Eccentricity Distribution of Coalescing Black Hole Binaries Driven by the Kozai Mechanism in Globular Clusters

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Outline

- Overview
  - Globular clusters
  - Binary black hole (BH) mergers in GC
  - Mergers driven by Kozai mechanism
- Evolution of merger systems under Kozai mechanism & others
  - Eccentricities
  - GW Frequencies
- Implications to LIGO detections

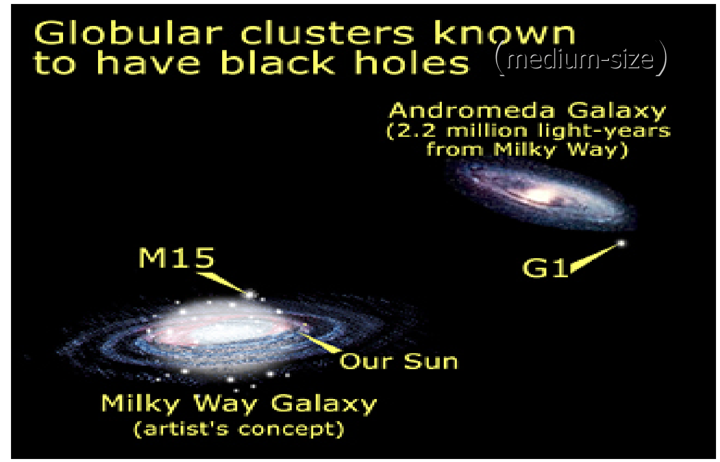
Globular Clusters

M_{BH}~4000 Msun 20,000 Msun

(stolen from the STScI web site)

Globular Clusters

- Very old star clusters
  e.g., ~12 billion yrs old
- N~10^6 objects
- All > 10 Msun stars evolves into BHs, NSs in ~10^7 yrs
- ~(6 e-4 N) > 20 Msun -> BHs
- Possible intermediate mass BHs in the Core
  * LIGO, LISA sources



M_{BH}~4000 Msun 20,000 Msun
(stolen and modified from the STScI web site)

Globular Clusters

- Excellent birthplace for BH binaries
  - after 10^7 yrs, > 20-25 Msun stars evolved into BHs :
  - BHs are the heaviest objects left

    2-body relaxation, energy equipartition, mass segregation:
  - BHs sink to the core
- BH binaries form in the core
- LIGO sources if merge within Hubble time

$$\tau_{\rm GR} \approx 6 \times 10^{10} \frac{(a/{\rm AU})^4 (\epsilon/0.01)^{3.5}}{(m_0+m_1) m_0 m_1/M_\odot{}^3} \ {\rm yr},$$

where $\epsilon = 1 - e^2$, m_0 and m_1 are the masses of each binary component.

- Binary Merger Time Scale

$$\tau_{\rm GR} \approx 6 \times 10^{10} \frac{(a/{\rm AU})^4 (\epsilon/0.01)^{3.5}}{(m_0+m_1)m_0 m_1/{M_\odot}^3} \quad {\rm yr},$$

* a: semi-major axis, $\varepsilon = 1 - e^2$, m_0, m_1: masses

* quadrupole approximation for gravitational radiation

* Need to reduce a or ε to have T_{GR} < Hubble time

$\frac{1)^{3.5}}{/M_\odot{}^3}$ yr,
he masses of

Central BH in Globular Clusters ?

- Binary-single interaction
  - throws out most BH binaries
  - ~ 8 % retained within lifetime
  - recoil velocity associated with hardening
  - major contribution to current BH-BH merger even rate
- Binary-binary interactions
  - produce hierarchical triple systems
  - ~ 20-50 %
- Kozai mechanism
  - drives inner binaries to extreme eccentricity
  - shortens T_{GR}
  - ~70 % inner binaries could merge successfully
    - » before interrupted by interactions with field stars
  - Subsequent merger ==> formation of Intermediate mass BHs (Miller & Hamilton 2002)

These merger systems are associated with extremely high eccentricities !

Why study eccentricities ?

- High e systems were not expected for LIGO detections
  - Gravitational radiation reaction is very efficient in circularizing the orbit
    - » e.g., Hulse Taylor NS-NS system
  - Current effort has been focused on GWs from circular orbits
- Circular templates might not be good enough for optimal detections of high -e systems
- Eccentricity distribution in these systems in LIGO band

