# Reducing the Newtonian Noise above and below the ground

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# Introduction

In future advanced interferometers Newtonian (gravity gradient) noise will be one of the fundamental limitations for the sensitivity in the low frequency region.

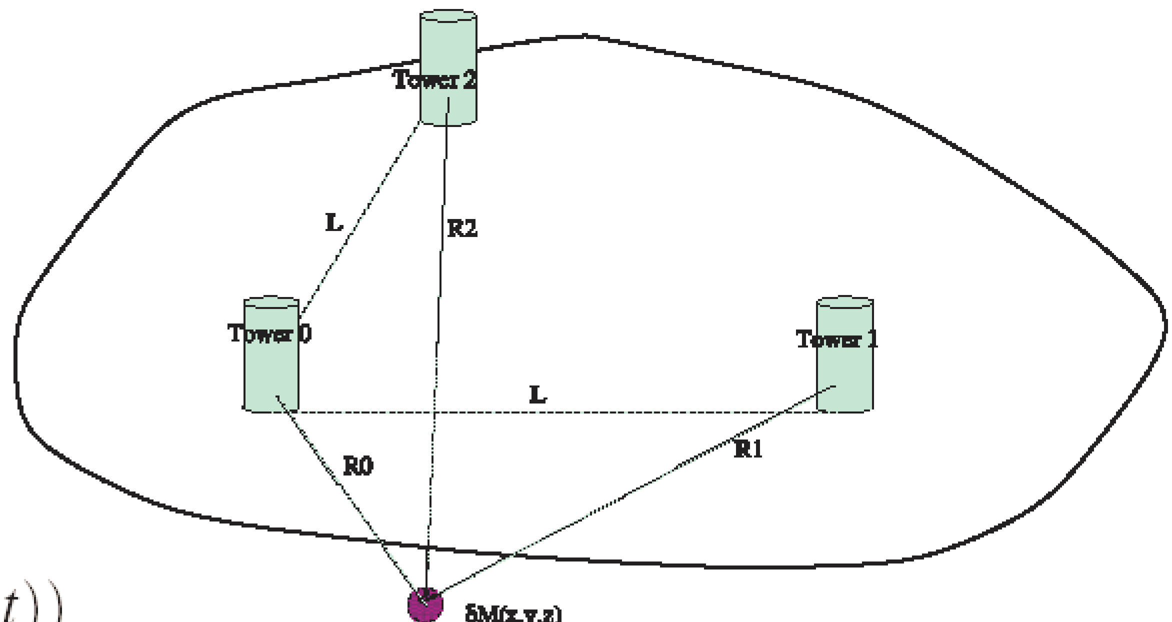
- Can it be estimated?
- What are the most important sources?
- Can it be reduced?

# Plan of the talk

- Generalities: estimation of NN
- Models for seismic NN
- Atmospheric NN
- "Software" reduction of NN: subtraction
- "Hardware" reduction of NN: going underground

# What is Newtonian Noise

Mass density fluctuations couple directly to the test masses:



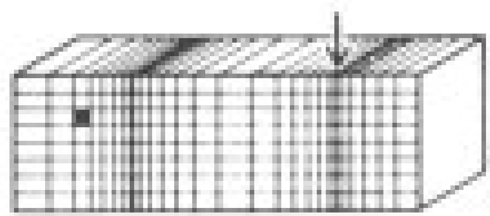

Example:

elastic

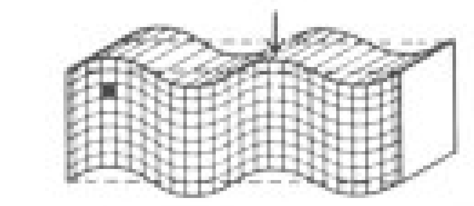
material:

$$\delta\rho(x,t) = -\vec{\nabla}(\rho(x,t)\,\vec{u}(x,t))$$

$$= -\rho\vec{\nabla}\cdot\vec{u}(x,t)$$

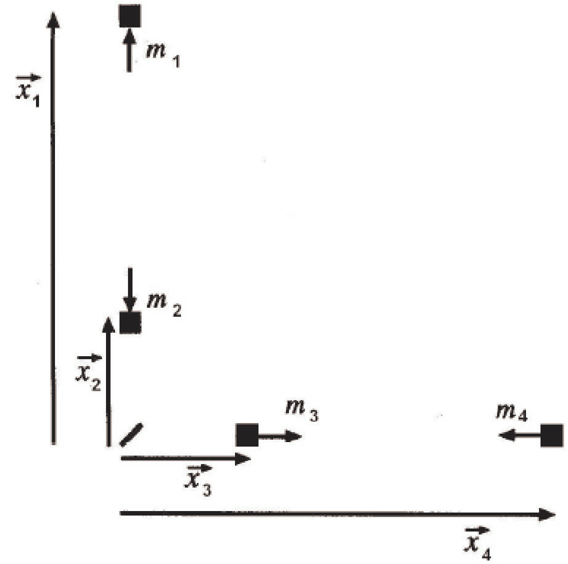


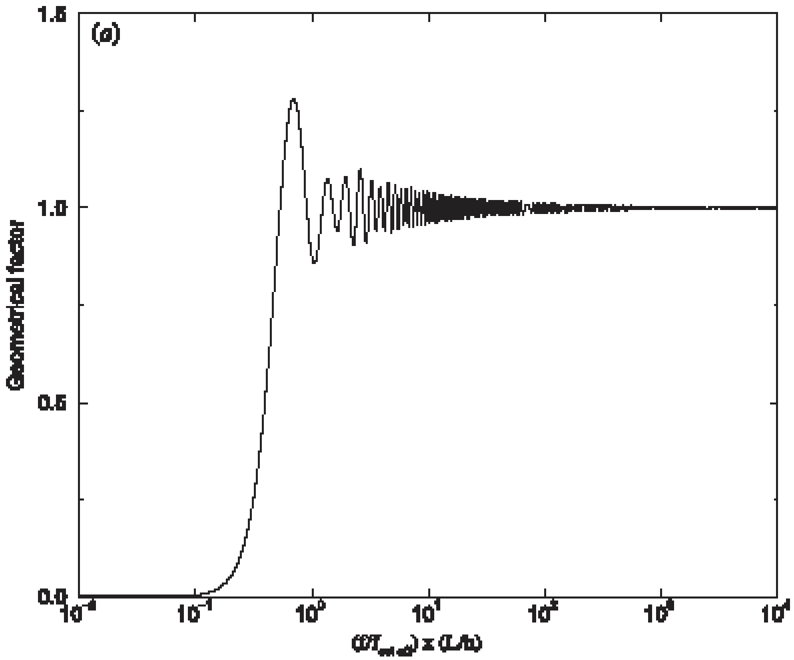
$$-\ \vec{u}(x,t)\cdot\vec{\nabla}\rho$$



# Density fluctuations

Effect depends on geometry.



