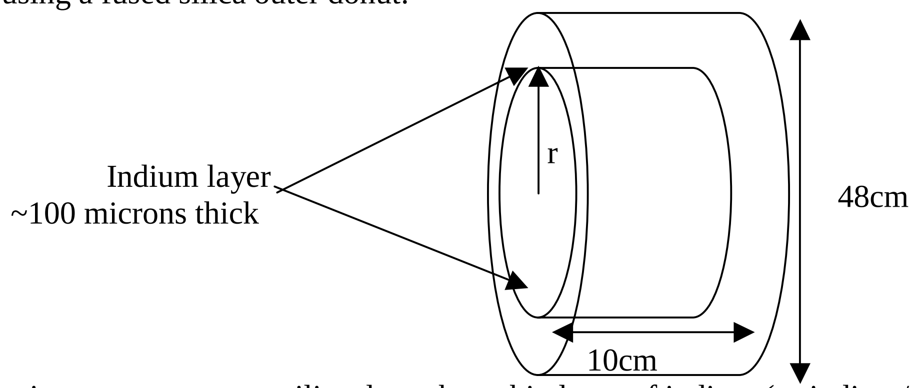
Thermal noise estimate for a compound test mass for Advanced LIGO using a fused silica or heavy glass cradle to hold a LIGO I silica test mass.

Simplest cradle to consider is the one that Dennis and I have both looked at – a "donut" shaped cylinder into which would fit a LIGO I size test mass. Total mass of this compound mass should be 40kg.

First consider using a fused silica outer donut:



Imagine joining inner mass to outer silica donut by a thin layer of indium (or indium/tin).

Total thermal noise in this combined test mass comes from adding in quadrature:

(a) the thermal noise from a 40kg monolithic silica test mass of the dimensions above assuming a homogeneous loss and a LIGO I size laser beam and

(b) the expected thermal noise due to the inhomogeneous loss associated with the layer of indium

(a) Thermal noise in a 40kg fused silica test mass

Using equation (59) in Liu and Thorne, the thermal noise in a half-infinite silica mass can be estimated. Using the following parameters:

ω_0 = 4.57cm ($\omega_0 = \sqrt{2}\, r_0$) σ = 0.17, E = 7.2e10 N/m^2, w = 2πf, f=10Hz, T = 290K

$\phi(w) = 1 \times 10^{-7}$

Gives: $x_{\text{Half infinite fused silica}}(10\text{Hz}) = 6.5 \times 10^{-20}$ m/$\sqrt{\text{Hz}}$

Using code from Marty Fejer which evaluates the Liu and Thorne correction factor for a finite size test mass of given aspect ratio and for appropriate beam size, the figure above becomes: $x_{\text{finite fused silica}}(10\text{Hz}) = 8.5 \times 10^{-20}$ m/$\sqrt{\text{Hz}}$

(b) Thermal noise from inhomogeneous loss associated with the indium layer

When there are inhomogeneous losses, far from the Gaussian beam spot, the thermal noise from these losses comes predominantly from the distortions of the mass caused by "inertial" forces. These are the distributed body forces whose resultant is equal and opposite to the resultant of the test Gaussian force applied to the front of the mass in Levin-type direct thermal noise calculations. Their existence is a way to get rid of the center of mass acceleration that would otherwise result from the unbalanced Gaussian pressure.

Calculations from Marty (following Liu and Thorne) suggest it seems possible to formulate fairly simple analytical expressions which allow estimation of the magnitude of the thermal noise resulting from inhomogeneous losses far from the Gaussian beam (eg: a lossy layer covering the barrel of a test mass). There is a draft of a publication in which this is worked through, but it isn't quite in shape for distribution yet. However, the calculation of the pure inertial terms is one of the simpler parts, and in some subset of test cases agrees reasonably with numerical results from K. Yamamoto's PhD thesis on inhomogeneous loss – see below**.

For our case - silica cradle:

For losses in a uniform skin of thickness $\delta \ll a$, some distance, $r > \omega_0$, from the center of the test mass the power spectral density of thermal noise, $S_{\text{lossy layer}}(f)$, normalized to the PSD for a half infinite mass $S_{\text{half infinite}}(f)$ is given by:

$$\frac{S_{\text{lossy layer}}(f)}{S_{\text{half infinite}}(f)} = \frac{2}{3\sqrt{\pi}} \frac{1}{1-\sigma^2} \frac{\omega_0}{a} \frac{H}{a} \frac{\delta}{a} \frac{r}{a} \frac{\phi_{lossylayer}}{\phi_{bulk}}$$

Where:

σ = Poissons ratio substrate, ω_0 = beam radius, δ = thickness of lossy layer
H = thickness of test mass, a = radius of test mass $\phi_{\text{lossy layer}}$ = loss factor of lossy layer
r = radial distance of lossy layer from center of mass ϕ_{bulk} = loss factor of substrate

Assuming a ϕ for the indium layer of 0.1 (literature suggests eg: indium-tin should be somewhere between 0.01 and 1), and a layer thickness of 100 microns, then the amplitude spectral density of the (purely inertial) noise from the indium layer is then:

$x_{\text{lossy layer}}(10\text{Hz}) \sim 1.7 \times 10^{-19}$ m/√Hz

Total expected thermal noise at 10Hz is then:

$$\left[x^2_{\text{finite fused silica}} + x^2_{\text{lossy layer}}\right]^{1/2}$$

ie: approximately 2×10^{-19} m/√Hz at 10Hz from one test mass

Taking: (a) $\phi_{\text{indium}} = 1$, total noise becomes $\sim 5.4 \times 10^{-19}$ m/√Hz at 10Hz
(b) $\phi_{\text{indium}} = 0.01$, total noise becomes $\sim 1 \times 10^{-19}$ m/√Hz at 10Hz

For comparison, in the Advanced LIGO Systems Design document (LIGO-T010075-00-D), figure 1 shows the baseline design for LIGO with sapphire test masses.
The thermo-elastically limited internal thermal noise is shown as having an equivalent strain sensitivity of $h \sim 2.5 \times 10^{-23}$/√Hz at 10Hz - corresponding to a displacement noise from each test mass (assuming equal contributions) of $\sim 5 \times 10^{-20}$ m/√Hz and decreasing as 1/w.
The thermal noise from the compound silica system above decreases more slowly as 1/√w (i.e. as standard intrinsic thermal noise).

NB: By definition, since the work described comes from a draft and is not yet quite publication ready, the above formulae and numbers come with a health warning and should be treated with some caution.

**The PhD thesis of Kazuhiro Yamamoto contains numerical results from finite element models of specific cases – essentially TAMA sized test masses with inhomogeneous loss of different types added.

His numerical results show that for inhomogeneous loss far from the beam, the resulting thermal noise is weakly dependent on beam radius, ω_0, reaching an asymptotic value for $\omega_0 << r$, where r is the distance of the loss from the center of the front face of the mass (i.e. center of position where laser hits the mass).

Taking the analytical estimate for the purely “inertial” thermal noise, i.e. the asymptotic value for $r >> \omega_0$, for

(a) a lossy layer on the barrel of a test mass and

(b) lossy standoffs on the side of a mass,

and using parameters from Yamamoto, the analytical formulae seem to give reasonable agreement with his numerical results. n.b. other terms become significant as $r \rightarrow \omega_0$.