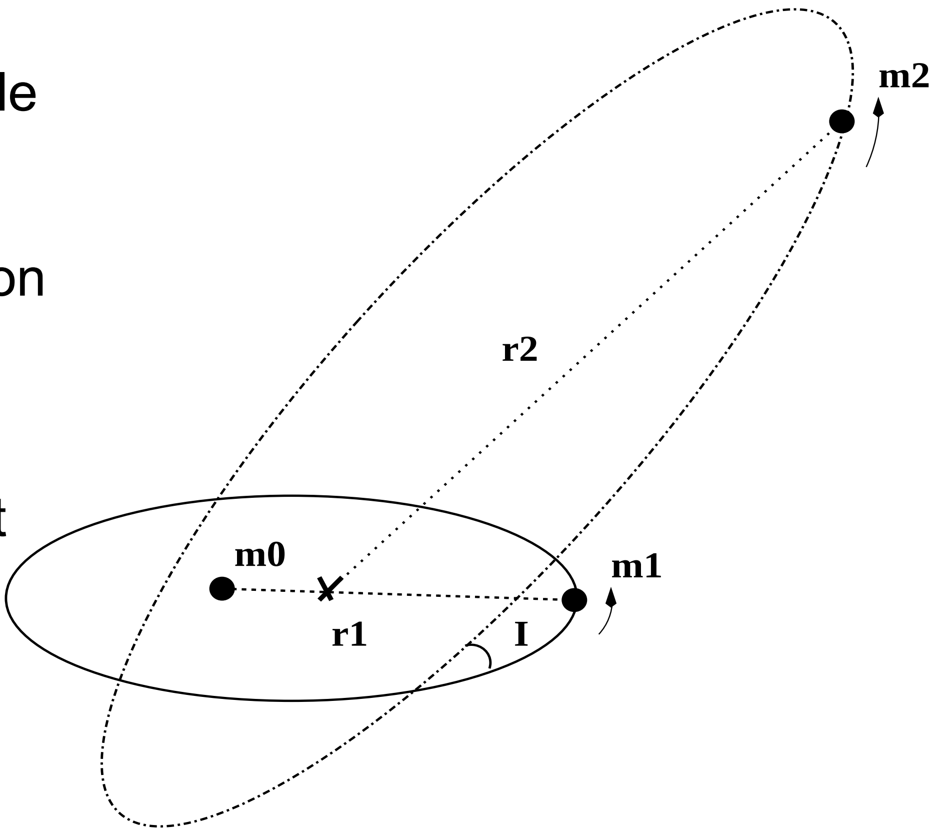
Procedure

- Study evolution of individual system
  - evolution of ε_{min}
  - $f^{m}_{GW} \sim (a\varepsilon)^{-1.5}$
- Find parameter space for successful mergers
  - consider a, e0, a1,a2,I0, g10, m's
  - merge before disrupted by a field star
- Derive eccentricity distribution in LIGO frequency band
  - f^{m}_{GW} =10, 40, 200 Hz

II. Evolution of Individual Triple System

Kozai Mechanism

- Operate in hierarchical Triple systems (r_2>> r_1)
  (Kozai 1962)
- Orbital averaged perturbation
  - $H=H_0 +H'_1 +H'_2$
  - Equations of motion
  - T_{evol} >> P_{orb}
- Oscillation of e_1 and g_1 w/ t
  - extreme e_1 for $I_0 \sim I_c$
  - e = 1 in classical limit
- a_1, a_2, e_2 fixed
  - ($E_{1,2,}$ $|J_2|$ conserved)
- Cyclic exchange of angular momentum between inner and outer binaries



$$\tau_{\rm GR} \approx 6\times 10^{10}\frac{(a/{\rm AU})^4(\epsilon/0.01)^{3.5}}{(m_0+m_1)m_0m_1/M_\odot^3}\ {\rm yr},$$

where $\epsilon = 1 - e^2$, m_0 and m_1 are the masses of each binary component.

$$\cos I = \frac{\alpha^2-\beta^2-\epsilon}{2\beta\sqrt{\epsilon}}.$$

Necessary condition for $e = 1$ ($\epsilon = 0$) is,

$$I_0 = I_c = \cos^{-1}\left(-\frac{\sqrt{\epsilon_0}}{2\beta}\right),$$

where $\beta = \mu_2/\mu_1\sqrt{(M_2a_2/M_1a_1)(1-e_2^2)}$ is the normalized magnitude of the angular momentum of the outer binary, α is the total angular momentum of the system with the same normalization.

- retrograde orbit
- $I_c \sim 90^\circ$

Three competing effects

- Kozai Mechanism
  - eccentricity enhancing
- Gravitational radiation reaction (GR effect)
  - extract energy and angular momentum
  - orbital decay
  - circularizing
  - important near ε_{min} , negligible otherwise
  - rapid transition once GR effect dominates
- Post-Newtonian periastron precession (PN effect)
  - mess up the phase relation
  - introduce fast oscillations to destroy Kozai cycle

PN Effect

First-order post-Newtonian approximation can be added to the doubly-averaged Hamiltonian as $H_{\rm PN} = -k\theta_{\rm PN}/\sqrt{\epsilon}$, where,

$$\theta_{\rm PN} = 8 \times 10^{-8} \frac{(M_1/M_\odot)^2}{m_2/M_\odot} \left(\frac{b_2}{a_1}\right)^3 \frac{1}{a_1/{\rm AU}},$$

and $k = 3Gm_0m_1m_2a_1^2/(8M_1b_2^3)$ is a quantity related to the evolution time scale.

Kozai mechanism + weak PN and GR effects predict

$$a_1 \propto \epsilon_{\min}^{-1}$$

The evolution of the parameters θ_{PN}, $\tau_{\rm evol}$, and $\tau_{\rm GR}$ with the decay of the orbit can then be summarized as follows.

$$\begin{aligned}\theta_{\rm PN} &\propto a_1^{-4} \propto \epsilon_{\min}^{4}\\ \tau_{\rm evol} &\propto a_1^{-3/2} \propto \epsilon_{\min}^{3/2}\\ \tau_{GR} &\propto a_1^{4}\epsilon_{\min}^{7/2} \propto \epsilon_{\min}^{-0.5}.\end{aligned}$$

