
10^{-22}
10^{-23}
i Boo
Closest resolved binaries
Interacting white dwarf binaries
WZ Sge
NS-NS
Resolved binaries at galactic center
Binary confusion noise estimate
LISA Threshold
1 year observation
S/N = 5
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
frequency (Hz)

The signal

$$h(t) = A_o \cos[\Phi_o + \omega t + \frac{\omega R}{c}\cos(\lambda)\cos(\Omega t - \beta - \varphi_o)]$$

A linear parameterization

$$h(t) = A_o \cos[\Phi_o + \omega t + A\cos(\Omega t) + B\sin(\Omega t)]$$

Phase is linear function of the parameters.

Frequency absorbed in parameters A and B

In the new parameterization the parameter space is a truncated cone whose base and top are discs of radii.

$$\frac{\omega_u R}{c} \quad \text{and} \quad \frac{\omega_l R}{c}$$

Cross-sections of constant frequency are discs of radii: $\dfrac{\omega R}{c}$

Each point of the disc corresponds to the two positions in the sky which differ by the sign in the ecliptic latitude angle λ.

Normalized reduced correlation function

$$C(\Delta w, \Delta A, \Delta B) =$$

$$\left(\int_0^1 \cos[\Delta wx + \Delta A\cos(2\pi nx) + \Delta B\sin(2\pi nx)]dx\right)^2 +$$

$$\left(\int_0^1 \sin[\Delta wx + \Delta A\cos(2\pi nx) + \Delta B\sin(2\pi nx)]dx\right)^2$$

Correlation function depends only on the difference between the signal parameters

We assume that the two signals are resolvable if their normalized correlation is less than 1/2

No. of resolvable signals per frequency bin:

$$N_r = \frac{\text{Area of constant frequency crossection of the parameter space}}{\text{Area of the correlation ellipse in the A - B plane}} = N_o \left(\frac{f}{f_o} \right)^2$$

Expected no. of detached wd-wd binaries in one frequency bin obtained from population studies:

$$N_s = \frac{1}{n} \left(\frac{f}{f_o} \right)^{-11/3}, \qquad \text{where} \qquad f_o = 10^{-2.8}\ \text{Hz}$$

Upper cutoff frequency of the background:

$$f_r = \frac{f_o}{(N_o n)^{3/17}}$$

$$f_r = 10^{-3.0}\ \text{Hz} \quad \text{for 1 yr of observation time}$$

There are around 68 thousand of wd-wd above frequency of $10^{-3.0}$ Hz.

Only those that exceed a certain detection threshold can be pulled out of the noise. There are two ways to calculate the threshold.

1. Assuming that we search the whole parameters space for unknown signals.
2. Assuming we search for a known signal in a resolution cell.

Formula relating threshold and false alarm probability:

$$p_F = 1 - [1 - \exp(-F)]^{N_c}$$

$T_1 = 7.6$ $N_c = 125$ billion $N_{dr} = 3.4 \times 10^3$

$T_2 = 2.7$ $N_c = 1$ $N_{dr} = 4.7 \times 10^4$

CLEANING LISA DATA

We tested our approach by adding two signals in the same frequency bin to LISA noise. We have performed all-sky search to see whether by matched filtering will be able to detect and resolve the two signals as predicted by our calculations.

Different approach to identifying and removing signals from LISA data base and CLEAN algorithm used in astronomy was recently presented by *N. Cornish and S. Larson*; astro-ph/0301548

