Thermoelastic dissipation in inhomogeneous media: loss measurements and thermal noise in coated test masses

Sheila Rowan, **Marty Fejer**
and
LSC Coating collaboration

Suspensions/Core Optics Session
LSC meeting
Livingston, LA
17th - 20th March 2003

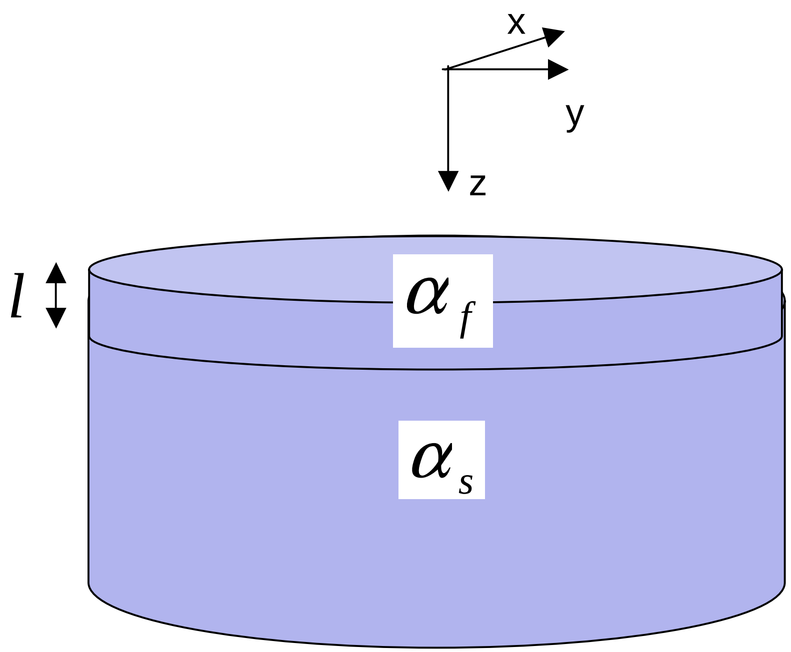
Introduction

- Sapphire or fused silica are the choices for Adv. LIGO test mass substrates
- Adding dielectric mirror coatings to the test masses will increase the thermal displacement noise sensed by the interferometer
- Recent work has studied the level of mechanical loss from ion-beam-sputtered coatings formed from layers of different materials
- Losses seen are large enough to be of relevance to Adv. LIGO
- Source of the mechanical losses as yet unknown

- Discussed here is analysis of one fundamental source of dissipation in thin coatings - thermoelastic damping associated with dissimilar thermal and elastic properties of coatings and test masses
- Needed for interpretation of measurements of coating loss and for calculations of resulting thermal noise in test masses
- (Learned yesterday - similar independent analysis of thermal noise component just carried out by Braginsky and Vyatchanin)

Formulation of problem

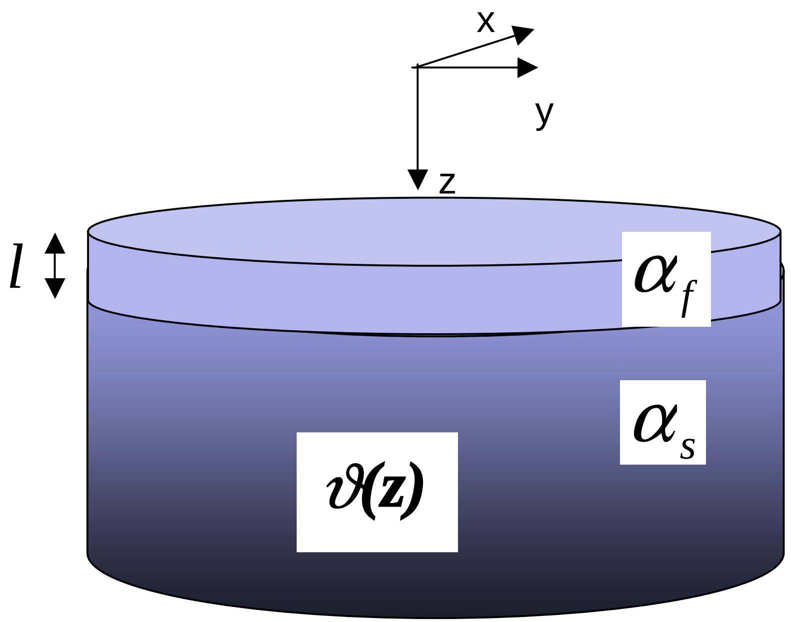
- First consider simplified case where coating and substrate have identical properties apart from coefficient of thermal expansion
- Applying a periodic (uniform) strain to the mass results in different amounts of heat being generated in regions with different α.
- Heat generated will flow between regions with different α.
- i.e. a thermal wave will be generated.