The evolution equations govern the four parameters ϵ, g_1, a_1, and α_0 and are given by

$$\frac{d\epsilon}{dt} = -10\kappa_E a_1^{1.5}\sqrt{\epsilon}(1-\epsilon)(1-\cos^2 I)\sin(2g_1) + \frac{\kappa_G}{a_1^4}\frac{1-\epsilon}{\epsilon^{2.5}}(\frac{425}{304} - \frac{121}{304}\epsilon),$$

$$\frac{dg_1}{dt} = \kappa_E a_1^{1.5}\left\{\frac{1}{\sqrt{\epsilon}}\left[4\cos^2 I + (5\cos 2g_1 - 1)(\epsilon - \cos^2 I)\right] + \frac{\cos I}{\beta}\left[2 + (1-\epsilon)(3 - 5\cos 2g_1)\right] + \frac{\theta_{\rm PN}}{\epsilon}\right\},$$

$$\frac{da_1}{dt} = -\frac{6}{19}\frac{\kappa_G}{a_1^3}\frac{1}{\epsilon^{3.5}}(\frac{425}{96} - \frac{61}{16}\epsilon + \frac{37}{96}\epsilon^2),$$

$$\frac{d\alpha_0}{dt} = -\frac{3}{19}\frac{\kappa_G}{a_1^{3.5}}\frac{1}{\epsilon^2}(\frac{15}{8} - \frac{7}{8}\epsilon)\frac{\sqrt{a_1\epsilon} + \beta_0\cos^2 I}{\alpha_0},$$

* Lidov (1976)+ Peters (1964)
+PN effect (Miller & Hamilton 2002

* $K_G << K_E$

* $\Theta_{PN} \sim a^{-4}$

where $\theta_{\rm PN}$ is defined in equation (), and

$$\kappa_{\rm E} = 7.4554\times 10^{-8}(\frac{m_2}{M_1})^{0.5}\frac{(m_2/M_\odot)^{0.5}}{(a_2/{\rm AU})^3(1-e_2^2)^{1.5}}\frac{1}{{\rm AU}^{1.5}},$$

$$\kappa_{\rm E} = 7.4554\times 10^{-8}(\frac{m_2}{M_1})^{0.5}\frac{(m_2/M_\odot)^{0.5}}{(a_2/{\rm AU})^3(1-e_2^2)^{1.5}}\frac{1}{{\rm AU}^{1.5}},$$

$$\kappa_G = 7.8218\times 10^{-26}\frac{m_0 m_1 (M/M_\odot)}{M_\odot^3}{\rm AU}^4.$$

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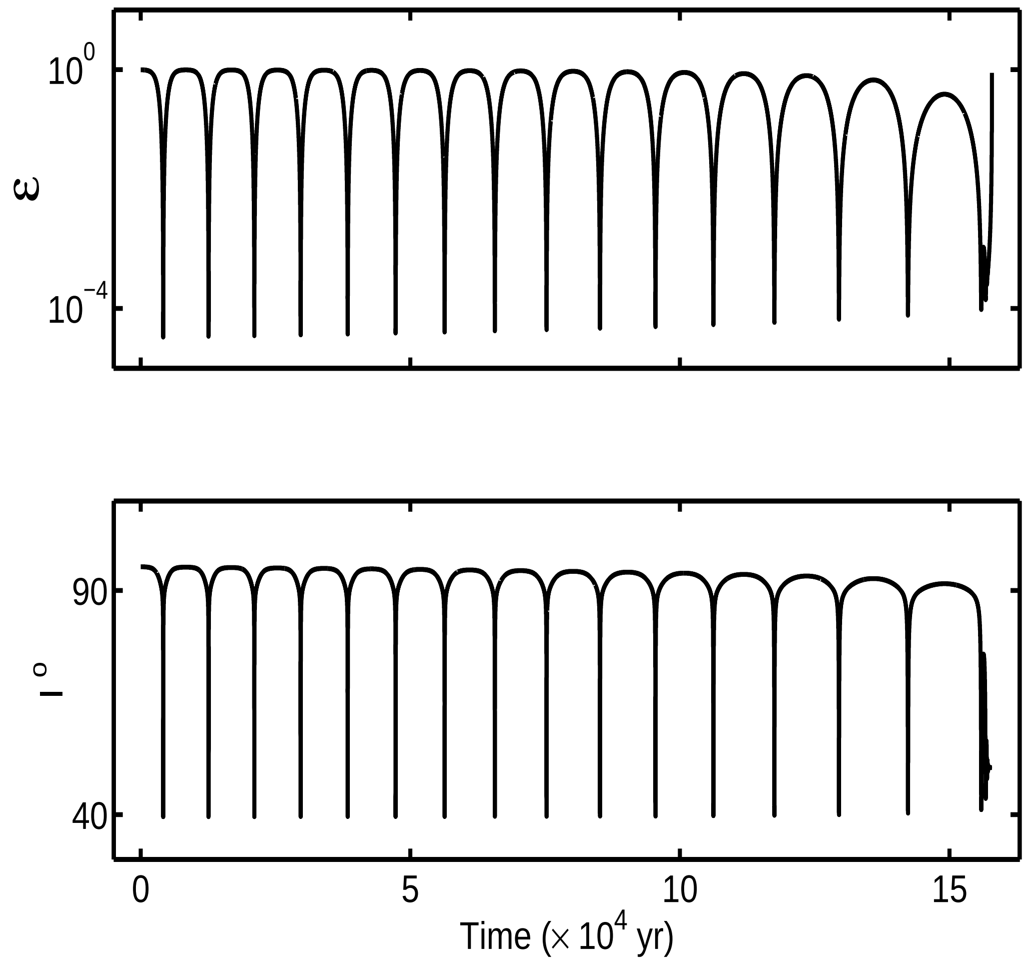
Estimate ε^0_{min}

