Detection Of High Frequency Gravitational Waves At LIGO

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1. Parametric Conversion “Picture”
2. Angle of Incidence of the G.W.
3. Results from H4K
4. Sensitivity
5. Possibilities with Advanced LIGO

PARAMETRIC CONVERSION

1. THE LIGO IFO ADMITS A SPECTRUM OF DISCRETE FREQUENCIES ν_n

THE FREQUENCIES ARE EQUALLY SPACED

$$\Delta\nu \equiv \nu_0 = 2L/c$$

ν_0 IS THE FREE SPECTRAL RANGE (fsr)

$$\nu_0 = 37.52 \text{ kHz}$$

n+1

n

n-1

WHEN THE IFO IS LOCKED ONLY ONE MODE IS OCCUPIED $n = \nu_n / \nu_0 \approx 10^{10}$

THE WIDTH OF THE MODES IS

$$\delta = \nu / Q \qquad Q = F\,(2L / \lambda) \approx 10^{12}$$

IN THE PRESENCE OF A PERTURBATION AT FREQUENCY

$$\Omega / 2\pi \approx \Delta\nu$$

THE (n+1) AND (n-1) MODES BECOME POPULATED

2. EXPECTED SIGNAL

E_n	FIELD IN MODE	n
$E_{n\pm1}$	FIELD IN MODE	$n\pm1$
η	DIMENSIONLESS PERTURBATION	

FOR $E_{n\pm1} << E_n$ AND $t >> Q / \omega$

$$E_{n\pm1} = 0.5\, E_n\, \eta\, Q$$

3. EXAMPLE: END MIRROR (ETM) MOTION

$x = x_0 \cos \Omega t$ $\qquad$ $\eta = x_0 / L$

$$E_{n\pm1} = E_n\, (x_0 / \lambda_0)\, F / [1 + (\Omega / \omega_\tau)^2]^{1/2}$$

$\omega_\tau = \nu_0\, (\pi / F)$

EFFECT OF A GRAVITATIONAL WAVE

- TRANSFER FUNCTION FOR "OPTIMAL INCIDENCE", $\theta = 0$

$$H_1(\Omega,\theta=0) = \mathrm{sinc}(\Omega\tau/2)\, e^{-i\Omega\tau/2}$$

$$\tau = 2L / c = 1 / \nu_0 \quad \therefore H_1(\Omega=2\pi\nu_0,\theta=0) = 0$$

