Spin-Orbit Resonance in Compact Binary LIGO Sources

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Outline

- Motivation
- Equations of motion, waveforms
- Characterizing spin-orbit/spin-spin resonances
- Resonance capture
- Consequences: template matching, upper limits
- Conclusions

We use Post-Newtonian equations of motion to calculate trajectories with spin effects. [1]

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_N + \vec{\mathbf{a}}_{1PN} + \vec{\mathbf{a}}_{2PN} + \vec{\mathbf{a}}_{2.5PN} + \vec{\mathbf{a}}_{SO} + \vec{\mathbf{a}}_{SS}$$

The spins also precess due to the Lens-Thirring effect:

$$\dot{\vec{\mathbf{S}}}_{\mathbf{1}} = \vec{\boldsymbol{\Omega}}_{\mathbf{1}} \times \vec{\mathbf{S}}_{\mathbf{1}} \qquad \dot{\vec{\mathbf{S}}}_{\mathbf{2}} = \vec{\boldsymbol{\Omega}}_{\mathbf{2}} \times \vec{\mathbf{S}}_{\mathbf{2}}$$

where

$$\vec{\boldsymbol{\Omega}}_{\mathbf{1}} \equiv \frac{1}{r^3}\left[\left(2 + \frac{3m_2}{2m_1}\right)\vec{\mathbf{L}}_{\mathbf{N}} - \vec{\mathbf{S}}_{\mathbf{2}} + 3(\hat{\mathbf{n}} \cdot \vec{\mathbf{S}}_{\mathbf{2}})\hat{\mathbf{n}}\right]$$

and

$$\vec{\boldsymbol{\Omega}}_{\mathbf{2}} \equiv \frac{1}{r^3}\left[\left(2 + \frac{3m_1}{2m_2}\right)\vec{\mathbf{L}}_{\mathbf{N}} - \vec{\mathbf{S}}_{\mathbf{1}} + 3(\hat{\mathbf{n}} \cdot \vec{\mathbf{S}}_{\mathbf{1}})\hat{\mathbf{n}}\right]$$