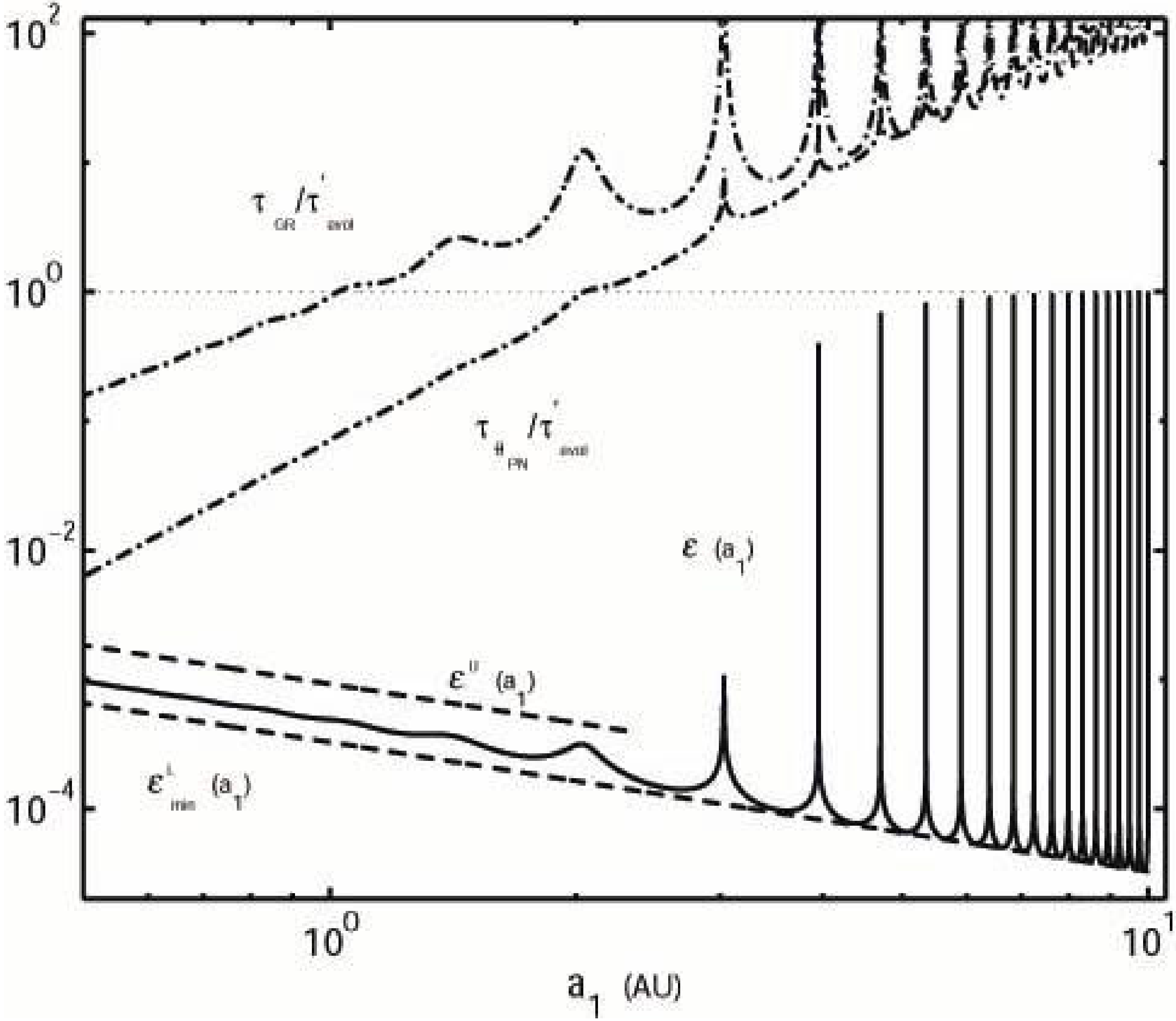
- evolution of f^m_{GW} depends on $a\varepsilon^0_{min}$
- $d\varepsilon/dt = 0$, solve equation implicitly
- assume energy conservation within one cycle
- initial guesses based on classical theory

I: Merge after many Kozai cycles

- Integrated from the ODE equations
- Typical case that the PN effects dominates before the GR effect
- System spends most time at low eccentricities
- Gradual change in the beginning
- Fast oscillations by PN effects
- GR effect dominates near ε_{min}
- Fast transition