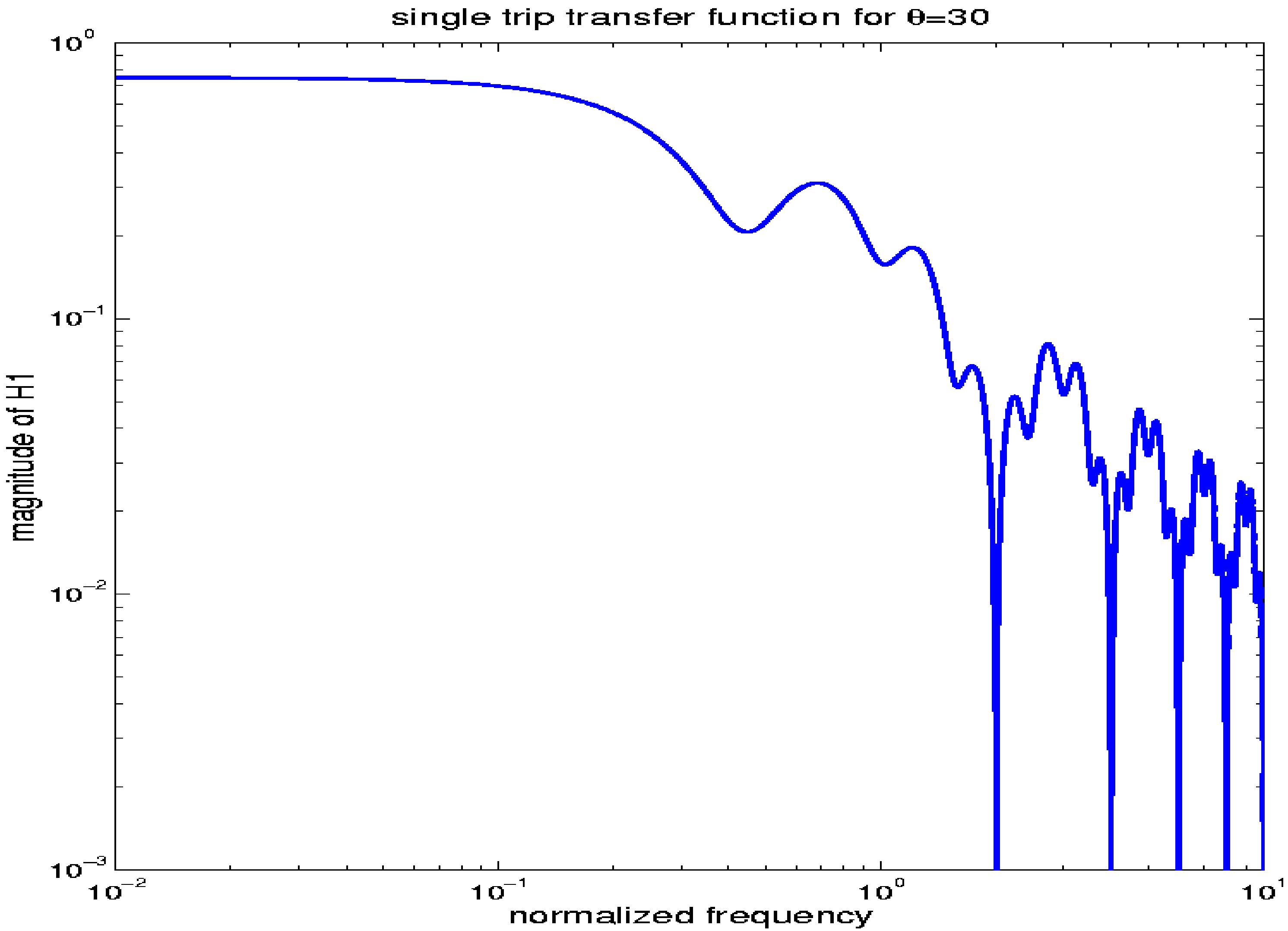
- TRANSFER FUNCTION FOR $\theta \neq 0$

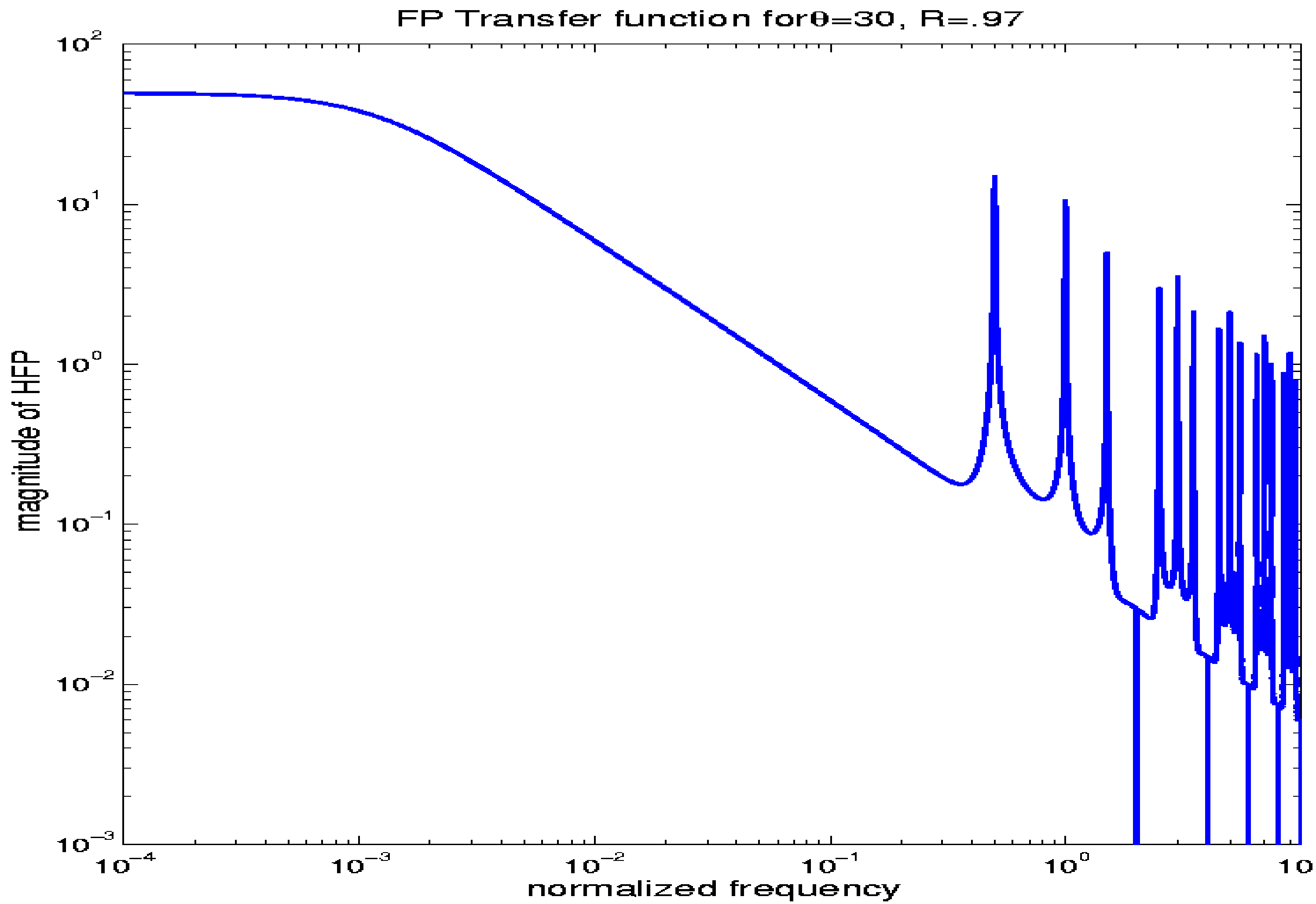
$$H_1(\Omega,\theta) = \cos^2\theta\ \{\mathrm{sinc}[\Omega\tau/4\ (1-\sin\theta)]e^{-i\Omega\tau/4} + \mathrm{sinc}[\Omega\tau/4\ (1+\sin\theta)]\ e^{i\Omega\tau/4}\}\ e^{-i(\Omega\tau/4)(2+\sin\theta)}$$

- FOR F-P CAVITY

$$H_{FP}(\Omega,\theta) = H_1(\Omega,\theta)\ /[(1- r_1)^2 + 4r_1 \sin^2(\Omega\tau/2)]^{1/2}$$

- AVERAGE OVER ANGLES, POLARIZATION $1/\sqrt{5}$
- SIDEBANDS AT $(\omega + \Omega)\ ,\ (\omega - \Omega)$


single trip transfer function for θ=30
magnitude of H1
normalized frequency
10^0
10^-1
10^-2
10^-3
10^-2
10^-1
10^0
10^1



FP Transfer function forθ=30, R=.97
magnitude of HFP
normalized frequency
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-3
10^-2
10^-1
10^0
10^1

RESULTS FROM H4K (2002-W.BUTLER)

1. SHAKE ITMX (SINGLE ARM)

EXTRAPOLATE MIRROR MOTION FROM D.C. CALIBRATION.
(FOR 1V DRIVE)

$$x_0(\nu_0) = x_{DC} / (1 + (\nu_0/\nu_p)^2) = 8 \times 10^{-16}\ \text{m} \qquad (1)$$

