Beyond the PSD: Discovering hidden nonlinearity

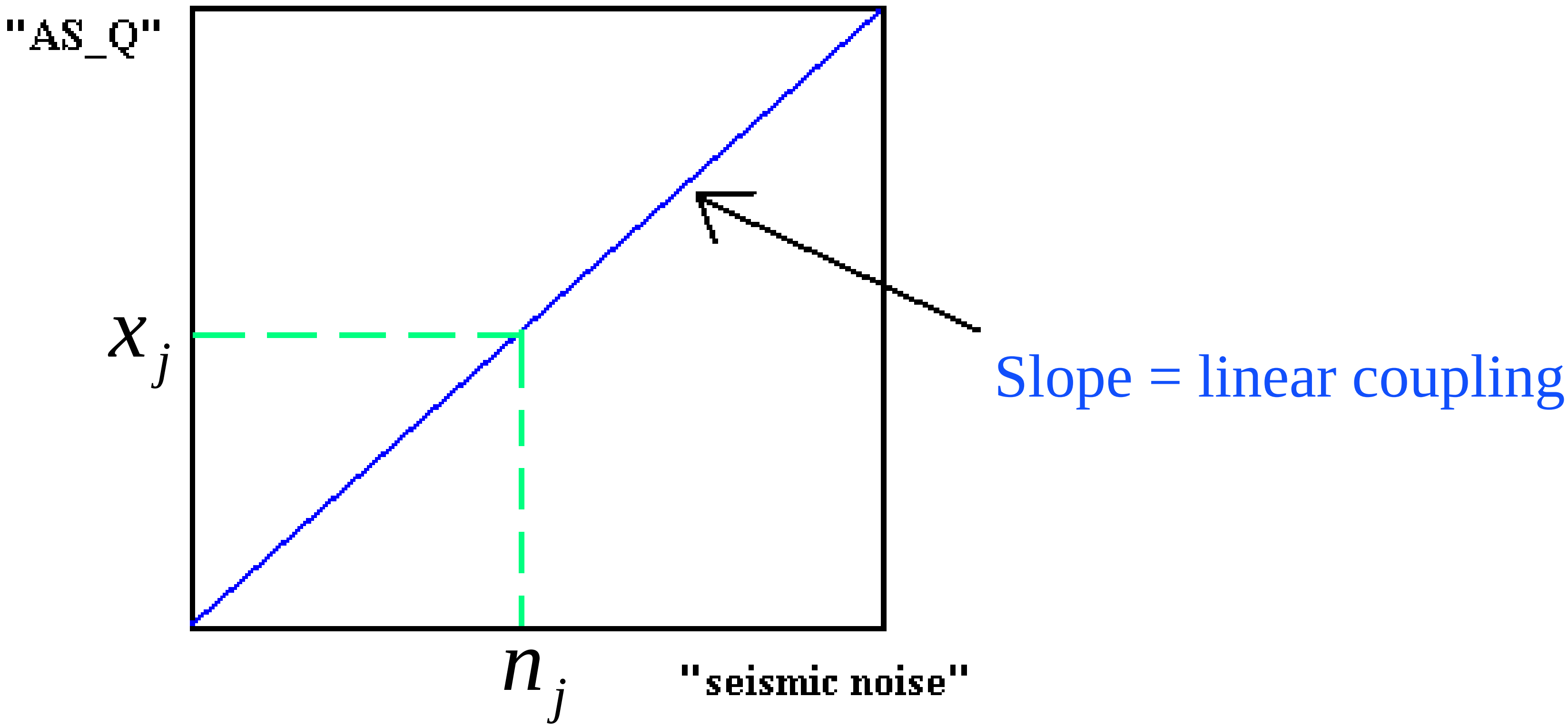
Tiffany Summerscales

Penn State University

LIGO-G020309-00-Z

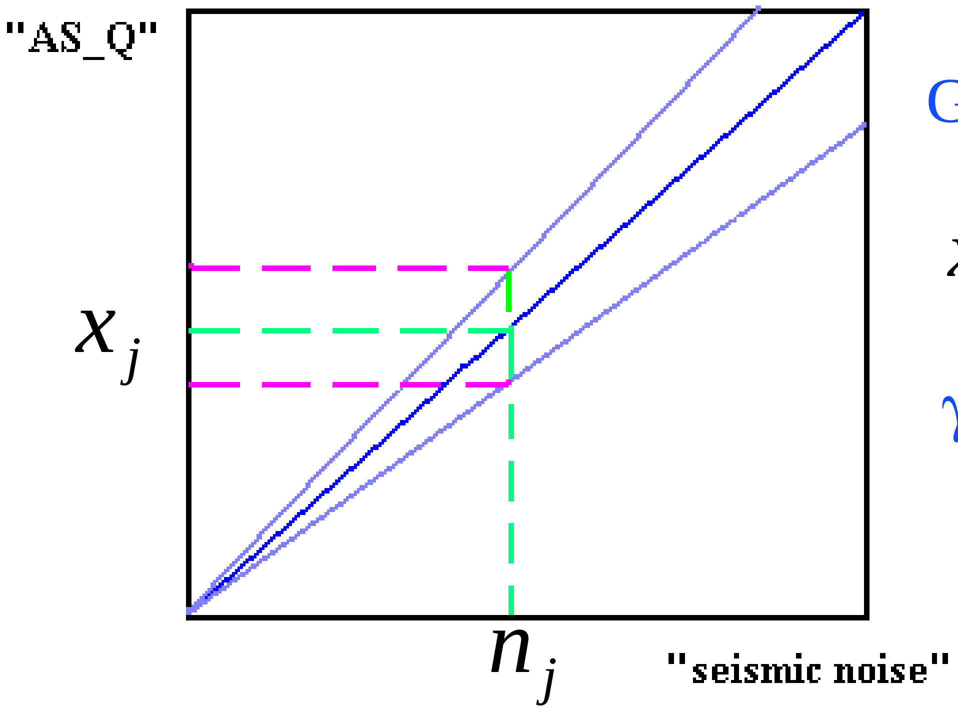
Coupling in LIGO data

Noise may couple to signal linearly . . .



Coupling in LIGO data

Or non-linearly: e.g., hysteresis



Gain = $(1+g\, n_{j-k})$

$$x_j = n_j(1+g\, n_{j-k})$$

γ: nonlinearity parameter

Coupling in LIGO data

- Model: $x_j = n_j(1 + g n_{j-k})$
- Suppose n_j white: what is autocorrelation (PSD) of x_j?
  - » C(l) = $< x_j x_{j-l} > \; = \; < n_j(1+gn_{j-k})\, n_{j-l}(1+gn_{j-k-l})> \; = \; (1+\gamma^2)\delta_{l,0}$
