Bursts: Statistical issues in UL setting

Erik Katsavounidis
MIT

LSC Meeting at LLO

March 19, 2003

DCC-G030119-00

Objectives

- Establish a measurement of the foreground

- Establish a measurement of the background

- Report on the search via setting confidence intervals that provide certain coverage
  - looking for excess of events: confronting number of signal and background events (Sylvestre et al.)
  - loudest event method (Brady et al.)
  - confronting statistical features of the signal and background event ensemble beyond their raw count (e.g., amplitude: Finn et al.)

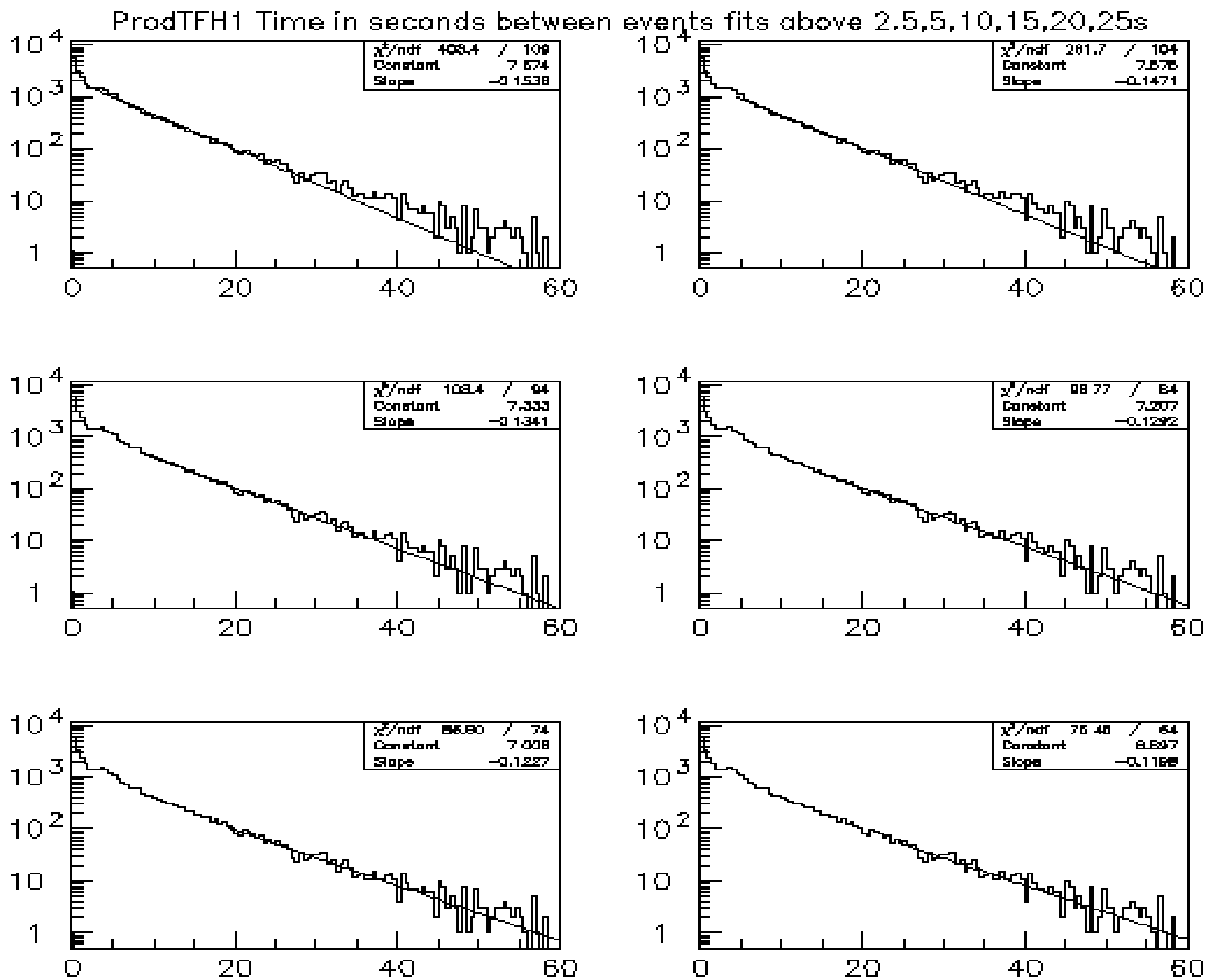
Background measurement

- ideally: turn off the source, repeat the experiment; not an option