OBSERVE AT	A.S.PORT	E_A
CARRIER FIELD	ON BS	E_2
SIDEBAND FIELD	ON BS	E_{RF}

$$|E_A / E_2| = 4\,(x_0 / \lambda_c)\,F \qquad (2)$$

PHOTODIODE VOLTAGE

$$V_A = k\,|E_2|^2\ |E_{RF} / E_2|\ |E_A / E_2| \cong 30\,|E_A / E_2|\ (V)$$

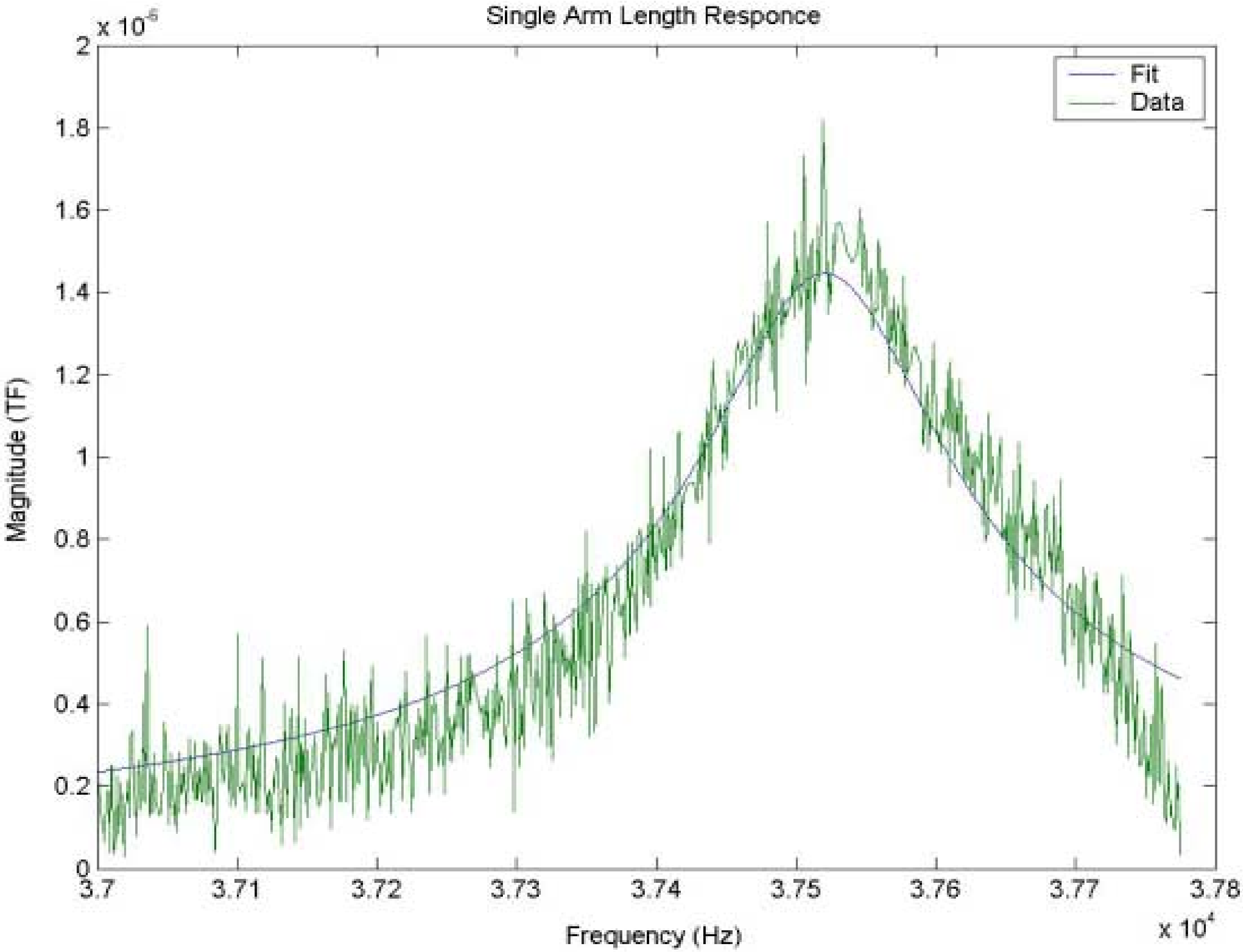
OBSERVE (FOR 1V DRIVE)

$$V_A = 3 \times 10^{-5}\ V \Rightarrow |E_A / E_2| \cong 10^{-6}$$

FIND USING (2)

$$x_0 \cong 10^{-15}\ \text{m}$$

IN AGREEMENT WITH (1).


Single Arm Length Responce
x 10^-6
Fit
Data
Magnitude (TF)
Frequency (Hz)
x 10^4
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
3.7
3.71
3.72
3.73
3.74
3.75
3.76
3.77
3.78

2. SHAKE ITMX (FULLY RECYCLED IFO)

PARAMETRIC RESONANCE MUCH NARROWER GOVERNED BY "DOUBLE CAVITY POLE"