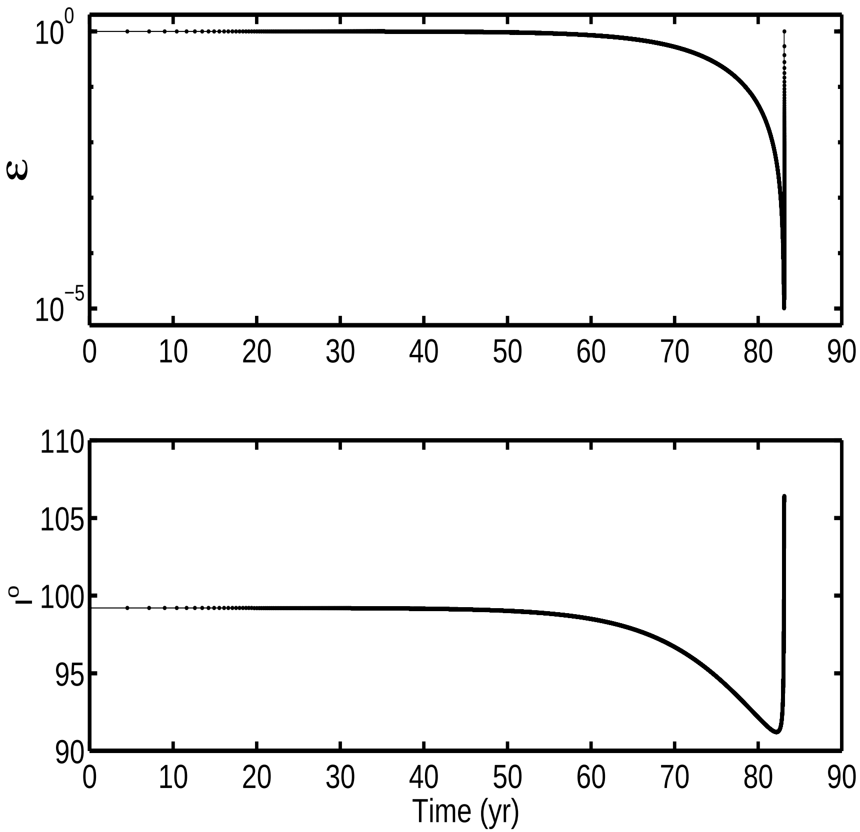


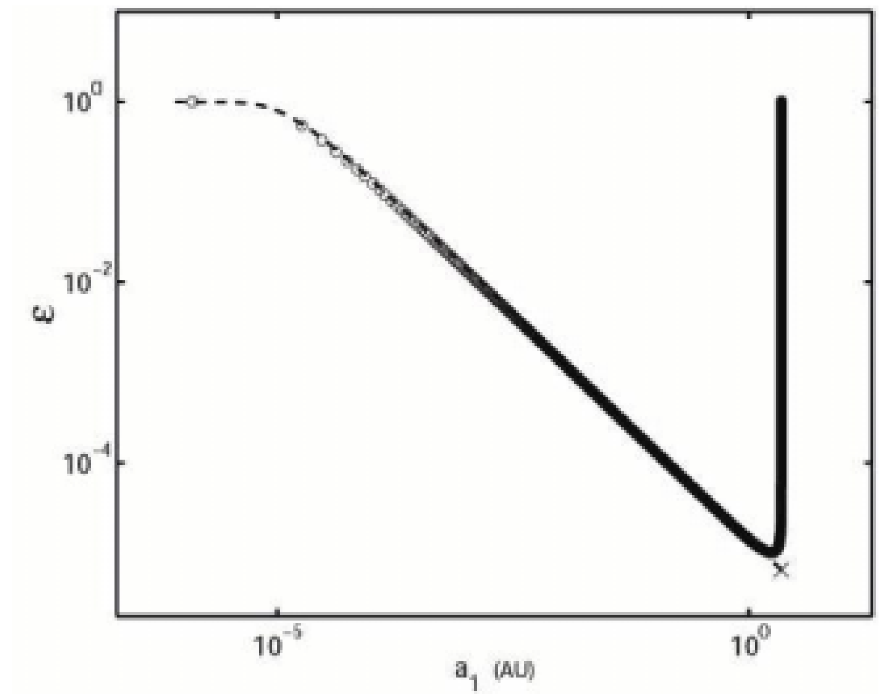
$* \varepsilon = 1 - e^2$



Dashed line: predicted lower and upper limits

II. Merge within one Kozai cycle





* $I_0 \sim I_c$
* x--- : predicted evolution
* typical case of extreme eccentricity