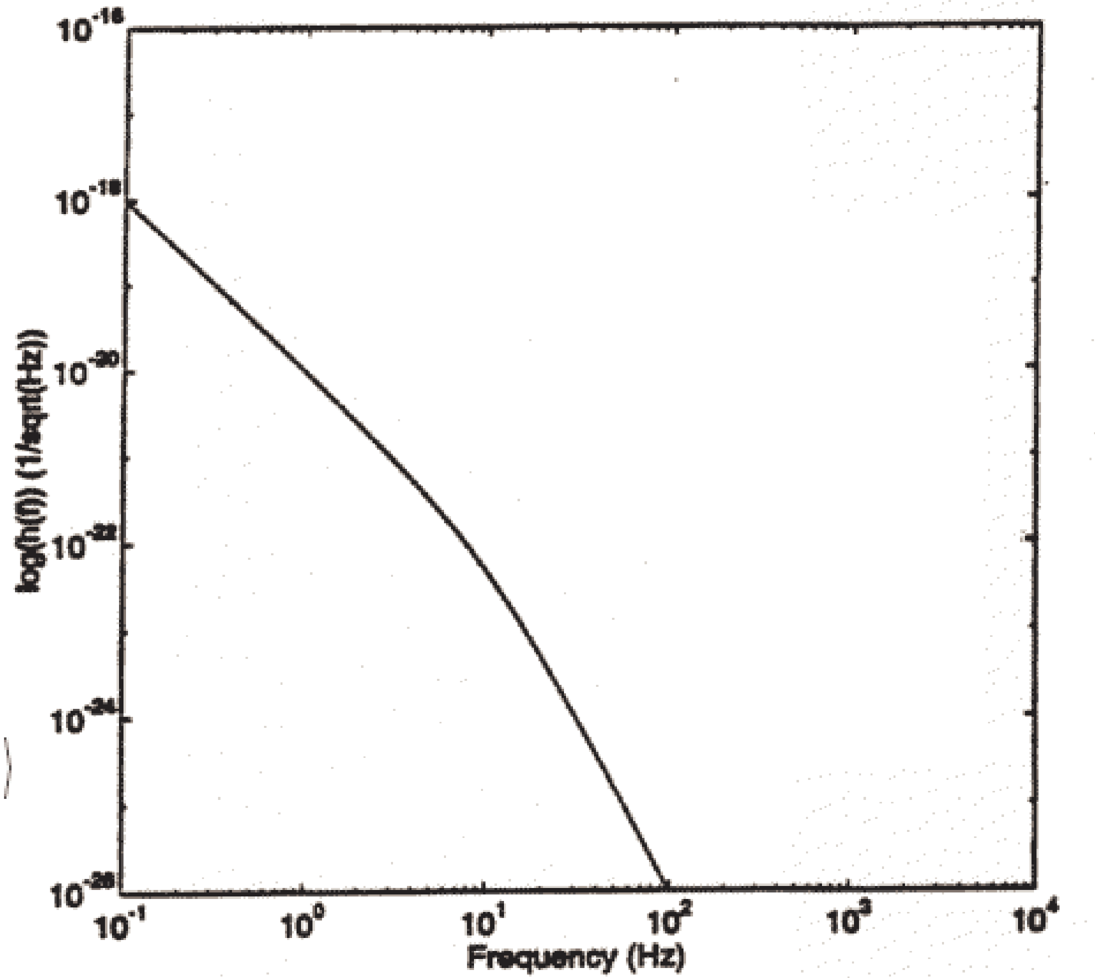

Coherently.

# Seismic NN: the Saulson's model

A simple model:

- ground partitioned in square blocks (size λ/2)
- mass fluctuation uncorrelated between different blocks

$$\left\langle |\delta h(\omega)|^2 \right\rangle = \frac{16}{3}\frac{\pi^3}{L^2}\frac{G^2\rho_0^2}{|H(\omega)|^2}\left\langle |\delta x(\omega)|^2 \right\rangle$$



# Seismic NN: elastic models

Elastic wave equation:

$$\partial_t^2 u_i(x,t) = c_T^2 \partial_k \partial_k u_i(x,t) + \left(c_L^2 - c_T^2\right) \partial_i \partial_k u_k(x,t)$$

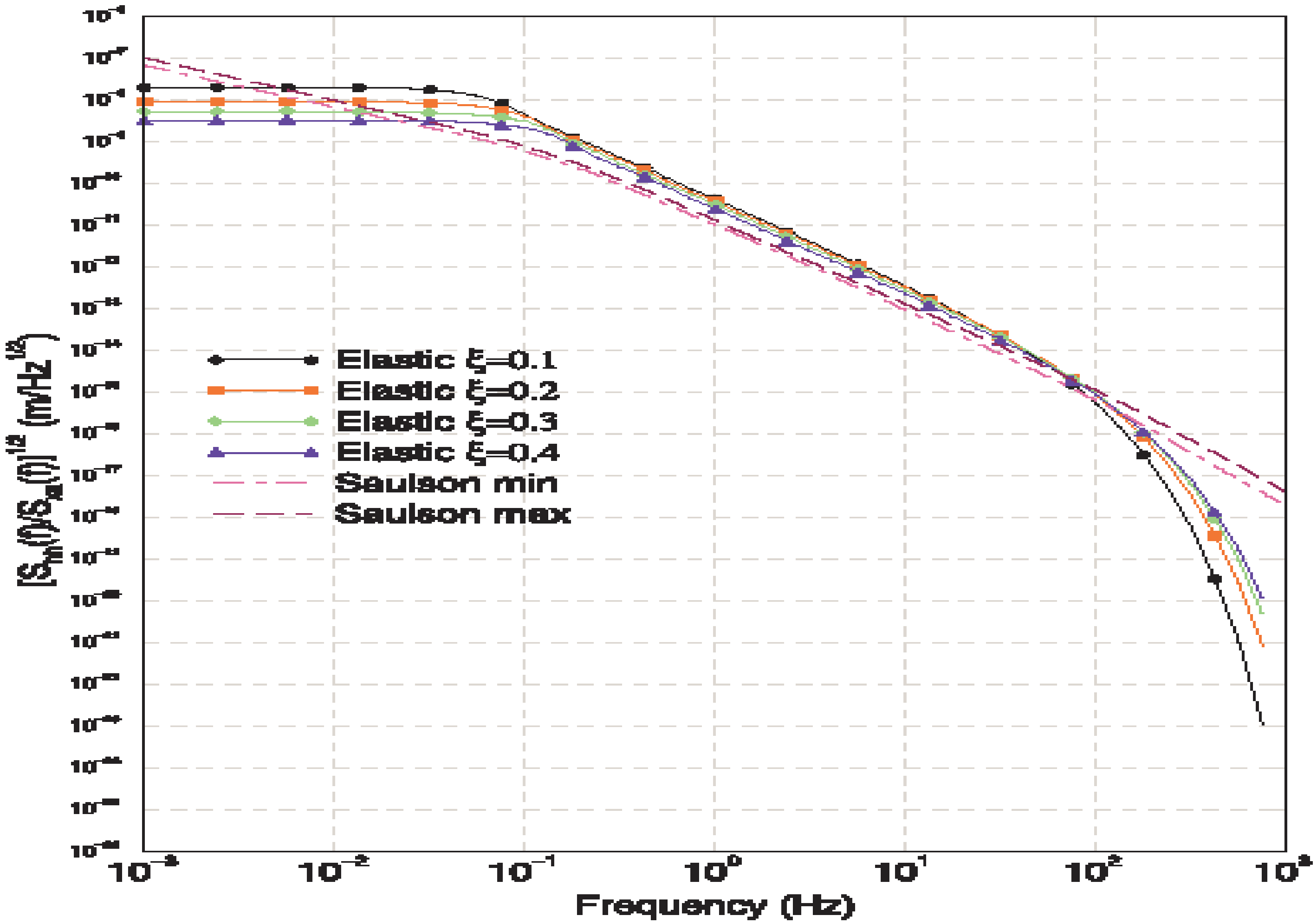
- Mode classification
- Evaluation of the contribution of each mode to NN