$$\omega_{cc} \cong \nu_0 (1 - r_{F*R})$$

r_{F*R} REFLECTIVITY OF FRONT CAVITY MIRROR WHEN RECYCLING CAVITY ON RESONANCE

3. SENSITIVITY

FOR $T_{INT} = 100$ s OBSERVE

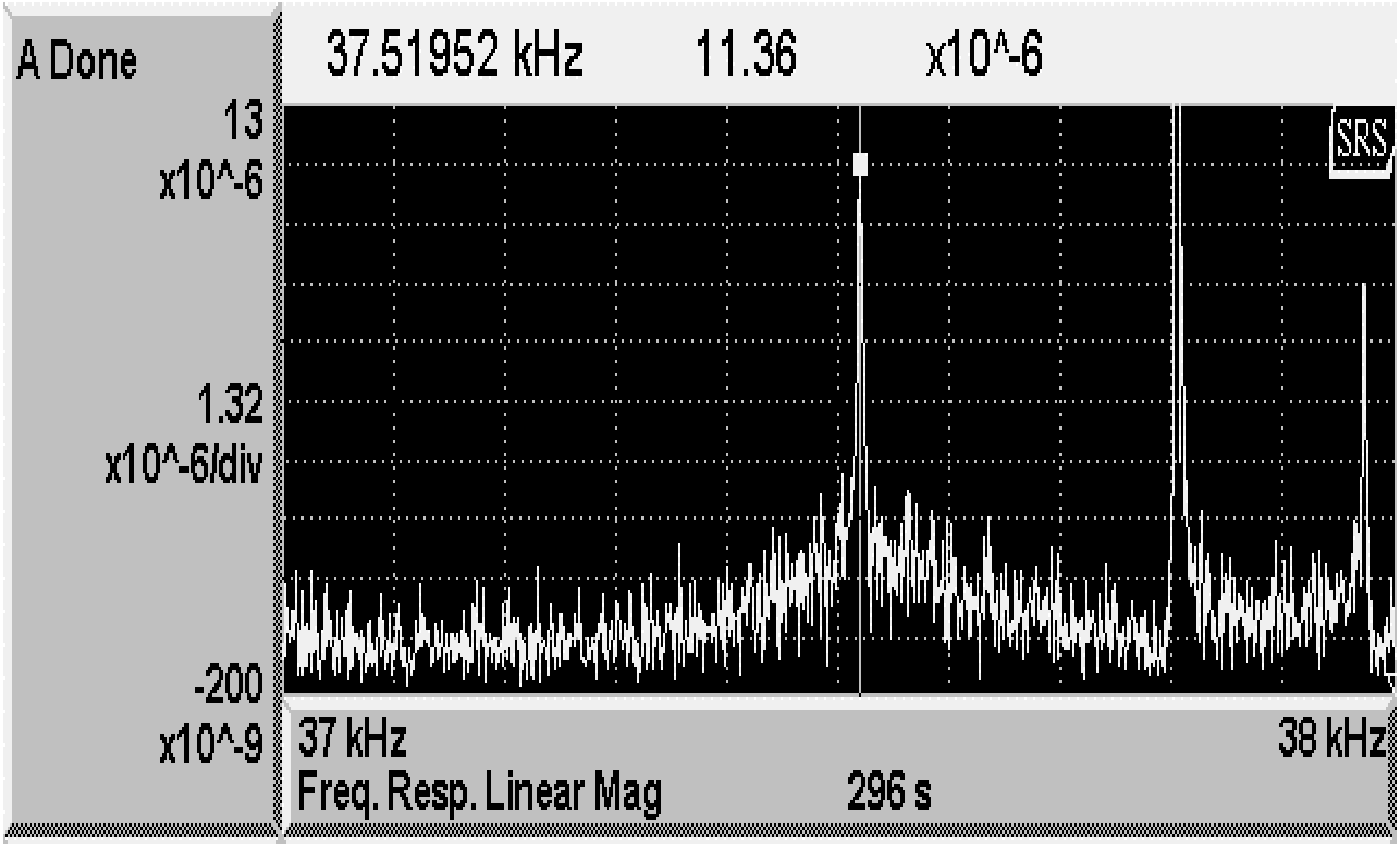
$V_N = 2 \times 10^{-7}$ V (S/N 150)