- estimation: time-shift the data coming from the individual detectors and repeat search for 'signal' but call it a measurement of background

  - time-shifts exceed maximum window of coincidence used (easy, since $\Delta T < 0.5s$)

  - gravitational wave events are a small fraction of the measured 'signal'


ProdTFH1 Time in seconds between events fits above 2.5,5,10,15,20,25s
χ²/ndf 408.4 / 109
Constant 7.674
Slope −0.1538
χ²/ndf 261.7 / 104
Constant 7.576
Slope −0.1471
χ²/ndf 103.4 / 94
Constant 7.333
Slope −0.1341
χ²/ndf 98.77 / 84
Constant 7.207
Slope −0.1292
χ²/ndf 86.80 / 74
Constant 7.008
Slope −0.1227
χ²/ndf 76.40 / 64
Constant 6.897
Slope −0.1168
0
20
40
60

Background estimation via time-shifts

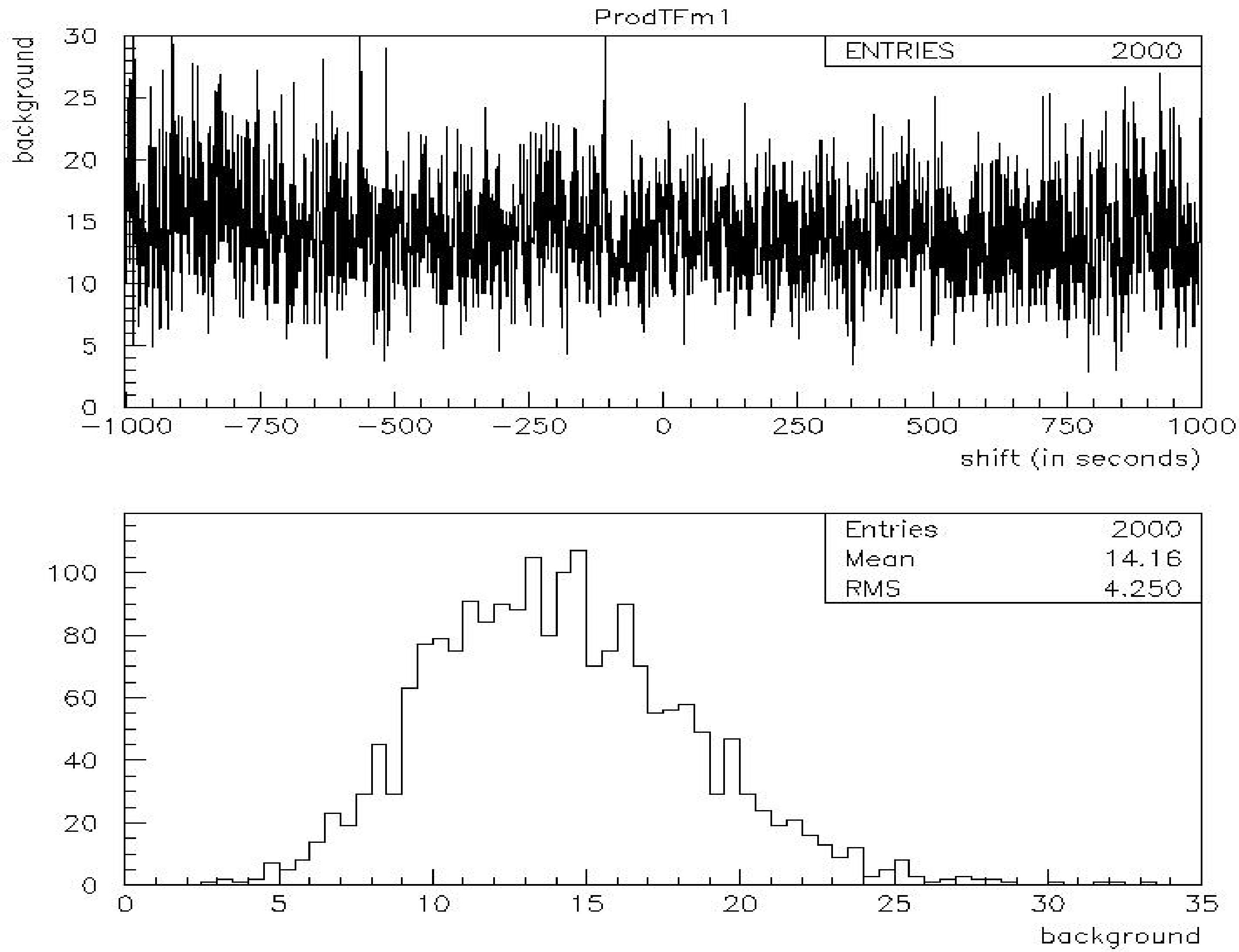
- test of independence of shift experiments