Transfer function between NN & seismic motion

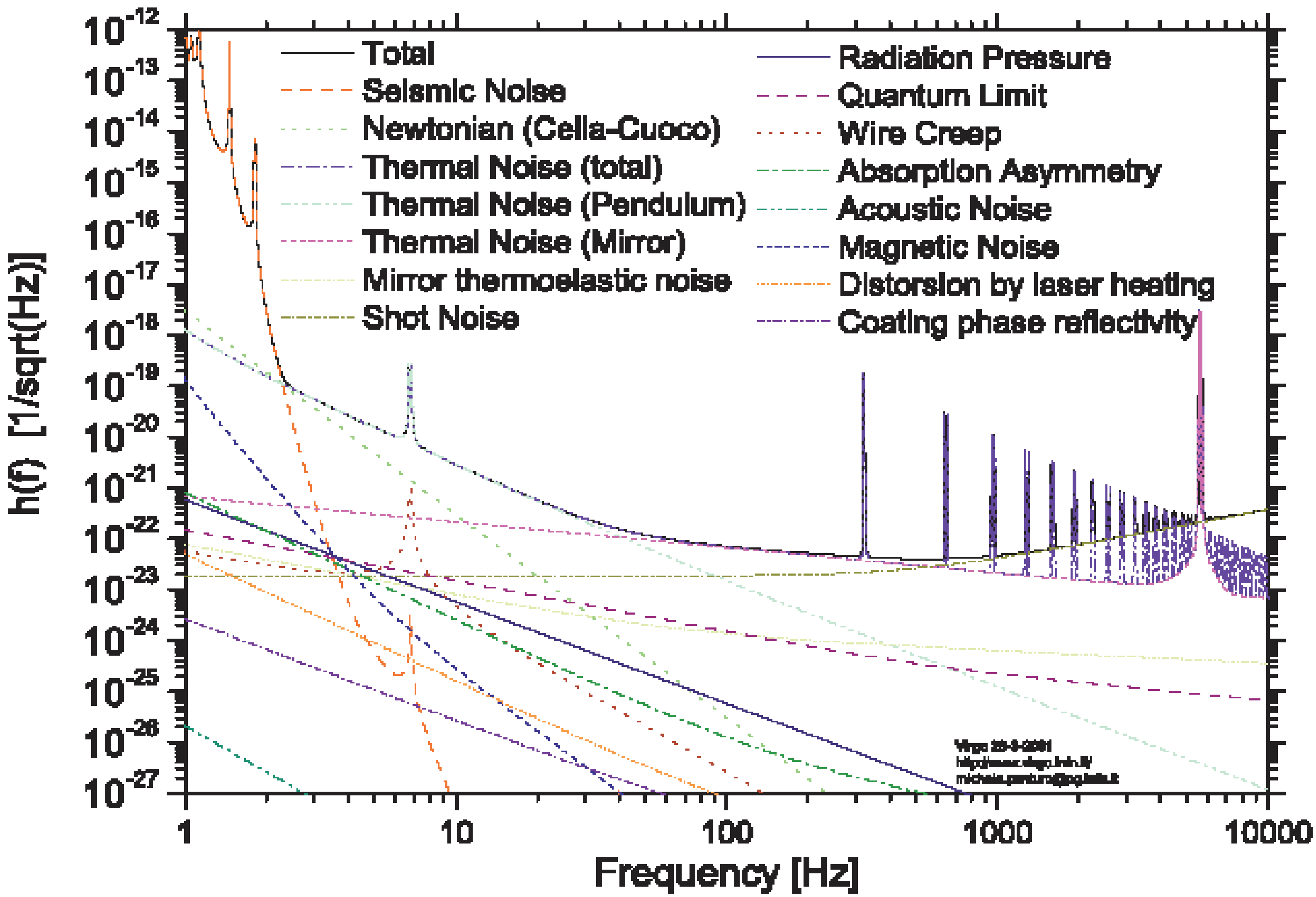
$$\begin{pmatrix} \delta\vec{a} \\ \delta a_z \end{pmatrix} (\vec{k}, z = h, \omega) = 4\pi G \rho_0 \frac{c_T^2}{\omega^2} \begin{pmatrix} \hat{k} \\ \mathrm{i} \end{pmatrix} \{k\vec{k} \cdot \delta\vec{x} + \mathrm{i}k^2 \delta z\} \mathrm{e}^{-hk}$$

Using some other assumptions…..

# Seismic NN: transfer function



# Seismic NN: noise estimate



# Atmospheric NN

- Saulson: effect of acoustic waves (negligible)
- Creighton: airborne objects, sonic booms,advection,…

Rayleigh Bernard scenarios (G.C., E. Cuoco, P. Tomassini)

$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\rho^{-1} \vec{\nabla} p + \nu \nabla^2 \vec{u} + \alpha \vec{g} \theta$$

$$\partial_t \theta + (\vec{u} \cdot \vec{\nabla}) \theta = \chi \nabla^2 \theta$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