GW Frequency

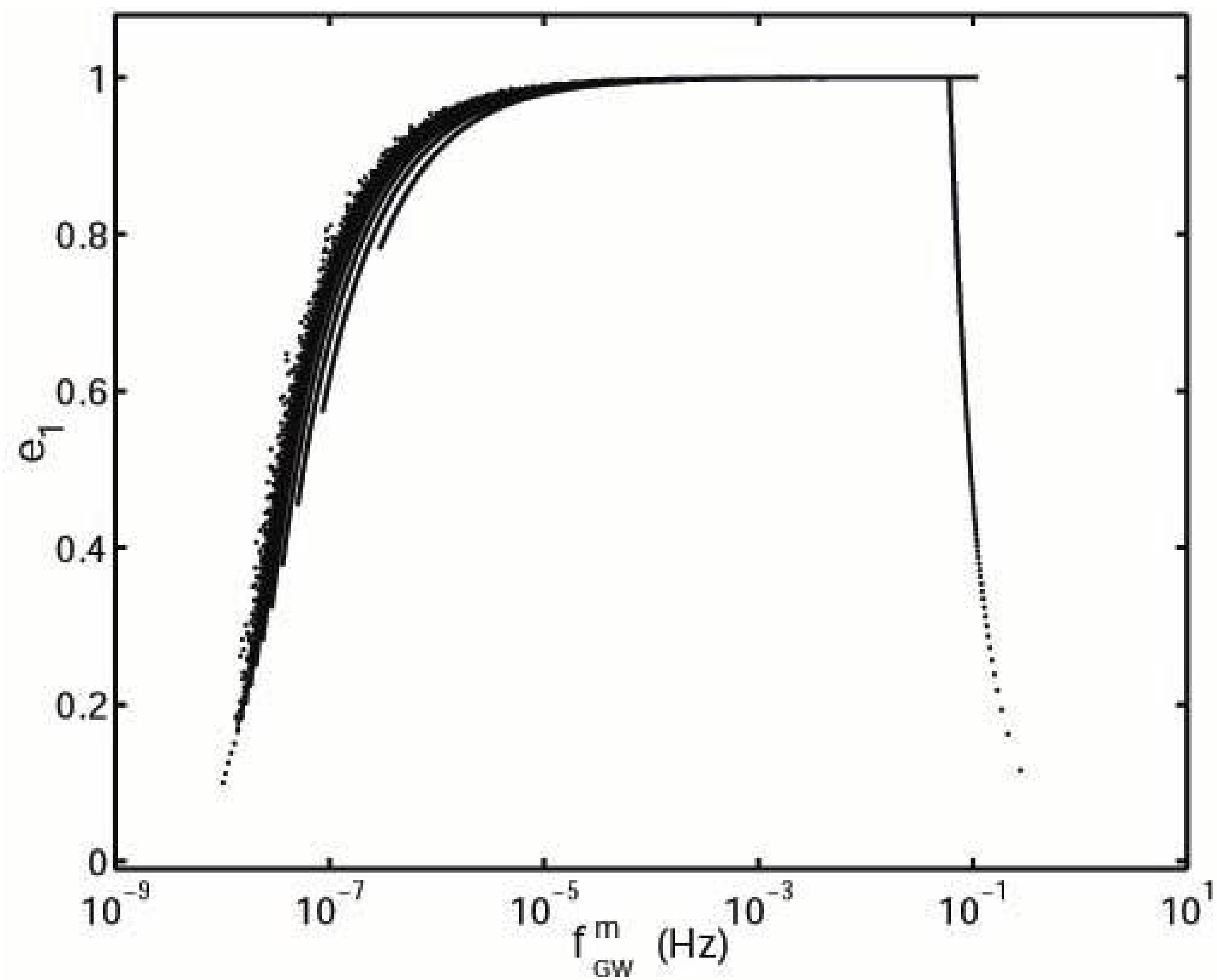
$f^{m}_{GW} = n_m f_{orb}$

$$f_{\rm orb} = \frac{\sqrt{GM_1}}{2\pi} a_1^{-1.5}.$$

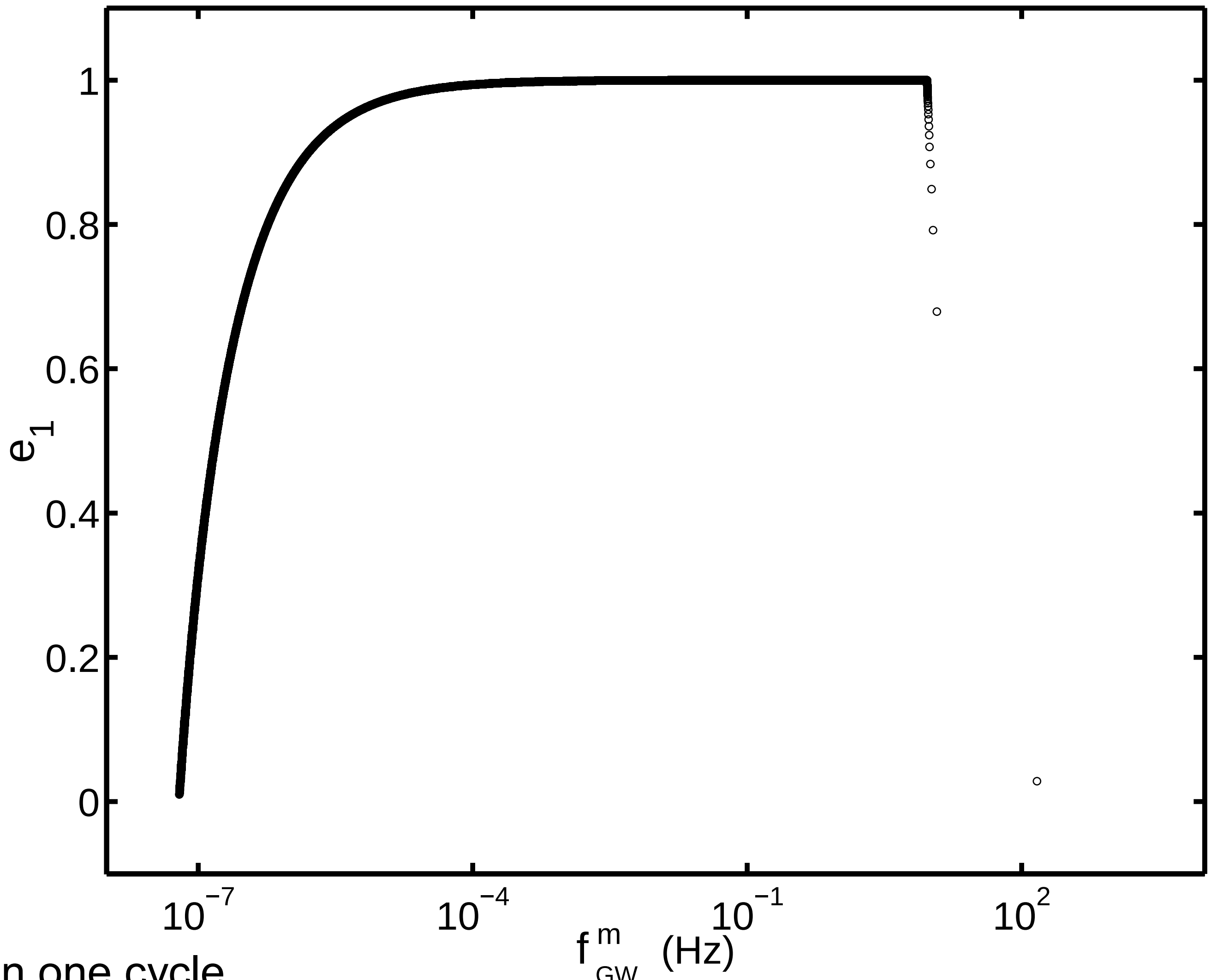
$$f^m_{\rm GW}(e_1) = \frac{\sqrt{GM_1}}{\pi}(1+e_1)^{1.1954}\frac{1}{(a_1\epsilon)^{1.5}},$$

$$a_1\epsilon = (a_{10}\epsilon^0_{\rm min})\left(\frac{e_1}{e_m}\right)^{12/19}\left(\frac{1+121/304 e_1^2}{1+121/304 e_m^2}\right)^{870/2299}.$$

- f^{m}_{GW} : peak GW frequency at maximum power
- its values depends on $a\varepsilon^{0}_{min}$
- $e \sim f^{m}_{GW}{}^{-19/18}$
- $n_m(e)$: peak harmonic, from Peters (1963)