- Assume here all strain fields vary slowly in x, y compared with film thickness
- Then thermal wave, $\vartheta(z)$ is a function only of z

Formulation of problem

- Periodic strains generate an oscillatory thermal wave, with spatially varying magnitude and sign
- Heat couples back, through α_f and α_s, into elastic deformations of the sample

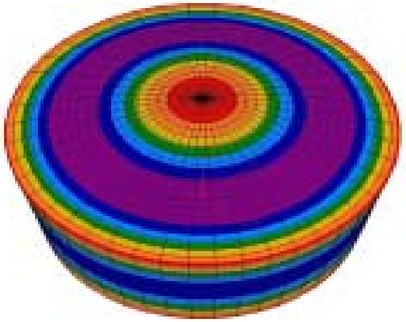


- The initial elastic fields (0th order) and subsequent thermally generated elastic fields (1st order) interact with a small phase difference to dissipate power
- Can then calculate the energy dissipated per cycle of applied strain

Relevance to loss measurements/thermal noise

Consider two different situations in which there are:

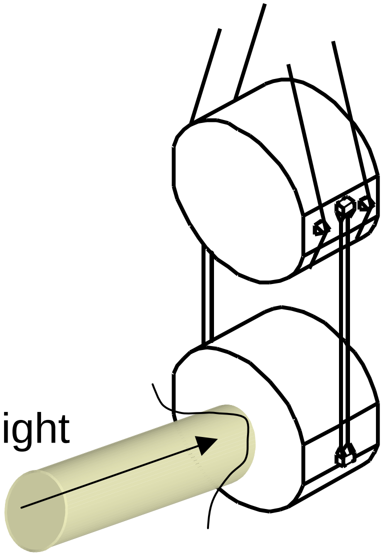
(a) a strain distribution in a test substrate during measurements of mechanical loss



Resonant mode of test substrate

and

(b) a stress 'applied' to a test mass by the Gaussian beam in an interferometer



laser light

Mechanical loss measurements

- In this case there are specified in-plane driving strains in the coating and substrate but no axial stress
- Write down appropriate driving elastic fields, calculate relevant temperature fluctuations, and induced elastic fields
- Calculate power dissipated

- By normalising energy dissipated to energy stored in coating - obtain an expression for thermoelastic dissipation, ϕ_{uni}

$$\boxed{\phi_{uni} = \frac{2E_f\alpha_f^2 T}{C_f(1-\nu_f)}\left[1-\frac{\alpha_s}{\alpha_f}\frac{E_s(1-\nu_f)}{E_f(1-\nu_s)}\frac{C_f}{C_s}\right]^2 g(\omega)}$$

Where the frequency dependence term $g(\omega)$ is given by:

$$g(\omega) = \mathrm{Im}\left[-\frac{1}{\sqrt{i\omega\tau_f}}\frac{\sinh\left(\sqrt{i\omega\tau_f}\right)}{\cosh\left(\sqrt{i\omega\tau_f}\right)+\sqrt{k_f C/k_s C_s}\,\sinh\left(\sqrt{i\omega\tau_f}\right)}\right] \quad ; \quad \tau_f = \frac{l^2 C_f}{k_f}$$