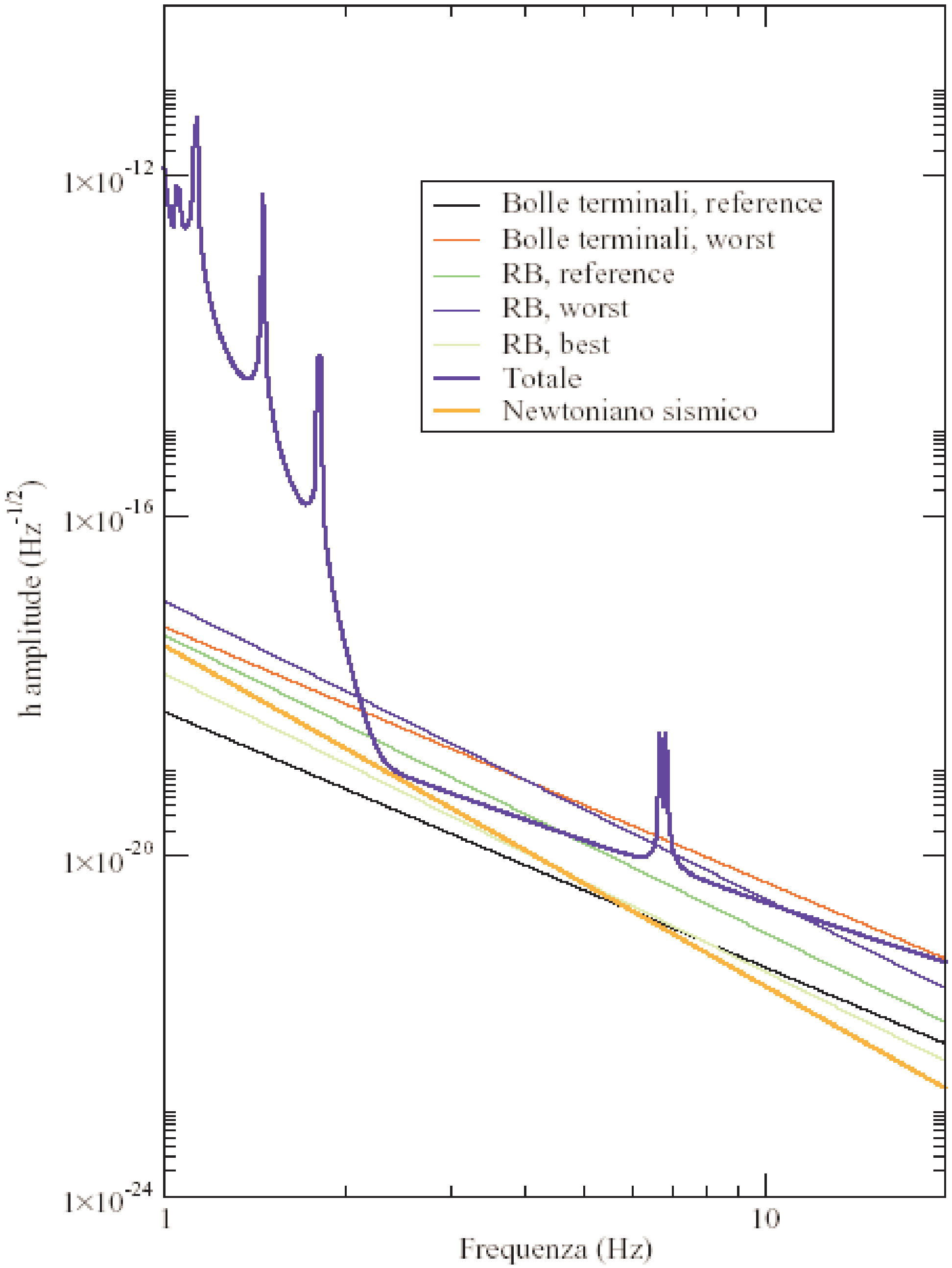
Different possibilities accordingly with the intensity of thermal gradient.

# Thermal bu

- Nucleation phase (slow)
- Ascension phase

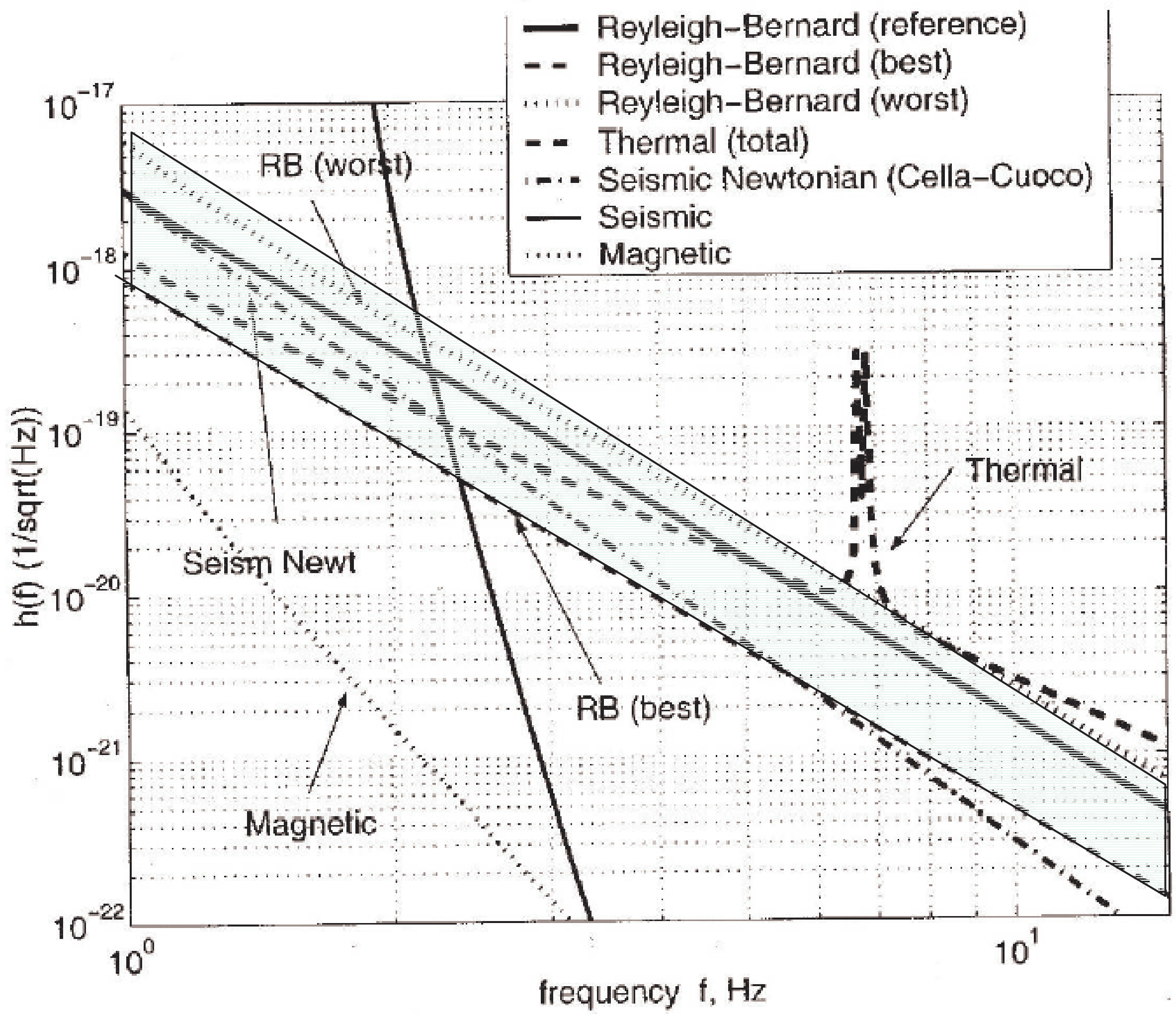
$$\partial_{tt} z = g\left(\frac{T_b(z) - T(z)}{T(z)}\right) - 6\pi v r z$$

$$T_b(z) = T_b(0) + \gamma_{ad} z$$



# Well developed turbulence

Method: extimation of the structure functions using simple scaling relations.