- merge after many cycles
- e~1 at LISA band
- e~0 at 10 Hz



- merge within one cycle
- e ~1 in LISA band
- e ~0.9 at 10 Hz
- e ~0.2 at 40 Hz

III. Eccentricity Distribution

Requirement for Successful Mergers

* the system should have enough time to reach extreme eccentricity
* merge should occur before the system is disrupted by encounters with field stars

$$\tau_{\rm evol} < \tau_{\rm enc},$$

$$\frac{\tau_{\rm GR}(a_1, \epsilon_{\rm min})}{\sqrt{\epsilon_{\rm min}}} < \tau_{\rm enc}.$$

The time scale for disruption (the same as the stellar encounter time scale) is given as

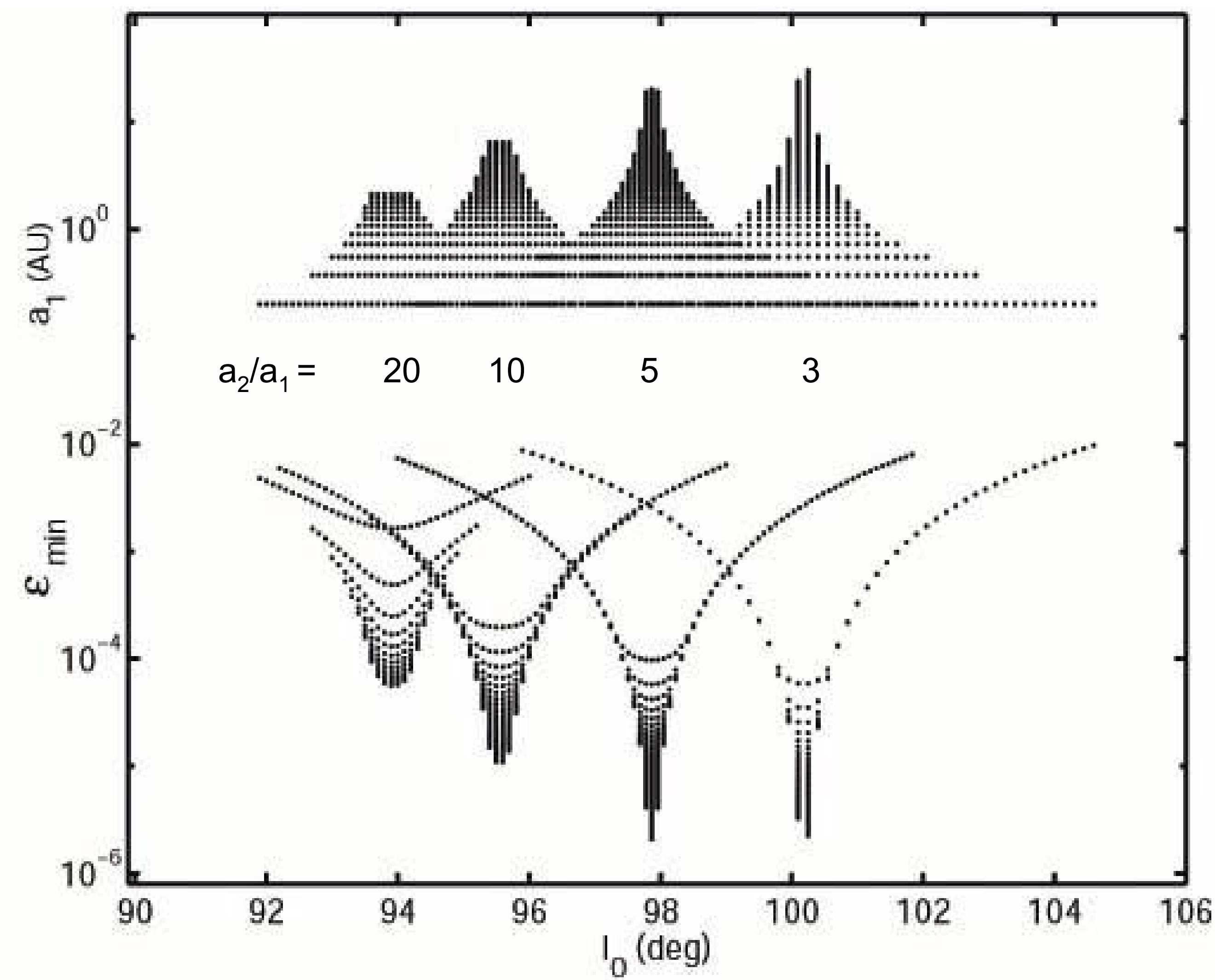
$$\tau_{\rm enc} \approx 6 \times 10^5 n_6^{-1} \frac{\rm AU}{a_2} \frac{10 M_\odot}{M_2}\ {\rm yr},$$