The optimum statistics to do all-sky search with matched filter

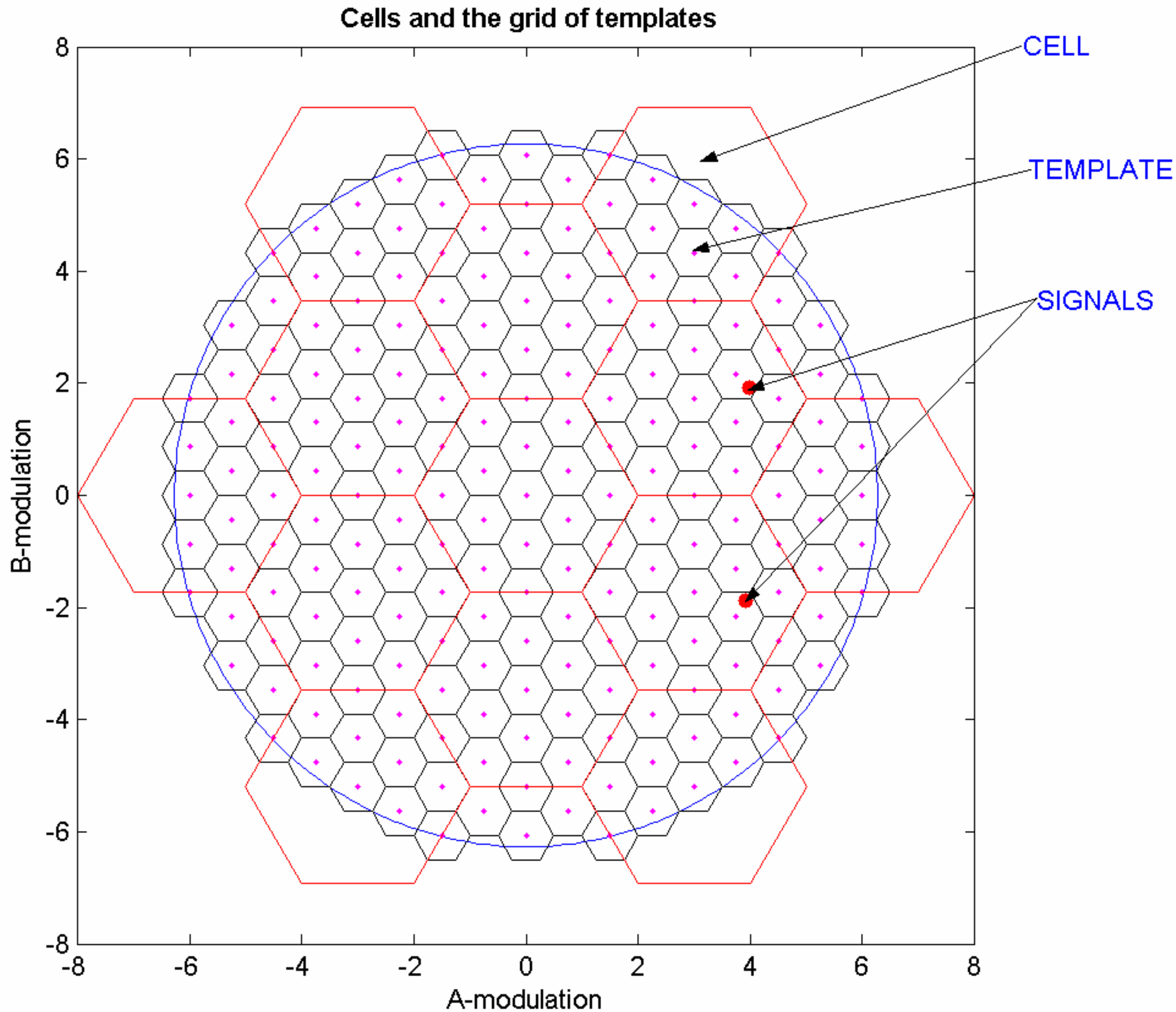
$$F = \frac{1}{T_o S_c} \left| \int_0^{T_o} x(t) \exp\{-i[A\cos(\Omega t) + B\sin(\Omega t)] - i\omega t\} \right|^2$$