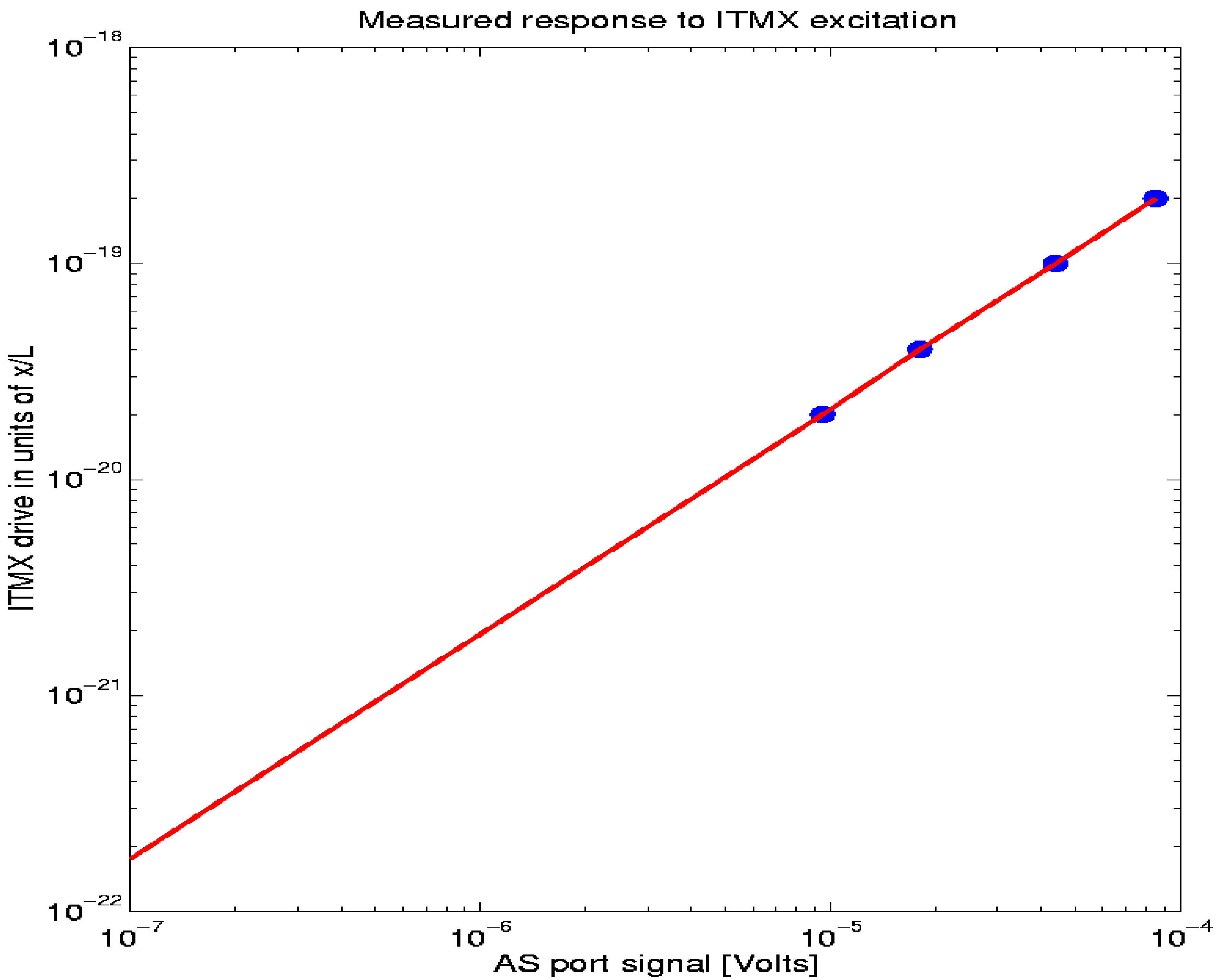
FOR $T_{INT} = 10^5$ s (1 DAY) EXPECT

$V_N = 0.6 \times 10^{-8}$ V

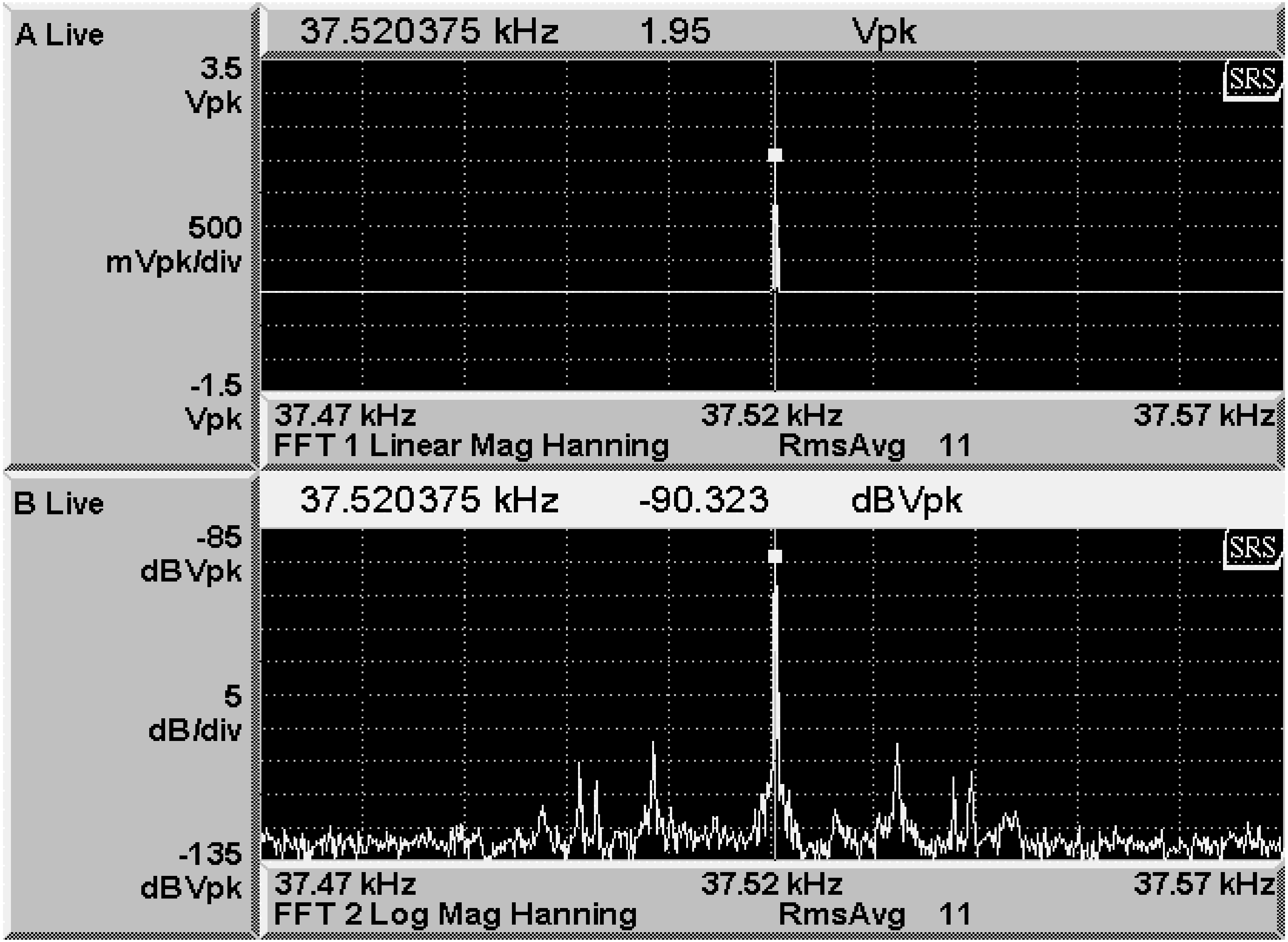
(S/N = 5) $x_0 = 10^{-18}$ m

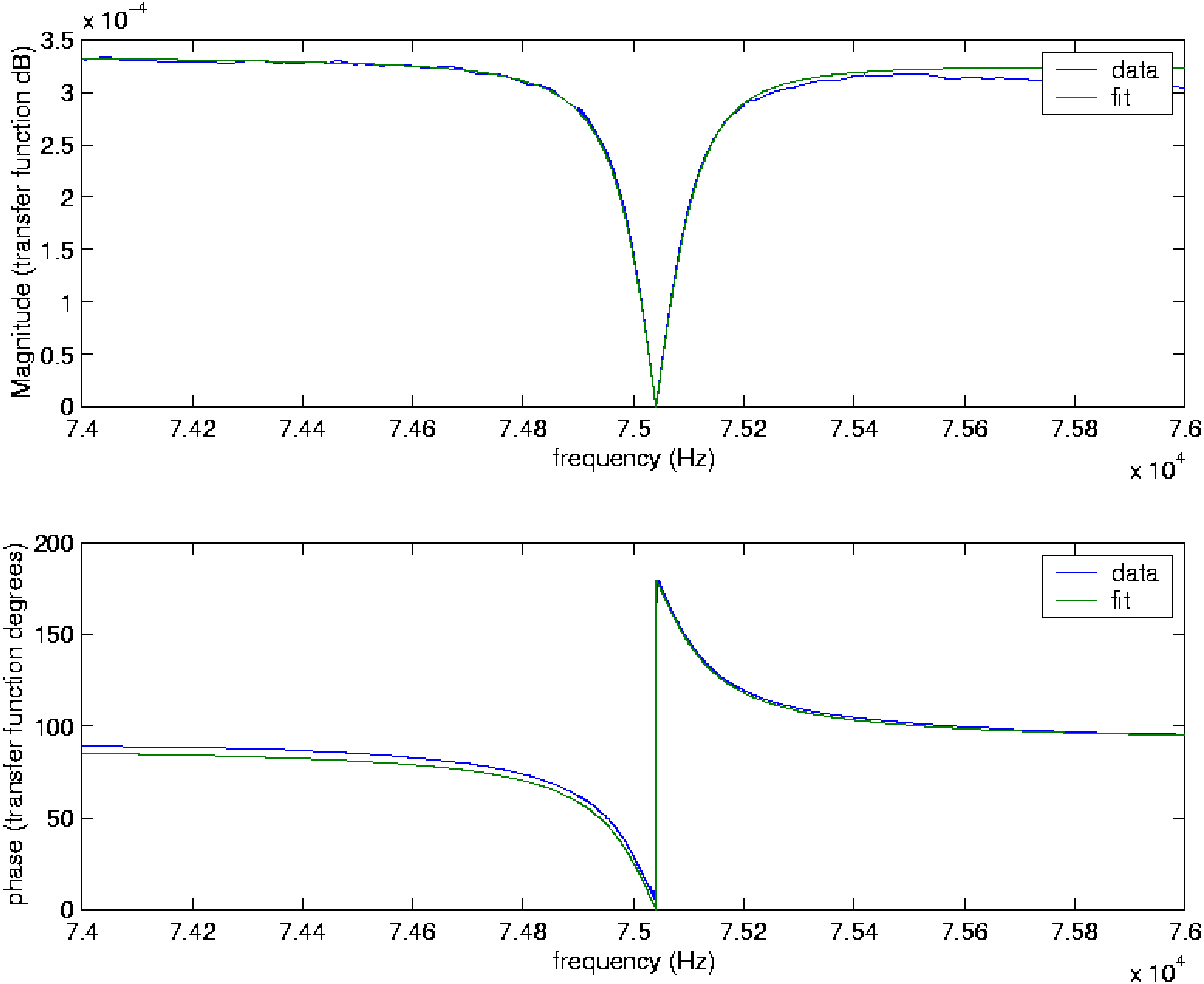
$x_0 / L = h = 2.5 \times 10^{-22}$


A Done
37.51952 kHz
11.36
x10^-6
13
x10^-6
1.32
x10^-6/div
-200
x10^-9
SRS
37 kHz
38 kHz
Freq. Resp. Linear Mag
296 s



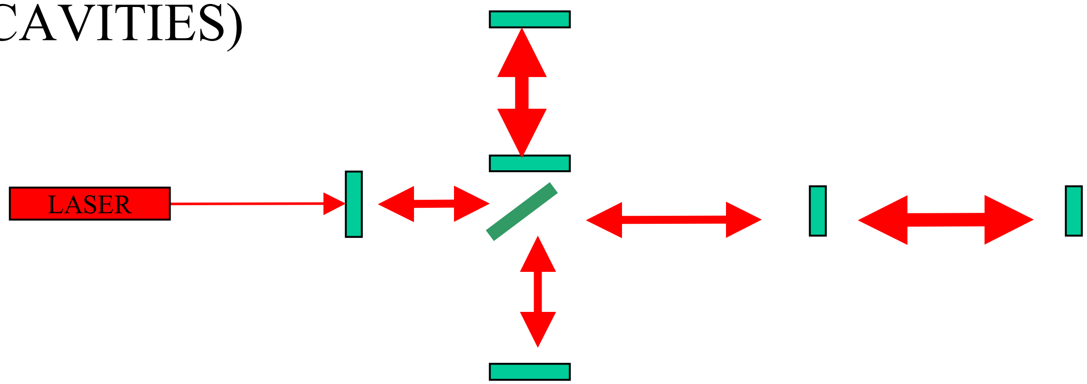
Measured response to ITMX excitation
ITMX drive in units of x/L
AS port signal [Volts]
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-7}
10^{-6}
10^{-5}
10^{-4}


A Live
37.520375 kHz 1.95 Vpk
3.5
Vpk
500
mVpk/div
-1.5
Vpk
SRS
37.47 kHz
37.52 kHz
37.57 kHz
FFT 1 Linear Mag Hanning
RmsAvg 11
B Live
37.520375 kHz -90.323 dBVpk
-85
dBVpk
5
dB/div
-135
dBVpk
SRS
37.47 kHz
37.52 kHz
37.57 kHz
FFT 2 Log Mag Hanning
RmsAvg 11


x 10^-4
Magnitude (transfer function dB)
3.5
3
2.5
2
1.5
1
0.5
0
7.4
7.42
7.44
7.46
7.48
7.5
7.52
7.54
7.56
7.58
7.6
frequency (Hz)
x 10^4
data
fit
phase (transfer function degrees)
200
150
100
50
0
7.4
7.42
7.44
7.46
7.48
7.5
7.52
7.54
7.56
7.58
7.6
frequency (Hz)
x 10^4
data
fit

LIGO WITH SIGNAL RECYCLING

THE "RECYCLING" MIRROR NOW COUPLES THE TWO ARMS (CAVITIES)