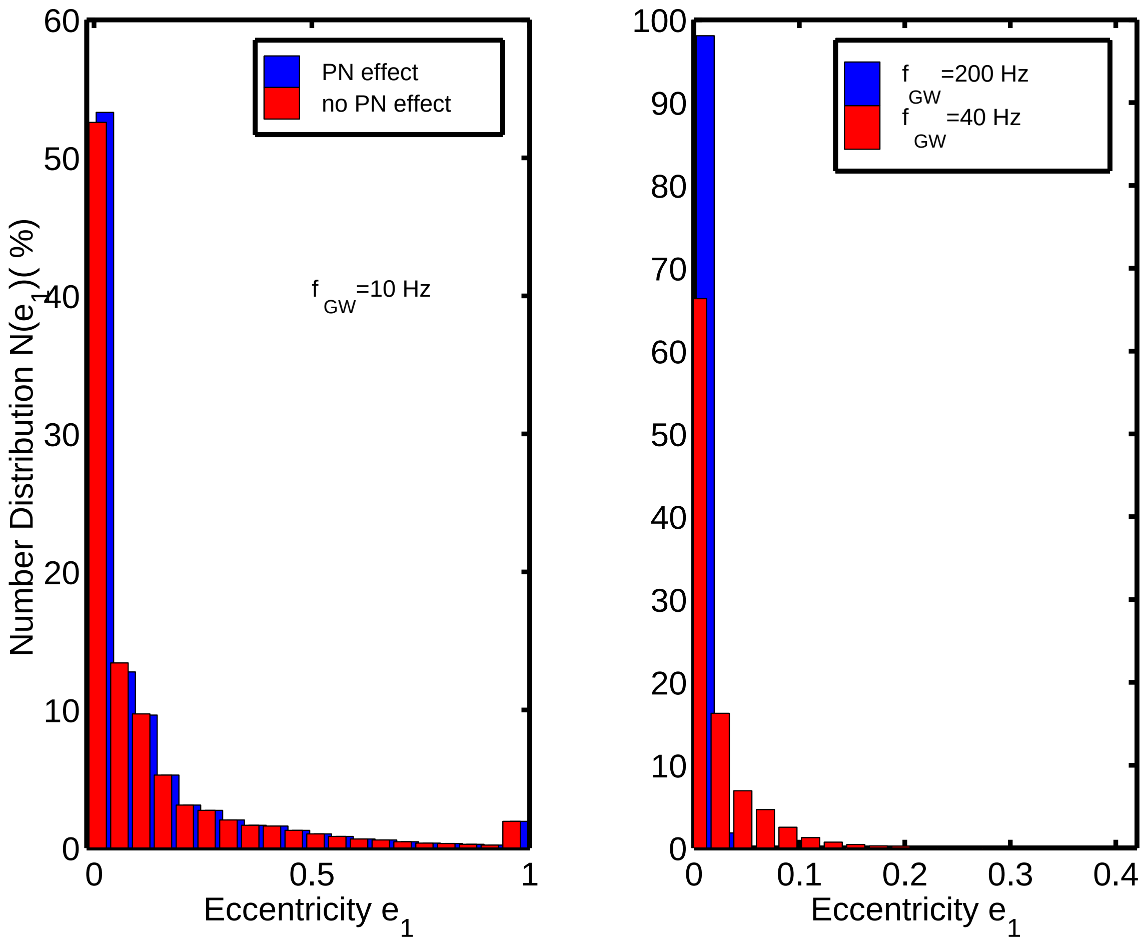
where the number of stars in the globular cluster is $N = 10^6 n_6$.

Parameter Space

- mass: all 10 M_{sun}
    - 3-20 Msun for known galactic stellar mass BHCs
- a_1 =0.2 - 30 AU
    - lower limit: not kicked out of GC
    - higer limit: disrupted by field stars
- a_2/a_1 =[3,5,10,20,30]
    - a_2/a_1 >3: required by the stability of the triple system
- e_{10},e_{20} =0.01-0.901
- g_{10}=0-90^o

Parameter Space





PN effect
no PN effect
f GW=10 Hz
Number Distribution N(e1)(%)
Eccentricity e1
f GW=200 Hz
f GW=40 Hz
Eccentricity e1

Conclusion

- At 10 Hz:
  $e > 0.1$: 30 %,
  $e \sim 1$: 2 %

  Eccentricity might be important for advanced LIGO

- At 40 Hz:
  $e<0.2$
- At 200 Hz:
  $e<0.04$

  * Consistent with $e \sim f^{m}_{GW}{}^{-19/18}$
  * Eccentricity is probably irrelevant for initial LIGO for this type of mergers.

$$\epsilon_{\rm min}^{1/2} \approx \frac{1}{2\Omega}\left\{\theta_{\rm PN} + \sqrt{\theta_{\rm PN}^2 + 20\Omega\epsilon_0\left[\cos I_0 + \sqrt{\epsilon_0}/(2\beta)\right]^2}\right\}.$$

$$\approx \frac{1}{2\Omega}\left[\theta_{\rm PN} + \sqrt{\theta_{\rm PN}^2 + 5\frac{\Omega}{a_1}\frac{(\alpha_0^2 - \beta_0^2)^2}{\beta_0^2}}\right].$$

Here

$$\Omega = 5-2\epsilon_0+\epsilon_0\cos^2 I_0+\frac{\theta_{\rm PN}}{\sqrt{\epsilon_0}}+4\epsilon_0(\cos I_0+\frac{\sqrt{\epsilon_0}}{2\beta})^2+5(1-\epsilon_0)(\cos^2 I_0-1)\sin^2(g_0).$$

The time scale for the system to swing from $e_1 \sim 0$ to $e_1 \sim 1$ is

$$\tau_{\rm evol} \approx 0.16 f \left(\frac{M_1}{m_2}\right)^{1/2}\left(\frac{a_2}{a_1}\right)^{3/2}\frac{(a_2/{\rm AU})^{3/2}}{(m_2/M_\odot)^{1/2}}(1-e_2^2)^{3/2}\ {\rm yr},$$

where $f \sim$ a few.

$$\tau_{\rm GR} \approx 6\times 10^{10}\frac{(a/{\rm AU})^4(\epsilon/0.01)^{3.5}}{(m_0+m_1)m_0 m_1/{M_\odot}^3}\quad {\rm yr},$$

where $\epsilon = 1 - e^2$, m_0 and m_1 are the masses of each binary component.