Thermoelastic noise

- Here use appropriate elastic fields for Gaussian pressure applied to front of test mass
- Calculate power dissipated per cycle, W_{diss}, then use approach of Levin to calculate power spectral density of thermal noise

$$\left(S(f) = \frac{2 k_b T}{\pi^2 f^2} \frac{W_{diss}}{F_o^2} \right)$$

$$\boxed{S_x(f) = \frac{8 k_b T^2}{\pi^2 f} \frac{l}{w^2} \frac{\alpha_s^2 C_f}{C_s^2} (1+\nu_s)^2 \Delta^2 g(\omega)}$$

w = beam radius
f = frequency

where

$$\Delta^2 \equiv \left\{ \frac{\alpha_f}{\alpha_s} \frac{C_s}{C_f} \frac{1}{2(1-\nu_f)} \left[\frac{1+\nu_f}{1+\nu_s} + (1-2\nu_s) \frac{E_f}{E_s} \right] - 1 \right\}^2$$

Note that like thermoelastic damping in bulk substrates (see Braginsky et al), thermoelastic noise is function of **material parameters of coating and substrate chosen - particularly the relative coefficients of thermal expansion**

Thermoelastic noise

- Substrate options for Advanced LIGO

  Sapphire

  Fused silica

- Coatings:

  SiO_2/Ta_2O_5 *(SiO_2/Nb_2O_5; SiO_2/Al_2O_3)*

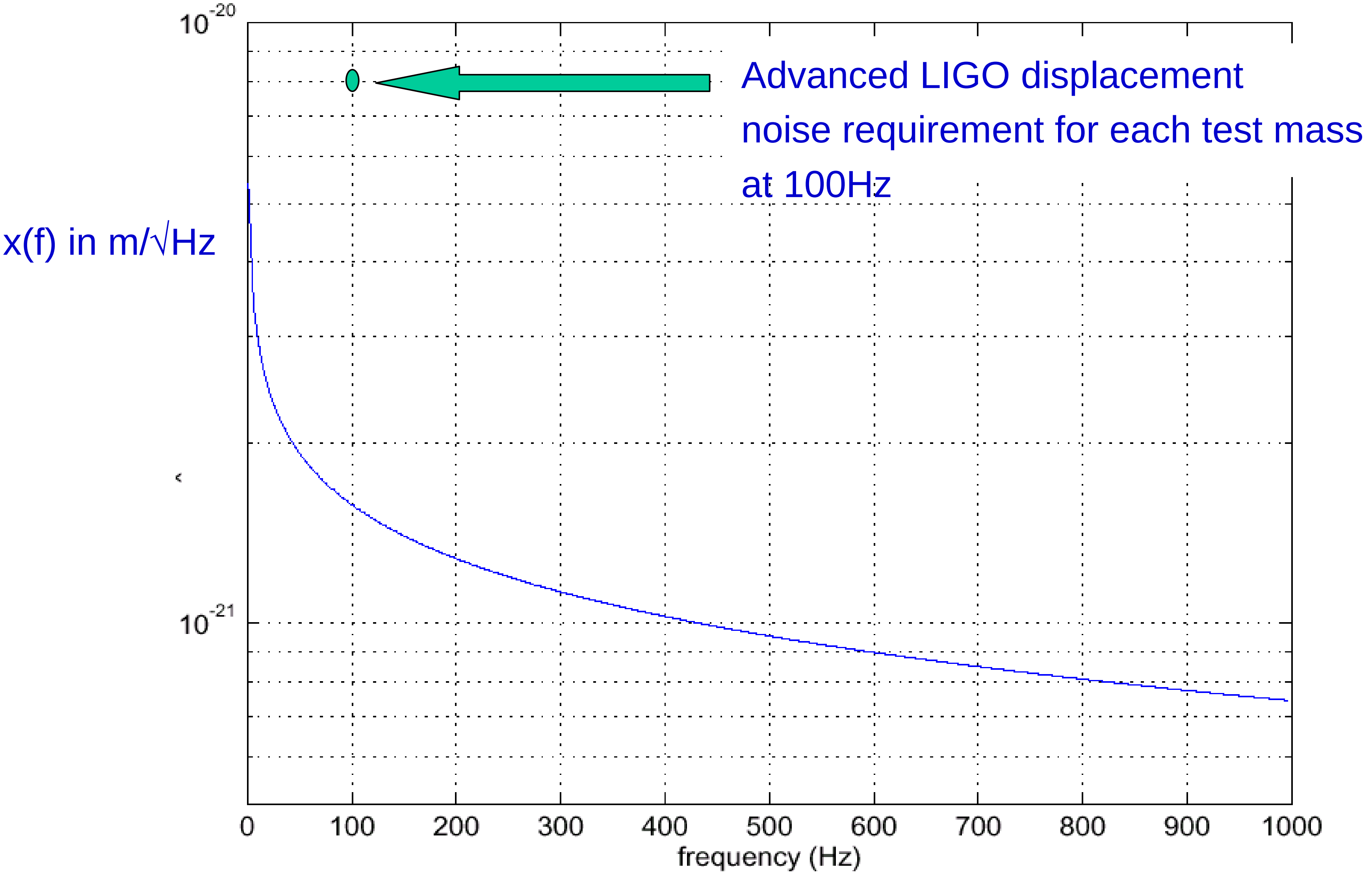
  Al_2O_3/Ta_2O_5

- Material parameters for bulk substrates reasonably well known
- Assume material parameters for SiO_2 and Al_2O_3 in coatings = same as bulk values