Demodulation of data on a grid of templates in A-B plane followed by FFT

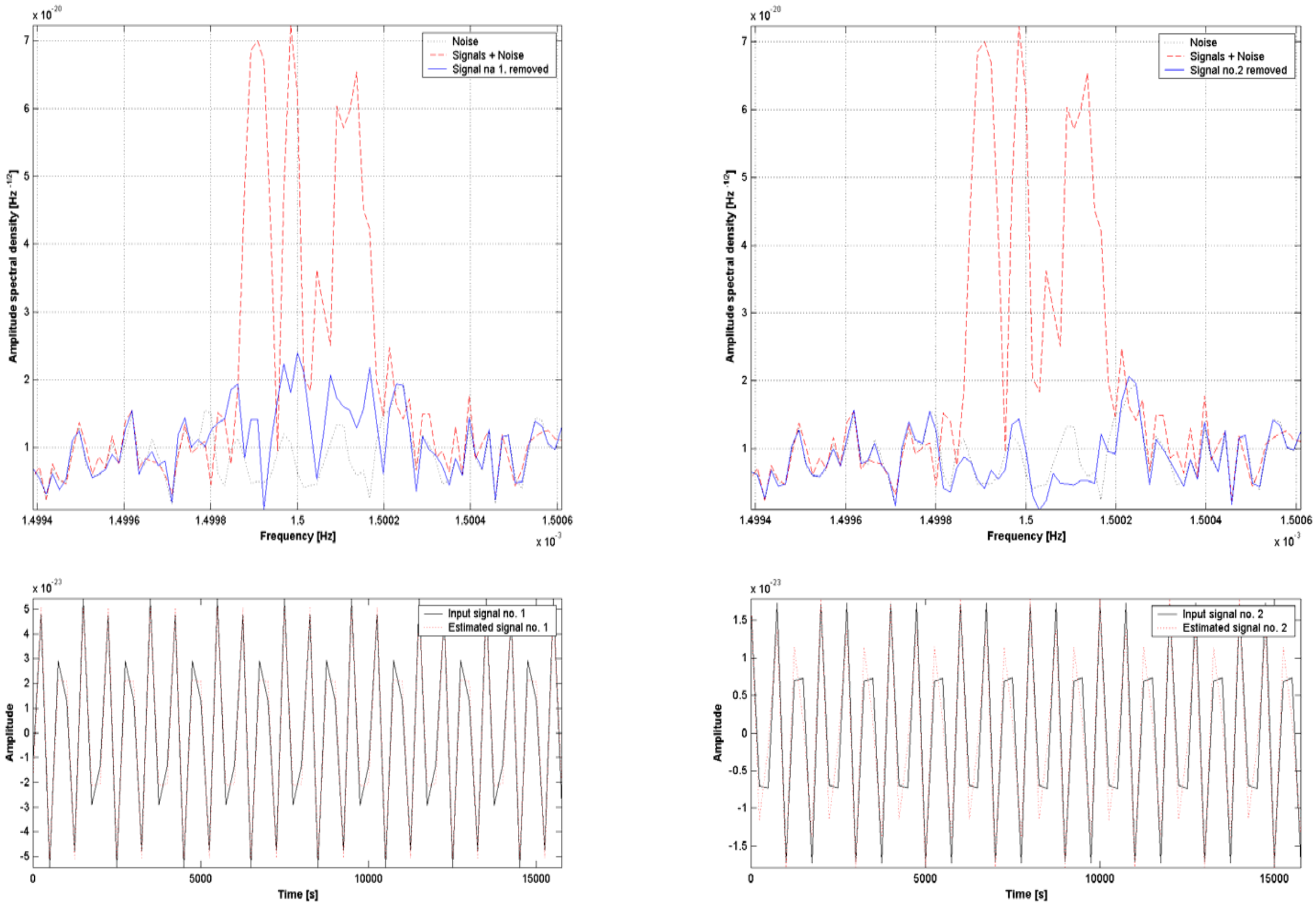
Two step procedure to find the maximum of F:

1. Coarse search: find maximum of F on a grid in A-B plane
2. Fine search: find accurate position of the maximum by Nelder-Mead optimization procedure.

ALL-SKY SEARCH OF LISA DATA



CLEANING LISA DATA



CLEANING LISA DATA