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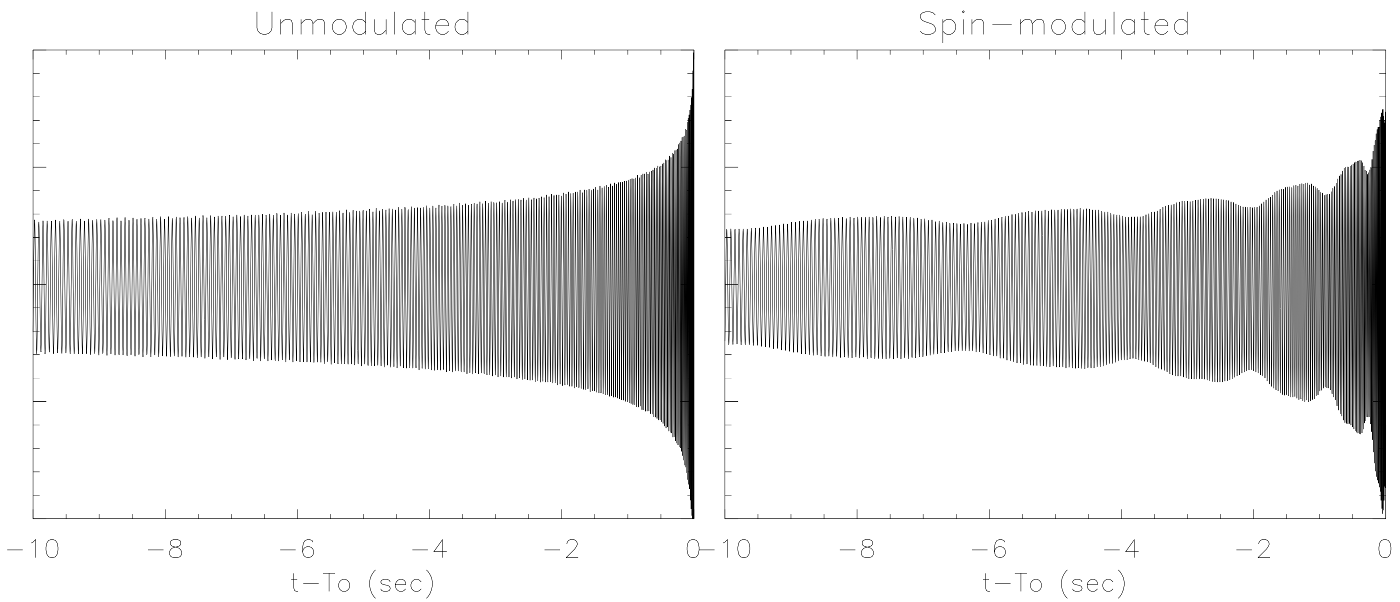
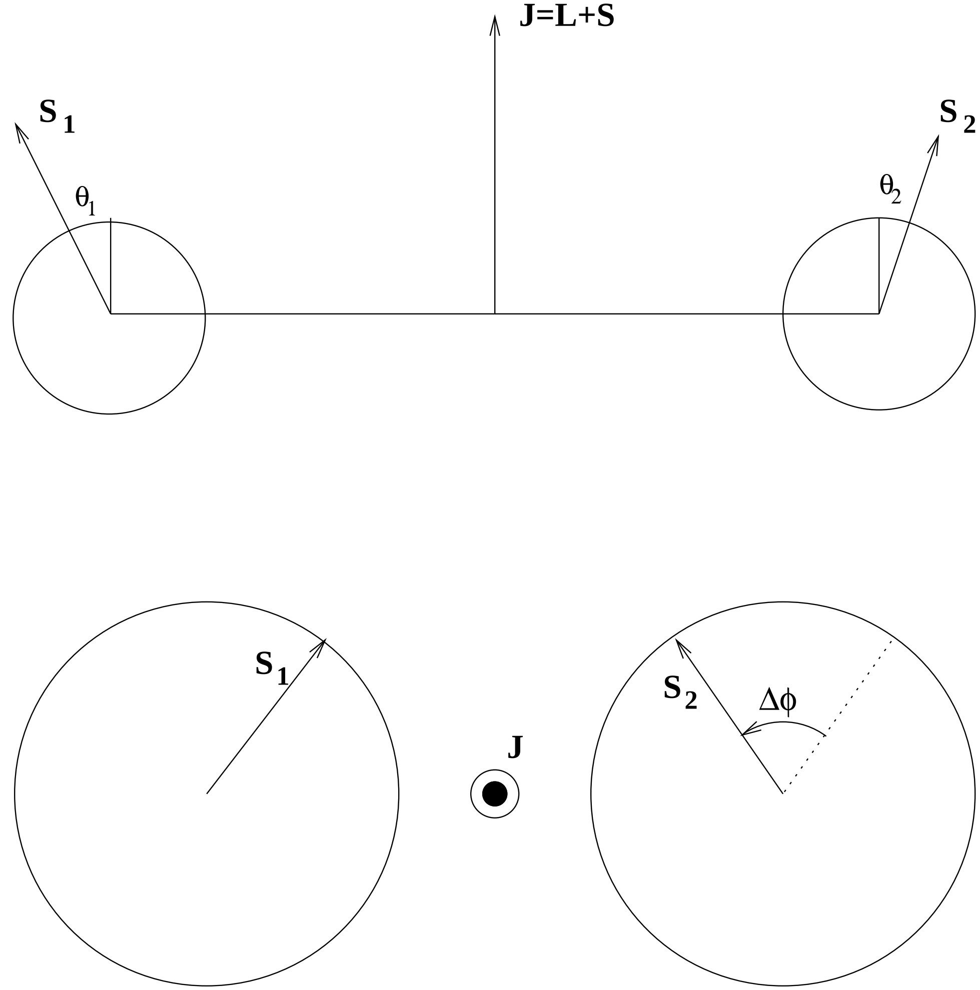
[1] L. E. Kidder, C. M. Will, and A. G. Wiseman, *Phys. Rev. D* **47**, R4183 (1993).