- Take values for Ta_2O_5 from literature where possible and otherwise assume to be same as Al_2O_3
- Note, values for α_f for Ta_2O_5 vary in literature -

  eg: - 4.4×10^{-5}/K

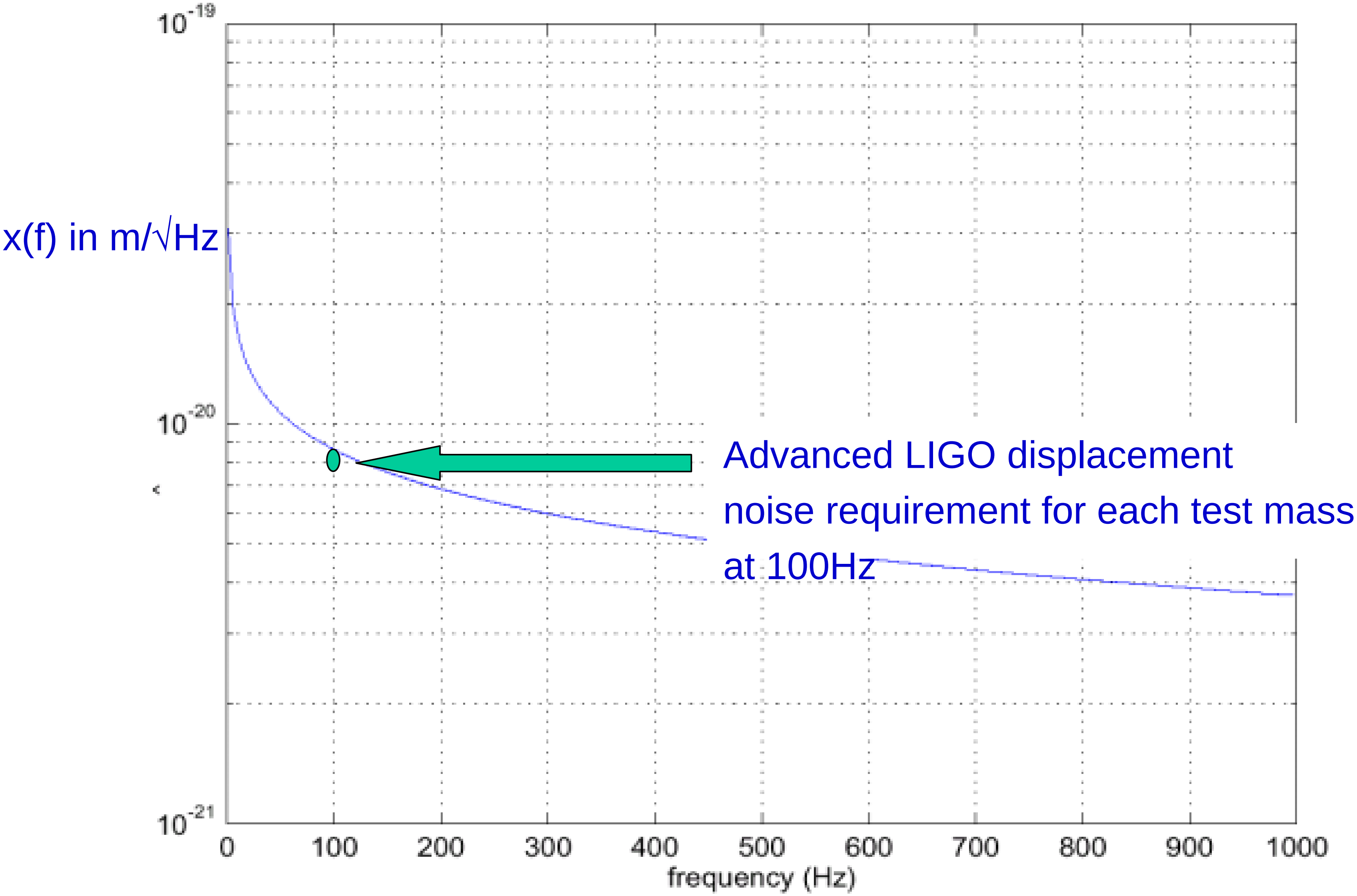
  3.6×10^{-6}/K use this value - closer to ‘typical’