THE CAVITY MODES SPLIT INTO

$\nu_S = \nu_0 + (c / 2L)(1/\pi) \tan\{\arctan [(1-r_1)/(1+r_1) \cot z/4]\}$

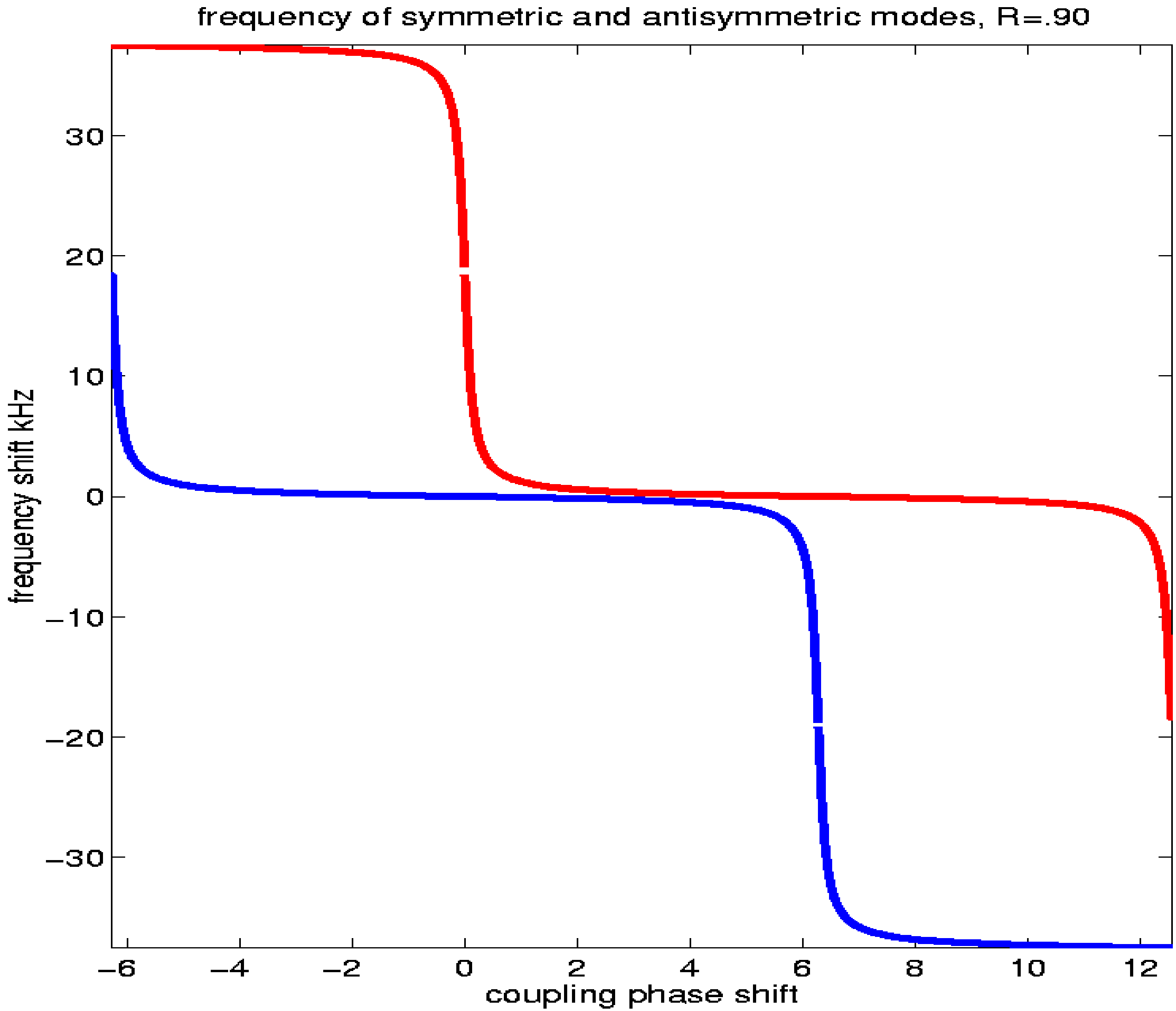
$\nu_A = \nu_0 - (c / 2L)(1/\pi) \tan\{\arctan [(1-r_1)/(1+r_1) \tan z/4]\}$

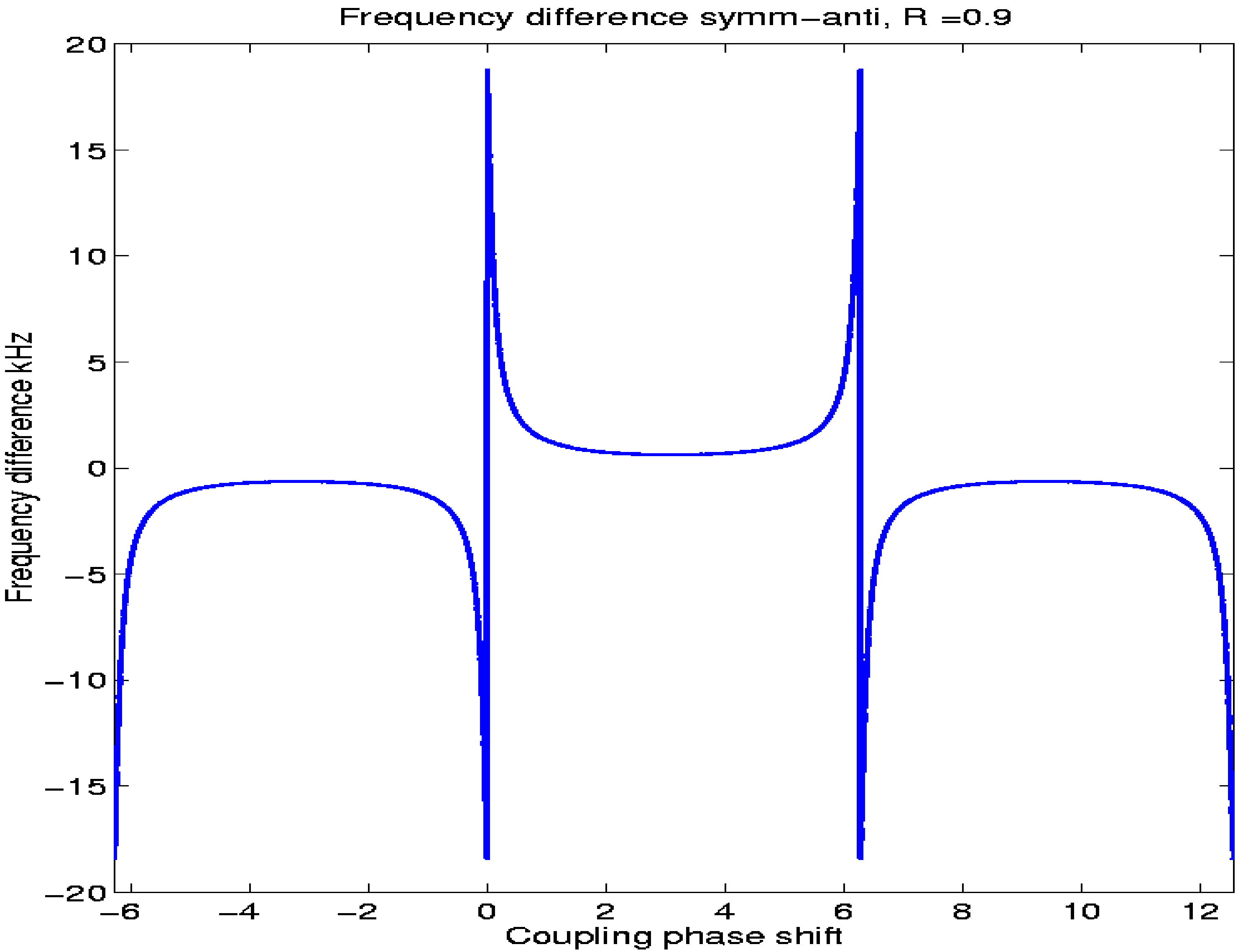
$z = 2\pi (2a / \lambda)$ $2a$ = distance between cavities