When the spins are not parallel to the orbital angular momentum, the orbital plane precesses, modulating the gravitational waveform and decreasing chance of detection. [2]

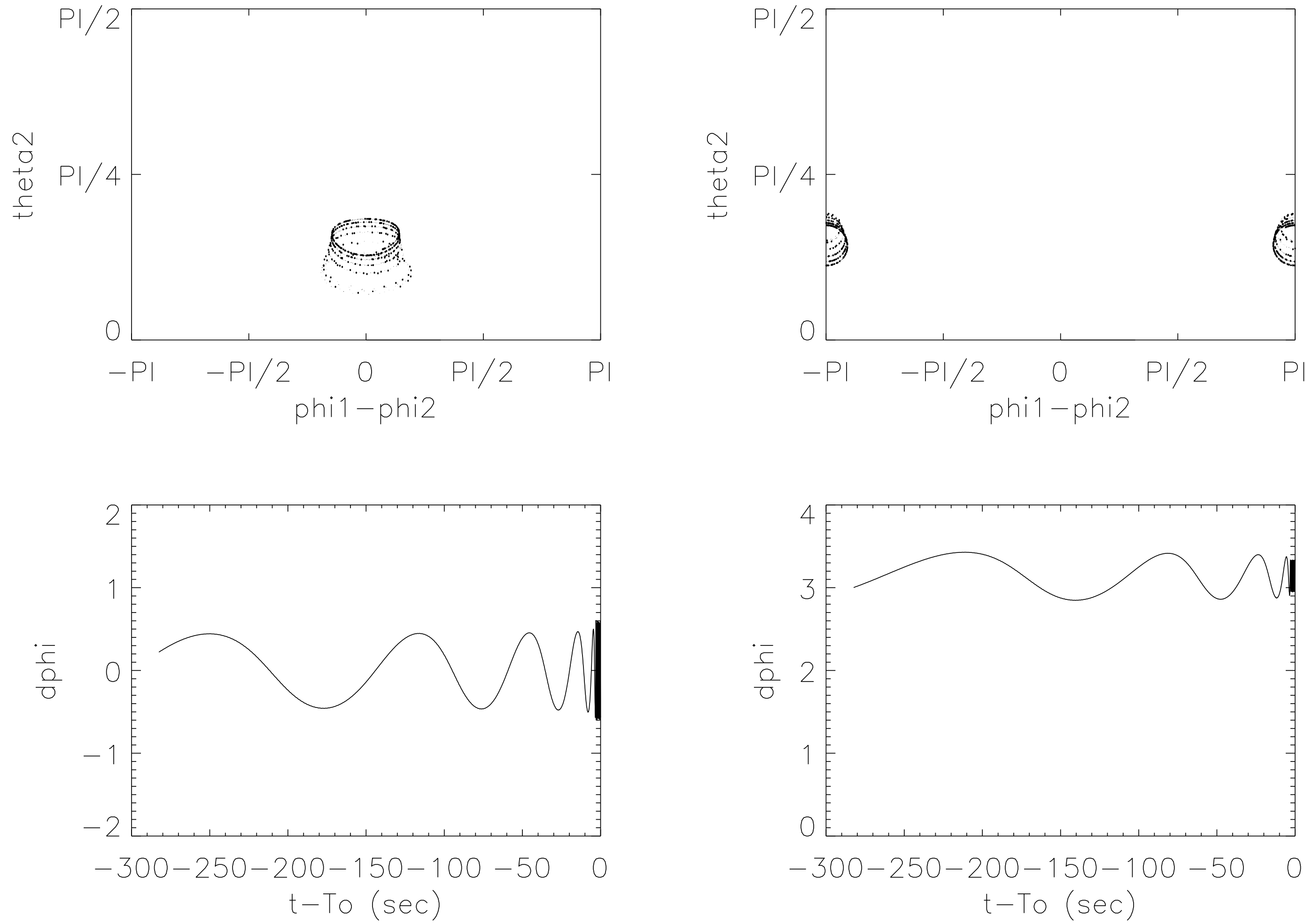


[2] T. A. Apostolatos *et. al.*, *Phys. Rev. D* **49**, 6274 (1994); T. A. Apostolatos, *Phys. Rev. D* **52**, 605 (1995).