  - » x are white!
- Conclusion: PSD inadequate tool for discovering non-linear couplings
- Question: How to discover non-linear couplings?

Non-linear couplings lead to correlations in time

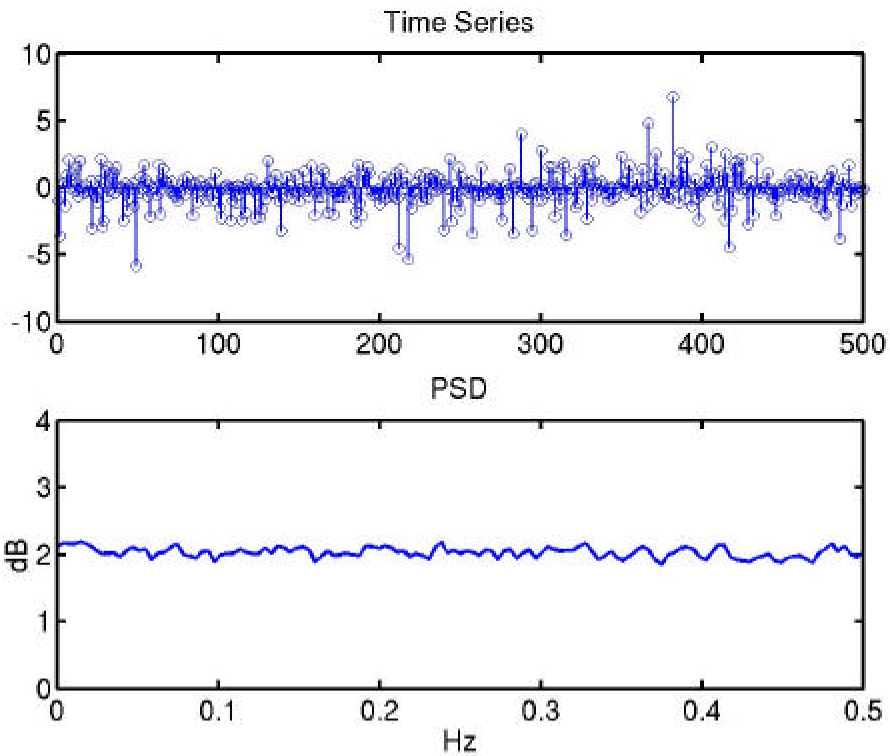
- Correlations still present, just non-linear and, so, hidden from linear tools
  - » $x_j=n_j(1+gn_{j-k})$: Signal now depends on noise now, and noise earlier
- Uncorrelated signals lead to Poisson distributed events
  - » Event? Sample above a threshold
- Correlated signals lead to non-Poissonian distribution
  - » Clustering or anti-clustering in time
- Discovery tool: test for Poisson distribution of event data