# Other sources

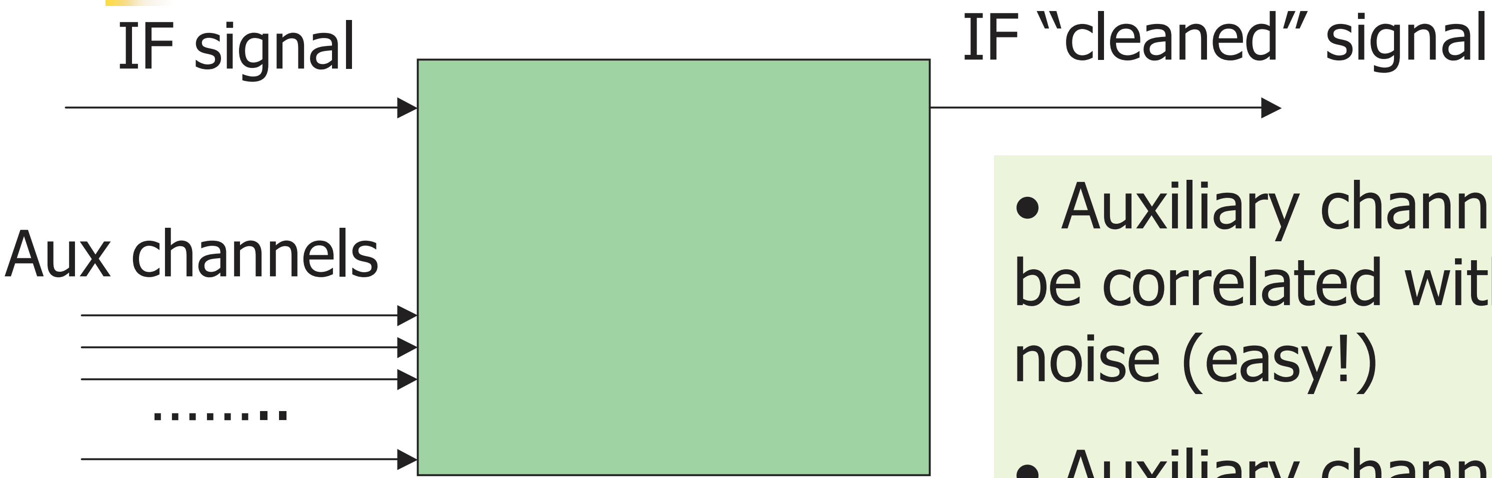
- Turbolent generation of acoustic waves (Lighthill process): **Negligible** (C. Cafaro, G. C.)

It seems that RB phenomenology is at least competitive with seismic NN

Models need validation:

- Atmospheric fluctuations should be well studied by astronomers (work in progress: reducing our ignorance in this field)
- Some data are available (balloons, airplanes)

# Subtraction of NN: strategy



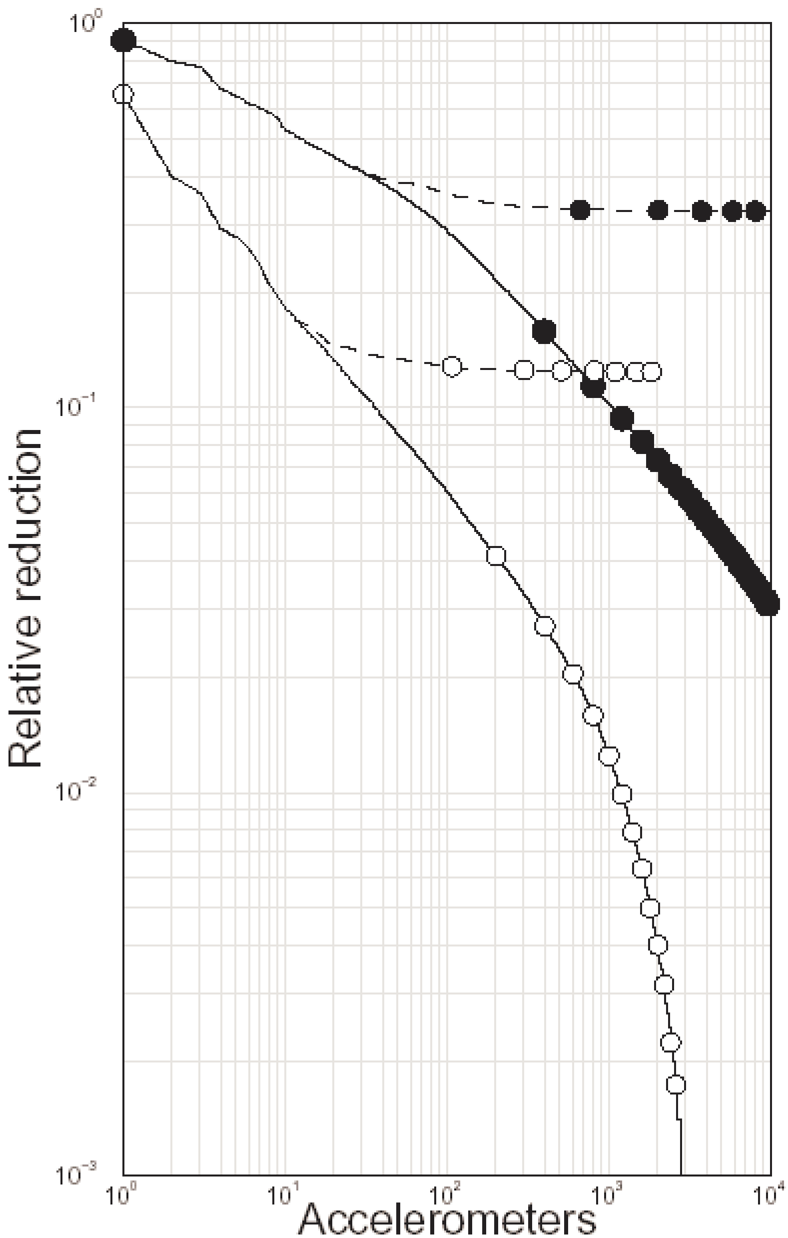

- Auxiliary channels should be correlated with the noise (easy!)
- Auxiliary channels should not be correlated with the useful signal (very easy!)

What we must put inside the box?