Coating thermoelastic noise: SiO_2/Ta_2O_5 on fused silica substrate



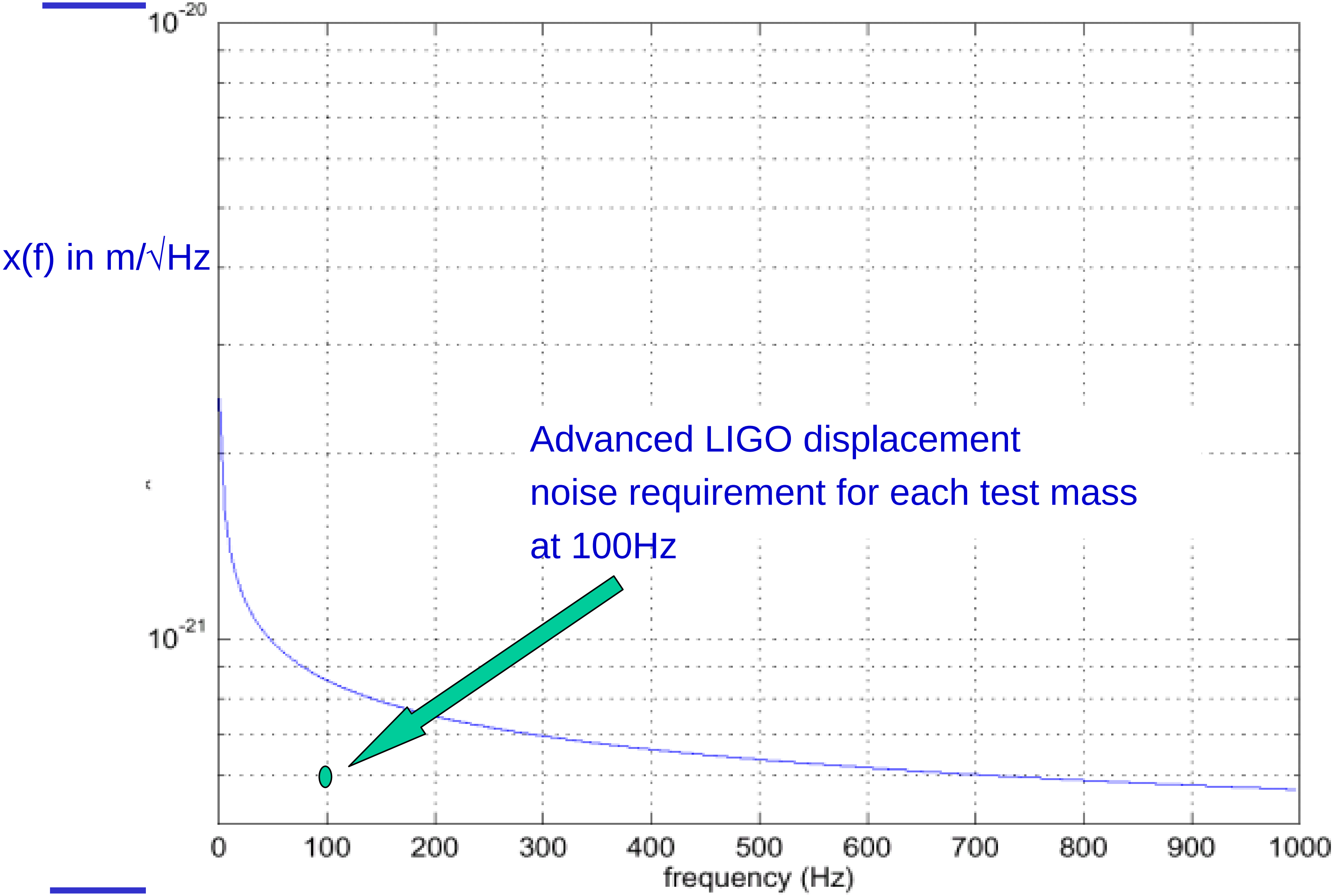
Advanced LIGO displacement noise requirement for each test mass at 100Hz