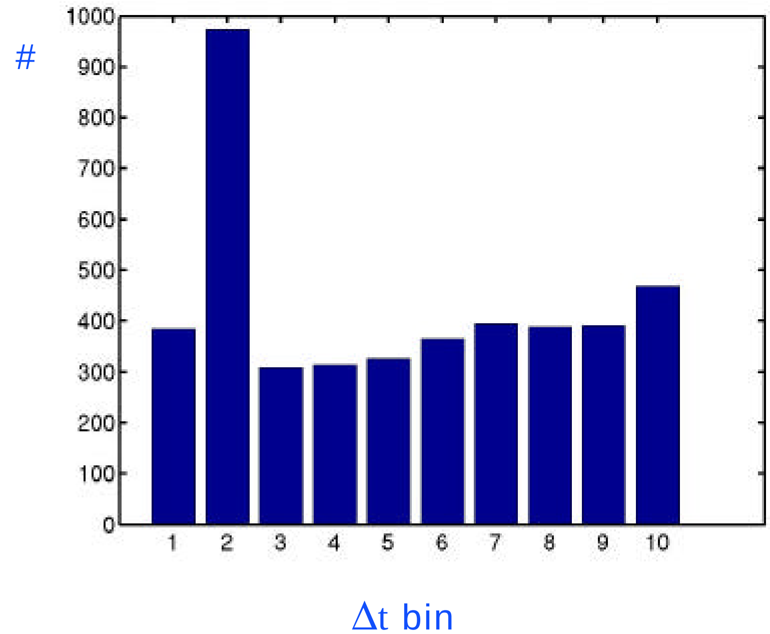
Test for non-linear coupling

- Identify events
  - » Set a threshold and classify above-threshold data points as "events"
    - – events for non-correlated data will be Poisson distributed in time
    - – events for data with correlations will be "bunched" and not be Poisson distributed
- Test for Poisson distribution
  - » Poisson distribution of events in interval T equivalent to exponential distribution of interval Dt between successive events
  - » Bin intervals between events
    - – Find mean rate
    - – Choose bins with exponentially increasing width so that - for Poisson data - expected number of events in each bin is same
  - » Evaluate χ^2 fit to exponential distribution
    - – Degrees of freedom ν? Number of bins less 2 (loose one d.o.f. because we calculated mean rate from data)
  - » Non-linear coupling? χ^2/ν statistically different from unity

Example

(k=5, γ=1)





No apparent evidence of nonlinearity

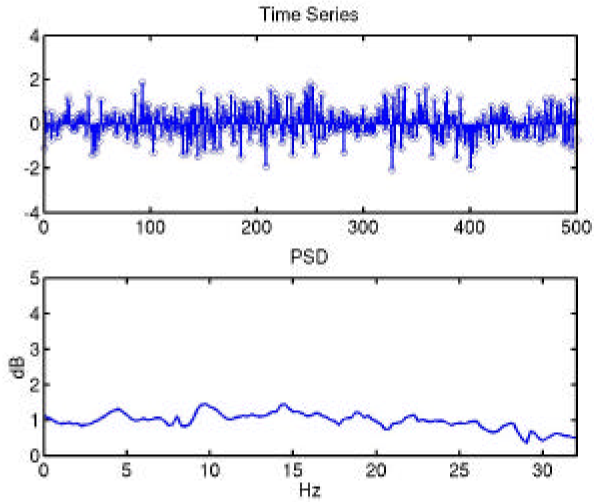
$\chi^2/\nu = 16.9 >> 1$!