1. model dependent strategy → Physical modelization
2. Model independent strategy → Adaptive strategy

# Qualitative discussion

- A seismic sensor give us complete information on a block
- Complete ignorance on blocks without sensors
- Blocks nearest to the test masses should be controlled first (for a given sensor budget)
- If the sensors are noisy, we can put two of them on the same block



# The linear case

- In the case of addictive, gaussian noise (on all the channels) we can find an explicit solution:

Subtracted signal:

$$X_{sub}(\omega) = X(\omega) - \sum_{i,j} < X(\omega)\, \hat{n}_i \cdot \vec{u}(x_i,\omega)^* > [C^{-1}(\omega)]_{ij}\, \hat{n}_j \cdot \vec{u}(x_j,\omega)$$

Relative noise power spectrum reduction:

$$1 - \sum_{i,j} \frac{< X(\omega)^*\, \hat{n}_i \cdot \vec{u}(x_i,\omega) > [C^{-1}(\omega)]_{ij} < \hat{n}_j \cdot \vec{u}(x_j,\omega)^*\, X(\omega) >}{< X(\omega)^* X(\omega) >}$$

Projector: noise can't increase

(if we use the correct C…)

# Optimization of sensor placement

•**If we have a physical model**, we can calculate all:

**Seismic correlations**

$$\langle \delta x_i(x_1)\, \delta x_j(x_2)\rangle = \pi \left(\frac{\omega}{c_T}\right)^2 |u_X|^2 \left\langle |A_4(\omega)|^2 \right\rangle G^T_{ij}(\vec{x}_1, \vec{x}_2) \qquad \langle \delta x_i(x_1)\, \delta z(x_2)\rangle = \pi \left(\frac{\omega}{c_T}\right)^2 \mathrm{Re}\,(u_X^\star u_Z) \left\langle |A_4(\omega)|^2 \right\rangle G^V_i(\vec{x}_1, \vec{x}_2)$$

$$\langle \delta z(x_1)\, \delta z(x_2)\rangle = \pi \left(\frac{\omega}{c_T}\right)^2 |u_Z|^2 \left\langle |A_4(\omega)|^2 \right\rangle G^S(\vec{x}_1, \vec{x}_2) \qquad \langle \delta x_i(x_1)\, \delta \theta_j(x_2)\rangle = \pi \left(\frac{\omega}{c_T}\right)^3 \mathrm{Re}\,(u_X^\star u_\theta) \left\langle |A_4(\omega)|^2 \right\rangle G^T_{ij}(\vec{x}_1, \vec{x}_2)$$

$$\langle \delta \theta_i(x_1)\, \delta \theta_j(x_2)\rangle = \pi \left(\frac{\omega}{c_T}\right)^4 |u_\theta|^2 \left\langle |A_4(\omega)|^2 \right\rangle G^T_{ij}(\vec{x}_1, \vec{x}_2) \qquad \langle \delta \theta_i(x_1)\, \delta z(x_2)\rangle = \pi \left(\frac{\omega}{c_T}\right)^3 \mathrm{Re}\,(u_Z^\star u_\theta) \left\langle |A_4(\omega)|^2 \right\rangle G^V_i(\vec{x}_1, \vec{x}_2).$$

**Newtonian correlations** **…mixed ones…**

$$\langle \delta a_i(x_1)\, \delta a_j(x_2)\rangle = \frac{1}{2} G^2 \rho_0^2 \left(\frac{\omega}{c_T}\right)^2 W^2 \left\langle |A_4(\omega)|^2 \right\rangle G^T_{ij}(\vec{x}_1, \vec{x}_2) \exp\left(-2h \frac{\omega}{c_T \sqrt{x}}\right)$$

$$\langle \delta a_i(x_1)\, \delta a_z(x_2)\rangle = \frac{1}{2} G^2 \rho_0^2 \left(\frac{\omega}{c_T}\right)^2 W^2 \left\langle |A_4(\omega)|^2 \right\rangle G^V_i(\vec{x}_1, \vec{x}_2) \exp\left(-2h \frac{\omega}{c_T \sqrt{x}}\right)$$

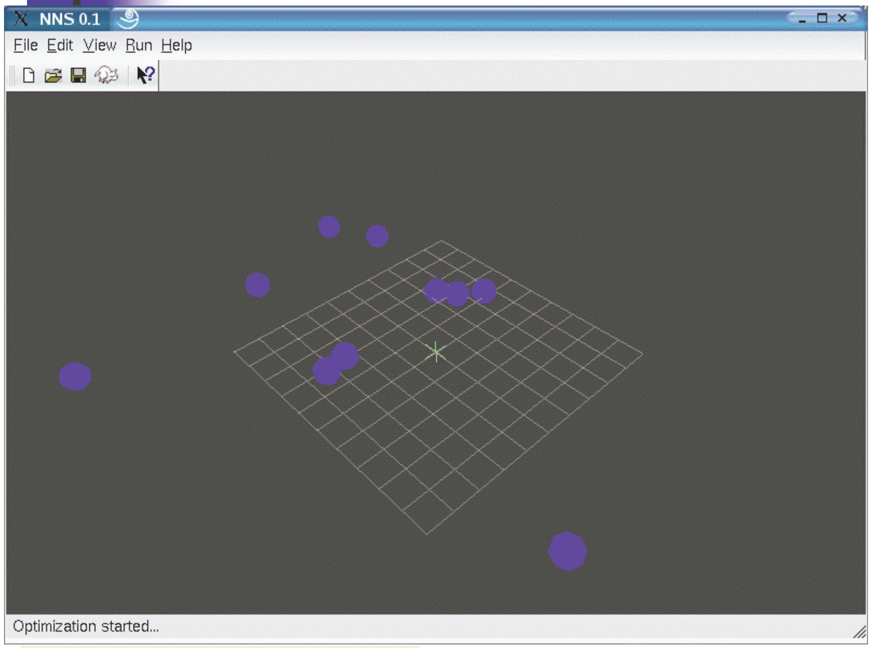
$$\langle \delta a_z(x_1)\, \delta a_z(x_2)\rangle = \frac{1}{2} G^2 \rho_0^2 \left(\frac{\omega}{c_T}\right)^2 W^2 \left\langle |A_4(\omega)|^2 \right\rangle G^S(\vec{x}_1, \vec{x}_2) \exp\left(-2h \frac{\omega}{c_T \sqrt{x}}\right)$$