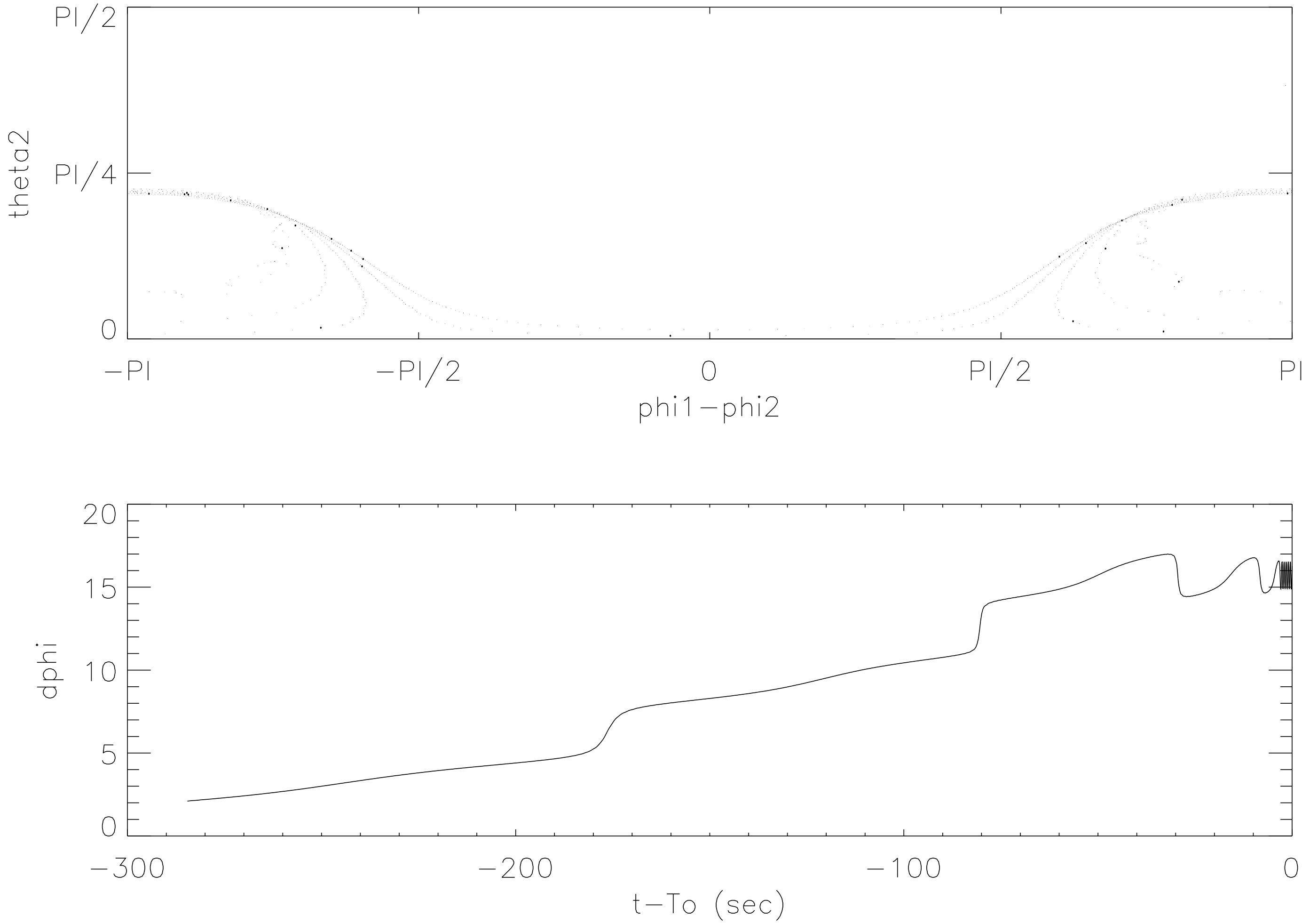
The spin angles are measured with respect to the total angular momentum vector $\vec{\mathbf{J}}$



To analyze the spin-orbital/spin-spin resonances, we follow the trajectory of the relative angles between the spin vectors



The evolution of the orbit due to radiation reaction can cause the binary to be captured into a resonance configuration.



Based on the binary separation, we estimate the number of orbits from resonance capture until merger.