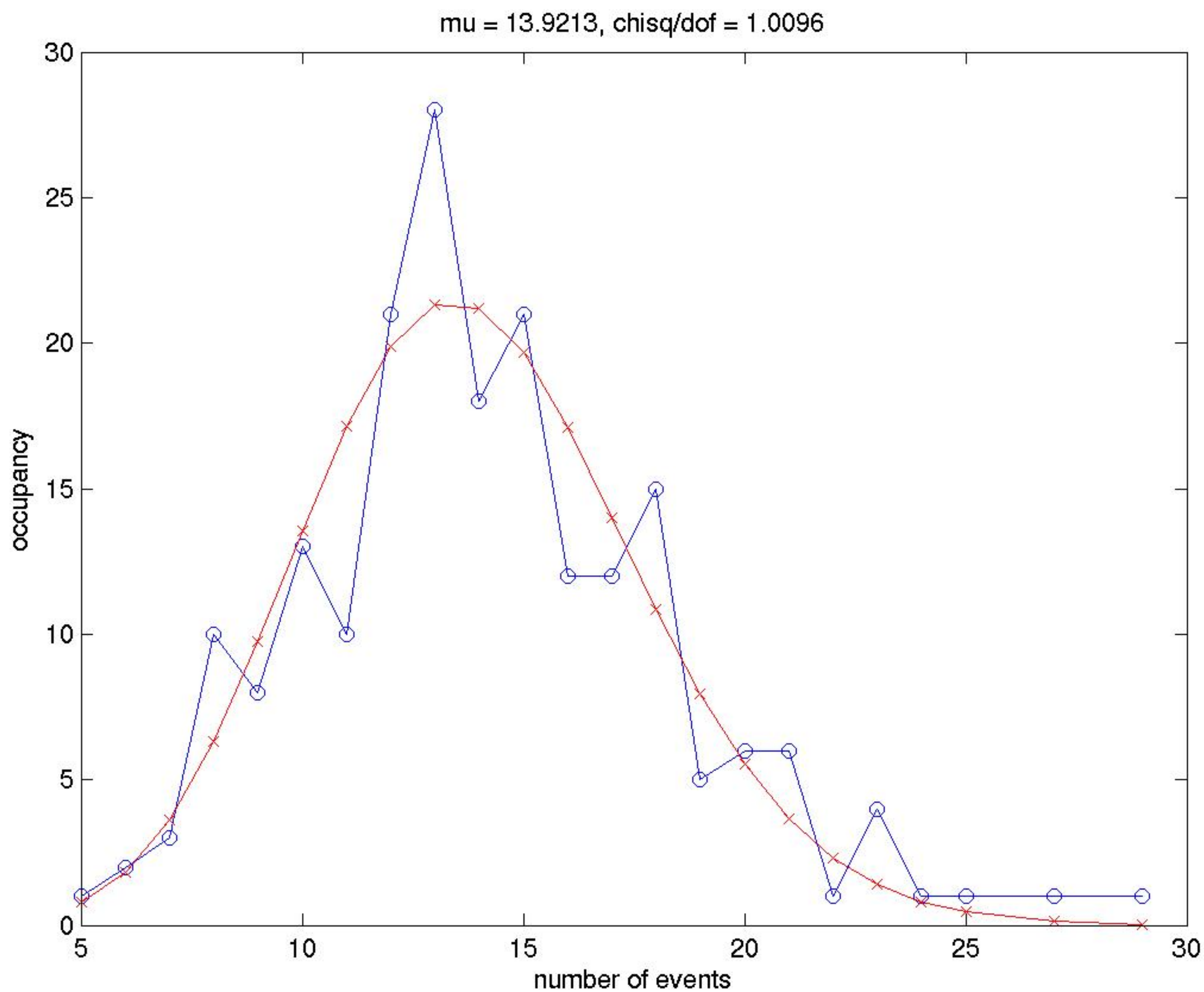
- background estimates are Poisson distributed:

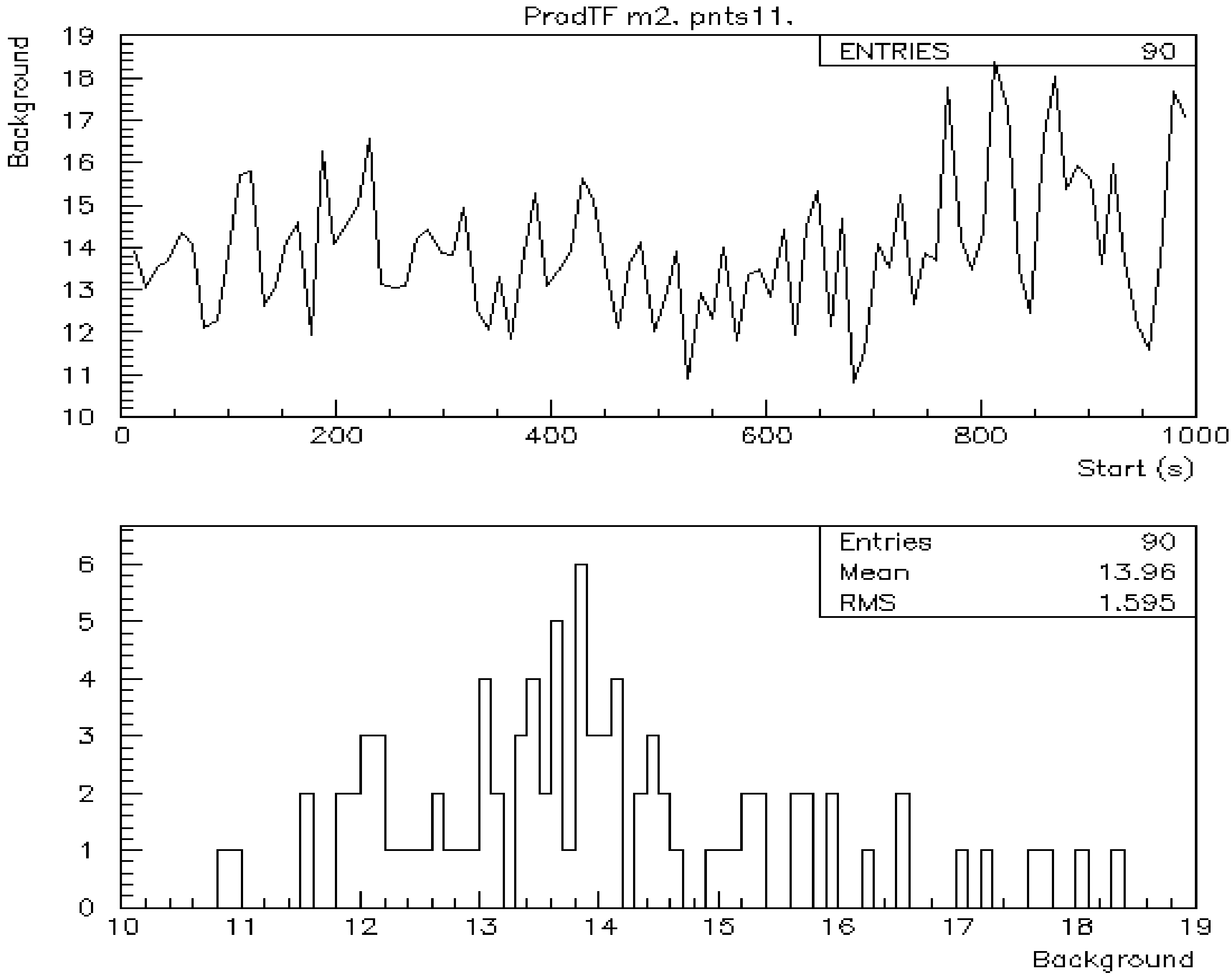
$$P(n|\mu_t) = \frac{{\mu_t}^n}{n!} e^{-\mu_t}$$