Coating thermoelastic noise: Al_2O_3/Ta_2O_5 on fused silica substrate



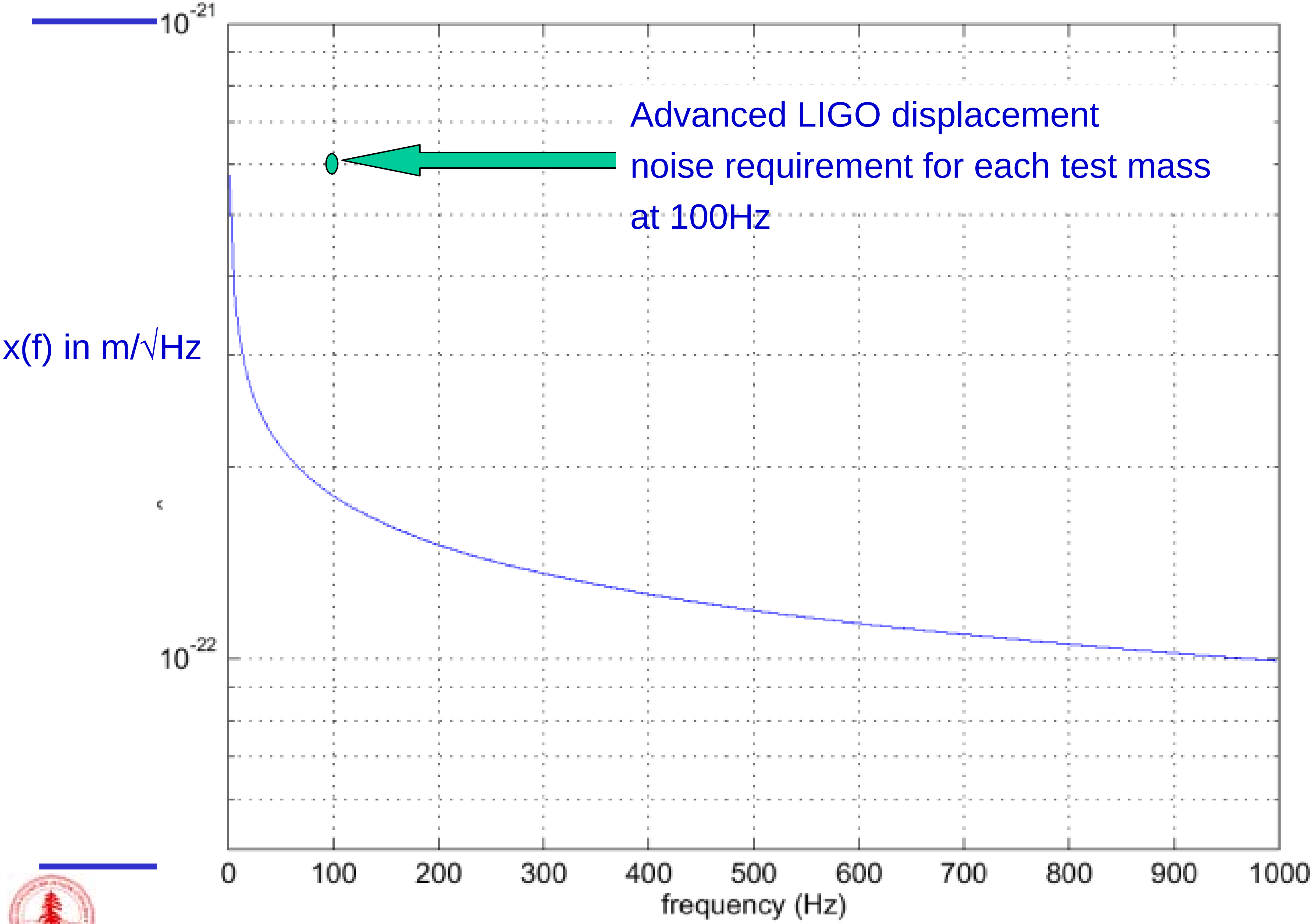
Advanced LIGO displacement noise requirement for each test mass at 100Hz

Coating thermoelastic noise: SiO_2/Ta_2O_5 on sapphire substrate



Advanced LIGO displacement noise requirement for each test mass at 100Hz

Coating thermoelastic noise: Al_2O_3/Ta_2O_5 on sapphire substrate



Advanced LIGO displacement noise requirement for each test mass at 100Hz

Interpretation of mechanical loss measurements

- Clear from previous graphs that coating thermoelastic noise can be significant at gravitational wave frequencies

- Need to also consider ‘intrinsic’ dissipation
- Experimental measurements of mechanical loss of coatings carried out at higher frequencies - kHz to 10’s of kHz
- Can use expression for ϕ_{uni} discussed earlier to calculate thermoelastic contribution to measured losses - smaller than measured losses
- So we are seeing intrinsic loss in addition to thermoelastic loss

- - use expression for ϕ_{uni} to aid interpretation of our experimental results - work ongoing

Summary

- Thermoelastic noise from coatings sets a limit to detector sensitivity
- Noise level depends on ***difference*** of material properties of coating and substrate
- Same coating will result in a different level of noise when applied to different substrates
- Of our current coating/substrate options

SiO_2/Ta_2O_5 applied to a fused silica substrate

or

Al_2O_3/Ta_2O_5 applied to a sapphire substrate

should give lowest levels of thermoelastic noise

- Must consider also 'intrinsic' dissipation of these coatings - measurements of mechanical loss factors of coatings important

Summary

- Noise level depends strongly on elastic and thermal properties of the ion-beam-sputtered mirror coatings, in particular the coefficient of thermal expansion of a coating
- These properties are typically are not well characterised
- Noise levels shown here are estimates only

- Worth investing some effort in characterising properties of Adv. LIGO coatings