Theoretical Evidence for the Sidebands Imbalance

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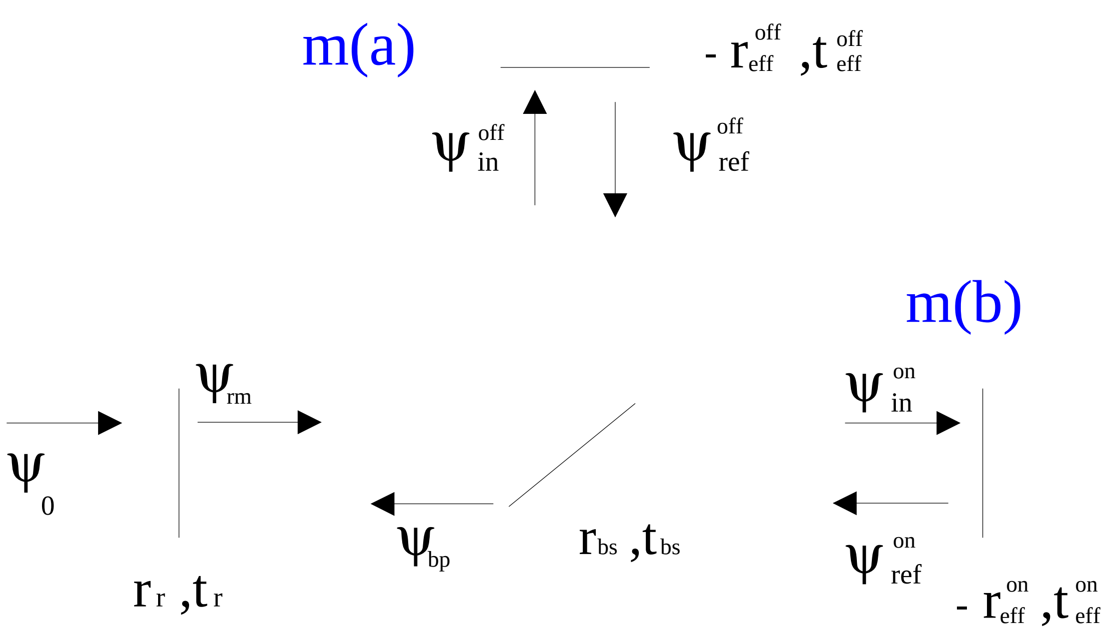
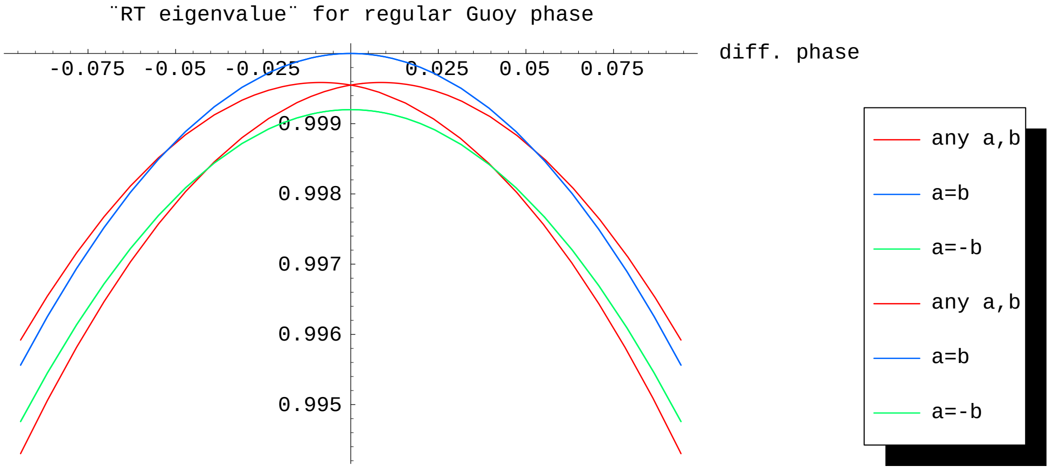
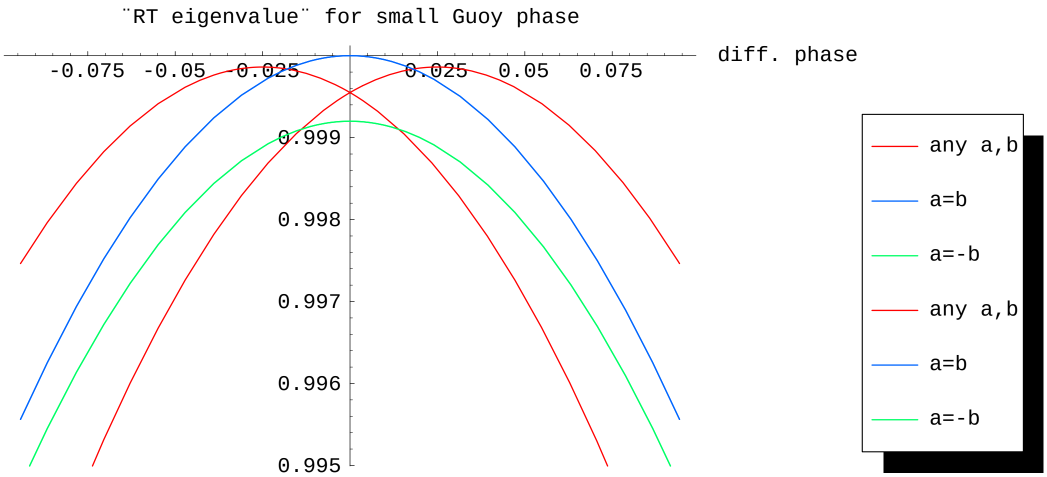
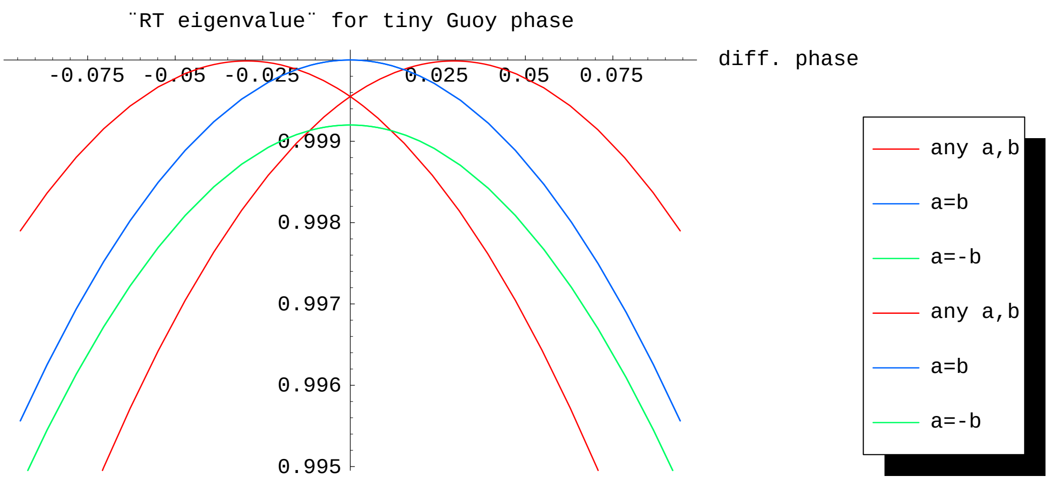
- Complete description of the effect of geometrical distortions that produces a sidebands imbalance by means of a common and differential perturbation in the two branches of the Michelson interferometer that couples different modes and gives an equivalent offset.
- Consequences on the tuning of the lengths in order to decrease such imbalance.
- Conclusions on the macroscopic lengths that are optimum for the sidebands and make perturbations less critical.