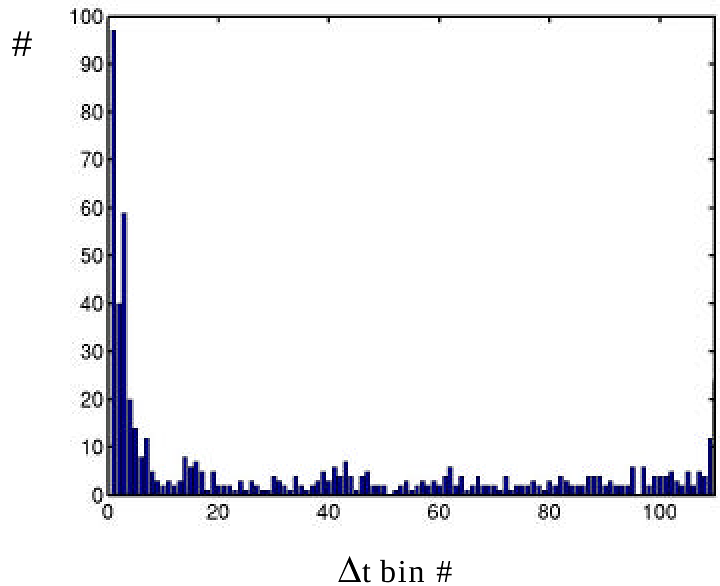
Nonlinearity clearly apparent

LIGO-G020309-00-Z

Application: E7 Playground

- H2:LSC-AS_Q in E7 playground data
  - » GPS start time: 694271952
  - » Duration: 1600s
- Demodulate, downsample, whiten to 20-84 Hz band





- $\chi^2/\nu=22.1$
  - » Non-linear correlation detected
  - » Use χ^2 to set CI on γ:

    $g \in [0.9, 2.1]$ with 90% confidence

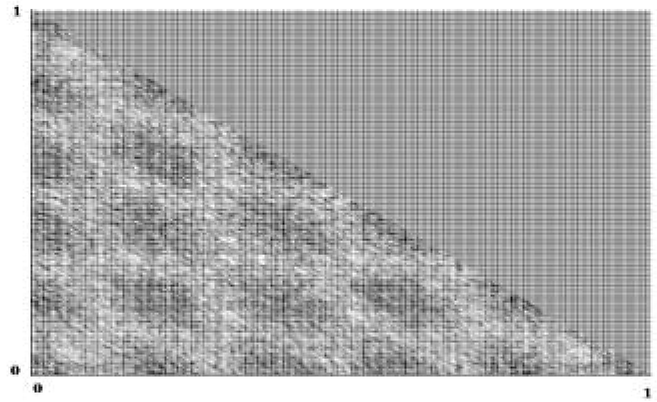
    $g \in [0.8, 2.8]$ with 98% confidence

LIGO-G020309-00-Z

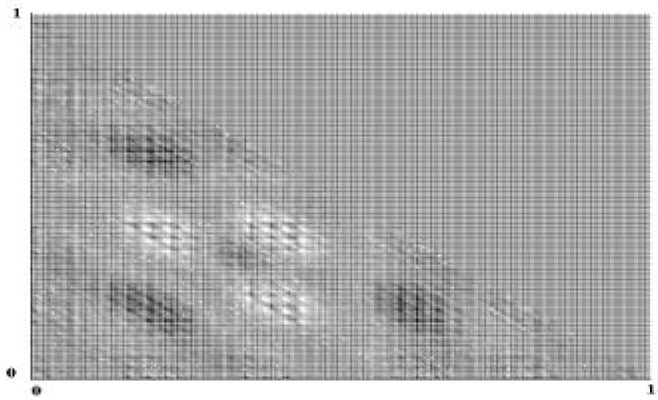
Bispectra

- Power Spectral Density:
  » $|X(\omega)|^2$
- Bispectrum:
  » $X(\omega_1)\ X(\omega_2)\ X(\omega_1+\omega_2)$
- Nonlinearity apparent in bispectra; however …
  » bispectra computationally expensive and difficult to interpret
  » χ^2 test inexpensive and simple to interpret
  ≈ χ^2 test sensitive to *any* non-linear coupling hysterisis-type correlation
    – bispectra zero for large γ in model problem

Model: k=5, γ=1



H2:LSC-AS_Q playground data



What's Next?

- Investigate S1 data
  - » Study data in sub-bands looking for frequency dependent non-couplings
- DMT Monitor
  - » Currently matlab tool