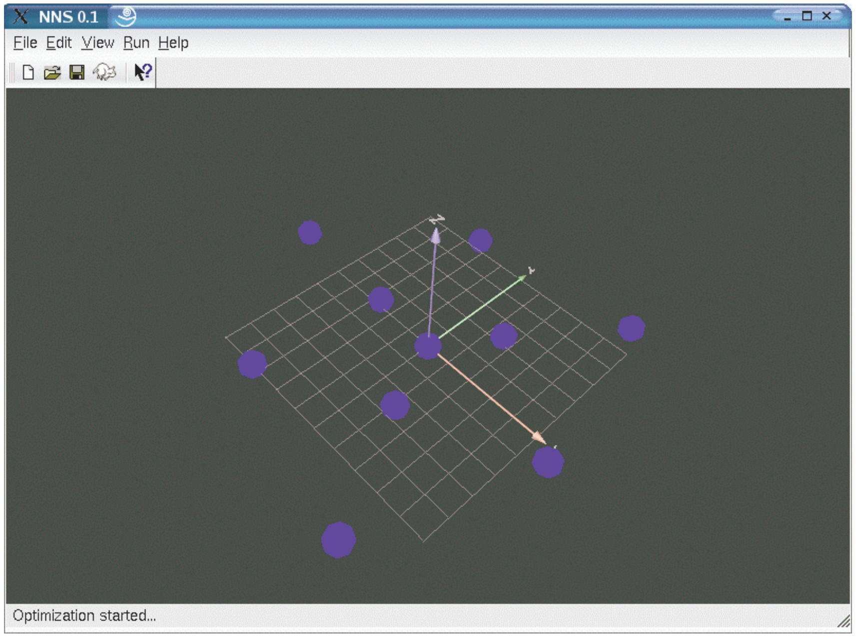
Numerically this is quite problematic…

Then we can find:

- the optimal set of sensors
- their optimal configuration

# Example of optimization




## Features:

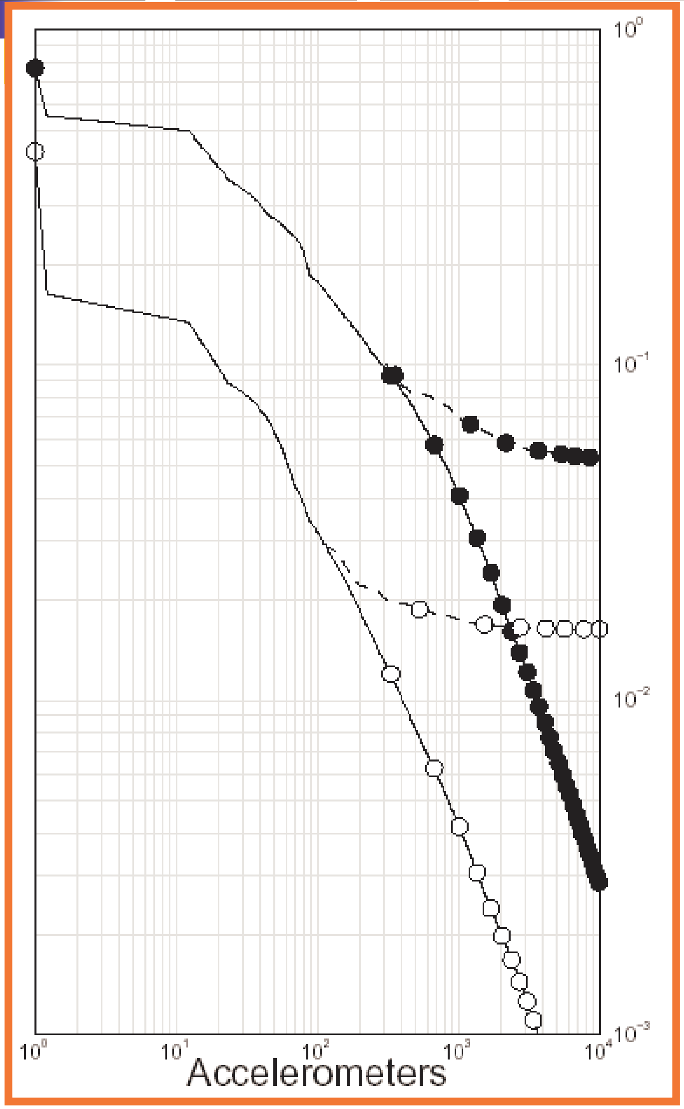
- In the frequency range of interest each test mass can be studied separately
- Optimal solution is quite robust (good to know, if we ignore the physical model)
- Sensors "cristallize" in quite regular configurations

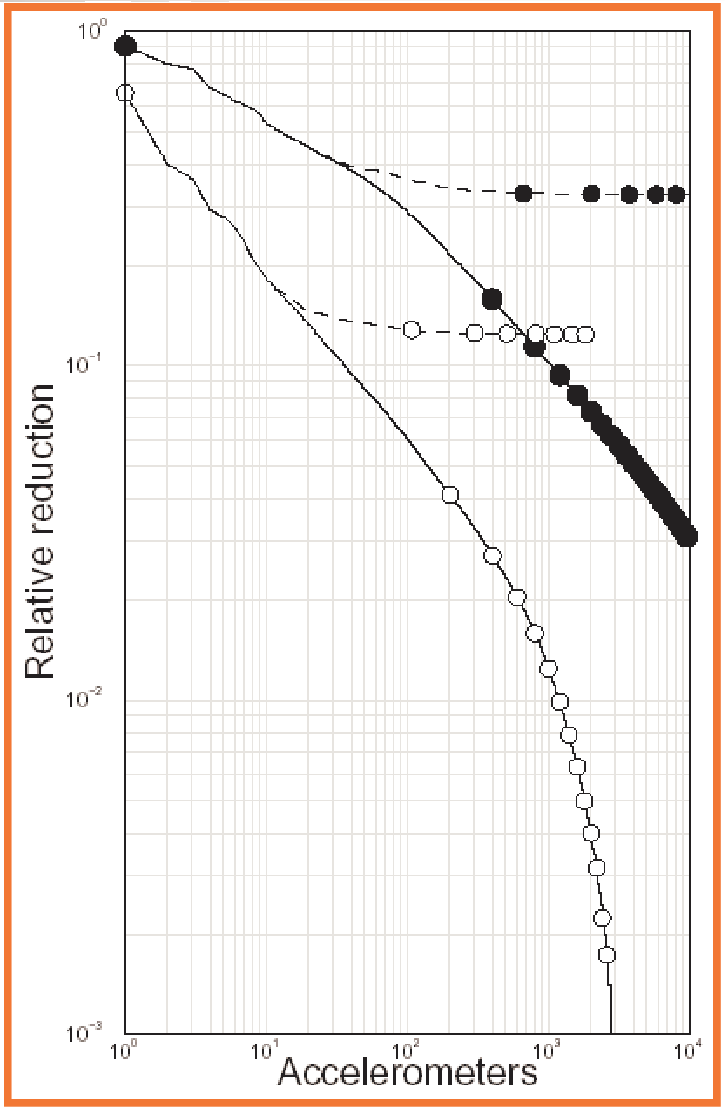
# Efficiency results

- Seismic correlations act as a repulsive potential (statistical independence)
- Mixed correlations "attract" the sensors near the test masses (maximize IF-aux correlations)
- Instrumental noise shift the eigenvalues of C, reducing the "repulsive potential"