Geometrical distortions are represented by matrices and if we consider one kind of perturbation at a time we only need to know how large its amplitude is so that the system is represented in terms of that parameter




¨RT eigenvalue¨ for regular Guoy phase
diff. phase
-0.075 -0.05 -0.025 0.025 0.05 0.075
0.999
0.998
0.997
0.996
0.995
any a,b
a=b
a=-b
any a,b
a=b
a=-b


¨RT eigenvalue¨ for small Guoy phase
diff. phase
-0.075 -0.05 -0.025 0.025 0.05 0.075
0.999
0.998
0.997
0.996
0.995
any a,b
a=b
a=-b
any a,b
a=b
a=-b


¨RT eigenvalue¨ for tiny Guoy phase
diff. phase
-0.075 -0.05 -0.025 0.025 0.05 0.075
0.999
0.998
0.997
0.996
0.995
any a,b
a=b
a=-b
any a,b
a=b
a=-b

Differential phase that maximizes the symmetric port power

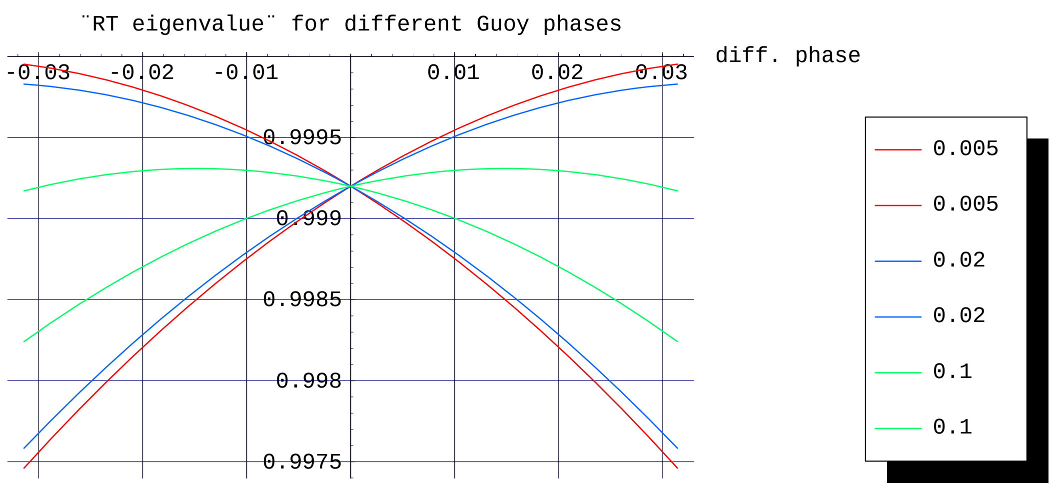
$$\phi_{opt} = \arctan\left[\tan\frac{a-b}{2}\cos\theta_g \sin\left(\arctan\frac{\tan\frac{a+b}{2}}{\sin\theta_g}\right)\right]$$

- $$\phi_{opt} \stackrel{a\to b}{\longrightarrow} 0$$
- $$\phi_{opt} \stackrel{a\to -b}{\longrightarrow} 0$$
- $$\phi_{opt} \stackrel{\theta_g\to 0}{\longrightarrow} \frac{a-b}{2}$$

Moreover for this choice the antisymmetric port is dark for the corresponding eigenvector that means that only the light of the non-resonating mode goes out at the beamsplitter.

This is a general feature that depends on **algebra** but not on the model nor the size of the vectors involved and I made a rigorous proof.

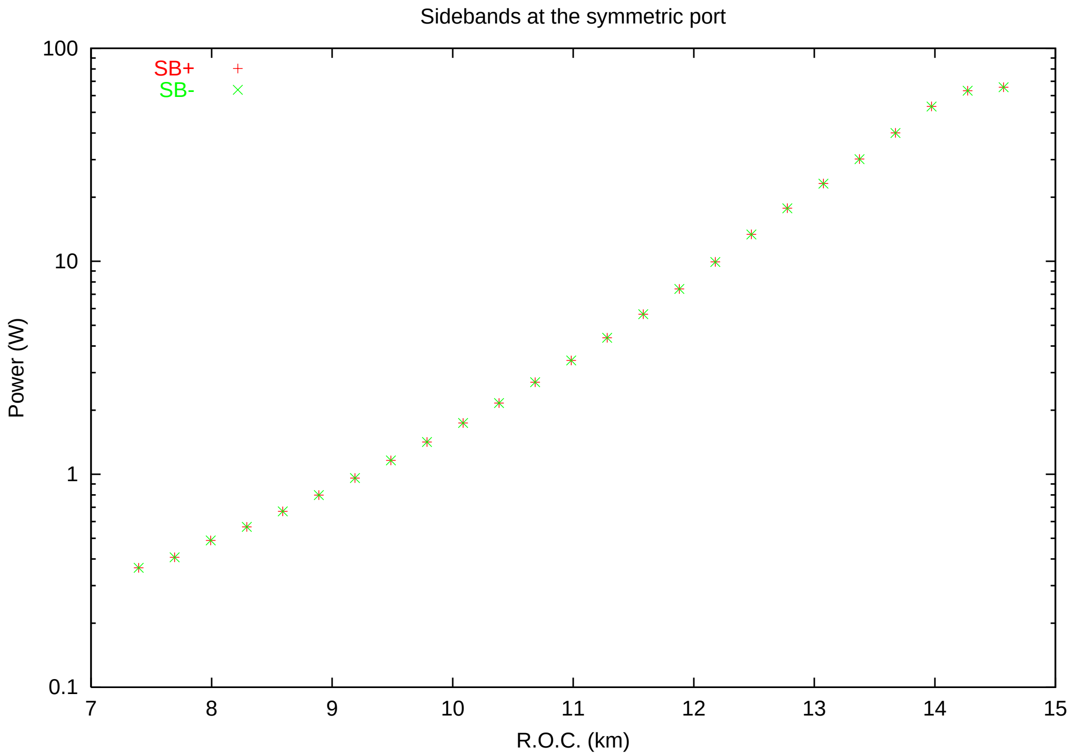
Imbalance at the bright port related with the differential phase of the sidebands