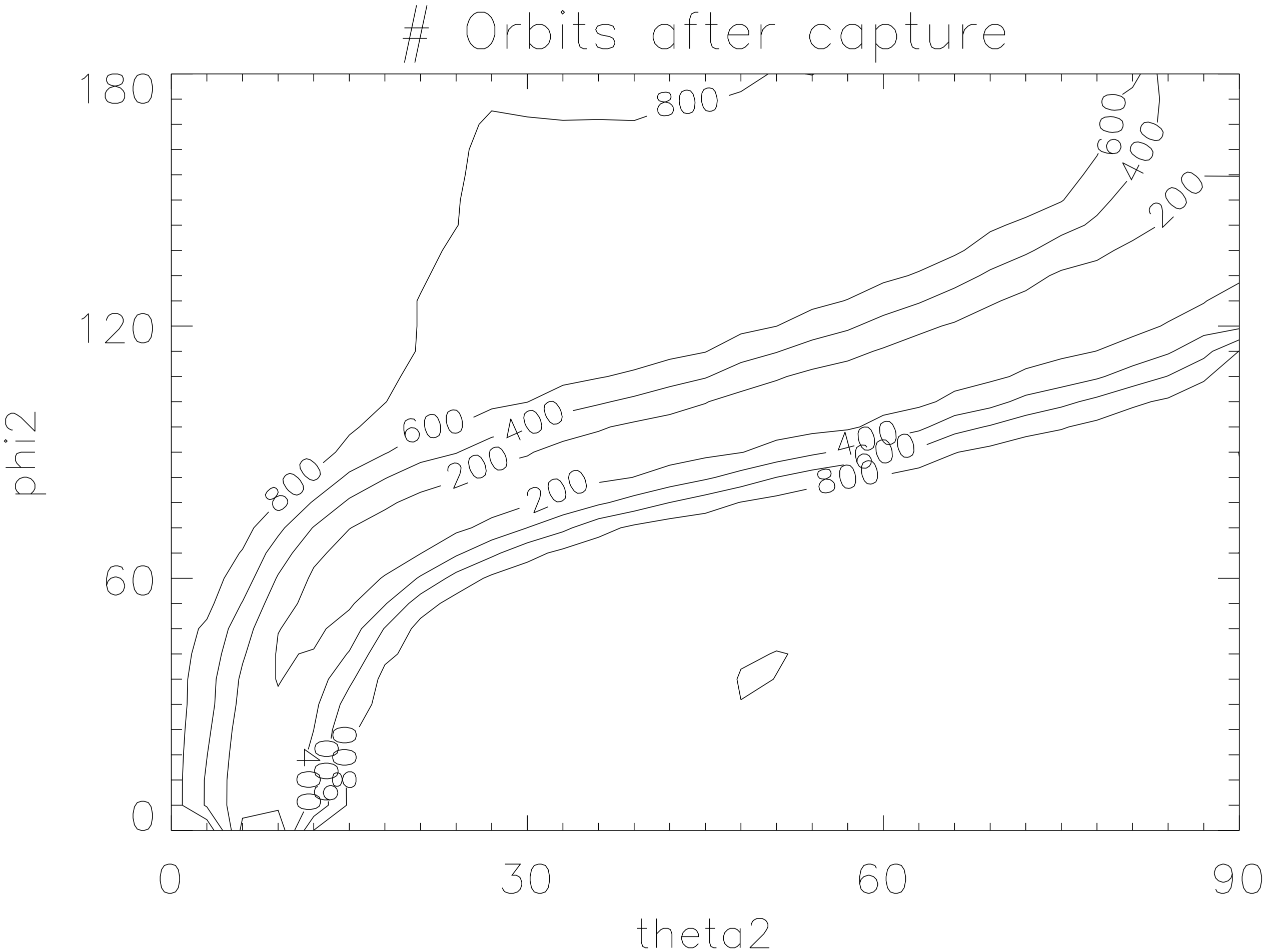
$$\#orbits(r \to 0) = \frac{r^{5/2}}{64\pi \mu m^{3/2}}$$

For the LIGO frequency range of 10-400 Hz, the corresponding range of binary separation is