Results: correlations are important

# Efficiency results

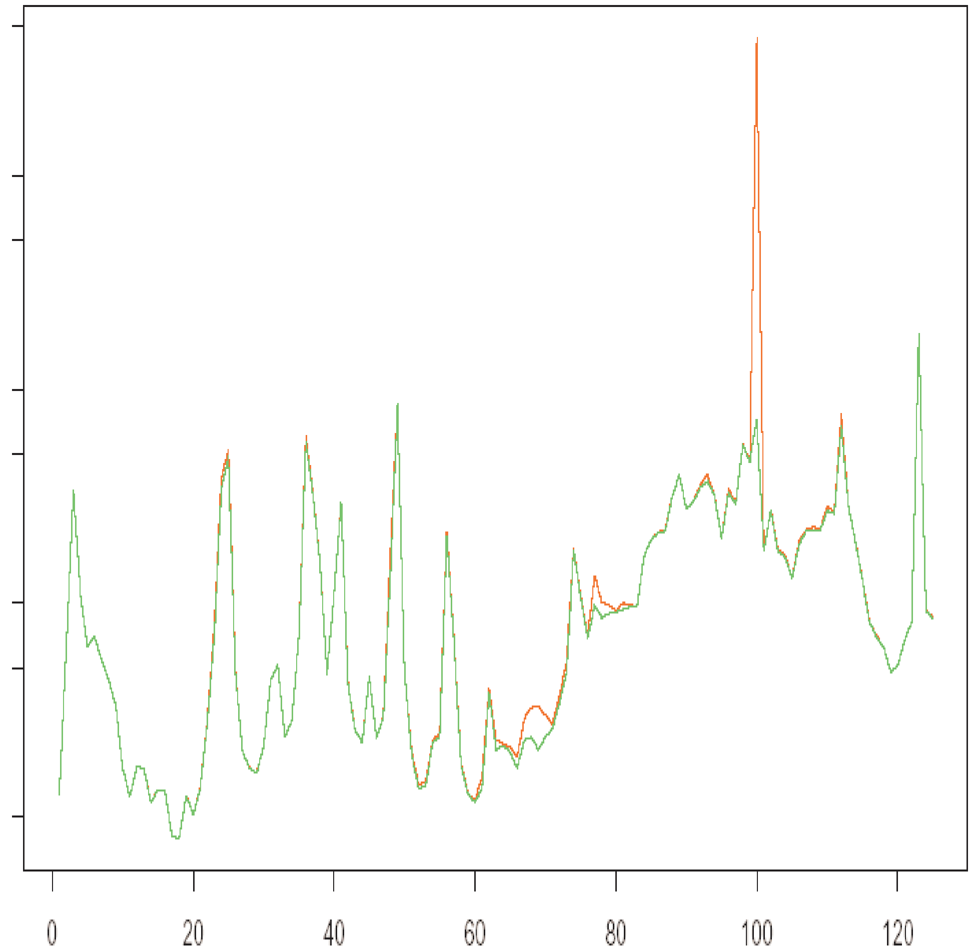


# Model independent subtraction

Can be seen as a "generalized" whitening:

- can be implemented "on line"
- can be made adaptive




Example: subtraction of acoustic channels from IF channel (VIRGO E2 data)

# Non linear case

What if the auxiliary channels contain non linear correlations with the IF channel?

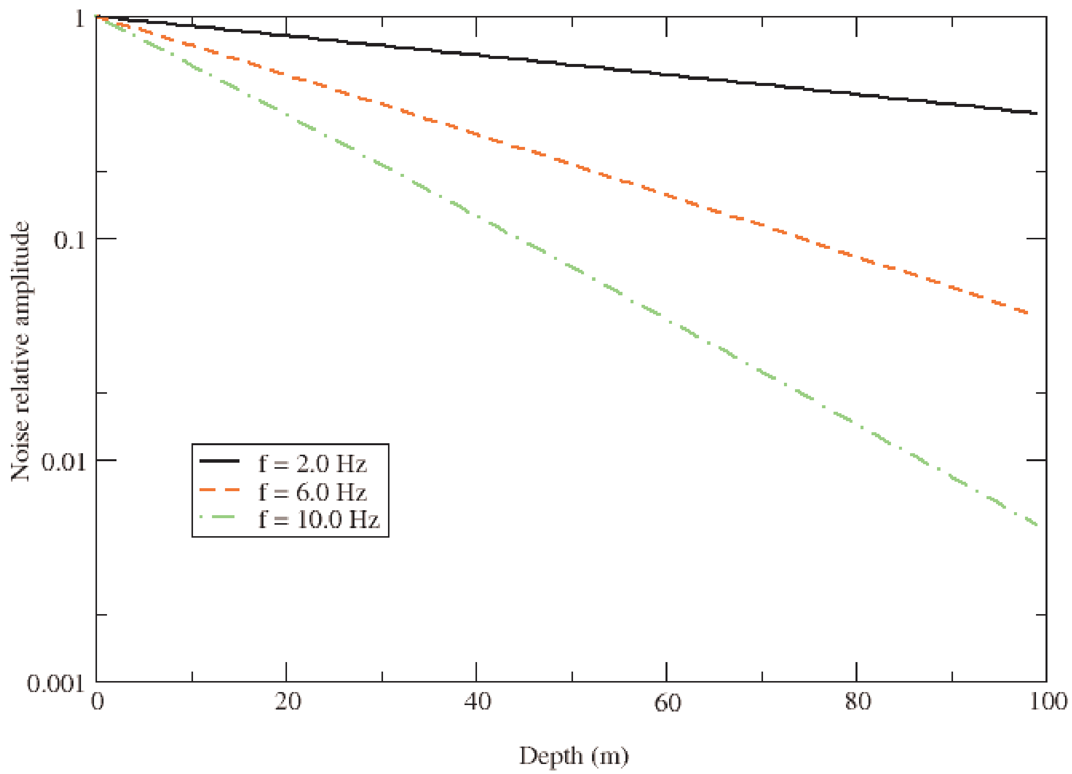
- not completely academic: a sensor for the modulus of the seismic motion

The problem can be formulated (and in some cases solved) as the design of the optimal Neyman-Pearson in presence of auxiliary channels

- can a nonlinear sensor do better?
- is the "subtraction" procedure independent by the detection problem? (There could be a well known answer (entropy?), I do not know).

# Going underground: seismic NN reduction

A simple fact: surface waves die exponentially with the depth




- Surface waves are probably the most important excitations for NN
- Surface movement dominate the bulk compression effect

# Other advantages:

- Horizontal seismic NN is self-regularized: weak dependence on empty space dimensions
- Atmospheric NN reduction: obvious
- Collective atmospheric effects should be damped exponentially
- Subtraction procedure should work well (but must be studied in detail)

# Going underground: some possible problems

- Localized seismic waves on the gallery
  - Small masses involved
  - Easily (???) monitorable
- Acoustic (pressure waves) resonances
  - Could be avoided (close the doors!)
  - Easily monitorable too

# Other hardware options

- Mechanical screen (es. cilindrical screen)
  - Delicate
  - Require hardware modifications
  - Not easily tunable
- Barriers for seismic noise
  - Should be carefully designed

# Conclusions

- NNS is promising: somebody tried it yet….
- NN will be relevant for advanced IF
- Going underground is a promising option