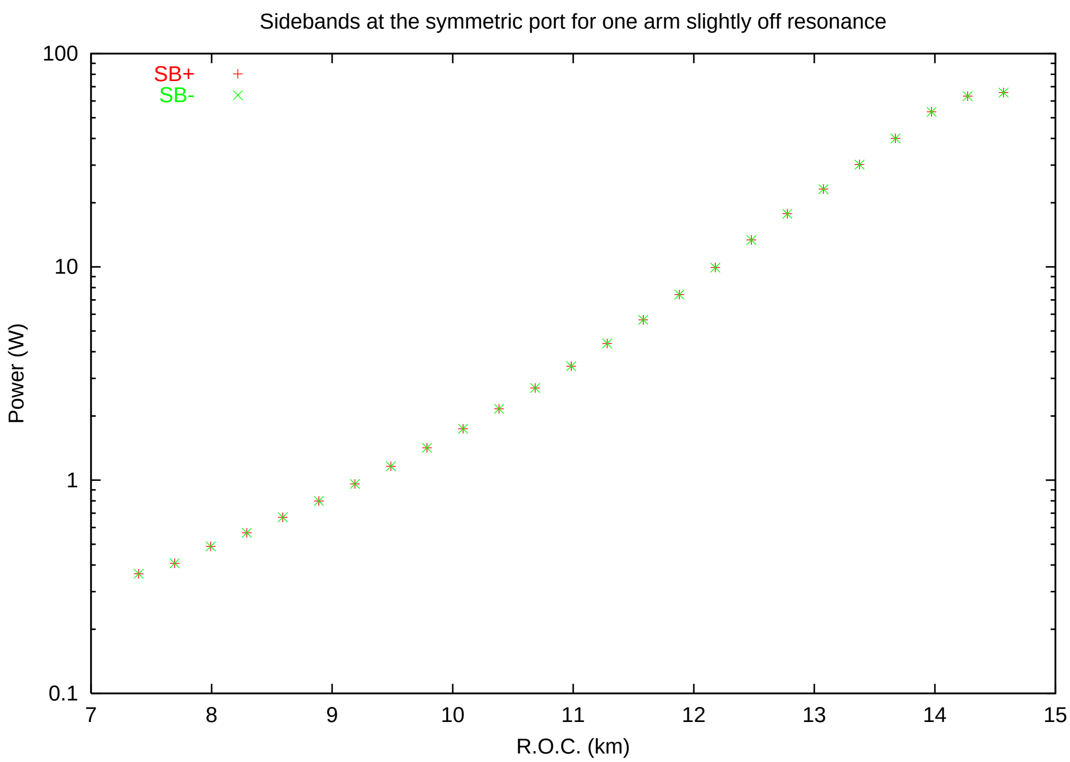


$$|\lambda_i(\phi)| - |\lambda_i(-\phi)| =$$

$$\pm 2 \sin\phi \sin\left(\frac{a-b}{2}\right) \cos\theta_g \sin\left(\arctan\frac{\tan\frac{a+b}{2}}{\sin\theta_g}\right)$$

$$\phi = \omega_{mod}\frac{2L_{Sch}}{c} \qquad L_{Sch} = \frac{l_2 - l_1}{2}$$


Sidebands at the symmetric port
SB+
SB-
Power (W)
R.O.C. (km)
100
10
1
0.1
7
8
9
10
11
12
13
14
15