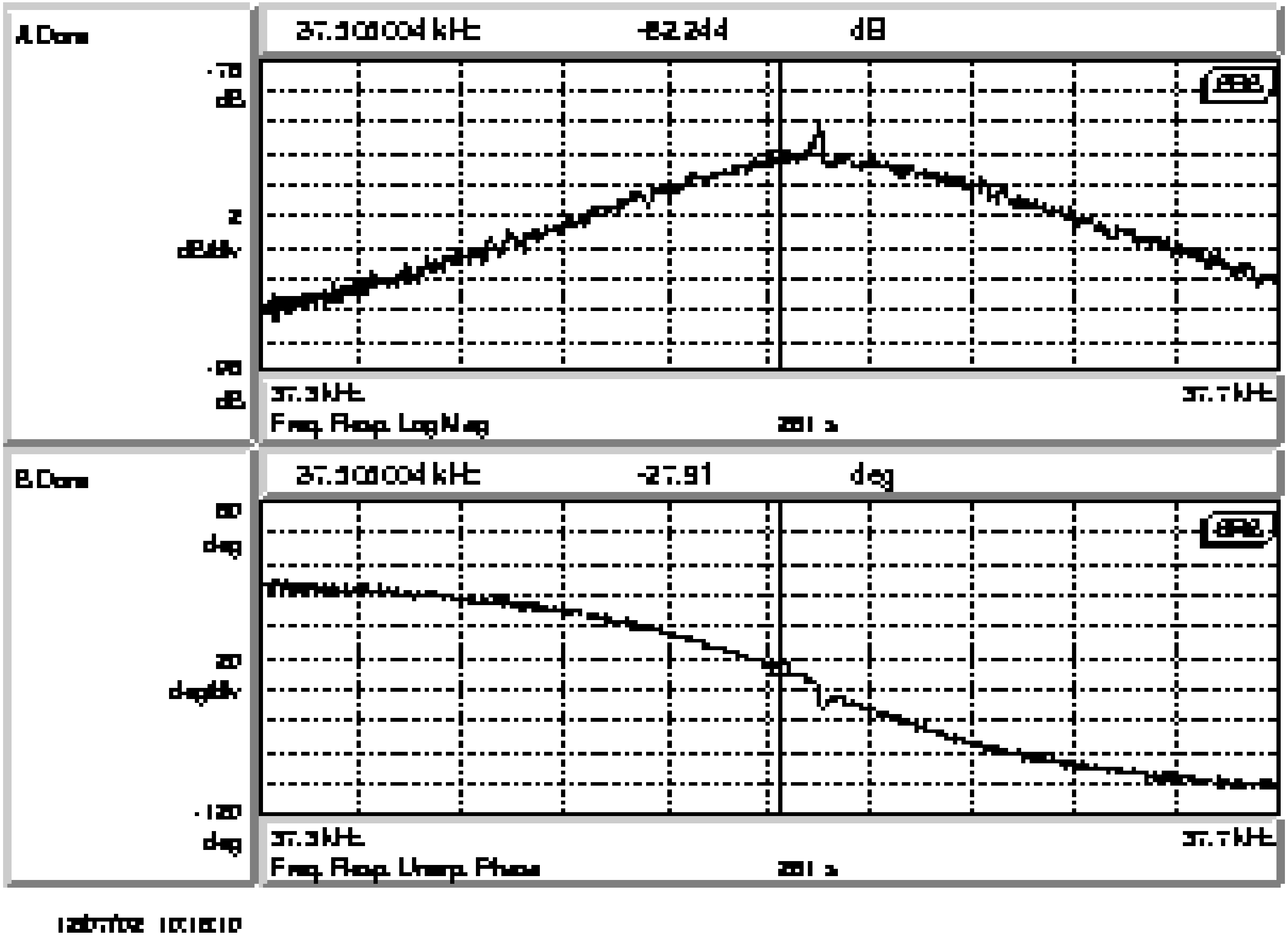
r_1 = cavity input mirror reflectivity

LOCK LASER AT ν_S

SIGNAL APPEARS AT ν_A


frequency of symmetric and antisymmetric modes, R=.90
frequency shift kHz
coupling phase shift
30
20
10
0
−10
−20
−30
−6
−4
−2
0
2
4
6
8
10
12


Frequency difference symm−anti, R =0.9
Frequency difference kHz
Coupling phase shift
20
15
10
5
0
−5
−10
−15
−20
−6
−4
−2
0
2
4
6
8
10
12


dB
37.3 kHz
37.7 kHz
Freq Resp Log Mag
-27.91
deg
Freq Resp Unwrap Phase