  where μ_t is the product of the true background rate times the livetime T

- time-shifts reduce coincident observation time between IFOs: $\mu_i = n_i T / T_i$


ProdTFm1
background
ENTRIES 2000
shift (in seconds)
Entries 2000
Mean 14.16
RMS 4.250
background


mu = 13.9213, chisq/dof = 1.0096
occupancy
number of events
0
5
10
15
20
25
30
5
10
15
20
25
30


ProdTF m2, pnts11,
Background
ENTRIES 90
Start (s)
Entries 90
Mean 13.96
RMS 1.595
Background

Confidence Intervals for GW Bursts

- treat detection and upper limits in a unified way; express result in terms of confidence intervals that include the true value of rate of GW bursts with a given probability [Feldman&Cousins, PRD 57, 3873 (1998)]

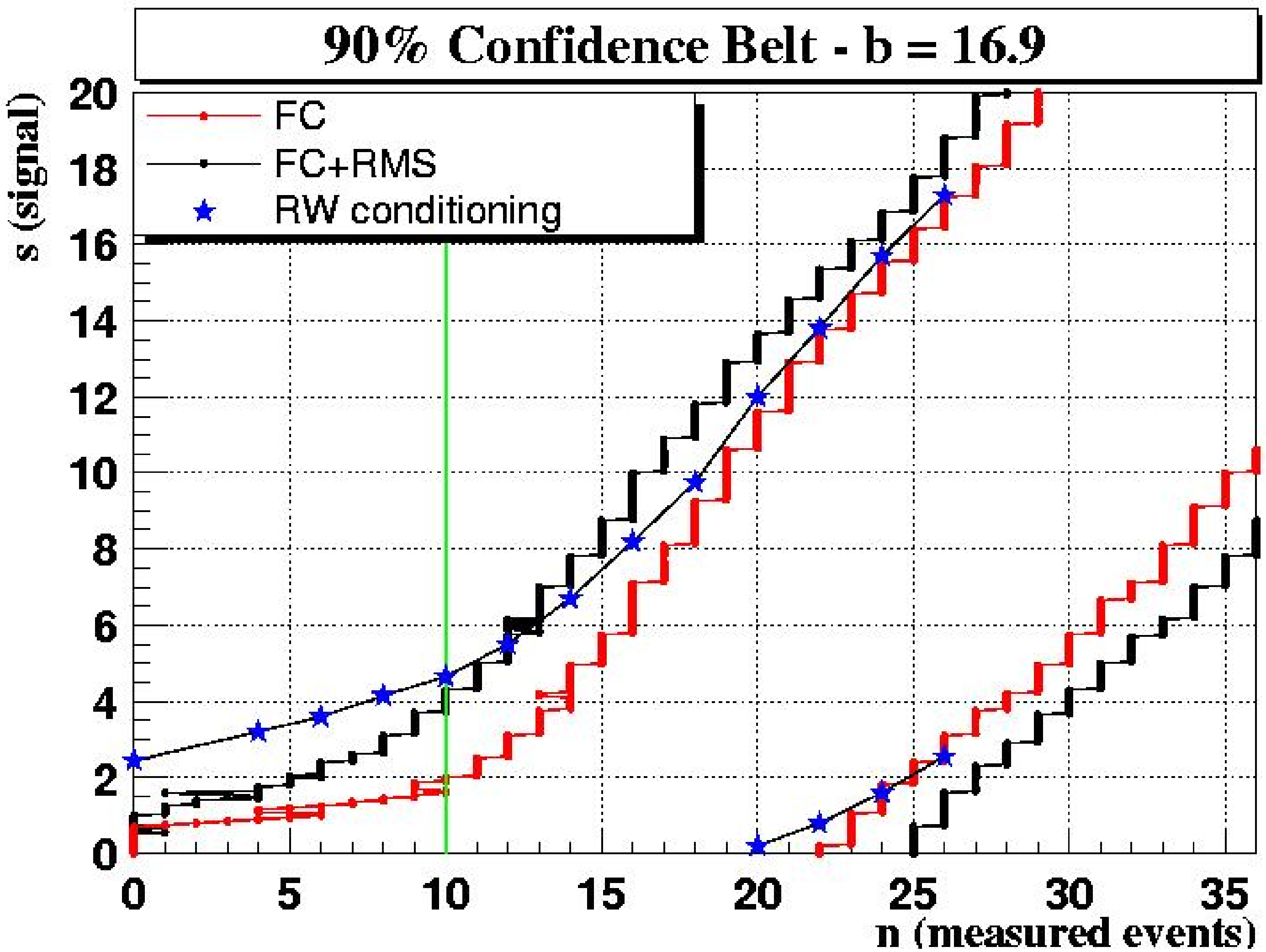
- assumptions: we have measured foreground N_f and background number N_b of events and we will test the hypothesis of the presence of a GW signal N_{gw}, then

$$P_{N_f}(N_{gw} + N_b) = \frac{e^{-(N_{gw}+N_b)}}{N_f!}(N_{gw} + N_b)^{N_f}$$

- result=upper limit/detection if confidence interval includes/does not include the null resutl

- while originally conceived to solve the unphysical limits reached by classical statistics in experiments with significantly fewer events than background (done!), method remains bothersome (to some!) in that measuring fewer events than expected will lower the UL respect to an identical experiment that measured a number of events equal to the background

- Roe-Woodroofe corrected the problem by introducing the concept of conditional coverage: at all times, background is less or equal to $n_{observed}$.



90% Confidence Belt - b = 16.9
FC
FC+RMS
RW conditioning
s (signal)
n (measured events)
0
2
4
6
8
10
12
14
16
18
20
0
5
10
15
20
25
30
35

Bursts UL in S1 and beyond

- Need robust statistic for establishing correlation and stationarity windows for performing background estimates.

- In S1, background systematics (step, start, duration) can not account for more than 0.8 events; rate-strength plot dominated by calibration systematics

- Feldman&Cousins set benchmark approach to UL for Poisson processes with background; still, be ready to modify according to Roe&Woodroofe if `foreground<background.and.foreground O(few).`