Sidebands at the symmetric port for one arm slightly off resonance
SB+
SB-
Power (W)
R.O.C. (km)
100
10
1
0.1
7
8
9
10
11
12
13
14
15

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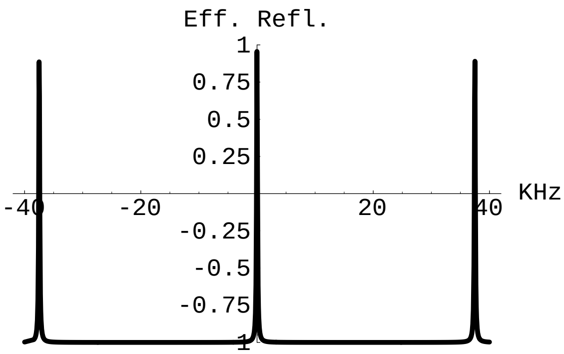

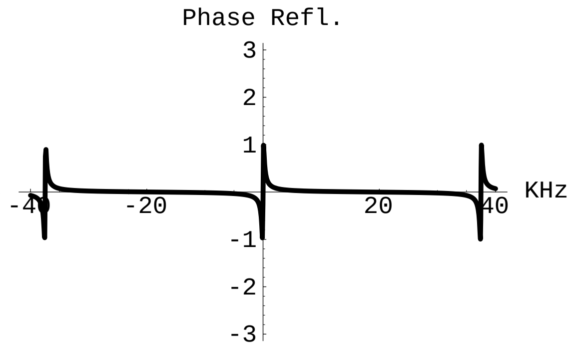

$$r_{eff}^{CR}(\delta\phi) \simeq +1\, e^{i(\frac{1+r}{1-r})\delta\phi}$$

$$r_{eff}^{SB}(\delta\phi) \simeq -1\, e^{i(\frac{1-r}{1+r})\delta\phi}$$

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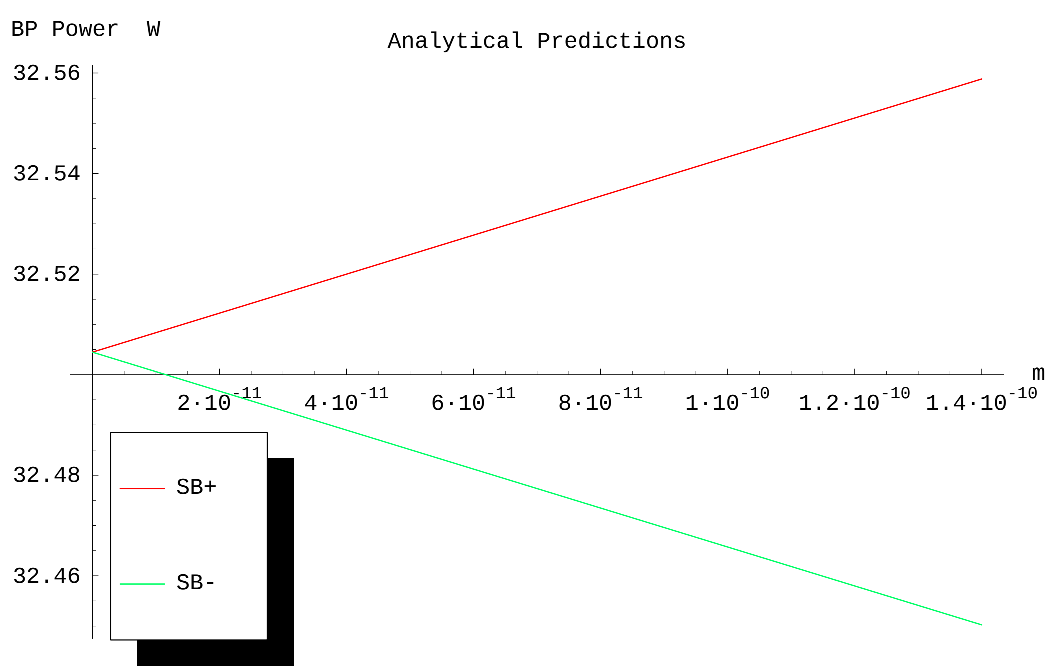
Let's choose a macroscopic length for the long arm cavity such that the phase gained by the sidebands is $\pm\pi/2$ in a round-trip.