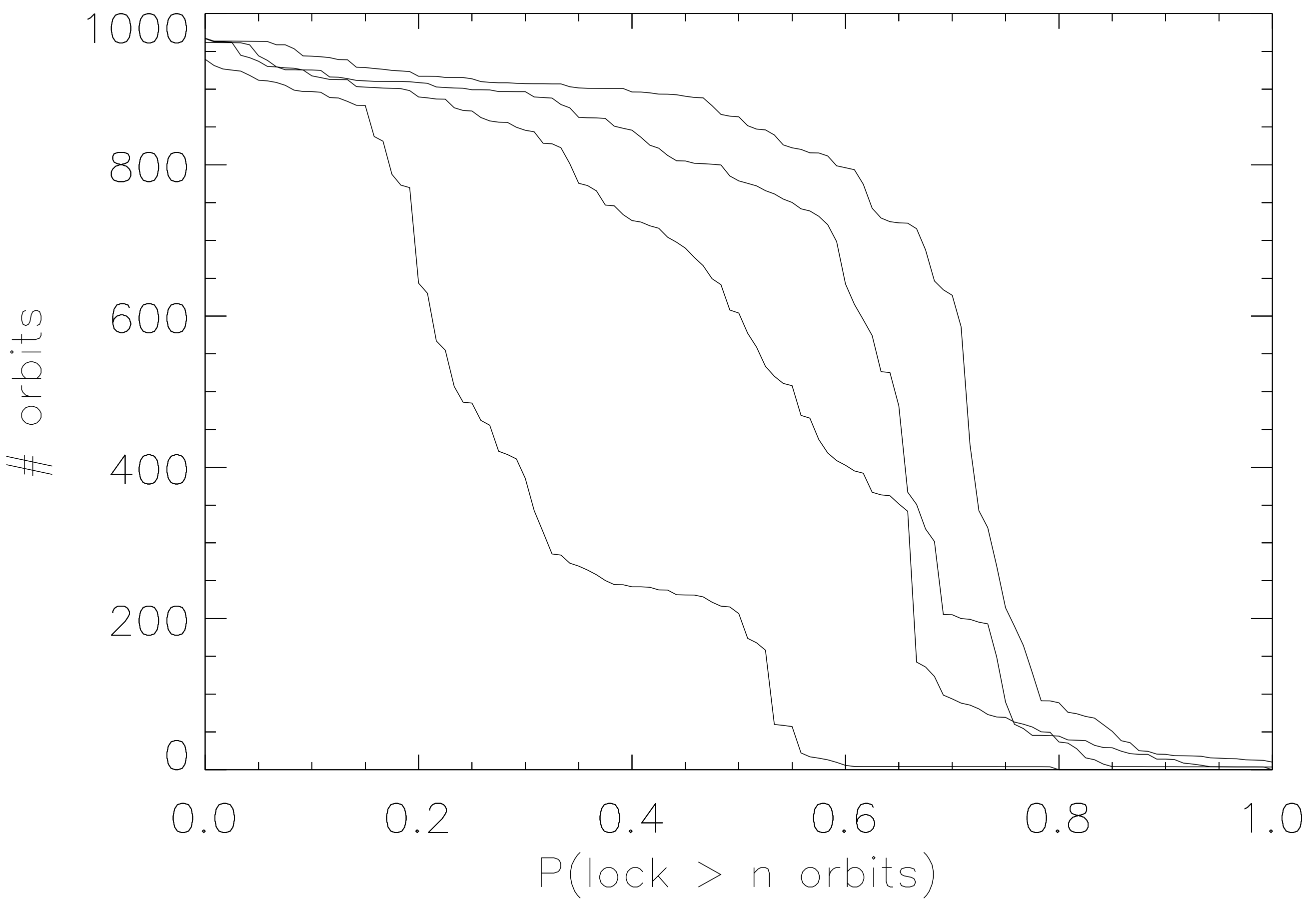
$$m = 20M_\odot \quad : \quad r/m = 50 \to 4 \quad (\sim 350 \text{ orbits})$$

$$m = 10M_\odot \quad : \quad r/m = 75 \to 6 \quad (\sim 1000 \text{ orbits})$$

For nearly equal masses, a large fraction of binary systems should be in resonance lock within the LIGO frequency band.



For a random distribution of spin orientations, the resonance lock is more likely for rapidly spinning black holes



Conclusions and future work

- We integrate the PN equations of motion for binary inspiral events including Lens-Thirring precession.
- The modulated waveforms may be undetectable with current unmodulated templates.
- By identifying spin-orbit resonances, we might expand the template library in a few small regions of parameter space while significantly increasing the ability to detect more inspiral events.
- By calculating the resulting fitting factor (FF) of the new template library, a better estimate of upper limits should be available for the early LIGO data.