Then for a small change in the cavity length:

$$r_{eff}^{SB+}(\delta\phi) \simeq -e^{i(\frac{1-r^2}{2r})} e^{i\delta\phi(\frac{1-r^4}{4r^2})}$$

$$r_{eff}^{SB-}(\delta\phi) \simeq -e^{-i(\frac{1-r^2}{2r})} e^{i\delta\phi(\frac{1-r^4}{4r^2})}$$



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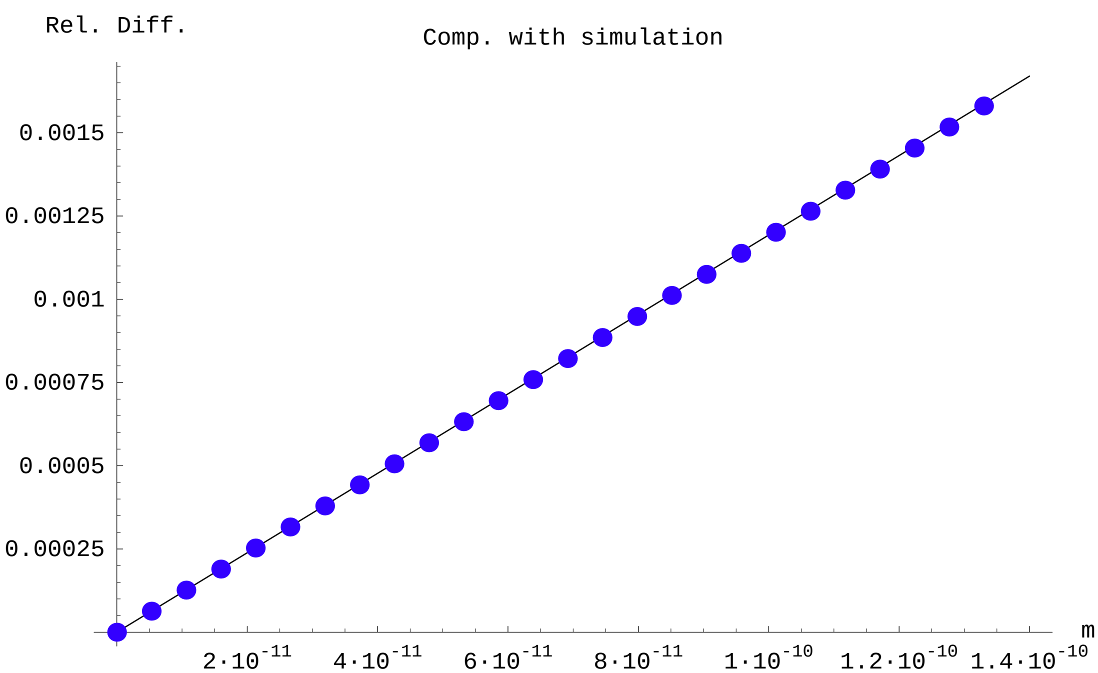
When the sidebands are not anti-resonant
a slight change of the length of the two arms
makes a difference in their recycling cavity power
although there is no other distortion and the
two Fabry-Perot cavities are completely identical.



Additional comments and conclusions

Sidebands exactly anti-resonant

This is the **optimal case** otherwise **significant consequences** are produced by a microscopic change of the lengths that makes the Carrier not exactly resonant.

Nice summary of effects

The Sidebands imbalance can be produced by not optimum built in configuration parameters although **microscopic offsets are automatically nulled** by servos so that the Carrier resonates in both the long arm cavities and the recycling one.

Necessary some specific measurements

Basically we would like to look for this "new" source of noise according to the model we have been constructing so far for simple geometrical distortions.